2. Prove the following by induction.
(i) $12 + 22 + \cdots n^2 = n(n+1)(2n+1)/6$.
Basis
$n=1$
$1^2= 1(1+1)(2x1+1)/6$
$1=1$

Assumption
$n=k$. $1^2 + 2^2 + 3^2 + \ldots\ldots + k^2= k(k+1)(2k+1)/6$
$n=k+1$ $1^2 + 2^2 + 3^2 + \ldots\ldots + k^2 + (k+1)^2 = (k+1)((k+1) +1)(2(k+1) +1)/6$

$k(k+1)(2k+1)/6 + (k+1)^2 = (k+1)(k+2)(2k+3)/6$
$k(k+1)(2k+1) + 6(k+1)^2 = (k+1)(k+2)(2k+3)$
$k(2k+1) + 6(k+1) = (k+2)(2k+3)$
$2k^2+k+6k+6 = 2k^2+3k+4k+6$
$2k^2+7k+6=2k+7k+6$

(ii) $1 + 2 + 22 + 23 \cdots + 2^n = 2^{(n+1)} - 1$.
Basis
$n=1$
$2^0+2^1 = 2^{(1+1)} - 1$.
$3=3$

Assumption
$n=k$. $1+2+2^2+\ldots+2^k= 2^{(k+1)}-1$
$n=k+1$ $1+2+2^2+\ldots+2^{(k+1)}= 2^{((k+1)+1)}-1$

$1+2+2^2+\ldots+2^{(k+1)}= 2^{((k+1)+1)}-1$
$2^{(k+1)} + 2^{(k+1)} = 2^{(k+2)}$
$2^1\ 2^k + 2^1\ 2^k = 2^2\ 2^k$
$2^k+ 2^k= 2^1\ 2^k$
$2(2^k)=2(2^k)$

(iii) 3 divides $7^{(n)} + 2$
Basis
$n=1$
$7^1+2=9$
$9/3=3$

RTP
$n=k$ 3 divides $(7^k + 2)$.
$n=k+1$ 3 divides $(7^{(k+1)} + 2)$.

Assumption
$n=k$
$7^k + 2 = 5m$
$7^k= 5m -2$

$7^{(k+1)} + 2=7(7^k) + 2$
$7^{(k+1)} + 2=6(7^k)+(7^k)+2$
6 is divisible by 3 / 3 is a factor of $(7^k)+2$ -Induction

3. Modifying the proof from lectures, prove that the square root of 3 is irrational. (You may assume that for any prime number p, and for any number $z \in Z$, if p is a factor of z^2 then p is a factor of z.)

√3 is irrational
Suppose not
√3 is rational

√3= a/b where a&b have no common factors

3= a^2/b^2
3b^2=a^2

If b is even then a must also be even, but this would then mean that z would be a common factor which is false

Therefore a&b must be odd

a=2n+1
b=2n+1

3(2m+1)^2=(2n+1)^2
3(4m^2+4m+1)=4n^2+4n+1
12m^2+12m+ 3 =4n^2+4n+1
12m^2+12m=4n^2+4n- 2
6m^2+ 6m=2n^2+2n- 1
2(3m^2+ 3m)=2(n^2+n)-1
Even / odd

This contradiction indicates that the preposition is false. Therefore √3 is irrational

The game of grippleball has only two methods of scoring, one that scores 2 points and one that scores 5 points. Show that any score of 5 or more is possible.

n=5
p(5)= 2(0) + 5(1)
= 5

Assumption
k>/ 5
K= 2a+5b where n=k

k>/5
When b>/1 -> a>/0
e.g
5=2(0)+5
5=5

k>/5
When a>/3 -> b>/0
e.g
6=2(3) + 5(0)
6=6

5. Translate into predicate logic using the given dictionary.
Let

- Px mean that x is a person.
- Wzq mean that z works at q.
- $F v$ mean that v is a factory.
- Sx mean that x lives in Sydney.

(i) Sydney has people.

$\exists x(Px \wedge Sx)$

(ii) Nobody lives in Sydney.

$\forall x(Px \rightarrow \neg Sx)$

(iii) Everyone works at a factory.

$\forall x(Px \rightarrow \exists y(Fy \wedge Wxy)$

(iv) Nobody in Sydney works at a factory.

$\forall x(Px \wedge Sx) \rightarrow \exists y(Fy \wedge xWy)$

(v) Factories have workers.

$\forall x(Fx \rightarrow \exists y(py \wedge wyx)$