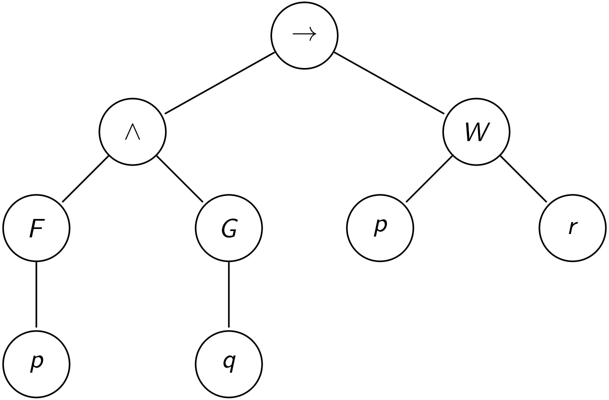
时态逻辑系统

作业参考答案

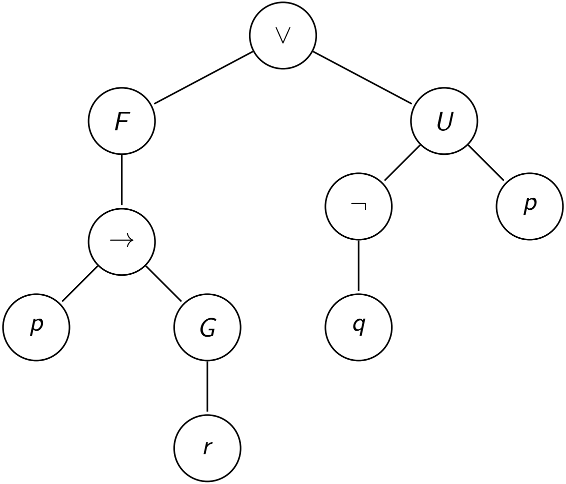
1.画出下面LTL公式的Parse树

(1)$Fp \wedge Gq \to pWr$



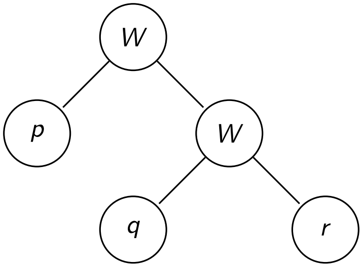
1.画出下面LTL公式的Parse树

$(2) F(p \to Gr) \vee \neg qUp$



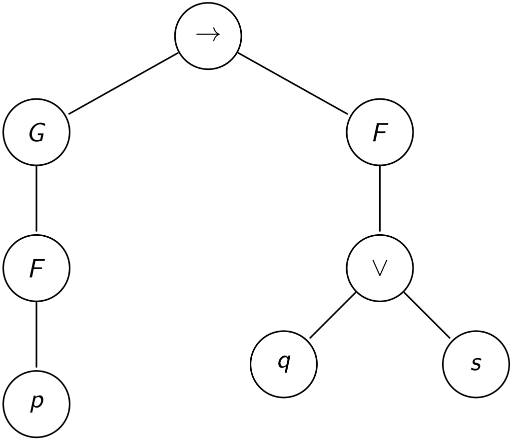
1.画出下面LTL公式的Parse树

(3)$pW(qWr)$



1.画出下面LTL公式的Parse树

(4) $GFp \to F(q \lor s)$



2.证明：$\phi U\psi \equiv \psi R(\phi \vee \psi) \wedge F\psi$

Proof.

设$A = \{\pi \mid \exists i,\ \pi^i \vDash \psi,\ \forall j = 1, 2, ..., i-1,\ \pi^j \vDash \phi\}$,
$B = \{\pi \mid$

$$((\exists k,\ \pi^k \vDash \psi,\ \forall j = 1, 2, ..., k,\ \pi^j \vDash \phi \vee \psi) \vee (\forall p,\ \pi^p \vDash \phi \vee \psi))$$

$$\wedge$$

$$(\exists s,\ \pi^s \vDash \psi)$$

$\}$.
1) $A \subseteq B$:
若$\pi \in A$, 则$\exists i_0,\ \pi^{i_0} \vDash \psi,\ \forall j = 1, 2, ..., i_0 - 1,\ \pi^j \vDash \phi$.
$\therefore \pi^{i_0} \vDash \phi \vee \psi$ 且$\forall j = 1, 2, ..., i_0 - 1$ 有$\pi^j \vDash \phi \vee \psi$
观察B, 令$k = i_0$, $s = i_0$, 得$\pi \in B$

Proof (Cont.)

2) $B \subseteq A$:
若$\pi \in B$，则$\exists s_0$，$\pi^{s_0} \vDash \psi$

(1) 若$\forall p$，$\pi^p \vDash \phi \vee \psi$，令$p_0$ 是使$\pi^{p_0} \vDash \psi$ 最小的数，令$m = min(p_0, s_0)$，m 总能取到. 因此观察A 令$i = m$，易知$\pi \in A$.

(2) 若$\exists k_0$，$\pi^{k_0} \vDash \psi$，$\forall j = 1, 2, ..., k_0$，$\pi^j \vDash \phi \vee \psi$，对比$A$，显然$\pi \in A$.

综上$A = B$，所以$\phi U \psi \equiv \psi R(\phi \vee \psi) \wedge F\psi$. □

3.依照下图的系统，考虑下面每个LTL公式ϕ

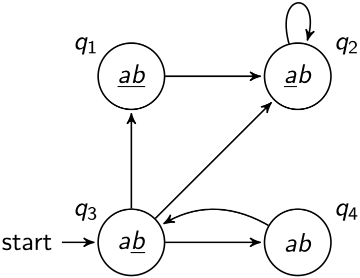
(1) Ga

(2) aUb

(3) $aUX(a \wedge \neg b)$

(4) $X\neg b \wedge G(\neg a \vee \neg b)$

(5) $X(a \wedge b) \wedge F(\neg a \wedge \neg b)$



(a) 找到一条从q_3出发的路，满足公式ϕ

Solution

(1) $Ga : q_3q_4q_3q_4q_3q_4\cdots$
(2) $aUb : q_3q_2q_2q_2\cdots$
(3) $aUX(a\wedge\neg b) : q_3q_4q_3q_2\cdots$
(4) $X\neg b\wedge G(\neg a\vee\neg b) : q_3q_1q_2q_2\cdots$
(5) $X(a\wedge b)\wedge F(\neg a\wedge\neg b) : q_3q_4q_3q_1q_2\cdots$

(b) 确定是否有$M, q_3 \models \phi$

Solution

(1) Ga: No
(2) aUb: No
(3) $aUX(a\wedge\neg b)$: No
(4) $X\neg b\wedge G(\neg a\vee\neg b)$: No
(5) $X(a\wedge b)\wedge F(\neg a\wedge\neg b)$: No

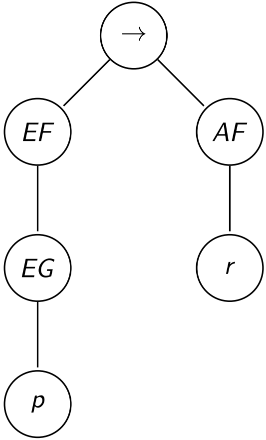
(c) 若将$\underline{a}$和$\underline{b}$解释为a和b的非，并表示通信协议中的发射信息，而a, b为接受信息，解释这些公式的具体含义。

Solution

(1) Ga: 任何状态下都接收a
(2) aUb: 一直接收a，直到某个状态，接收b
(3) $aUX(a \wedge \neg b)$: 一直接收a，直到某个状态，它的下一个状态发射b
(4) $X\neg b \wedge G(\neg a \vee \neg b)$: 下一个状态发射$b$，并且对任何状态，发射$a$或者$b$
(5) $X(a \wedge b) \wedge F(\neg a \wedge \neg b)$: 下一个状态接收$a$和$b$，并且存在某个状态，发射$a$和$b$

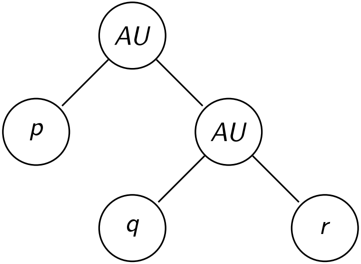
1.画出下面CTL公式的Parse树

(1)$EFEGp \to AFr$



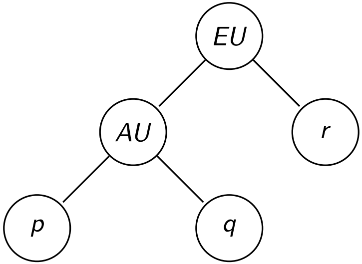
1.画出下面CTL公式的Parse树

(2)$A[pUA[qUr]]$



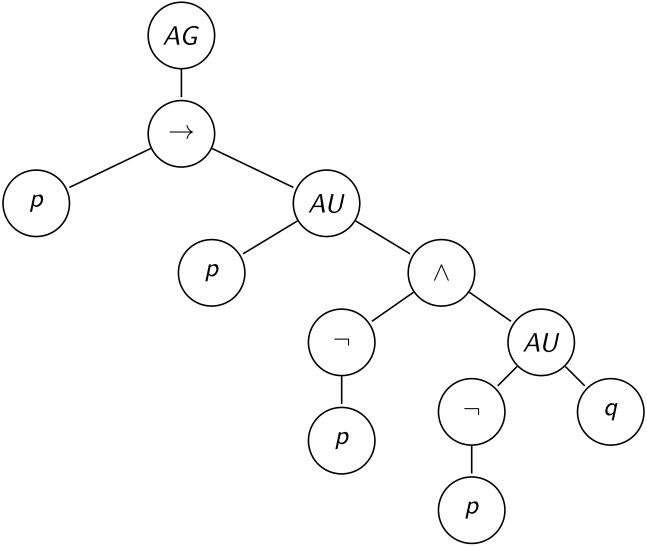
1.画出下面CTL公式的Parse树

(3)$E[A[pUq]Ur]$

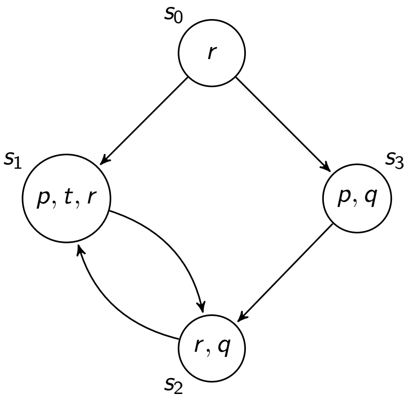


1.画出下面CTL公式的Parse树

(4)$AG(p \rightarrow A[pU[\neg p \wedge A[\neg pUq]]])$



2.依照下图的系统



(a)从s_0开始，将这个系统展开成一个无穷树，并画出所有长度为4的计算路.

Solution

$$
\begin{tikzcd}
 & s_0 \arrow[dl] \arrow[dr] & \\
s_1 \arrow[d] & & s_3 \arrow[d] \\
s_2 \arrow[d] & & s_2 \arrow[d] \\
s_1 \arrow[d] & & s_1 \arrow[d] \\
\vdots & & \vdots
\end{tikzcd}
$$

(b) 确定是否有$M, s_0 \models \phi$ 以及$M, s_2 \models \phi$成立，其中ϕ 是LTL或CTL公式

Solution

1.$\neg p \to r$

$r \in L(s_0) \quad \therefore M, s_0 \models \phi$

$r \in L(s_2) \quad \therefore M, s_2 \models \phi$

2.Ft

所有从s_0出发的路一定经过s_1 $\quad \therefore M, s_0 \models \phi$

所有从s_2出发的路一定经过s_1 $\quad \therefore M, s_2 \models \phi$

3.$\neg EGr$

所有从s_0出发的路，不满足$\forall M, s_i \models r$ $\quad \therefore M, s_0 \not\models \phi$

所有从s_2出发的路，不满足$\forall M, s_i \models r$ $\quad \therefore M, s_2 \not\models \phi$

4.$E(tUq)$

$t \notin L(s_0) \quad \therefore M, s_0 \not\models \phi$

$t \in L(s_2) \quad \therefore M, s_2 \models \phi$

Solution (Cont.)

$5.EFq$

$s_0 \to s_3 \to \cdots$且$q \in L(s_3) \qquad \therefore M, s_0 \models \phi$

$q \in L(s_2) \qquad \therefore M, s_2 \models \phi$

$6.EGr$

$s_0 \to s_1 \to s_2 \to s_1 \to \cdots \qquad \therefore M, s_0 \models \phi$

$s_2 \to s_1 \to s_2 \to \cdots \qquad \therefore M, s_2 \models \phi$

$7.G(r \vee q)$

对于从任意s_i出发的π 满足$\pi \models (r \vee q) \qquad \therefore M, s_0 \models \phi$且$M, s_2 \models \phi$

$\mathcal{M} = (S, \rightarrow, L)$是任何CTL模型，用符号$[|\phi|]$表示集合$\{s \mid s \in S, \mathcal{M}, s \vDash \phi\}$. 证明:

a) $[|\top|] = S$.

Proof.

$\because \forall s \in S, s \vDash \top$

$\therefore [|\top|] = S$ $\square$

b) $[|\bot|] = \phi$.

Proof.

$\because \forall s \in S, s \nvDash \bot$

$\therefore [|\bot|] = \emptyset$ $\square$

c) $[|\neg\phi|] = \{s \mid s \vDash \neg\phi\}$.

Proof.

$[|\neg\phi|] = \{s \mid s \vDash \neg\phi\} = \{s \mid s \nvDash \phi\} = S - [|\phi|]$ $\square$

d) $[|\phi \wedge \psi|] = [|\phi|] \cap [|\psi|]$.

Proof.

$\forall s \in [|\phi \wedge \psi|]$
iff $s \vDash \phi \wedge \psi$
iff $s \in [|\phi|]$ and $s \in [|\psi|]$
iff $s \in [|\phi|] \cap [|\psi|]$

$\square$

e) $[|\phi \vee \psi|] = [|\phi|] \cup [|\psi|]$.

Proof.

$\forall s \in [|\phi \vee \psi|]$
iff $s \vDash \phi \vee \psi$
iff $s \vDash \phi$ or $s \vDash \psi$
iff $s \in [|\phi|]$ or $s \in [|\psi|]$
iff $s \in [|\phi|] \cup [|\psi|]$

$\square$

f) $[|\phi \to \psi|] = (S - [|\phi|]) \cup [|\psi|]$.

Proof.

$\forall s \in [|\phi \to \psi|]$
iff $s \vDash \phi \to \psi$
iff $s \vDash \neg\phi \vee \psi$
iff $s \vDash \neg\phi$ or $s \vDash \psi$
iff $s \in (S - [|\phi|])$ (由(c)) or $s \in [|\psi|]$
iff $s \in (S - [|\phi|]) \cup [|\psi|]$

$\square$

g) $[|AX\phi|] = S - [|EX\neg\phi|]$.

Proof.

$\forall s \in [|AX\phi|]$
iff $s \vDash AX\phi$
iff $s \vDash \neg EX\neg\phi$ (由$AX\phi \equiv \neg EX\neg\phi$)
iff $s \vDash S - [|EX\neg\phi|]$ (由(c))

$\square$

h) $[|A(\phi U\psi)|] = [|\neg(E(\neg\phi U(\neg\phi \wedge \neg\psi)) \vee EG\neg\psi)|]$.

Proof.

$\forall s \in [|A(\phi U\psi)|]$
iff $s \models A(\phi U\psi)$
iff $s \models \neg(E(\neg\phi U(\neg\phi \wedge \neg\psi)) \vee EG\neg\psi)$
(由$A(\phi U\psi) \equiv \neg(E(\neg\phi U(\neg\phi \wedge \neg\psi)) \vee EG\neg\psi)$)

$\square$