Project Description

Applicant:	Michael Vogt, University of Bonn
Project Title:	New Methods and Theory for the Comparison of Nonparametric Trend Curves

1 State of the art and preliminary work

The comparison of nonparametric curves is a classical topic in econometrics and statistics. Depending on the application context, the curves of interest are densities, distribution functions, time trends or regression curves. The problem of testing for equality of densities has been studied for example in ??, ?? and ??. Tests for equality of distribution functions can be found in ??, ?? or ??. Tests for equality of trend and regression curves have been developed in ??, ?? and ?? among many others. In the proposed project, we focus on the comparison of nonparametric trend curves.

Moreover, the comparison of trend curves is an important topic in many statistical applications. Economists, for example, are interested in comparing the trends of long-term interest rates for different countries. Moreover, they may want to assess whether the trends in real GDP growth differ across countries. In finance, massive amounts of data on thousands of stocks are available today. One question of interest is to compare how the volatility of different stocks evolves over time. Finally, in climatotology, large spatial data sets have been collected which comprise long temperature time series for many different locations. Climatologists are very much interested in analyzing the trending behaviour of these time series. In particular, they would like to know how the temperature trend varies across locations.

The statistical problem of comparing trends has a wide range of applications in economics, finance and other fields such as climatology and biology. In economics, a common issue is to compare trends in real GDP across different countries. It is highly debated whether real GDP growth is faster in economies in transition (such as Brazil, China or India) than in developed countries (cp. ??). Another example concerns the dynamics of long-term interest rates. To better understand these dynamics, researchers aim to compare the yields of US Treasury bills at different maturities over time (cp. ??). In finance, it is also of interest to compare the volatility trends of different stocks (cp. ??). A final example comes from climatotology. In recent years, large spatial data sets have been collected which comprise long temperature time series for many different locations. Climatologists are very much interested in analyzing the trending behaviour of these time series (cp. ??). In particular, they would like to know how the temperature trend varies across locations.

Classically, time trends are modelled stochastically in econometrics; see e.g. Stock and Watson (1988). Recently, however, there has been a growing interest in econometric models with deterministic time trends; see Cai (2007), Atak et al. (2011), Robinson (2012) and Chen et al. (2012) among others. Non- and semiparametric trend modelling has attracted particular interest in a panel data context. Li et al. (2010), Atak et al. (2011), Robinson (2012) and Chen et al. (2012) considered panel models where the observed time series have a common time trend. In many applications, however, the assumption of a common time trend is quite harsh. In particular when the number of observed time series is large, it is quite natural to suppose that the time trend may differ across time series. More flexible panel settings with heterogeneous trends have been studied, for example, in Zhang et al. (2012) and Hidalgo and Lee (2014).

In what follows, we consider a general panel framework with heterogeneous trends which is useful for a number of economic and financial applications: Suppose we observe a panel of n time series $\mathcal{Z}_i = \{(Y_{it}, X_{it}) : 1 \le t \le T\}$ for $1 \le i \le n$, where Y_{it} are real-valued random variables and $X_{it} = (X_{it,1}, \ldots, X_{it,d})^\top$ are d-dimensional random vectors. Each time series $\mathcal{Z}_i$ is modelled by the equation

$$Y_{it} = m_i\Big(\frac{t}{T}\Big) + \beta_i^\top X_{it} + \alpha_i + \varepsilon_{it} \tag{1}$$

for $1 \le t \le T$, where $m_i : [0,1] \to \mathbb{R}$ is a nonparametric (deterministic) trend function, X_{it} is a vector of regressors or controls and β_i is the corresponding parameter vector. Moreover, α_i are so-called fixed effect error terms and ε_{it} are standard regression errors with $\mathbb{E}[\varepsilon_{it}|X_{it}] = 0$ for all t. Model (1) nests a number of panel settings which have recently been considered in the literature. Special cases of model (1) with a nonparametric trend specification are for example considered in Atak et al. (2011), Zhang et al. (2012) and Hidalgo and Lee (2014). Versions of model (1) with a parametric trend are studied in Vogelsang and Franses (2005), Sun (2011) and Xu (2012) among others.

As usual in nonparametric regression, the trend functions m_i in model (1) depend on rescaled time t/T rather than on real time t; cp. ??, ?? and ?? for the use and some discussion of the rescaled time argument. The functions m_i are only identified up to an additive constant in model (1): One can reformulate the model equation as $Y_{it} = [m_i(t/T) + c_i] + \beta_i^\top X_{it} + [\alpha_i - c_i] + \varepsilon_{it}$, that is, one can freely shift additive constants between the trend $m_i(t/T)$ and the error component α_i. In order to obtain identification, one may impose different normalization constraints on the trends m_i. One possibility is to normalize them such that $\int_0^1 m_i(u)du = 0$ for all i. In what follows, we take for granted that the trends m_i satisfy this constraint. Within the general framework of model (1), we can formulate a number of interesting statistical questions concerning the trend functions m_i for $1 \leq i \leq n$.

(a) Testing for equality of nonparametric trend curves

Most of the current literature on deterministic trends in time series heavily relies on the critical assumption of the common trend structure. This assumption means that each individual in the panel exhibits the same trend behavior. The majority of the works in this topic can not be easily generalized to the setting where the trend functions are not the same for different individuals. Therefore, it is vital to be able to test the assumption of the common trend before imposing it.

This question can formally be adressed by a statistical test of the null hypothesis

$$H_0 : \text{There exists a function } m : [0,1] \to \mathbb{R} \text{ such that } m_i = m \text{ for all } 1 \leq i \leq n.$$

A closely related question is whether all time trends have the same parametric form. To formulate the corresponding null hypothesis, let $m(\theta, \cdot) : [0,1] \to \mathbb{R}$ be a function which is known up to the finite-dimensional parameter $\theta_0 \in \Theta$, where Θ denotes the parameter space. The null hypothesis of interest now reads as follows:

$$H_{0,\text{para}} : \text{ There exists } \theta \in \Theta \text{ such that } m_i(\cdot) = m(\theta, \cdot) \text{ for all } 1 \leq i \leq n.$$

If $m(\theta, w) = a + bw$ with $\theta = (a, b)$, for example, then H_0 is the hypothesis that all trends m_i are linear with the same intercept a and slope b. A somewhat simpler but yet important hypothesis is given by

$$H_{0,\text{const}} : m_i \equiv 0 \text{ for all } 1 \leq i \leq n.$$

Under this hypothesis, there is no time trend at all in the observed time series. Put differently, all the time trends m_i are constant. (Note that under the normalization constraint $\int_0^1 m_i(w)dw = 0$, m_i must be equal to zero if it is a constant function.) A major aim of our project is to develop new tests for the hypotheses H_0, $H_{0,\text{para}}$ and $H_{0,\text{const}}$ in model (1). In order to keep the exposition focused, we restrict attention to the hypothesis H_0 in what follows. Tests of $H_{0,\text{para}}$, $H_{0,\text{const}}$ and related hypotheses were for example studied in Lyubchich and Gel (2016) and Chen and Wu (2018).

In recent years, a number of different approaches have been developed to test the hypothesis H_0. Degras et al. (2012) considered the problem of testing H_0 within the model framework

$$Y_{it} = m_i\Big(\frac{t}{T}\Big) + \alpha_i + \varepsilon_{it} \qquad (1 \le t \le T,\, 1 \le i \le n), \tag{2}$$

where $\mathbb{E}[\varepsilon_{it}] = 0$ for all i and t and the terms α_i are assumed to be deterministic. Obviously, (2) is a special case of (1) which does not include additional regressors. Degras et al. (2012) construct an L_2-type statistic to test H_0. The statistic is based on nonparametric kernel estimators $\hat{m}_{i,h}$ and $\hat{m}_h$ of the functions m_i and m, where h denotes the bandwidth parameter. With the help of these estimators, the authors define the statistic

$$\Delta_{n,T} = \sum_{i=1}^{n} \int_0^1 \big(\hat{m}_i(u) - \hat{m}(u)\big)^2 du,$$

which measures the L_2-distance between the estimators $\hat{m}_i$ and $\hat{m}$. In the theoretical part of their paper, they derive the limit distribution of $\Delta_{n,T}$. Chen and Wu (2018) develop theory for test statistics closely related to those from Degras et al. (2012), however, under more general conditions on the error processes $\mathcal{E}_i = \{\varepsilon_{it} : 1 \le t \le T\}$.

Zhang et al. (2012) investigate the problem of testing the hypothesis H_0 in a slightly restricted version of model (1), where $\beta_i = \beta$ for all i. The regression coefficients β_i are thus assumed to be homogeneous in their setting. They construct a residual-based test of H_0 as follows: First, the authors estimate the model under the null by profile least squares and obtain the estimates $\widehat{\beta}$ and $\widehat{m}_i(t/T) \equiv \widehat{m}(t/T)$. Then, they obtain the augmented residual $\widehat{u}_{it}$ under H_0 by $\widehat{u}_{it} = Y_{it} - \widehat{\beta}^T X_{it} - \widehat{m}(t/T)$, which consistently estimates $u_{it} = \alpha_i + \varepsilon_{it}$. Afterwards, they run the local linear regression of $\widehat{u}_{it}$ on t/T and from this regression, the authors calculate the non-parametric goodness-of-fit R_i^2 for each i. The test statistic for testing H_0 is then defined by averaging the non-parametric goodness-of-fit R_i^2:

$$\bar{R}^2 = \frac{1}{n} \sum_{i=1}^{n} R_i^2,$$

which is proved to be asymptotically normal both under the null hypothesis and under a sequence of local alternatives.

The tests of Zhang et al. (2012), Degras et al. (2012) and Chen and Wu (2018) are based on nonparametric estimators of the trend functions m_i or other parameters of the model. They thus depend on one or several bandwidth parameters. It is however far from clear how to choose these bandwidths in an appropriate way. This is a quite general problem which concerns essentially all tests that are based on nonparametric curve estimators. There are of course many theoretical results on the optimal choice of bandwidth for estimation purposes. However, the optimal bandwidth for curve estimation is usually not optimal for testing. Optimal bandwidth choice for tests is indeed a quite open problem, and only little theory for simple cases is available, cp. Gao and Gijbels (2008). Since tests based on nonparametric curve estimators are commonly quite sensitive to the choice of bandwidth and theory for optimal bandwidth selection is not available, it appears preferable to work with bandwidth-free tests.

A classical way to obtain a bandwidth-free test of the hypothesis H_0 is to use CUSUM-type statistics which are based on partial sum processes. This approach is taken in Hidalgo and Lee (2014). **[Add details!]**

A more modern way to obtain a test statistic which is free of classical bandwidth parameters is to use multiscale methods. The general idea is as follows: Let S_h be a test statistic for the null hypothesis of interest, which depends on the bandwidth h. Rather than considering only a single statistic S_h for a specific bandwidth h, a multiscale approach simultaneously considers a whole family of statistics $\{S_h : h \in \mathcal{H}\}$, where $\mathcal{H}$ is a set of bandwidth values. The multiscale test then proceeds as follows: For each bandwidth or scale h, one checks whether $S_h > q_h(\alpha)$, where $q_h(\alpha)$ is a bandwidth-dependent critical value (for given significance level α). The multiscale test rejects if $S_h > q_h(\alpha)$ for at least one scale h. The main theoretical difficult in this approach is of course to derive appropriate critical values $q_h(\alpha)$.

One of the first multiscale methods proposed in the literature is the SiZer approach of Chaudhuri and Marron (1999, 2000). In recent years, this approach has been extended in various directions; see Hannig and Marron (2006) and ?? among others. Park et al. (2009) developed SiZer methods for the comparison of nonparametric trend curves in a simplified version of model (1). Their analysis, however, is mainly methodological and only partly backed up by theory. Indeed, theory is only derived for the special case $n = 2$, that is, for the case that only two time series are observed. Moreover, the theoretical results are only valid under very severe restrictions on the set of bandwidths $\mathcal{H}$ that is taken into account. In particular, the bandwidths in the set $\mathcal{H}$ are assumed to be bounded away from zero. Put differently, they are not allowed to converge to zero as the sample size grows, which is obviously a very severe limitation.

A major aim of our project is to develop new multiscale tests of the hypothesis H_0 in the general model (1) which do not have the limitations of the SiZer methods discussed above. Importantly, we do not only intend to develop new test methodology but also to back up the methods by a general asymptotic distribution theory. To achieve this, we plan to build on a multiscale approach pioneered by Dümbgen and Spokoiny (2001). This general approach has been very influential in recent years and has been further developed in numerous directions; see for example Dümbgen (2002), Rohde (2008) and Proksch et al. (2018) for multiscale methods in the regression context and Dümbgen and Walther (2008), Rufibach and Walther (2010), Schmidt-Hieber et al. (2013) and Eckle et al. (2017) for methods in the context of density estimation. Importantly, all of these studies are limited to the case of independent data. It turns out that it is highly non-trivial to extend the methods to the case of dependent data. To do so, markedly different technical tools are needed. A first step to provide such tools has recently been made in Khismatullina and Vogt (2018). They developed multiscale methods for testing shape restrictions of the nonparametric trend function m in the univariate time series model $Y_t = m(t/T) + \varepsilon_t$. In our project, we aim to make further progress in this direction. Please see Section ?? on objectives for the details.

(b) Clustering of nonparametric trend curves

Consider the general panel data model (1) and suppose that the null hypothesis $H_0 : m_1 = \ldots = m_n$ is violated in this model. Even though some of the trend functions m_i are different in this case, there may still be groups of time series with the same time trend. Formally, a group stucture can be defined as follows within the framework of model (1): There exist sets or groups of time series $G_1, \ldots, G_{K_0}$ with $\{1, \ldots, n\} = \dot{\bigcup}_{k=1}^{K_0} G_k$ such that for each $1 \leq k \leq K_0$,

$$m_i = m_j \quad \text{for all } i, j \in G_k. \tag{3}$$

Hence, the time series of a given group G_k all have the same time trend. An interesting statistical problem is how to estimate the unknown groups $G_1, \ldots, G_{K_0}$ and their unknown number K_0 from the data.

There are several approaches to this problem in the context of models closely related to (1). Degras et al. (2012) used a repeated testing procedure based on the methods described in part (a) of this section to estimate the unknown group structure in model (2). Zhang (2013) developed a clustering method within the same model framework which makes use of information criteria. **[Add details!]** Vogt and Linton (2017) constructed a thresholding method to estimate the unknown group structure in the panel model $Y_{it} = m_i(X_{it}) + \alpha_i + \varepsilon_{it}$, where X_{it} are random regressors. Their approach can also be adapted to the case of fixed regressors $X_{it} = t/T$.

The problem of estimating the unknown groups $G_1, \ldots, G_{K_0}$ and their unknown number K_0 in model (1) has close connections to functional data clustering. There, the aim is to cluster smooth random curves that are functions of (rescaled) time and that are observed with or without noise. A number of different clustering approaches have been proposed in the context of functional data models; see for example Abraham et al. (2003), Tarpey and Kinateder (2003) and Tarpey (2007) for procedures based on k-means clustering, James and Sugar (2003) and Chiou and Li (2007) for model-based clustering approaches and Jacques and Preda (2014) for a recent survey.

The problem of finding the unknown group structure in model (1) is also closely related to a developing literature in econometrics which aims to identify unknown group structures in parametric panel regression models. In its simplest form, the panel regression model under consideration is given by the equation $Y_{it} = \beta_i^\top X_{it} + u_{it}$ for $1 \le t \le T$ and $1 \le i \le n$, where the coefficient vectors β_i are allowed to vary across individuals i. Similarly as the trend functions in model (1), the coefficients β_i are assumed to belong to a number of groups: there are K_0 groups $G_1, \ldots, G_{K_0}$ such that $\beta_i = \beta_j$ for all $i, j \in G_k$ and all $1 \le k \le K_0$. The problem of estimating the unknown groups and their unknown number has been studied in different versions of this modelling framework; cp. Su et al. (2016), Su and Ju (2018) and Wang et al. (2018) among others. Bonhomme and Manresa (2015) considered a related model where the group structure is not imposed on the regression coefficients but rather on the unobserved time-varying fixed effects.

Virtually all of the proposed procedures to cluster nonparametric curves in panel and functional data models related to (1) have the following drawback: they depend on a number of bandwidths or smoothing parameters required to estimate the nonparametric functions m_i. In general, nonparametric curve estimators strongly depend on the chosen bandwidth parameters. A clustering procedure which is based on such estimators can be expected to be strongly influenced by the choice of bandwidths as well. Moreover, as in the context of statistical testing, there is no theory available on how to pick the bandwidths optimally for the clustering problem. Hence, as in the context of testing, it is desirable to construct a clustering procedure which is free of classical bandwidth parameters.

There are different ways to move into the direction of a bandwidth-free clustering algorithm. One possibility is to employ Wavelet methods. A Bayesian Wavelet-based method to cluster nonparametric curves has been developed in Ray and Mallick (2006). There, the simple model $Y_{it} = m_i(t/T) + u_{it}$ is considered, where m_i are smooth functions of rescaled time t/T and the error terms u_{it} are restricted to be i.i.d. Gaussian noise.

Another possibility is to use multiscale methods. This approach has recently been taken in Vogt and Linton (2018). They develop a clustering approach within the framework of the panel regression model $Y_{it} = m_i(X_{it}) + u_{it}$, where X_{it} are random regressors and u_{it} are general error terms that may include fixed effects. Imposing the same group structure as in (3) on their model, they construct estimators of the unknown groups and their unknown number as follows: In a first step, they develop a multiscale statistic $\hat{d}_{ij}$ which measures the distance between any two functions

m_i and m_j. In a second step, the distance measures $\hat{d}_{ij}$ are used as the basis of a hierarchical clustering algorithm. In the theoretical part of their paper, they derive some consistency results for their estimators. Letting $\hat{K}_0$ be the estimator of K_0 and $\{\hat{G}_1, \ldots, \hat{G}_{\hat{K}_0}\}$ the estimator of the group structure $\{G_1, \ldots, G_{K_0}\}$, they in particular show that under appropriate regularity conditions,

$$\mathbb{P}\big(\hat{K}_0 = K_0\big) \to 1 \quad \text{and} \quad \mathbb{P}\Big(\{\hat{G}_1, \ldots, \hat{G}_{\hat{K}_0}\} = \{G_1, \ldots, G_{K_0}\}\Big) \to 1 \tag{4}$$

as the sample size goes to infinity. Even though promising, the consistency result (4) is only a first step into the direction of a complete asymptotic theory. A more refined theory would comprise results on convergence rates and confidence statements about the estimators.

Building on the work of Vogt and Linton (2018), we aim to develop multiscale clustering methods in model (1). We in particular aim to go beyond the basic theory developed in Vogt and Linton (2018) and to provide results on convergence rates and confidence statements. We give more details on these objectives in Section ??.

1.1 Project-related publications

1.1.1 Articles published by outlets with scientific quality assurance, book publications, and works accepted for publication but not yet published

Vogt, M. and Linton, O. (2017). Classification of non-parametric regression functions in longitudinal data models. *Journal of the Royal Statistical Society: Series B*, **79** 5-27.

1.1.2 Other publications

Khismatullina, M. and Vogt, M. (2018). Multiscale inference and long-run variance estimation in nonparametric regression with time series errors. *Preprint.*

Vogt, M. and Linton, O. (2018). Multiscale clustering of nonparametric regression curves. *Preprint.*

2 Objectives and work programme

2.1 Anticipated total duration of the project

2 years from 01.10.2019 to 30.09.2021

2.2 Objectives

The main aim of the project is to develop new methods and theory for the comparison and clustering of nonparametric trend curves. As a modelling framework, we will consider the general panel setting (1) which was briefly introduced in Section ??: Suppose we observe a panel of n time series $\mathcal{Z}_i = \{(Y_{it}, X_{it}) : 1 \le t \le T\}$ for $1 \le i \le n$. Each time series $\mathcal{Z}_i$ is modelled by the equation

$$Y_{it} = m_i\Big(\frac{t}{T}\Big) + \beta_i^\top X_{it} + \alpha_i + \varepsilon_{it} \tag{5}$$

for $1 \le t \le T$, where m_i is a nonparametric time trend curve, X_{it} is a vector of regressor or control variables, α_i are unobserved fixed effects and ε_{it} are idiosyncratic error terms with $\mathbb{E}[\varepsilon_{it}|X_{it}] = 0$. For each i, $\mathcal{P}_i = \{(X_{it}, \varepsilon_{it})\}$ is assumed to be a general time series process which fulfills some weak dependence conditions (e.g. conditions formulated in terms of strong mixing coefficients or in terms of the physical dependence measure introduced by Wu ??). We will not only allow for time series dependence in the data, but also for some forms of cross-sectional dependence. Put differently, we will allow the time series $\mathcal{P}_i$ to be dependent across i. To derive our theoretical results, we will assume that the time series length T tends to infinity. The number of time series n, in contrast, may either be bounded or diverging.

(a) Contributions to statistical multiscale testing

The first main contribution of the project is to develop a novel multiscale test for the comparison of the trend curves m_i ($1 \le i \le n$). More specifically, we aim to develop multiscale tests forr the hypothesis $H_0 : m_1 = \ldots = m_n$ and for the related hypotheses discussed in part (b) of Section ??. To keep the exposition focused, we restrict attention to H_0 in what follows. For any interval $[u-h, u+h] \subseteq [0,1]$, consider the hypothesis

$$H_0^{[i,j]}(u,h) : m_i(w) = m_j(w) \text{for all } w \in [u-h, u+h].$$

Obviously, the hypothesis H_0 can be reformulated as

H_0 : The hypothesis $H_0^{[i,j]}(u,h)$ holds true for all intervals $[u-h, u+h] \subseteq [0,1]$ and for all $1 \le i < j \le n$.

We construct a multiscale method to simultaneously test the hypothesis $H_0^{[i,j]}(u,h)$ for all possible points (u,h) and all pairs (i,j) with $i < j$.[1] We plan to construct

[1]Obviously, in practice, we can not consider all points $u \in [0,1]$ and all $h > 0$ but have to restrict

such a method along the following lines:

Step 1: Construct nonparametric estimators $\hat{m}_{i,h}$ of the trend functions m_i, where h denotes the bandwidth parameter.

Step 2: Construct a test statistic $\hat{S}_{ij}(u,h)$ for the hypothesis $H_0^{[i,j]}(u,h)$. A simple choice is a statistic of the form $\hat{S}_{ij}(u,h) = \sqrt{Th}(\hat{m}_{i,h}(u) - \hat{m}_{j,h}(u))/\hat{\nu}_{ij,h}(u)$, where $\hat{\nu}_{ij,h}(u)$ is chosen such that the asymptotic variance of the statistic is normalized to 1.

Step 3: Aggregate the statistics $\hat{S}_{ij}(u,h)$ into a multiscale statistic. As already discussed in Section ??, we will use the aggregation scheme of Dümbgen and Spokoiny (2001) to do so. The resulting multiscale statistic has the form

$$\hat{\Psi}_{n,T} = \max_{1 \le i < j \le n} \sup_{u,h} \big\{ |\hat{S}_{ij}(u,h)| - \lambda(h) \big\},$$

where $\lambda(h)$ are (appropriately chosen) additive corrections terms. As one can see, the multiscale statistic is not obtained by simply aggregating the individual statistics $\hat{\Psi}_{n,T}$. We rather take the supremum of the additively corrected statistics $|\hat{S}_{ij}(u,h)| - \lambda(h)$. This idea of this correction scheme goes back to Dümbgen and Spokoiny (2001).

Given the statistic $\hat{\Psi}_{n,T}$ the multiscale test is carried out as follows: Suppose for a moment we could compute the $(1-\alpha)$-quantile $q^*_{n,T}(\alpha)$ of the statistic $\hat{\Psi}_{n,T}$ under the null H_0. Then we proceed as follows:

Reject the overall null hypothesis H_0 if $\hat{\Psi}_{n,T} > q^*_{n,T}(\alpha)$.

By construction, the decision rule in ?? is a rigorous level α test. However, the quantile $q^*_{n,T}(\alpha)$ is gihgly complicated and not know in practicve. The main theoretical challenge is to come up with an (asymptotic) apprioximation $q_{n,T}(\alpha)$ of this quantule which is computable in practice such that

$$\mathbb{P}(\hat{\Psi}_{n,T} > q_{n,T}(\alpha)) = 1 - \alpha + o(1).$$

As already mention in Section ??, we try to build on the methods dervie in ?? to derive such an approximation. However, the panel setting differs in various respect from the univeratie time series setting in ??, which is why the technical details ??. / substantial amont of work to extend / adapt the techniques to the setting at hand.

Compared to existing methods, the multiscale test ptroposed above has the following important advantages:

attention to a finite subset of points. We ignore this in our presentation for simplicity.

(i) Unlike many other methods (cp. ?? and the discussion in Section ??), it does not depend on a specific bandwidth parameter. It rather takes into account multiple scales or bandwidths h simulteneously.

(ii) it is much more informative than non-multiscale tests: the method can be reagrded as a simutelenous test of the family of hypothesis $H_0^{[i,j]}(u,h)$ for all points (u,h) and all i,j. Looking at the test in this way, one may define the following decision rule:

For each interval $[u-h,u+h]$, reject the hypopthesis $H_0^{[i,j]}(u,h)$ if the corrected test statistic $|\hat{S}_{ij}(u,h)| - \lambda(h)$ is above the critical value $q_{n,T}(\alpha)$, that is, if $|\hat{S}_{ij}(u,h)| - \lambda(h) > q_{n,T}(\alpha)$.

We aim to prove the following result: With asymptotci probability $1-\alpha$, the hypothesis $H_0^{[i,j]}(u,h)$ is violated for all intervals $[u-h,u+h]$ and for all (i,j) for which $|\hat{S}_{ij}(u,h)| - \lambda(h) > 0$. hence, we can make the following simulteneous confidence statement: With statistical conficende $1-\alpha$, there is a violaten of the hypothesis $H_0^{[i,j]}(u,h)$ for all intervals and all time series (i,j) where the test finds something. Hence, the multiscale test does not only tell us whether H_0 is vioated, it also allows us to make rigorous statistical confidence statement about (i) which trends are different and (ii) in which time regions $[u-h,u+h]$ they differ. This is valuable information in many applications.

(b) Contributions to curve clustering

The second main contribution is to develop a clustering approach which is based on the multiscale test from the first main part of the project. Our ojectives are as follows:

(i) We adapt the multiscale clustering methods from Vogt and Linton (2018) to the setting at hand. We in particular use the multiscale statistics constructed in part (a) as distance / dissimilarityx measures in a hierarchical clustering algoruthm.

(ii) The main challenge is to derive theory for this clustering approach which goes beyond the basic asymptotic results of Vogt and Linton (2018). Using the theoretical results on the multiscale statistic $\hat{\Psi}_{n,T}$ enables us to do so and to derive much more precise theroetical statements about the clustering method. We give an example to make this claim more precise: The estimator of the unknown number of groups K_0 depends aon athreshold parameter. There is only a hezuristic rule for choosing this paramater. However, we can construct threshols with the help of the quantiles $q_{n,T}(\alpha)$. This should allow us to make confidence statements about the estimator of K_0. Denoting this estimator by $\hat{K}_0$, we conjecture that we are able to derive a result of the following form (under appropriate regularity conditions):

$$\mathbb{P}(\hat{K}_0 = K_0) = (1 - \alpha) + o(1).$$

Hence, by picking the significance level equal to α and constructing the estimator $\hat{K}_0$ on the basis of $q_{n,T}(\alpha)$, we get that $\hat{K}_0$ is eqaul to the true number of groups K_0 with (asymptotic) probability $1 - \alpha$. We can thus tune the clusteering procedure such that the erro probability of misestimating the number of groups K_0 is (asymptotically) controlled.

(c) Empirical applications

We intend to explore some empirical applications with the help of the new mutlscale testing and clustering methods.

Model (5) and the proposed testing/clustering method are useful in a number of application contexts which we aim to explore. We here give some examples:

Example 1. *Short-term risk-free interest rates are one of the main topics of interest in the financial markets. For example, it is a key component of the capital asset pricing model, which describes the relationship between risk and return. Furthermore, the risk-free rate is also a required input in financial calculations regarding the pricing of bonds. There is an evergrowing amount of literature on the dynamics of interest rate. US Treasury bills are the real-world investment that serve as the proxy for these rates. Park et al. (2009) analyze the yields of the 3-month, 6-month, and 12-month Treasury bills in the context of comparing nonparametric curves. The authors assume that the yields come from the following model:*

$$Y_{it} = m_i(t) + \sigma_i \varepsilon_{it}, \quad i = 1, \ldots, n, \quad t = 1, \ldots, T, \tag{6}$$

which is a simplification of our model (5)*. Park et al. (2009) apply SiZer method to the data and come to the conclusion that the underlying structure for different time periods is almost identical. They could not find any significant difference between any pair of the time periods, which concides with the results from applying other methods, see, for example, Fan and Yao (2008).*

Example 2. *Another example of comparison of time series with nonparametric trend functions described in Park et al. (2009) involves the long-term rates for US, Canada, and Japan from January 1980 to December 2000. The data is assumed to follow the same model* (6)*. The authors perform pairwise comparison of the curves* *as well as comparison of the three time series at the same time using the proposed SiZer method. In both cases their method was able to detect significant differences nd indicate "suspicious" regions. However, since SiZer is a graphical device that is mainly designed for data exploration rather than for rigorous statistical inference, they do not make simultaneous confidence statements with a predetermined confidence level about the regions where these differences were most probable to occur. Our proposed multiscale method, in contrast, is a rigorous level-α-test of the hypothesis H_0 which is aimed specifically at that.*

Example 3. *Economic growth has been a key topic in marcoeconomics over many decades. Economists are very much interested in the question whether gross domestic product (GDP) growth has been faster in some countries than in others. One of the ways to model the source of economic growth is to incorporate a nonparametric deterministic time trend in the model. For example, Zhang et al. (2012) consider such a model for the OECD economic growth data. Specifically, they investigate the following model for growth rates:*

$$\Delta \ln GDP_{it} = \beta_1 \Delta \log L_{it} + \beta_2 \Delta \log K_{it} + \beta_3 \Delta \log H_{it} + f_i(t/T) + \alpha_i + \varepsilon_{it}, \qquad (7)$$

where $i = 1, \ldots, n$, $t = 1, \ldots, T = 140$, GDP is gross domestic product, K is capital stock, L is labour input, H is human capital, α_i is a fixed effect, $f_i(\cdot)$ is an unknown smooth time trend furnction and ε_{it} are idiosyncratic errors. The errors are allowed to be dependent cross-sectionally, but not serially over t. The data comes from $n = 16$ OECD countries.

Zhang et al. (2012) estimate the common component of time trends which appears to be significantly different from zero over a wide range its support. Moreover, they test the null hypothesis that there are no significant differences in the time trends for the 16 OECD countries. Based on the bootstrap p-values the authors are able to reject the null hypothesis of all the trends being equal at the 10% confidence level. Hence, it can be interesting to be able to further cluster the OECD countries based on their economic growth rates.

Example 4. *The issue of global warming has been a vital topic for many scientists over the last few decades. Since the late 1970, different models that describe the global temperature have been published. In the current literature it is common to assume that the temperate time series (global as well as local) follow a model that can be decomposed into a deterministic trend component and a noise component, see, for example, Ghil and Vautard (1991) and Mudelsee (2018). In order to estimate and attribute the trends in climate variables, a variety of econometric methods have been employed, starting from the simple linear (Yue et al. (2013)) and quadratic* *regression (??) to the empirical mode decompoistion (Wu et al. (2011)), spectrum analysis (Ghil and Vautard (1991)) and semi- and fully non-parametric methods (Gao and Hawthorne (2006)). Parametric and change points methods are mostly suited to quantify the magnitude of the warming trend or to determine the change points, whereas nonparametric methods are best designed to describe the trend over the full time interval without imposing any additional structure on it. However, most of these papers apply nonparametric methods to analyze only one time series or the authors assume that the trend function is common for different time series (Atak et al. (2011)). To our knowledge, only a few papers regarding the comparison of warming trends in different cities or countries have been published (Zhang et al. (2012)).*

Zhang et al. (2012) propose the following semiparametric panel model for unbalanced data to describe the trend in UK regional temperatures:

$$y_{it} = \beta^T D_t + m_i(t/T) + \alpha_i + \varepsilon_{it}, \quad i = 1, \dots, n, \quad t = 1, \dots, T \tag{8}$$

where y_{it} are the monthly mean maximum temperature, monthly mean minimum temperature or total rainfall in millimeters at a station i in month t, D_t is a 11-dimensional vector of monthly dummy variables, α_i is the fixed effect for station i, $m_i(\cdot)$ is an unknown trend function and ε_{it} are idiosyncratic errors. The dataset used is the balanced panel data set for $n = 26$ stations in UK for $T = 382$ months from October 1978 to July 2010. This model is a special case of our proposed model (5) *with dummy variables as covariates.*

Zhang et al. (2012) are interested in testing the null hypothesis $m_i = m$ for all $i = 1, 2, \dots, n$. In order to do this, they apply a non-parametric R^2-based test for common trends that was developed in their paper. Based on the obtained p-values, they reject the null hypothesis of common trend at 5% *level for the monthly mean maximum temperature and the monthly mean minimum temperature. However, they do not reject the null hypothesis for the total rainfall eve at the significance level of* 10%. *As before, it would be interesting to further cluster the UK stations based on the common trend in order to be able to detect the causes of this warming trend. Moreover, it can also be of particular interest to see in which time regions the trends are significantly different from each other.*

2.3 Work programme incl. proposed research methods

The first part of the research period will be devoted to derive the multiscale test methods described in part (a) of Section ??. The second part will focus on the multiscale clustering methods described in part (b) of Section ??.

Milestone	2019	2020	2021
Multiscale testing	Oct–Dec	Jan–Dec	
Multiscale clustering			Jan–Oct

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4 Requested modules/funds

Explain each item for each applicant (stating last name, first name).

4.1 Basic Module

4.1.1 Funding for Staff

Nr.	Position	2019	2020	2021
1	Research staff U. Bonn (EGr. 13 TV-L 75 %)	11.869 €	47.475 €	35.606 €
2	Student Assistant Bonn	2.700 €	10.800 €	8.100€
	Required Amount	14.569€	58.275€	43.706€

Job description of staff payed from auxiliary support for the funding period requested

1. Marina Khismatullina already possesses considerable experience in the study of nonparametric models with time series error. Moreover, she is a co-author of the paper "Multiscale Inference and Long-Run Variance Estimtor in Non-parametric Regression with Time Series Friends" by Khismatullina and Vogt, which is currently submitted to JRSSB. She will be capable to develop computational software taylored to assess the empirical performance of the proposed multiscale test.

2. At the onset of the project a student assistent position should be available in order to support stuff with exploratory data analysis, data mining and organisational issues. The prerequisities are strong analytical and programming skills.

4.1.2 Direct Project Costs

[Text]

4.1.2.1 Equipment up to Euro 10,000, Software and Consumables

[Text]

4.1.2.2 Travel Expenses

[Text]

4.1.2.3 Visiting Researchers (excluding Mercator Fellows)

[Text]

4.1.2.4 Expenses for Laboratory Animals

[Text]

4.1.2.5 Other Costs

[Text]

4.1.2.6 Project-related publication expenses

[Text]

5 Project requirements

5.1 Employment status information

For each applicant, state the last name, first name, and employment status (including duration of contract and funding body, if on a fixed-term contract).

[Text]

5.2 First-time proposal data

Only if applicable: Last name, first name of first-time applicant

[Text]

5.3 Composition of the project group

List only those individuals who will work on the project but will not be paid out of the project funds. State each person's name, academic title, employment status, and type of funding.

[Text]

5.4 Cooperation with other researchers

5.4.1 Researchers with whom you have agreed to cooperate on this project

[Text]

5.4.2 Researchers with whom you have collaborated scientifically within the past three years

[Text]

5.5 Scientific equipment

The University of Bonn has a sufficient infrastructure in hard- and software. Personal computers are available and can be used within the project. Equipment like printer, fax and copier can be used as well.

6 Additional information

If applicable, please list proposals requesting major instrumentation and/or those previously submitted to a third party here.

[Text]