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Testing for common deterministic trend slopes☆

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Abstract

We propose tests for hypotheses on the parameters for deterministic trends. The model framework assumes a multivariate structure for trend-stationary time series variables. We derive the asymptotic theory and provide some relevant critical values. Monte Carlo simulations suggest which tests are more useful in practice than others. We apply our tests to examine real GDP convergence for a sample of seven European countries.

JEL classification: C12; C32

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1. Introduction

In several empirical situations it is found that time series data contain a deterministic trend, while they are otherwise stationary. An example in macroeconomics concerns differences between real output for pairs of countries (see Hobijn and Franses (2000) among others) or pairs of regions within the U.S. (see Carlino and Mills (1993), Loewy and Papell (1995) and Tomljanovich and Vogelsang (2002) among others), where such a trend-stationary series indicates some degree of convergence. Other examples can be found in disciplines such as tourism and marketing, where tourist arrivals and sales

☆ The computer programs used for all calculations in this paper can be obtained from the corresponding author. Additional simulation results can also be obtained upon request. Work on this paper was initiated when the first author visited the Tinbergen Institute Rotterdam (August 1998) and the second author visited Cornell University (May 1999).

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often display upward trending patterns. Finally, environmental data like temperatures may also display trends, and if these are upward moving this can be taken as evidence of global warming, see Bloomfield (1992), Woodward and Gray (1993), Zheng and Basher (1999) and Fomby and Vogelsang (2002) among many others.

In some of the above cases it may be of interest to examine if two or more trend-stationary time series have the same slope. This would allow for testing whether a pair of countries are converging with the same speed as another pair. In the empirical portion of this paper, we ask whether gross domestic product (GDP) growth in Italy has been faster than in other European countries. Are the differences in growth between Italy and other countries the same? These are just some examples of empirically interesting tests that involve joint restrictions on the slopes of trend functions of multiple time series. While there has been recent research on univariate trend function inference and modeling (see Perron, 1991; Canjels and Watson, 1997; Vogelsang, 1997, 1998), multivariate trend modeling and inference has received little attention.[1] The goal of this paper is to propose and analyze tests regarding the slopes of multiple trend-stationary time series including null hypotheses that involve linear cross equation restrictions.

The outline of our paper is as follows. In Section 2, we discuss the model representation, parameter estimation, and the test statistics of interest. A key issue is the estimation of the asymptotic covariance matrix, for which we aim to compare three different approaches, amongst which is the familiar heteroskedasticity autocorrelation consistent (HAC) estimator. The other two approaches are new and are based on extensions of the approach proposed by Kiefer and Vogelsang (2002a). In Section 3, we derive the relevant asymptotic theory. We tabulate useful critical values. Additionally, we discuss asymptotic power of the tests in a special case. In Section 4, we use Monte Carlo simulations to examine the finite sample performance of the test statistics. We observe that the tests work best if the number of restrictions being tested is small relative to the sample size. Additionally, we find that the HAC-based tests have serious size distortions, while the new tests perform satisfactorily. In Section 5, we apply our tests to six European real per capita GDP series relative to Italy. In 1950 all six countries had levels of real per capita GDP higher than Italy. Our tests indicate that from 1950 to 1992 Italy has grown faster than those six countries although the null hypothesis that the relative rates are equal can be rejected. Our results suggest that Italy has been catching up to much of Europe on a systematic basis.

2. The model and test statistics

In this section we present the model, parameter estimation and the relevant test statistics.

[1] It should be noted that the asymptotic theory for multivariate time series regressions developed by Park and Phillips (1988) includes our model as a special case where no covariates are included. However, they do not consider hypotheses involving cross equation restrictions on the linear trend parameters as we do here.

2.1. Representation

Consider n trend-stationary time series denoted by $y_{1,t}$ to $y_{n,t}$ with $t = 1, 2, \ldots, T$, and assume that they can be represented by

$$y_{i,t} = \mu_i + \beta_i t + u_{i,t} \quad i = 1, \ldots, n. \tag{1}$$

Define the three $n \times 1$ vectors u_t, μ and β by $(u_{1,t}, u_{2,t}, \ldots, u_{n,t})'$, $(\mu_1, \mu_2, \ldots, \mu_n)'$ and $(\beta_1, \beta_2, \ldots, \beta_n)'$, respectively. It is assumed that a functional central limit theorem applies to u_t, that is,

$$T^{-1/2} \sum_{t=1}^{[rT]} u_t \Rightarrow \Lambda W_n(r), \tag{2}$$

where $\Rightarrow$ denotes weak convergence, $W_n(r)$ is an $n \times 1$ vector of standard independent Wiener processes, and $[rT]$ is the integer part of rT. See, for example, Phillips and Durlauf (1986) for conditions under which (2) holds. When u_t is covariance stationary it follows that Λ is the matrix square root of the matrix Ω defined as

$$\Omega = \Lambda \Lambda' = \sum_{j=-\infty}^{\infty} \Gamma_j,$$

where $\Gamma_j = E[u_t u'_{t-j}]$. It is well known from the time series literature that Ω is equal to 2π times the zero-frequency spectral density matrix of the vector, u_t. Note that for a functional central limit theorem of the form given by (2) to hold, Ω is required to be finite and positive definite. This rules out stationary time series with long memory.[2]

2.2. Estimation

The parameters in (1) can be estimated by applying ordinary least-squares (OLS) equation by equation, which results in $\hat{\mu}$ and $\hat{\beta}$. If the errors are second-order stationary (a typical condition under which (2) will hold), then from the classic results of Grenander and Rosenblatt (1957), OLS is asymptotically equivalent to GLS (and equivalent to MLE under Gaussian errors). In addition, because (1) is a seemingly unrelated regression (SUR) with the same regressors in each equation, OLS is equivalent to the SUR estimator, which is the GLS estimator that accounts for contemporaneous correlation across the series. Thus, OLS has some nice optimality properties.

It will be convenient to express $\hat{\beta}_i$ as follows. Define $\bar{t} = T^{-1} \sum_{t=1}^{T} t$ and $\tilde{t} = t - \bar{t}$, then

$$\hat{\beta}_i = \left(\sum_{t=1}^{T} \tilde{t}^2 \right)^{-1} \sum_{t=1}^{T} \tilde{t} y_{i,t},$$

[2] In the case of stationary long memory processes, alternative functional central limit theorems could be used. However, the distribution theory obtained would differ substantially from what is obtained in this paper.

for $i = 1, 2, \ldots, n$. These estimators can be summarized into the $n \times 1$ vector $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_n)'$. As usual we obtain

$$\hat{\beta} - \beta = \left(\sum_{t=1}^{T} \tilde{t}^2 \right)^{-1} \sum_{t=1}^{T} \tilde{t} u_t. \tag{3}$$

To derive the asymptotic distribution of (3), note that

$$T^{-3} \sum_{t=1}^{T} \tilde{t}^2 \to \frac{1}{12}, \tag{4}$$

where $\to$ denotes convergence, and that

$$T^{-3/2} \sum_{t=1}^{T} \tilde{t} u_t \Rightarrow \Lambda \int_0^1 \left(r - \frac{1}{2} \right) \mathrm{d}W_n(r) = \Lambda \left[\frac{1}{2} W_n(1) - \int_0^1 W_n(r)\,\mathrm{d}r \right]. \tag{5}$$

For later it will be useful to define the process

$$V_n(r) = \left(r - \frac{1}{2} \right) W_n(r) - \int_0^r W_n(s)\,\mathrm{d}s - W_n(1) \int_0^r \left(s - \frac{1}{2} \right) \mathrm{d}s. \tag{6}$$

Note that because $\int_0^1 (s - \frac{1}{2})\,\mathrm{d}s = 0$, it follows that

$$V_n(1) = \left[\frac{1}{2} W_n(1) - \int_0^1 W_n(r)\,\mathrm{d}r \right]. \tag{7}$$

Using (4), (5) and (2), we obtain

$$T^{3/2}(\hat{\beta} - \beta) = \left(T^{-3} \sum_{t=1}^{T} \tilde{t}^2 \right)^{-1} T^{-3/2} \sum_{t=1}^{T} \tilde{t} u_t \Rightarrow 12 \Lambda V_n(1). \tag{8}$$

It is easy to show that $12\Lambda V_n(1)$ is distributed as $\mathrm{N}(0, 12\Omega)$. Therefore, inference regarding β can be carried out in the usual way provided a consistent estimator of Ω is available. It is well known from the time series literature that Ω is equal to 2π times the zero-frequency spectral density matrix of the vector, u_t. Therefore, Ω can be consistently estimated using well known spectral estimation methods. In this paper we focus on nonparametric estimators of Ω. Under regularity conditions similar to those required for (2) to hold, Ω can be consistently estimated using the class of estimators based on smoothing sums of the sample autocovariances of OLS residuals. See Priestley (1981) for a comprehensive discussion of these estimators and Grenander and Rosenblatt (1957) for the specific case of detrended data. In the econometrics literature, zero-frequency spectral density estimators are usually labeled HAC estimators following Newey and West (1987) and Andrews (1991) who extended consistency results for estimates of Ω to more general settings.

Here we focus on the Bartlett kernel estimator (Bartlett, 1950; Newey and West, 1987) defined as

$$\hat{\Omega}_{\mathrm{HAC}} = \hat{\Gamma}_0 + \sum_{j=1}^{M} \left(1 - \frac{j}{M}\right)(\hat{\Gamma}_j + \hat{\Gamma}_j'), \tag{9}$$

where $\hat{\Gamma}_j = T^{-1}\sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j}'$ and M is the truncation lag or bandwidth. For $\hat{\Omega}_{\mathrm{HAC}}$ to be consistent, M must increase as the sample increases but at a slower rate. Andrews (1991) showed that the rate $T^{1/3}$ minimizes the approximate MSE for $\hat{\Omega}$. In the Monte Carlo simulations and empirical work that follows, we choose M using the data-dependent method suggested by Andrews (1991) based on the VAR(1) plug-in formula.

We now consider alternatives to using consistent estimates of $\hat{\Omega}$. Following Kiefer et al. (2000) and Kiefer and Vogelsang (2002b), suppose we set $M = T$ in (9). Let $\hat{\Omega}_{M=T}$ denote this estimator of Ω. Although $M = T$ does not result in a consistent estimator of Ω, valid testing is still possible because $\hat{\Omega}_{M=T}$ is asymptotically proportional to Ω as is shown below. The advantage of $\hat{\Omega}_{M=T}$ is that it uses a specific bandwidth and this choice of bandwidth is captured by the first-order asymptotics.

The asymptotic behavior of $\hat{\Omega}_{M=T}$ is as follows. Following Kiefer and Vogelsang (2002a) and using the fact that $\sum_{j=1}^{T} \hat{u}_t = 0$, it holds that

$$\hat{\Omega}_{M=T} = \hat{\Gamma}_0 + \sum_{j=1}^{T-1} \left(1 - \frac{j}{T}\right)(\hat{\Gamma}_j + \hat{\Gamma}_j') = 2T^{-2}\sum_{t=1}^{T} \hat{S}_t \hat{S}_t', \tag{10}$$

where $\hat{S}_t = \sum_{j=1}^{t} \hat{u}_j$. In the appendix we prove that

$$T^{-1/2}\hat{S}_{[rT]} \Rightarrow \Lambda \hat{V}_n(r), \tag{11}$$

where $\hat{V}_n(r) = W_n(r) - rW_n(1) - 12V_n(1)\int_0^r (s - \frac{1}{2})\,\mathrm{d}s$. It directly follows from (10), (11) and the continuous mapping theorem that

$$\hat{\Omega}_{M=T} = 2T^{-1}\sum_{t=1}^{T} T^{-1/2}\hat{S}_t T^{-1/2}\hat{S}_t' \Rightarrow 2\Lambda\left(\int_0^1 \hat{V}_n(r)\hat{V}_n(r)'\,\mathrm{d}r\right)\Lambda'. \tag{12}$$

We also consider an alternative to $\hat{\Omega}_{M=T}$ which is constructed using $\tilde{t}\hat{u}_t$ instead of $\hat{u}_t$. In a standard application of HAC estimators to the regressions given by (1), $\tilde{t}\hat{u}_t$ would be used. Because $\tilde{t}\hat{u}_t$ is not a vector of stationary time series, establishing consistency of a HAC estimator in this case would be difficult if even feasible. However, if we use $M = T$, the asymptotic behavior of the HAC estimator can be easily derived. Specifically, define

$$\tilde{\Omega}_{M=T} = \tilde{\Gamma}_0 + \sum_{j=1}^{T-1} \left(1 - \frac{j}{T}\right)(\tilde{\Gamma}_j + \tilde{\Gamma}_j'), \tag{13}$$

where

$$\tilde{\Gamma}_j = T^{-1}\sum_{t=j+1}^{T} [(t - \bar{t})\hat{u}_t][(t - j - \bar{t})\hat{u}_{t-j}']. \tag{14}$$

Again, using Kiefer and Vogelsang (2002a), we can write

$$\tilde{\Omega}_{M=T} = 2T^{-2}\sum_{t=1}^{T}\tilde{S}_t\tilde{S}_t', \tag{15}$$

where $\tilde{S}_t = \sum_{j=1}^{t}(j-\bar{t})\hat{u}_j$. In the appendix we prove that

$$T^{-3/2}\tilde{S}_{[rT]} \Rightarrow \Lambda\tilde{V}_n(r), \tag{16}$$

where $\tilde{V}_n(r) = V_n(r) - 12V_n(1)\int_0^r (s-\frac{1}{2})^2\,ds$. It directly follows from (15), (16) and the continuous mapping theorem that

$$T^{-2}\tilde{\Omega}_{M=T} = 2T^{-1}\sum_{t=1}^{T}T^{-3/2}\tilde{S}_t T^{-3/2}\tilde{S}_t' \Rightarrow 2\Lambda\left(\int_0^1 \tilde{V}_n(r)\tilde{V}_n(r)'\,dr\right). \tag{17}$$

2.3. Test statistics

The hypotheses of interest in this paper are

$$\mathrm{H}_0 : R\beta = r,$$

$$\mathrm{H}_1 : R\beta \neq r, \tag{18}$$

where R and r are $q\times n$ and $q\times 1$ matrices of known constants. To test the null hypothesis in (18) against the relevant alternative hypothesis, we consider three tests. The first two tests are F-tests which we compute using the Bartlett HAC estimator with bandwidth T. These tests rely either on $\tilde{\Omega}_{M=T}$ or on $\hat{\Omega}_{M=T}$. The first test statistic is

$$F_1^* = T(R\hat{\beta}-r)'\left[R\left(T^{-1}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1}\tilde{\Omega}_{M=T}\left(T^{-1}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1}R'\right]^{-1}(R\hat{\beta}-r)/q,$$

where $\tilde{\Omega}_{M=T}$ is defined in (15). The second F-test we consider is

$$F_2^* = (R\hat{\beta}-r)'\left[R\left(\sum_{t=1}^{T}\tilde{t}^2\right)^{-1}\hat{\Omega}_{M=T}R'\right]^{-1}(R\hat{\beta}-r)/q,$$

where $\hat{\Omega}_{M=T}$ is defined in (10).

Naturally, if there is only a single restriction to test, that is $q=1$, then one can consider a t-test. In the present notation, these tests are

$$t_1^* = \frac{T^{1/2}(R\hat{\beta}-r)}{\left[R\left(T^{-1}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1}\tilde{\Omega}_{M=T}\left(T^{-1}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1}R'\right]^{1/2}}$$

and

$$t_2^* = \frac{(R\hat{\beta}-r)}{\left[R\left(\sum_{t=1}^{T}\tilde{t}^2\right)^{-1}\hat{\Omega}_{M=T}R'\right]^{1/2}},$$

respectively.

The standard alternative to F_1^* and F_2^* is a Wald test based on $\hat{\Omega}_{\mathrm{HAC}}$ defined as

$$W_{\mathrm{HAC}} = (R\hat{\beta} - r)' \left[R \left(\sum_{t=1}^{T} \tilde{t}^2 \right)^{-1} \hat{\Omega}_{\mathrm{HAC}} R' \right]^{-1} (R\hat{\beta} - r).$$

Likewise, we defined the standard HAC t-test as

$$t_{\mathrm{HAC}} = \frac{(R\hat{\beta} - r)}{\left[R \left(\sum_{t=1}^{T} \tilde{t}^2 \right)^{-1} \hat{\Omega}_{\mathrm{HAC}} R' \right]^{1/2}}.$$

3. Asymptotic theory and critical values

In this section we develop the asymptotic theory for the tests. We also compute useful critical values. Finally, we discuss asymptotic power.

3.1. Asymptotic distributions

Assumption (2) is sufficient to obtain the asymptotic null distributions of F_1^* and F_2^*. The asymptotic distributions of these tests are summarized in the following theorem and corollary.

Theorem 1. *Let the data be generated by* (1) *and suppose assumption* (2) *holds. Then, as* $T \to \infty$,

$$F_1^* \Rightarrow V_q(1)' \left[2 \int_0^1 \tilde{V}_q(r) \tilde{V}_q(r)' \, \mathrm{d}r \right]^{-1} V_q(1)/q,$$

$$F_2^* \Rightarrow V_q(1)' \left[\frac{1}{6} \int_0^1 \hat{V}_q(r) \hat{V}_q(r)' \, \mathrm{d}r \right]^{-1} V_q(1)/q.$$

Corollary 1. *Let the data be generated by* (1) *and suppose assumption* (2) *holds. If* $q = 1$, *then as* $T \to \infty$:

$$t_1^* \Rightarrow \frac{V_1(1)}{\left(2 \int_0^1 \tilde{V}_1(r)^2 \, \mathrm{d}r \right)^{1/2}}, \qquad t_2^* \Rightarrow \frac{V_1(1)}{\left(\frac{1}{6} \int_0^1 \hat{V}_1(r)^2 \, \mathrm{d}r \right)^{1/2}}.$$

Note that F_1^* and F_2^* are quadratic forms involving the normal random vector $V_q(1)$ and random matrices that reflect the use of $\tilde{\Omega}_{M=T}$ and $\hat{\Omega}_{M=T}$.

Finally, if $\hat{\Omega}_{\mathrm{HAC}}$ is a consistent estimator of Ω, then the asymptotic distribution of the Wald test W_{HAC} is χ^2 with q degrees of freedom. When $q = 1$, t-tests based on $\hat{\Omega}_{\mathrm{HAC}}$ are asymptotically distributed as $\mathrm{N}(0, 1)$.

Table 1
Asymptotic critical values for t_1^* and t_2^*

	0.90	0.95	0.975	0.99
t_1^*	3.315	4.566	5.820	7.416
t_2^*	3.898	5.222	6.482	8.100

Left tail critical values follow from symmetry around zero.

3.2. Critical values

The critical values for the t_1^*, t_2^*, F_1^* and F_2^* can be obtained through Monte Carlo simulation. The asymptotic critical values were simulated using 50,000 replications. The Wiener processes were approximated by normalized sums of i.i.d. N(0, 1) errors using 1000 steps. The critical values for the t_1^* and t_2^* tests are given in Table 1. Right tail critical values are given. The left tail critical values follow from symmetry around zero. The critical values for the F_1^* and F_1^* tests are given in Table 2, where we tabulate the critical values for tests for q restrictions, where q runs from 1 to 30.

3.3. Asymptotic power

One way to compare and contrast the new tests with each other and with standard HAC-based tests is to examine asymptotic power for local alternatives. To keep the analysis transparent, we consider the case of $q=1$ and focus on the regression

$$y_{1,t} = \mu_1 + \beta_1 t + u_{1,t}. \tag{19}$$

Let $\sigma_1^2 = \gamma_0 + \sum_{j=-\infty}^{\infty} \gamma_j$, where $\gamma_j = \mathrm{Cov}(u_{1,t}, u_{1,t-j})$. Then, under assumption (2),

$$T^{-1/2} \sum_{t=1}^{[rT]} \Rightarrow \sigma_1 W_1(r).$$

Consider the one-sided hypothesis $\mathrm{H}_0 : \beta_1 \leqslant \beta_0$ against $\mathrm{H}_1 : \beta_1 > \beta_0$. We can obtain nondegenerate limiting distributions for the t-tests under the local alternative $\beta_1 = \beta_0 + cT^{-3/2}$. Thus, β_1 converges to β_0 at rate $T^{-3/2}$.

The following theorem gives the limiting distribution of the tests under the local alternative. The proof is given in the appendix.

Theorem 2. *Suppose the data are generated by* (19) *and that assumption* (2) *holds. Suppose t-tests are constructed for testing* $\mathrm{H}_0 : \beta_1 \leqslant \beta_0$ *against* $\mathrm{H}_1 : \beta_1 > \beta_0$. *Suppose* $\beta_1 = \beta_0 + cT^{-3/2}$, *then, as* $T \to \infty$:

$$t_{\mathrm{HAC}} \Rightarrow \frac{\delta + 12V_1(1)}{\sqrt{12}}, \quad t_1^* \Rightarrow \frac{\delta + 12V_1(1)}{(288 \int_0^1 \tilde{V}_1^2(r)\,\mathrm{d}r)^{1/2}}, \quad t_2^* \Rightarrow \frac{\delta + 12V_1(1)}{(24 \int_0^1 \hat{V}_1^2(r)\,\mathrm{d}r)^{1/2}},$$

where $\delta = c/\sigma_1$.

Table 2
Asymptotic critical values of F_1^* and F_2^* for q restrictions

F_1^*					F_2^*				
q	0.90	0.95	0.975	0.99	q	0.90	0.95	0.975	0.99
1	20.81	33.63	48.42	72.23	1	20.14	41.53	58.57	83.96
2	26.27	38.10	51.08	71.04	2	28.90	40.68	53.58	73.50
3	30.97	42.38	54.66	73.40	3	30.95	41.45	52.86	68.67
4	34.90	46.75	59.35	76.75	4	33.26	43.84	54.60	69.30
5	38.63	49.82	61.88	78.29	5	35.51	45.43	55.86	70.14
6	42.76	54.68	67.53	83.98	6	38.26	48.39	58.91	73.36
7	47.29	59.32	71.77	88.54	7	41.22	51.35	61.62	75.87
8	50.74	62.87	74.33	90.29	8	43.50	53.25	63.18	76.71
9	54.63	67.17	80.14	95.76	9	46.36	56.86	67.24	80.55
10	58.26	70.99	83.31	100.1	10	49.05	58.90	68.92	93.06
11	61.82	74.51	87.45	103.3	11	51.56	62.08	71.92	85.85
12	66.02	79.17	92.49	109.2	12	54.54	65.01	74.95	88.65
13	69.26	82.45	95.61	113.1	13	56.49	67.07	77.41	92.66
14	72.73	86.02	98.94	115.0	14	59.43	69.98	80.32	93.61
15	75.98	88.70	102.5	119.8	15	61.65	72.32	82.74	97.02
16	79.34	93.04	106.7	124.1	16	64.33	74.83	85.63	99.79
17	82.98	96.55	110.4	127.1	17	66.95	77.89	88.15	102.1
18	86.52	101.3	114.9	133.0	18	69.69	80.75	91.93	106.0
19	90.36	105.5	119.6	138.4	19	72.52	84.24	95.18	109.2
20	93.16	108.1	122.3	141.3	20	74.35	86.17	97.20	112.2
21	96.39	111.1	125.1	144.3	21	76.74	87.99	99.32	113.2
22	99.96	114.6	129.7	147.4	22	79.45	91.12	102.6	116.1
23	103.1	117.9	133.2	150.9	23	81.77	93.27	104.6	118.3
24	107.3	122.3	137.3	156.8	24	84.53	96.56	108.5	123.7
25	110.4	125.9	140.9	160.7	25	86.94	99.07	110.8	125.3
26	114.1	129.4	144.1	163.6	26	89.57	102.0	113.7	128.6
27	117.4	133.5	148.8	167.9	27	91.88	104.5	116.5	131.6
28	120.3	136.3	151.5	171.6	28	94.40	106.8	119.2	134.2
29	123.6	139.6	155.7	175.5	29	96.89	109.4	121.3	136.7
30	126.7	143.5	158.9	179.2	30	99.41	111.8	124.2	140.0

Using Theorem 2, asymptotic power can be computed by simulating the distributions under the local alternative for various values of δ and computing rejection probabilities using the asymptotic null critical values. Using the same simulation methods used to compute null critical values, we computed asymptotic power for $\delta \in [0, 20]$. Power with a nominal level of 5% is plotted in Fig. 1. Clearly, power is highest for t_{HAC}, followed by t_2^* and then t_1^*. The reason that t_1^* and t_2^* have slightly lower power is because they use "standard errors" with sampling variability that does not vanish as T increases. However, as we will show in the next section, t_{HAC} is generally more size distorted in finite samples. Thus, t_1^* and t_2^* trade off power in exchange for better size.

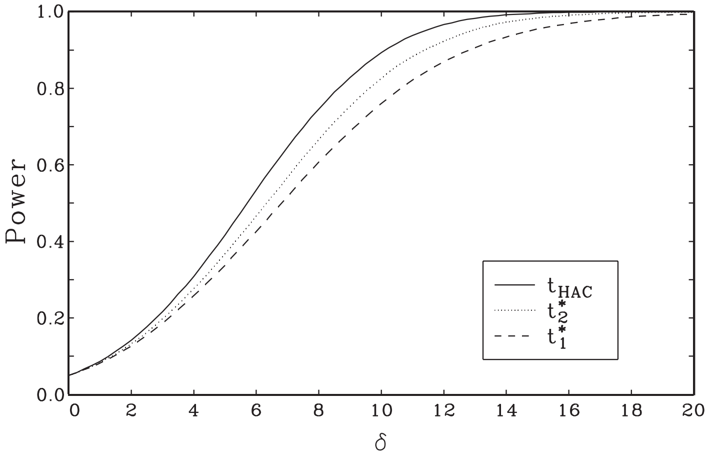

Fig. 1. Asymptotic power of $t_{\mathrm{HAC}}, t_1^*, t_2^*$; labeled in descending order.

4. Finite sample performance

In this section we examine the finite sample performance of the F_1^* and F_2^* tests and compare them to W_{HAC}. We generate data according to (1) with $n=6$ (to match the empirical application) where the errors are modeled as a VAR(2) process, that is

$$u_t = A_1 u_{t-1} + A_2 u_{t-2} + \varepsilon_t$$

with ε_t a Gaussian multivariate white noise process with $\mathrm{E}(\varepsilon_t \varepsilon_t') = \Sigma$ and $\varepsilon_0 = \mathbf{0}$. For one set of simulations, we focus on the following VAR(1) specification where $A_1 = \rho I_6$, $A_2 = \mathbf{0}$, $\Sigma = (\sqrt{1-\rho^2}) I_6$, I_6 is a 6×6 identity matrix and ρ is a scalar. Under this specification, the six time series all follow the same AR(1) process and the series are uncorrelated with each other. The variance–covariance matrix, Σ, is normalized so that the variance of each series is equal to one for all values of $|\rho| < 1$. We consider the following null hypotheses:

$$\mathrm{H}_0^1 : \beta_1 = 0,$$

$$\mathrm{H}_0^2 : \beta_1 = \beta_2 = 0,$$

$$\mathrm{H}_0^3 : \beta_1 = \beta_2 = \beta_3 = 0,$$

$$\mathrm{H}_0^4 : \beta_1 = \beta_2 = \cdots = \beta_6 = 0,$$

where $q = 1, 2, 3, 6$, respectively. To explore the finite sample size of the tests, we generate data under these null hypotheses and we set the intercepts and other slopes to zero as the three tests are exactly invariant to those parameters. To compare power

Table 3
Empirical null rejection probabilities and size-corrected power of the three tests, without and with prewhitening. Testing for a single zero restriction on β ($q=1$) ($T=100$; 10,000 replications were used)

ρ	β	No prewhitening			VAR(1) prewhitening			VAR(2) prewhitening		
		F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}
0.0	0.000	0.047	0.050	0.062	0.042	0.043	0.062	0.041	0.039	0.078
	0.005	0.233	0.252	0.295	0.229	0.251	0.289	0.229	0.251	0.274
	0.010	0.640	0.713	0.809	0.642	0.717	0.800	0.637	0.710	0.780
	0.020	0.978	0.996	1.000	0.980	0.996	1.000	0.979	0.997	1.000
0.2	0.000	0.052	0.056	0.088	0.041	0.042	0.068	0.041	0.040	0.082
	0.005	0.172	0.186	0.205	0.168	0.184	0.202	0.166	0.184	0.196
	0.010	0.488	0.548	0.630	0.490	0.553	0.621	0.491	0.546	0.603
	0.020	0.927	0.972	0.997	0.934	0.975	0.996	0.930	0.973	0.992
0.4	0.000	0.058	0.063	0.112	0.042	0.043	0.074	0.042	0.041	0.087
	0.005	0.127	0.133	0.147	0.126	0.136	0.144	0.125	0.131	0.143
	0.010	0.345	0.389	0.446	0.352	0.393	0.442	0.349	0.383	0.428
	0.020	0.812	0.894	0.951	0.827	0.902	0.948	0.823	0.891	0.940
0.6	0.000	0.067	0.073	0.141	0.044	0.044	0.084	0.045	0.044	0.098
	0.005	0.094	0.099	0.104	0.096	0.100	0.105	0.095	0.097	0.105
	0.010	0.231	0.250	0.279	0.232	0.254	0.283	0.230	0.250	0.275
	0.020	0.625	0.707	0.789	0.637	0.718	0.780	0.624	0.704	0.771
0.8	0.000	0.091	0.101	0.199	0.048	0.045	0.112	0.053	0.049	0.125
	0.005	0.071	0.071	0.076	0.070	0.075	0.075	0.070	0.073	0.073
	0.010	0.140	0.146	0.160	0.136	0.153	0.155	0.132	0.146	0.149
	0.020	0.375	0.419	0.472	0.372	0.437	0.452	0.361	0.418	0.438

The cells contain the finite sample empirical rejection probabilities (at a 5% significance level), where the power is corrected for the empirical size. All series have AR(1) errors with parameter ρ. The Wald test is implemented using a Bartlett kernel with Andrews VAR(1) data-dependent bandwith. The critical values for the F_1^* and F_2^* tests can be found in Table 2 for $q=1$, respectively.

of the tests, we generate data under the alternatives

$$\mathrm{H}_1^1 : \beta_1 = \beta,$$

$$\mathrm{H}_1^2 : \beta_1 = \beta_2 = \beta,$$

$$\mathrm{H}_1^3 : \beta_1 = \beta_2 = \beta_3 = \beta,$$

$$\mathrm{H}_1^4 : \beta_1 = \beta_2 = \cdots = \beta_6 = \beta,$$

respectively. The results are summarized in Tables 3–6. We use $T=100$ and 10,000 replications in all cases. We only report results for $\rho = 0, 0.2, 0.4, 0.6, 0.8$ and $\beta = 0, 0.05, 0.10, 0.20$. We implement the tests without prewhitening and with VAR(1) and VAR(2) prewhitening (see Andrews and Monahan, 1992). Empirical rejection probabilities under the null hypotheses were computed using 5% asymptotic critical values.

Table 4
Empirical null rejection probabilities and size-corrected power of the three tests, without and with prewhitening. Testing for two zero restrictions on β ($q = 2$) ($T = 100$; 10,000 replications were used)

ρ	β	No prewhitening			VAR(1) prewhitening			VAR(2) prewhitening		
		F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}
0.0	0.000	0.052	0.051	0.067	0.042	0.039	0.078	0.041	0.040	0.100
	0.005	0.280	0.317	0.409	0.283	0.316	0.398	0.271	0.304	0.375
	0.010	0.752	0.839	0.953	0.755	0.840	0.945	0.738	0.827	0.928
	0.020	0.992	1.000	1.000	0.993	1.000	1.000	0.991	0.999	1.000
0.2	0.000	0.060	0.061	0.115	0.043	0.042	0.087	0.045	0.043	0.107
	0.005	0.199	0.220	0.282	0.204	0.220	0.273	0.198	0.213	0.260
	0.010	0.598	0.672	0.823	0.601	0.676	0.805	0.581	0.655	0.785
	0.020	0.964	0.995	1.000	0.970	0.996	1.000	0.967	0.994	1.000
0.4	0.000	0.072	0.073	0.155	0.047	0.044	0.099	0.048	0.046	0.121
	0.005	0.145	0.155	0.190	0.143	0.157	0.187	0.145	0.150	0.178
	0.010	0.426	0.486	0.615	0.430	0.490	0.602	0.413	0.471	0.574
	0.020	0.892	0.959	0.997	0.903	0.966	0.997	0.888	0.957	0.994
0.6	0.000	0.093	0.093	0.214	0.052	0.047	0.122	0.059	0.052	0.148
	0.005	0.103	0.109	0.128	0.104	0.111	0.127	0.103	0.104	0.121
	0.010	0.277	0.309	0.378	0.279	0.316	0.374	0.272	0.300	0.352
	0.020	0.729	0.814	0.927	0.734	0.829	0.921	0.711	0.807	0.901
0.8	0.000	0.147	0.148	0.338	0.071	0.060	0.187	0.081	0.069	0.219
	0.005	0.079	0.077	0.081	0.074	0.082	0.082	0.076	0.080	0.079
	0.010	0.162	0.167	0.188	0.165	0.179	0.186	0.161	0.169	0.182
	0.020	0.460	0.504	0.585	0.456	0.526	0.578	0.442	0.502	0.557

The cells contain the finite sample empirical rejection probabilities (at a 5% significance level), where the power is corrected for the empirical size. All series have AR(1) errors with parameter ρ. The Wald test is implemented using a Bartlett kernel with Andrews VAR(1) data-dependent bandwith. The critical values for the F_1^* and F_2^* tests can be found in Table 2 for $q = 2$, respectively.

Empirical rejection probabilities under the alternatives were computed using 5% empirical finite sample critical values (obtained from the null distributions). Thus, finite sample power is size corrected so that power comparisons are meaningful.

In Table 3 we give the results for H_0^1. Several patterns emerge. First, in nearly all cases, empirical null rejection probabilities of F_1^* and F_2^* are closer to 0.05 than W_{HAC}. This is especially true as ρ increases. Second, prewhitening usually improves the size of the tests when ρ is not close to zero. Note however, that for W_{HAC} using VAR(2) prewhitening leads to more size distortions than VAR(1) prewhitening. Thus, size distortions of W_{HAC} are least when the prewhitening filter matches the DGP. For the F^* statistics, increasing the prewhitening lag length from one to two lags does not lead to an increase in size distortions and only a minimal reduction in power. Third, and as expected given finite sample results in Kiefer et al. (2000) and Kiefer and Vogelsang (2002b), the size-corrected power of W_{HAC} is higher than F_1^* and F_2^*.

Table 5
Empirical null rejection probabilities and size-corrected power of the three tests, without and with prewhitening. Testing for three zero restrictions on β ($q = 3$) ($T = 100$; 10,000 replications were used)

ρ	β	No prewhitening			VAR(1) prewhitening			VAR(2) prewhitening		
		F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}
0.0	0.000	0.055	0.054	0.070	0.044	0.043	0.091	0.050	0.044	0.128
	0.005	0.337	0.375	0.507	0.330	0.368	0.488	0.312	0.357	0.451
	0.010	0.846	0.909	0.991	0.844	0.907	0.984	0.810	0.891	0.976
	0.020	0.997	1.000	1.000	0.997	1.000	1.000	0.996	1.000	1.000
0.2	0.000	0.070	0.067	0.130	0.049	0.046	0.102	0.056	0.048	0.139
	0.005	0.234	0.255	0.344	0.230	0.254	0.333	0.217	0.251	0.305
	0.010	0.693	0.762	0.920	0.689	0.763	0.907	0.653	0.746	0.877
	0.020	0.985	0.998	1.000	0.984	0.999	1.000	0.981	0.998	1.000
0.4	0.000	0.088	0.085	0.192	0.055	0.050	0.121	0.063	0.053	0.161
	0.005	0.163	0.176	0.218	0.160	0.175	0.215	0.157	0.170	0.203
	0.010	0.512	0.569	0.719	0.498	0.574	0.709	0.482	0.550	0.674
	0.020	0.941	0.979	1.000	0.943	0.984	1.000	0.928	0.979	0.999
0.6	0.000	0.122	0.120	0.285	0.067	0.058	0.162	0.080	0.066	0.210
	0.005	0.117	0.123	0.134	0.111	0.121	0.136	0.109	0.119	0.133
	0.010	0.331	0.362	0.435	0.312	0.362	0.445	0.306	0.347	0.423
	0.020	0.816	0.882	0.968	0.802	0.892	0.966	0.777	0.870	0.952
0.8	0.000	0.212	0.213	0.479	0.110	0.090	0.276	0.138	0.108	0.328
	0.005	0.083	0.088	0.091	0.081	0.090	0.088	0.079	0.087	0.089
	0.010	0.181	0.196	0.209	0.181	0.204	0.214	0.176	0.195	0.211
	0.020	0.540	0.583	0.648	0.524	0.599	0.666	0.501	0.569	0.645

The cells contain the finite sample empirical rejection probabilities (at a 5% significance level), where the power is corrected for the empirical size. All series have AR(1) errors with parameter ρ. The Wald test is implemented using a Bartlett kernel with Andrews VAR(1) data-dependent bandwith. The critical values for the F_1^* and F_2^* tests can be found in Table 2 for $q = 3$, respectively.

This higher power comes at the expense of greater size distortion. Fourth, the size of F_1^* and F_2^* are very similar while F_2^* clearly has higher power. This suggests that F_2^* would be preferred over F_1^* in practice.

The dominance of F_2^* over F_1^* is continued in Tables 4–6. F_2^* almost always has higher power than F_1^* and when prewhitening is used, F_2^* tends to be less size distorted.

Table 5 shows that while F_1^* and F_2^* have reasonable size, especially for $\rho \leqslant 0.6$ when prewhitening is used, W_{HAC} tends to over-reject even for ρ close to zero. All the tests have over-rejection problems for large q as Table 6 shows. W_{HAC} is particularly bad even when the data has weak serial correlation. F_1^* and F_2^* are less distorted, but still over-reject quite substantially when ρ is large. VAR(1) prewhitening usually improves things, but VAR(2) prewhitening often leads to more size distortions. A telling result from this table is that F_2^* clearly dominates F_1^* when $q = 6$. F_2^* is less size distorted than F_1^* yet F_2^* has greater size-adjusted power.

Table 6
Empirical null rejection probabilities and size-corrected power of the three tests, without and with prewhitening. Testing for six zero restrictions on β ($q=6$) ($T=100$; 10,000 replications were used)

ρ	β	No prewhitening			VAR(1) prewhitening			VAR(2) prewhitening		
		F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}
0.0	0.000	0.054	0.053	0.099	0.055	0.044	0.151	0.089	0.057	0.246
	0.005	0.498	0.526	0.716	0.458	0.511	0.662	0.421	0.476	0.604
	0.010	0.962	0.978	1.000	0.948	0.976	1.000	0.915	0.963	0.997
	0.020	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
0.2	0.000	0.082	0.076	0.207	0.067	0.051	0.180	0.110	0.070	0.281
	0.005	0.333	0.354	0.482	0.309	0.347	0.453	0.289	0.322	0.411
	0.010	0.871	0.909	0.989	0.839	0.902	0.983	0.792	0.870	0.968
	0.020	0.998	1.000	1.000	0.998	1.000	1.000	0.995	0.999	1.000
0.4	0.000	0.128	0.117	0.344	0.089	0.063	0.236	0.147	0.089	0.342
	0.005	0.218	0.231	0.283	0.201	0.230	0.281	0.190	0.210	0.258
	0.010	0.694	0.740	0.874	0.651	0.729	0.871	0.603	0.681	0.822
	0.020	0.990	0.996	1.000	0.986	0.997	1.000	0.973	0.992	1.000
0.6	0.000	0.224	0.197	0.557	0.143	0.099	0.340	0.232	0.140	0.463
	0.005	0.142	0.146	0.157	0.133	0.149	0.163	0.128	0.135	0.151
	0.010	0.456	0.484	0.547	0.430	0.484	0.585	0.393	0.440	0.535
	0.020	0.928	0.956	0.991	0.913	0.958	0.994	0.877	0.931	0.986
0.8	0.000	0.462	0.439	0.867	0.331	0.227	0.603	0.466	0.320	0.709
	0.005	0.091	0.094	0.088	0.090	0.095	0.100	0.087	0.096	0.094
	0.010	0.240	0.249	0.222	0.231	0.255	0.283	0.217	0.242	0.262
	0.020	0.701	0.727	0.680	0.663	0.723	0.813	0.621	0.681	0.767

The cells contain the finite sample empirical rejection probabilities (at a 5% significance level), where the power is corrected for the empirical size. All series have AR(1) errors with parameter ρ. The Wald test is implemented using a Bartlett kernel with Andrews VAR(1) data-dependent bandwith. The critical values for the F_1^* and F_2^* tests can be found in Table 2 for $q=6$, respectively.

Because the simulation design in Tables 3–6 is simplistic, we also report simulations results using some more empirically relevant specifications. Because there are a very large number of parameters in a six dimensional VAR we used parameter values based on fitting VAR(1) and VAR(2) models to the six relative GDP series (in logarithms) analyzed in the next section. We first detrended the series and then fit the VAR models using OLS. The Σ matrix was estimated using the sample variance and covariances of the OLS residuals. To illustrate how contemporaneous correlation across the series affects the tests, we also report results for two white noise error ($A_1=A_2=\mathbf{0}$) specifications. WN(1) has $\Sigma=0.0005I_6$ and WN(2) has $\Sigma=\hat{\Sigma}$ where $\hat{\Sigma}$ is computed using the sample variances and covariances from the detrended time series. The point estimates of the WN(2), VAR(1) and VAR(2) models are available upon request.

We report results for the null hypothesis that all six slopes are the same:

$$\mathrm{H}_0:\beta_1=\beta_2=\cdots=\beta_6$$

Table 7
Empirical null rejection probabilities and size-corrected power of the three tests, without and with prewhitening. Test that all of six slopes are equal ($q = 5$) ($T = 50$; 10,000 replications were used)

DGP	β	No prewhitening			VAR(1) prewhitening			VAR(2) prewhitening		
		F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}
WN(1)	0.00000	0.062	0.057	0.140	0.091	0.058	0.238	0.220	0.101	0.424
	0.00025	0.081	0.086	0.091	0.077	0.081	0.085	0.075	0.078	0.080
	0.00050	0.196	0.215	0.264	0.169	0.191	0.225	0.154	0.174	0.202
	0.00075	0.412	0.441	0.576	0.343	0.399	0.492	0.298	0.349	0.421
	0.00100	0.642	0.700	0.852	0.553	0.634	0.770	0.473	0.563	0.679
WN(2)	0.00000	0.061	0.060	0.135	0.090	0.056	0.237	0.219	0.101	0.427
	0.00025	0.101	0.101	0.110	0.091	0.098	0.098	0.083	0.088	0.100
	0.00050	0.274	0.288	0.369	0.239	0.266	0.314	0.205	0.230	0.283
	0.00075	0.557	0.595	0.744	0.475	0.547	0.656	0.402	0.465	0.578
	0.00100	0.798	0.844	0.955	0.706	0.794	0.907	0.607	0.710	0.828
VAR(1)	0.00000	0.211	0.195	0.623	0.299	0.195	0.557	0.506	0.321	0.718
	0.00025	0.065	0.064	0.067	0.061	0.069	0.063	0.062	0.065	0.062
	0.00050	0.118	0.126	0.129	0.109	0.127	0.123	0.105	0.119	0.112
	0.00075	0.234	0.248	0.250	0.203	0.252	0.248	0.189	0.219	0.219
	0.00100	0.404	0.432	0.424	0.353	0.424	0.451	0.316	0.355	0.387
VAR(2)	0.00000	0.124	0.109	0.447	0.172	0.092	0.364	0.424	0.243	0.650
	0.00025	0.088	0.092	0.090	0.078	0.083	0.077	0.096	0.094	0.094
	0.00050	0.257	0.270	0.257	0.202	0.229	0.196	0.242	0.256	0.280
	0.00075	0.554	0.592	0.572	0.427	0.490	0.446	0.482	0.527	0.619
	0.00100	0.801	0.861	0.856	0.665	0.762	0.750	0.700	0.765	0.885

The cells contain the finite sample empirical rejection probabilities (at a 5% significance level), where the power is corrected for the empirical size. The data is generated according to either the WN(1), WN(2), VAR(1) or VAR(2) models with parameters based on estimates from the six relative GDP series in the empirical application. The Wald test is implemented using a Bartlett kernel with Andrews VAR(1) data-dependent bandwith. The critical values for the F_1^* and F_2^* tests can be found in Table 2 $q = 5$, respectively.

and we consider the alternative hypothesis

$$\mathrm{H}_1 : \beta_1 = \beta, \;\; \beta_2 = \beta_3 = \cdots = \beta_6 = 0.$$

We report results for $\beta = 0.0, 0.00025, 0.0005, 0.00075, 0.0001$ and $T = 50$ in Table 7. In Table 8 we report results for $T = 200$ for $\beta = 0.0$. The general patterns in these tables are similar to the patterns in Tables 3–6. F_2^* has the least size distortions while W_{HAC} can severely over-reject under the null especially for the VAR(1) and VAR(2) data generating models. VAR(1) pre-whitening delivers slight improvements for F_2^* but magnifies distortions for F_1^* and W_{HAC}. VAR(2) prewhitening generally causes all three tests to over-reject much more compared to not using prewhitening. This even holds when the true data is generated according to a VAR(2) model. These results suggest that when the sample size is small relative to the number of restrictions being tested, over-rejections can be a problem with moderately persistent data and prewhitening

Table 8
Empirical null rejection probabilities of the three tests, without and with prewhitening. Test that all of six slopes are equal ($q = 5$) ($T = 200$; 10,000 replications were used)

DGP	β	No prewhitening			VAR(1) prewhitening			VAR(2) prewhitening		
		F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}	F_1^*	F_2^*	W_{HAC}
WN(1)	0.0	0.050	0.052	0.068	0.041	0.041	0.083	0.044	0.038	0.113
WN(2)	0.0	0.054	0.053	0.065	0.041	0.041	0.082	0.045	0.041	0.111
VAR(1)	0.0	0.068	0.061	0.224	0.060	0.046	0.155	0.075	0.048	0.193
VAR(2)	0.0	0.042	0.038	0.154	0.031	0.021	0.074	0.064	0.039	0.164

The cells contain the finite sample empirical rejection probabilities (at a 5% significance level). The data is generated according to either the WN(1), WN(2), VAR(1) or VAR(2) models with parameters based on estimates from the six relative GDP series in the empirical application. The Wald test is implemented using a Bartlett kernel with Andrews VAR(1) data-dependent bandwith. The critical values for the F_1^* and F_2^* tests can be found in Table 2 for $q = 5$, respectively.

may not improve matters. In some sense, these problems are small sample problems as Table 8 illustrates. With $T = 200$, null rejection probabilities are close to 0.05 for F_1^* and F_2^*. W_{HAC} still tends to over-reject but much less so compare to when $T = 50$. Interestingly, VAR(2) prewhitening continues to do worse than VAR(1) prewhitening when the true data follows a VAR(2).

Finally, comparing the results for the two white noise specifications, we see that null rejection probabilities are nearly identical in the two cases. This suggests that contemporaneous correlation across series has little effect on the performance of the tests. The strength of the serial correlation in the series is the much more relevant factor.

5. European economic convergence: Italy

In this section we apply our tests to examine possible convergence across European GDP. There are many ways to study convergence, but it seems that a useful approach is to see if the log ratios of real per capita GDP are trend-stationary. If there is a significant positive trend, then the numerator country grows faster than the benchmark country. When there is no trend, then convergence has happened already.

The data are taken from the Penn World Tables, and concern annual data ranging from 1950 to 1992. Initially, we considered all European countries, but it turns out that often tests for unit roots do not give a clear picture, that is, are not decisive as to whether individual series have a unit root while their log ratios are trend-stationary. Indeed, these two results are mandatory for a proper application and interpretation of our own tests. Eventually, we arrive at a selection of seven countries, Austria, Denmark, France, Italy, the Netherlands, Sweden and Germany, which all have log real GDP series which have a unit root, while the log ratios of those six countries to the benchmark Italy (with Italy in the numerator) appear to be trend-stationary. This finding is summarized in Table 9, where we report unit root test results using the GLS

Table 9
Unit root tests for European log GDP data, 1950–1992. The test regression includes a constant and a trend. The tests are the ADF-GLS t-tests with $\bar{c} = -13.5$

Variable	k	kmax	t-test
Austria	5	5	−0.93
Denmark	0	5	−1.04
France	1	5	−0.42
Italy	0	5	−0.90
Netherlands	2	5	−1.29
Sweden	5	5	−1.04
Germany	3	5	−1.48
Austria–Italy	1	5	−3.73[a]
Denmark–Italy	0	5	−4.06[a]
France–Italy	1	5	−3.66[a]
Netherlands–Italy	1	5	−3.54[a]
Sweden–Italy	0	5	−3.04[b]
Germany–Italy	1	5	−3.79[a]

The lag length was chosen using the general to specific data dependent method used by Perron (1989), Perron (1990) and Perron and Vogelsang (1992) where the coefficient on last lagged first difference in the ADF regression is tested using a two-sided 10% t-test based on asymptotic $N(0,1)$ critical values.

[a]Significant at the 0.01 level.

[b]Significant at the 0.05 level.

detrended version of the familiar Dickey Fuller test following Elliott et al. (1996). In Fig. 2, we display the seven real per capita GDP series, where we observe that Italy has the lowest real per capita GDP in 1950, and hence it would be interesting to test whether Italy has been catching up in the subsequent four decades.

In Table 10, we report on the individual trend slope OLS estimates for the log ratio series (Italy in the numerator). The third column gives the sum of the estimated autoregressive parameters that we used to carry out the unit root tests, and it is clear that there is mild but relevant autocorrelation in these trend-stationary series. This provides relevance for the use of our pre-whitened tests. The next six columns give the t-tests for the trend parameters, and it is clear that all tests point towards significantly positive trend parameters. This result is reinforced by the joint test results in Table 11.

It might be that all trend parameters are positive, and hence that there is convergence occurring between Italy and other European countries, but that the degree of convergence is common to the six countries under scrutiny. This can be checked by testing if the trend parameters are the same across the six log-ratio series. The test results in Table 12 convincingly indicate that this is not the case. To see if there are perhaps pairs or groups of countries to which Italy is converging with the same speed, we consider pairwise tests for the equality of trend parameters. The results in Tables 13–15 show that common rates of convergence is mostly rejected, with the exception that France/Germany and Denmark/Netherlands respectively are converging to Italy at the same rate.

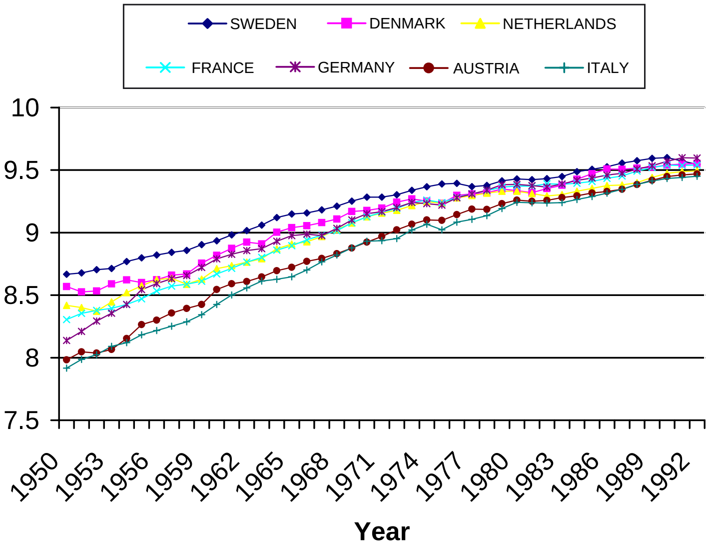

Fig. 2. Logarithm of real per capita GDP.

Table 10
Single equation estimation results for log ratios of DGP all relative to Italy with Italy in the numerator

Month	$\hat{\beta}$	$\hat{\alpha}$	No prewhitening			VAR(1) prewhitening		
			t_1^*	t_2^*	t_{HAC}	t_1^*	t_2^*	t_{HAC}
Austria	0.0013	0.531	4.830[b]	6.637[b]	2.493[a]	4.564[b]	3.704	2.375[a]
Denmark	0.0108	0.579	12.948[a]	22.795[a]	12.085[a]	16.102[a]	15.219[a]	13.299[a]
France	0.0059	0.584	20.757[a]	25.931[a]	11.276[a]	11.011[a]	8.496[a]	9.452[a]
Netherlands	0.0095	0.617	22.452[a]	29.847[a]	12.279[a]	16.197[a]	11.703[a]	13.151[a]
Sweden	0.0139	0.699	18.176[a]	30.674[a]	17.113[a]	26.336[a]	22.010[a]	18.503[a]
Germany	0.0054	0.664	9.329[a]	12.989[a]	5.586[a]	5.292[c]	5.049[c]	5.628[a]

$\hat{\alpha}$ is the sum of autoregressive parameters taken from the unit root tests reported in Table 9.
[a]Significant at the 0.01 level.
[b]Significant at the 0.05 level.
[c]Significant at the 0.1 level.

6. Conclusions and direction for future research

In this paper, we develop tests that can be used to test general linear hypotheses regarding the linear trend slope parameters of a vector of trend stationary time series. As an alternative to traditional Wald tests based on HAC robust standard errors, we recommend the use of a F-tests based on the approach of Kiefer et al. (2000) and Kiefer and Vogelsang (2002a). One of the new tests has much better finite sample

Table 11
Joint tests for the restriction that all GDP series relative to Italy have slope equal to zero

Test statistic	No prewhitening	VAR(1) prewhitening
F_1^*	925.51[a]	1911.25[a]
F_2^*	801.89[a]	1066.01[a]
W_{HAC}	907.07[a]	1001.86[a]

The critical values for the F_1^* and F_2^* tests can be found in Table 2 for $q=6$, respectively.
[a]Significant at the 0.01 level.

Table 12
Joint tests for the restriction that all GDP series relative to Italy have the same slope

Test statistic	No prewhitening	VAR(1) prewhitening
F_1^*	242.92[a]	1737.63[a]
F_2^*	385.72[a]	794.60[a]
W_{HAC}	330.47[a]	749.62[a]

The critical values for the F_1^* and F_2^* tests can be found in Table 2 for $q=5$, respectively.
[a]Significant at the 0.01 level.

Table 13
Pairwise two-sided t_1^* tests for equality of trend parameters for six log GDP series relative to Italy. Below (above) the diagonal are the results without (with) VAR(1) prewhitening

Month	Austria	Denmark	France	Netherlands	Sweden	Germany
Austria	n.a.	−8.950[a]	−8.858[a]	−12.710[a]	−12.802[a]	−10.569[a]
Denmark	−11.757[a]	n.a.	7.327[b]	2.257	−19.868[a]	4.317[c]
France	−5.793[b]	6.987[b]	n.a.	−19.132[a]	−13.150[a]	0.592
Netherlands	−9.881[a]	2.013	−24.344[a]	n.a.	−8.704[a]	4.466[c]
Sweden	−19.777[a]	−15.851[a]	−11.836[a]	−6.925[b]	n.a.	7.324[b]
Germany	−5.393[c]	3.795[c]	0.321	2.609	6.805[b]	n.a.

The critical values for the t_1^* and test can be found in Table 1.
[a]Significant at the 0.01 level.
[b]Significant at the 0.05 level.
[c]Significant at the 0.10 level.

size with comparable but slightly lower power than the traditional HAC robust Wald test.

We applied our tests to 43 years of annual data on log real per capita GDP for seven European countries. Relative to Italy, we find that log ratios (with Italy in the numerator) are all trend-stationary, where the trend is positive. Hence, Italy has been catching up with at least six European countries, and it also seems that France/Germany and Denmark/Netherlands have converged respectively to Italy at the same rate. It should be stressed here that we do not see our exercise as the most decisive study on European growth, although all of our tests do show coherence in their implications. Our tests are only relevant to trend-stationary series, and hence our decision to consider

Table 14
Pairwise two-sided t_2^* tests for equality of trend parameters for six log GDP series relative to Italy. Below (above) the diagonal are the results without (with) VAR(1) prewhitening

Month	Austria	Denmark	France	Netherlands	Sweden	Germany
Austria	n.a.	-15.640^a	-13.060^a	-19.251^a	-22.030^a	-13.255^a
Denmark	-9.564^a	n.a.	10.646^a	2.772	-24.121^a	7.584^b
France	-4.556^c	8.415^a	n.a.	-27.815^a	-18.430^a	0.816
Netherlands	-7.335^b	2.125	-22.383^a	n.a.	-10.269^a	6.265^b
Sweden	-13.845^a	-24.122^a	-13.551^a	-7.045^b	n.a.	12.602^a
Germany	-5.403^c	3.308	0.283	2.216	5.532^b	n.a.

The critical values for the t_2^* and test can be found in Table 1.
[a]Significant at the 0.01 level.
[b]Significant at the 0.05 level.
[c]Significant at the 0.10 level.

Table 15
Pairwise two-sided t^*_{HAC} tests for equality of trend parameters for six log GDP series relative to Italy. Below (above) the diagonal are the results without (with) VAR(1) prewhitening

Month	Austria	Denmark	France	Netherlands	Sweden	Germany
Austria	n.a.	-7.558^a	-5.495^a	-8.147^a	-11.207^a	-5.981^a
Denmark	-9.403^a	n.a.	4.998^a	1.156	-9.141^a	3.647^a
France	-4.994^a	4.773^a	n.a.	-10.904^a	-9.053^a	0.362
Netherlands	-9.007^a	1.111	-11.446^a	n.a.	-4.458^a	2.767^a
Sweden	-14.004^a	-8.617^a	-8.175^a	-4.032^a	n.a.	6.359^a
Germany	-6.140^a	4.250^a	0.340	2.852^a	7.263^a	n.a.

[a]Significant at the 0.01 level.

seven countries only. In order to be able to consider all European series, one would need tests for trend slopes that are robust to errors with an autoregressive root equal or close to unity.

Given that little research has been done on multivariate trend function inference, there are many directions to extend the approaches in this paper. The most obvious and simple extension would be to consider kernels other than the Bartlett kernel. More interesting extensions include models with unit root or near unit root errors, models with cointegrated errors, and models with higher-order trends or trends with structural change.

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Appendix A.

This appendix contains the proofs of the various results derived in this paper.

Proofs of (11) and (16). Simple algebra along with (2) and (8) gives

$$\begin{aligned}
T^{-1/2}\hat{S}_{[rT]} &= T^{-1/2}\sum_{t=1}^{[rT]}\hat{u}_t \\
&= T^{-1/2}\sum_{t=1}^{[rT]}(u_t-\bar{u}) - T^{3/2}(\hat{\beta}-\beta)T^{-2}\sum_{t=1}^{[rT]}\tilde{t} \\
&\Rightarrow \Lambda\left[W_n(r) - rW_n(1) - 12V_n(1)\int_0^r\left(s-\frac{1}{2}\right)\mathrm{d}s\right] \\
&= \Lambda\hat{V}_n(r),
\end{aligned}$$

which proves (11). Similarly, using (2)–(5) and (8) it follows that

$$\begin{aligned}
&T^{-3/2}\tilde{S}_{[rT]} \\
&= T^{-3/2}\sum_{t=1}^{[rT]}\tilde{t}\hat{u}_t \\
&= T^{-3/2}\sum_{t=1}^{[rT]}\tilde{t}(u_t-\bar{u}) - T^{3/2}(\hat{\beta}-\beta)T^{-3}\sum_{t=1}^{[rT]}\tilde{t}^2 \\
&\Rightarrow \Lambda\left[\int_0^r\left(s-\frac{1}{2}\right)\mathrm{d}W_n(s) - \left(\int_0^r\left(s-\frac{1}{2}\right)\mathrm{d}s\right)W_n(1)\right. \\
&\qquad \left. -12V_n(1)\int_0^r\left(s-\frac{1}{2}\right)^2\mathrm{d}s\right] \\
&= \Lambda\tilde{V}_n(r),
\end{aligned}$$

which proves (16).

Proof of Theorem 1. Simple algebra under H_0 gives

$$\begin{aligned}
F_1^* = T^{3/2}(R(\hat{\beta}-\beta))'\left[R\left(T^{-3}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1}T^{-2}\tilde{\Omega}_{M=T}\left(T^{-3}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1}R'\right]^{-1} \\
\times T^{3/2}(R(\hat{\beta}-\beta))/q. \qquad \text{(A.1)}
\end{aligned}$$

Using (4), (8) and (12) it follows that

$$F_1^* \Rightarrow [R\Lambda V_n(1)]'\left[2R\Lambda\int_0^1\tilde{V}_n(r)\tilde{V}_n(r)'\,\mathrm{d}r\Lambda'R'\right]^{-1}[R\Lambda V_n(1)]/q.$$

We can write $R\Lambda W_n(r)$ as $\Lambda^* W_q(r)$, where Λ^* is a $q\times q$ matrix with $\Lambda^*\Lambda^{*'} = R\Lambda\Lambda'R'$ because $W_n(r)$ is a vector of independent Gaussian random variables. Therefore, direct algebra allows us to write $R\Lambda V_n(r)$ as $\Lambda^* V_q(r)$, and $R\Lambda\tilde{V}_n(r)$ as $\Lambda^*\tilde{V}_q(r)$. Using these representations gives

$$F_1^* \Rightarrow [\Lambda^* V_q(1)]' \left[2\Lambda^* \int_0^1 \tilde{V}_q(r)\tilde{V}_q(r)' \mathrm{d}r \Lambda^{*'}\right]^{-1} [\Lambda^* V_q(1)]/q$$

$$= V_q(1)' \left[2 \int_0^1 \tilde{V}_q(r)\tilde{V}_q(r)' \, \mathrm{d}r\right]^{-1} V_q(1)/q.$$

Using similar arguments as for F_1^*, it follows from (4), (8) and (17) that

$$F_2^* = T^{3/2}[R(\hat{\beta} - \beta)]' \left[R\left(T^{-3}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1} \hat{\Omega}_{M=T} R'\right]^{-1} T^{3/2}[R(\hat{\beta} - \beta)]/q$$

$$\Rightarrow [R\Lambda V_n(1)]' \left[\frac{1}{6} R\Lambda \int_0^1 \hat{V}_n(r)\hat{V}_n(r)' \, \mathrm{d}r \Lambda' R'\right]^{-1} [R\Lambda V_n(1)]/q$$

$$= V_q(1)' \left[\frac{1}{6} \int_0^1 \hat{V}_q(r)\hat{V}_q(r)' \, \mathrm{d}r\right]^{-1} V_q(1)/q.$$

Proof of Theorem 2. Under the local alternative we have

$$\hat{\beta}_1 - \beta_0 = cT^{-3/2} + \left(\sum_{t=1}^{T}\tilde{t}^2\right)^{-1} \sum_{t=1}^{T}\tilde{t}u_t. \tag{A.2}$$

Therefore, using (4) and (5) it immediately follows that

$$T^{3/2}(\hat{\beta}_1 - \beta_0) = c + \left(T^{-3}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1} T^{-3/2}\sum_{t=1}^{T}\tilde{t}u_t \Rightarrow c + 12\sigma_1 V_1(1). \tag{A.3}$$

Simple algebra gives

$$t_1^* = \frac{T^{3/2}(\hat{\beta}_1 - \beta_0)}{\left[\left(T^{-3}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1} T^{-2}\tilde{\Omega}_{M=T}\left(T^{-3}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1}\right]^{1/2}},$$

$$t_2^* = \frac{T^{3/2}(\hat{\beta}_1 - \beta_0)}{\left[\left(T^{-3}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1} \hat{\Omega}_{M=T}\right]^{1/2}},$$

and

$$t_{\mathrm{HAC}} = \frac{T^{3/2}(\hat{\beta}_1 - \beta_0)}{\left[\left(T^{-3}\sum_{t=1}^{T}\tilde{t}^2\right)^{-1} \hat{\Omega}_{\mathrm{HAC}}\right]^{1/2}}.$$

Because $\tilde{\Omega}_{M=T}$, $\hat{\Omega}_{M=T}$ and $\hat{\Omega}_{\text{HAC}}$ are exactly invariant to β_1, their limits are the same as under the null hypothesis. Therefore, using (A.3), we have

$$t_1^* \Rightarrow \frac{c + 12\sigma_1 V_1(1)}{[288\sigma_1^2 \int_0^1 \tilde{V}_1(r)^2 \,\mathrm{d}r]^{1/2}},$$

$$t_2^* \Rightarrow \frac{c + 12\sigma_1 V_1(1)}{[24\sigma_1^2 \int_0^1 \hat{V}_1(r)^2 \,\mathrm{d}r]^{1/2}}$$

and

$$t_{\text{HAC}} \Rightarrow \frac{c + 12\sigma_1 V_1(1)}{[12\sigma_1^2]^{1/2}}.$$

Simple algebra completes the proof.

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