Fuzzy Control

Motivation for Fuzzy Control

Question:

- Can you read the text?
- Can you read it now?
- Now?

Fundamentals of fuzzy logic

Fuzzy Set Theory

- 1965 by L. A. Zadeh
- *Membership function*
- $\mu_A : x \rightarrow [0,1]$

Example: Let $X \in \mathbb{R}$ and A be the fuzzy set of approximately 50-year-old persons.

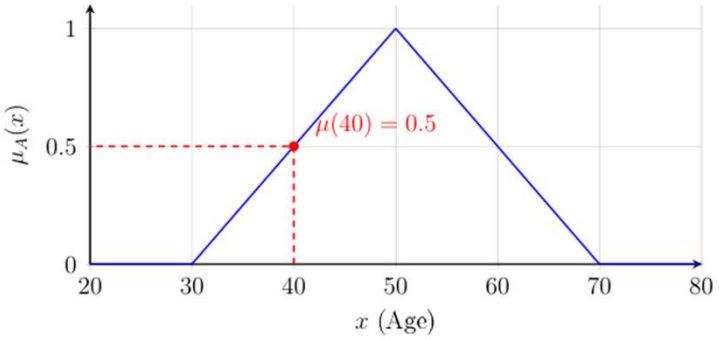


Figure 2.1: Membership function for: 'approximately 50 years old'

- *Fuzzyfication*
- $\mu_{50\ years}(40) = 0.5$

Fundamentals of fuzzy logic

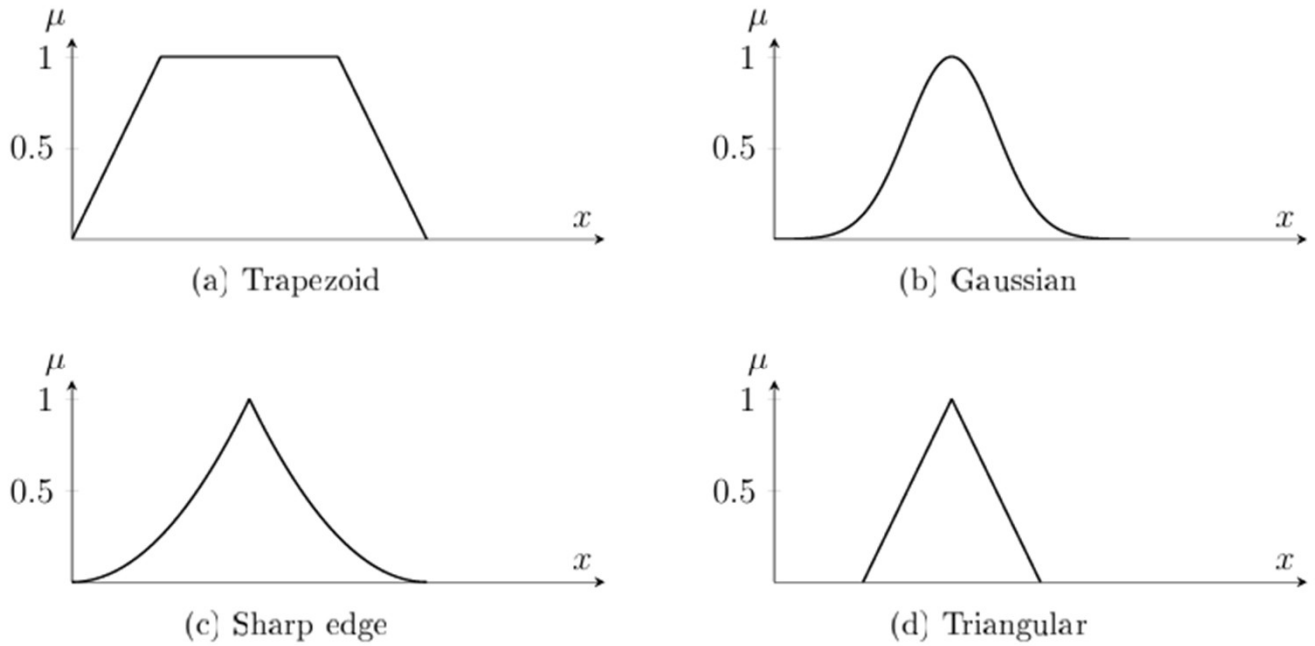
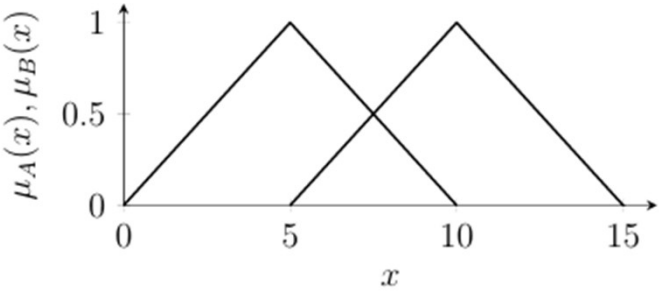


Figure 2.2: Examples of membership functions

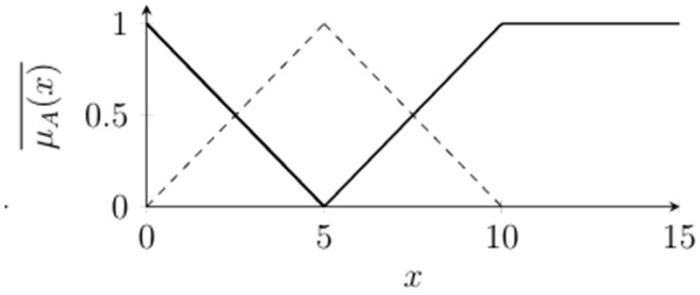
Fundamentals of fuzzy logic

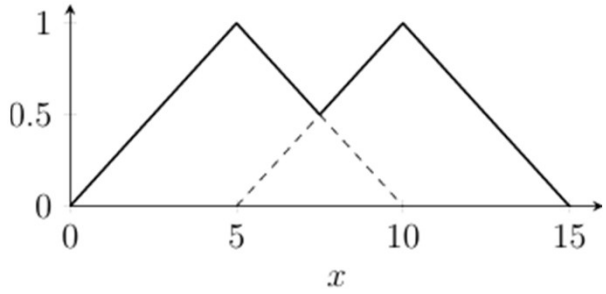
Fuzzy Set Theory

1. $f_{A'}(x) = 1 - f_A(x)$ (Complement)
2. $f_{A \cup B}(x) = \max(f_A(x), f_B(x))$ (Union)
3. $f_{A \cap B}(x) = \min(f_A(x), f_B(x))$ (Intersection)

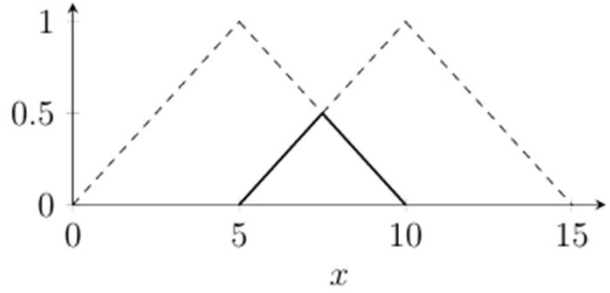


(a) Original fuzzy sets $\mu_A(x)$ and $\mu_B(x)$





Union Zadeh s-norm



Intersection Zadeh t-norm

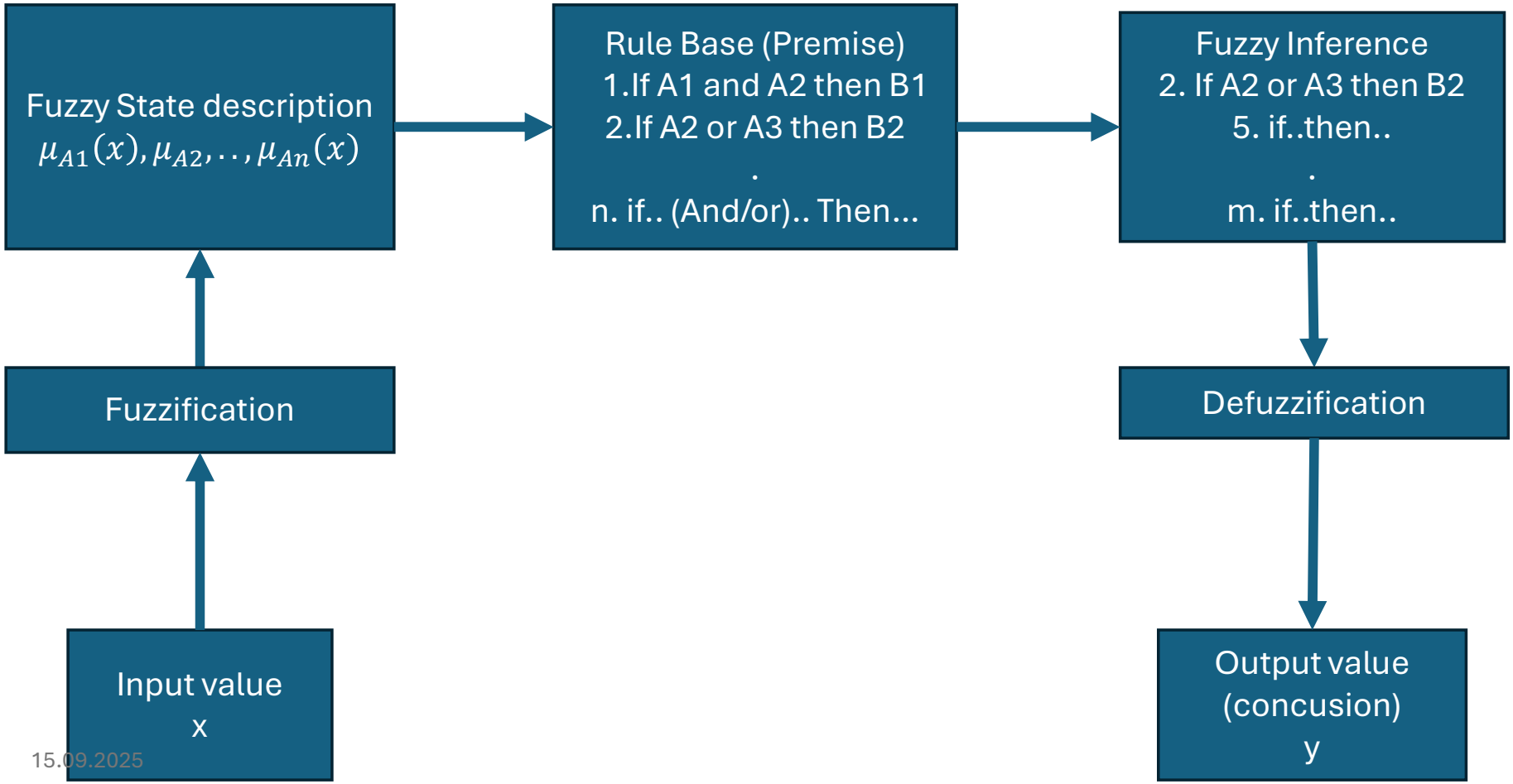
From Fuzzy Logic to Fuzzy Control

- 1975 Ebrahim Mamdani
- fuzzy control system
- rule-based reasoning and defuzzication
- *Mamdani model*

Example: Temperature

If the temperature is too high, then slightly reduce the heating power.

Mamdani Model Overview



Fuzzy Conclusion

- Evaluate fuzzy inference
- using Zaseh‘s s-norm or t-norm
- Output membership function is clipped

Example: Statement: If A1 and A2 then B1

$\mu_{A1} = 1, \mu_{A2} = 0.8$

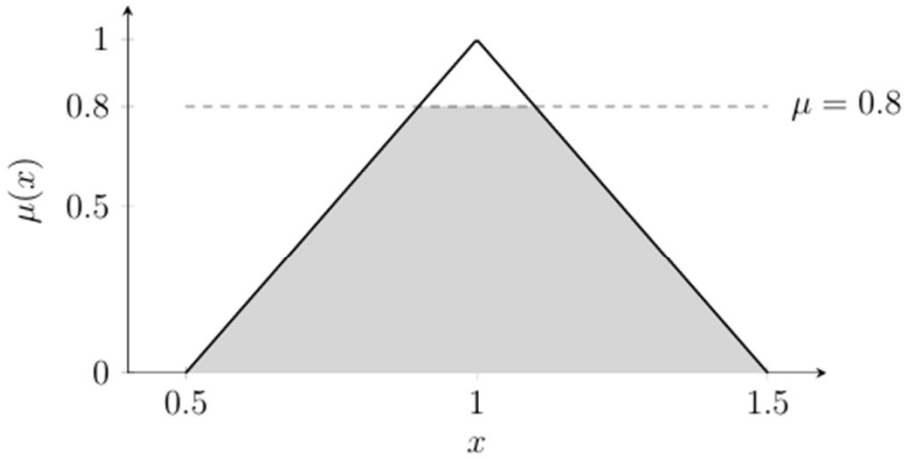


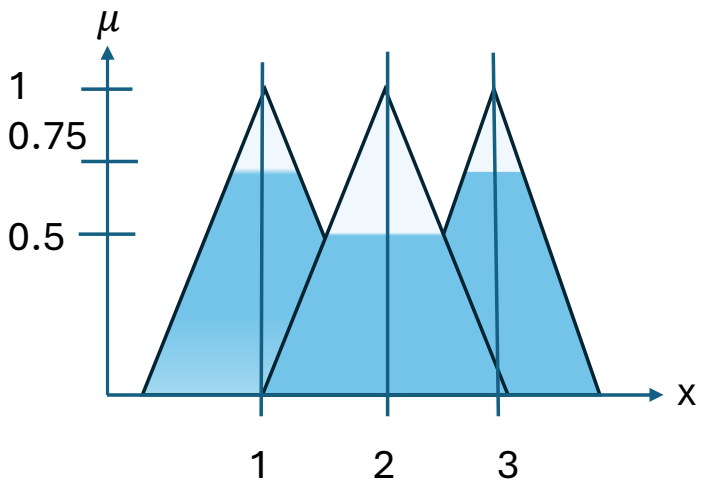
Figure 2.5: Example of an output membership function which is clipped at $\mu = 0.8$

Center of Gravity Method

- „Point of balance“
- Area under the clipped Graph is calculated
- Center if the area is calculated
- Drop the plumb line from the center of gravity

- For triangular shapes:

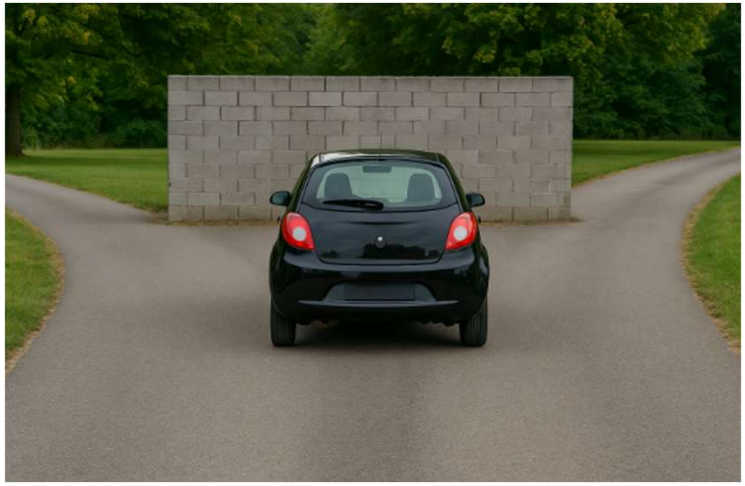
$$u_0 = \frac{\sum u_i \mu_{premise_i}}{\sum \mu_{premise_i}}$$

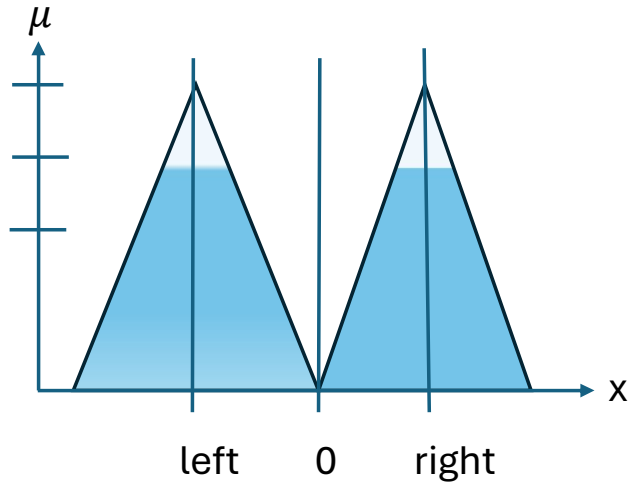


Example:

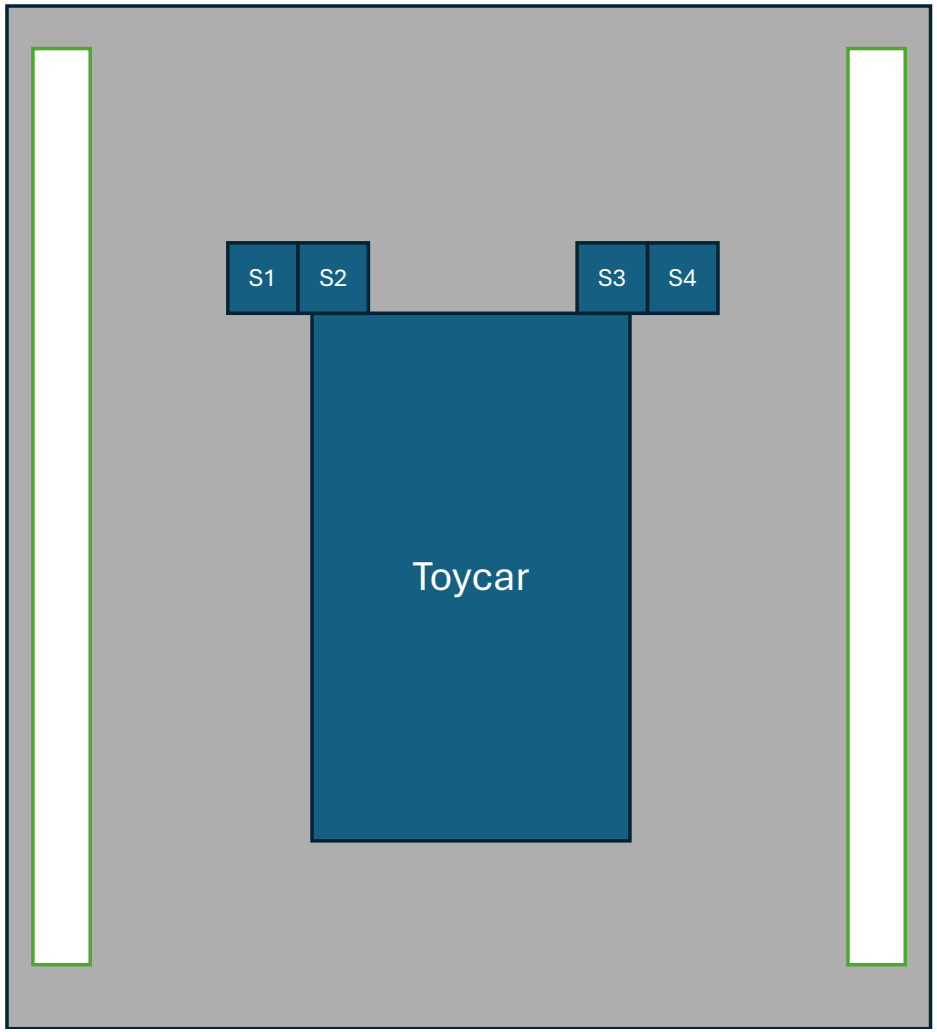
$$u_0 = \frac{1 * 0.75 + 2 * 0.5 + 3 * 0.75}{0.75 + 0.5 + 0.75} = \frac{4}{2} = 2$$

Center of Gravity Method





Example: Lane Keeping Assistent

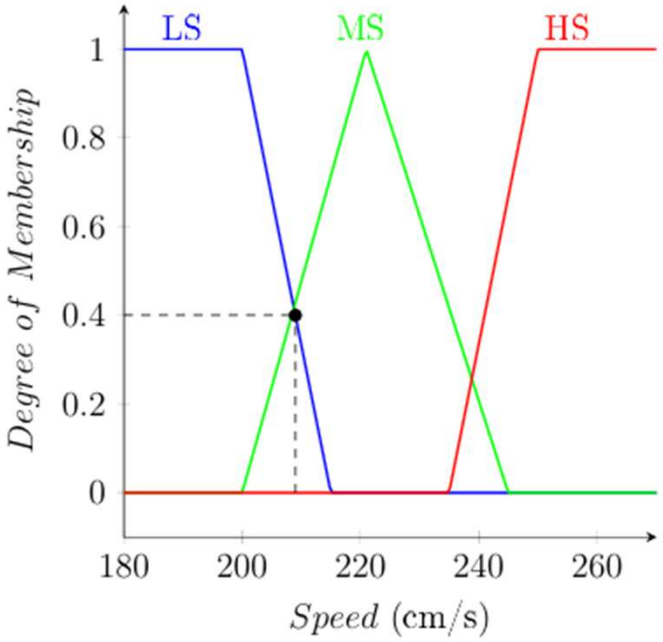


TCRT5000 (Bild: amazon.de)

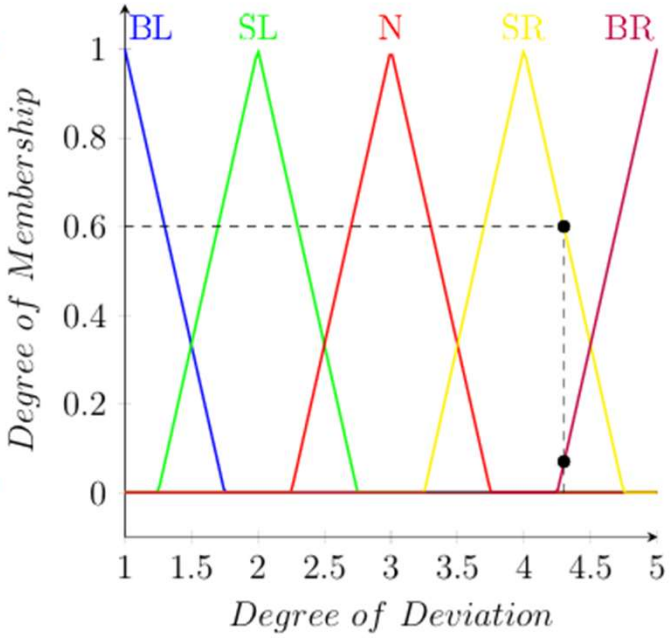
- Deviation measured by four TCRT5000L sensors
- According to the number of sensors crossing the line: Small, Big or Neutral Deviation

Source: N. C. Basjaruddin, Kuspriyanto, Suhendar, D. Saefudin, and S. A. Aryani, Lane keeping assist system based on fuzzy logic, in 2015 International Electronics Symposium (IES)

Example: Lane Keeping Assistent



LS: Low Speed
MS: Medium Speed
HS: High Speed



BL/BR: Big Left/Big Right
SL/SR: Small Left /Small Right
N: Neutral

- Speed three membership functions
- Degree of Deviation five membership functions
- Deviation assigned numbers from 1 (Big Left Deviation) to 5 (Big Right Deviation)

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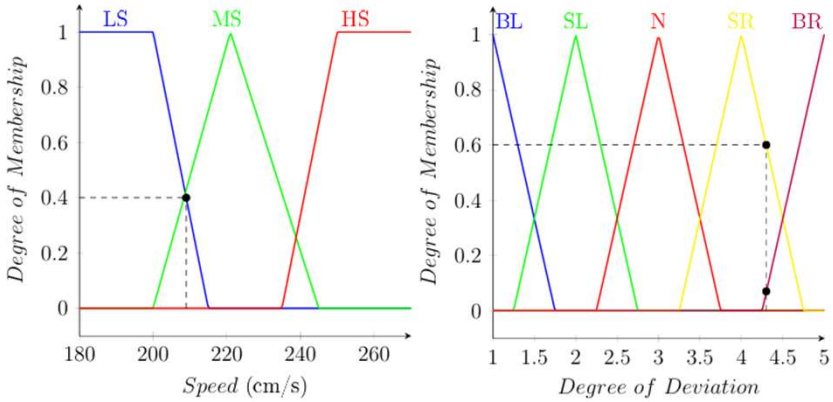

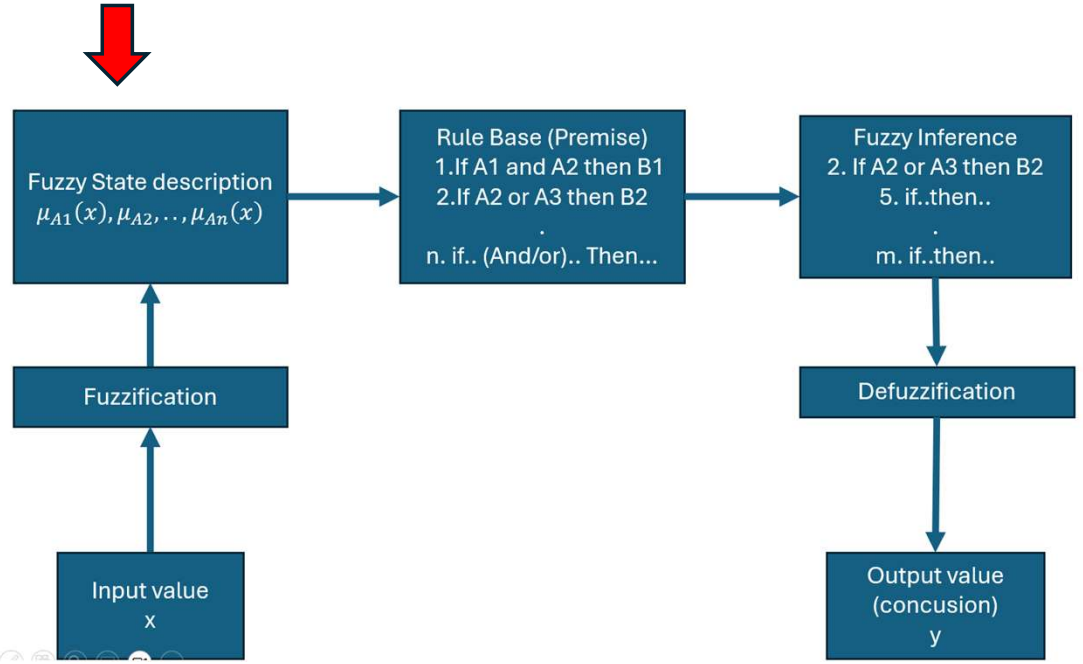

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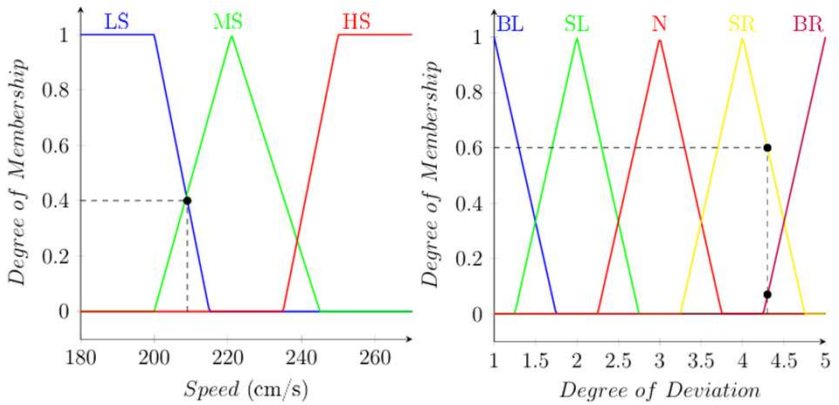
Fuzzy State description
Speed: 209 cm/s
Deviation: 4.3



In lingustic terms: The car is between low speed an medium speed with a little bit more than small deviation to the right.

Mathematical:

$$\mu_{LS}(209) = 0.43,\ \mu_{MS}(209) = 0.4,\ \mu_{HS}(209) = 0.0$$
$$\mu_{BL}(4.3) = 0.0,\ \mu_{SL}(4.3) = 0.0,\ \mu_{N}(4.3) = 0.0,\ \mu_{SR}(4.3) = 0.6,\ \mu_{BR}(4.3) = 0.067$$

Short (Membership vector):

$$\mu_{Speed}(209) = (0.43,0.4,0.0)$$
$$\mu_{Deviation}(4.3) = (0.0,0.0,0.0,0.6,0.067)$$

Example: Lane Keeping Assistent

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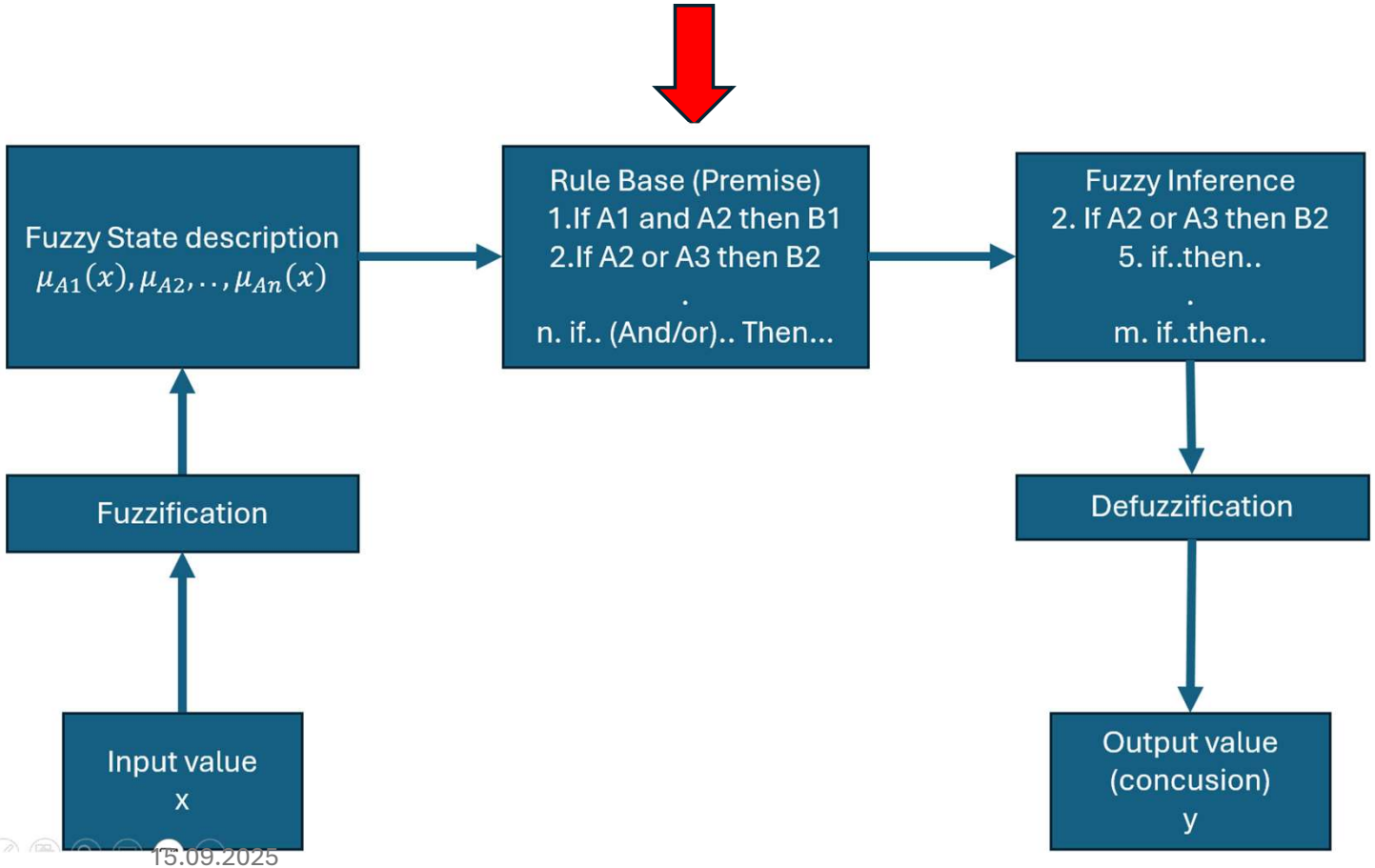

Rule Base is needed:

1. if speed is medium speed and deviation is normal then zero steering angle
2. if speed is low speed and deviation is small right then medium steering angle
3. if speed is low speed and deviation is big left then medium steering angle
4. ...

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Array of the Rule Base (Premise)

Speed		Deviation				
		SR	BR	Normal	BL	SL
	HS	MSA	BSA	ZSA	BSA	MSA
	MS	MSA	BSA	ZSA	BSA	MSA
	LS	SSA	MSA	ZSA	MSA	SSA

SR/L: Small Right/Left
BR/L : Big Right/Left

HS: High Speed
MS: Medium Speed
LS: Low Speed

SSA: Small Steering Angle
MSA: Medium Steering Angle
BSA: Big Steering Angle

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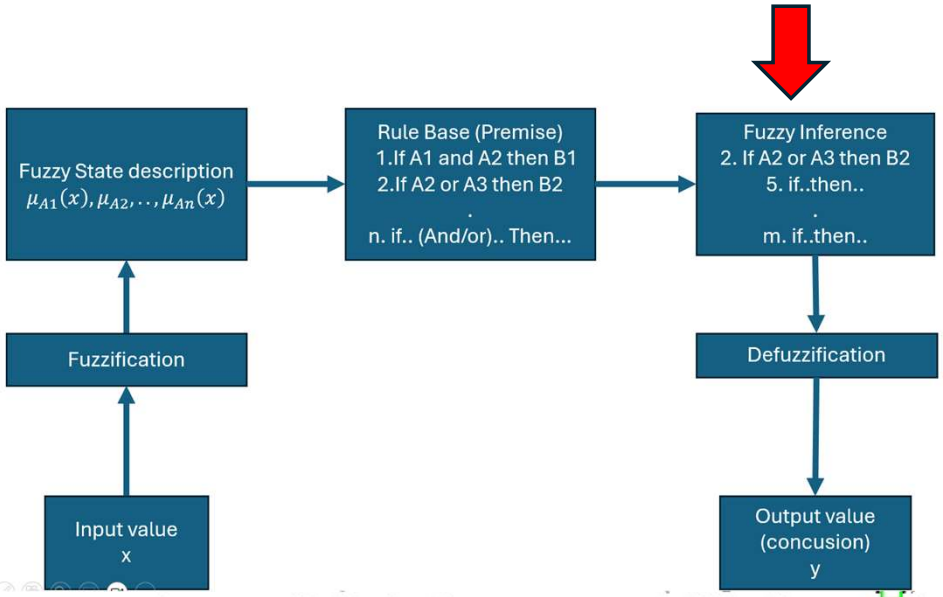
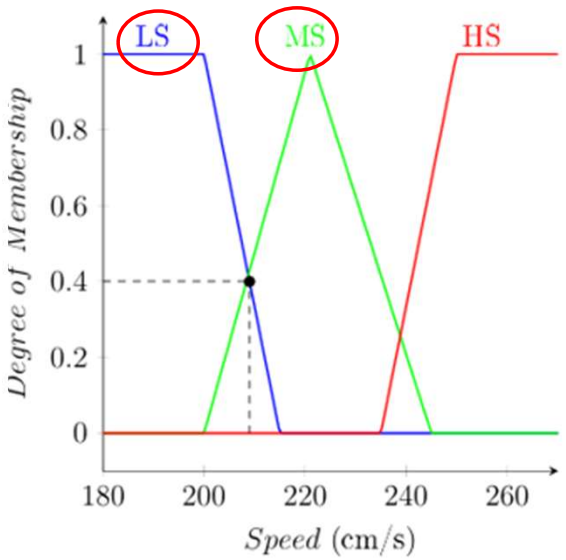
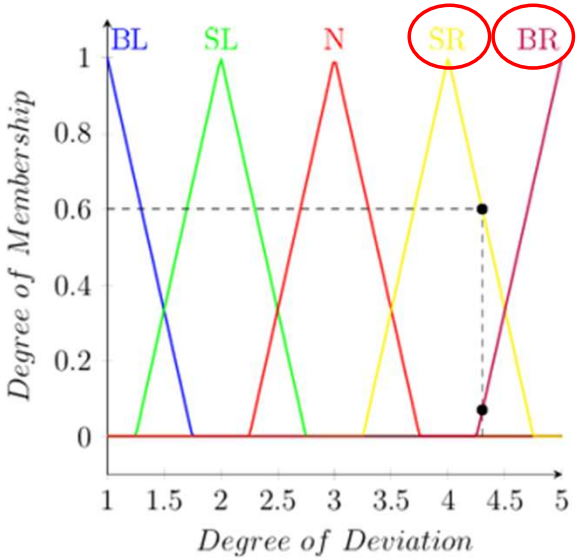


Speed		Deviation				
		SR	BR	Normal	BL	SL
	HS	MSA	BSA	ZSA	BSA	MSA
	MS	MSA	BSA	ZSA	BSA	MSA
	LS	SSA	MSA	ZSA	MSA	SSA





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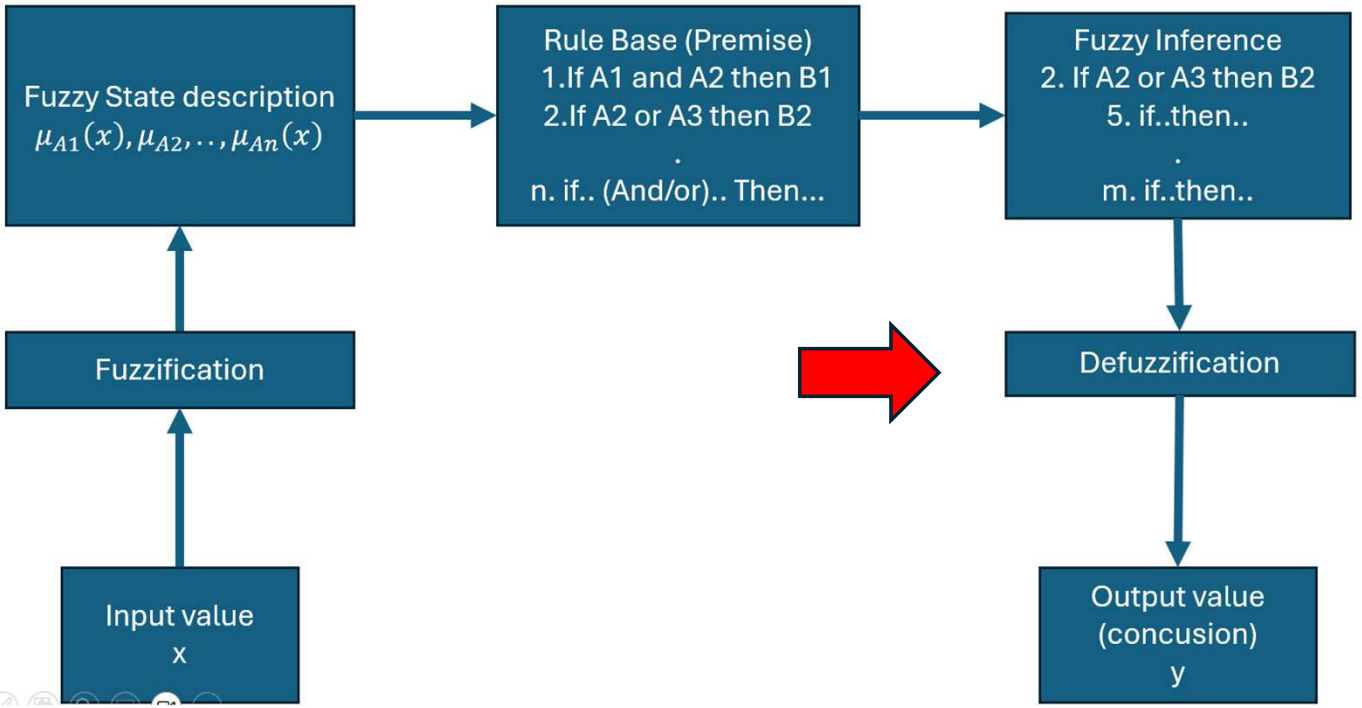

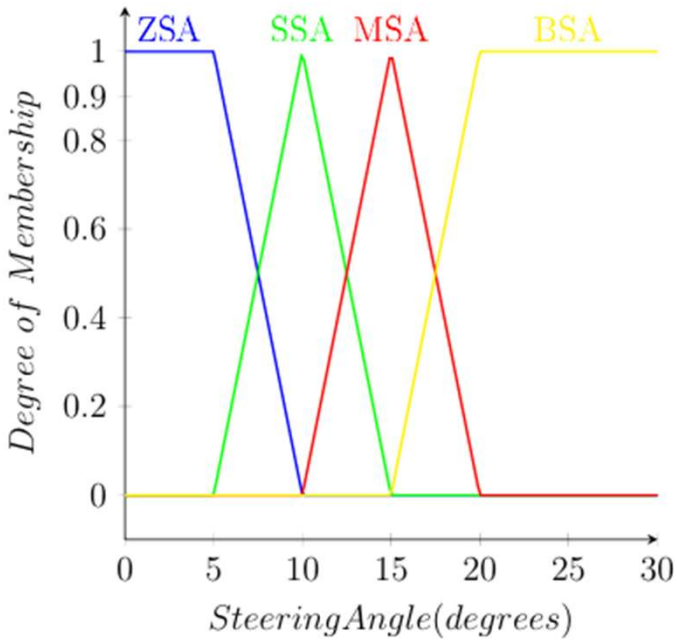

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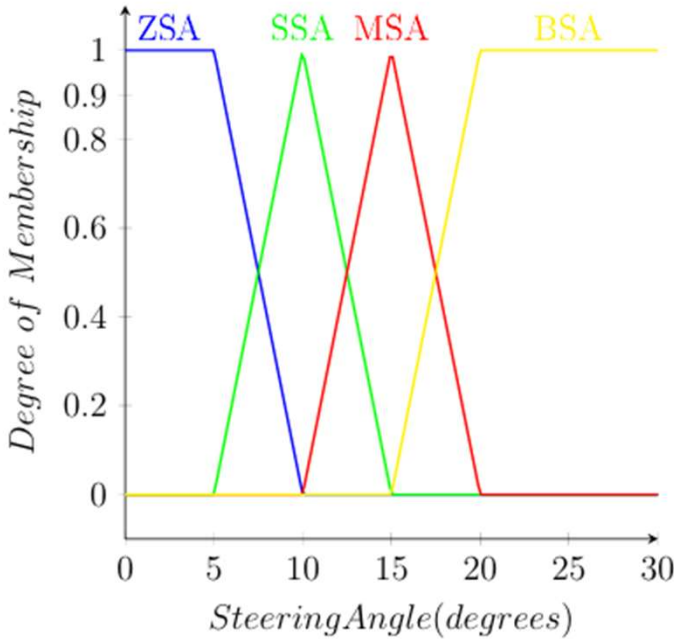

$$\mu_{Speed}(209) = (0.43, 0.4, 0.0)$$
$$\mu_{Deviation}(4.3) = (0.0, 0.0, 0.0, 0.6, 0.067)$$

		Deviation				
		SR	BR	Normal	BL	SL
Speed	HS	MSA	BSA	ZSA	BSA	MSA
	MS	MSA	BSA	ZSA	BSA	MSA
	LS	SSA	MSA	ZSA	MSA	SSA

Using the t-norm:

1. $f_{LS \cap SR}(x) = \min(0.43, 0.6) = 0.43$ SSA
2. $f_{LS \cap BR}(x) = \min(0.43, 0.067) = 0.067$ MSA
3. $f_{MS \cap SR}(x) = \min(0.4, 0.6) = 0.43$ BSA
4. $f_{MS \cap BR}(x) = \min(0.4, 0.067) = 0.067$ MSA

Example: Lane Keeping Assistent

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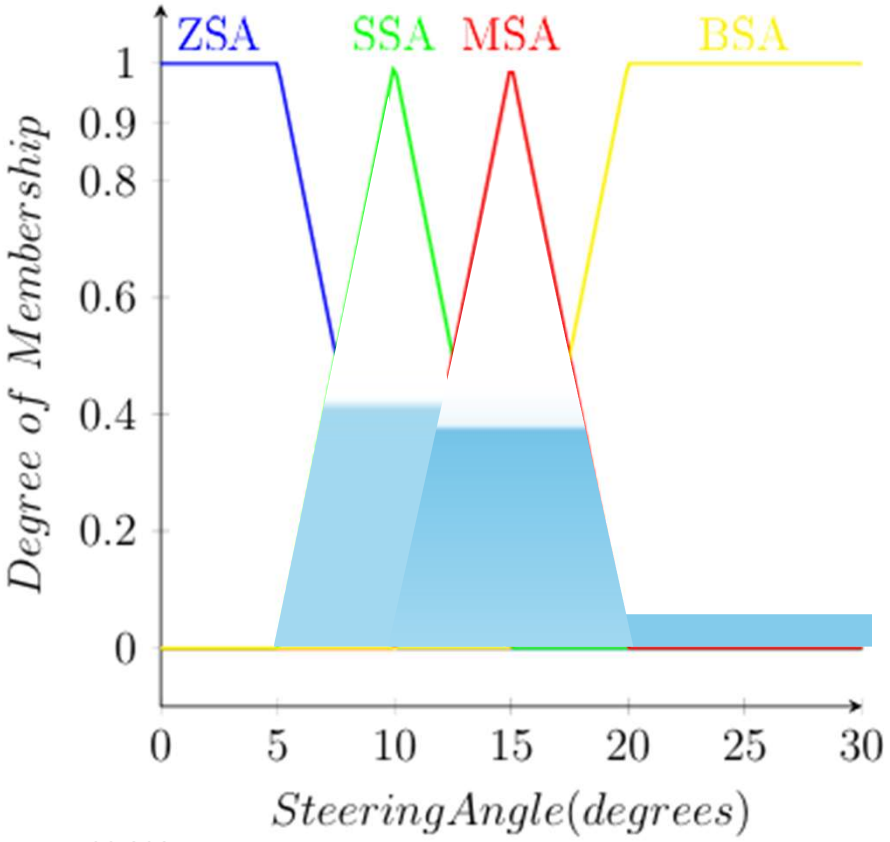

		Deviation				
		SR	BR	Normal	BL	SL
Speed	HS	MSA	BSA	ZSA	BSA	MSA
	MS	MSA	BSA	ZSA	BSA	MSA
	LS	SSA	MSA	ZSA	MSA	SSA

Using the t-norm:

1. $f_{LS \cap SR}(\mathrm{x}) = \min(0.43, 0.6) = 0.43$ SSA
2. $f_{LS \cap BR}(\mathrm{x}) = \min(0.43, 0.067) = 0.067$ MSA
3. $f_{MS \cap SR}(\mathrm{x}) = \min(0.4, 0.6) = 0.4$ MSA
4. $f_{MS \cap BR}(\mathrm{x}) = \min(0.4, 0.067) = 0.067$ BSA

Using the s-norm:

$f_{LS \cap BR}(\mathrm{x})$ OR $f_{MS \cap SR}(\mathrm{x})$

$$f_{f_{LS \cap BR} \cup f_{MS \cap SR}}(\mathrm{x}) = \max(f_{LS \cap BR}(\mathrm{x}), f_{MS \cap SR}(\mathrm{x})) = \max(0.067, 0.4) = 0.4 \text{ MSA}$$

Example: Lane Keeping Assistent

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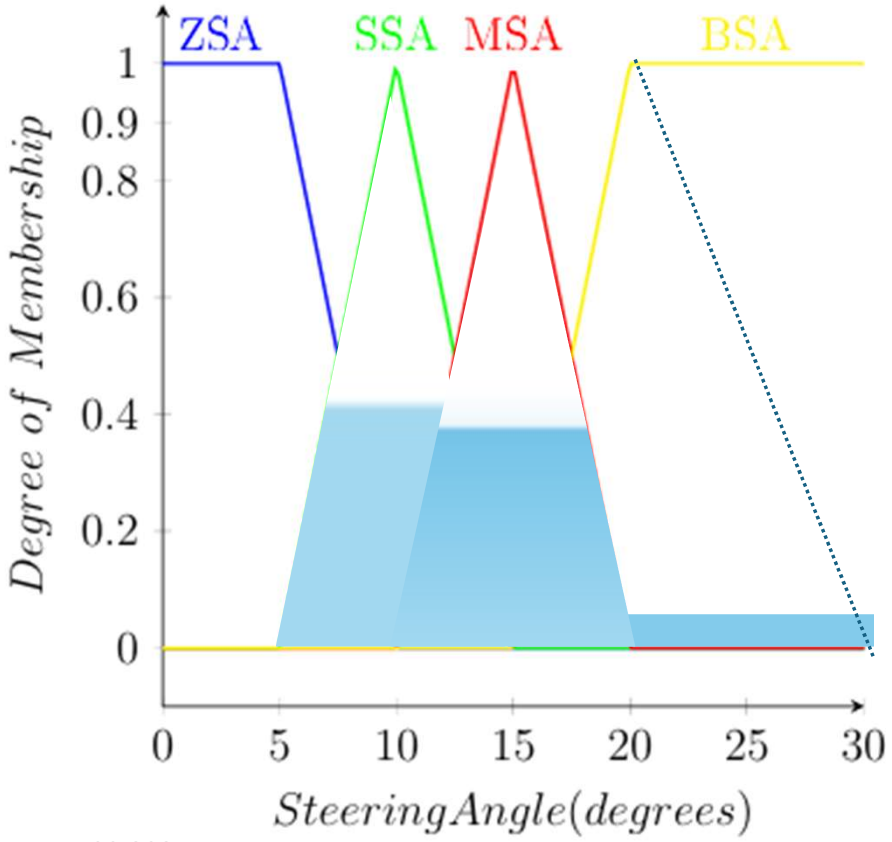

1. $f_{LS \cap SR}(\mathrm{x}) = 0.43$ SSA
2. $f_{f_{LS \cap BR} \cup f_{MS \cap SR}}(\mathrm{x}) = 0.4$ MSA
3. $f_{MS \cap BR}(\mathrm{x})$ 0.067 BSA

$$u_0 = \frac{\sum u_i \mu_{premise_i}}{\sum \mu_{premise_i}}$$

$$u_0 = \frac{10*0.43+1\ *0.4+20*0.067}{0.43+0.4+0.067} \approx 13.16°$$

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- Easy set up
- Two small sensor types
- One activator
- Low computational effort
- Simple Line Keeping assistent was achieved