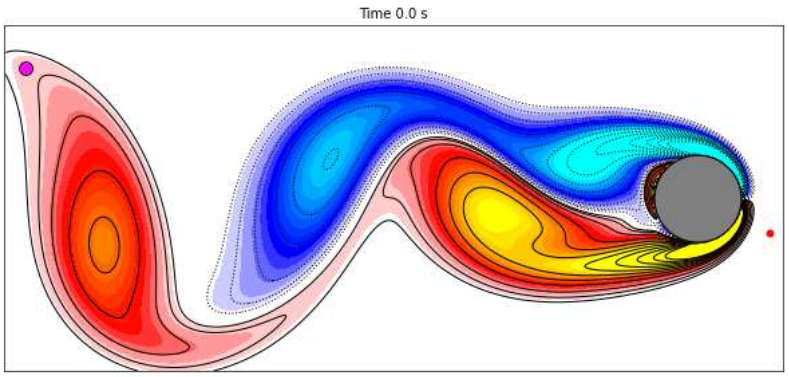
# Fish Navigation in Turbulent Flow




## Install L4CasADi and *Dependencies*

```
import sys


# @title
!pip install torch --index-url https://download.pytorch.org/whl/cpu
!pip install scikit-build cmake ninja
!pip install git+https://github.com/Tim-Salzmann/l4casadi --no-build-isolation
```

```
Looking in indexes: https://download.pytorch.org/whl/cpu
Requirement already satisfied: torch in /usr/local/lib/python3.10/dist-packages (2.3.0+cu121)
Requirement already satisfied: filelock in /usr/local/lib/python3.10/dist-packages (from torch) (3.14.0)
Requirement already satisfied: typing-extensions>=4.8.0 in /usr/local/lib/python3.10/dist-packages (from torch) (4.11.0)
Requirement already satisfied: sympy in /usr/local/lib/python3.10/dist-packages (from torch) (1.12)
Requirement already satisfied: networkx in /usr/local/lib/python3.10/dist-packages (from torch) (3.3)
Requirement already satisfied: jinja2 in /usr/local/lib/python3.10/dist-packages (from torch) (3.1.4)
Requirement already satisfied: fsspec in /usr/local/lib/python3.10/dist-packages (from torch) (2023.6.0)
INFO: pip is looking at multiple versions of torch to determine which version is compatible with other requirements. This could take
Collecting torch
  Downloading https://download.pytorch.org/whl/cpu/torch-2.3.0%2Bcpu-cp310-cp310-linux_x86_64.whl (190.4 MB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 190.4/190.4 MB 6.7 MB/s eta 0:00:00
Requirement already satisfied: MarkupSafe>=2.0 in /usr/local/lib/python3.10/dist-packages (from jinja2->torch) (2.1.5)
Requirement already satisfied: mpmath>=0.19 in /usr/local/lib/python3.10/dist-packages (from sympy->torch) (1.3.0)
Installing collected packages: torch
  Attempting uninstall: torch
    Found existing installation: torch 2.3.0+cu121
    Uninstalling torch-2.3.0+cu121:
      Successfully uninstalled torch-2.3.0+cu121
Successfully installed torch-2.3.0+cpu
Collecting scikit-build
  Downloading scikit_build-0.17.6-py3-none-any.whl (84 kB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 84.3/84.3 kB 2.6 MB/s eta 0:00:00
Requirement already satisfied: cmake in /usr/local/lib/python3.10/dist-packages (3.27.9)
Collecting ninja
  Downloading ninja-1.11.1.1-py2.py3-none-manylinux1_x86_64.manylinux_2_5_x86_64.whl (307 kB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 307.2/307.2 kB 8.0 MB/s eta 0:00:00
Requirement already satisfied: distro in /usr/lib/python3/dist-packages (from scikit-build) (1.7.0)
Requirement already satisfied: packaging in /usr/local/lib/python3.10/dist-packages (from scikit-build) (24.0)
Requirement already satisfied: setuptools>=42.0.0 in /usr/local/lib/python3.10/dist-packages (from scikit-build) (67.7.2)
Requirement already satisfied: tomli in /usr/local/lib/python3.10/dist-packages (from scikit-build) (2.0.1)
Requirement already satisfied: wheel>=0.32.0 in /usr/local/lib/python3.10/dist-packages (from scikit-build) (0.43.0)
Installing collected packages: ninja, scikit-build
Successfully installed ninja-1.11.1.1 scikit-build-0.17.6
Collecting git+https://github.com/Tim-Salzmann/l4casadi
  Cloning https://github.com/Tim-Salzmann/l4casadi to /tmp/pip-req-build-oabxkm88
  Running command git clone --filter=blob:none --quiet https://github.com/Tim-Salzmann/l4casadi /tmp/pip-req-build-oabxkm88
  Resolved https://github.com/Tim-Salzmann/l4casadi to commit c135d4f069e0b11f2a837005c329b088914bdfc3
  Preparing metadata (pyproject.toml) ... done
Requirement already satisfied: torch in /usr/local/lib/python3.10/dist-packages (from l4casadi==1.4.0) (2.3.0+cpu)
Collecting casadi>=3.6 (from l4casadi==1.4.0)
  Downloading casadi-3.6.5-cp310-none-manylinux2014_x86_64.whl (72.3 MB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 72.3/72.3 MB 7.9 MB/s eta 0:00:00
Requirement already satisfied: jinja2>=3.1 in /usr/local/lib/python3.10/dist-packages (from l4casadi==1.4.0) (3.1.4)
```

```
Requirement already satisfied: numpy in /usr/local/lib/python3.10/dist-packages (from casadi>=3.6->l4casadi==1.4.0) (1.25.2)
Requirement already satisfied: MarkupSafe>=2.0 in /usr/local/lib/python3.10/dist-packages (from jinja2>=3.1->l4casadi==1.4.0) (2.1.5)
Requirement already satisfied: filelock in /usr/local/lib/python3.10/dist-packages (from torch->l4casadi==1.4.0) (3.14.0)
Requirement already satisfied: typing-extensions>=4.8.0 in /usr/local/lib/python3.10/dist-packages (from torch->l4casadi==1.4.0) (4.1
Requirement already satisfied: sympy in /usr/local/lib/python3.10/dist-packages (from torch->l4casadi==1.4.0) (1.12)
Requirement already satisfied: networkx in /usr/local/lib/python3.10/dist-packages (from torch->l4casadi==1.4.0) (3.3)
Requirement already satisfied: fsspec in /usr/local/lib/python3.10/dist-packages (from torch->l4casadi==1.4.0) (2023.6.0)
Requirement already satisfied: mpmath>=0.19 in /usr/local/lib/python3.10/dist-packages (from sympy->torch->l4casadi==1.4.0) (1.3.0)
Building wheels for collected packages: l4casadi
  Building wheel for l4casadi (pyproject.toml) ... done
  Created wheel for l4casadi: filename=l4casadi-1.4.0-cp310-cp310-linux_x86_64.whl size=51731 sha256=2006bc65050d51fc77505ee3bd43f6a3
  Stored in directory: /tmp/pip-ephem-wheel-cache-heuvagk6/wheels/cd/54/8a/b8796f827085bb3682fe49796c0f4fe19ddeec1a76ab4187d1
Successfully built l4casadi
```

```
# @title
!git clone https://github.com/Tim-Salzmann/l4casadi /tmp/l4casadi
sys.path.append('/tmp/l4casadi/examples/fish_turbulent_flow')
```

```
fatal: destination path '/tmp/l4casadi' already exists and is not an empty directory.
```

```
# @title
from trajectory_generation import trajectory_generator_solver
from utils import plot_velocity_field_particle
```

## Import

```
import os
from matplotlib.animation import FuncAnimation
import matplotlib.pyplot as plt
import casadi as cs
import torch
import numpy as np
```

```
import l4casadi as l4c
```

## Optimization

### Set Fish Start Point Position

```
# @title Set Fish Start Point Position
y_start_pos = 1.1 # @param {type:"slider", min:-1.8, max:1.8, step:0.
p_start = np.array([7.75, y_start_pos])
p_goal = np.array([-0.85, -0.4])
u_lim = 1
T = 20
N = 151
dt = T / N
```

**y_start_pos:** 1.1

### Load PyTorch Turbulent Flow Model

```
checkpoint = torch.load(
    "/tmp/l4casadi/examples/fish_turbulent_flow/models/turbolent_flow_model.pt",
    map_location=torch.device('cpu'),
)

# Standardization
model = checkpoint["model"]
meanX = checkpoint["mean"]["x"]
stdX = checkpoint["std"]["x"]
meanY = checkpoint["mean"]["y"]
stdY = checkpoint["std"]["y"]
```

### Create L4CasADi Model from PyTorch Model

```
x = cs.MX.sym("x", 3)
xn = (x - meanX) / stdX

y = l4c.L4CasADi(model, name="turbulent_model")(xn)

y = y * stdY + meanY
fU = cs.Function("fU", [x], [y[0]])
fV = cs.Function("fV", [x], [y[1]])
```

```
/usr/local/lib/python3.10/dist-packages/torch/jit/_check.py:177: UserWarning: The TorchScript type system doesn't support instance-level
  warnings.warn(
```

## Optimization for Energy Efficiency

(This can take 1-2 minutes on Colab CPU)

```
# Generate solver
nlp = trajectory_generator_solver(
    fU=fU, fV=fV, dt=dt, N=N, T=T, u_lim=u_lim, GT=False)

# Set Initial Guess and Parameters
params = np.vstack([p_start, np.tile(p_goal[:, None], N).T])
u_init = np.zeros((N - 1, 2))
p_init = np.zeros((N, 2))
p_init[:, :] = p_start
x_init = np.vstack([p_init, u_init])

# Solve NLP
x_init_flat = cs.reshape(x_init, 4 * N - 2, 1)
params_flat = cs.reshape(params, (N + 1) * 2, 1)
sol = nlp["solver"](x0=x_init_flat, p=params_flat, lbg=nlp["lbg"], ubg=nlp["ubg"])

# extract solution
p_sol = np.squeeze(sol["x"])[: N * 2].reshape(2, N).T
u_sol = np.squeeze(sol["x"])[N * 2 :].reshape(2, N - 1).T
```

```
This is Ipopt version 3.14.11, running with linear solver MUMPS 5.4.1.

Number of nonzeros in equality constraint Jacobian...:     1204
Number of nonzeros in inequality constraint Jacobian.:      904
Number of nonzeros in Lagrangian Hessian.............:     1498

Total number of variables............................:      602
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:      304
Total number of inequality constraints...............:      753
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:      753
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  0.0000000e+00 8.60e+00 0.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  6.4898837e-02 8.14e+00 2.08e+00  -1.0 1.33e+02    -  1.72e-02 5.32e-02f  1
   2  5.8596360e-02 7.87e+00 2.01e+00  -1.0 1.21e+02    -  4.97e-02 3.40e-02h  1
   3  5.0090107e-02 7.43e+00 1.90e+00  -1.0 1.15e+02    -  5.25e-02 5.57e-02H  1
   4  5.0147752e-02 7.20e+00 1.84e+00  -1.0 1.08e+02    -  6.11e-02 3.11e-02h  2
   5  5.2546091e-02 7.03e+00 3.43e+00  -1.0 1.03e+02    -  7.22e-02 2.32e-02h  3
   6  5.8183212e-02 6.82e+00 6.37e+00  -1.0 1.00e+02    -  8.31e-02 3.04e-02h  3
   7  6.4802461e-02 6.58e+00 1.04e+01  -1.0 9.73e+01    -  9.39e-02 3.45e-02h  3
   8  6.4053990e-02 6.46e+00 1.70e+01  -1.0 9.57e+01    -  1.03e-01 1.87e-02h  4
   9  6.3103281e-02 6.31e+00 2.48e+01  -1.0 9.53e+01    -  1.16e-01 2.23e-02h  4
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  10  6.6390195e-02 6.15e+00 3.35e+01  -1.0 9.39e+01    -  1.27e-01 2.58e-02h  4
  11  8.4861792e-02 6.02e+00 4.08e+01  -1.0 9.21e+01    -  1.00e-01 2.03e-02h  4
  12  1.6512066e-01 5.86e+00 4.96e+01  -1.0 8.98e+01    -  1.25e-01 2.71e-02h  3
  13  3.9527336e-01 5.63e+00 5.80e+01  -1.0 8.60e+01    -  1.49e-01 4.01e-02h  2
  14  9.5918057e-01 5.20e+00 5.76e+01  -1.0 7.96e+01    -  1.35e-01 7.66e-02w  1
  15  1.3086806e+00 4.89e+00 5.32e+01  -1.0 6.42e+01    -  1.05e-01 5.82e-02w  1
  16  1.3692397e+00 4.67e+00 4.95e+01  -1.0 5.35e+01    -  1.02e-01 4.62e-02w  1
  17  4.6060108e-01 5.52e+00 6.49e+01  -1.0 4.99e+01    -  1.35e-01 1.92e-02h  2
  18  5.2248492e-01 5.36e+00 7.30e+01  -1.0 7.74e+01    -  1.48e-01 2.78e-02h  3
  19  5.9142931e-01 5.23e+00 8.02e+01  -1.0 7.27e+01    -  1.46e-01 2.57e-02h  3
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  20  6.7071571e-01 5.09e+00 8.46e+01  -1.0 6.80e+01    -  1.28e-01 2.54e-02h  3
  21  7.5423469e-01 4.98e+00 8.76e+01  -1.0 6.29e+01    -  1.32e-01 2.32e-02h  3
```

```
  22  8.3411131e-01 4.86e+00 8.99e+01  -1.0 5.87e+01    -  1.60e-01 2.43e-02h  3
  23  8.9727798e-01 4.74e+00 9.31e+01  -1.0 5.51e+01    -  2.08e-01 2.46e-02h  3
  24  1.0947544e+00 4.49e+00 1.01e+02  -1.0 5.31e+01    -  2.64e-01 5.15e-02h  2
  25  1.2367957e+00 4.31e+00 1.17e+02  -1.0 5.24e+01    -  1.17e-01 4.11e-02h  2
  26  1.2905738e+00 4.27e+00 1.16e+02  -1.0 5.09e+01    -  1.20e-02 9.26e-03h  4
  27  4.7817932e+00 4.00e+00 1.09e+02  -1.0 5.00e+01    -  6.32e-02 6.20e-02w  1
  28  4.8766958e+00 4.00e+00 1.22e+02  -1.0 4.45e+01    -  9.40e-02 1.91e-03w  1
  29  4.8260313e+00 3.71e+00 9.74e+01  -1.0 4.61e+01    -  1.74e-02 7.16e-02w  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  30  1.4130211e+00 4.23e+00 1.25e+02  -1.0 3.87e+01    -  6.32e-02 7.75e-03h  3
  31  1.4192510e+00 4.23e+00 1.33e+02  -1.0 4.95e+01    -  4.15e-02 4.23e-04h  8
  32  1.4199018e+00 4.23e+00 1.51e+02  -1.0 4.96e+01    -  9.03e-02 5.40e-05h 11
  33  1.5299571e+00 4.16e+00 1.53e+02  -1.0 5.00e+01    -  3.45e-02 1.68e-02h  3
  34  1.5969076e+00 4.14e+00 1.62e+02  -1.0 4.85e+01    -  4.98e-02 6.26e-03h  4
  35  1.7359933e+00 4.09e+00 1.92e+02  -1.0 4.81e+01    -  1.36e-01 1.19e-02h  3
  36  1.7665989e+00 4.06e+00 2.30e+02  -1.0 4.77e+01    -  1.19e-01 7.53e-03h  4
  37  1.8201424e+00 3.98e+00 2.52e+02  -1.0 4.78e+01    -  7.51e-02 1.74e-02h  3
```

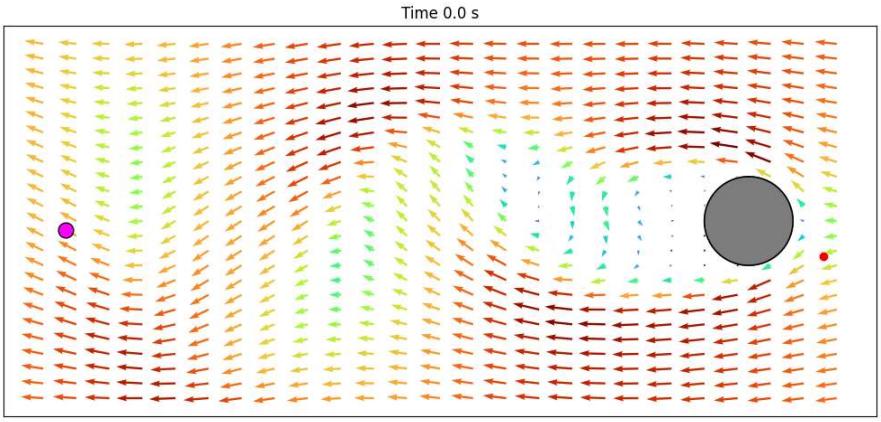
## Visualize Result

```
# Generate velocity field for visualization
neval = 25
Xgrid, Ygrid = np.meshgrid(np.linspace(-1, 8, neval), np.linspace(-2, 2, neval))
U = np.zeros((N, neval, neval))
V = np.zeros((N, neval, neval))
for t in range(0, N):
    for i in range(neval):
        for j in range(neval):
              U[t, i, j] = np.squeeze(fU([t * T / (N - 1), Xgrid[i, j], Ygrid[i, j]]))
              V[t, i, j] = np.squeeze(fV([t * T / (N - 1), Xgrid[i, j], Ygrid[i, j]]))
```

```
%%capture
# Create Animation
fig, ax = plt.subplots(figsize=(10, 5.))
frames = N
anim = FuncAnimation(
    fig,
    lambda frame_num: plot_velocity_field_particle(
        Xgrid,
        Ygrid,
        U[frame_num],
        V[frame_num],
        p_sol[max(0, frame_num - 10): frame_num + 1, 0],
        p_sol[max(0, frame_num - 10): frame_num + 1, 1],
        p_start,
        p_goal,
        round(frame_num / frames * T, 3),
    ),
    frames=frames,
    interval=100,
)

anim.save('anim.gif')
```

```
from IPython.display import Image
Image('./anim.gif')
```


Time 0.0 s