Quantitative Methods for Finance

2025 Marina Dolfin

QMF2_3_Modelling_Regression

Estimating a Linear Regression Model with OLS

This activity introduces ordinary least squares (OLS) linear regression in MATLAB using a tiny, didactic house-pricing dataset. We'll keep the workflow practical.

(A short theoretical background is provided at the end of the Live Script.)

Workflow (the basic one):

- Load a small table of house prices and basic features
- Fit a linear model with fitlm
- Inspect fit quality with the built-in diagnostic plot
- Make a point prediction for a new house profile using predict

Dataset

HousePrices is a table file (XLSX) with a numeric response variable Price and a couple of predictors (Size and a simple indicator feature).

This example shows the typical workflow for linear regression analysis using `fitlm`. The general workflow should include preparing a data set, fitting a linear regression model, evaluating and improving the fitted model, and predicting response values for new predictor data.

In the present example we perform only the very basic steps

```
% Import data from spreadsheet
HousePrices2 = readtable("C:\Users\mdolf\OneDrive\Documents\Didattica 2026\London\Job\GitHub Materials\HousePrices.xlsx");

% Display results
HousePrices2
```

HousePrices2 = 47×3 table

	Sfeet	Bedrooms	Price
1	2104	3	399900

	Sfeet	Bedrooms	Price
2	1600	3	329900
3	2400	3	369000
4	1416	2	232000
5	3000	4	539900
6	1985	4	299900
7	1534	3	314900
8	1427	3	198999
9	1380	3	212000
10	1494	3	242500
11	1940	4	239999
12	2000	3	347000
13	1890	3	329999
14	4478	5	699900

⋮

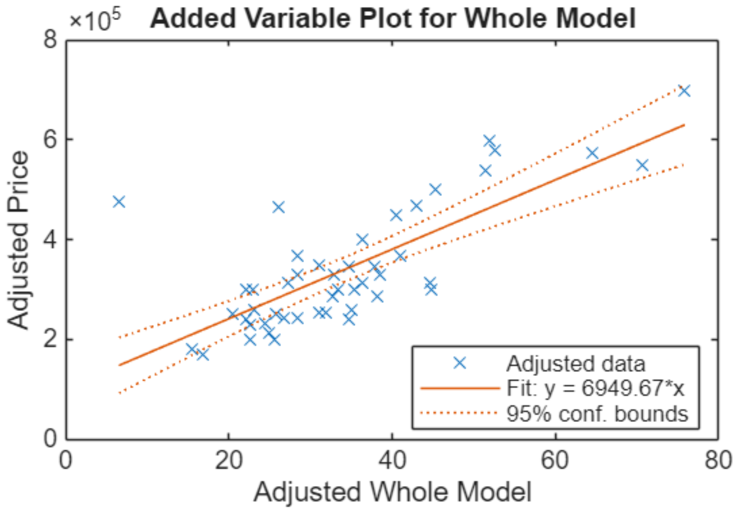
Fit an OLS linear model using the table; by default the **last column** is treated as the response (set `'ResponseVar','Price'` to be explicit).

```
model=fitlm(HousePrices)
```

```
model = 
Linear regression model:
    Price ~ 1 + Sfeet + Bedrooms

Estimated Coefficients:
                   Estimate       SE       tStat       pValue  
                  __________    ______    _______    __________

    (Intercept)   1.0293e+05     52587     1.9573      0.056669
    Sfeet             110.01    17.521     6.2784    1.3143e-07
    Bedrooms          6948.8     19233    0.36129       0.71961


Number of observations: 47, Error degrees of freedom: 44
Root Mean Squared Error: 8.33e+04
R-squared: 0.576,  Adjusted R-Squared: 0.556
F-statistic vs. constant model: 29.9, p-value = 6.44e-09
```

Show the built-in diagnostic plot: with **one predictor it's data + fitted line; with multiple predictors it's an added-variable** (partial regression) plot.

```
plot(model)
```



Predict for a new case; when you pass a **matrix/row, columns must match the model's predictor order**. (Using a table with named variables avoids order mistakes.)

```
estimatedPrice= predict(model,[5000,1]);

fprintf('Estimated price: $%.0f\n', estimatedPrice);
```

```
Estimated price: $659908
```

Theoretical background (example of unconstrained quadratic optimization).

Model and assumptions

Let $y \in \mathbb{R}^n$ be the response (or regressand), $X \in \mathbb{R}^{n\times p}$ the matrix of regressors, $\beta \in \mathbb{R}^p$ the coefficients, and $\varepsilon \in \mathbb{R}^n$the errors.

$$y = X\beta + \varepsilon, \qquad \mathbb{E}[\varepsilon] = 0, \qquad \mathrm{Var}(\varepsilon) = \sigma^2 I_n.$$

OLS minimizes the residual sum of squares (RSS):

$$\mathrm{RSS}(\beta) = \|y - X\beta\|_2^2 = (y - X\beta)^\top (y - X\beta) = y^\top y - 2\beta^\top X^\top y + \beta^\top X^\top X\,\beta.$$

Since $X^\top X \succcurlyeq 0$, $\mathrm{RSS}(\beta)$ is a convex quadratic in β. If $\mathrm{rank}(X) = p$, the minimizer is unique.

Setting the gradient to zero yields the equations:

$$X^\top X\,\widehat{\beta} = X^\top y \quad \Rightarrow \quad \widehat{\beta} = (X^\top X)^{-1} X^\top y \quad (X^\top X \text{ is invertible under Gauss-Markov assumptions}).$$

Under Gauss-Markov assumptions:

$$\widehat{\sigma}^2 = \frac{\mathrm{RSS}(\widehat{\beta})}{n-p}, \qquad \mathrm{Var}(\widehat{\beta}) = \sigma^2 (X^\top X)^{-1} \approx \widehat{\sigma}^2 (X^\top X)^{-1}.$$

"Linear" refers to linearity in the coefficients, not necessarily in the raw predictors. A degree-d polynomial regression augments predictors:

$$\phi(x) = \left[1,\ x,\ x^2, \ldots, x^d\right],$$

and fits $y = \phi(x)\beta + \varepsilon$ by OLS. The model is linear in β even though it is nonlinear in x. (In practice, center/scale predictors before adding higher-order terms to mitigate multicollinearity.)

RESET (linearity/specification)

To probe functional-form misspecification, augment the baseline model with powers of the fitted values, e.g. $\widehat{y}^2, \widehat{y}^3$, and test their joint significance. For nested models M_0 (baseline) and M_1 (augmented) with $p_0 < p_1$ parameters and residual sums of squares $\mathrm{RSS}_0, \mathrm{RSS}_1$, the usual nested F-statistic is

$$F = \frac{(\mathrm{RSS}_0 - \mathrm{RSS}_1)/(p_1 - p_0)}{\mathrm{RSS}_1/(n - p_1)}.$$

A small p-value suggests misspecification/nonlinearity in the baseline specification.