Bisimulation in CCS

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Syntax and Semantics of CCS

Syntax

$P ::= 0 \mid \alpha.P \mid \sum_{i\in I} P_i \mid P|P \mid P\backslash L \mid P[f]$

Operational Semantics

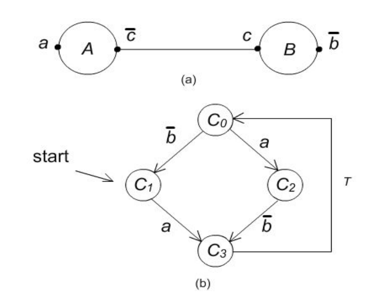
- Act: $\dfrac{}{\alpha.P \xrightarrow{\alpha} P}$ Sum$_j$: $\dfrac{P_j \xrightarrow{\alpha} P_j'}{\sum_{i\in I} P_i \xrightarrow{\alpha} P_j'} (j \in I)$
- Com: $\dfrac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$ $\dfrac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$ $\dfrac{P \xrightarrow{l} P' \quad Q \xrightarrow{\bar{l}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$
- Res: $\dfrac{P \xrightarrow{\alpha} P'}{P\backslash L \xrightarrow{\alpha} P'\backslash L} (\alpha, \alpha' \notin L)$ Rel: $\dfrac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$
- Con: $\dfrac{P \xrightarrow{\alpha} P'}{A \xrightarrow{\alpha} P'}$ $(A \overset{def}{=} P)$

Definition of Strong Bisimulation

A binary relation $\mathcal{S} \subseteq \mathcal{P} \times \mathcal{P}$ over agents is a strong bisimulation if $(P, Q) \in \mathcal{S}$ implies, for all $\alpha \in Act$,
(i) Whenever $P \xrightarrow{\alpha} P'$ then, for some Q', $Q \xrightarrow{\alpha} Q'$ and $(P', Q') \in \mathcal{S}$
(ii) Whenever $Q \xrightarrow{\alpha} Q'$ then, for some P', $P \xrightarrow{\alpha} P'$ and $(P', Q') \in \mathcal{S}$
Denoted by $P \sim Q$.

Example for Strong Bisimulation

- Definition of (a):
  - $A \stackrel{def}{=} a.A'$
    $A' \stackrel{def}{=} \bar{c}.A$
  - $B \stackrel{def}{=} c.B'$
    $B' \stackrel{def}{=} \bar{b}.B$
  - $(a) \stackrel{def}{=} (A|B)\backslash c$
- Definition of (b):
  - $C_0 \stackrel{def}{=} \bar{b}.C_1 + a.C_2$
    $C_1 \stackrel{def}{=} a.C_3$
    $C_2 \stackrel{def}{=} \bar{b}.C_3$
    $C_3 \stackrel{def}{=} \tau.C_0$
  - $(b) \stackrel{def}{=} C_1$



Checking Strong Bisimulation

- (a) behaves like (b)
- $\mathcal{S} = \{((A|B)\backslash c, C_1), ((A'|B)\backslash c, C_3),$
  $((A|B')\backslash c, C_0), ((A'|B')\backslash c, C_2)\}$

Example

- $(A|B')\backslash c \xrightarrow{a} (A'|B')\backslash c$
  $(A|B')\backslash c \xrightarrow{\bar{b}} (A|B)\backslash c$
- $C_0 \xrightarrow{a} C_2$
  $C_0 \xrightarrow{\bar{b}} C_1$

- $(a) \sim (b)$

Definition of Weak Bisimulation

Preliminary Definitions

- If $t \in Act^*$, then $\hat{t} \in \mathcal{L}^*$ is the sequence gained by deleting all occurrences of τ from t.
  In particular, $\widehat{\tau^n} = \varepsilon$
- If $t = \alpha_1 \cdots \alpha_n \in Act^*$, then we write $E \xrightarrow{t} E'$ if $E \xrightarrow{\alpha_1} \cdots \xrightarrow{\alpha_n} E'$.
- If $t = \alpha_1 \cdots \alpha_n \in Act^*$, then $E \overset{t}{\Rightarrow} E'$ if
  $$E(\xrightarrow{\tau})^* \xrightarrow{\alpha_1} (\xrightarrow{\tau})^* \cdots (\xrightarrow{\tau})^* \xrightarrow{\alpha_n} (\xrightarrow{\tau})^* E'$$

Definition of Weak Bisimulation

A binary relation $\mathcal{S} \subseteq \mathcal{P} \times \mathcal{P}$ over agents is a weak bisimulation if $(P, Q) \in \mathcal{S}$ implies, for all $\alpha \in Act$,

(i) Whenever $P \xrightarrow{\alpha} P'$ then, for some Q', $Q \overset{\hat{\alpha}}{\Rightarrow} Q'$ and $(P', Q') \in \mathcal{S}$

(ii) Whenever $Q \xrightarrow{\alpha} Q'$ then, for some P', $P \overset{\hat{\alpha}}{\Rightarrow} P'$ and $(P', Q') \in \mathcal{S}$

Denoted by $P \approx Q$.

Example for Weak Bisimulation

- Definition of (b):
  - $C_0 \overset{def}{=} \bar{b}.C_1 + a.C_2$

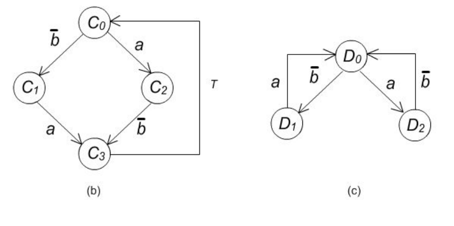
    $C_1 \overset{def}{=} a.C_3$

    $C_2 \overset{def}{=} \bar{b}.C_3$

    $C_3 \overset{def}{=} \tau.C_0$
  - $(b) \overset{def}{=} C_1$
- Definition of (c):
  - $D_0 \overset{def}{=} a.D_2 + \bar{b}.D_1$

    $D_1 \overset{def}{=} a.D_0$

    $D_2 \overset{def}{=} \bar{b}.D_0$
  - $(c) \overset{def}{=} D_1$



Checking Weak Bisimulation

- (c) behaves like (b) in spite of internal actions
- $\mathcal{S} = \{(C_0, D_0), (C_1, D_1),$
  $(C_2, D_2), (C_3, D_0)\}$

Example

- $C_3 \xrightarrow{\tau} C_0$
  $C_0 \xrightarrow{a} C_2 \qquad C_3 \stackrel{a}{\Rightarrow} C_2$
  $C_0 \xrightarrow{\bar{b}} C_1 \qquad C_3 \stackrel{\bar{b}}{\Rightarrow} C_1$
- $D_0 \xrightarrow{a} D_2$
  $D_0 \xrightarrow{\bar{b}} D_1$

- $(b) \approx (c)$