Individual Claims Reserving using Activation Patterns

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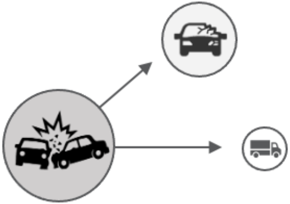
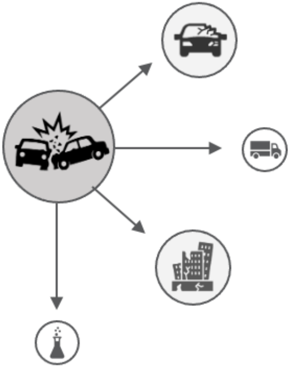
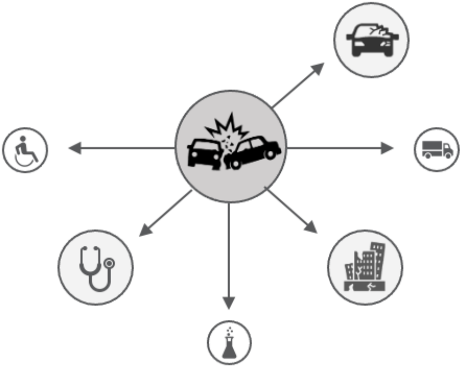
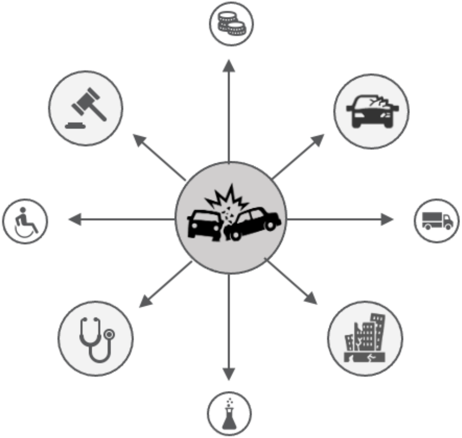
UQÀM | co-operators
Chaire Co-operators en analyse des risques actuariels

You just had a car accident...what could go wrong?









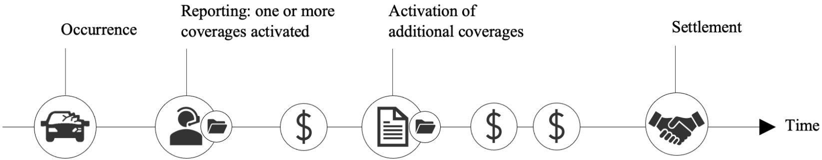


Figure 1: Typical development process of a claim.

- Examples of dependence modelling in the (granular) **claims reserving litterature**:
  - Zhou, X. and Zhao, X. (2010), Applying copula models to individual claim loss reserving methods, *Insurance: Mathematics and Economics*
  - Lopez, O. (2016), A censored copula model for micro-level claim reserving, *Insurance: Mathematics and Economics*

- Examples of correlated risks modelling in the **insurance pricing litterature**:
  - Frees, E. and Valdez, E. (2008), Hierarchical insurance claims modeling, *Journal of the American Statistical Association*
  - Shi, P., Feng, X. and Boucher, J.P. (2016), Multilevel modeling of insurance claims using copulas, *The Annals of Applied Statistics*

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Notation

- $\boldsymbol{A}_{i,j} \in \{0,1\}^C$: activation pattern vector of the C insurance coverages, of dimension $1 \times C$ for claim i, $i = 1, ..., n$ in development year j, $j = 1, ..., J$.
  $\mathcal{V}$ is the set of the $V = 2^C - 1$ patterns possible in development year j.

- $(P_{i,j,c}|A_{i,j,c} = 1) \in \{0,1\}$: payment pattern

$$(P_{i,j,c}|A_{i,j,c} = 1) = \begin{cases} 1, & \text{if a payment has been made for claim } i \\ & \quad \text{and coverage } c \text{ in year } j \text{ given } A_{i,j,c} = 1, \\ 0, & \text{otherwise.} \end{cases}$$

- $(Y_{i,j,c}|P_{i,j,c} = 1) \in \mathbb{R}^+$: severity associated to claim i in development year j for coverage c.
  - $\tilde{Y}_{i,j,c}$: total remaining severity of claims i for coverage c from development year j until settlement of the claim.

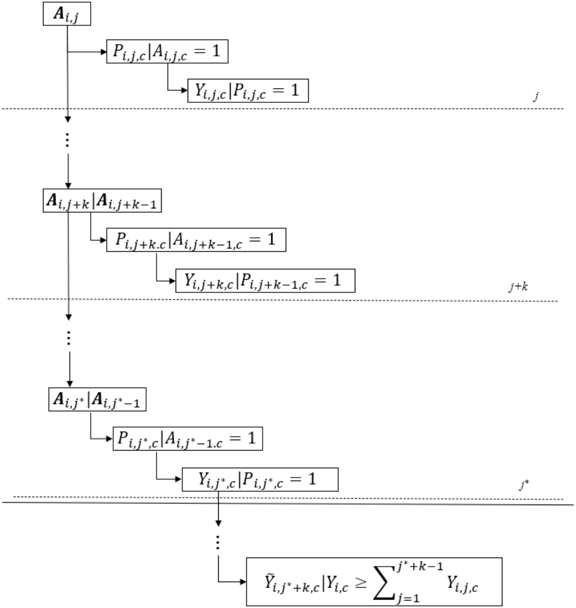

Figure 2: Summary of the activation patterns model

Development year j

a) Activation patterns: $\boldsymbol{A}_{i,j}$

Let $\boldsymbol{a}_{i,j}^{v} \in \mathcal{V}$ be a possible realisation of $\boldsymbol{A}_{i,j}$.

$$\mathrm{P}[\boldsymbol{A}_{i,j} = \boldsymbol{a}_{i,j}^{v} | \boldsymbol{x}_{i,j}, \boldsymbol{\beta}_{j,v}] = \frac{\exp\left(\boldsymbol{x}_{i,j}' \boldsymbol{\beta}_{j,v}\right)}{\sum_{k=1}^{V} \exp\left(\boldsymbol{x}_{i,j}' \boldsymbol{\beta}_{j,k}\right)}.$$

b) Payment patterns: $P_{i,j,c}|A_{i,j,c} = 1$

$$(P_{i,j,c}|A_{i,j,c} = 1) \sim \text{Bernoulli}\big(\pi_{j,c}(\boldsymbol{x}_{i,j}, \gamma_{j,c})\big)$$

$$\pi_{j,c}(\boldsymbol{x}_{i,j}, \gamma_{j,c}) = \frac{\exp\left(\boldsymbol{x}'_{i,j}\gamma_{j,c}\right)}{1 + \exp\left(\boldsymbol{x}'_{i,j}\gamma_{j,c}\right)}$$

c) Claim severity: $Y_{i,j,c}|P_{i,j,c} = 1$

$$\mathrm{E}[Y_{i,j,c}|P_{i,j,c} = 1] = g^{-1}(\boldsymbol{x}'_{i,j}\boldsymbol{\alpha}_{\boldsymbol{j},\boldsymbol{c}}).$$

Development years $j+k, j+k+1, ..., j^*$, $k \geq 1$

Hypothesis:

- $(A_{i,j+k,c} | A_{i,j+k-1,c} = 1) = 1$, for all $k \geq 1 \iff \mathcal{V}^* \subset \mathcal{V}$
- Stabilization of the activation patterns after development year j^*

a) Activation patterns: $\boldsymbol{A}_{i,j+k} | \boldsymbol{A}_{i,j+k-1}$

$$\mathrm{P}[\boldsymbol{A}_{i,j+k} = \boldsymbol{a}^{v*}_{i,j+k-1} | \boldsymbol{A}_{i,j+k-1}, \boldsymbol{x}_{i,j+k}, \beta_{j+k,v^*}]$$

$$= \begin{cases} \dfrac{\mathrm{P}[\boldsymbol{A}_{i,j+k} = \boldsymbol{a}^{v*}_{i,j+k-1} | \boldsymbol{x'}_{i,j+k}, \beta_{j+k}]}{\sum_{\boldsymbol{a}^{v*}_{i,j+k-1} \in \mathcal{V}^*} \mathrm{P}[\boldsymbol{A}_{i,j+k} = \boldsymbol{a}^{v*}_{i,j+k-1} | \boldsymbol{x'}_{i,j+k}, \beta_{j+k}]}, & \text{if } \boldsymbol{a}^{v*}_{i,j+k-1} \in \mathcal{V}^* \\ 0, & \text{otherwise,} \end{cases}$$

b) Payment patterns: $P_{i,j+k,c}|A_{i,j+k,c} = 1$

$$(P_{i,j+k,c}|A_{i,j+k,c} = 1) \sim \text{Bernoulli}\big(\pi_{j+k,c}(\boldsymbol{x}_{i,j+k}, \gamma_{j+k,c})\big)$$

$$\pi_{j+k,c}(\boldsymbol{x}_{i,j+k}, \gamma_{j+k,c}) = \frac{\exp\left(\boldsymbol{x}'_{i,j+k}\gamma_{j+k,c}\right)}{1 + \exp\left(\boldsymbol{x}'_{i,j+k}\gamma_{j+k,c}\right)}$$

c) Claim severity: $Y_{i,j+k,c}|P_{i,j+k,c} = 1$

$$\text{E}[Y_{i,j+k,c}|P_{i,j+k,c} = 1] = g^{-1}(\boldsymbol{x}'_{i,j+k}\boldsymbol{\alpha}_{j+k,c})$$

When we reach development year j^*, $Y_{i,j^*,c}$ is the last severity calculated for claim i and coverage c such that the total severity for that claim is equal to $Y_i = \sum_{j=1}^{j^*-1}\sum_{c=1}^{C} Y_{i,j,c} + \sum_{c=1}^{C}\tilde{Y}_{i,j^*,c}$.

Development years $j^* + k$, $k \geq 1$

When claim i has reached development year $j^* + k$ with $k \geq 1$, its remaining severity for coverage c starting from year $j^* + k$ is given by

$$Y_{i,j^*+k,c} | Y_{i,c} \geq \sum_{j=1}^{j^*+k-1} Y_{i,j,c}.$$

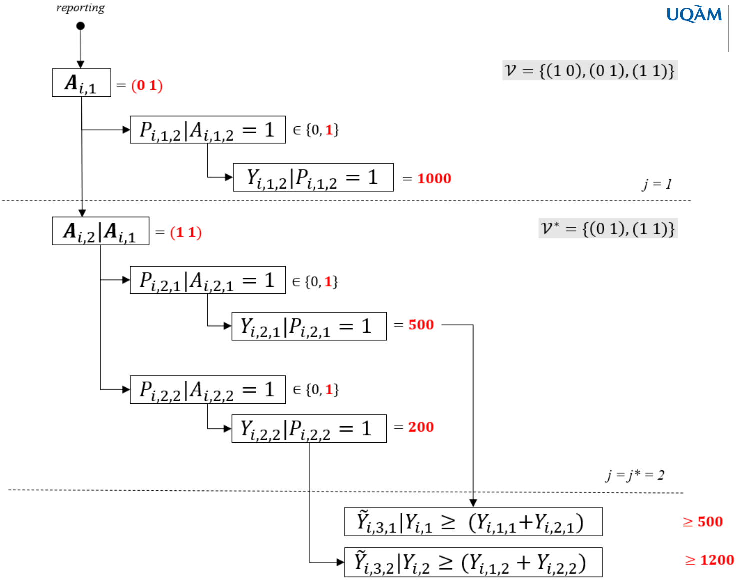

Figure 3: Illustration of the activation patterns model

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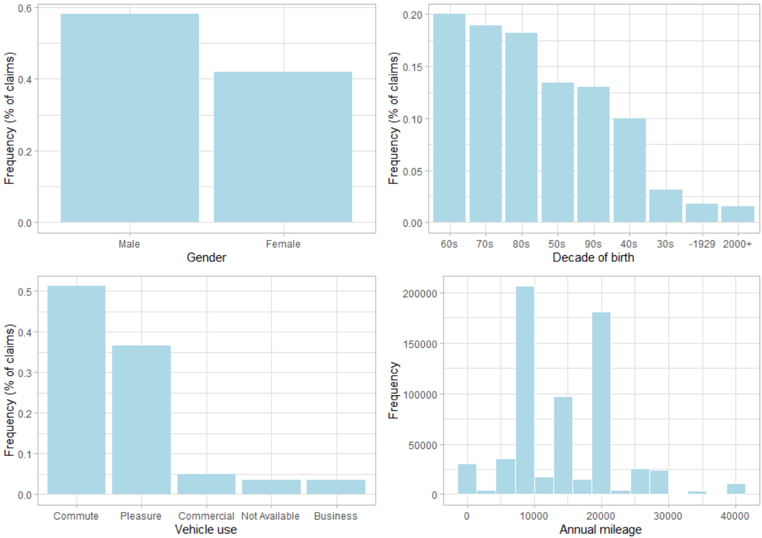
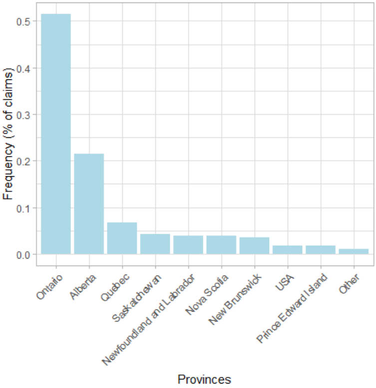
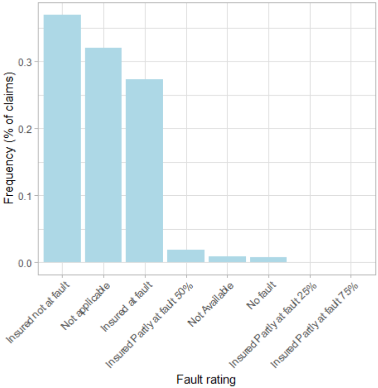
Data exploration - Risk factors




Frequency (% of claims)
0.0
0.1
0.2
0.3
0.4
0.5
Ontario
Alberta
Quebec
Saskatchewan
Newfoundland and Labrador
Nova Scotia
New Brunswick
USA
Prince Edward Island
Other
Provinces


Frequency (% of claims)
0.0
0.1
0.2
0.3
Insured not at fault
Not applicable
Insured at fault
Insured Partly at fault 50%
Not Available
No fault
Insured Partly at fault 25%
Insured Partly at fault 75%
Fault rating

Data exploration - insurance coverages

Table 1: Weight of each coverage in the portfolio

Coverage	% of claims	% of the total cost	% of the reserve
Accident Benefits	9.42	12.82	30
Bodily Injury	5.70	13.13	55
Vehicle Damage	96.39	70.44	14
Loss of Use	51.89	3.61	>1

Data exploration - activation and payment delays

Table 2: Percentage of claims with different activation delays for the four coverages

	Activation delays		
Coverage	No delay	1 year	≥2 years
Accident Benefits	96.65	3.18	0.11
Bodily Injury	89.89	7.70	2.11
Vehicle Damage	98.20	1.73	0.07
Loss of Use	97.80	2.17	0.03

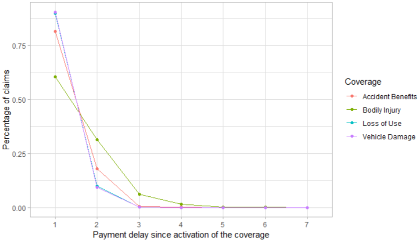


Figure 4: Payment delays per coverage

Estimation

- $C = 4$ insurance coverages.
- $j^* = 2$
- Activation of the coverages at any time in any given year until the 30^{th} of December, subsequent payments are recorded on the 31^{st} of December.
- Estimation by maximum likelihood.

Table 3: Choice of the distributions for the payment severities.

Coverage	Model	$j = 1$		$j = 2+$	
		AIC	BIC	AIC	BIC
Accident Benefits	Log-Normal	266,408	266,651	181,577	181,827
	Gamma	264,411	264,654	183,670	183,921
	Pareto	262,903	263,146	181,281	181,532
	Generalized Beta II	**261,110**	**261,368**	**181,222**	**181,487**
	Weibull	264,101	264,344	182,545	182,796
Bodily Injury	Log-Normal	100,523	100,735	106,985	107,214
	Gamma	99,675	99,887	107,096	107,325
	Pareto	**98,971**	**99,183**	106,691	106,919
	Generalized Beta II	99,966	100,185	106,884	107,119
	Weibull	99,187	99,400	**106,673**	**106,902**
Vehicle Damage	Log-Normal	5,375,907	5,376,246	620,136	620,431
	Gamma	5,385,180	5,385,519	619,456	619,751
	Pareto	5,342,022	5,342,361	617,247	**617,542**
	Generalized Beta II	**5,330,989**	**5,331,349**	**617,245**	617,557
	Weibull	5,371,787	5,372,126	618,531	618,827
Loss of Use	Log-Normal	1,874,185	1,874,498	193,705	193,967
	Gamma	1,868,867	1,869,180	192,929	193,191
	Pareto	1,894,927	1,895,241	194,376	194,638
	Generalized Beta II	**1,861,585**	**1,861,918**	**192,212**	**192,489**
	Weibull	1,877,508	1,877,821	193,459	193,721

Predictive distributions and model comparisons

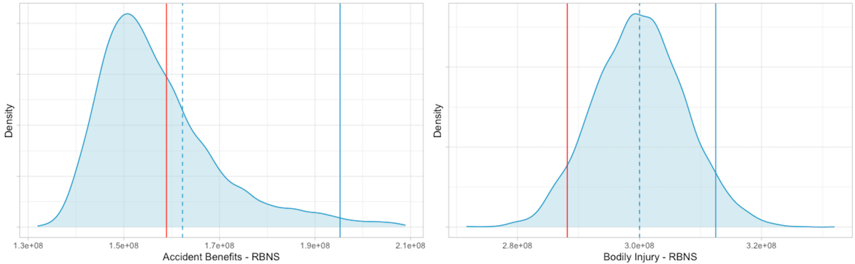


Figure 5: Simulated reserves for the Accident Benefits and Bodily Injury coverages as of 01/01/2019.

Comparison with 3 models:

- Overdispersed Poisson Chain Ladder
- Independence model
- Individual claims reserving model proposed by K. Antonio and R. Plat (2014) [1]

Coverage	Observed	Simul.	Act. pat.	ODP	Ind.	[1]
Accident Benefits	>158.90	Mean	162.15	71.01	143.41	143.46
		$\text{VaR}_{0.95}$	196.64	81.66	173.50	169.30
Bodily Injury	>288.17	Mean	300.02	177.64	278.18	-
		$\text{VaR}_{0.95}$	312.58	204.63	290.31	-
Global reserve	>524.91	Mean	536.86	493.14	495.32	-
		$\text{VaR}_{0.95}$	574.58	537.16	526.93	-

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4. Conclusion

- Separate modelling of activation and payments, allowing for a more complete modelling of the development of claims.
- Use of aggregate models requires larger quantities of data.
- Heavier-tailed, more conservative predictive distribution.
- More prudent predictions at the coverage-level, leading to a better understanding of the underlying dynamics of the portfolio.

Selected references

Antonio, K. and Plat, R. (2014), Micro-level stochastic loss reserving for general insurance, *Scandinavian Actuarial Journal*, 2014:7, 649-669.

Côté, M-P.,Genest, C. and Stephens, D.A. (2021), A Bayesian Approach to Modeling Multivariate Multilevel Insurance Claims in the Presence of Unsettled Claims, *Bayesian Analysis, Bayesian Anal. Advance Publication*, 1-27.

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