Svodjenje krive drugog reda na kanonski oblik

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```
Kriva = 5 - 26 * a + 5 * a^2 - 4 * x - 4 * a * x + 8 * x^2 - 26 * y + 10 * a * y - 4 * x * y + 5 * y^2;
```

Za parametar a uzimamo da je a = (broj indeksa) mod 5.

```
a = Mod[286, 5]
```

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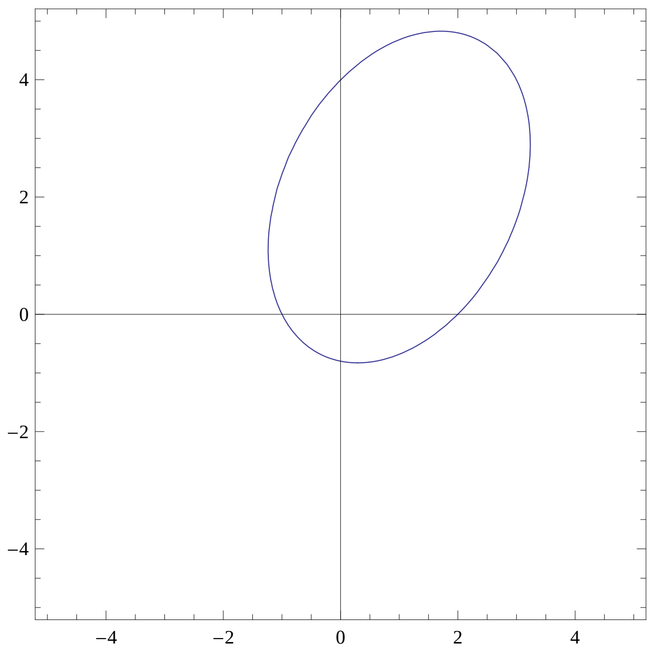
Crtamo krivu.

```
KrivaCrtez = ContourPlot[Kriva == 0, {x, -5, 5}, {y, -5, 5}, Axes → True]
```

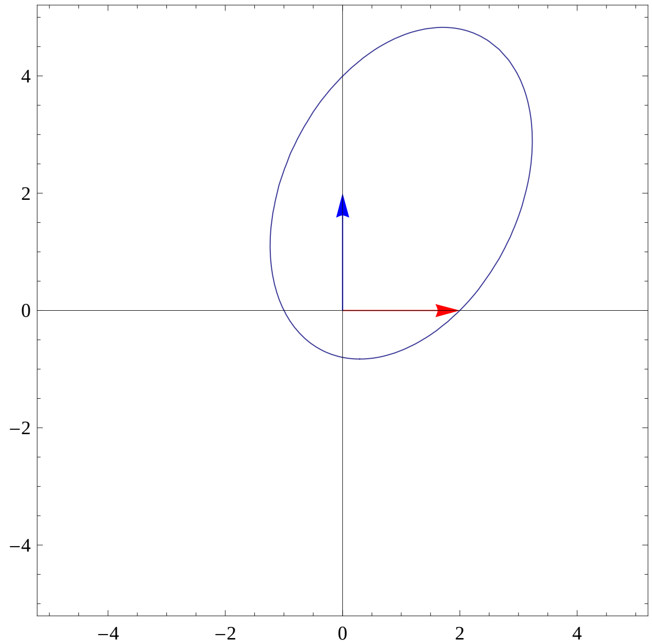


Vektori e1 i e2 su vektori repera u kom kriva ima dati oblik.

```
e1 = {Red, Arrow[{{0, 0}, {2, 0}}]};
```

```
e2 = {Blue, Arrow[{{0, 0}, {0, 2}}]};
```

```
Show[KrivaCrtez, Graphics[{e1, e2}]]
```



Matrica krive :

```
AA = {{8, -2}, {-2, 5}};
```

```
MatrixForm[AA]
```

$$\begin{pmatrix} 8 & -2 \\ -2 & 5 \end{pmatrix}$$

Vektor sopstvenih vrednosti i matrica cije su vrste sopstveni vektori koji odgovaraju tim sopstvenim vrednostima :

```
eiSistem = Eigensystem[AA]
```

```
{{9, 4}, {{-2, 1}, {1, 2}}}
```

```
MatrixForm /@ eiSistem
```

$$\left\{\begin{pmatrix} 9 \\ 4 \end{pmatrix}, \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}\right\}$$

```
CC = eiSistem[[2]]
```

```
{{-2, 1}, {1, 2}}
```

```
MatrixForm[CC]
```

$$\begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$$

Kako su sopstveni vektori ortogonalni,
treba da ih normiramo. Pravimo funkciju za normiranje na vrste matrice CC.

```
CC = (# / Norm[#]) & /@ CC
```

$$\left\{\left\{-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}, \left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}\right\}$$

Matrica prelaska na novu bazu treba da ima sopstvene vektore kao kolone, pa transponujemo matricu.Dobijamo ortogonalnu matricu.

```
CC = Transpose[CC]
```

$$\left\{\left\{-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}, \left\{\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}\right\}$$

```
MatrixForm[CC]
```

$$\begin{pmatrix} -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

Matrica CC dijagonalizuje matricu AA.

```
Inverse[CC].AA.CC // Simplify // MatrixForm
```

$$\begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}$$

Proveravamo da li ova matrica cuva orijentaciju.

```
Det[CC]
```

-1

Dobijamo da je menja.

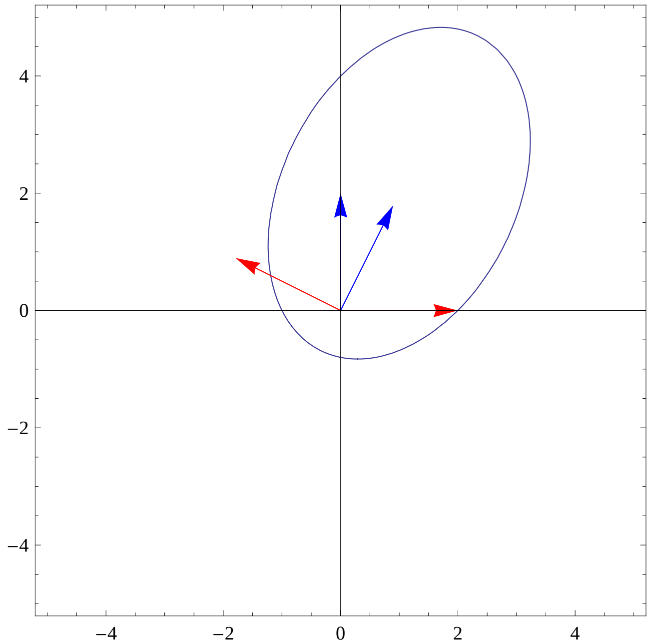
Crtamo nove bazne vektore. Oni su sada kolone matrice CC.ali nam je lakse da uzmemo vrste od transponovane CC.

```
f1v = Transpose[CC][[1]];
f2v = Transpose[CC][[2]];
```

Pravimo odgovarajuce strelice.

```
f1 = {Red, Arrow[{{0, 0}, 2 f1v}]};
f2 = {Blue, Arrow[{{0, 0}, 2 f2v}]};
```

```
Show[KrivaCrtez, Graphics[{e1, e2}], Graphics[{f1, f2}]]
```



Vidimo da su stara i nova baza raznih orijentacija. Zamenicemo kolone.

$$CC = CC.\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\left\{\left\{\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right\}, \left\{\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}\right\}$$

```
MatrixForm[CC]
```

$$\begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

Matrica sada ima determinantu 1.

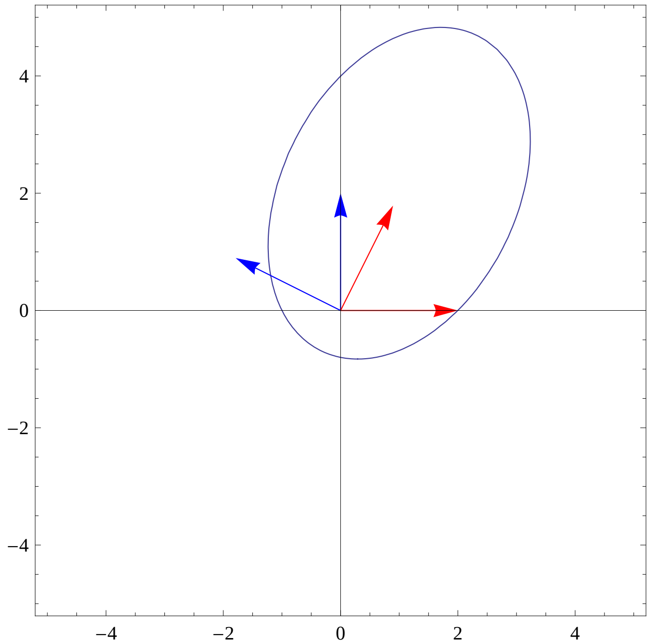
```
Det[CC]
```

1

Promenicemo i vektore koje cemo da crtamo.

```
f1v = Transpose[CC][[1]];
f2v = Transpose[CC][[2]];
f1 = {Red, Arrow[{{0, 0}, 2 f1v}]};
f2 = {Blue, Arrow[{{0, 0}, 2 f2v}]};
```

```
Show[KrivaCrtez, Graphics[{e1, e2}], Graphics[{f1, f2}]]
```



Trazimo transformaciju koordinata koja je odredjena matricom CC.Stare koordinate (x, y) dobijamo mnozenje novih koordinata (xp, yp) matricom CC.

```
CC.{xp, yp}
```

$$\left\{\frac{xp}{\sqrt{5}}-\frac{2\,yp}{\sqrt{5}},\ \frac{2\,xp}{\sqrt{5}}+\frac{yp}{\sqrt{5}}\right\}$$

U novim, zarotiranim koordinatama, nema clana xp , yp.

$$\text{Kriva /. }\left\{x\to\frac{xp}{\sqrt{5}}-\frac{2\,yp}{\sqrt{5}},\ y\to\frac{2\,xp}{\sqrt{5}}+\frac{yp}{\sqrt{5}}\right\}\text{ // Simplify}$$

$$-16-8\sqrt{5}\,xp+4\,xp^2+9\,yp^2$$

$$\text{Kriva1 = 2 Kriva /. }\left\{x\to\frac{xp}{\sqrt{5}}-\frac{2\,yp}{\sqrt{5}},\ y\to\frac{2\,xp}{\sqrt{5}}+\frac{yp}{\sqrt{5}}\right\}\text{ // Simplify}$$

$$-32-16\sqrt{5}\,xp+8\,xp^2+18\,yp^2$$

Sada je potrebno da uradimo translaciju, $xs = xp-\sqrt{5}$, $ys=\frac{3\,yp}{2}$.

$$\text{Kriva1 /. }\left\{xp\to xs+\sqrt{5},\ yp\to\frac{2\,ys}{3}\right\}\text{ // Simplify}$$

$$8\left(-9+xs^2+ys^2\right)$$

Vidimo da je kriva $\frac{xs^2}{9}+\frac{ys^2}{9}=1$, **elipsa.**

Odredjujemo centar krive, iz translacije stavljajuci da je xs = ys = 0.

```
CentarKrive = CC.{√5, 0}
```

```
{1, 2}
```

Crtamo reper (xs, ys) u kom kriva ima kanonski oblik.Potrebno je uraditi translaciju do centra. Transliramo pocetnu i krajnju tacku vektora.

```
g1 = {Red, Arrow[(# + CentarKrive) & /@ {{0, 0}, 2 f1v}]};
```

```
g2 = {Blue, Arrow[(# + CentarKrive) & /@ {{0, 0}, 2 f2v}]};
```

Crtamo kako izgleda konacni koordinatni sistem gde je crvena osa nova xs osa, a plava nova ys osa.

```
Show[KrivaCrtez, Graphics[{e1, e2}], Graphics[{g1, g2}]]
```