3.Let the universe be N = {0, 1, 2, . . .}.
Let
P x mean “x is prime”
Exy mean that “x equals y”
Lxy mean “x is less than y”
Ex mean “x is even”

Write in predicate logic:
(i) 2 is prime.
P2
(ii) Every prime number is odd.
∀x(Px->¬Ex)
(iii) Every prime number except 2 is odd.
∀x((Px^¬Ex2)->¬Ex
(iv) There are infinitely many prime numbers.
∀x∃y((Px^Py)->(¬Exy^Lxy)

4. Let U = {2, 3, 4, 5, 6, 7, 8, 9}, A = {x ∈ U : x is prime}, B = {2, 4, 7} C = {x ∈ U : 3 ≤ x ≤ 8}, D = {8, 9}
List all members of the sets A∪B, A∩C, A∩D, C−B, −C, A×D, D×A, D×B×D, P(B)
(where P(B) is the power set of B).
A∪B= 2, 3, 4, 5, 7
A∩C= 3, 5, 7
A∩D= -
C−B= 3, 5, 6, 8
-C= 2, 9
A×D=(2,8),(2,9),(3,8),(3,9),(5,8),(5,9),(7,8),(7,9)
D×A=(8,2),(8,3),(8,5),(8,7),(9.2), (9,3), (9,5), (9,7)
D×B×D=(8,2,8), (8,2,9) (8,3,8), (8,3,9) ,(8,5,8),(8,5,9),(8,7,8), (8,7,9), (9.2,8), (9,2,9), (9,3,8), (9,3,9), (9,5,8), (9,5,9) (9,7,8) (9,7,9)
P(B)= 2, 4, 7, (2,4), (2,7), (4,7), (2,4,7)

5. Prove that for any sets A, B, C,
A ∩ (B − C) = (A ∩ B) − (A ∩ C).

A ∩ (B − C)
Distributive
(A ∩ B) − (A ∩ C)

6. Suppose set A has 9 elements, and B has 11. Calculate the size of A ∪ B in the two cases
(a) A and B are disjoint;
P(A)+P(B)=P(AuB)
P(9)+P(11)=20
(b) A and B have 3 elements in common.
P(A)+P(B)-P(AnB)=P(AuB)
P(9)+P(11)-P(3)=17

7. How many subsets of {3, 7, 11, 12} have 11 as a member? (Calculate this without listing the subsets.)

Subset=2^n
{3, 7, 12}=2^3
{3, 7, 12}=8 Subsets have 11 as member

Let A1, A2, A3, A4 be sets with cardinalities given by $|A_i| = 2i + 1$ for $1 \leq i \leq 4$.
Calculate the cardinality of the Cartesian product set $A_1 \times A_2 \times A_3 \times A_4$

$|A_1 \text{ x } A_2| = (2(1)+1) \text{ x } (2(2)+1)$
$|A_1 \text{ x } A_2| = 3 \text{x} 5$
$|A_1 \text{ x } A_2| = 15$

$|A_1 \times A_2 \times A_3 \times A_4| = (2(1)+1) \text{ x } (2(2)+1) \text{ x } (2(3)+1) \text{ x } (2(4)+1)$
$|A_1 \times A_2 \times A_3 \times A_4| = 3 \text{ x } 5 \text{ x } 7 \text{ x } 9$
$|A_1 \times A_2 \times A_3 \times A_4| = 945$