Recasting Monolithic Event-B Specifications into Modular \mathcal{EVT}

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1 Cars on a Bridge

This is a formalisation in \mathcal{EVT} using specification-building operators of the cars on a bridge example outlined in Chapter 2 of Abrial's book [?]. For all steps in this development we provide the Event-B machine and relevant contexts to show what we are trying to emulate and we also provide an associated \mathcal{EVT} specification that is more modular. The approaches to modularisation that we have taken are by no means the only ones available. There are many ways to write the same specification. The Event-B specifications below were all obtained directly (as a .zip file) from the Event-B wiki page which contains specifications corresponding to all of the systems described in the book.

1.1 The Abstract Machine, M0

Figure 1 contains an Event-B specification corresponding to the first abstract machine in this development. It describes the behaviour of cars entering and leaving the mainland. This abstract model is comprised of a context specifying a natural number constant and a basic machine description. We form the corresponding modular \mathcal{EVT} specification that is contained in Figure 2 as follows:

- **Lines 1–5:** This is a \mathcal{CASL} specification that describes the context from Figure 1. The constant d is represented as an operation of the appropriate type and axioms are included as predicates.
- Lines 6–13: This is a specification that describes the data contained in the machine and also contains the Initialisation event. Separating the data component of the machine is a development strategy that we have used throughout this document but it is not the only approach that could have been taken. It also would have been possible to parametrise the specification of the machine by it's data. This would be a more elegant approach, however, it is also more complex for the unfamiliar reader so we have opted for a strictly specification-building approach in this case study.
- Lines 14–19 and 20–25: These contain specifications that describe the behaviour of leaving and entering, we have chosen to include these as separate specifications so that they will be easy to reuse later.
- **Lines 26–29:** This is the full machine specification that takes a copy of the leave and enter specifications using with to rename the events appropriately.

```
1 MACHINE mO
                                                               SEES cd
                                                          3
                                                                VARIABLES
                                                               INVARIANTS
                                                                   inv1: n \in \mathbb{N}
                                                                   inv2: n \leq d
inv3: n < 0 \lor n < d
  CONTEXT cd
                                                               EVENTS
                                                          9
     CONSTANTS
                                                         10
                                                                  Initialisation
       d
                                                         11
                                                                    then
     AXIOMS
                                                                       act1: n := 0
                                                         12
        axm1: d \in \mathbb{N}
                                                                  Event ML_out @ordinary
                                                         13
        axm2: d > 0
                                                         14
                                                                    when
7 END
                                                                       grd1: n < d
                                                         15
                                                         16
                                                                     then
                                                                       act1: n := n + 1
                                                         17
                                                         18
                                                                   Event ML_in \widehat{=}ordinary
                                                         19
                                                                     when
                                                         20
                                                                        grd1: n > 0
                                                         21
                                                                     then
                                                                        act1: n := n - 1
                                                         22
                                                         23 END
```

Fig. 1. Event-B abstract machine with context.

Even though this is quite a small and simple Event-B model, it is easy to see that the corresponding \mathcal{EVT} specification is, by far, more modular. This will be further evidenced by the reuse of the leave and enter specifications in what follows.

1.2 The First Refinement, M1

3

4

5

6

In the first refinement step, as can be seen in Figure 3, new events are added to describe cars entering and leaving the island. New variables are added to record the number of cars on the island, cars on the mainland and those on the bridge. The events ML_in and ML_out are also refined to utilise these new variables. Figure 4 contains an \mathcal{EVT} specification corresponding to this Event-B machine specification. It is comprised of the following modular specifications:

Lines 1–11: As was the approach taken with the abstract machine, this specification describes the data to be used and also a specification of the Initialisation event. Note that we have omitted those invariants that are labelled as theorems because they should be derived from the specification. It is possible to include them using the %implied annotation that is made available in Hets but we have omitted them for simplicity.

Lines 12–21: This specification describes the island events by combining two copies of the Enter specification that was constructed as part of the abstract \mathcal{EVT} specification in Figure 2. We use with to provide appropriate renaming of events and variables.

Lines 22–30: This describes the refined mainland events. Note that where multiple events with the same name appear in a specification, they are assumed to be merged. This is the basic \mathcal{CASL} notion of "same name, same thing".

```
1 spec CD over CASL =
     sort N
 3
      ops d : ℕ
 4
      d > 0
 5 end
                                                         20 spec Enter over \mathcal{EVT} =
 6 spec DATAMO
                        \mathcal{EVT} =
                                                         21
                                                               DATAMO then
                  over
                                                               event in ordinary =
      CD with \rho
 8
                                                         23
                                                                 when n > 0
      then
 9
        \mathtt{sort}\ \mathbb{N}
                                                         24
                                                                 thenAct n := n - 1
10
        ops n:N
                                                         25 end
        event Init ordinary =
11
                                                            spec MOML over \mathcal{EVT} =
12
          thenAct n := 0
                                                         26
13 end
                                                         27
                                                               (LEAVE with out → ML_out) and
                                                         28
                                                               (ENTER with in → ML_in)
14 spec Leave over \mathcal{EVT} =
                                                         29 end
15
      DATAMO then
16
      event out ordinary =
17
        when n < d
18
        thenAct n:= n+1
19 end
```

Fig. 2. This figure contains four specifications, one for the data (and the Init event), one for the act of leaving, one for the act of entering and the specification corresponding to the abstract Event-B machine. The separation of the abstract machine into these primitive component specifications was a design decision that we chose to make at the beginning in order to make the example more modular and to illustrate how the specification-building operators can combine these in a coherent way.

This specification contains a copy of the abstract mainland events and adds new behaviour by defining these events again here.

Lines 31–33: This is the \mathcal{EVT} specification that describes the full behaviour of the Event-B machine contained in Figure 3.

It is clear that the \mathcal{EVT} specification in Figure 4 corresponds to a modular version of the Event-B machine in Figure 3 but the question remains as to whether this \mathcal{EVT} specification is a refinement of the abstract \mathcal{EVT} specification in Figure 2. This can easily be checked using the institution theoretic notion of refinement as model class inclusion along the reduct. This broadly means that if we were to restrict the concrete specification to only contain signature elements of the abstract specification then all models of the concrete specification must also be models of the abstract. Removing the relevant signature items actually results in exactly the abstract specification given in Figure 2 and so the refinement relation holds.

1.3 The Second Refinement, M2

The next refinement step was not as gradual as the last in that quite a lot of new behaviour was added to the Event-B model as shown in Figure 5. The first big addition was that of a context that contains colours which will be used as values for the variables that are used to control the behaviour of a pair of traffic lights. Events for these lights were added to the machine and the current events were modified to account for this behaviour.

```
22
                                                         Event ML_out ≘ordinary
                                                 23
                                                           refines ML_out
                                                 24
                                                           when
                                                 25
                                                               \mathbf{grd1}\colon a \ + \ b \ < \ d
 1 MACHINE m1
                                                 26
                                                               grd2: c = 0
     refines m0
                                                 27
                                                            then
 3
     SEES cd
                                                 28
                                                              act1: a := a + 1
 4
      VARIABLES
                                                 29
                                                         Event IL_in ≘convergent
 5
        a, b, c
                                                 30
                                                           when
 6
     INVARIANTS
                                                 31
                                                              grd1: a > 0
         inv1: a \in
                                                 32
                                                           then
         inv2: b \in
 8
                                                              act1: a := a - 1
act2: b := b + 1
                                                 33
 9
         inv3: c \in \mathbb{N}
                                                 34
         inv4: n = a + b + c
inv5: a = 0 \lor c = 0
10
                                                 35
                                                         11
        thm1: a + b + c \in \mathbb{N} theorem thm2: c > 0 \lor a > 0
                                                 36
12
                                                 37
                                                               grd1: 0 < b
13
                                                  38
                                                               grd2: a = 0
               14
                                                 39
15
                                                 40
                                                              act1: b := b - 1
     VARIANT 2*a + b
16
                                                 41
                                                               act2: c := c + 1
     EVENTS
17
                                                         Event ML_in ≘ordinary
                                                 42
        Initialisation
18
                                                 43
                                                           refines ML_in
19
          then
                                                 44
                                                            when
             act2: a := 0
20
                                                 45
                                                              grd1: c > 0
             act3: b := 0
21
                                                 46
22
             \mathtt{act4} \colon \ c \ := \ 0
                                                 47
                                                               act2: c := c - 1
                                                 48 END
```

Fig. 3. Event-B machine m1

```
1 spec DataM1 over \mathcal{EVT}
      DATAMO
 3
      then
        ops a, b, c : \mathbb{N}
 5
        . n = a + b + c
         a = 0 \lor c = 0
                                                         22 spec M1ML over \mathcal{EVT} =
 6
                                                               DATAM1 and MOML and
        event Init ordinary =
                                                         23
                                                         24
          thenAct a := 0
                                                                 (Enter with in \mapsto Ml_in, n \mapsto c)
 9
                  b := 0
                                                         25
                                                               then
10
                                                         26
                                                                event ML_out ordinary =
                  c := 0
                                                         27
                                                                  when a + b < d
11 end
                                                                      c = 0
                                                         28
12 spec M1IL over \mathcal{EVT} =
                                                         29
                                                                  thenAct a := a + 1
      DATAM1 and (ENTER with in \mapsto IL_in, n \mapsto a)
                                                         30 end
13
        and (ENTER with in \mapsto IL_out, n \mapsto b)
14
                                                         31 spec M1 over \mathcal{EVT} =
15
      then
16
        event IL_in convergent =
                                                         32
                                                              M1ML and M1IL
          thenAct b := b + 1
                                                         33 end
17
        event IL_out convergent =
18
19
          when a = 0
          thenAct c := c + 1
20
21 end
```

Fig. 4. This figure contains a modularised version of the \mathcal{EVT} -specification corresponding to the first machine at the first refinement step (m1).

- **Lines 1–5:** This is a \mathcal{CASL} specification that specifies the new data type Color. This corresponds to the context described in the Event-B model in Figure 5.
- **Lines 6–24:** As in the previous specifications we have separated the data and Initialisation event from the rest of the specification.
- **Lines 25–37:** This specification describes the behaviour of the refined mainland entry and exit events.
- **Lines 38–52:** This specification describes the behaviour of the refined island entry and exit events.
- Lines 53–63: This specification contains the specification of a traffic light being set to green, we separate this from the rest of the model so that we can use to account for the behaviour of two traffic lights without the need to repeat the specification twice.
- Lines 64–74: This specification creates the two traffic lights that we need in accordance with the Event-B specification in Figure 5. The basic light specification that we described on lines 53–63 above is included twice here, with appropriate renamings carried out via signature morphism. This basic light was missing some information and these details have been added to each event by providing new definitions of them such that their guards and actions are merged with their previous definitions.
- **Lines 75–77:** This is the \mathcal{EVT} specification that corresponds to the full Event-B model contained in Figure 5.

1.4 The Third Refinement, M3

The third refinement step results in quite a large Event-B model as can be seen in Figures 7 and 8. We have translated this into the corresponding \mathcal{EVT} specification described in Figures 9 and 10.

```
Event ML_out2 ≘ordinary
 1 CONTEXT Color
                                                                    2
                                                                             refines ML_out
 2
      SETS Color
                                                                    3
      CONSTANTS
 3
                                                                    4
                                                                                   grd1: ml\_tl = green
 4
       red, green
                                                                                   grd2: a + b + 1 = d
                                                                    5
 5
      AXIOMS
                                                                                then
         \verb"axm4: Color = \{green, red\}
 6
                                                                                   act1: a := a + 1
         axm3: green \neq red
                                                                    8
                                                                                   act2: ml\_tl := red
 8 END
                                                                                   act3: ml\_pass := 1
                                                                   10
                                                                            Event IL_out1 = ordinary
 9 MACHINE m2
                                                                               refines IL_out
                                                                   11
10
      refines m1
                                                                   12
                                                                               when
11
      SEES cd, Color
                                                                                 grd1: il\_tl = green
                                                                   13
12
      VARIABLES
                                                                                  grd2: b > 1
                                                                   14
13
        a, b, c,
                                                                   15
                                                                               then
14
        ml_tl, il_tl,
                                                                                  act1: b := b - 1
                                                                   16
15
        il_pass, ml_pass
                                                                                  act2: c := c + 1
                                                                   17
      INVARIANTS
16
                                                                                  act3: il_pass := 1
                                                                   18
17
         inv1: ml\_tl \in \{red, green\}
                                                                             Event IL_out2 = ordinary
                                                                   19
18
         inv2: il\_tl \in \{red, green\}
                                                                               refines IL_out
                                                                   20
19
         inv3: ml_{-}tl = green \Rightarrow c = 0
                                                                   21
                                                                               when
20
         inv12: ml\_tl = green \Rightarrow a + b + c < d
                                                                                  grd1: il\_tl = green
                                                                   22
21
         inv4: il_{-}tl = green \Rightarrow a = 0
                                                                   23
                                                                                  grd2: b = 1
22
         inv11: il_{-}tl = green \Rightarrow b > 0
                                                                   24
                                                                               then
         inv6: il\_pass \in \{0,1\}
inv7: ml\_pass \in \{0,1\}
23
                                                                   25
                                                                                  act1: b := b - 1
24
                                                                                  act2: il\_tl := red
                                                                   26
25
         inv8: ml\_tl = red \Rightarrow ml\_pass = 1
                                                                   27
                                                                                  act3: c := c + 1
26
         inv9: il_{-}tl = red \Rightarrow il_{-}pass = 1
                                                                                  act4: il\_pass := 1
                                                                   28
27
         inv5: il_{-}tl = red \lor ml_{-}tl = red
                                                                             Event ML_tl_green \( \hat{\text{\text{\convergent}}} \)
                                                                   29
         thm2: 0 \ge a \Rightarrow a = 0 theorem thm3: 0 \ge b \Rightarrow b = 0 theorem thm4: 0 \ge c \Rightarrow c = 0 theorem thm5: \neg(d \le 0) theorem
                                                                   30
                                                                               when
29
                                                                                  grd1: ml\_tl = red
                                                                   31
                                                                                  grd2: a + b < d
                                                                   32
31
                                                                                  grd3: c = 0
                                                                   33
         thm6: b + 1 \ge d \land \neg (b + 1 = d) \Rightarrow \neg (b < d)
32
                                                                                  grd4: il\_pass = 1
                                                                   34
    theorem
                                                                   35
                                                                               then
         thm7: b \leq 1 \wedge \neg (b = 1) \Rightarrow \neg (b > 0) theorem
                                                                                  act1: ml\_tl := green
         thm1: (m\overline{l}_{-}tl = green \land a + b + 1 < d)
34
                                                                   37
                                                                                  act2: il\_tl := red
                \forall (ml\_tl = green \land a + b + 1 = d)
35
                                                                                  act3: ml\_pass := 0
                                                                   38
36
                \vee (il\_tl = green \land b > 1)
                39
                                                                             Event IL_tl_green ≘convergent
37
                                                                             1<sup>when</sup>
38
                                                                                  grd1: il\_tl = red
                \forall (il\_tl = red \land 0 < b \land a = 0 \land ml\_pass_{42}^{41} = 1)
39
                                                                                  grd2: 0 < b
                \vee \stackrel{.}{0} < a \vee 0 < c theorem
40
                                                                   43
                                                                                  grd3: a = 0
      VARIANT ml_pass + il_pass
41
                                                                                  grd4: ml\_pass = 1
42
      EVENTS
                                                                   45
                                                                               then
43
        Initialisation
                                                                   46
                                                                                  act1: il\_tl := green
44
          then
                                                                   47
                                                                                  act2: ml\_tl := red
             act2: a := 0
45
                                                                   48
                                                                                  act3: il\_pass := 0
             act3: b := 0
46
                                                                   49
                                                                              Event IL_in êordinary
47
             act4: c := 0
                                                                   50
                                                                                refines IL_in
             act1: ml\_tl := red
48
                                                                   51
                                                                                when
49
             act5: il\_tl := red
                                                                                  grd11: 0 < a
50
             act6: ml_pass := 1
                                                                   53
51
             act7: il\_pass := 1
                                                                                  act11: a := a - 1
                                                                   54
        Event ML_out1 = ordinary
52
                                                                   55
                                                                                   act12: b := b + 1
53
          refines ML_out
                                                                              Event ML_in ≘ordinary
54
          when
                                                                   57
                                                                                refines ML_in
             grd1: ml\_tl = green
55
                                                                                when
             grd2: a + b + 1 < d
56
                                                                   59
                                                                                   grd1: 0 < c
57
          then
                                                                                then
             act1: a := a + 1
58
                                                                   61
                                                                                   act1: c := c + 1
59
             act2: ml\_pass := 1
```

Fig. 5. Event-B machine m2 and context.

```
38 spec M2IL over \mathcal{EVT} =
                                                         39
                                                                DATAM2 and
                                                          40
                                                                (M1IL hide via IL_out → IL_out1) and
 1 spec Color over \mathcal{CASL}=
                                                         41
                                                                (M1IL with IL_out with IL_out2)
     sort Color
                                                          42
     ops red, green : Color
. green ≠ red
 3
                                                         43
                                                                  event IL_out1 ordinary =
 4
                                                          44
                                                                    when il_tl = green
 5 end
                                                         45
                                                                         b > 1
                                                          46
                                                                    thenAct il_pass := 1
 6 spec DataM2 over \mathcal{EVT} =
                                                          47
                                                                  event IL_out2 ordinary =
      DATAM1 and COLOR
                                                          48
                                                                    when il_tl = green
 8
      then
                                                          49
                                                                         b = 1
        ops ml_tl, il_tl:Color
 9
                                                          50
                                                                    thenAct il_pass := 1
10
            il_pass, ml_pass : {0,1}
                                                          51
                                                                             il_tl := red
11
        . ml_tl = green \Rightarrow c = 0
12
          ml_tl = green \Rightarrow a + b + c < d
          il_tl = green \Rightarrow a = 0
13
                                                         53 spec TLGREEN over \mathcal{EVT} =
          il_tl = green \Rightarrow b > 0
14
                                                                DataM2
          ml_tl = red \Rightarrow ml_pass = 1
15
                                                                then
          il_tl = red \Rightarrow il_pass = 1
16
                                                          56
                                                                  event lgreen convergent =
          il_tl = red \times ml_tl = red
17
                                                                    when ml_tl = red
                                                         57
18
        variant ml_pass + il_pass
                                                         58
                                                                         c = 0
19
        event Init ordinary =
                                                                          il_pass = 1
20
          thenAct ml_tl : = red
                                                          60
                                                                    thenAct ml_tl := green
21
                   il_{-}tl : = red
                                                                            il_tl := red
                                                         61
                   ml_pass : = 1
22
                                                          62
                                                                             ml_pass := 0
23
                   il_pass : = 1
                                                         63 end
24\ {\rm end}
                                                         64 spec M2green over \mathcal{EVT} =
25 spec M2ML over \mathcal{EVT} =
                                                         65
                                                                (TLGREEN with lgreen \mapsto ML_tl_green) and
26
      DATAM2 and
                                                                (TLGREEN with lgreen \mapsto IL_tl_green, ml_tl \mapsto il_tl,
                                                          66
^{27}
      (M1ML hide via ML_out \mapsto ML_out1) and
                                                          67
                                                                il_pass \mapsto ml_pass, il_tl \mapsto ml_tl, ml_pass \mapsto il_pass,
28
      (M1ML with ML_out with ML_out2)
                                                         68
                                                                c \mapsto a)
29
                                                          69
                                                                then
30
        event ML_out1 ordinary =
                                                          70
                                                                  event ML_tl_green convergent =
31
           when ml_tl = green
                                                          71
                                                                    when a + b < d
32
           thenAct ml_pass := 1
                                                          72
                                                                  event IL_tl_green convergent =
33
        event ML_out2 ordinary =
                                                          73
                                                                    when 0 < b
34
           when ml_tl = green
                                                         74 end
35
           thenAct ml_pass := 1
36
                   ml_tl := red
                                                          75 spec M2 over \mathcal{EVT} =
37 end
                                                          76
                                                               M2GREEN and M2IL and M2ML
                                                          77 end
```

Fig. 6. Full description of the m2 Event-B machine using modular \mathcal{EVT} .

```
1 CONTEXT Sensor
      SETS Sensor
                                                                       EVENTS
                                                                1
 3
      CONSTANTS on, off
                                                                2
                                                                         Initialisation
                                                                3
                                                                           then
 5
         axm1: Sensor = \{on, off\}
                                                                4
                                                                              act2: a := 0
 6
         axm2: \neg on = off
                                                                5
                                                                              act3: b := 0
 7 END
                                                                6
                                                                              act4: c := 0
                                                                              act1: ml\_tl := red
 8 MACHINE m3
                                                                8
                                                                              act5: il\_tl := red
 9
      refines m2
                                                                9
                                                                              act6: ml_pass := 1
      SEES cd, Color, Sensor
10
                                                                10
                                                                              act7: il\_pass := 1
      VARIABLES
11
                                                                              act15: \hat{ml}_out_10 := FALSE
                                                               11
12
        a, b, c,
                                                               12
                                                                              act16: il\_out\_10 := FALSE
        ml_tl, il_tl,
13
                                                                13
                                                                              act17: ml_in_10 := FALSE
14
        ml_pass, il_pass,
                                                               14
                                                                              \verb"act18: il\_in\_10 := FALSE"
15
        A. B. C.
                                                                15
                                                                              act8: A := 0
        ML_OUT_SR. ML_IN_SR.
16
                                                                              act9: B := 0

act10: C := 0
                                                               16
17
        IL_OUT_SR, IL_IN_SR,
                                                               17
        ml_out_10, ml_in_10,
18
                                                                               {\tt act11} \colon \ ML\_IN\_SR \ := \ off
                                                                18
19
        ml_in_10, il_in_10
                                                                19
                                                                               act12: ML\_OUT\_SR := off
      INVARIANTS
20
                                                               20
                                                                               act13: IL\_OUT\_SR := off
         inv1: IL_{-}IN_{-}SR = on \Rightarrow A > 0
21
                                                                              act14: IL\_IN\_SR := off
                                                                21
         inv2: IL\_OUT\_SR = on \Rightarrow B > 0
inv3: ML\_IN\_SR = on \Rightarrow C > 0
22
                                                               22
                                                                           Event ML_out1 ≘ordinary
23
                                                                             refines ML_out1
         inv4: ml\_out\_10 = TRUE \Rightarrow ml\_tl = green
24
                                                               24
                                                                                 grd1: ml\_out\_10 = TRUE
         inv5: il\_out\_10 = TRUE \Rightarrow il\_tl = green
25
         inv6: il.out.10 = IRUE \Rightarrow il.il = green

inv6: IL_IN\_SR = on \Rightarrow il.in\_10 = FALSE_{27}^{26}
                                                                                 grd2: a + b + 1 < d
26
                                                                                 act1: a := a + 1
         inv7: IL\_OUT\_SR = on
27
                                                                                 act2: ml-pass := 1
         \Rightarrow il\_out\_10 = FALSE \\ \text{inv8: } ML\_IN\_SR = on
28
                                                                30
                                                                                 act3: ml\_out\_10 := FALSE
29
                                                               31
                                                                           Event ML_out2 ≘ordinary
                 \Rightarrow \ ml\_in\_10 \ = \ FALSE
30
                                                                             refines ML_out2
31
         inv9: ML\_OUT\_SR = on
                                                               33
                                                                              when
                 \Rightarrow ml\_out\_10 = FALSE
32
                                                                34
                                                                                 grd1: ml\_out\_10 = TRUE
         inv10: il_in_10 = TRUE
33
                                                                                 grd2: a + b + 1 = d
                 \land \ ml\_out\_10 \ = \ TRUE \ \Rightarrow \ A \ = \ a
34
                                                               36
                                                                              then
35
         inv11: il_in_10 = FALSE
                                                                37
                                                                                 act1: a := a + 1
                  \land \ ml\_out\_10 \ = \ TRUE \ \Rightarrow \ A \ = \ a \ + \ 1 
36
                                                                                 act2: ml\_tl := red
                                                                38
37
         inv12: il_in_10 = TRUE
                                                                39
                                                                                 act3: ml\_pass := 1
38
                 \land ml\_out\_10 = FALSE \Rightarrow A = a
                                                                40
                                                                                 act4: ml\_out\_10 := FALSE
39
         inv13: il_in_10 = FALSE
                                                                           Event IL_out1 ≘ordinary
                                                               41
                 \land \ ml\_out\_10 \ = \ FALSE \ \Rightarrow \ A \ = \ a
40
                                                               42
                                                                             refines IL out1
41
         inv14: il_in_10 = TRUE
                                                               43
                                                                              when
                 \land \ il\_out\_10 \ = \ TRUE \ \Rightarrow \ B = b
42
                                                                                 grd1: il\_out\_10 = TRUE
                                                               44
43
         inv15: il_in_10 = TRUE
                                                                                 grd2: b > 1
                                                                45
44
                 \land \ il\_out\_10 \ = \ FALSE \ \Rightarrow \ B \ = \ b \, + \, 1
                                                                46
                                                                              then
45
         inv16: il_in_10 = FALSE
                                                               47
                                                                                act1: b := b - 1
46
                 \land il\_out\_10 = TRUE \Rightarrow B = b 1
                                                                                 act2: c := c + 1
                                                               48
47
         inv17: il\_in\_10 = FALSE
                                                                                 act3: il\_pass := 1
                                                               49
48
                 \land \ il\_out\_10 \ = \ FALSE \ \Rightarrow \ B \ = \ b
                                                                                 act4: il\_out\_10 := FALSE
                                                               50
         inv18: il\_out\_10 = TRUE
49
                                                                           Event IL_out2 ≘ordinary
                                                               51
                 \land ml\_in\_10 = TRUE \Rightarrow C = c
                                                                             refines IL_out2
                                                               52
         inv19: il\_out\_10 = TRUE
                                                                53
                                                                              when
                 \land \ ml\_in\_10 \ = \ FALSE \ \Rightarrow \ C \ = \ c \ + \ 1
                                                                                 grd1: il\_out\_10 = TRUE
                                                                54
53
         inv20: il\_out\_10 = FALSE
                                                                55
                                                                                 grd2: b = 1
54
                \land ml\_in\_10 = TRUE \Rightarrow C = c \ 1
                                                                              then
                                                                56
55
         inv21: il\_out\_10 = FALSE
                                                               57
                                                                                 act1: b := b - 1
                \land ml\_in\_10 = FALSE \Rightarrow C = c
56
                                                                                 \verb"act2": il\_tl := red"
                                                                58
         inv22: A = 0 \lor C = 0
57
                                                               59
                                                                                 act3: c := c + 1
         inv23: A + B + C \leq d
58
                                                                60
                                                                                 act4: il\_pass := 1
         inv24: A \in \mathbb{N}
59
                                                               61
                                                                                 act5: il\_out\_10 := FALSE
60
         inv25: B \in \mathbb{N}
61
         inv26: C \in \mathbb{N}
```

Fig. 7. Event-B machine description of the third refinement M3.

```
Event ML_tl_green \( \hat{\text{\text{\convergent}}} \)
 1
 2
             refines ML_tl_green
 3
             when
                grd1: ml\_tl = red
 4
                \verb"grd2: $a + b < d"
 5
                grd3: c = 0
 6
                grd4: il_pass = 1
grd5: il_out_10 = FALSE
 7
 8
                grd6: ML\_OUT\_SR = on
 9
10
             then
                \verb"act1: ml\_tl := green"
11
12
                act2: il\_tl := red
                                                                 Event IL_OUT_ARR ≘ordinary
13
                act3: ml\_pass := 0
                                                                   when
           Event IL_tl_green ≘convergent
14
                                                       3
                                                                      grd1: IL\_OUT\_SR = off
15
             refines IL_tl_green
                                                       4
                                                                      grd2: il\_out\_10 = FALSE
16
                                                                      grd3: B > 0
                                                       5
17
                {\tt grd1}\colon\thinspace il\_tl \ = \ red
                                                       6
                                                                   then
18
                grd2: 0 < b
                                                                      act1: IL\_OUT\_SR := on
19
                grd3: a = 0
                                                                 Event ML_OUT_DEP ≘ordinary
                                                       8
20
                grd4: ml\_pass = 1
                                                       9
                                                                   when
21
                grd5: ml\_out\_10 = FALSE
                                                                      grd1: ML\_OUT\_SR = on
                                                      10
22
                grd6: IL\_OUT\_SR = on
                                                                      grd2: ml\_tl = green
                                                      11
23
             then
                                                      12
                                                                   then
24
               act1: il\_tl := green
                                                                      act1: ML\_OUT\_SR := off
act2: ml\_out\_10 := TRUE
                                                      13
25
                act2: ml_{-}tl := red
                                                      14
26
               act3: il_pass := 0
                                                                      \mathtt{act3} \colon A \ := \ A \ + \ 1
                                                      15
27
           Event ML_in =ordinary
                                                                 Event ML_IN_DEP \( \hat{\text{a}}\) ordinary
                                                      16
28
             refines ML_in
                                                      17
                                                                   when
29
                                                                      grd1: ML\_IN\_SR = on
                                                      18
               grd1: ml_in_10 = TRUE
30
                                                      19
                                                                   then
31
                grd2: c > 0
                                                                      act1: ML_IN_SR := off
                                                      20
32
             then
                                                                      act2: ml.in.10 := TRUE
act3: C := C - 1
                                                      21
33
               act1: c := c - 1
                                                      22
               act2: ml\_in\_10 := FALSE
34
                                                                 Event IL_IN_DEP \(\hat{=}\) ordinary
                                                      23
35
           Event IL_in ≘ordinary
                                                      24
                                                                   when
36
             refines IL_in
                                                      25
                                                                      grd1: IL\_IN\_SR = on
37
             when
                                                      26
                                                                   then
               grd1: il_in_10 = TRUE
38
                                                      27
                                                                      act1: IL\_IN\_SR := off
39
                grd2: 0 < a
                                                      28
                                                                      act2: il\_in\_10 := TRUE
40
             then
                                                                      act3: A := A - 1
act4: B := B + 1
                                                      29
41
               act1: a := a - 1
                                                      30
                act2: b := b + 1
42
                                                                 Event IL_OUT_DEP \( \hat{=} \) ordinary
                                                      31
                act3: il\_in\_10 := FALSE
43
                                                      32
           Event ML_OUT_ARR ≘ordinary
44
                                                      33
                                                                      grd1: IL\_OUT\_SR = off
45
             when
                                                      34
                                                                      grd2: il_{-}tl = green
                grd1: ML\_OUT\_SR = off
46
                                                      35
                grd2: ml\_out\_10 = FALSE
47
                                                      36
                                                                      \verb"act1: IL-OUT-SR" := off
48
             then
                                                      37
                                                                      act2: il\_out\_10 := TRUE
               act1: ML\_OUT\_SR := on
49
                                                                      act3: B := B - 1
act4: C := C + 1
                                                      38
           50
                                                      39
51
             when
                                                      40 end
               grd1: ML\_IN\_SR = off
52
                grd2: ml_in_10 = FALSE
53
54
                grd3: C > 0
             then
55
               act1: ML\_IN\_SR := on
56
           Event IL_IN_ARR ≘ordinary
57
58
                grd1: IL\_IN\_SR = off
59
                grd2: il_in_10 = FALSE
60
                grd3: A > 0
61
62
             then
                \verb"act1: IL\_IN\_SR := on
63
```

Fig. 8. Event-B m3 continued.

```
1 spec ToggLe10 over \mathcal{EVT} =
                                                                     ops t: BOOL
                                                                3
                                                                     event toggle ordinary =
 1 spec Sensor over CASL =
                                                                          when
     sort Sensor
                                                                5
                                                                            t = TRUE
 3
     ops on, off : Sensor
                                                                6
                                                                          thenAct
       \neg on = off
                                                                            t = FALSE
 5 end
                                                                8 end
 6
    spec DataM3 over \mathcal{EVT} =
                                                                9 spec InOut over \mathcal{EVT} =
      DATAM2 and (SENSOR with \rho)
                                                               10
                                                                     M2ML and M2IL
 8
                                                                     and (ToggLe10 with toggle \mapsto ML_out1, t \mapsto ml_out_10)
                                                               11
 9
         ops A, B, C : \mathbb{N}
                                                                     and (ToggLE10 with toggle → ML_out2, t → ml_out_10)
                                                               12
10
             ML_OUT_SR, ML_IN_SR : Sensor
                                                                     and (ToggLe10 with toggle \mapsto IL_out1, t \mapsto il_out_10)
                                                               13
11
              IL_OUT_SR, IL_IN_SR : Sensor
                                                               14
                                                                     and (ToggLe10 with toggle \mapsto IL_out2, t \mapsto il_out_10)
12
              ml_out_10, ml_in_10 : Bool
                                                                     and (ToggLe10 with toggle \mapsto ML_in, t \mapsto ml_in_10)
                                                               15
13
              il_out_10, il_in_10 : Bool
                                                               16
                                                                     and (ToggLE10 with toggle \mapsto IL_in, t \mapsto il_in_10)
         . IL_IN_SR = on \Rightarrow A > 0
14
                                                               17 end
15
           IL_OUT_SR = on \Rightarrow B > 0
16
           ML_IN_SR = on \Rightarrow C > 0
                                                               18 spec TLGREEN over \mathcal{EVT} =
           ml_out_10 = TRUE \Rightarrow ml_tl = green
17
                                                               19
                                                                     SENSOR then
           il_out_10 = TRUE \Rightarrow il_tl = green
18
                                                                        op sensor: Sensor, b: Bool
                                                               20
           IL_IN_SR = on \Rightarrow il_in_10 = FALSE
19
                                                               21
                                                                        eventsetgreenconvergent
           IL_OUT_SR = on \Rightarrow il_out_10 = FALSE
20
                                                               22
                                                                          when
21
           ML_IN_SR = on \Rightarrow ml_in_10 = FALSE
                                                                            b = FALSE
                                                               23
22
           ML_OUT_SR = on \Rightarrow ml_out_10 = FALSE
                                                               24
                                                                            sensor = on
23
           il_in_10 = TRUE \wedge ml_out_10 = TRUE
                                                               25
                                                                          thenAct
24
              \Rightarrow A = a
                                                               26 end
25
           il_in_10 = FALSE \land ml_out_10 = TRUE
26
              \Rightarrow A = a + 1
                                                               27 spec M3green over \mathcal{EVT} =
           il_in_10 = TRUE \( \text{ml_out_10} = FALSE \)
                                                               28
                                                                     M2green ikwand DataM3 and
28
              \Rightarrow A = a 1
                                                               29
                                                                      (TLGREEN with setgreen \mapsto ML_tl_green, b \mapsto il_out_10,
29
           il_in_10 = FALSE \land ml_out_10 = FALSE
                                                               30
                                                                        sensor → ML_OUT_SR) and
30
              \Rightarrow A = a
                                                               31
                                                                      (TLGREEN with setgreen \mapsto IL_tl_green, b \mapsto ml_out_10,
           il_in_10 = TRUE \land il_out_10 = TRUE
31
                                                               32
                                                                        \mathtt{sensor} \; \mapsto \; \mathtt{IL\_OUT\_SR})
32
              ⇒ B=b
                                                               33 end
33
           il_in_10 = TRUE \wedge il_out_10 = FALSE
34
              \Rightarrow B = b + 1
                                                               34 spec ARR over \mathcal{EVT} =
35
           il_in_10 = FALSE \( \tilde{\) il_out_10 = TRUE
                                                               35
                                                                     SENSOR then
36
              \Rightarrow B = b 1
                                                               36
                                                                        ops sensor : Sensor, b: Bool
37
           il_in_10 = FALSE \( \til_out_10 = FALSE \)
                                                               37
                                                                        event Arr ordinary =
38
              \Rightarrow B = b
                                                               38
                                                                          when
           il_out_10 = TRUE \wedge ml_in_10 = TRUE
39
                                                               39
                                                                             sensor = off
40
              \Rightarrow C = c
                                                               40
                                                                             b = FALSE
           il_out_10 = TRUE \wedge ml_in_10 = FALSE
41
                                                               41
                                                                           thenAct
42
              \Rightarrow C = c + 1
                                                               42
                                                                            sensor := on
           il_out_10 = FALSE \wedge ml_in_10 = TRUE
43
                                                               43 end
44
              \Rightarrow C = c 1
           il_out_10 =FALSE \wedge ml_in_10 = FALSE
45
                                                               44 spec EXTARR over \mathcal{EVT} =
46
             \Rightarrow C = c
                                                               45
                                                                     ARR then
           A = 0 \lor C = 0
47
                                                                      {\tt op \ num :} \ \mathbb{N}
48
           A + B + C \leq d
                                                               47
                                                                     event Arr ordinary
49
           event Init ordinary =
                                                                        when
50
              thenAct ml_out_10 := FALSE
                                                               49
                                                                          num > 0
51
                       il_out_10 := FALSE
                                                              50 end
52
                       ml_in_10 := FALSE
53
                       il_in_10 := FALSE
                                                               51 spec ALLARR over \mathcal{EVT} =
54
                       A := 0
                                                                      (ARR with Arr \mapsto ML_OUT_ARR, sensor \mapsto ML_out_SR,
                                                               52
55
                       B := 0
                                                               53
                                                                         b \mapsto ml_out_10) and
56
                       C := 0
                                                               54
                                                                      (EXTARR with Arr \mapsto ML_IN_ARR, sensor \mapsto ML_in_SR,
57
                       ML_IN_SR := off
                                                                         b \mapsto ml_in_10, num \mapsto C) and
                                                               55
58
                       IL_IN_SR := off
                                                                      (EXTARR with Arr \mapsto IL_IN_ARR, sensor \mapsto IL_in_SR,
                                                               56
59
                       ML_OUT_SR := off
                                                                         b \mapsto il_in_10, num \mapsto A) and
                                                               57
60
                       IL_OUT_SR := off
                                                                      (EXTARR with Arr \mapsto IL_OUT_ARR, sensor \mapsto iL_out_SR,
                                                               58
61 end
                                                                         b \mapsto il\_out\_10, num \mapsto B)
                                                               59
                                                               60 end
```

Fig. 9. Full description of the m3 Event-B machine using modular \mathcal{EVT} .

```
1 spec ILDEP over \mathcal{EVT} =
 2
      SENSOR then
 3
        \ensuremath{\mathsf{ops}} s : sensor, b : Bool, n1,n2 : \ensuremath{\mathbb{N}}
 4
        event ildep ordinary
 5
          when
 6
             s = on
           {\tt thenAct}
 8
             s := off
 9
             b := TRUE
             n1 := n1 - 1
10
11
             n2 := n2 + 1
12 end
13 spec ALLLILDEP over \mathcal{EVT} =
      DATAM3 and
      (ILDEP with ildep \mapsto IL_IN_DEP, s \mapsto IL_IN_SR,
        b \mapsto il_in_10, n1 \mapsto A, n2 \mapsto B) and
17
      (ILDEP with ildep \mapsto IL_OUT_DEP, s \mapsto IL_OUT_SR,
       b \mapsto il\_out\_10, n1 \mapsto B, n2 \mapsto C)
19
20
        event IL_OUT_DEP ordinary
21
          when
             il_tl = green
23 end
24 spec MLDEP over \mathcal{EVT} =
25
      SENSOR then
26
        ops s : Sensor, b : Bool
        event mldep ordinary
          when
29
            s = on
30
           thenAct
31
            s := off
             b : =TRUE
32
33 end
34 spec ALLMLDEP over \mathcal{EVT} =
35
      DATAM3 and
      (MLDEP with mldep \mapsto ML_IN_DEP, s \mapsto ML_IN_SR, b \mapsto ml_in_10) and
36
      (MLDEP with mldep \mapsto ML_OUT_DEP, s \mapsto ML_OUT_SR, b \mapsto ml_out_10)
37
38
39
        event ML_IN_DEP ordinary
40
          thenAct
          C := C - 1
event ML_OUT_DEP ordinary
41
42
43
            when
44
             ml_tl = green
45
            thenAct
              A := A + 1
46
47 end
48 spec ALLDEP over \mathcal{EVT} =
49 ALLILDEP and ALLMLDEP
50 end
51 spec M3 over \mathcal{EVT} =
52 INOUT and M3GREEN and ALLARR and ALLDEP
53 end
```

Fig. 10. The \mathcal{EVT} specification corresponding to M3 continued from Figure 9