Timed Temporal Logics

Outline

1. Background
2. Timed temporal logics
3. Metric temporal logic (MTL)
4. Translating $MTL_{0,\infty}$ into non-deterministic timed Büchi automata

Background

实时性质

Bounded reachability: p will occur within d time units.

- 例：10天之内，必会下雨
- $\Diamond_{[0,d]} p$

Bounded safety properties: p will not occur within d time units.

- 例：10天之内，不会下雨
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Bounded response properties:: Every p should be followed by an occurrence of a q, not later than d time units.

- 例：任何时候打110报警，警察将在10分钟内赶到
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$$\Box(p \rightarrow \Diamond_{[0,d]} q)$$

Minimum separation properties: Once an event p happens, it will not happen again within d time units.

- 例：打流感疫苗后，2个月内不能再打
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$$\Box(p \rightarrow \hat{\Box}_{[0,d)} \neg p)$$

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Timed Temporal Logics

Linear-Time Temporal Logics:

- MTL (Metric Temporal Logic)

$$\Box(p \to \Diamond_{[0,d]} q)$$

- TPTL (Timed Propositional Temporal Logic)

$$\Box(p \to x.\Diamond(q \wedge x \leq d))$$

Branching-Time Temporal Logics:
TCTL (Timed CTL)

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$$A\Box(p \to A\Diamond_{[0,d]} q)$$

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Interval Temporal Logics:
DC(Duration Calculus) , ...

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The syntax of TCTL is given by the following grammar:

$$\varphi ::= a \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid E(\varphi_1 \,\mathsf{U}_I\, \varphi_2) \mid A(\varphi_1 \,\mathsf{U}_I\, \varphi_2)$$

where $a \in AP$, and I is an interval of $\mathbb{R}^{\geq 0}$ with integral bounds.

Metric temporal logic (MTL)

Metric Temporal Logic(MTL), proposed by Ron Koymans in 1990, is a timed extension of LTL.

Action-based MTL: Let Σ be a finite set of actions, the MTL-formulas over Σ are defined by the following grammar.

$$\varphi ::= \mathit{true} \mid a \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \mathrm{O}\varphi \mid \varphi_1 \, \mathsf{U}_I \, \varphi_2$$

where $a \in \Sigma$ is an action, $I \subseteq [0, \infty)$ is an open, closed, or half-open interval with endpoints in $\mathbb{N} \cup \{\infty\}$.

Metric temporal logic (MTL)

Some *abbreviations*:

$$\varphi\, R_I\, \phi := \neg(\neg\varphi\, \mathrm{U}_I\, \neg\phi) \qquad (\textit{the dual of } U_I)$$

$$\Diamond_I\, \varphi := \mathit{true}\, \mathrm{U}_I\, \varphi$$

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Examples:

$p\, \mathrm{U}_{[1,5)}\, q$,

$\Diamond_{[3,5]}\, p$, $\Box_{(10,25]}\, p$,

$\Box_{[0,1000]}(p \rightarrow \Diamond_{[5,10]} q)$

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Metric temporal logic (MTL)

Pointwise semantics for MTL

Let Σ be a set of actions, and $\tau =< t_0, t_1, \ldots, t_i, \cdots >$ be an unbounded nondecreasing sequence of time values $t_i \in \mathbb{R}^{\geq 0}$.

Then $\omega = (a_1, t_1), (a_2, t_2), (a_3, t_3), \ldots$ is called a *timed word* in pointwise semantics, where $a_i \in \Sigma$ for all $i \in \mathbb{N}$.

We can interpret all *MTL*-formulas(including O operator) over such a timed word.

Definition. Given a timed word $\omega = (a_1, t_1), (a_2, t_2), (a_3, t_3), \ldots$ and a MTL formula φ, then the satisfaction relation $(\omega, i) \models \varphi$ (ω satisfies φ at position i) is defined as follows:

$(\omega, i) \models a$ iff $a = a_i$

$(\omega, i) \models O\varphi$ iff $(\omega, i+1) \models \varphi$

$\ldots$

$(\omega, i) \models \varphi_1 \mathrm{U}_I \varphi_2$ iff there exists $j \geq i$ such that $(\omega, j) \models \varphi_2$, $t_j - t_i \in I$, and $(\omega, k) \models \varphi_1$ for all k with $i \leq k < j$.

$(\omega, i) \models \varphi_1 \mathrm{R}_I \varphi_2$ iff for all $j \geq i$ such that $t_j - t_i \in I$, either $(\omega, j) \models \varphi_2$, or there exists k with $i \leq k < j$ and $(\omega, k) \models \varphi_1$.

Metric temporal logic (MTL)

Continuous Semantics:

Timed state sequence (TSS): Let P be a finite set of propositions. A timed state sequence $\pi = (I_0, \sigma_0), (I_1, \sigma_1), (I_2, \sigma_2), (I_3, \sigma_3), \ldots$ over P is called timed state sequence if the following coditions are satisfied:

- For each $i \geq 0$, σ_i is a subset of P;
- $I_0,\ I_1,\ I_2,\ I_3, \ldots$ is an infinite sequence of bounded intervals such that $\bigcup_{i \geq 0} I_i = \mathbb{R}^{\geq 0}$, and for all $i >= 0$, the intervals I_i and I_{i+1} are adjacent.

For example, let $P = \{p, q\}$, then $([0, 2], \{p\})$, $((2, 3), \{p, q\})$, $([3, 3], \{q\})$, $((3, 8.5], \{\})$, $((8.5, 20), \{p\})$, $\ldots$ is a TSS.

Equivalently, a TSS π can be viewed as a function from $\mathbb{R}^{\geq 0}$ to 2^P, indicating for each time $t \in \mathbb{R}^{\geq 0}$ a subset of P, that is, $\pi(t) \subseteq P$ for all $t \in \mathbb{R}^{\geq 0}$.

Metric temporal logic (MTL)

Continuous Semantics for MTL

Definition. Given a timed state sequence π and a next-free MTL formula φ, φ holds at time $t \in \mathbb{R}^{\geq 0}$, denoted $(\pi, t) \models \varphi$, can be defined as follows.

$(\pi, t) \models a$ iff $a \in \pi(t)$;

...

$(\pi, t) \models \varphi_1 \mathsf{U}_I \varphi_2$ iff there exists a $t' \geq t$ such that $t' - t \in I$, $(\pi, t') \models \varphi_2$, and for all reals $t'' \in (t, t')$, we have $(\pi, t'') \models \varphi_1$

MTL is a powerful specification language for timed systems.

Model checking and satisfiability for *MTL*:

- Continuous Semantics, infinite run: undecidable
- Continuous Semantics, finite run: undecidable
- Pointwise Semantics, infinite run: undecidable
- Pointwise Semantics, finite run: decidable

Metric Interval Temporal Logic(MITL) is proposed by Rajeev Alur, Tomas Feder, Thomas A. Henzinger in 1991.

$$\varphi ::= a \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \mathrm{O}\varphi \mid \varphi_1 \mathsf{U}_I \varphi_2$$

where I is a non-singular interval with endpoints in $\mathbb{N} \cup \{\infty\}$.

Singular intervals: $[d, d]$, where $d \in \mathbb{N}$

MITL is decidable.

Metric temporal logic (MTL)

Model checking and satisfiability for *MITL*:

- Continuous Semantics, infinite run: EXPSPACE-complete
- Continuous Semantics, finite run: EXPSPACE-complete
- Pointwise Semantics, infinite run: EXPSPACE-complete
- Pointwise Semantics, finite run: EXPSPACE-complete

$MTL_{0,\infty}$ is a subset of MITL, defined by the following syntax.

$$\varphi ::= a \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \mathrm{O}\varphi \mid \varphi_1 \mathrm{U}_{\sim d} \varphi_2$$

where $a \in \Sigma$, $d \in \mathbb{N}$, $\sim \in \{<, \leq, >, \geq\}$ and $< d$, $\leq d$, $> d$, $\geq d$ denote the intervals $[0, d)$, $[0, d]$, (d, ∞), $[d, \infty)$ respectively.

Model checking and satisfiability for $MTL_{0,\infty}$:

Continuous Semantics, infinite run: PSPACE-complete

Continuous Semantics, finite run: PSPACE-complete

Pointwise Semantics, infinite run: PSPACE-complete

Pointwise Semantics, finite run: PSPACE-complete