Confidence Intervals (CI)

A **confidence interval** is a range of plausible values for a population parameter. In this class, we will be constructing confidence intervals for two population parameters:

- population mean, μ
- population proportion, p

While there are many kinds of confidence intervals in statistics (which you can learn about in more advanced classes), we are focusing on confidence intervals with the following form:

$$\text{point estimate} \pm z^* \times SE$$

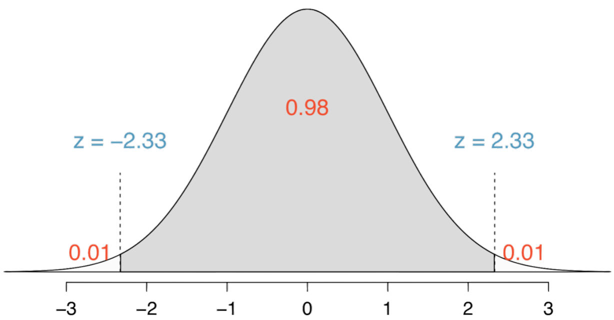
This kind of interval is a Wald confidence interval. **It relies on an assumption of approximate normality. Before we construct such confidence intervals, we must assess whether approximate normality is an appropriate assumption (i.e. is the parent distribution normal? is the sample size "sufficiently large"?).**

The **point estimate** is a

- sample mean, $\bar{x}$ (if we wish to estimate the population mean), or
- sample proportion, $\hat{p}$ (if we wish to estimate the population proportion).

The z^* is a number which is sometimes referred to as the critical value. This comes from a standard normal distribution and changes based on the confidence level we choose for our interval. For example, for a 95% confidence interval, z^*=1.96; this corresponds to cutoff points between which 95% of the probability lies.

The figure below gives an illustration of this for a 98% confidence interval; z^*=2.33.



The standard error (SE) is

- $\frac{\sigma}{\sqrt{n}}$ (if we wish to estimate the population mean), or
- $\sqrt{\frac{p(1-p)}{n}}$ (if we wish to estimate the population proportion).

Example R Code

To find z^* for a 98% confidence interval:

- `qnorm(p=0.01, mean=0, sd=1, lower.tail=FALSE)`

Interpretation

Confidence intervals are:

- always about the population
- are not probability statements
- only about population parameters, not individual observations
- only reliable if the sample statistic they're based on is an unbiased estimator of the population parameter

The general interpretation for a 95% confidence interval for the population mean is as follows:

We are 95% confident that the population mean is between $\bar{x} - \underbrace{1.96}_{z^* \text{ for a 95\% CI}} \times \sigma/\sqrt{n}$ *(the lower bound of the interval) and* $\bar{x} + 1.96 \times \sigma/\sqrt{n}$ *(the upper bound of the interval).*

The general interpretation for a 95% confidence interval for the population proportion is as follows:

We are 95% confident that the population proportion is between $\bar{x} - 1.96 \times \sqrt{p(1-p/)n}$ *(the lower bound of the interval) and* $\bar{x} + 1.96 \times \sqrt{p(1-p)/n}$ *(the upper bound of the interval).*

What does it mean to be 95% confident?

Imagine going out and collecting 100 different samples, and then constructing a separate confidence interval for each one. We expect that about 95 of these hundred confidence intervals will contain the true population parameter (either the population mean or the population proportion, depending on which one you were interested in, given the data).

Practice Problems

1. According to official census figures, 8% of couples living together are not married (in the United States). A researcher took a random sample of 400 couples and found that 9.5% of them are not married. Estimate the proportion of couples in the United States that are not married. Show all your work. (Note, estimate here indicates that you should create a confidence interval.)

   (a) Is this problem about a mean or a proportion?

   (b) What is the value of (i) n; (ii) $\hat{p}$?

   (c) Check that the conditions of the relevant Central Limit Theorem are satisfied for this problem.

   (d) Estimate the proportion of couples in the United States that are not married. Show all your work. (Note, estimate here indicates that you should create a confidence interval.)

   (e) Interpret your result from (d).

2. The board of a major credit card company requires that the mean wait time for customers for service calls is at most 3.00 minutes. To make sure that the mean wait time is not exceeding the requirement, an assignment manager tracks the wait times of 45 randomly selected calls. The mean wait time was calculated to be 3.4 minutes. Assume the population standard deviation is 1.45 minutes.

   (a) Is this problem about a mean or a proportion?

   (b) Check that the conditions of the relevant Central Limit Theorem are satisfied for this problem.

   (c) Estimate the mean wait time for customers for service calls. Show all your work.

   (d) Interpret your result from (c).

3. The population standard deviation for waiting times to be seated at a restaurant is know to be 10 minutes. An expensive restaurant claims that the average waiting time for dinner is approximately 1 hour, but we suspect that this claim is inflated to make the restaurant appear more exclusive and successful. A random sample of 30 customers yielded a sample average waiting time of 50 minutes.

   (a) Is this problem about a mean or a proportion?

   (b) Check that the conditions of the relevant Central Limit Theorem are satisfied for this problem.

   (c) Estimate the mean wait time for dinner. Show all your work.

   (d) Interpret your result from (c).