Comments on the paper "Multiscale Inference and Long-Run Variance Estimation in Nonparametric Regression with Time Series Errors"

**Summary**

The paper develops a multiscale method to test qualitative hypotheses of trends in a mean signal plus noise model. Their approach allows one to test constant or monotonic or other patterns of the mean trends, which is of significant scientific interest. Theoretical properties of their approach is discussed and a simulation study is provided. Also they applied it to the real data of yearly mean temperature in Central England.

**Comments** I have the following list of detailed comments.

- A consistency result of Proposition 3.3.

  I believe that the following type of result can be obtained: $P(E_T^\ell) \to 1$. Theorem 3.1 is for testing purpose. In certain application one might be interested in such consistency result. Basically one needs to study the behavior of $q_T(\alpha)$ when $\alpha \to 0$.

- Estimation of long run variance using autoregressive processes.

  The authors considered estimating $\sigma^2$ using AR processes. A limitation is that the order $p$ is fixed and finite. It appears that the latter limitation can be relaxed. For a stationary process $\varepsilon_t$ (not necessarily linear), one can fit an AR process with large $p$

  $$\varepsilon_t = \sum_{j=1}^{p} a_j \varepsilon_{t-j} + \eta_t, \tag{1}$$

  properties of fitted $\hat{a}_1, \dots, \hat{a}_p$ can be obtained from the rsults in the following papers

  - W. B. Wu and Mohsen Pourahmadi (2009): Banding Sample Covariance Matrices of Stationary Processes, Statistica Sinica 19 1755-1768
  - H. Xiao and W. B. Wu (2012). Covariance Matrix Estimation for Stationary Time Series. Annals of Statistics, Volume 40, Number 1 (2012), 466-493.

  A similar version of the authors estimate (4.14) can be used. Rate of convergence (cf. Proposition 4.1) can be derived with rate $T^{-1/2}$ therein possibly replaced by a larger term of the form $T^{-c}$ with $c < 1/2$.

- Real data application.

  The authors analyzed the yearly mean Central England temperature data. It will be interesting to apply their approach to the global temperature data. In the paper " Isotonic regression: another look at the change point problem. Biometrika, 88, 793-804, 2001", an increasing trend function is fitted. It will be important to know which period the sequence in increasing/decreasing.

- Simulation Study

  In the simulation study the authors considered AR(1) processes with relatively weaker dependence: $a_1 \in \{-0.5, -0.25, 0.25, 0.5\}$. One should consider the stronger positive/negative dependence case with $a = \pm 0.9$ (say). How does the strength of dependence affect the performance of the procedure?