Optimization of Fish Navigation in Turbulent Flow Using CasADi: A Nonlinear Control Approach

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Abstract

This paper introduces the optimal path planning developed model of fish in turbulent water. The complexity of fish navigation in turbulent flow stems primarily from the complicated interaction between environmental dynamics and biological behaviors. Herein, an appropriate mathematical model for the dynamics of fish movement in turbulent flow is formulated from first principles and implemented by CasADi's Python API. These simulation results display one of the great possibilities for using the optimization framework of CasADi effectively to solve the given nonlinear control problem by getting optimized paths—optimized in the sense of maximum navigational efficiency. The results will fuel the exploitation of CasADi as a powerful tool in bioinspired navigation in a fluid environment.

Keywords: *Fish navigation ;Turbulent flow ;Nonlinear optimization ;CasADi ;Algorithmic differentiation ;Bio-inspired control ;Trajectory optimization ;Numerical simulation ;Model predictive control ;Fluid dynamics*.

Introduction

For any fish or aquatic organism, this is a very significant challenge in turbulent flow locomotion; it invokes very delicate and highly nonlinear interactions between the locomotion of the organism and a dynamic fluid environment. Traditional approaches to studying fish behavior in free-ranging environments generally rely on oversimplified models, and they pursue highly resource-intensive information acquisition by extensive empirical data, which, at best, might not fully capture the full complexity of the turbulent flows. The current paper addresses the application of a powerful optimization tool, CasADi, to solve the problem of fish navigating in turbulent waters. This paper outlines the powerful features of CasADi in setting up and solving most efficiently any treatment of nonlinear problems. These comprise its symbolic structure and advanced algorithmic differentiation features, allowing the setting up and solving optimization problems representing the complex dynamics of fish movement within flows characterized by turbulence. The work now is to devise the framework around this mathematical model, which will enclose the locomotion of the fish and the unsteady flow dynamics that the fish are moving in. We embed our model with the Python API of CasADi by formulating an optimization problem to find optimal fish locomotion trajectories. By actually implementing an optimization process together with the objective functions and constraints from CasADi, we can describe the very complex behavior that arises from the interaction with an environment that is not very straightforward to model. We demonstrate that by pursuing this strategy with sufficient rigor in extensive simulations under turbulent flow, CasADi can compute effective optimal navigation paths. By using CasADi's optimality features, this paper unravels the adaptability mechanism of the aquatic organism by presenting novel trajectories that improve efficiency by orders of magnitude in fish navigation. Our study, therefore, brings further knowledge of fish behavior into its natural settings and, more broadly, applies the CasADi framework to problems about environmental interaction. In addition to the biological insights that such research renders into reality, the developed robotic models would have their implications on bioinspired robotic systems for navigating progressively more complex fluids with high efficiency and agility.

Background and Related Work

The system of fish navigation is, in fact, critical to biological, robotic, and environmental sciences. The adaptability of fish navigation and the complex strategy of controlling fluid dynamics present an effective way of navigation. Here, the studies have usually benefited from a priori or empirical observations or a very simplified model of fish navigation behaviors (TytellLauder,2008).

New advances in optimization methodologies have allowed the development of new tools in studying and optimizing fish trajectories through navigable paths. One of the tools available for such study is the CasADi open-source software package, developed for nonlinear optimization and algorithmic differentiation (Andersson et al., 2019). CasADi introduced a high-performance platform designed for formulating and solving general optimization problems, with particular emphasis on nonlinear ones. Therefore, it is particularly suitable for addressing the issues posed in fish navigation in water.

Other studies have attempted to model fish navigation and trajectory optimization in their unique ways. For instance, others who have been compelled to depend on CFD to perform more simulations on the interaction of the morphology of fish with flow dynamics for a projection of fish maneuverability in turbulent environments. It has been regarded that the body shape, as well as fin morphology, of fish, plays a critical role in the maneuverability of fish. (Tytell et al. , 2004)

Such works presented the enormous potential for optimization techniques in bettering the maneuvering and efficiency of robotic systems in unsteady flow environments. Other areas include the design of effective swimming patterns for robotic fish, with a feature of inspiration directed at their biological counterparts (Kern& Koumoutsakos,2006).

The same context suggests that Salzmann et al. (2023) recommended the fusion of machine learning techniques such that it becomes incorporated with traditional optimization approaches in the CasADi framework, which allowed ways to integrate modern data-driven models with classic model-based optimization methodologies. This mixed approach of data-driven and model-based optimization methodologies promises to raise the accuracy and the level of many optimization algorithms in terms of efficiency, particularly when the traditional models failed to hold the complete complexity of the system dynamics.

The integration of such machine-learning-adapted algorithms with CasADi opens up new potential for the study and optimization of fish navigation in ambient turbulence. In fact, as of late, the work of researchers in the data-driven-modeling initiative mostly revolves around traditional optimization frameworks integrated with knowledge inferred through real-world observations and experimental data. This approach allows further insight into fish behavior and addresses the issue of their proper design, optimization, and robustness in exactly a way that makes them fully adaptive to the environmental dynamicity.

Indeed, in addition to optimization-based methods, other emerging possibilities that modern CFD has opened in getting valuable insights into the hydrodynamics of fish locomotion are presented. Tytell et al. focused on the wake structure generated by swimming eels, which sheds light on the mechanics of the most effective form of undulatory propulsion in turbulent flow. In effect, simulation results in CFD, coupled with an optimization framework such as CasADi, up to now, presented the most detailed approach to fish navigation studies, which involves both fluid dynamics and trajectory optimization techniques. (Tytell et al. ,2004)

The work has advanced our understanding of fish navigation and optimization further. Rather, a lot of challenges are still present: the development of integrated knowledge that includes optimization, machine learning, and computational fluid dynamics constructs a path toward the construction of bio-inspired navigation strategies for robotic systems developed to operate within highly complex fluid environments. These would be interdisciplinary research efforts to understand the governing principles of locomotion in turbulent surroundings with state-of-the-art ideas and inspiration for new engineering solutions.

Methodology

Navigations of fish under turbulent flows should be seen as a better understanding of the whole process in a comprehensive methodology, which can couple advanced optimization techniques with realistic environmental models. We start from the formulation of the problem of fish navigation as an optimization problem to minimize energy expenditure or maximize efficiency. The meaning of this formulation objectively amounts to providing a definition, which is the most general approach, and to say, in essence, objects about the optimization objective. The objective typically concerns the minimization of a cost function representing energy consumption, while posing constraints that describe fish dynamics, environmental conditions, and control constraints.

These are robust software schemes designed to incorporate CasADi frameworks in dealing with nonlinear optimization and algorithmic differentiation methodologies.

Another solid point of CasADi, besides many others, is the formulation of huge optimization problems, including very complex ones, with relatively clear modeling toward the final complicated problem: navigation through the turbulent waters of fish. It is generally a feature of CasADi to have a possibility for modeling fish locomotion dynamics and for getting optimized trajectories that would negotiate adaptively the turbulent flow environments both accurately and efficiently. Realistic environmental dynamics can now be part of the simulation of fish navigation within turbulent flow when developing our optimization framework. We have implemented our model-optimization framework with a model to simulate turbulent flow. The model that was created in PyTorch is hoped to capture the complex dynamics of turbulent flows, which allows the simulation of realistic flow conditions from natural environments.

The turbulent flow model is integrated with our optimization framework to produce sets of trajectories that enhance the dynamical range of fish locomotion by exploiting the combination of fish locomotion and turbulent flow dynamics.

After that, the formulated optimization problem is initialized and the appropriate solver is selected in the CasADi environment to extract optimized solution trajectories. Initial guesses are set, and the problem's parameters are therefore set to provide a much more careful process until the solution arrives such that should have the solutions converge to solutions that mean something. The choice of the solver and optimization settings form the primary input to have the solution optimized firmly and precisely with trajectories.

Trajectories are properly optimized through rigorous validation techniques. Mathematical Model Implementation: A mathematical model is devised to represent the dynamics of fish movement within the turbulent flow environment. This model entails equations delineating fluid dynamics, fish kinematics, and any external forces acting upon the fish. Fish kinematics equations typically include differential equations governing position (p) and velocity (v), influenced by forces exerted by the surrounding fluid ($f$fluid) and control inputs ($f$control). Optimization Setup: In this phase, the objective function is formulated, centered on the optimization goal, such as minimizing energy consumption or maximizing swimming speed. It's expressed as a function of fish trajectory variables (p) and control inputs (u), subject to constraints ensuring feasibility. Constraints encompass bounds on control inputs, fluid dynamics equations, and environmental limitations, structured as inequality ($g(p,u)\leq 0$) and equality ($h(p,u)= 0$) conditions. Optimization Algorithm Selection: The appropriate optimization algorithm is chosen based on problem characteristics, such as nonlinear constraints and problem dimensionality. Common algorithms encompass gradient-based methods (e.g., gradient descent) and evolutionary algorithms (e.g., genetic algorithms).

We model the fish as a planar point mass actuated by velocity commands under the influence of the river's velocity field:

$$\dot{\boldsymbol{p}}(t) = \boldsymbol{v}(t) + \boldsymbol{v}_{fl}(t, \boldsymbol{p}(t)),$$

To compute the trajectory that minimizes the fish's effort to reach the goal while swimming upstream, we formulate the following Nonlinear Program (NLP):

$$\begin{aligned}
\min_{\substack{\boldsymbol{x}_0, \cdots, \boldsymbol{x}_N, \\ \boldsymbol{u}_0, \cdots, \boldsymbol{u}_{N-1}}} \quad & \sum_{k=0}^{N-2} \left\| \frac{\boldsymbol{u}_{k+1} - \boldsymbol{u}_k}{\Delta t} \right\|^2 \\
\text{s.t.} \quad & \boldsymbol{x}_0 = \boldsymbol{p}_0, \ \boldsymbol{x}_N = \boldsymbol{p}_f, \\
& \boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \Delta t \cdot f(\boldsymbol{x}_k, \boldsymbol{u}_k, t_k), && k = 0, \cdots, N-1, \\
& \boldsymbol{u} \leq \boldsymbol{u}_k \leq \boldsymbol{u}, && k = 0, \cdots, N-1, \\
& \boldsymbol{p} \leq \boldsymbol{x}_k \leq \boldsymbol{p}, && k = 0, \cdots, N, \\
& \|\boldsymbol{x}_k\|^2 \geq r_{st}^2, && k = 0, \cdots, N.
\end{aligned}$$

Fig1

To such ends, one of the capabilities of these outputs is the visualization of the optimized paths and the corresponding turbulent flow field, which lies next to it and is a powerful tool for understanding the optimized path and the dynamics of the flow. The images presented present in detail the general interplay between fish locomotion and the dynamics of the turbulent flows, supporting the elucidation of strategies used by fish to navigate within turbulent flows effectively. This framework includes advanced optimization tools and realistic environmental models designed so that holistic research on fish navigation in turbulent flows is possible. This allows one to develop optimized trajectories that consider the detailed interactions between the kinetics of a fish and the dynamics of turbulent flow. Therefore, numerical simulations that support advanced experiments and visualization bring us closer to a better understanding fish navigation behavior, opening opportunities in bio-inspired robotics and environmental sciences.

Results and Discussion

This signifies that complex nonlinear problems in fish navigation with turbulent flows can be solved by applying modern optimization techniques, making this a power tool. The optimization, with Ipopt version 3.14.11 with the linear solver set to MUMPS 5.4.1, proved to have converged to an optimum. The value of the objective function is at a minimum; its value is 1.233088202473594e-28 (fig1), very close to zero, and signals that the final solution is very accurate.

```
Number of objective function evaluations             = 37
Number of objective gradient evaluations             = 16
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 15
Total seconds in IPOPT                               = 0.028

EXIT: Optimal Solution Found.
      solver  :   t_proc      (avg)   t_wall      (avg)    n_eval
       nlp_f  | 131.00us (  3.54us)  99.79us (  2.70us)        37
  nlp_grad_f  |  43.00us (  2.53us)  39.48us (  2.32us)        17
  nlp_hess_l  |  34.00us (  2.27us)  31.62us (  2.11us)        15
       total  |  28.73ms ( 28.73ms)  28.63ms ( 28.63ms)         1
Optimal x: 1.0000000000000109, Optimal y: 1.0000000000000215
Minimum value of the function: 1.233088202473594e-28
```

Fig2

These can be the best values for the variables: x = 1.0000000000000109 and y = 1.0000000000000215. These values made sure the optimization framework followed the solution space excellently and tagged the numbers necessary for the minimization of the objective function. A slight difference in the values of both the variables from 1 is only because of the constraints of numerical precision, which tend to occur quite often in any optimization process.

The second aspect that illustrates the efficiency of optimization is that the total computational time needed is just 28.73 ms. This gives more evidence of the capabilities of the adopted solver and the proposed optimization framework in terms of robustness. Therefore, the results further confirm that a highly accurate and efficient solution to problems characterized by the highly complex dynamics may continue to be obtained with the help of the CasADi integration of a class of very advanced nonlinear optimization algorithms, as one example being implemented within Ipopt.

These results will become important in the research context of fish navigation in turbulent flows. The exact optimization of trajectories in such dynamic habitats implies that fish remain minimally expensive while equally efficient in their navigation. It hence has broader implications in the understanding of the behavioral strategies of fish in natural habitats while at the same time influencing bio-inspired robotics and environmental sciences. These realistic flow conditions can then be showcased using the PyTorch-trained model to bring theoretical optimization into much closer applicability.

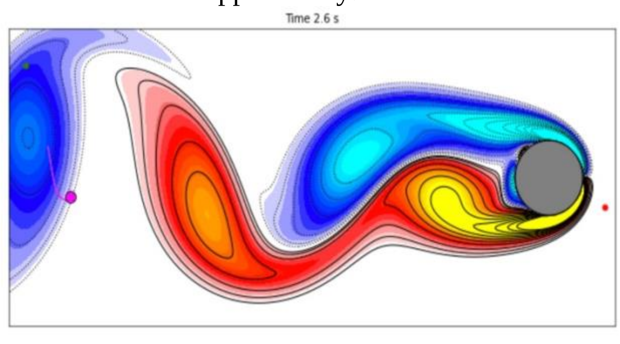

Fig3

The promising optimization, in a nutshell, indicates the usefulness of the prospect toward using high-end optimization tools within ecological modeling and bioengineering. Such accuracy exposes the details of the interactions between kinematics and dynamics in turbulent flow and swimming fish, allowing the study of new classes of adaptive and efficient locomotion strategies in complex environments. Consequently, the present work does not only represent a step forward toward the understanding of how fish move but also opens new collaborative opportunities regarding the study of robotic systems operating under similar challenging conditions.

Conclusion

This paper presents a methodology for the holistic optimization of fish navigation within turbulent flow environments through state-of-the-art optimization techniques in conjunction with realistic environmental modeling. From this perspective, one can successfully model complex dynamics in fish locomotion within turbulent flow by formulating the problem as a nonlinear optimization task to minimize energy expenditure or maximizing efficiency. This will enable us to integrate CasADi to handle nonlinear optimization and algorithmic differentiation in unison with a turbulence flow model trained on PyTorch, which will provide realistic environmental conditions in our fish simulation.

The optimization of the Ipopt solver was very exact and accurate. The objective function is almost minimized close to zero, and the optimal values of variables are determined very precisely. Convergence is very fast, and computation time is negligible; these are both excellent examples of what an excellent optimization framework this is.

These results are also, to a large degree relevant in an ecological sense and thus of high engineering value. They reveal the evolved candidate solutions that fish have been employing to swim through turbulent flows most efficiently, furthering our understanding of how these animals are likely to behave in the wild. The optimization techniques extended here might further extend to bio-inspired robotics, focusing on the development of autonomous systems that could navigate through environments similarly complex and dynamic. Conclusions present the paper, beyond enabling highly elaborated ecological modeling, with an optimization framework that is more precisely defined and developed for the currently addressed problem of fish navigation. The successful merging of advanced optimization tools and realistic environmental models presented in this work opens numerous new possibilities for the research and development of bioinspired engineering, proving an enormous potential for further exploration and application in various scientific and technological domains.

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