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Development Economics – HA2

Question 1 - Praying for Rain: The Welfare Cost of Seasons

Notation:
Individual stochastic component = $e^{-\sigma_\varepsilon^2}\varepsilon_t$
Stochastic seasonal component = $e^{-\sigma_m^2}\varepsilon_m$
Deterministic Seasonal component = $e^{g(m)}$
Individual component = z

Part 1

On the one hand, we are given that household *i* of age *t* in season *m* obtains a level consumption defined as the following random variable:

$$c_{m,t} = z\left[e^{g(m)}e^{-\sigma_\epsilon^2/2}\epsilon_t\right]$$

On the other hand, we are also given the household *i* lifetime utility is:

$$W(z) = \sum_{t=1}^{40}\beta^{12t}\left[\sum_{m=1}^{12}\beta^{m-1}\frac{c_{m,t}^{1-\eta}}{1-\eta}\right]$$

a) Compute the welfare gains of removing the seasonal component from the stream of consumption separately for each degree of seasonality in Table 1.

Here, we are going to compare the welfare gains of changing from the consumption with all the shocks (individual stochastic component and seasonal component) with the consumption after removing the seasonal component following the degree of seasonality given in Table 1.

Remember from class that we are interested in finding the amount of consumption across all periods and states that individuals living in a reference scenario will demand to remain indifferent between their current scenario and the counterfactual. Applied to this case, we want to find the amount of consumption in all months and all years that should be given to an individual facing seasonality to make him/her as happy as being in a situation without seasonality in terms of welfare:

$$\sum_{t=1}^{40}\beta^{12t}\left[\sum_{m=1}^{12}\beta^{m-1}u(c_{m,t}^{allshocks}(1+g))\right] = \sum_{t=1}^{40}\beta^{12t}\left[\sum_{m=1}^{12}\beta^{m-1}u(c_{m,t}^{noseasonal})\right]$$

Find the results for the different levels of risk-aversion (η) and different degrees of seasonality in Table 1:

Table 1. Mean welfare gains of removing seasonality			
	Low seasonality	Medium seasonality	High seasonality
η=1	0.0042	0.0086	0.0171
η=2	0.0066	0.0185	0.0601
η=4	0.0118	0.0426	0.1867

The first thing to notice is that the higher is the degree of seasonality, the higher is the compensation that needs to be given to individuals from removing seasonality.

The second thing that is worth to remark is that the higher is the level of risk-aversion, the higher is the compensation that needs to be given to individuals when facing seasonality. In other words, the higher is the uncertainty of shocks, the more is the amount of compensation we should give to him/her (since they want to smooth consumption).

b) Compute the welfare gains of removing the nonseasonal consumption risk.

As we did in Part a), we want to find the amount of consumption in all months and all years that should be given to an individual facing seasonality to make him/her as happy as being in a situation without nonseasonal consumption risk in terms of welfare.

$$\sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{allshocks}(1+g)) \right] = \sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{noseasonalconsumptionrisk}) \right]$$

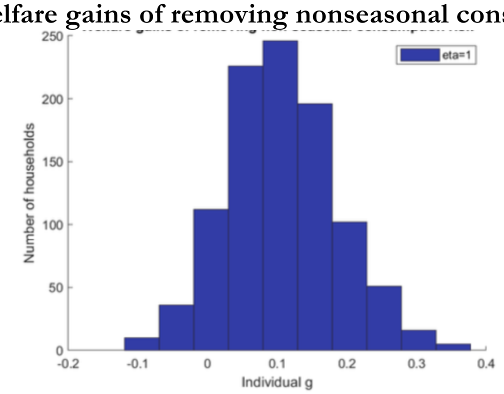
Find the results for the different levels of risk-aversion (η) and different degrees of seasonality in Table 2:

Table 2. Mean welfare gains of removing the nonseasonal consumption risk			
	Low seasonality	Medium seasonality	High seasonality
η=1	0.1046	0.1046	0.1046
η=2	0.2182	0.2182	0.2182
η=4	0.4689	0.4689	0.4689

First, notice that since we have removed the individual stochastic component (or nonseasonal consumption risk), the welfare gains are the same for all the degrees of seasonality.

Second, given that the realization of individual shocks (ε_t) are different across individuals, the welfare gains are not the same across individuals. This can be observed in Figure 1, where there is plotted the distribution of welfare gains. Notice that the majority of the individuals receive positive welfare gains (g>0) when we remove the nonseasonal consumption risk.

Figure 1. Welfare gains of removing nonseasonal consumption risk

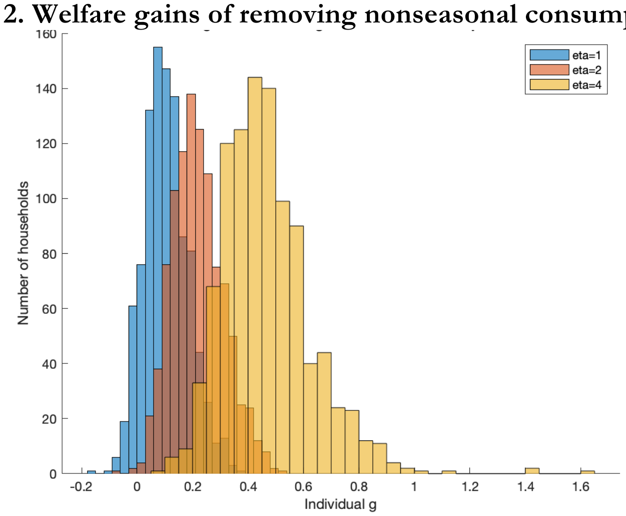


c) Compare and discuss your results in (a) and (b).
The comparison is explained in part a) and b).

d) Redo for η={2,4}.

As illustrated before, the welfare gains are the same for any level of seasonality, but differ across individuals when we remove the nonseasonal consumption risk. By observing Figure 2 and following the same reasoning as in previous parts, it can be seen that the higher is η the higher are the welfare gains of removing non-seasonal consumption risk. Remember that the higher is η implies a more risk-averse individual and, therefore, the more he/she needs to be compensated when there is present an individual stochastic component.

Figure 2. Welfare gains of removing nonseasonal consumption risk



Part 2

a) Compute the welfare gains of removing the seasonal component (all combinations of deterministic and stochastic) from the stream of consumption separately for each degree of seasonality in Table 1 and 2.

In order to compute the welfare gains, it is going to be considered that the deterministic seasonal component has a degree of seasonality "middle".

Here, we are going to compare the welfare gains of changing from the consumption with all the shocks (individual stochastic component, deterministic and stochastic seasonal component) with the consumption after removing the (deterministic or stochastic, or both) seasonal component following the degree of seasonality given in Table 1 and Table 2.

Mean welfare gains of removing the deterministic seasonal component

$$\sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{allshocks}(1+g)) \right] = \sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{deterministicseasonal}) \right]$$

Find the results for the different levels of risk-aversion (η) and different degrees of seasonality in Table 3:

Table 3. Mean welfare gains of removing the deterministic seasonal component			
	Low seasonality	Medium seasonality	High seasonality
η=1	0.0086	0.0086	0.0086
η=2	0.0185	0.0185	0.0185
η=4	0.0426	0.0426	0.0426

Notice the results are the same as in Part 1a): the higher is the degree of seasonality, the higher is the compensation that needs to be given to individuals when facing seasonality. Moreover, the higher is the risk-averse parameter, the higher has to be the compensation for the individual in order to bear risk.

Mean welfare gains of removing the stochastic seasonal components

$$\sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{allshocks}(1+g)) \right] = \sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{stochasticseasonal}) \right]$$

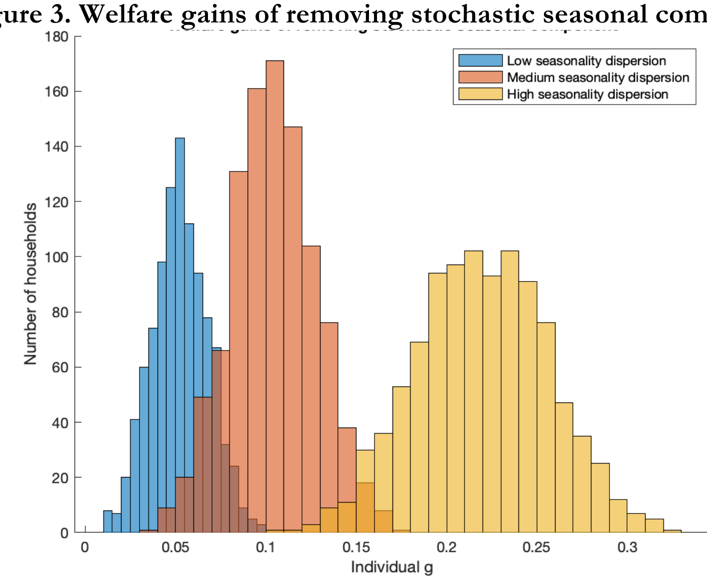
Find the results for the different levels of risk-aversion (η) and different degrees of seasonality in Table 4:

Table 4. Mean welfare gains of removing the stochastic seasonal components			
	Low seasonality	Medium seasonality	High seasonality
η=1	0.0513	0.1050	0.2208
η=2	0.1007	0.2135	0.4826
η=4	0.1913	0.4455	1.1901

It can be seen that the higher is the level of the stochastic seasonal component, the higher are the welfare gains of removing stochastic seasonality. This again is explained by the fact that individuals are risk-averse and prefer to smooth consumption rather than having consumption paths that vary a lot, which is what happens with the stochastic seasonal component. Moreover, the higher is the risk-averse parameter, the higher has to be the compensation for the individual in order to bear risk.

Again, the welfare gains differ across individuals when we remove the stochastic seasonal component. By observing Figure 3, it can be observed that the higher is the seasonality dispersion, the higher are the welfare gains.

Figure 3. Welfare gains of removing stochastic seasonal component



Mean welfare gains of removing the both seasonal components

$$\sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{allshocks}(1+g)) \right] = \sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{bothseasonal}) \right]$$

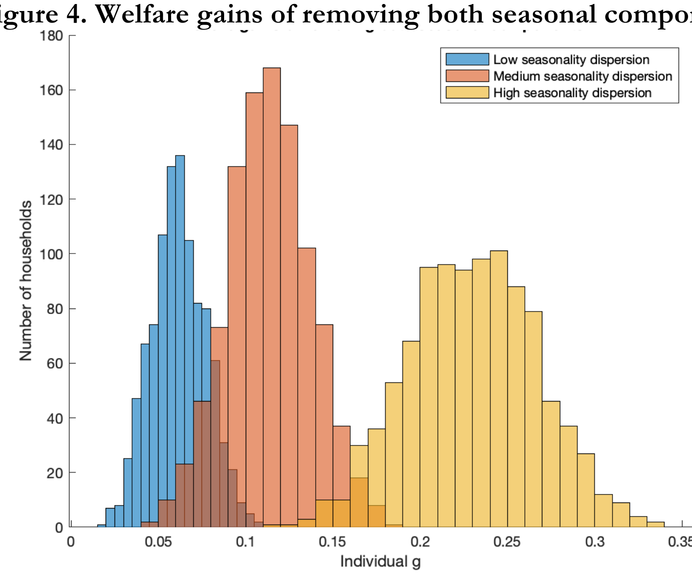
Find the results for the different levels of risk-aversion (η) and different degrees of seasonality in Table 5:

Table 5. Mean welfare gains of removing the both seasonal components			
	Low seasonality	Medium seasonality	High seasonality
η=1	0.0603	0.1145	0.2314
η=2	0.1210	0.2359	0.5099
η=4	0.2421	0.5071	1.2834

The results in Table 5 constitute a sum of the welfare gains of removing the stochastic and the deterministic components separately. On the one hand, there is the deterministic component of seasonality that make individuals worse off given that they are risk-averse. Remember that this effects are stronger the higher is the seasonality. On the other hand, there is the stochastic component of seasonality that adds more uncertainty to the consumption path. Notice that the higher is the stochastic component, the higher has to be the compensation for bearing risk. The last point can be also observed in Figure 4.

Once more, the higher is the risk-averse parameter, the higher has to be the compensation for the individual in order to bear risk.

Figure 4. Welfare gains of removing both seasonal components



b) Compute the welfare gains of removing the nonseasonal consumption risk.

Here, we are going to compare the welfare gains of changing from the consumption with all the shocks (individual stochastic component, deterministic and stochastic seasonal component) with the consumption after removing the nonseasonal consumption risk following the degree of seasonality given in Table 1 and Table 2.

$$\sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{allshocks}(1+g)) \right] = \sum_{t=1}^{40} \beta^{12t} \left[\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t}^{noseasonalconsumptionrisk}) \right]$$

Find the results for the different levels of risk-aversion (η) and different degrees of seasonality in Table 6:

Table 6. Mean welfare gains of removing the nonseasonal consumption risk			
	Low seasonality	Medium seasonality	High seasonality
η=1	0.1096	0.1096	0.1096
η=2	0.2168	0.2171	0.2165
η=4	0.4684	0.4664	0.4499

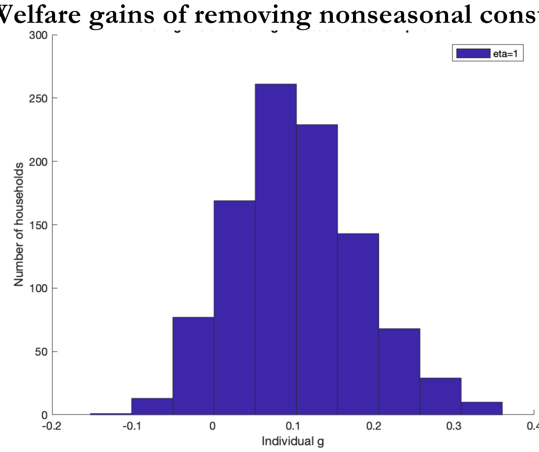
As is Part 1b):

First, notice that since we have removed the individual stochastic component (or nonseasonal consumption risk), the welfare gains are the same for all the degrees of seasonality.

Second, given that the realization of individual shocks (ε_t) are different across individuals, the welfare gains are not the same across individuals. This can be observed in Figure 5, where there is plotted the distribution of welfare gains. Notice that the majority of the individuals receive positive welfare gains (g>0) when we remove the nonseasonal consumption risk.

Moreover, the higher is the risk-averse parameter, the higher has to be the compensation for the individual in order to bear risk.

Figure 5. Welfare gains of removing nonseasonal consumption risk



c) Compare and discuss your results in (a) and (b).
The comparison is explained in part a) and b).

d) Redo for η={2,4}.
Done in part a) and b).

Question 2 - Adding Seasonal Labour Supply

On the one hand, we are given that household *I* of age *t* in season *m* obtains a level consumption defined as the following random variable:

$$c_{m,t} = z\left[e^{g(m)}e^{-\sigma_m^2/2}\epsilon_m e^{-\sigma_\epsilon^2/2}\epsilon_t\right]$$

On the other hand, we are also given the household *i* lifetime utility is:

$$W(z) = \sum_{t=1}^{40}\beta^{12t}\left[\sum_{m=1}^{12}\beta^{m-1}\left(\log c_{m,t} + \kappa\frac{h_{m,t}^{1+\frac{1}{v}}}{1+\frac{1}{v}}\right)\right]$$

In order to generate highly positively correlated stochastic components for consumption and labour it is assumed that the seasonal shocks $ln\ \varepsilon_m^c$ and $ln\ \varepsilon_m^h$ follow the following process:

$$\begin{pmatrix} ln\ \varepsilon_{m,i}^c \\ ln\ \varepsilon_{m,i}^h \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & 0.03 \\ 0.03 & \sigma_m^2 \end{pmatrix}\right)$$

In order to generate highly positively correlated deterministic seasonal components it is assumed that $g(m)$ takes the same values for consumption and labour.

In order to check the different effects, I add or remove the shocks that are relevant for each part of the exercise in the same way I proceeded in the previous question. However, if I want to understand what are the effects in particular for consumption, I just add or remove the shocks in the part of the utility function that corresponds to consumption, while keeping the part of labour with all the shocks. If instead I want to analyse labour, I just add or remove the shocks in the part of the utility function that corresponds to labour, while keeping the part of consumption with all the shocks.

a) Assume a deterministic seasonal component and a stochastic seasonal component for labour supply both of which are highly positively correlated with their consumption counterparts. Then, compute the welfare gains of removing seasons isolating the effects of consumption and leisure.

Here, we are going to compare the welfare gains of changing from the consumption with all the shocks (individual stochastic component, deterministic and stochastic seasonal component) with the consumption after removing the seasonal risks following the degree of seasonality given in Table 1 and Table 2.

Find the results for the different effects and different degrees of seasonality in Table 7:

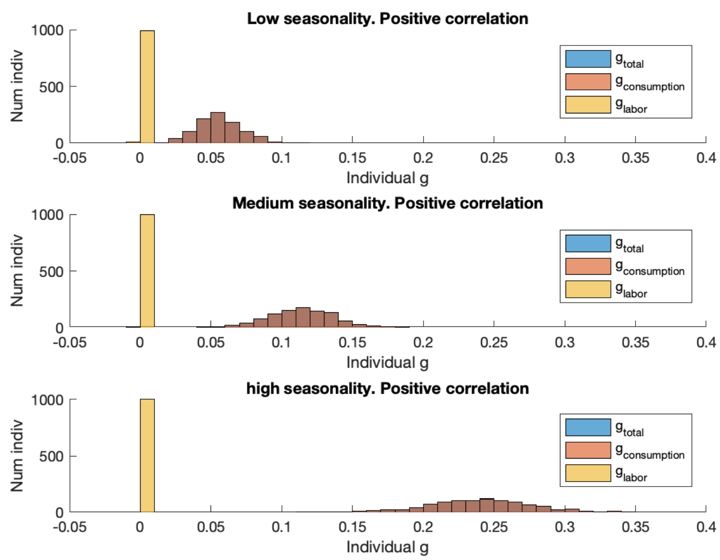
Table 7. Mean welfare gains of removing seasonal risks (positive correlation)			
	Low seasonality	Medium seasonality	High seasonality
Total effects	0.0559	0.1148	0.2412
Consumption effects	0.0559	0.1147	0.2411
Labour effects	0.0134	0.0323	0.0873

The first thing to notice is that labour effects are very small in comparison to the consumption (or total) effects. Moreover, the consumption effects are almost equal to the total effects (I was expecting that the total effects were going to be equal to the sum of consumption and labour effects).

Second, the distribution of welfare gains is different across individuals since the seasonal stochastic shocks realization vary across individuals. This fact can be seen in Figure 6, where it is clear that the consumption effects dominate and are almost overlapping the total effects.

Third, notice that the higher is the seasonality, the higher are the welfare gains of removing the seasonal risk, again reflecting the risk-aversion of the individual of not smoothing consumption.

Figure 6. Welfare gains of removing both seasonal risks (positive correlation)



b) Assume a deterministic seasonal component and a stochastic seasonal component for labour supply both of which are highly negatively correlated with their consumption counterparts. Then, compute the welfare gains of removing seasons isolating the effects of consumption and leisure.

In order to generate highly negatively correlated stochastic components for consumption and labour it is assumed that the seasonal shocks $ln\ \varepsilon_m^c$ and $ln\ \varepsilon_m^h$ follow the following process:

$$\begin{pmatrix} ln\ \varepsilon_{m,i}^c \\ ln\ \varepsilon_{m,i}^h \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & -0.03 \\ -0.03 & \sigma_m^2 \end{pmatrix}\right)$$

In this case, to generate highly negatively correlated deterministic seasonal components it is assumed that $g^c(m) = -g^h(m)$. The two values for the deterministic seasonal components are going to be the same, but the ones for labour will have a negative sign.

Here, we are going to compare the welfare gains of changing from the consumption with all the shocks (individual stochastic component, deterministic and stochastic seasonal component) with the consumption after removing the seasonal risks following the degree of seasonality given in Table 1 and Table 2.

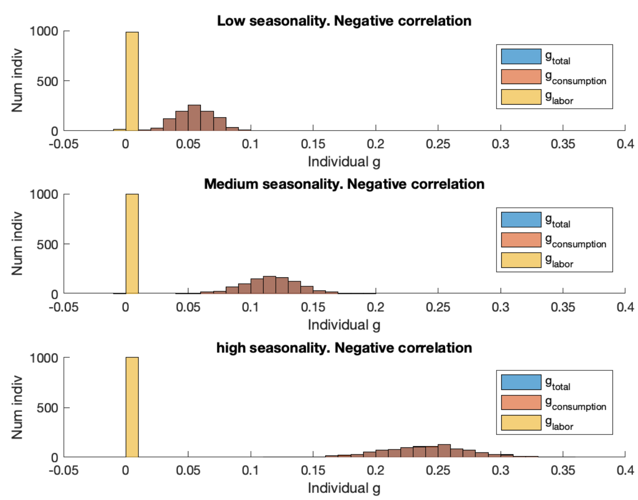
Find the results for the different effects and different degrees of seasonality in Table 8:

Table 8. Mean welfare gains of removing seasonal risks (negative correlation)

	Low seasonality	Medium seasonality	High seasonality
Total effects	0.0554	0.1163	0.2407
Consumption effects	0.0554	0.1163	0.2406
Labour effects	0.1351	0.3275	0.8526

The results and comments are roughly the same as in Part a).

Figure 7. Welfare gains of removing both seasonal risks (negative correlation)



c) How do your answers to (a) and (b) change if the nonseasonal stochastic component of consumption and leisure are correlated?

As can be observed in the figures and tables presented in Part a) and b), it seems that there is no difference in results when we assume either positive or negative correlation between the seasonal components of consumption and labour. The reason why we found these results is probably due to the fact that the utility function we are using does not account for any complementarity between consumption and labour.