

Algorithm

1. Calculate $\gamma(k) = \frac{\sigma_\eta^2}{1-a_1^2} \cdot a_1^{|k|}$.
2. Calculate $\text{Var}(\bar{Y}) = \frac{\gamma(0)}{T} + \frac{2}{T} \sum_{k=1}^{T-1} (1 - \frac{k}{T})\gamma(k)$.
3. Calculate $T^* = \frac{\gamma(0)}{\text{Var}(\bar{Y})}$.
4. For each location $i = 1/T, 2/T, \dots, 1$ and each bandwidth $h = 3/T, 8/T, \dots, 1/4 + 3/T$ we calculate the following values:

- $ESS(i, h) = \frac{\frac{1}{h} \sum_{t=1}^T K(\frac{i-t/T}{h})}{\frac{1}{h} K(0)}$;
- $ESS^*(i, h) = \frac{T^*}{T} \cdot ESS$;
- $l(i, h) = \frac{T}{ESS^*(i, h)}$;
- $q(i, h) = \Phi^{-1}\left(\frac{1+(1-\alpha)^{\frac{1}{l(i, h)}}}{2}\right)$;
- $(X^T W X)^{-1} X^T W$, where $W = \{diag(\frac{1}{h} K(\frac{i-t/T}{h}))\}$ and

$$X = \begin{pmatrix} 1 & (1/T - i) \\ 1 & (2/T - i) \\ \vdots & \vdots \\ 1 & (1 - i) \end{pmatrix}; \quad (1)$$

- $sd(\widehat{m}'_h(i)) = \sqrt{((X^T W X)^{-1} (X^T \Sigma X) (X^T W X)^{-1})_{2,2}}$, where Σ is the kernel weighted covariance matrix of errors with generic element

$$\sigma_{kl} = \gamma(|k-l|) \frac{1}{h} K\left(\frac{i-k/T}{h}\right) \frac{1}{h} K\left(\frac{i-l/T}{h}\right).$$

5. Discard all pairs of location and bandwidth (i, h) where $ESS^* < 5$.
6. Based on the grid from 5, calculate Gaussian quantile for our method.