Question 1
Find the prime factorisations of 980 and 3675. Use these factorisations to determine the greatest common denominator and least common multiple of these two numbers.

Prime factorisation
980=2^2 x 5 x 7^2
(2+1)x(1+1)x(2+1)=18 distinct factor

3875=3 x 5^2 x7^2
(1+1)x(2+1)x(2+1)=18 distinct factor

GCD: 5 x 7^2
LCM: 5^2 x 7^2

Question 2
Now use the Euclidean algorithm and Lemma 15 of the notes to find the greatest common divisor and least common multiple of 980 and 3675.

E.g
GCD (980,3675.) a=980 b=3675
3675= (980 x 3) + 735
980=(735x1)+245 <- GCD
735=(245x3)+0
Therefore 245 is GCD for 980, AND 3675

LCD (980,3675) a=980 b=3675
LCD=(axb)/gcd
LCD=(980x3675)/245
LCD=14700

Question 3
Use the Euclidean algorithm to find the greatest common divisor of a and b, then express it in the form am+bn for some m,n $\in \mathbb{Z}$ where
(i)a = 55, b = 216.
216=(55x3)+51
55=(51x1)+4
51=(4x12)+3
12=(3x4)+0
GCD=3

3=55m+216n

3=51-(4x12)
4=55-(51x1)
51=216-(55x3)

(ii) a = 330, b = 72.
330=(72x4)+42
72=(42x1)+30
42=(30x1)+12
30=(12x2)+6
12=(6x2)+0
GCD=6

6=330m+72n

6=30-(12x2)
12=42-(30x1)
30=72-(42x1)
42=330-(72x4)

Question 4
Prove that if $a \mid b$ and $c \mid d$ then $ac \mid bd$.
If a divides b and c divides d then axc divides bxd

Question 5
A number p is called a Mersenne prime if it is prime and $p = 2\ n - 1$ for some $n \in N$. For example, $7 = 2\ 3 - 1$ is a Mersenne prime. Prove that if $n > 2$ is even, then $2n - 1$ cannot be prime

Question 6
Prove that if p is prime and $p \mid a\ k$, then $p\ k \mid a\ k$

Question 7
(i) Prove that if x and y are odd, then 4 ∤ (x^2 +y^2)
(ii) Use the previous part to show that if x^2 + y^2 = z^2 where x, y,z ∈ N, then one of x and y must be even.