FYS3150 - PROJECT 2 - AUTUMN 2015

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Abstract

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Introduction

Theory

Similarity transformations

In the following we assume that the matrix $\mathbf{A}$ is a real and symmetric matrix, $\mathbf{A} \in \mathbb{R}^{n \times n}$. Then, there exists a set of real orthogonal matrices $\mathbf{S_i}$, such that

$$\mathbf{S_n^T} ... \mathbf{S_1^T} \mathbf{A} \mathbf{S_1} ... \mathbf{S_n} = \mathbf{D}, \tag{1}$$

where $\mathbf{S_i^T S_i} = \mathbf{S_i S_i^T} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix, and $\mathbf{D}$ is given by

$$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 & 0 & \dots & 0 \\ 0 & \lambda_2 & 0 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \dots & \dots & \dots & \lambda_n \end{bmatrix},$$

where $\lambda_1, \lambda_2, ..., \lambda_n$ is the eigenvalues of $\mathbf{A}$. If we look at a single similarity transformation, where $\mathbf{B} = \mathbf{S^T A S}$ is a similarity transform of $\mathbf{A}$, we can show that $\mathbf{B}$ and $\mathbf{A}$ have the same eigenvalues, but in general not the same eigenvectors. We starts out with the eigenvalue equation of $\mathbf{A}$.

$$\mathbf{Ax} = \lambda \mathbf{x} \quad \Rightarrow \quad \mathbf{S^T A I x} = \lambda \mathbf{S^T x}$$

$$\Rightarrow \quad (\mathbf{S^T A S})(\mathbf{S^T x}) = \lambda(\mathbf{S^T x}) \quad \Rightarrow \quad \mathbf{B}(\mathbf{S^T x}) = \lambda(\mathbf{S^T x})$$

We then see that $\mathbf{B}$ and $\mathbf{A}$ have the same eigenvaules, but when $\mathbf{A}$ has the eigenvector $\mathbf{x}$, $\mathbf{B}$ has the eigenvector $\mathbf{S^T x}$. We then know for sure that the matrix $\mathbf{D}$ in Equation (1) gives the eigenvalues of $\mathbf{A}$.

Jacobi's method

This is a method one can use to obtain the eigenvalue matrix $\mathbf{D}$ in Equation (1).

Method

Results and discussion

Conclusion