Table 1: Size of our multiscale test for different AR parameters a_1 and a_2 , sample sizes T and nominal sizes α .

)	$a_1 = -0.5$).	a	$a_1 = -0.25$	2		$a_1 = 0.25$			$a_1 = 0.5$		(a_1, a_2)	$(a_1, a_2) = (0.167, 0.178)$	7, 0.178)
	non	nominal size	e α	nor	nominal size	α	noi	nominal size α	α	noı	nominal size α	e α	no	nominal size α	σe
	0.01	0.01 0.05 0.1	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.01 0.05 0.1	0.1	0.01	0.05	0.1
T = 250	0.015	0.050	0.127	0.014	0.057	0.120	0.011	0.046	0.116	0.013	0.042	0.108	0.011	0.052	0.117
T = 350	0.009	0.067	0.120	0.010	0.055	0.095	0.009	0.055	0.096	0.010	0.049	0.090	0.010	0.059	0.114
T = 500	0.015	0.053	0.128	0.015	0.047	0.100	0.018	0.048	0.101	0.015	0.042	0.106	0.015	0.056	0.107

Table 2: Power of our multiscale test for different AR parameters a_1 and a_2 , sample sizes T and nominal sizes α . The three panels (a)–(c) corresponds to different slope parameters β of the broken line m.

	$(a_1, a_2) = (0.167, 0.178)$	nominal size α 0.05 0.1	0.460 0.612 0.654 0.770 0.815 0.907		$(a_1, a_2) = (0.167, 0.178)$	nominal size α 0.05 0.1	0.724 0.851 0.922 0.958 0.983 0.994		$(a_1, a_2) = (0.167, 0.178)$	nominal size α 0.05 0.1	0.918 0.958 0.988 0.997 0.999 1.000
	$(a_1, a_2) =$	nomi 0.01	0.269 C 0.390 C 0.623 C		$(a_1, a_2) =$	nomi 0.01	0.549 C 0.759 C 0.933 C		$(a_1, a_2) =$	nomi 0.01	0.804 C 0.950 C 0.994 C
	$a_1 = 0.5$	inal size α 0.05 0.1	0.097 0.181 0.141 0.221 0.162 0.285		$a_1 = 0.5$	nominal size α 1 0.05 0.1	0.143 0.259 0.231 0.334 0.309 0.451		= 0.5	inal size α 0.05 0.1	0.224 0.367 0.361 0.481 0.473 0.649
	$a_1 =$	nominal size 0.01 0.05	0.036 0.0 0.050 0.1 0.060 0.1		$a_1 =$	nomine 0.01 0.	0.062 0.1 0.092 0.2 0.137 0.3		$a_1 =$	$\begin{array}{cc} \text{nominal size} \\ 0.01 & 0.05 \end{array}$	0.100 0.5 0.162 0.5 0.285 0.4
(a) $\beta = 1.5$	$a_1 = 0.25$	nominal size α 0.05 0.1	77 0.177 0.324 16 0.273 0.385 95 0.389 0.551	(b) $\beta = 2.0$	$a_1 = 0.25$	nominal size α 0.05 0.1	0.164 0.340 0.520 0.262 0.483 0.615 0.469 0.716 0.821	(c) $\beta = 2.5$	$a_1 = 0.25$	nominal size α 0.05 0.1	0.322 0.543 0.703 0.470 0.737 0.833 0.773 0.919 0.968
(a) β	2	$\frac{\alpha}{0.1}$ 0.01	0.702 0.077 0.834 0.116 0.972 0.195	β (q)	2	α 0.1 0.01	0.916 0.164 0.986 0.262 0.999 0.469	(c) <i>\beta</i>	5	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.993 0.322 1.000 0.470 1.000 0.773
	$a_1 = -0.25$	nominal size α 0.01 0.05 (0.319 0.548 0.463 0.753 0.775 0.925		$a_1 = -0.25$	nominal size α 0.01 0.05 (0.663 0.846 0.863 0.969 0.983 0.997		$a_1 = -0.25$	nominal size α 0.01 0.05 (0.901 0.971 0.990 1.000 0.999 1.000
	$a_1 = -0.5$	nominal size α 0.01 0.05 0.1	0.484 0.726 0.853 0.735 0.913 0.955 0.945 0.988 0.997		$a_1 = -0.5$	nominal size α 0.01 0.05 0.1	0.869 0.961 0.985 0.979 0.997 1.000 1.000 1.000 1.000		$a_1 = -0.5$	nominal size α 0.01 0.05 0.1	0.989 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000
			T = 250 $T = 350$ $T = 500$				T = 250 $T = 350$ $T = 500$				T = 250 $T = 350$ $T = 500$

Table 3: Size of our multiscale test WITHOUT LAMBDA for different AR parameters a_1 and a_2 , sample sizes T and nominal sizes α .

)	$a_1 = -0.5$	5	a_1	$_{1} = -0.25$	5)	$a_1 = 0.25$,,,	-	$a_1 = 0.5$		(a_1, a_2)	$(a_1, a_2) = (0.167, 0.178)$, 0.178)
	Ion	nominal size α	ie α	nomir	ninal size	ε α	nor	nominal size α	ε α	nor	nominal size	α	non	nominal size α	ε α
	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
250	0.028	0.160	0.266	0.015	0.077	0.142	0.003	0.035	0.069	0.002	0.026	0.055	0.003	0.025	0.049
350	0.033	0.174	0.266	0.018	0.086	0.155	0.007	0.030	0.064	0.000	0.019	0.044	0.006	0.022	0.051
200	0.051	0.172	0.287	0.016	0.080	0.161	0.008	0.034	0.075	0.008	0.031	0.048	0.005	0.025	0.065

Table 4: Power of our multiscale test WITHOUT LAMBDA for different AR parameters a_1 and a_2 , sample sizes T and nominal sizes α . The three panels (a)–(c) corresponds to different slope parameters β of the broken line m.

						ř								010)	1100
	a_1 :	e.u.=		$a_{\scriptscriptstyle m I}$	$a_1 = -0.25$	- -		$a_1 = 0.25$			$a_1 = 0.5$		(a_1, a_2)	$(a_1, a_2) = (0.10i, 0.1i\delta)$	(, 0.110)
0.	nomin 0.01	nominal size α 1 0.05 (0.1	non 0.01	nominal size $lpha$ 1 0.05 (e α 0.1	non 0.01	nominal size α 1 0.05 ($\alpha = \alpha$ 0.1	no 0.01	nominal size $lpha$	ie α 0.1	no 0.01	nominal size α	α α 0.1
0.5		0.533	0.664 0.826	0.125	0.334	0.475	0.026	0.096	0.165	0.007	0.049	0.083	0.110	0.257	0.348
0.7	0.725 0	0.895	0.948	0.451	0.699	0.812	090.0	0.157	0.237	0.019	0.058	0.097	0.319	0.514	0.631
							(b) $\beta = 2.0$	0.4							
	a_1	$a_1 = -0.5$		a_1	$a_1 = -0.25$	35		$a_1 = 0.25$	20		$a_1 = 0.5$		(a_1, a_2)	$(a_1, a_2) = (0.167, 0.178)$	7, 0.178)
0.	nomin 0.01	nominal size α 1 0.05 ($\alpha = 0.1$	non 0.01	nominal size α 1 0.05 (e α 0.1	noi 0.01	nominal size α 1 0.05 ($lpha \alpha = 0.1$	no 0.01	nominal size α 1 0.05 (ie α 0.1	no 0.01	nominal size $lpha$	se α 0.1
0.0	0.618 0	0.869	0.926	0.367	0.662	0.759	0.056	0.160	0.256	0.018	0.068	0.111	0.283	0.513	0.619
0.6		1.000	1.000	0.898	0.969	0.986	0.184	0.360	0.486	0.038	0.101	0.172	0.753	0.889	0.926
							(c) $\beta = 2.5$.5							
	a_1 :	= -0.5		a_1	$_{\rm L} = -0.25$	35	1	$a_1=0.25$	2		$a_1 = 0.5$	9	(a_1,a_2)	$(a_1, a_2) = (0.167, 0.178)$	7, 0.178)
0.	nomin 0.01	nominal size $lpha$	α 0.1	nom 0.01	nominal size $lpha$ 1 0.05 (e α 0.1	noi 0.01	nominal size $lpha$	$\alpha = \alpha$ 0.1	no 0.01	nominal size $lpha$ 1 0.05 (ie α 0.1	no 0.01	nominal size $lpha$	α α 0.1
9:0	0.918 C	0.982	0.996	0.704	0.983	0.931	0.121	0.310	0.431	0.032	0.096	0.157	0.542	0.788	0.840
1.(1.000	1.000	0.993	0.999	0.999	0.454	0.686	0.779	0.088	0.212	0.307	0.963	0.989	0.993