Postgraduate Course: Process Algebra

Communicating Sequential Process

Operational Semantics and Refinement

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Outline

1. Brief introduction of the operational semantics for GCL
2. Operational semantics for CSP in detail.
3. The relationship of simulation and refinement for CSP

Operational Semantics for GCL

1. Assignment

$$\langle x := e,\, \sigma\rangle \;\longrightarrow\; \langle \varepsilon,\, \sigma[e/x]\rangle$$

2. Sequential Composition

If $\langle P,\, \sigma\rangle \longrightarrow \langle \epsilon,\, \sigma'\rangle$, then $\langle P \;;\; Q,\, \sigma\rangle \longrightarrow \langle Q,\, \sigma'\rangle$

If $\langle P,\, \sigma\rangle \longrightarrow \langle P',\, \sigma'\rangle$, then $\langle P \;;\; Q,\, \sigma\rangle \longrightarrow \langle P' \;;\; Q,\, \sigma'\rangle$

3. Conditional

$$\langle P \lhd b \rhd Q,\, \sigma\rangle \;\longrightarrow\; \langle P,\, \sigma\rangle, \qquad \text{if} \quad b(\sigma)$$

$$\langle P \lhd b \rhd Q,\, \sigma\rangle \;\longrightarrow\; \langle Q,\, \sigma\rangle, \qquad \text{if} \quad \neg b(\sigma)$$

4. Iteration

$$\langle b * P,\, \sigma\rangle \;\longrightarrow\; \langle P \;;\; b * P,\, \sigma\rangle, \qquad \text{if} \quad b(\sigma)$$

$$\langle b * P,\, \sigma\rangle \;\longrightarrow\; \langle \mathbf{Skip},\, \sigma\rangle, \qquad \text{if} \quad \neg b(\sigma)$$

Prefix and Choice

1. Event Prefix

$$(a \rightarrow P) \xrightarrow{a} P$$

2. Choosing between Events

$$(x : A \rightarrow P(x)) \xrightarrow{a} P(a), \qquad \text{Here, } a \in A$$

Input and Output

1. **Input**

$$(c!v \to P) \xrightarrow{c.v} P$$

2. **Output**

$$(c?x : T \to P(x)) \xrightarrow{c.v} P(v), \qquad v \in T$$

Choice

1. Internal Choice

$$P \sqcap Q \xrightarrow{\tau} P$$

$$P \sqcap Q \xrightarrow{\tau} Q$$

2. Recursion

$$\mu X \bullet F(X) \xrightarrow{\tau} F(\mu X \bullet F(X)/X)$$

3. External choice

(1) If $P \xrightarrow{\tau} P'$, then $P \Box Q \xrightarrow{\tau} P' \Box Q$

$$Q \Box P \xrightarrow{\tau} Q \Box P'$$

(2) If $P \xrightarrow{a} P'$, then $P \Box Q \xrightarrow{a} P'$

$$Q \Box P \xrightarrow{a} P'$$

Parallel Composition

(1) If $P \xrightarrow{\tau} P'$, then $P \,{}_B\|_C\, Q \xrightarrow{\tau} P' \,{}_B\|_C\, Q$

(2) If $Q \xrightarrow{\tau} Q'$, then $P \,{}_B\|_C\, Q \xrightarrow{\tau} P \,{}_B\|_C\, Q'$

(3) If $P \xrightarrow{a} P'$, then $P \,{}_B\|_C\, Q \xrightarrow{a} P' \,{}_B\|_C\, Q$ $(a \in B - C)$

(4) If $Q \xrightarrow{a} Q'$, then $P \,{}_B\|_C\, Q \xrightarrow{a} P \,{}_B\|_C\, Q'$ $(a \in C - B)$

(5) If $P \xrightarrow{a} P'$ and $Q \xrightarrow{a} Q'$

then $P \,{}_B\|_C\, Q \xrightarrow{a} P \,{}_B\|_C\, Q'$ $(a \in B \cap C)$

Interleaving and Sequential Composition

1. Interleaving

If $P \xrightarrow{x} P'$, then $P|||Q \xrightarrow{x} P'|||Q$

If $Q \xrightarrow{x} Q'$, then $P|||Q \xrightarrow{x} P|||Q'$

2. Sequential Composition

If $P \xrightarrow{x} P'$, then $P \; ; \; Q \xrightarrow{x} P' \; ; \; Q \; (x \neq \surd)$

If $\exists P' \bullet P \xrightarrow{\surd} P'$, then $P \; ; \; Q \xrightarrow{\surd} Q$

Concealment Renaming

1. Concealment

If $P \xrightarrow{x} P'$, then $P \setminus B \xrightarrow{x} P' \setminus B \ (x \notin B)$

If $P \xrightarrow{a} P'$, then $P \setminus B \xrightarrow{\tau} P' \setminus B \ (a \in B)$

2. Renaming

If $P \xrightarrow{x} P'$, then $f[P] \xrightarrow{y} f[P'] \quad (y = f(x))$

If $P \xrightarrow{x} P'$, then $f^{-1}[P] \xrightarrow{y} f^{-1}[P'] \quad (f(y) = x)$

Divergence

1. Definition of Divergence

A process P can diverge (denoted as $\uparrow P$) if it is infinitely often capable of τ transition.

$$P \xrightarrow{\tau} P_0 \xrightarrow{\tau} \ldots \xrightarrow{\tau} P_n \xrightarrow{\tau} \ldots$$

2. Definition of Stableness

A process P is called stable if it cannot perform τ transition currently.

$$Stable(P) =_{df} (\neg \exists P' \bullet P \xrightarrow{\tau} P')$$

Failure and Divergence Revisited (1)

(1) Some Definitions

(a) $P \xrightarrow{*} P' =_{df} P(\xrightarrow{\tau})^* P'$

(b) $P \overset{a}{\Longrightarrow} P' =_{df} \exists P_1 \bullet P \xrightarrow{*} P_1 \;\wedge\; P_1 \xrightarrow{a} P'$

(c) Assume $s = a_1 a_2 ... a_n$ and $s' = a_2 ... a_n$. Then

$$P \overset{s}{\Longrightarrow} P' =_{df} \exists P_1 \bullet P \overset{a_1}{\Longrightarrow} P_1 \;\wedge\; P_1 \overset{s'}{\Longrightarrow} P'$$

Failure and Divergence Revisited (2)

(2) Failure

$$failure(P)$$

$$=_{df} \{(s, X) \mid \exists P_1, P_2 \bullet \; P \overset{s}{\Longrightarrow} P_1 \;\wedge\; P_1 \overset{*}{\longrightarrow} P_2 \;\wedge stable(P_2) \;\wedge \\ \forall c \in X \bullet \neg(P_2 \overset{c}{\longrightarrow}) \;\}$$

(3) Divergence

$$divergence(P)$$

$$=_{df} \{s \mid \exists P_1 \bullet P \overset{s}{\Longrightarrow} P_1 \;\wedge\; \uparrow P_1\}$$

Simulation

Definition

A simulation is defined as a binary relation $\mathcal{R}$ satisfying that

(1) If $\uparrow P$, then $\uparrow Q$,

Otherwise,

(2) If $P \;\mathcal{R}\; Q$ and $P \stackrel{a}{\Longrightarrow} U$, then there exists a process W

such that $\quad Q \stackrel{a}{\Longrightarrow} W$ and $U \;\mathcal{R}\; W$

(3) If $P \stackrel{*}{\longrightarrow} P_1$ and $stable(P_1)$ and $\neg(P \stackrel{a}{\longrightarrow})$, then

$\exists Q_1 \bullet Q \stackrel{*}{\longrightarrow} Q_1$ and $stable(Q_1)$ and $\neg(Q \stackrel{a}{\longrightarrow})$,

We use $\leq$ to denote the largest simulation relation.

Compositional Properties

Theorem 1 If $P \le Q$, then

(a) $a \to P \le a \to Q$

(b) $P \sqcap R \le Q \sqcap R$

(c) $P \Box R \le Q \Box R$

(d) $P || R \le Q || R$

(e) $P \ ; \ R \le Q \ ; \ R$ and $R \ ; \ P \le R \ ; \ Q$

Theorem 2

For all $x \in B$, if $P(x) \le Q(x)$, then $x : B \to P(x) \le x : B \to Q(x)$

Relationship between Simulation and Refinement

(1) Refinement

$$\begin{aligned} & P \ \mathbf{refine} \ Q \\ =_{df} \ & failue(P) \subseteq failure(Q) \ \wedge \\ & divergence(P) \subseteq divergence(Q) \end{aligned}$$

(2) Theorem

If $P \leq Q$, then P **refine** Q