Group A

(1) Evaluate the following integral by first switching the order of integration from $dy\,dx$ to $dx\,dy$ and then integrating.

$$\int_0^3 \int_{x^2}^9 x^3 e^{y^3}\, dy\, dx$$

(2) Evaluate whether the following series coverge or diverge:

a) $\sum_{n=1}^{\infty} \frac{1+2^n}{1+3^n}$

b) $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$

Group B

(1) Evaluate the following integral over the unit circle by first transforming to polar coordinates and then integrating.

$$\iint_R (x^2+y^2)e^{\sqrt{x^2+y^2}}dA \text{ where } R=\{(x,y)|x^2+y^2 \leqslant 1\}$$

Hint: The polar coordinate transformation is given by $x = r\cos\theta$ and $y = r\sin\theta$. Also $\cos^2\theta + \sin^2\theta = 1$, in case you missed it in class.

(2) Using Lagrange multipliers, solve the following minimization problem:

$$f(x,y) = \sqrt{x^2+y^2+1} \text{ subject to } g(x,y) = xy - 1$$