Age-Dependent Compartmental Models and Vaccincation

STAT 244NF: Infectious Disease Modeling

Incorporating age into a stochastic SEIR model[1]

This method assumes that we have a separate SEIR model for each age, but that everyone randomly contacts each other in a population, regardless of their age, so any infectious person can contact any susceptible person and has a chance of infecting them.

Notation for SEIR model for a single age, a at time t

- $S[a]_t$: number of individuals of age a that are susceptible at time t
- $E[a]_t$: number of individuals of age a that are pre-infectious at time t
- $I[a]_t$: number of individuals of age a that are infectious at time t
- $R[a]_t$: number of individuals of age a that are recovered at time t

Notation for random variables representing individuals moving between compartments of age a at time t

- $E^{(SE)}[a]_t$: number of individuals of age a that are newly pre-infectious (E) at time t
  - $\lambda_t = 1 - \exp\left\{-c_e \times \frac{I_{t-1}}{N_{t-1}}\right\}$ (I_{t-1} is everyone who is infectious at time $t-1$, regardless of their age. This assumes everyone randomly contacts each other and there is no age-dependent contact structure)
  - $E^{(SE)}[a]_t \sim Binomial(S[a]_{t-1}, \lambda_t)$
- $I^{(EI)}[a]_t$: number of individuals of age a that are newly infectious (I) at time t
  - $I^{(EI)}[a]_t \sim Binomial(E[a]_{t-1}, \pi = 1/D')$
- $R^{(IR)}[a]_t$: number of individuals of age a that are newly recovered (R) at time t
  - $R^{(IR)}[a]_t \sim Binomial(I[a]_{t-1}, \rho = 1/D)$

Difference Equations

For $t-1$ in a given year (but not the last day/week/month of the year, depending on the time scale for t):

$$\begin{aligned}
S[a]_t &= S[a]_{t-1} - E^{(SE)}[a]_t \\
E[a]_t &= E[a]_{t-1} + E^{(SE)}[a]_t - I^{(EI)}[a]_t \\
I[a]_t &= I[a]_{t-1} + I^{(EI)}[a]_t - R^{(IR)}[a]_t \\
R[a]_t &= R[a]_{t-1} + R^{(IR)}[a]_t
\end{aligned}$$

[1] Stochastic adaptation to method of Schenzle presented in Vynnycky and White, Ch. 5.3

Any births (random variable B) will be accounted for at the end of the year, and in this formulation we will not include deaths. Also, first we will assume that there is no vaccination of newborns. Therefore, when $t-1$ is at the end of the year,

$$
\begin{aligned}
S[a]_t &= S[a-1]_{t-1} - E^{(SE)}[a-1]_t \\
E[a]_t &= E[a-1]_{t-1} + E^{(SE)}[a-1]_t - I^{(EI)}[a-1]_t \\
I[a]_t &= I[a-1]_{t-1} + I^{(EI)}[a-1]_t - R^{(IR)}[a-1]_t \\
R[a]_t &= R[a-1]_{t-1} + R^{(IR)}[a-1]_t
\end{aligned}
$$

which is the same as what we had before, but we also need to add new births for age 0. We will define B as the number of new births (I am omitting t because we are updating annually.) We can assume one of two possible probability distributions for this:

- $B \sim Binomial(N_{t-1}, \beta)$ where β is the per capita annual birthrate, or
- $B \sim Poisson(\beta \times N_{t-1})$

$$
\begin{aligned}
S[0]_t &= B \\
E[0]_t &= 0 \\
I[0]_t &= 0 \\
R[0]_t &= 0
\end{aligned}
$$

This assumes all newborns are born susceptible and that there is no vaccination of newborns. We easily can incorporate vaccination.

Let's assume that some proportion ν of newborns are vaccinated a particular infection (babies are vaccinated for a variety of infections and diseases in their first year of life, including Hepatitis B, Rotavirus, Diptheria, Tetanus, Pertussis and Influenza[2]).

In this case, what is the average number of newborns that are vaccinated? What compartment would they go into?

What is the average number of newborns that are not vaccinated? What compartment would they go into?

[2] https://www.cdc.gov/vaccines/schedules/hcp/imz/child-adolescent.html

How would you modify the equations for age 0 to accommodate this?

$$
\begin{aligned}
S[0]_t &= \\
E[0]_t &= \\
I[0]_t &= \\
R[0]_t &=
\end{aligned}
$$

These averages are not guaranteed to be integers. What modifications could you make to guarantee that these are integers? Hint: You can use a similar approach to what we did with the individuals moving between compartments. Make your modifications below.

$$
\begin{aligned}
S[0]_t &= \\
E[0]_t &= \\
I[0]_t &= \\
R[0]_t &=
\end{aligned}
$$

Implementation

Sketch out pseudo code for implementation.

What do you see as potential challenges for implementing this model?

Are there any modifications you can think of that might be reasonable? If so, what assumptions are you making?

```r
# library(numbers)
SEIR_age_simulation <- function(S0, E0, I0, R0, RN, pD, D, b, v, maxAge, time){
  S <- matrix(nrow=(time+1), ncol=(maxAge+1))
  E <- matrix(nrow=(time+1), ncol=(maxAge+1))
  I <- matrix(nrow=(time+1), ncol=(maxAge+1))
  R <- matrix(nrow=(time+1), ncol=(maxAge+1))
  N <- rep(NA, time+1)
  lambda <- rep(NA, time+1)

  ## Initialize
  S[1,] <- S0 ## now a vector
  E[1,] <- E0 ## now a vector
  I[1,] <- I0 ## now a vector
  R[1,] <- R0 ## now a vector
```

```r
  for (i in 2:time){
    N[i-1] <- sum(c(S[i-1,],E[i-1,],I[i-1,],R[i-1,]))
    lambda[i] <- 1-exp(-RN/D*sum(I[i-1,])/N[i-1])
    if (mod(n=i, m=52)!=0){
      S[i,] <- S[i-1,]-lambda[i]*S[i-1,]
      E[i,] <- E[i-1,]+lambda[i]*S[i-1,]-(1/pD)*E[i-1,]
      I[i,] <- I[i-1,]+(1/pD)*E[i-1,]-(1/D)*I[i-1,]
      R[i,] <- R[i-1,]+(1/D)*I[i-1,]
    }
    else{
      ## Moving everyone up by one year (in age)
      S[i,2:maxAge] <- S[i-1,1:(maxAge-1)]-lambda[i]*S[i-1,1:(maxAge-1)]
      E[i,2:maxAge] <- E[i-1,1:(maxAge-1)]+lambda[i]*S[i-1,1:(maxAge-1)]-(1/pD)*E[i-1,1:(maxAge-1)]
      I[i,2:maxAge] <- I[i-1,1:(maxAge-1)]+(1/pD)*E[i-1,1:(maxAge-1)]-(1/D)*I[i-1,1:(maxAge-1)]
      R[i,2:maxAge] <- R[i-1,1:(maxAge-1)]+(1/D)*I[i-1,1:(maxAge-1)]
      ## births
      S[i,1] <- (b*N[i-1])*(1-v)
      E[i,1] <- 0
      I[i,1] <- 0
      R[i,1] <- (b*N[i-1])*v
    }
  }
  return(list(S=S, E=E, I=I, R=R))
}
```

```r
test <- SEIR_age_simulation(S0=rep(100, 70), E0=rep(0, 70), I0=c(rep(0,4), 1, rep(0,65)), R0=rep(0,70),
                            RN=7, pD=5, D=6, b=1/(70*52), v=0.2, maxAge=69, time=104)

str(test)
```

```
## List of 4
##  $ S: num [1:105, 1:70] 100 100 100 100 99.9 ...
##  $ E: num [1:105, 1:70] 0 0.0167 0.0272 0.0372 0.049 ...
##  $ I: num [1:105, 1:70] 0 0 0.00333 0.00822 0.01429 ...
##  $ R: num [1:105, 1:70] 0 0 0 0.000555 0.001925 ...
```

What does this function do so far?

Brainstorm: What would be a useful way to reorganize the results from this simulation to better visualize the results?