AN AUTOPILOT DESIGNED TO MAINTAIN THE PITCH ATTITUDE OF AN AIRCRAFT

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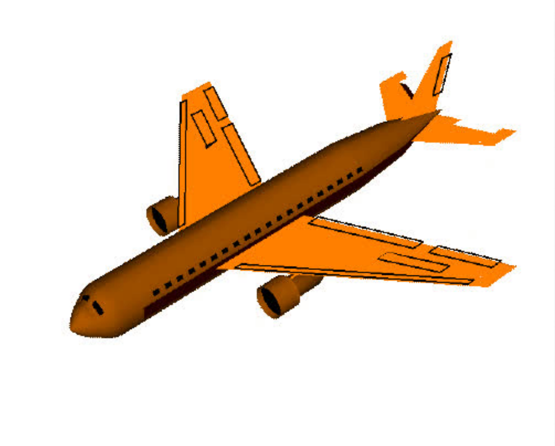
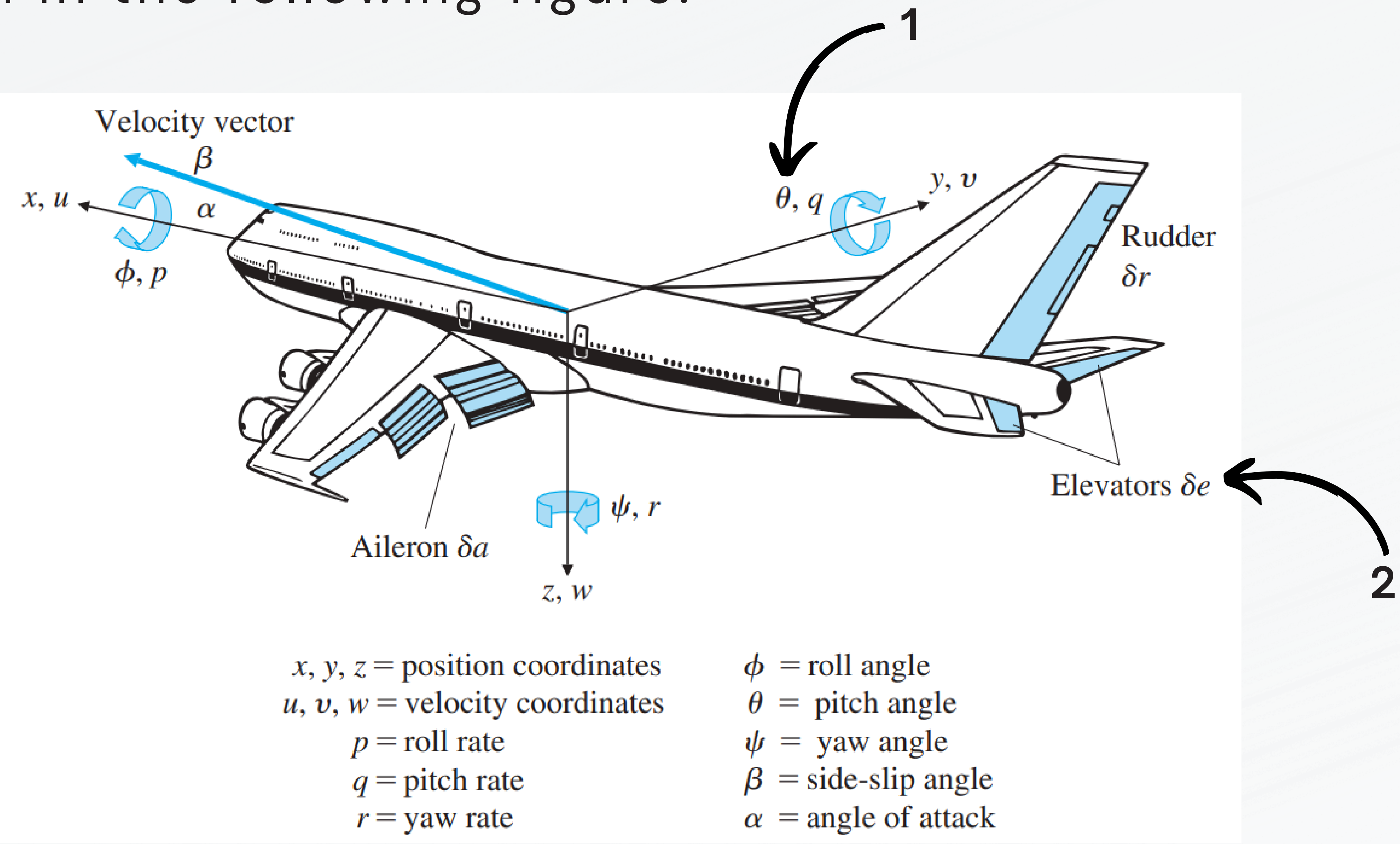
Problem Formulation

- **Boeing 747** motion is typically modeled using linearized equations of motion.
- Our focus is on the context of an **autopilot system** designed to maintain the **aircraft's pitch attitude** .
- The pitch attitude is defined as the angular orientation of the aircraft's nose relative to the horizon. The autopilot ensures that the aircraft's pitch angle remains stable and within desired limits despite external disturbances or changes in flight conditions.



Problem Formulation

- A schematic with the relevant coordinates that move with the Boeing 747 airplane is shown in the following figure.



MATHEMATICAL MODELLING

Considering the following parameters:

1. u = Forward velocity perturbation in the aircraft in x direction,
2. q = Angular rate (pitch rate) about the positive y-axis,
3. θ = Pitch-angle from the reference value,

System Input: δe = movable tail-section, or “elevator,” angle for pitch control.

System Output: θ

MATHEMATICAL MODELLING

From these parameters, we derived equations that provide the following state-space representation of the system.

$$\dot{x} = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & -5 \\ 0 & 56.7 & 0 \end{bmatrix} [x] + \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix} [\delta_e]$$

Therefore, the state matrix is as follows:

$$A = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & -5 \\ 0 & 56.7 & 0 \end{bmatrix}$$

The output we are interested in is the pitch angle. The output equation is:

$$y = [0 \quad 0 \quad 1][x]$$

MATHEMATICAL MODELLING

Therefore, the Continuous-time transfer function is:

$$\frac{\theta(s)}{\delta_e(s)} = G(s) = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 284.4215s + 88.7355}$$

SYSTEM ANALYSIS

Stability Analysis

$$|A-\lambda I| = \left|\begin{bmatrix} -0.313-\lambda & 56.7 & 0 \\ -0.0139 & -0.426-\lambda & -5 \\ 0 & 56.7 & -\lambda \end{bmatrix}\right|$$

eigenvalues:
-0.3121 + 0.0000i
-0.2134 +16.8595i
-0.2134 -16.8595i

The system is internally stable, which implies BIBO stability.

Controllability Analysis

$$CO = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

CO = 3×3

0.2320	1.0784	-1.0107
0.0203	-0.0119	-5.7650
0	1.1510	-0.6732

The controllability matrix has rank of 3 which is full row rank. Thus, the system is controllable

Observability Analysis

$$OB = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

OB = 3×3

0	0	1.0000
0	56.7000	0
-0.7881	-24.1542	-283.5000

The observability matrix has full column rank of 3. So, the system is observable

STATE FEEDBACK CONTROLLER

$$\Delta_f(s) = s^3 + \bar{\alpha}_1 s^2 + \bar{\alpha}_2 s + \bar{\alpha}_3 = s^3 + 0.739s^2 + 284.4215s + 88.7355$$

$$\bar{K} = \left[\bar{a}_1 - a_1 \quad \bar{a}_2 - a_2 \quad \bar{a}_3 - a_3\right] = [0.0024 \quad 0.7390 \quad 0.3126]$$

The actual feedback gain applied to the original system is:

$$K = \bar{K}P = [0.0030 \ 0.0857 \ 0.6385]$$

TRACKING REFERENCE SIGNAL FEEDFORWARD GAIN

$$\dot{X} = (A - \mathrm{BK})X + BK_f \quad = \begin{pmatrix} \frac{29\,\mathrm{kf}}{125} - \frac{313\,x_1}{1000} + \frac{567\,x_2}{10} - \frac{1467004581238105\,x_3}{9903520314283042199192993792} \\ \frac{203\,\mathrm{kf}}{10000} - \frac{139\,x_1}{10000} - \frac{213\,x_2}{500} - 5\,x_3 \\ \frac{567\,x_2}{10} \end{pmatrix}$$

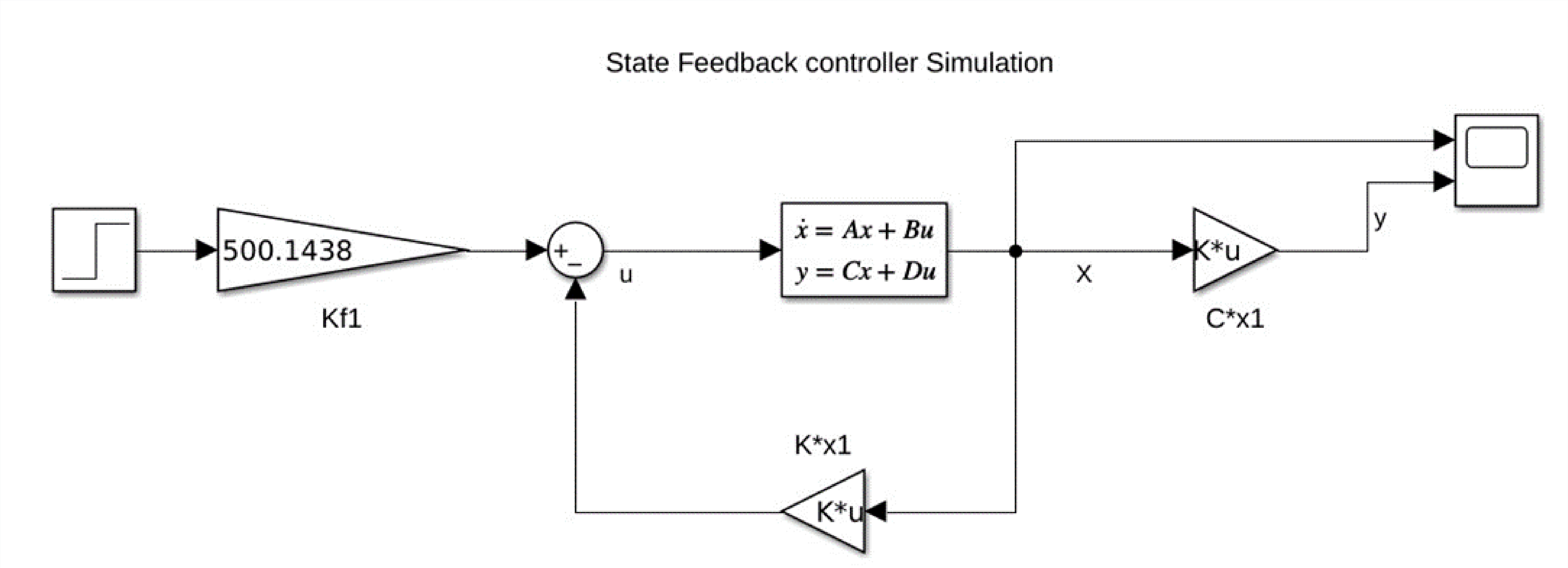
$$G(s)_{CL} = C\big(sI - (A - BK)\big)^{-1} BK_f$$

At $s = 0$

$$K_f = 500.1438$$

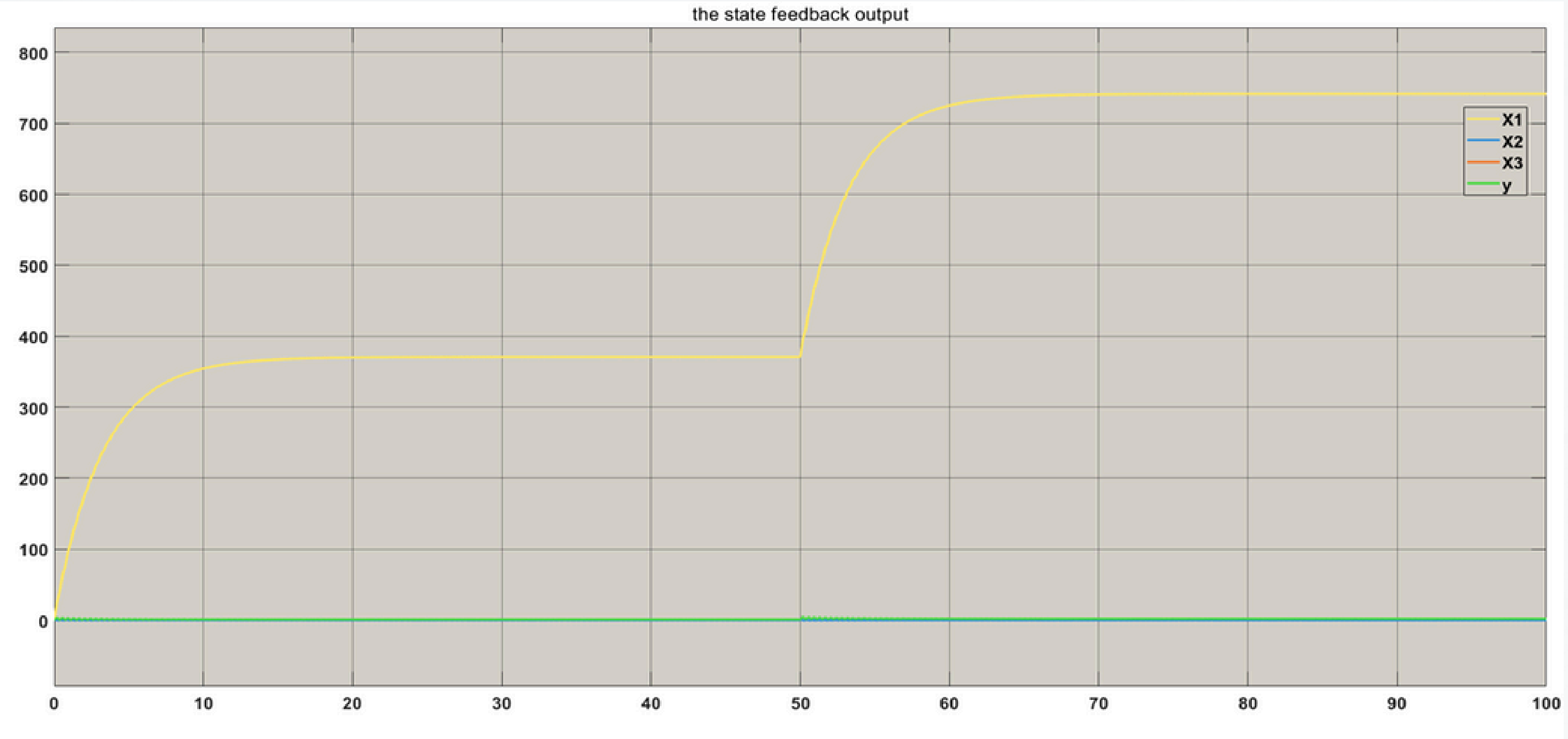
TRACKING REFERENCE SIGNAL FEEDFORWARD GAIN

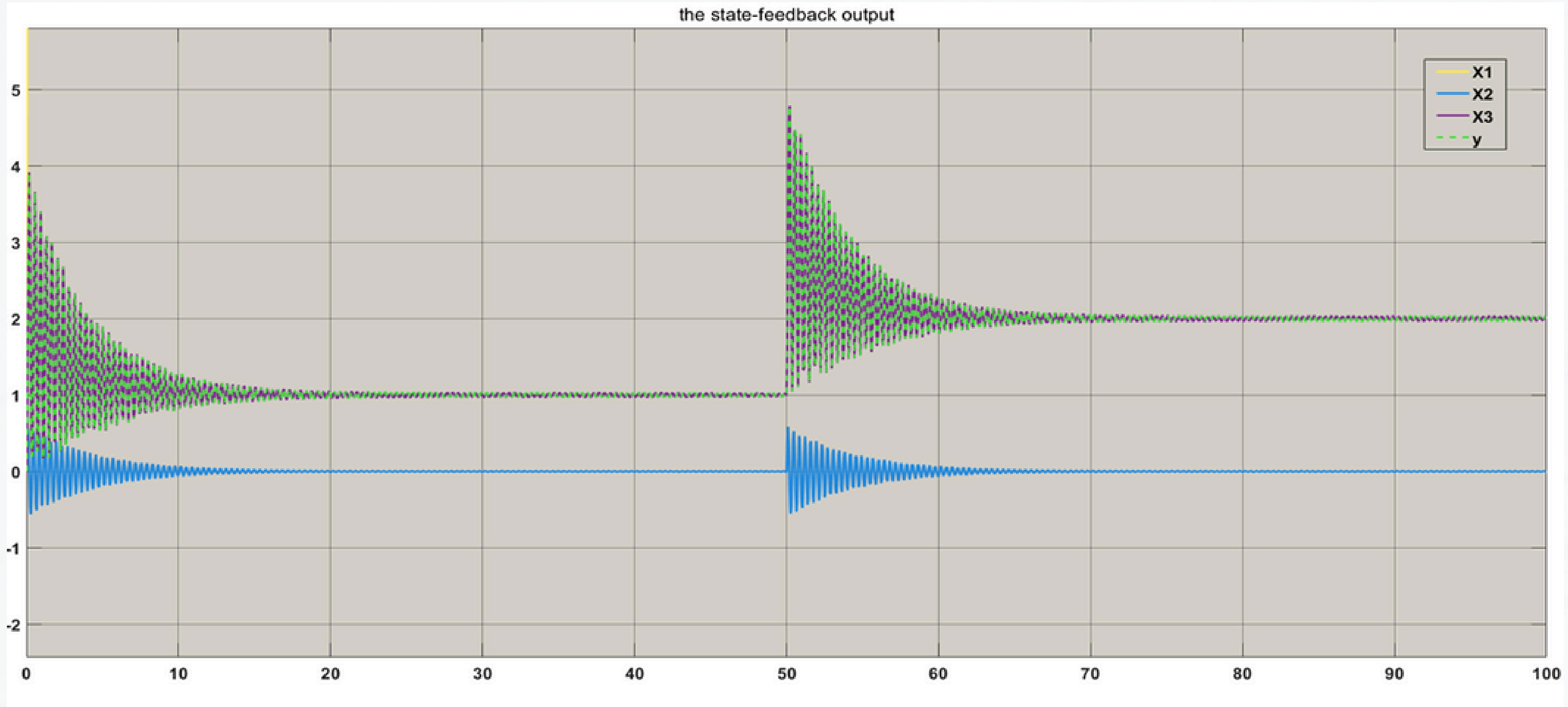
The following picture shows the State feedback controller simulation:



TRACKING REFERENCE SIGNAL FEEDFORWARD GAIN

The state feedback output is shown in the following two pictures:



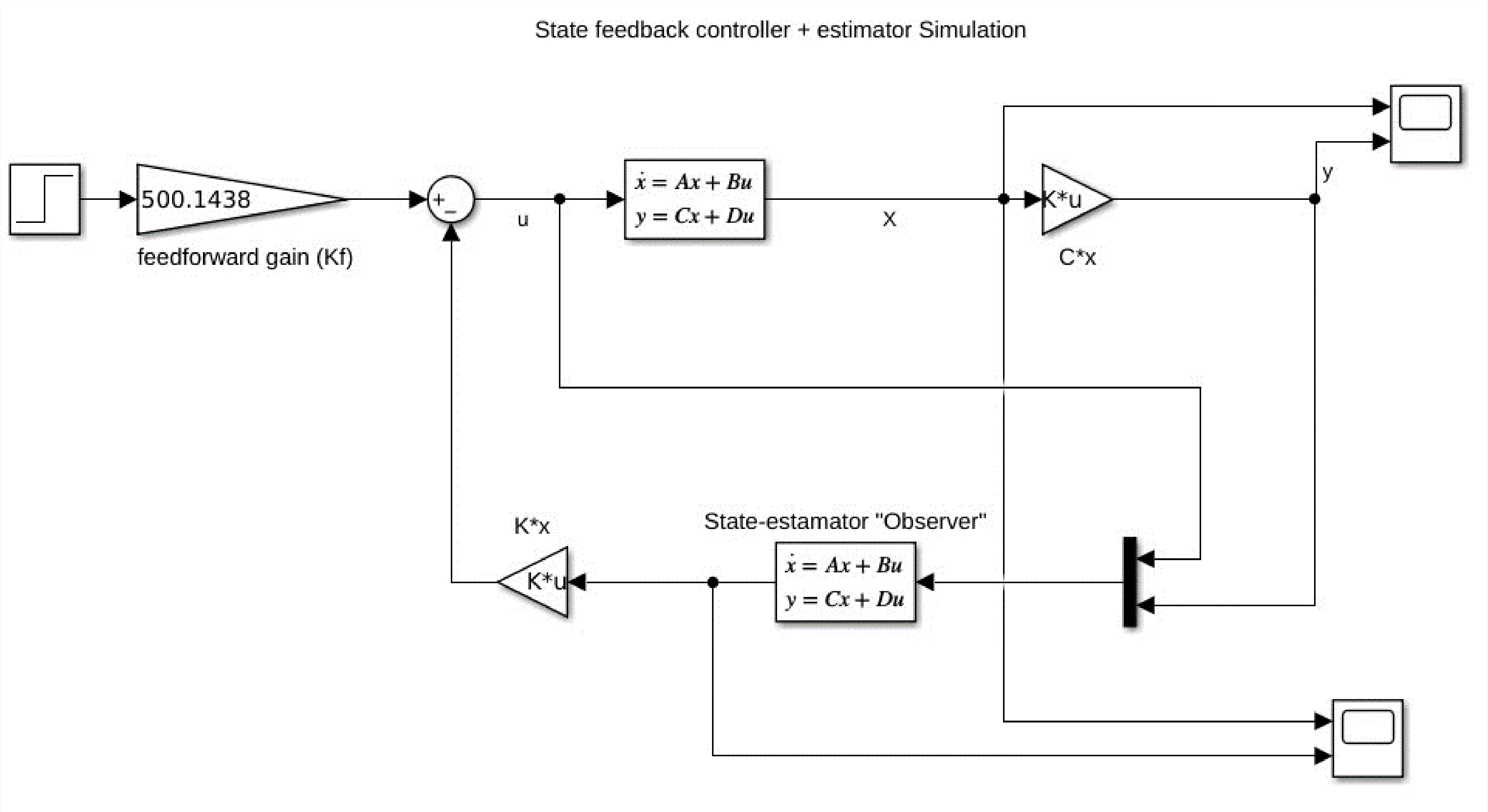


- The reference begins at 1 when time is 0, then changes to 2 at time 50.
- The output tracks the reference, and the output y matches the value of , which represents .

STATE ESTIMATOR (OBSERVER)

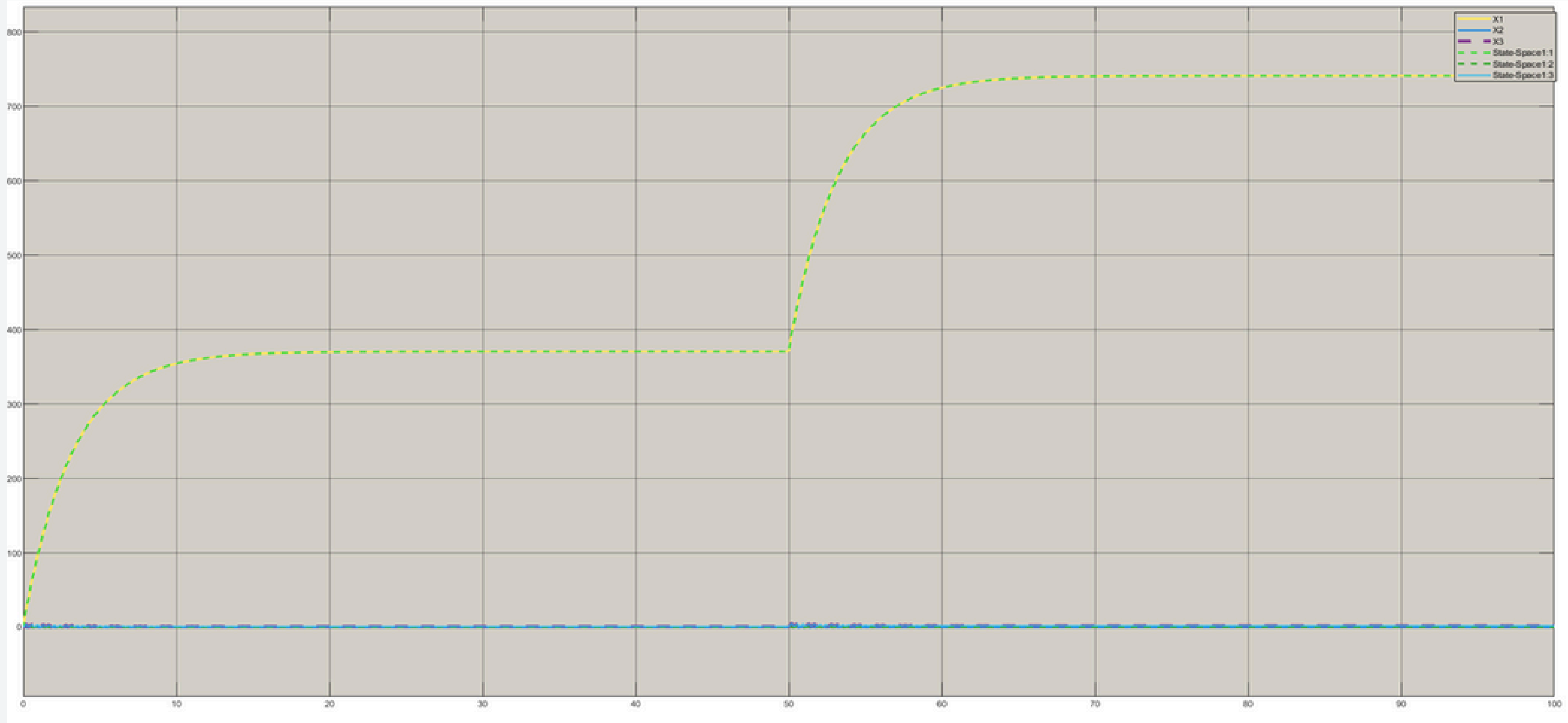
$$sI - (A - LC) = 0$$

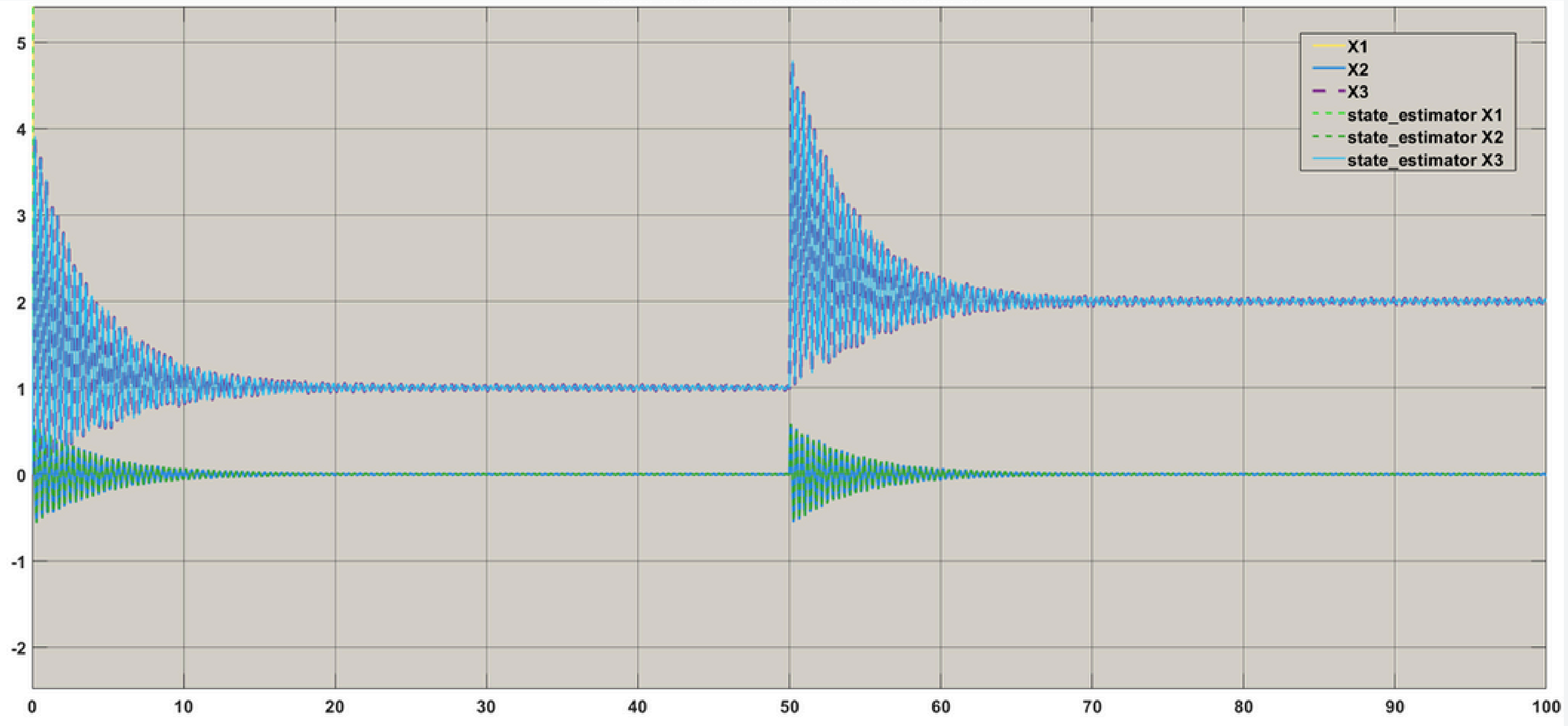
Getting the **L** to design of state feedback control and estimator is shown in the following picture:



STATE ESTIMATOR (OBSERVER)

The results of the state estimator with the feedback control:





- The estimator design is verified because the output of the state estimator tracks the reference, and matches the values of the system.

REFERENCES

[1] Bryson, A. E., Jr., Control of Spacecraft and Aircraft. Princeton, NJ: Princeton University Press, 1994.
[2] Franklin, Gene F., J. David Powell, Abbas Emami-Naeini, and J. David Powell. Feedback control of dynamic systems. Vol. 4. Upper Saddle River: Prentice hall, 2002.

Questions