Lecture 5: Sequences & Series I

September 10, 2019

What is a Sequence?

A sequence is a set of numbers written in some order

$$a_1, a_2, a_3, \ldots, a_{n-1}, a_n$$

An infinite sequence can be thought of as a function whose domain is the set of positive integers $i \in \mathbb{I}$ such that

$$a_i = f(i)$$

Examples:
Give the first four terms of the following sequences

(a) $a_n = \frac{n}{n+1}$

(b) $a_n = 2 + (.1)^n$

(c) $a_n = (-1)^{n+1} \frac{n^2}{3n-1}$

(d) $a_n = 4$

Limits of Sequences

A sequence $\{a_n\}$ has the limit

$$\lim_{n\to\infty}\{a_n\} = L$$

if for all $\epsilon > 0$ there exits a positive N such that $|a_n - L| < \epsilon$ whenever $n > N$. If the limit exists then the sequence converges, otherwise the sequence diverges.

Examples:
If possible, find the limit of the following sequences:

(a) $\{\frac{n}{2n+1}\}$

(b) $\{\frac{(-1)^n n^2}{1+n^3}\}$

(c) $\{(1+\frac{c}{n})^n\}$

What is a Series?

If we add all the terms of a sequence