Table 1: Size of our multiscale test for different AR parameters a_1 and a_2, sample sizes T and nominal sizes α.

	$a_1 = -0.5$			$a_1 = -0.25$			$a_1 = 0.25$			$a_1 = 0.5$			$(a_1, a_2) = (0.167, 0.178)$		
	nominal size α			nominal size α			nominal size α			nominal size α			nominal size α		
	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
$T = 250$	0.015	0.050	0.127	0.014	0.057	0.120	0.011	0.046	0.116	0.013	0.042	0.108	0.011	0.052	0.117
$T = 350$	0.009	0.067	0.120	0.010	0.055	0.095	0.009	0.055	0.096	0.010	0.049	0.090	0.010	0.059	0.114
$T = 500$	0.015	0.053	0.128	0.015	0.047	0.100	0.018	0.048	0.101	0.015	0.042	0.106	0.015	0.056	0.107

Table 2: Power of our multiscale test for different AR parameters a_1 and a_2, sample sizes T and nominal sizes α. The three panels (a)–(c) corresponds to different slope parameters β of the broken line m.

(a) $\beta = 1.5$

	$a_1 = -0.5$			$a_1 = -0.25$			$a_1 = 0.25$			$a_1 = 0.5$			$(a_1, a_2) = (0.167, 0.178)$		
	nominal size α			nominal size α			nominal size α			nominal size α			nominal size α		
	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
$T = 250$	0.484	0.726	0.853	0.319	0.548	0.702	0.077	0.177	0.324	0.036	0.097	0.181	0.269	0.460	0.612
$T = 350$	0.735	0.913	0.955	0.463	0.753	0.834	0.116	0.273	0.385	0.050	0.141	0.221	0.390	0.654	0.770
$T = 500$	0.945	0.988	0.997	0.775	0.925	0.972	0.195	0.389	0.551	0.060	0.162	0.285	0.623	0.815	0.907

(b) $\beta = 2.0$

	$a_1 = -0.5$			$a_1 = -0.25$			$a_1 = 0.25$			$a_1 = 0.5$			$(a_1, a_2) = (0.167, 0.178)$		
	nominal size α			nominal size α			nominal size α			nominal size α			nominal size α		
	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
$T = 250$	0.869	0.961	0.985	0.663	0.846	0.916	0.164	0.340	0.520	0.062	0.143	0.259	0.549	0.724	0.851
$T = 350$	0.979	0.997	1.000	0.863	0.969	0.986	0.262	0.483	0.615	0.092	0.231	0.334	0.759	0.922	0.958
$T = 500$	1.000	1.000	1.000	0.983	0.997	0.999	0.469	0.716	0.821	0.137	0.309	0.451	0.933	0.983	0.994

(c) $\beta = 2.5$

	$a_1 = -0.5$			$a_1 = -0.25$			$a_1 = 0.25$			$a_1 = 0.5$			$(a_1, a_2) = (0.167, 0.178)$		
	nominal size α			nominal size α			nominal size α			nominal size α			nominal size α		
	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
$T = 250$	0.989	1.000	1.000	0.901	0.971	0.993	0.322	0.543	0.703	0.100	0.224	0.367	0.804	0.918	0.958
$T = 350$	1.000	1.000	1.000	0.990	1.000	1.000	0.470	0.737	0.833	0.162	0.361	0.481	0.950	0.988	0.997
$T = 500$	1.000	1.000	1.000	0.999	1.000	1.000	0.773	0.919	0.968	0.285	0.473	0.649	0.994	0.999	1.000

Table 3: Size of our multiscale test WITHOUT LAMBDA for different AR parameters a_1 and a_2, sample sizes T and nominal sizes α.

	$a_1 = -0.5$			$a_1 = -0.25$			$a_1 = 0.25$			$a_1 = 0.5$			$(a_1, a_2) = (0.167, 0.178)$		
	nominal size α			nominal size α			nominal size α			nominal size α			nominal size α		
	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
250	0.028	0.160	0.266	0.015	0.077	0.142	0.003	0.035	0.069	0.002	0.026	0.055	0.003	0.025	0.049
350	0.033	0.174	0.266	0.018	0.086	0.155	0.007	0.030	0.064	0.006	0.019	0.044	0.006	0.022	0.051
500	0.051	0.172	0.287	0.016	0.080	0.161	0.008	0.034	0.075	0.008	0.031	0.048	0.005	0.025	0.065

Table 4: Power of our multiscale test WITHOUT LAMBDA for different AR parameters a_1 and a_2, sample sizes T and nominal sizes α. The three panels (a)–(c) corresponds to different slope parameters β of the broken line m.

(a) $\beta = 1.5$

	$a_1 = -0.5$			$a_1 = -0.25$			$a_1 = 0.25$			$a_1 = 0.5$			$(a_1, a_2) = (0.167, 0.178)$		
	nominal size α			nominal size α			nominal size α			nominal size α			nominal size α		
	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
250	0.234	0.533	0.664	0.125	0.334	0.475	0.026	0.096	0.165	0.007	0.049	0.083	0.110	0.257	0.348
350	0.471	0.739	0.826	0.251	0.443	0.572	0.036	0.120	0.191	0.014	0.058	0.100	0.194	0.337	0.446
500	0.725	0.895	0.948	0.451	0.699	0.812	0.060	0.157	0.237	0.019	0.058	0.097	0.319	0.514	0.631

(b) $\beta = 2.0$

	$a_1 = -0.5$			$a_1 = -0.25$			$a_1 = 0.25$			$a_1 = 0.5$			$(a_1, a_2) = (0.167, 0.178)$		
	nominal size α			nominal size α			nominal size α			nominal size α			nominal size α		
	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
250	0.618	0.869	0.926	0.367	0.662	0.759	0.056	0.160	0.256	0.018	0.068	0.111	0.283	0.513	0.619
350	0.885	0.970	0.986	0.618	0.826	0.901	0.113	0.238	0.336	0.032	0.090	0.138	0.500	0.713	0.796
500	0.991	1.000	1.000	0.898	0.969	0.986	0.184	0.360	0.486	0.038	0.101	0.172	0.753	0.889	0.926

(c) $\beta = 2.5$

	$a_1 = -0.5$			$a_1 = -0.25$			$a_1 = 0.25$			$a_1 = 0.5$			$(a_1, a_2) = (0.167, 0.178)$		
	nominal size α			nominal size α			nominal size α			nominal size α			nominal size α		
	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1	0.01	0.05	0.1
250	0.918	0.982	0.996	0.704	0.889	0.931	0.121	0.310	0.431	0.032	0.096	0.157	0.542	0.788	0.840
350	0.992	0.999	1.000	0.926	0.983	0.996	0.250	0.420	0.528	0.070	0.144	0.216	0.811	0.930	0.962
500	1.000	1.000	1.000	0.993	0.999	0.999	0.454	0.686	0.779	0.088	0.212	0.307	0.963	0.989	0.993