(1) $\mathcal{F}_g = \{ m : \mathbb{R} \to \mathbb{R}_{\geq 0} \mid$

$m(u) = c\, g_s\left(\frac{u-a}{b}\right)$ with $a, b, c > 0$

and g_s a density with mean 0, sd $1 \}$

$$m_i(u) = c_i\, g_s\left(\frac{u-a_i}{b_i}\right) \quad \forall i \in \mathcal{G}_s .$$

$$\int_{-\infty}^{\infty} m_i(u)\, du = c_i \int g_s\left(\frac{u-a_i}{b_i}\right) du$$
$$= c_i \int b_i\, g_s(\rho)\, d\rho = c_i b_i$$

$$\int_{-\infty}^{\infty} u\, m_i(u)\, du = c_i \int u\, g_s\left(\frac{u-a_i}{b_i}\right) du$$
$$= c_i b_i \int (a_i + b_i \rho)\, g_s(\rho)\, d\rho$$
$$= c_i b_i a_i$$

$$\int_{-\infty}^{\infty} u^2 m_i(u)\, du = c_i \int u^2 g_s\left(\frac{u-a_i}{b_i}\right) du$$
$$= c_i b_i \int (a_i + b_i \rho)^2 g_s(\rho)\, d\rho$$
$$= c_i b_i a_i^2 + c_i b_i b_i^2$$

Define
$$m_i^*(u) = \frac{m_i(u)}{\int_{-\infty}^{\infty} m_i(v)\, dv} .$$

Then

$$a_i = \int_{-\infty}^{\infty} u\, m_i^*(u)\, du$$

$$b_i = \Big\{ \underbrace{\int_{-\infty}^{\infty} u^2 m_i^*(u)\, du - \Big(\int_{-\infty}^{\infty} u\, m_i^*(u)\, du \Big)^2}_{=:(+)} \Big\}^{1/2}$$

$$c_i = \int_{-\infty}^{\infty} m_i(u)\, du \,/\, (+).$$

(2) Define

$$\hat{m}_i(u) = \frac{\sum_{t=1}^{T} K\left(\frac{u - t/\sqrt{T}}{h}\right) y_{it}}{\sum_{t=1}^{T} K\left(\frac{u - t/\sqrt{T}}{h}\right)}$$

(K rectangular, $h = 3.5/\sqrt{T}$ (7 days), $h = 7/\sqrt{T}$ (14 days), ...) and

$$\hat{m}_i^*(u) = \frac{\hat{m}_i(u)}{\int_{-\infty}^{\infty} \hat{m}_i(v)\, dv}.$$

Note that

$$\int_{-\infty}^{\infty} v^{\ell} \hat{m}_i(v)\, dv \approx \sum_{s=1}^{T} \frac{1}{\sqrt{T}} \left(\frac{s}{\sqrt{T}}\right)^{\ell} \hat{m}_i\left(\frac{s}{\sqrt{T}}\right). \quad (*)$$

Then

$$\hat{a}_i = \int_{-\infty}^{\infty} u \, \hat{m}_i^*(u) \, du$$

$$\hat{l}_i = \left\{ \int_{-\infty}^{\infty} u^2 \hat{m}_i^*(u) \, du - \left(\int_{-\infty}^{\infty} u \, \hat{m}_i^*(u) \, du \right)^2 \right\}^{1/2}$$

$$\hat{c}_i = \int_{-\infty}^{\infty} \hat{m}_i(u) \, du \, / \, \hat{l}_i \, ,$$

where we use the approximation (*) to compute the above integrals numerically.

(3) By definition,

$$m_i(a_i + l_i v) / c_i = g_k(v) \quad \forall i \in G_k .$$

Hence, compute

$$\hat{p}_i(v) = \hat{m}_i(\hat{a}_i + \hat{l}_i v) / \hat{c}_i \quad \forall i = 1, \dots, n$$

as well as

$$\hat{p}_i^*(v) = \frac{\hat{p}_i(v)}{\int_{-\infty}^{\infty} \hat{p}_i(w) \, dw} \quad \forall i = 1, \dots, n$$

(which is a proper density).

Now calculate the (squared) Hellinger distances

$$\Delta_{ij} = \int \left(\sqrt{\hat{P}_i^*(w)} - \sqrt{\hat{P}_j^*(w)} \right)^2 dw$$

and define the dissimilarity measure

$$\mathcal{D}(S, S') = \max_{i \in S,\, j \in S'} \Delta_{ij}$$

for $S, S' \subseteq \{1, \dots, n\}$. Run a HAC algorithm with this dissimilarity measure.