A Calculus of Broadcasting Systems

December 21, 2012

Overview

Communication

- point-to-point message passing (CSP, CCS, ACP)
  - $$\frac{B_1 \xrightarrow{\lambda v} B_1' \quad B_2 \xrightarrow{\bar{\lambda} v} B_2'}{B_1 | B_2 \xrightarrow{\tau} B_1' | B_2'}$$ (CCS Com)
- broadcast
  1. processes speak one at a time and are heard instantaneously by all others
  2. contention between speakers is nondeterministically
  3. ...

Reference

1. K. V. S. Prasad, A Calculus of Broadcasting Sytems. TAPSOFT. Vol.1 1991, 338-358, 1991.
2. K. V. S. Prasad, A Calculus of Broadcasting Systems. Sci. Comput. Program. 25(2-3), 285-327, 1995.

Calculus of Broadcasting Systems

Informal models of broadcasting

1. Local area networks (physical model)
   - Message (header, value), the header specifies who is to read the message
   - Each message is received by all
   - Each receiver examines the header of message
     If the header is one it is monitoring, receiver reads it
     Otherwise it discards it
   - Only one can broadcast at a time
2. Speech as broadcast communication (intuitive model)
   - The public address system

Syntax of CBS

The syntax of CBS

$$E ::= X \mid \mathbf{0} \mid \tau!E \mid a(v)!E \mid a(x)?E \mid E + E \mid E|E \mid$$
$$E\backslash\mathcal{N} \mid E[\phi] \mid recX.E$$

where $a(v)$ stands for the message with name a and value v.

- τ: a special name
- X: a variable, x: a variable internal to an agent
- $\mathcal{N} \subseteq \mathcal{S}$ a set of message names
- $\phi : \mathcal{S} \cup \{\tau\} \to \mathcal{S} \cup \{\tau\}$ and $\phi(\tau) = \tau$

where $\mathcal{S}$ is the set of message names

Operational Semantics of CBS

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Actions

- $a(v)!$ is the transmit prefix.

$$a(v)!E \xrightarrow{a(v)!} E$$
$$\tau!E \xrightarrow{\tau!} E \qquad \text{(Transmit-Transmit)}$$

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$$a(x)?E(x) \xrightarrow{a(v)?} E(v) \qquad \text{(Monitor-Read)}$$

e.g. $a(x)?b(y)?c(x+y)!F$

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- $\mathbf{0}$ has nothing to transmit or monitor.

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Discards

- $a(v)!E$ will only transmit and discards all messages it receives.

$$a(v)!E \xrightarrow{b(u):} a(v)!E$$
$$\tau!E \xrightarrow{a(v):} \tau!E \qquad \text{(Transmit-Discard)}$$

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- $a(x)?E(x)$ monitors only a and discards messages with any other name.

$$a(x)?E \xrightarrow{b(v):} a(x)?E \quad a \neq b \qquad \text{(Monitor-Discard)}$$

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- $\mathbf{0}$ discards everything it receives.

$$\mathbf{0} \xrightarrow{a(v):} \mathbf{0} \qquad (\mathbf{0}\text{-discard})$$

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- $E \xrightarrow{\tau:} E$ (Expr-Discard)

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Rules of Sum

- $$\frac{E \xrightarrow{\mu} E'}{E + F \xrightarrow{\mu} E'} \qquad \frac{E \xrightarrow{\mu} E'}{F + E \xrightarrow{\mu} E'}$$

(Sum-Transmit/Read)

where $\mu \in Act = (\mathcal{S} \times V \times \{!, ?\}) \cup \{\tau!\}$

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Rules of Sum

- $$\frac{E \xrightarrow{\mu} E'}{E + F \xrightarrow{\mu} E'} \quad \frac{E \xrightarrow{\mu} E'}{F + E \xrightarrow{\mu} E'}$$ (Sum-Transmit/Read)
- $$\frac{E \xrightarrow{a(v):} E \quad F \xrightarrow{a(v):} F}{E + F \xrightarrow{a(v):} E + F}$$ (Sum-Discard)

where $\mu \in Act = (\mathcal{S} \times V \times \{!, ?\}) \cup \{\tau!\}$

Operational Semantics of CBS

Example: a cat, her owner, and his friend

$$MEIOSIS \stackrel{def}{=\!=} meiosis?miao!MEIOSIS$$

$$OWNER \stackrel{def}{=\!=} meiosis!miao?ha?SUCC$$

$$FRIEND \stackrel{def}{=\!=} meiosis?miao?ha!\mathbf{0}$$

$$CATSYSTEM \stackrel{def}{=\!=} MEIOSIS \mid OWNER \mid FRIEND$$

where
MEIOSIS is a cat who answers to her name.
OWNER proves this by doing a test with *FRIEND*.
SUCC denotes the successful state of the test.

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Intuitive transition

$$\begin{aligned} CATSYSTEM &\xrightarrow{meiosis!} miao!MEIOSIS|miao?ha?SUCC|miao?ha!\mathbf{0} \\ &\xrightarrow{miao!} MEIOSIS \mid ha?SUCC \mid ha!\mathbf{0} \\ &\xrightarrow{ha!} MEIOSIS \mid SUCC \mid \mathbf{0} \end{aligned}$$

Operational Semantics of CBS

Rules of Parallel Composition

- $$\frac{E \xrightarrow{a(v)!} E' \quad F \xrightarrow{a(v)?} F'}{E|F \xrightarrow{a(v)!} E'|F'}$$ (Communication)

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Rules of Parallel Composition

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- $$\frac{E \xrightarrow{a(v)?} E' \quad F \xrightarrow{a(v)?} F'}{E|F \xrightarrow{a(v)?} E'|F'}$$ (Join-read)

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- $$\frac{E \xrightarrow{a(v)?} E' \quad F \xrightarrow{a(v)?} F'}{E|F \xrightarrow{a(v)?} E'|F'}$$ (Join-read)
- $$\frac{E \xrightarrow{a(v)!} E' \quad F \xrightarrow{a(v):} F}{E|F \xrightarrow{a(v)!} E'|F}$$ (Interleave)

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- $$\frac{E \xrightarrow{a(v)?} E' \quad F \xrightarrow{a(v)?} F'}{E|F \xrightarrow{a(v)?} E'|F'}$$ (Join-read)
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- $$\frac{E \xrightarrow{a(v)?} E' \quad F \xrightarrow{a(v):} F}{E|F \xrightarrow{a(v)?} E'|F}$$ (Read-discard)

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- $$\frac{E \xrightarrow{a(v)?} E' \quad F \xrightarrow{a(v):} F}{E|F \xrightarrow{a(v)?} E'|F}$$ (Read-discard)
- $$\frac{E \xrightarrow{a(v):} E \quad F \xrightarrow{a(v):} F}{E|F \xrightarrow{a(v):} E|F}$$ (Join-discard)

Operational Semantics of CBS

One-to-many Communication (Broadcasting)

- $$\frac{E \xrightarrow{a(v)!} E' \quad F \xrightarrow{a(v)?} F' \qquad G \xrightarrow{a(v)?} G'}{E|F \xrightarrow{a(v)!} E'|F' \qquad (E|F)|G \xrightarrow{a(v)!} E'|F'|G'}$$

- $$\frac{E \xrightarrow{a(v)!} E' \quad F \xrightarrow{a(v)?} F' \quad G \xrightarrow{a(v)?} G'}{F|G \xrightarrow{a(v)?} F'|G' \qquad E|(F|G) \xrightarrow{a(v)!} E'|F'|G'}$$

Commutative and Associative

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Applying Transition Rules (Step 1)

$$\dfrac{\dfrac{MEIOSIS \xrightarrow{meiosis?} miao!MEIOSIS \quad OWNER \xrightarrow{meiosis!} miao?ha?SUCC}{MEIOSIS|OWNER \xrightarrow{meiosis!} miao!MEIOSIS|miao?ha?SUCC} \quad FRIEND \xrightarrow{meiosis?} miao?ha!\mathbf{0}}{MEIOSIS|OWNER|FRIEND \xrightarrow{meiosis!} miao!MEIOSIS|miao?ha?SUCC|miao?ha!\mathbf{0}}$$

Corresponding Rule

$$\dfrac{E \xrightarrow{a(v)!} E' \quad F \xrightarrow{a(v)?} F'}{E|F \xrightarrow{a(v)!} E'|F'}$$

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Corresponding Rule

$$\dfrac{E \xrightarrow{a(v)!} E' \quad F \xrightarrow{a(v)?} F'}{E|F \xrightarrow{a(v)!} E'|F'}$$

Operational Semantics of CBS

Step 2

$$\dfrac{\dfrac{miao!MEIOSIS \xrightarrow{miao!} MEIOSIS \quad miao?ha?SUCC \xrightarrow{miao?} ha?SUCC}{miao!MEIOSIS|miao?ha?SUCC \xrightarrow{miao!} miao!MEIOSIS|miao?ha?SUCC} \quad miao?ha!\mathbf{0} \xrightarrow{miao?} ha!\mathbf{0}}{miao!MEIOSIS|miao?ha?SUCC|miao?ha!\mathbf{0} \xrightarrow{miao!} MEIOSIS|ha?SUCC|ha!\mathbf{0}}$$

Corresponding Rules

$$\dfrac{E \xrightarrow{a(v)!} E' \quad F \xrightarrow{a(v)?} F'}{E|F \xrightarrow{a(v)!} E'|F'}$$

Operational Semantics of CBS

Step 3

$$\dfrac{\dfrac{MEIOSIS \xrightarrow{ha:} MEIOSIS \quad ha?SUCC \xrightarrow{ha?} SUCC}{MEIOSIS|ha?SUCC \xrightarrow{ha?} MEIOSIS|SUCC} \quad ha!\mathbf{0} \xrightarrow{ha!} \mathbf{0}}{MEIOSIS|ha?SUCC|ha!\mathbf{0} \xrightarrow{ha!} MEIOSIS|SUCC|\mathbf{0}}$$

Corresponding Rules

$$\dfrac{E \xrightarrow{a(v):} E \quad F \xrightarrow{a(v)?} F'}{E|F \xrightarrow{a(v)?} E|F'} \qquad \dfrac{E \xrightarrow{a(v)?} E' \quad F \xrightarrow{a(v)!} F'}{E|F \xrightarrow{a(v)!} E'|F'}$$

Operational Semantics of CBS

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Operational Semantics of CBS

Step 3

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Rules of Hide and Rename

- $\dfrac{E \xrightarrow{\nu} E'}{E\backslash\mathcal{N} \xrightarrow{\nu} E'\backslash\mathcal{N}} \quad name(\nu) \notin \mathcal{N}$

- $\dfrac{E \xrightarrow{a(v)!} E'}{E\backslash\mathcal{N} \xrightarrow{\tau!} E'\backslash\mathcal{N}} \quad a \in \mathcal{N}$

- $\dfrac{}{E\backslash\mathcal{N} \xrightarrow{a(v):} E'\backslash\mathcal{N}} \quad a \in \mathcal{N}$

- $\dfrac{E \xrightarrow{\nu} E'}{E[\phi] \xrightarrow{\phi(\nu)} E'[\phi]}$

where $\nu \in (S \times V \times \{!, ?, :\}) \cup \{\tau!, \tau :\}$

应该是E\N

Operational Semantics of CBS

Rules of Recursion

- $$\frac{E[recX.E/X] \xrightarrow{\mu} E'}{rexX.E \xrightarrow{\mu} E'}$$
- $$\frac{E[recX.E/X] \xrightarrow{a:} E[recX.E/X]}{rexX.E \xrightarrow{a:} recX.E}$$

where $\mu \in Act = (\mathcal{S} \times V \times \{!, ?\}) \cup \{\tau!\})$