INSTITUTO FEDERAL DE EDUCAÇÃO, CIÊNCIA E TECNOLOGIA

REGRAS DE DERIVAÇÃO

16) Se $f(x) = (g \circ l)(x) = g(l(x))$,

então $f'(x) = g'(l(x)) . l'(x)$.

Ex.: Se $f(x) = \text{Sen}\, x^7 = \text{Sen}(x^7)$

então $f'(x) = \text{Cos}(x^7) . 7x^6$

Sol: Tome $l(x) = x^7$ e $g(x) = \text{Sen}\,x$.

Temos $(g \circ l)(x) = g(l(x)) = g(x^7) = \text{Sen}(x^7) = f(x)$.

logo, $g'(x) = \text{Cos}\,x$ e $l'(x) = 7.x^6$.

Daí, $f'(x) = \text{Cos}(x^7) . 7x^6$.

Ex: Se $f(x) = tg(\text{Sen}\,x)$, então

$f'(x) = \text{Sec}^2(\text{Sen}\,x) . \text{Cos}\,x$.

Ex: Se $f(x) = \text{Cos}(tg(x^6 + x + 1))$, então

$f'(x) = -\text{Sen}(tg(x^6 + x + 1)) . \text{Sec}^2(x^6 + x + 1) . (6.x^5 + 1)$.

17) Seja $f : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x \longmapsto y = f(x)$

onde $f(x) = \text{arc Sen}\, x = \text{Sen}^{-1} x$.

Temos que $f'(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

obs: $D(f) = [-1, 1]$, mas $D(f') = (-1, 1)$

Obs.: $\text{arc Sen}\, x = w \iff \text{Sen}\, w = x$

18) Seja $f : [-1, 1] \longrightarrow [0, \pi]$

$x \longmapsto y = f(x)$

onde $f(x) = \text{arc Cos}\, x = \text{Cos}^{-1} x$

Temos que $f'(x) = \dfrac{-1}{\sqrt{1 - x^2}}$.

obs: $D(f) = [-1, 1]$, mas $D(f') = (-1, 1)$.

Obs: $\text{arc Cos}\, x = w \iff \text{Cos}\, w = x$.

19) Seja $f : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$x \longmapsto y = f(x)$

onde $f(x) = \text{arc}\,tg\, x = tg^{-1} x$.

Temos que $f'(x) = \dfrac{1}{1 + x^2}$.

obs: $D(f) = \mathbb{R}$ e $D(f') = \mathbb{R}$

Obs: $\text{arc}\,tg\, x = w \iff tg\, w = x$

Ex: Se $f(x) = \text{arc}\,tg(2x)$, então

$f'(x) = \dfrac{1}{1 + (2x)^2} . 2 = \dfrac{2}{1 + 4x^2}$.

REGRAS DE DERIVAÇÃO

16) Se $f(x) = (g \circ l)(x) = g(l(x))$,

então $f'(x) = g'(l(x)) . l'(x)$.

Ex.: Se $f(x) = \operatorname{Sen} x^7 = \operatorname{Sen}(x^7)$

então $f'(x) = \operatorname{Cos}(x^7) . 7x^6$

Sol: Tome $l(x) = x^7$ e $g(x) = \operatorname{Sen} x$.

Temos $(g \circ l)(x) = g(l(x)) = g(x^7) = \operatorname{Sen}(x^7) = f(x)$.

Logo, $g'(x) = \operatorname{Cos} x$ e $l'(x) = 7.x^6$.

Daí, $f'(x) = \operatorname{Cos}(x^7) . 7x^6$.

Ex: Se $f(x) = \operatorname{tg}(\operatorname{Sen} x)$, então

$f'(x) = \operatorname{Sec}^2(\operatorname{Sen} x) . \operatorname{Cos} x$.

Ex: Se $f(x) = \operatorname{Cos}(\operatorname{tg}(x^6 + x + 1))$, então

$f'(x) = -\operatorname{Sen}(\operatorname{tg}(x^6 + x + 1)) . \operatorname{Sec}^2(x^6 + x + 1) . (6.x^5 + 1)$.

17) Seja $f : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x \longmapsto y = f(x)$

onde $f(x) = \operatorname{arcSen} x = \operatorname{Sen}^{-1} x$.

Temos que $f'(x) = \dfrac{1}{\sqrt{1 - x^2}}$

obs: $D(f) = [-1, 1]$, mas $D(f') = (-1, 1)$

Obs.: $\operatorname{arcSen} x = w \iff \operatorname{Sen} w = x$

18) Seja $f : [-1, 1] \longrightarrow [0, \pi]$

$x \longmapsto y = f(x)$

onde $f(x) = \operatorname{arcCos} x = \operatorname{Cos}^{-1} x$

Temos que $f'(x) = \dfrac{-1}{\sqrt{1 - x^2}}$.

obs: $D(f) = [-1, 1]$, mas $D(f') = (-1, 1)$.

Obs: $\operatorname{arcCos} x = w \iff \operatorname{Cos} w = x$.

19) Seja $f : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$x \longmapsto y = f(x)$

onde $f(x) = \operatorname{arctg} x = \operatorname{tg}^{-1} x$.

Temos que $f'(x) = \dfrac{1}{1 + x^2}$.

obs: $D(f) = \mathbb{R}$ e $D(f') = \mathbb{R}$

Obs: $\operatorname{arctg} x = w \iff \operatorname{tg} w = x$

Ex: Se $f(x) = \operatorname{arctg}(2x)$, então

$f'(x) = \dfrac{1}{1 + (2x)^2} . 2 = \dfrac{2}{1 + 4x^2}$.

INSTITUTO FEDERAL DE EDUCAÇÃO, CIÊNCIA E TECNOLOGIA

REGRAS DE DERIVAÇÃO

20) Se $f(x) = [g(x)]^{l(x)}$, então

$$f'(x) = [g(x)]^{l(x)} \cdot \left[l'(x) \cdot \ln g(x) + l(x) \cdot \frac{g'(x)}{g(x)} \right].$$

Sol: $f(x) = [g(x)]^{l(x)}$

$$\ln f(x) = \ln [g(x)]^{l(x)}$$

$$\ln f(x) = l(x) \cdot \ln g(x).$$ (DERIVANDO)

$$\frac{f'(x)}{f(x)} = l'(x) \cdot \ln g(x) + l(x) \cdot \frac{g'(x)}{g(x)}$$

$$f'(x) = f(x) \left[l'(x) \ln g(x) + l(x) \cdot \frac{g'(x)}{g(x)} \right].$$

Ex: Se $f(x) = x^{\operatorname{sen} x}$, então

$$f'(x) = x^{\operatorname{sen} x} \cdot \left[\cos x \cdot \ln x + \operatorname{sen} x \cdot \frac{1}{x} \right]$$

$$= x^{\operatorname{sen} x} \left(\cos x \cdot \ln x + \frac{\operatorname{sen} x}{x} \right).$$

1	7	13	~~19~~
2	8	14	~~20~~
3	9	~~15~~	
4	10	~~16~~	
5	~~11~~	~~17~~	
~~6~~	12	~~18~~	

$f'(x) = 0$