Solving the power flow problem on integrated transmission-distribution networks: a review and numerical assessment

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Abstract

Power flow simulations form an essential tool for electricity network analysis but conventional models are designed to work on a separated transmission or distribution network only. The continuing growth of electricity consumption, demand side participation and renewable resources makes the electricity networks co-dependent. Integrated models incorporate the coupling of the networks and interaction that they have on each other, representing the power flow within this changing environment accurately. Several numerical methods are available to solve the power flow problem on integrated networks. They can be categorized as a unified or as a splitting method and networks can be modeled as a homogeneous or hybrid network. In this paper, we review and assess these methods on the network models by running simulations on small test networks and comparing the outcome on accuracy, computational time, and convergence. The review shows that all the methods solve the power flow problem accurately. Methods running on hybrid networks have a slight advantage in computational time. Realistic network models, running on millions of buses and with large distribution networks, should give a better insight into the speed of the computations.

1 Introduction

Power flow computations are used to simulate the transport, generation, and consumption of power in electricity networks. The simulations are important for safe operation and planning of the electricity grid. An electricity grid consists of one transmission network, responsible for transport of power over large distances, and several distribution networks, responsible for the transport to end-consumers. Transmission networks transport high-voltage power to substations from where power is converted to a lower voltage level. The distribution network transports low-voltage power from substations to end-consumers. These transmission and distribution domains are separately analyzed where one domain is modeled in detail and the other is simplified in this model to one bus. Eg: in the analysis of the transmission system, the distribution network is simplified to one load in the transmission model. In the analysis of the distribution system, the transmission network is simplified as a generator.

The power system is changing: more renewable energy is entering the grid at distribution level, we see an increase of demand-side participation as a mechanism to balance frequency, and we see an increase of electricity consumption. The changing environment requires a more detailed analysis of the electricity network and a more complex model of the interaction between the networks. Integrated transmission-distribution network models study the interaction that these networks have on each other. But it is not straight-forward to integrate these separate domains. The networks have different characteristics what has resulted in different network models that cannot be easily integrated. The most important difference is that the transmission network is balanced and therefore modeled in single-phase while the distribution network is in general not balanced and modeled in three-phase. Furthermore, the different characteristics have lead to the development of different algorithms to solve the power flow problem. The power flow problem is a nonlinear problem which is solved using iterative methods. Well-known algorithms to solve transmission systems are: Gauss-Seidel, Newton-Raphson, Decoupled loadflow, and DC loadflow [1]. These solvers have been adapted to distribution solvers, among which the most well-known are: Newton-Raphson current mismatch [2] [3], implicit Z_{bus} [4] [5], BIBC/BCBV [6] [7], and Forward/Backward Sweep [8] [9] methods. Besides the technical complexity of integrating network models, we have the issue of confidentiality. System operators are not willing or not allowed to share network information with other system operators. *One can reason that conventional models will at a certain moment not suffice anymore. Lagging legislation will then follow the necessary change towards integrating network models.*

De volgende zin moet er helemaal niet in of vriendelijker verwoord worden

Integrated networks can be modeled as homogeneous and as hybrid networks. A homogeneous network model is a three-phase representation of both the transmission and distribution network. A hybrid network model consists of a single-phase transmission and a three-phase distribution network. Several numerical methods have been developed to solve integrated networks. They can be classified as a unified [10] or as a splitting method [11]. The unified methods solve the integrated system as a whole. The splitting method divides the networks into a master and a slave and defines an extra iterative scheme between them. At every iteration, the separated systems are solved on its own. The integrated system is solved once convergence on the boundary between the systems have been reached. This article reviews the proposed integration methods and analyzes them by running simulations on small test-cases. We pay special attention to their convergence behaviour: CPU-time and the number of iterations.

2 Characterization of the power flow problem

The steady-state power flow problem is the problem of determining the voltages V in a network, given the specified power[1] $S = P + \iota Q$ and current I. V and I are related by Ohm's Law, $I = YV$, and S and V are related by $S = VI^*$. Y represents the admittance of a power cable. Power is generated in three phases leading to three sinusoidal functions that describe phase a, b, and c of the voltage and of the current. The voltage and current are both expressed in phasor notation:

$$V_p = |V|_p \exp{(\iota\delta_{V_p} - \phi_p)}, \tag{1}$$

$$I_p = |I|_p \exp{(\iota\delta_{I_p} - \phi_p)}, \qquad p \in \{a, b, c\}, \tag{2}$$

where $|\cdot|$ describes the phasor magnitude, δ_* the phase angle, and ϕ_* the phase shift. Phasors are often expressed as $V = \{|V|, \delta\}$. Current is never specified in an electricity system, therefore we substitute Ohm's Law into $S = VI^*$ and get a nonlinear equation for S, the three-phase nonlinear power flow equation, described as follows:

$$S_p = V_p(\mathbf{Y}V)_p^*, \qquad p \in \{a, b, c\}. \tag{3}$$

We represent an electricity network as a graph consisting of buses $i = 1, .., N$ and branches (named after the two surrounding buses). These buses are either a load bus (PQ bus), a generator bus (PV bus), or a reference bus, depending on the information we know at that point. The loads in a network are modeled as PQ buses, loads consume power and at this bus, the active (P) and reactive power (Q) are specified. Generators are modeled as PV buses, except for the first generator in a network, this bus is modeled as the slack bus. Generators supply power and at this bus, the active power (P) and voltage magnitude ($|V|$) are specified. At a slack bus, the voltage magnitude and angle are specified. This is summarized in table 1. We scale all the engineering quantities in the

Table 1: Bustypes in a network and the information we know and not know at each bus i.

bus type	known	unknown
PQ-bus	$P_i,\ Q_i$	$\delta_i,\ \lvert V_i\rvert$
PV-bus	$P_i,\ \lvert V_i\rvert$	$Q_i,\ \delta_i$
slack bus	$\delta_i,\ \lvert V_i\rvert$	$P_i,\ Q_i$

power system to per-unit (pu) quantities by dividing them by their base value. In this way, we bring the voltage in a narrow range close to unity to eliminate erroneous values [1].

Equation (3) is a nonlinear equation so we solve it using iterative methods. At

[1] We use the subscript ι as the imaginary unit.

each node i, we solve the following equation for the unknown quantities:

$$S_i = V_i \sum_{k=1}^{N} \mathbf{Y}^*_{ik} V^*_k \tag{4}$$

The nodal admittance matrix $\mathbf{Y}_{ij}$ consists of admittance y_{ij} of a line between node i and j and nodal shunt susceptance b_c. Line admittance consists of a real and imaginary part: $y_{ij} = 1/z_{ij} = 1/(r_{ij} + \iota x_{ij})$, z being impedance, r being resistance and x being reactance. The nodal admittance matrix relating node i and j looks as the following:

$$\mathbf{Y}_{ij} = \begin{bmatrix} (y_{ij} + \iota \frac{b_c}{2}) \frac{1}{\tau^2} & -y_{ij} \frac{1}{\tau \exp(-\iota \theta_s)} \\ -y_{ij} \frac{1}{\tau \exp(\iota \theta_s)} & y_{ij} + \iota \frac{b_c}{2} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_{ij}$$

The parameters θ_s and τ are the transformer's phase-shift angle and tap ratio, when there is no transformer between two nodes but just a normal power cable, $\theta_s = 0$ and $\tau = 1$.

3 Electricity networks

An electricity network consists of one transmission and several distribution networks. The design and characteristics of these types are different and therefore require different approaches to solve equation (3).

3.1 Transmission networks: single-phase representation

The transmission network is the high-voltage network, responsible for the transportation of power over large distances. It is a balanced system which means that the three phases a, b, and c of the generated power are equal in magnitude and equal in phase-shift (ϕ). For a voltage V this means that $|V|_a = |V|_b = |V|_c$ and $\phi_{ab} = \phi_{bc} = \phi_{ca} = \frac{2}{3}\pi$. To simplify and speed-up the computations in the transmission network, they only calculate V_a and deduct the other two phases from here. This changes (3) into the following:

$$S_p = V_p (YV)^*_p, \quad p \in \{a, b, c\} \quad \Rightarrow \quad S_a = V_a (YV)^*_a. \tag{5}$$

We use Newton-Raphson power mismatch (NR-power) to compute unknown quantities at each bus i. NR-power computes V_i using the following power mismatch formulation:

$$\Delta S_i = S_{s,i} - S(V_i) \approx 0. \tag{6}$$

S_s is the specified power, the known information at generator and load nodes: $S_s = S_g - S_d$, subscript g representing the generator buses and d the load buses. $S(V)$ is the injected power, $S(V) = V(\mathbf{Y}V)^*$. The complex power S is split into an active and reactive part and combined to form the power mismatch vector F,

$$F(V) = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} P_s - P(V) \\ Q_s - Q(V) \end{bmatrix}. \tag{7}$$

We denote the state variables V by x, $x := V = \{|V|, \delta\}$. We compute V in an iterative manner using the Jacobian J of the power mismatch vector:

$$\Delta x^{\nu} = -\mathbf{J^{-1}}(x^{\nu})F(x^{\nu}), \tag{8}$$

$$x^{\nu+1} = x^{\nu} + \Delta x^{\nu}, \tag{9}$$

where the Jacobian is represented as follows:

$$\mathbf{J}(x) = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} \end{bmatrix}.$$

We repeat this until the norm of the power mismatch vector $|F|_{\infty}$ is lower than a certain tolerance value ϵ. We choose $\epsilon = 10^{-8}$ and start with a flat profile as initial guess: $V^0 = 0$. The Newton-Raphson algorithm is as follows:

Algorithm 3.1 The Newton-Raphson iterative method

1: Set $\nu = 0$ and choose appropriate starting value $\mathbf{x}^0$;
2: Compute $F(x^{\nu})$;
3: Test convergence: If $|F(x^{\nu})|_{\infty} \leq \epsilon$ then x^{ν} is the solution, Otherwise continue.
4: Compute the Jacobian matrix $\mathbf{J}(x^{\nu})$;
5: Update the solution:

$$\Delta x^{\nu} = -\mathbf{J^{-1}}(x^{\nu})F(x^{\nu})$$
$$x^{\nu+1} = x^{\nu} + \Delta x^{\nu}$$

6: Update iteration counter $\nu + 1 \rightarrow \nu$, go to step 2.

3.2 Distribution networks: three-phase representation

Distribution systems are unbalanced: the three phases are not equal in magnitude nor in phase-shift. This requires us to compute all the three phases in every iteration when solving equation (3). We use Newton-Raphson Three-Phase Current Injection Method (NR-TCIM) [2] to solve distribution networks. Instead of applying the standard Newton-Raphson method to power mismatches, Ohm's Law is directly used resulting in the current mismatch vector:

$$F(x) = \begin{bmatrix} \Delta I^{Re,abc}(x) \\ \Delta I^{Im,abc}(x) \end{bmatrix} = \begin{bmatrix} I_s^{Re,abc} - I^{Re,abc}(x) \\ I_s^{Im,abc} - I^{Im,abc}(x) \end{bmatrix}. \tag{10}$$

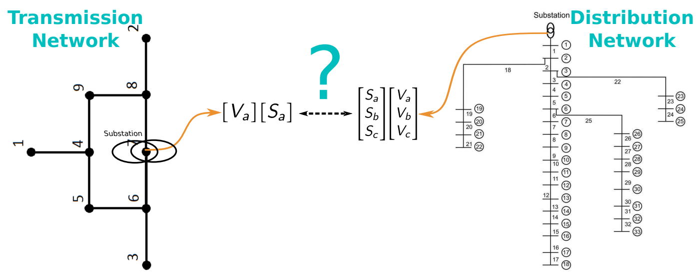

Figure 1: Information mismatch at the substation between a transmission and distribution network

The specified current I_s and computed current $I(V)$ are calculated using the injected complex power and Ohm's Law:

$$I_{s,i} = \left(\frac{\overline{S_s}}{V}\right)_i \quad \text{and} \quad I(V)_i = \mathbf{Y}V_i \tag{11}$$

The Jacobian is formed by taking the derivative of the real and imaginary current mismatch with respect to the real and imaginary voltage.

4 Integrating transmission and distribution networks

Transmission networks and distribution networks are connected to each other via a substation. In separate network models, this substation is modeled as a single-bus containing the information of the other system. In transmission network models, this substation is modeled as a load bus. In distribution network models, the substation is modeled as the slack bus. The total load in the transmission model is equal to the total generated power of the slack bus in the distribution model. The difference of the substation as load bus, is that the parameters S and V are modeled as single-phase quantities, while in the distribution model as three-phase quantities. Figure 1 shows this information mismatch. Table 2 shows the single-phase and three-phase representation of S, V and Y in the network models. We need integration methods to resolve this mismatch at the substation and solve the power flow problem on the integrated network. We conducted a literature study to investigate which numerical methods exist to solve this problem.

The literature suggests two approaches to run computations on integrated networks: a unified and a splitting approach, and it suggests two ways of modeling integrated networks: as a homogeneous or as a hybrid network. Two integrated three-phase networks form a homogeneous network[2]: both the transmission and

[2]two single-phase networks can also make a homogeneous network, but we regard only unbalanced three-phase distribution networks.

Table 2: Representation of parameters in transmission and distribution network models

parameter	Transmission	Distribution
S_i	$[S_a]_i$	$[S_a\ S_b\ S_c]_i^T$
V_i	$[V_a]_i$	$[V_a\ V_b\ V_c]_i^T$
Y_{ij}	$\begin{bmatrix} \underset{1\times1}{Y_{11}^a} & \underset{1\times1}{Y_{12}^a} \\ \underset{1\times1}{Y_{21}^a} & \underset{1\times1}{Y_{22}^a} \end{bmatrix}_{ij}$	$\begin{bmatrix} \underset{3\times3}{Y_{11}^{abc}} & \underset{3\times3}{Y_{12}^{abc}} \\ \underset{3\times3}{Y_{21}^{abc}} & \underset{3\times3}{Y_{22}^{abc}} \end{bmatrix}_{ij}$

distribution network are modeled in three phases. A hybrid network consists of a single-phase transmission network and a three-phase distribution network. The unified methods first integrates the two network into one system and then solves the entire system at once. This has the advantage that it requires only one iterative process to solve the system, but also implies that the system has to be solved with either NR-power or NR-TCIM[3], being disadvantageous for a part of the system. The splitting approach appoints the transmission network as the master and the distribution network as the slave. It iterates between the two networks and at each iteration, it solves the networks separately. The master-slave splitting uses an extra iterative scheme besides the transmission and distribution schemes, but this has the advantage that the separate systems can be solved with the preferred Newton-Raphson algorithm. We call the unified approach applied to homogeneous networks the full three-phase (F3P) method and applied to hybrid networks the interconnected (IC) method. We call all splitting methods Master-Slave splitting (MSS) methods, as described in the literature [12], and define them based on their option, eg: the splitting approach applied to hybrid networks is called the MSS-hybrid method. Figure 2 gives an overview of the methods.

4.1 Unified methods

Unified methods solve the integrated system using one iterative scheme applied to the entire integrated network. Both the separate network models include the substation in their system. The integrated network is created by converting this substation to two buses connected via a transformer. Figure 3 clarifies this idea. This transformer is modeled as a distribution cable with the same admittance $\mathbf{Y}_{ij}$ as the first cable in the distribution network. The bus at the transmission side of the transformer is modeled as a load bus, just as in the original network. The bus at the distribution side of the transformer is modeled as generator bus. This bus is the former slack bus of the distribution network, but as only one

[3] In het geval van dit paper dan

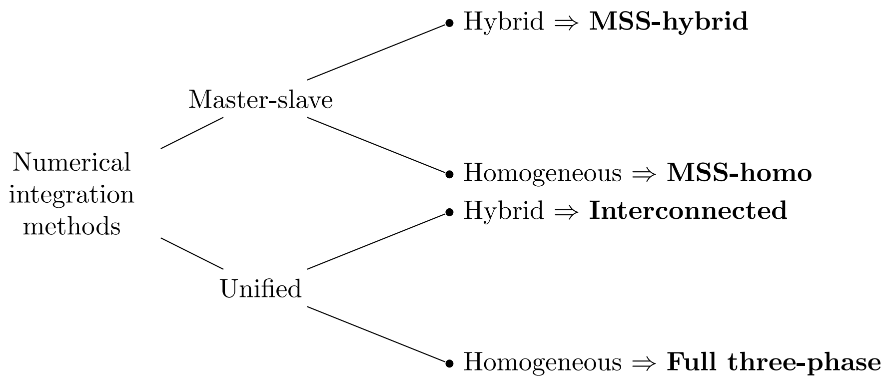

Figure 2: Classification of numerical methods to solve integrated systems.

reference bus is allowed per system, it is changed to a generator bus. Unified methods require us to choose one algorithm to solve the entire network. We will compare both NR-power and NR-TCIM to solve the system.

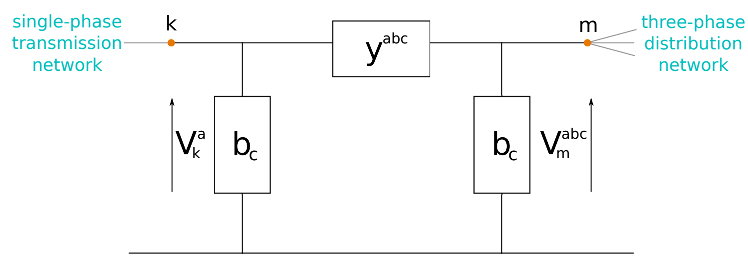

Figure 3: The original substation replaced by two buses k and m which are connected by a transformer.

4.1.1 The full three-phase method

The full three-phase method is the unified method applied on homogeneous networks and a homogeneous network consists of two three-phase networks. Unbalanced distribution networks are by default modeled in three-phase, the transmission networks requires a transformation. This transformation is based on the assumption that the transmission system is balanced: we deduct the phases b and c from the first phase a and we transform the voltage V_a, the power S_a, and the admittance Y_a of all the buses $i = 1, ..., N$ to their three-phase equivalents. We transform all the buses $i = 1, .., N$ in the transmission system using transformer matrices:

$$\mathbf{T_1} = \begin{bmatrix}1 & a & a^2\end{bmatrix}^T \quad \text{and} \quad \mathbf{T_2} = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T, \quad a = e^{\frac{2}{3}\pi\iota},$$

and identity matrix $\mathbf{I}_{3\times 3}$. This results into the following:

$$\mathbf{T_1}\left[V_a\right]_i = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix}_i^T, \tag{12}$$

$$\mathbf{T_2}\left[S_a\right]_i = \begin{bmatrix} S_a & S_b & S_c \end{bmatrix}_i^T, \tag{13}$$

$$\begin{bmatrix} Y_{11}^a \otimes \mathbf{I}_{3\times 3} & Y_{12}^a \otimes \mathbf{I}_{3\times 3} \\ Y_{21}^a \otimes \mathbf{I}_{3\times 3} & Y_{22}^a \otimes \mathbf{I}_{3\times 3} \end{bmatrix}_{ij} = \begin{array}{c} \\ 3 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 3 & \quad 3 \end{array} \\ \begin{bmatrix} \mathbf{Y}_{11}^{abc} & \mathbf{Y}_{12}^{abc} \\ \mathbf{Y}_{21}^{abc} & \mathbf{Y}_{22}^{abc} \end{bmatrix}_{ij} \end{array}. \tag{14}$$

4.1.2 The interconnected method

The interconnected method is the unified method applied to hybrid networks. A hybrid network consists of a single-phase transmission part and a three-phase distribution part. The transformer in between load bus k of the transmission network and the generator bus m of the distribution network connects the information between the two networks. It couples the single-phase quantities at the transmission side to the three-phase quantities at distribution side by transforming the nodal admittance matrix $\mathbf{Y}_{km}$. We use three transformer matrices

$$\mathbf{T_1}, \quad \mathbf{T_3} = \frac{1}{3}[1\ a\ a^2], \quad \text{and} \quad \mathbf{T_4} = \frac{1}{3}\left[1\ 1\ 1\right], \quad a = e^{\frac{2}{3}\pi\iota},$$

to establish the connection of bus k and m via the admittance matrix $\mathbf{Y}_{km}$. This transformation is based on the assumption that the connecting bus k is balanced. This means that:

$$\begin{bmatrix} V_a & V_b & V_c \end{bmatrix}_k^T = \mathbf{T_1}\left[V_a\right]_k, \tag{15}$$

$$\left[I_a\right]_k = \mathbf{T_3}\begin{bmatrix} I_a & I_b & I_c \end{bmatrix}_k^T, \tag{16}$$

$$\left[S_a\right]_k = \mathbf{T_4}\begin{bmatrix} S_a & S_b & S_c \end{bmatrix}_k^T. \tag{17}$$

Using current injections The NR-TCIM method uses Ohm's law directly. The relation between node k and m is expressed as follows:

$$I = \mathbf{Y}V \quad \Leftrightarrow \quad \begin{bmatrix} I_k \\ I_m \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_k \\ V_m \end{bmatrix} \tag{18}$$

If node k and m we're both modeled in three-phase, we know the following:

$$I_k^{abc} = \mathbf{Y}_{11}^{abc} V_k^{abc} + \mathbf{Y}_{12}^{abc} V_m^{abc}, \tag{19}$$

$$I_m^{abc} = \mathbf{Y}_{21}^{abc} V_k^{abc} + \mathbf{Y}_{22}^{abc} V_m^{abc}. \tag{20}$$

Replacing $\mathbf{V}_k^{abc}$ by $\mathbf{T_1}\left[V_a\right]_k$ (equation 15) and multiplying (20) with $\mathbf{T_3}$ to obtain $\left[I_a\right]_k$ (equation (16)) results in:

$$I_k^a = \mathbf{T_3}\mathbf{I}_k^{abc} = \mathbf{T_3}\mathbf{Y}_{11}^{abc}\mathbf{T_1}V_k^a + \mathbf{T_3}\mathbf{Y}_{12}^{abc}\mathbf{V}_m^{abc}, \tag{21}$$

$$\mathbf{I}_m^{abc} = \mathbf{Y}_{21}^{abc}\mathbf{T_1}V_k^a + \mathbf{Y}_{22}^{abc}V_m^{abc}. \tag{22}$$

From (21) and (22) we see that our new nodal admittance matrix becomes:

$$\mathbf{Y}_{km} = \begin{array}{cc} & \begin{array}{cc} 1 & 3 \end{array} \\ \begin{array}{c} 1 \\ 3 \end{array} & \left[\begin{array}{cc} \mathbf{T_3}[\mathbf{Y^{abc}}_{11}]\mathbf{T_1} & \mathbf{T_3}[\mathbf{Y}^{abc}_{12}] \\ [\mathbf{Y}^{abc}_{21}]\mathbf{T_1} & \mathbf{Y}^{abc}_{22} \end{array}\right]_{km} \end{array} \tag{23}$$

Using power injections The NR-power methods uses power injections. The relation between node k and m is expressed as follows:

$$S = VI^* \quad \Leftrightarrow \quad \begin{bmatrix} S_k \\ S_m \end{bmatrix} = \begin{bmatrix} V_k \\ V_m \end{bmatrix} \begin{bmatrix} I_k \\ I_m \end{bmatrix}^* \tag{24}$$

In the same manner as current injections, we can write this relation in three-phase:

$$S_k^{abc} = V_k^{abc} I_k^{abc*} + V_k^{abc} I_m^{abc*}, \tag{25}$$

$$S_m^{abc} = V_m^{abc} I_k^{abc*} + V_m^{abc} I_m^{abc*}. \tag{26}$$

Multiplying the first line from the left by $\mathbf{T_4}$, replacing V_k^{abc} and I_k^{abc} using (15) and (16), and substituting $I^* = (\mathbf{Y}V)^*$ we obtain:

$$S_k^a = \mathbf{T_4} S_k^{abc} = \mathbf{T_4}\mathrm{diag}(\mathbf{T_1} V_k^a) \cdot \mathbf{Y}_{11}\mathbf{T_1} V_k^a + \mathbf{T_4}\mathrm{diag}(\mathbf{T_1} V_k^a) \cdot \mathbf{Y}_{12} V_m^{abc}, \tag{27}$$

$$S_m^{abc} = V_m^{abc}\mathbf{Y}_{21}\mathbf{T_1} V_k^a + V_m^{abc}\mathbf{Y}_{22} V_m^{abc}. \tag{28}$$

We can rewrite the first part of the rhs of (27) as:

$$\mathbf{T_4}\mathrm{diag}(\mathbf{T_1} V_k^a) = \mathbf{T_4}\mathrm{diag}(\mathbf{T_1})\mathrm{diag}(V_k^a) \tag{29}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a \end{bmatrix} \mathrm{diag}(V_k^a) \tag{30}$$

$$= \underbrace{\frac{1}{3}\begin{bmatrix} 1 & a^2 & a \end{bmatrix}}_{\mathbf{T_5}} \mathrm{diag}(V_k^a) \tag{31}$$

$$\Leftrightarrow \mathrm{diag}(V_k^a)\mathbf{T_5}. \tag{32}$$

Equations (27), (28), and (32) results in a new admittance matrix $\mathbf{Y}_{km}$:

$$\mathbf{Y}_{km} = \begin{array}{cc} & \begin{array}{cc} 1 & 3 \end{array} \\ \begin{array}{c} 1 \\ 3 \end{array} & \left[\begin{array}{cc} \mathbf{T_5}[\mathbf{Y^{abc}}_{11}]\mathbf{T_1} & \mathbf{T_5}[\mathbf{Y}^{abc}_{12}] \\ [\mathbf{Y}^{abc}_{21}]\mathbf{T_1} & \mathbf{Y}^{abc}_{22} \end{array}\right]_{km} \end{array} \tag{33}$$

4.2 Master-Slave splitting methods

In contrast to the unified methods, the MSS-methods keeps two separate domains, the transmission and distribution network (or the master and the slave), and introduces an extra iterative scheme between the domains. The two domains have one overlapping bus: the substation. In the unified methods, the substation is converted to two buses connected by an extra transformer. The MSS-methods keeps the substation as one bus, which they call the boundary bus B. As the two domains are solved separately, the boundary bus becomes the slack bus for the distribution system and a load bus for the transmission system. In one MSS-iteration, we solve the slave, injects the solution of the boundary bus into the master, and then solve the master. This iterative process continues until the difference between the boundary bus of the slave and the master is smaller than a certain tolerance value ϵ.As the boundary bus is the slack bus of the slave, it requires the voltage V_B as known information. In the first iteration, we initialize the voltage as $V_B = 1.0$ pu. In the following iterations, we take the voltage from the output of the master. The boundary bus is a load bus for the master and thus requires the complex power as known. We use the output from the slave S_B as input for the master. Algorithm 4.1 shows how the iterative scheme of the MSS-method works. As the MSS-method

define ϵ

Algorithm 4.1 General algorithmic approach of the Master-Slave splitting method

1: Set iteration counter $\nu = 0$. Initialize the voltage V_B^0 of the Slave.
2: Solve the slave system. Output: $S_B^{\nu+1}$.
3: Inject $S_B^{\nu+1}$ into the Master.
4: Solve the Master. Output: $V_B^{\nu+1}$.
5: Is $|V_B^{\nu+1} - V_B^{\nu}|_1 > \epsilon$? Repeat step 2 till 5.

solves the master and slave separately, it allows for using different algorithms per domain. In this way, we choose to solve the slave with the advantageous NR-TCIM method and the master with the NR-power method.

The MSS-method can be applied to homogeneous networks and to hybrid networks. The first one requiring a transformation of the entire master domain, the latter requiring a transformation of the boundary bus only.

4.2.1 The MSS-homogeneous method

The MSS-method applied to homogeneous networks requires a transformation of the single-phase transmission system. The balanced transmission system is transformed in the same was as in the F3P-method. The voltage, power, and admittance of all the buses $i = 1, .., N$ are transformed to three-phase equivalents. This idea is summarized in equations (12) - (14) of section 4.1.1. Note that there is no connecting transformer between the transmission and distribution system: The output S_B of the slave can be directly injected into the master.

4.2.2 The MSS-hybrid method

The MSS-method applied to hybrid systems keeps the transmission system in single-phase. Only a transformation of the boundary bus is then required.

As we first solve the slave, we receive the complex power S_B as three-phase output, which we have to transform to a single-phase quantity. Once we have solved the master, we have to transform the single-phase output of the voltage V_B of the master. Here again, we assume that the boundary bus B is balanced. Balanced three-phase power in pu is related to single-phase power in (17):

$$[S_a] = \mathbf{T_4} \begin{bmatrix} S_a & S_b & S_c \end{bmatrix}^T . \tag{34}$$

We transform the complex power of the boundary bus $[S_{abc}]_B$ to $[S_a]_B$ using equation (34).
The voltage of the boundary bus has the same relation as in (15):

$$\begin{bmatrix} V_a & V_b & V_c \end{bmatrix}_B^T = \mathbf{T_1} \begin{bmatrix} V_a \end{bmatrix}_B , \quad \mathbf{T_1} = [1 \; a^2 \; a]^T \quad \text{and} \quad a = e^{\frac{2}{3}\pi\iota}. \tag{35}$$

The MSS-methods do not require a transformation of the nodal admittance matrix. We transform the necessary boundary parameters directly. At every MSS-iteration, we make transformation (34) and (35) after step 2 and step 4 of algorithm 4.1, respectively.

4.2.3 The Master-Slave iterative schemes

Two iterative schemes of the Master-Slave splitting are defined [13]. The first is the Convergence Alternating Iterative (CAI) scheme and the other is the Multistep Alternating Iterative (MAI) scheme. In the CAI-scheme, we define an explicit convergence condition for the slave and for the master. The slave is solved with NR-TCIM, for which we define a tolerance value ϵ_D. Once this system has converged, we inject its boundary output into the master. We solve the master using NR-power, for which we also define a (not necessarily) different tolerance value ϵ_T. Once the master has converged, we inject its boundary output into the slave. The integrated network is converged once we have met the convergence condition of the MSS-algorithm.

Give ϵ

In the MAI-scheme, we define a maximum number of iterations per separate system, ie: $I_{max,D}$ and $I_{max,T}$. We inject the output of one system into the other as soon this maximum number of iterations has been reached. The convergence of the integrated network is still based on the convergence condition of the MSS-algorithm.

Speeding up the CAI-scheme At every MSS iteration, we solve the separated domains on its own. To solve this separated system, we need to initialize the voltages in order to have an initial guess to start Newton-Raphson. In the current suggested schemes, we initialize all the buses, except the boundary bus B, to $V = 1.0$ pu. We can reduce the number of required iterations for the separate systems if we initialize the voltages to its last obtained solution in the previous MSS-iteration, ie $V_0^{\nu+1} = V_{I,D}^{\nu}$. This will probably be beneficial for the CAI-scheme, where a separated domain requires a couple of iterations before it is converged.

Clarify the coming phrase

5 Numerical assessment of integration methods

We are testing the unified and splitting methods on several test-cases. We implemented the methods in the Matpower[4] library. We created several integrated test cases from existing transmission and distribution test networks. We used the IEEE 9-bus, 118-bus, and 3120-bus data as transmission network test cases. We used the IEEE 13-bus and 37-bus data as distribution test cases. We modified the 13-bus Distribution network to a 10-bus network by deleting the buses connected to a regulator. The 37-bus network is originally a balanced distribution network. We changed it to an unbalanced network by shifting 20% of the loads of phase b equally to phase a and c, like the original authors [10].
To connect the load buses of the transmission network to the slack bus of the distribution network, we replaced the load-bus by a transformer with the same line impedance as the first distribution cable. We also changed the D-slack bus, the right hand side of the new transformer, to a load bus having the load of the former connecting load bus [10].
From these separated test cases, we created the following combinations: T9-D13, T9-D37, T118-D13, D118-D37, and T3120-D37. All distribution cases are connected by their reference bus. The 9-bus transmission network is connected via bus 7, the 118-bus network via bus 89, and the 3120-bus network via bus 3. We created two new test-cases to see how the integration methods respond when we connect multiple distribution networks to it: T9-2D13 and T9-3D13, respectively 2 and 3 D13-networks connected to the T9-network.

	IC		*MS-hybrid-CAI*				*MS-hybrid-MAI*			
	its	*CPU*	I_{MSS}	I_T	I_D	*CPU*	I_{MSS}	I_T	I_D	*CPU*
	#	*sec*	#	#	#	*sec*	#	#	#	*sec*
T9-D13 (7)	4	0.455	4	4	4	0.308	6	2	2	0.323
T9-D37 (7)	4	0.464	3	4	4	0.325	6	2	2	0.352
T118-D13 (89)	4	0.522	3	7	5	0.348	4	2	2	0.312*
T118-D37 (89)	4	0.518	3	7	5	0.356	4	2	2	0.358
T3120-D37 (3003)	5	143	3	6	4	383	6*	2*	2*	235*

Table 3: Comparison on number of iterations for hybrid networks (for the MSS-method (I_{MSS}) and the necessary iterations per system (I_T and I_D)), and CPU-time of the four different MSS-methods, applied on five test-cases. The numbers marked with an asterisk converged to wrong results. In this case, the MAI method required more than 2 subiterations to converge correctly. The slowest CPU times are printed in bold.

[4]MATPOWER is a package of free, open-source Matlab-language M-files for solving steady-state power system simulation and optimization problems [14]

	F3P		*MS-homo-CAI*				*MS-homo-MAI*			
	its	*CPU*	I_{MSS}	I_T	I_D	*CPU*	I_{MSS}	I_T	I_D	*CPU*
	#	*sec*	#	#	#	*sec*	#	#	#	*sec*
T9-D13 (7)	4	**0.610**	4	4	4	0.237	6	2	2	**0.257**
T9-D37 (7)	4	**0.587**	3	4	4	0.262	6	2	2	**0.299**
T118-D13 (89)	5	**0.763**	3	7	5	0.324	4	2	2	0.297
T118-D37 (89)	5	**0.920**	3	7	5	0.360	4	2	2	0.325
T3120-D37 (3003)	5	**2438**	3	6	4	1213	6	2	2	841

Table 4: Comparison on number of iterations for homogeneous networks (for the MSS-method (I_{MSS}) and the necessary iterations per system (I_T and I_D)), and CPU-time of the four different MSS-methods, applied on five test-cases. The numbers marked with an asterisk converged to wrong results. In this case, the MAI method required more than 2 subiterations to converge correctly. The slowest CPU times are printed in bold.

Table 5: Test case T118-D37 with different MAI-iterative schemes applied on a homogeneous network

	Infinity norm							
	$\|V\|$			δ				
I_T,I_D	a	b	c	a	b	c	sec	I_{MSS}
2	2.017e-01	2.017e-01	2.017e-01	1.993e+01	1.993e+01	1.993e+01	0.325	4
3	3.436e-02	3.436e-02	3.436e-02	7.153e+00	7.150e+00	7.156e+00	0.368	5
4	9.069e-03	9.005e-03	9.100e-03	7.426e-01	7.393e-01	7.448e-01	0.323	3
5	8.984e-03	8.911e-03	9.005e-03	4.507e-01	4.474e-01	4.530e-01	0.351	3
6	8.984e-03	8.911e-03	9.005e-03	4.501e-01	4.468e-01	4.524e-01	0.347	3

The results show that most of the methods are good options to solve integrated networks. They converge in a reasonable amount of time and number of iterations and this number is not increasing when the size of the network increases. Only the MS-homogeneous-MAI method is not working properly. The method does not reach convergence. If we take a closer look at the voltage profiles, Figure ... , we see a difference in how this is distributed according to the separated networks. We start with the unified methods. We see a drop at bus 7, this is the bus connected to the distribution network via a transformer. In the Master-Slave methods we see a different profile at bus 7: the voltage here is equal to the voltage at the first bus of the distribution network (bus 10 in the plot). But bus 2 is significantly lower. The results differ, because the MSS-algorithm does not put a transformer in between the two connecting nodes: it uses the input and output directly. So bus 7 IS bus 1 of the distribution network. We then must see the voltage drop somewhere else in the transmission network, which we do: mainly at bus 2. This bus is directly connected to bus

Table 6: Test case T118-D37 with different MAI-iterative schemes applied on a hybrid network

	Infinity norm							
	$\lvert V \rvert$			δ				
I_T,I_D	a	b	c	a	b	c	sec	I_{MSS}
2	2.017e-01	2.017e-01	2.017e-01	1.993e+01	1.993e+01	1.993e+01	0.358	4
3	3.436e-02	3.436e-02	3.436e-02	7.153e+00	7.153e+00	7.153e+00	0.380	5
4	9.058e-03	9.058e-03	9.058e-03	7.422e-01	7.422e-01	7.422e-01	0.353	3
5	8.963e-03	8.963e-03	8.963e-03	4.504e-01	4.504e-01	4.504e-01	0.354	3
6	8.963e-03	8.963e-03	8.963e-03	4.497e-01	4.497e-01	4.497e-01	0.351	3

Table 7: Test cases F3P norms

	Infinity norm					
	$\lvert V \rvert$			δ		
test-case	a	b	c	a	b	c
T9-D13	3.854e-04	6.851e-04	2.000e-04	-6.119e-03	-5.376e-03	-6.475e-03
T9-D37	4.000e-04	5.153e-04	4.100e-04	3.769e-01	-1.230e-02	1.529e-01
T118-D37	9.791e-05	6.000e-05	7.000e-05	3.979e-01	3.600e-01	4.237e-01
T3120-D37	2.409e-04	7.524e-04	1.293e-04	2.271e+00	2.121e-01	2.495e-01

Table 8: Test cases IC norms

	Infinity norm					
	$\lvert V \rvert$			δ		
test-case	a	b	c	a	b	c
T9-D13	2.895e-04	6.851e-04	2.000e-04	-6.000e-03	-6.000e-03	-6.000e-03
T9-D37	4.000e-04	5.247e-04	4.000e-04	3.769e-01	3.769e-01	3.769e-01
T118-D37	9.791e-05	7.000e-05	7.000e-05	3.939e-01	3.939e-01	3.939e-01
T3120-D37	2.409e-04	7.180e-04	1.593e-04	3.434e-01	2.320e-01	2.320e-01

Table 9: Test cases N2I1 norms

	Infinity norm					
	$\lvert V \rvert$			δ		
test-case	a	b	c	a	b	c
T9-D13	2.934e-04	7.982e-04	2.874e-04	4.794e-02	4.246e-02	5.097e-02
T9-D37	1.916e-04	5.394e-04	1.916e-04	2.271e+00	1.571e-02	3.297e-02
T118-D37	8.984e-03	8.911e-03	9.005e-03	4.501e-01	4.468e-01	4.524e-01
T3120-D37	2.440e-04	7.296e-04	1.303e-04	3.434e-01	1.951e-02	4.082e-02
MaxT9-D13	14	16	6	3	3	3
MaxT9-D37	6	41	6	23	3	3
MaxT118-D37	117	117	117	92	92	92
MaxT3120-D37	3137	3152	3126	3133	2017	2017

Table 10: Test cases N1I1 norms

	Infinity norm					
	$\lvert V \rvert$			δ		
test-case	a	b	c	a	b	c
T9-D13	2.934e-04	7.982e-04	2.874e-04	4.712e-02	4.712e-02	4.712e-02
T9-D37	1.916e-04	5.299e-04	1.916e-04	3.434e-01	2.487e-02	1.529e-01
T118-D37	8.963e-03	8.963e-03	8.963e-03	4.497e-01	4.497e-01	4.497e-01
T3120-D37	2.440e-04	7.296e-04	1.303e-04	2.271e+00	3.085e-02	1.529e-01
MaxT9-D13	14	16	6	3	3	3
MaxT9-D37	6	40	6	22	3	17
MaxT118-D37	117	117	117	92	92	92
MaxT3120-D37	3137	3152	3126	3134	2017	3128

Table 11: Test cases N2I2 norms

	Infinity norm					
	$\lvert V \rvert$			δ		
test-case	a	b	c	a	b	c
T9-D13	5.749e-04	7.037e-04	5.749e-04	6.702e-02	1.231e+00	1.261e+00
T9-D37	4.364e-04	4.796e-04	3.958e-04	2.587e+01	2.724e-02	1.011e+01
T118-D37	2.017e-01	2.017e-01	2.017e-01	1.993e+01	1.993e+01	1.993e+01
T3120-D37	1.048e-01	1.048e-01	1.048e-01	4.504e+02	1.535e+01	8.085e+01
MaxT9-D13	6	16	6	8	11	11
MaxT9-D37	8	19	8	22	8	31
MaxT118-D37	82	82	82	92	92	92
MaxT3120-D37	32	32	32	3134	2009	3128

Table 12: Test cases N1I2 norms

	Infinity norm					
	$\lvert V\rvert$			δ		
test-case	a	b	c	a	b	c
T9-D13	5.749e-04	7.037e-04	5.749e-04	6.932e-02	1.295e+00	1.261e+00
T9-D37	4.262e-04	5.400e-04	4.262e-04	2.721e+01	3.459e-02	1.011e+01
T118-D37	2.017e-01	2.017e-01	2.017e-01	1.993e+01	1.993e+01	1.993e+01
T3120-D37	1.048e-01	1.048e-01	1.048e-01	4.504e+02	1.536e+01	8.085e+01
MaxT9-D13	6	16	6	8	11	11
MaxT9-D37	8	41	8	22	8	31
MaxT118-D37	82	82	82	92	92	92
MaxT3120-D37	32	32	32	3134	2009	3128

7, so now gets less power.
Distribution networks are radial, so power flows into the first node and is then distributed to the rest of the network. So all these buses are only influenced by the previous bus. That is why the voltage profiles of distribution networks remains the same, only the voltage magnitude of the entire network is higher, because they receive more power from the connected transmission network, then they would get from the generator when the network is modeled separately.

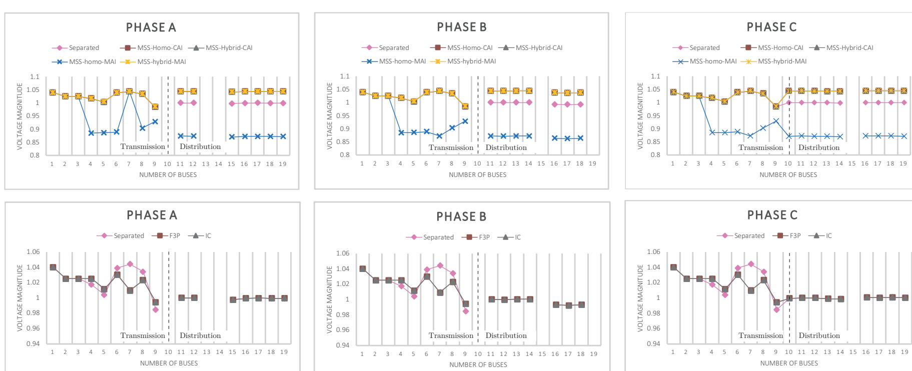

Figure 4: Voltage magnitudes of the different integration methods and separated method. Up: The MSS-methods. Down: The unified methods. Bus 7 and 11 are the buses that are connected.

5.1 Speeding-up the computations

Real transmission and distribution networks, in the Netherlands, have millions of buses. The time to run simulations then becomes really important. In the

test-simulations, it is difficult to draw any conclusions from the CPU-time. Only the T2383-D13 network gives a visible difference in CPU-time. As expected, the F3P-method is somewhat slower than the IC-method, and the MSS-methods are even slower. This is logical, because in the F3P-method, the size of the Jacobian increases of being 2n by 2n, to 6n by 6n, n the number of buses in the network. The MSS-method requires 4 iterations, thus solving each the transmission network and distribution network 4 times, instead of one integrated network 4 times.
But real networks have another difference, compared to the T2383 network: real distribution networks are much larger than real transmission networks. So the size of transmission networks is not really influencing the speed of computations. Furthermore, very large networks, must be solved in parallel and the MSS-method is some kind of domain-decomposition approach: multiple distribution networks can run on parallel cores and then inject the result to the main core, where the transmission network is solved. Doing new simulations using these speed-up techniques on realistically sized networks, should give a better idea which method is most favourable to solve the integrated power flow problem.

6 Discussion

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