6,7,9,10

1. Modifying the proof from lectures, prove that the cube root of 2 is irrational.
Marks 3
Suppose $3\sqrt{2}$ is rational
Let $3\sqrt{2}=p/q$ (be and q are positive integers and relatively prime)
$2=p^3/q^3$
$2q^3=p^3$ let $p=2n$ (n positive integer)
$2q^3=2^3n^3$
$q^3=2^2n^3$ let $q=2m$ (m positive integer)
$2^3m^3=2^2n^3$ <-contradiction
Therefore $3\sqrt{2}$ is irrational

2. How many words of 6 letters can be chosen from a group of 200 letters (a strange alphabet) given that
(i) Repetition is allowed
$(200)^6=64000000000000$
(ii) Repetition is not allowed
(200) x (194) x (188) x (182) x (176) x (170)=39721218000000
(iii) Given that the group is broken into two groups of 100 letters, and the first 3 must be chosen from the first group, and the last three from the second; repetion is not allowed.
(100)x(97)x(94)x(100)x(97)x(94)= 831379240000
Marks 1,1,1

3.An imaginary country is considering possible formats for its number plates. Which of the following formats would produce the largest number of possible number plates? Explain your reasoning.
Number of digits: 10
Number of letters: 26
Assuming repitition is allowed
(a) Three letters, followed by three digits.
(26)x(26)x(26)x(10)x(10x(10)=
17,576,000
(b) Seven digits.
(10)x(10)x(10)x(10)x(10)x(10)x(10)=
10000000
(c) Five letters.
(26)x(26)x(26)x(26)x(26)=
11881376
Marks 3

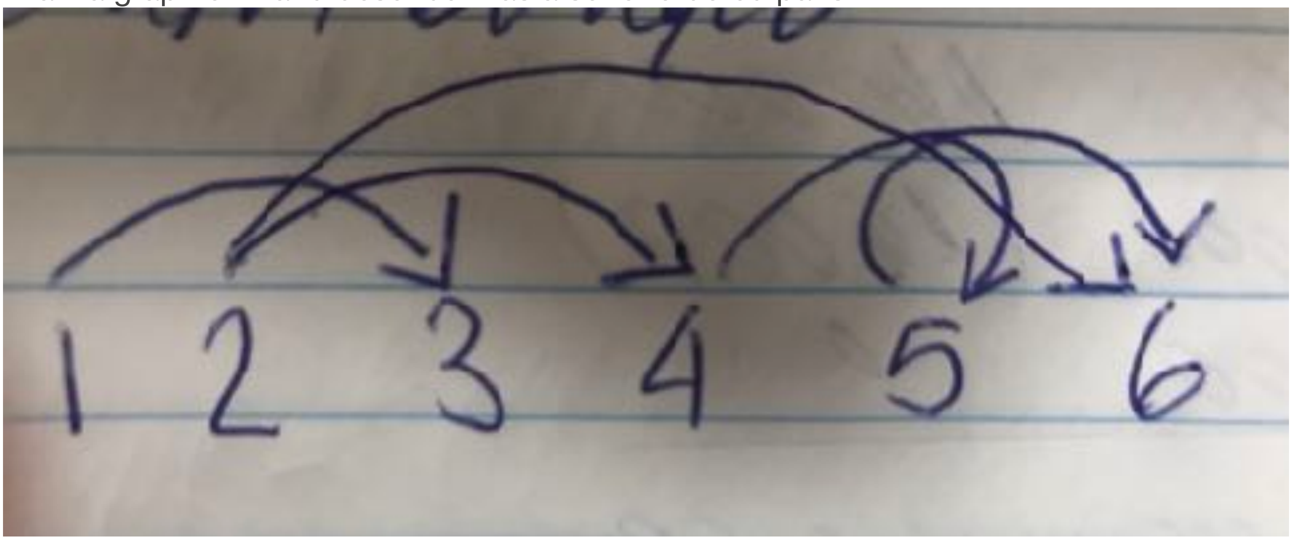
4.How many positive integers between 1 and 600 are divisible by either 6 or 10?
$|D6|=100/\lfloor 6 \rfloor$ <- symbol means smallest integer (whole number)
$|D6|=16$
$|D10|=100/\lfloor 10 \rfloor$
$|D10|=10$
$D6 \cap D10$= Smallest number divisible by both 6 & 10 = $|D30|$
$|D30|=100/\lfloor 30 \rfloor$
$|D30|=3$
$|D6 \cup D10|= |D6| + |D10| - |D30|$
$|D6 \cup D10|= 16 + 10 - 3$

$|D6 \cup D10| = 23$
23 numbers are divisible by either 6 or 10
Marks 2

5. Use the Binomial Theorem to find the coefficient of x^4 in $(x + 1/x)^{10}$

6. Let $A = \{1, 2, 3, 4\}$, and consider the following three relations on A:
id_A, $R = \{(x, y) : x \geq y\}$, $S = \{(x, y) : x + y \text{ is odd}\}$.
(i) For each relation draw a coordinate diagram, a set diagram, and a directed graph.
(ii) List the members of $\{x : xR\,3\}$ and $\{y : 2Sy\}$.
(iii) List the members of R^{-1}, RS, $R \cap S$.
Marks 1,1,1

7. Let R be the equivalence relation on the set $\{1, 2, \ldots, 7\}$ given by the partition
$\{ \{2, 4, 6\}, \{1, 3\}, \{5\}, \{7\} \}$
Draw a graph of R and describe R as a set of ordered pairs.



R=(2,4),(2,6),(4,6),(1,3),(5,5),(6,6)
Marks 2

8. Let R be a relation on a set A. Prove that if $RR \subseteq R$, then R is transitive.
Marks 1
$RR \subseteq R$
Assume R is transitive
let
$(a,b) \in R^2$
Therefore $c \in A$
Where
$(a,c) \in R^2$ and $(c,b) \in R$
Since R is transitive, $(a,b) \in R$
$(a,b) \in R^2$ and $(a,b) \in R$
$R^2 \in R$

9. Prove that the relation
$R = \{(x, y) : 4 \text{ divides } (x - y)\}$
is an equivalence relation on the set $A = \{0, 1, \ldots, 20\}$, and list all its equivalence classes
Marks 2

10.Let R be an equivalence relation on a set A. Prove

$$[a]_R \neq [b]_R \quad \text{implies} \quad [a]_R \cap [b]_R = \emptyset.$$

(Hint: give a proof by contrapositive, making use of the result proved in lectures that aRb implies [a]R = [b]R.)
Marks 1