Quantitative Methods for Finance

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QMF3_1_MachineLearning_Supervised

Logistic Regression for Binary Classification

This example illustrated how **logistic regression** can be used for **binary classification** problems.

Starting from raw exam score data, we:

- explore and visualized the dataset,
- train a logistic regression model using `fitglm`,
- interpret the model coefficients to understand the impact of each variable,
- compute predicted probabilities and training accuracy, and
- visualize the **decision boundary** separating the two classes.

The fitted model provides a simple yet powerful way to **estimate probabilities of categorical outcomes** and to **visualize the separating surface** in the predictor space — a key idea that extends naturally to higher-dimensional classification problems and more advanced machine learning models.

Data

The data_classification.txt contains scores for two exams in addition to a binary variable which denotes whether students were admitted to a university.

We obtain a logistic regression model to **predict admission probability** using the `fitglm` function.

Load the data into a `table` and preview the data

The first two columns (variables) will contain the exam scores and the third column the admission labels which we convert to `logical` values.

We also compute some summary statistics on the three variables.

```
clear;
data = readtable('data_classification.xlsx');

data.Properties.VariableNames = {'Exam1','Exam2','Admitted'};

data.Admitted = logical(data.Admitted)
```

data = 88×3 table

	Exam1	Exam2	Admitted
1	34	78	false
2	30	43	false
3	35	72	false
4	60	86	true
5	79	75	true
6	45	56	false
7	61	96	true
8	75	46	true
9	76	87	true
10	84	43	true
11	95	38	false
12	75	30	false
13	82	30	false
14	69	97	true

⋮

```
summary(data)
```

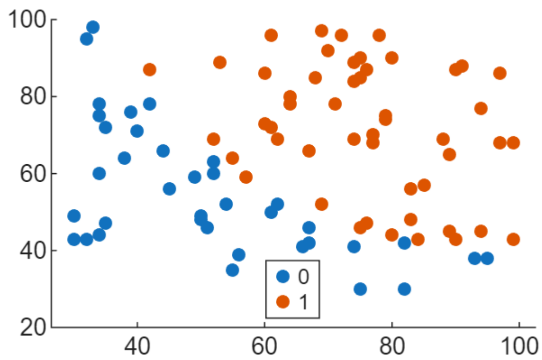
```
data: 88×3 table

Variables:

    Exam1: double
    Exam2: double
    Admitted: logical (50 true)

Statistics for applicable variables:

             NumMissing      Min        Median        Max        Mean          Std

    Exam1        0            30            67         99       65.2727       19.5093
    Exam2        0            30       65.5000         98       64.2955       18.7673
```

We create a **grouped scatter plot** of the two exam scores, using the variable `Admitted` to define the groups.

- Each point represents a student.
- The x-axis shows **Exam 1** scores and the y-axis shows **Exam 2** scores.
- MATLAB automatically assigns different colors or markers for the two groups (`Admitted = 1` and `Admitted = 0`).

This allows you to **visually separate admitted and non-admitted students** and observe whether a boundary between the two classes might exist in the exam-score space.

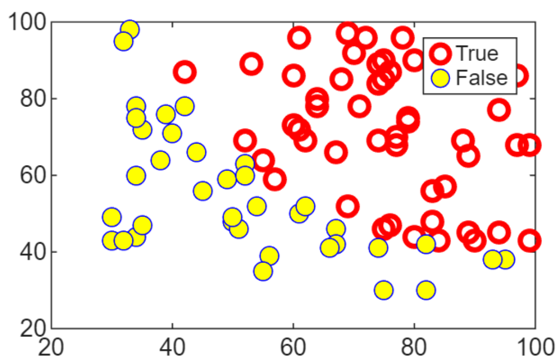
```
gscatter(data.Exam1,data.Exam2,data.Admitted)
```



We now plot the data manually to have full control over the style and prepare for adding the decision boundary.

Unlike **gscatter**, this approach lets us customize marker colors, sizes, and legend entries — using red circles for admitted students and yellow-filled blue circles for those not admitted.

```
varNames = data.Properties.VariableNames;
% Plot the data with + for true and 0 for false examples
inds = data.(varNames{3}) == 1;
plot(data.(varNames{1})(inds), data.(varNames{2})(inds), 'ro','LineWidth', 2,
'MarkerSize', 7);
inds = data.(varNames{3}) == 0;
hold on

plot(data.(varNames{1})(inds), data.(varNames{2})(inds), 'bo', 'MarkerFaceColor',
'y','MarkerSize', 7);
legend('True','False')
hold off
```



Logistic Regression

We train the logistic regression model using the `fitglm` function, which fits a **generalized linear model** (GLM) to the data.

By setting the **`'Distribution'`** parameter to **`'binomial'`**, we specify that the response variable (`Admitted`) is **binary** — taking only the values 0 or 1.

This tells MATLAB to use the **logistic (logit) link function**, so the model estimates the **probability of admission** as a function of the two exam scores.

```
logMdl = fitglm(data,'Distribution','binomial')
```

```
logMdl = 
Generalized linear regression model:
    logit(Admitted) ~ 1 + Exam1 + Exam2
    Distribution = Binomial

Estimated Coefficients:
                   Estimate       SE        tStat       pValue  
                   ________    ________    _______    __________

    (Intercept)    -24.469       5.8101    -4.2115    2.5365e-05
    Exam1          0.20664     0.049113     4.2074    2.5835e-05
    Exam2          0.19073      0.04872     3.9147    9.0499e-05


88 observations, 85 error degrees of freedom
Dispersion: 1
Chi^2-statistic vs. constant model: 83.4, p-value = 7.89e-19
```

Note the form of the model displayed in the output above. This is short-hand for

$$\text{logit}(Admitted) = 1 * \theta_0 + Exam1 * \theta_1 + Exam2 * \theta_2$$

Since $\text{logit}(x)$ is the *inverse function* of $\text{sigmoid}(x)$, this model is equivalent to logistic regression model form for the probability of admission used in ex2:

$$Admitted = h_\theta(x) = \text{sigmoid}(\theta^T x),$$

where x includes the two exam scores and a bias term. A bias term is added automatically by `fitglm`.

Predict the training accuracy and probability of admission

Recall that a prediction of admission corresponds to a predicted probability > 0.5. Run the code below to extract the θ values from the trained model, predict the probability of admission, and compute the training accuracy.

```
theta = logMdl.Coefficients.Estimate
```

```
theta = 3×1
  -24.4693
    0.2066
    0.1907
```

```
% Predict the probability for a student with scores of 45 and 85

prob = predict(logMdl,[45 85]);

fprintf('For a student with scores 45 and 85, we predict an admission probability
of %f\n\n', prob);
```

```
For a student with scores 45 and 85, we predict an admission probability of 0.739104
```

```
% Compute the training accuracy

Admitted = predict(logMdl,data) > 0.5;

fprintf('Train Accuracy: %f\n', mean(double(Admitted == data.Admitted)) * 100);
```

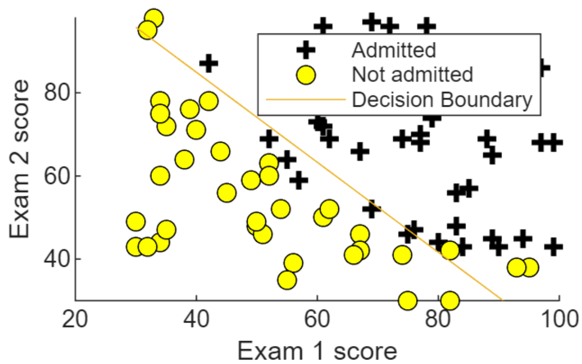
```
Train Accuracy: 88.636364
```

Visualize the decision boundary

Ceate a grid of exam scores and create the decision boundary plot.

```
figure; hold on;
% Plot the positive and negative examples

varNames = data.Properties.VariableNames;
% Plot the data with + for true and 0 for false examples
inds = data.(varNames{3}) == 1;
plot(data.(varNames{1})(inds), data.(varNames{2})(inds), 'k+','LineWidth', 2,
'MarkerSize', 7);
inds = data.(varNames{3}) == 0;


plot(data.(varNames{1})(inds), data.(varNames{2})(inds), 'ko', 'MarkerFaceColor',
'y','MarkerSize', 7);
% Plot the decision boundary
xvals = [min(data.Exam1), max(data.Exam1)];
```

```
yvals = -(theta(1)+theta(2)*xvals)/theta(3);
plot(xvals,yvals); hold off;
ylim([min(data.Exam2),max(data.Exam2)]);

% Labels and Legend
xlabel('Exam 1 score')
ylabel('Exam 2 score')
legend('Admitted','Not admitted','Decision Boundary')
hold off;
```



Next Step – Exercise on Classification (Exam Scores)

- Upload into the workspace the data contained into the spreadsheet data_classification.xlsx
- In the MATLAB Apps tab, select the Classification Learner app from the Machine Learning section
- Select 'New Session -> From Workspace' to start
- Under 'Workspace Variable', select 'data' and you will automatically find Exam1 and Exam2

- The app should correctly detect Admitted as the response variable to predict.

- Choose the default validation option.
- Select a model (e.g., logistic regression, decision tree, or SVM) and click the **Train** button.
- Experiment with several standard models using default settings and see if you can achieve at least 80% accuracy.

This exercise helps you compare the manual logistic regression workflow with MATLAB's interactive machine learning tools, reinforcing how supervised learning models can be trained and evaluated in different ways.