



# REGRAS DE DERIVAÇÃO

16) Se  $f(x) = (g \circ l)(x) = g(l(x))$ ,  
então  $f'(x) = g'(l(x)) \cdot l'(x)$

Ex: Se  $f(x) = \sin x^2 = \sin(x^2)$

então  $f'(x) = \cos(x^2) \cdot 2x$

Sol: Tome  $l(x) = x^2$  e  $g(x) = \sin x$ .

Temos  $(g \circ l)(x) = g(l(x)) = g(x^2) = \sin(x^2) = f(x)$ .

Logo,  $g'(x) = \cos x$  e  $l'(x) = 2x$ .

Dai,  $f'(x) = \cos(x^2) \cdot 2x$ .

Ex: Se  $f(x) = \frac{1}{2}g(\sin x)$ , então

$$f'(x) = \frac{1}{2}g'(\sin x) \cdot \cos x$$

Ex: Se  $f(x) = \cos(\frac{1}{2}g(x^2 + x + 1))$ , então

$$f'(x) = -\sin(\frac{1}{2}g(x^2 + x + 1)) \cdot \frac{1}{2}g'(x^2 + x + 1) \cdot (2x + 1)$$

17) Se  $f: [-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

$$x \mapsto y = f(x)$$

onde  $f(x) = \arcsin x = \sin^{-1}x$ .

Temos que  $f'(x) = \frac{1}{\sqrt{1-x^2}}$

Obs:  $D(f) = [-1, 1]$ , mas  $D(f') = (-1, 1)$

Obs:  $\arcsin \sin x = x \iff \sin w = x$

18) Se  $f: [-1, 1] \rightarrow [0, \pi]$

$$x \mapsto y = f(x)$$

onde  $f(x) = \arccos x = \cos^{-1}x$

Temos que  $f'(x) = \frac{-1}{\sqrt{1-x^2}}$

Obs:  $D(f) = [-1, 1]$ , mas  $D(f') = (-1, 1)$

Obs:  $\arccos \cos x = x \iff \cos w = x$

19) Se  $f: \mathbb{R} \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

$$x \mapsto y = f(x)$$

onde  $f(x) = \arctan x = \tan^{-1}x$ .

Temos que  $f'(x) = \frac{1}{1+x^2}$

Obs:  $D(f) = \mathbb{R}$  e  $D(f') = \mathbb{R}$

Obs:  $\arctan \tan x = x \iff \tan w = x$

Ex: Se  $f(x) = \arctan(2x)$ , então

$$f'(x) = \frac{1}{1+(2x)^2} \cdot 2 = \frac{2}{1+4x^2}$$



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$$f'(x) = -\sin(\frac{1}{2}(x^2 + x + 1)) \cdot \sec^2(x^2 + x + 1) \cdot (x^2 + x + 1)'$$

17) Seja  $f: [-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$   
 $x \mapsto y = f(x)$

onde  $f(x) = \arcsin x = \sin^{-1} x$ .

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Obs:

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19) Seja  $f: \mathbb{R} \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$   
 $x \mapsto y = f(x)$

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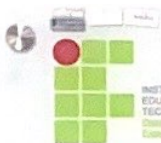
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## REGRAS DE DERIVAÇÃO

20) Se  $f(x) = [g(x)]^{l(x)}$ , então

$$f'(x) = [g(x)]^{l(x)} \cdot \left[ l'(x) \cdot \ln g(x) + l(x) \cdot \frac{g'(x)}{g(x)} \right]$$

sol:  $f(x) = [g(x)]^{l(x)}$

$$\ln f(x) = \ln [g(x)]^{l(x)}$$

$$\ln f(x) = l(x) \cdot \ln g(x) \quad (\text{DERIVANDO})$$

$$\frac{f'(x)}{f(x)} = l'(x) \cdot \ln g(x) + l(x) \cdot \frac{g'(x)}{g(x)}$$

$$f'(x) = f(x) \cdot \left[ l'(x) \cdot \ln g(x) + l(x) \cdot \frac{g'(x)}{g(x)} \right]$$

Ex: Se  $f(x) = x^{\sin x}$ , então

$$f'(x) = x^{\sin x} \cdot \left[ \cos x \cdot \ln x + \sin x \cdot \frac{1}{x} \right]$$

$$= x^{\sin x} \cdot \left( \cos x \cdot \ln x + \frac{\sin x}{x} \right)$$

①	7	13	<del>11</del>
②	8	14	<del>20</del>
3	9	<del>15</del>	
4	10	<del>16</del>	
5	<del>11</del>	<del>17</del>	
<del>6</del>	12	<del>18</del>	

$$f'(x) = 0$$