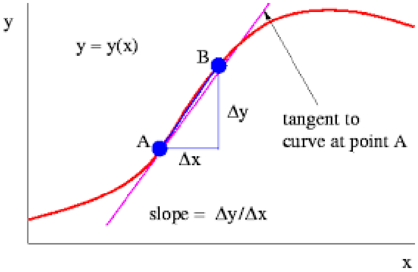
Lecture 2: Derivatives

August 29, 2019

Formal Definition of a Derivative



labman.phys.uk.edu

$f'(a)$ is the derivative of a function f at a if

$$f'(a) = \lim_{x \to a} \frac{f(a+h) - f(a)}{h}$$

Example:

(a) Given $f(x)\,\frac{x}{2x-1}$ find $f'(a)$.

Derivative Laws

1. $f(x) = c \implies f'(x) = 0$
2. $f(x) = x^n \implies f'(x) = nx^{n-1}$
3. $F(x) = f(x)g(x) \implies F'(x) = f(x)g'(x) + g(x)f'(x)$
4. $F(x) = \frac{f(x)}{g(x)} \implies F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ if $g(x) \neq 0$
5. $F(x) = f(x) + g(x) \implies f'(x) + g'(x)$
6. $F(x) = f(x) - g(x) \implies f'(x) - g'(x)$
7. $f(x) = e^x \implies f'(x) = e^x$
8. $f(x) = \log(x) \implies f'(x) = 1/x$

Examples:

Find the derivative of the following functions

(a) $f(t) = \frac{6}{\sqrt[3]{t^5}}$

(b) $f(x) = \frac{x}{x+c/x}$

Using Derivatives to Find Max & Min

Let F be a function of x and A a set of numbers contained on the domain of F.

(i) A point x in A is a maximum for F on A if $F(x) \geqslant F(y)$ for every y on A. $F(x)$ is called the maximum of F on A.

(ii) A point x in A is a minimum for F on A if $F(x) \leqslant F(y)$ for every y on A. $F(x)$ is called the minimum of F on A.

Theorem

Let F be any function defined on (a, b). If x is a max or min point for F on (a, b) and F is differentiable, then $F'(x) = 0$

Using Derivatives to Find Max & Min

A critical or stationary point of a function F is a number x such that $F'(x) = 0$. $F(x)$ itself is called the critical value of F.

Finding a Max or Min

Consider a function on a closed interval $[a, b]$. In order to locate the max and/or min three kinds of points must be considered.

1. The critical points of F in $[a, b]$.
2. The end points of $[a, b]$.
3. The points in $[a, b]$ such that F is not differentiable at x.

Examples:

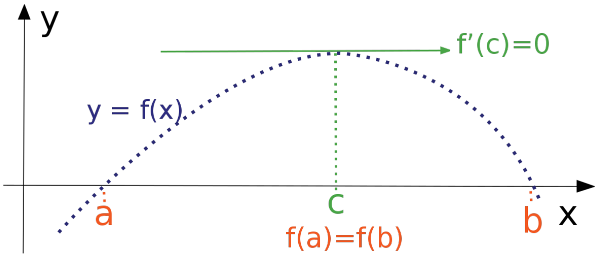
Find the max and min of each of the following

(a)

(b)

(c)

Rolle's Theorem

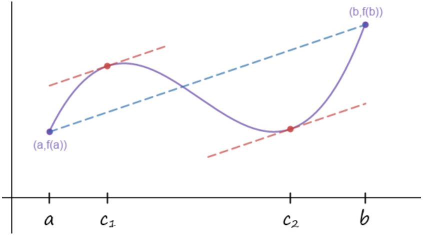


wikipedia

Theorem (Rolle's Theorem)

If F is continuous on $[a, b]$ and differentiate on (a, b) and $F(a) = F(b)$ then there is a number x in (a, b) such that $F'(x) = 0$.

Mean Value Theorem



expii

Theorem (Mean Value Theorem)

If F is continuous on $[a, b]$ and differentiable on (a, b) then there is a number in (a, b) such that

$$F'(x) = \frac{F(b) - F(a)}{b - a}$$

Cauchy Mean Value Theorem

Theorem (Cauchy Mean Value Theorem)

If F and g are continuous on $[a,b]$ and differentiable on (a,b) then there is a number x in (a,b) such that

$$\frac{F'(x)}{g'(x)} = \frac{F(b)-F(a)}{g(b)-g(a)} \implies [F(b)-F(a)]g'(x) = [g(b)-g(a)]F'(x)$$

L'Hopitals Rule

Theorem (L'Hopitals Rule)

Suppose that $\lim_{x\to a} F(x) = 0$ *and* $\lim_{x\to a} g(x) = 0$ *and*

(i) F' *and* g' *exist for each* x *of an interval about* $x = a$ *except possibly* a *itself.*

(ii) $g' \neq 0$ *for* $x \neq a$ *in this interval.*

(iii) $\lim_{x\to a} \frac{F'(x)}{g'(x)} = A$

Then $\lim_{x\to a} \frac{F(x)}{g(x)} = A.$

Example:

(a) $\lim_{x\to 0} \frac{e^x - 1}{\ln(1+x)}$