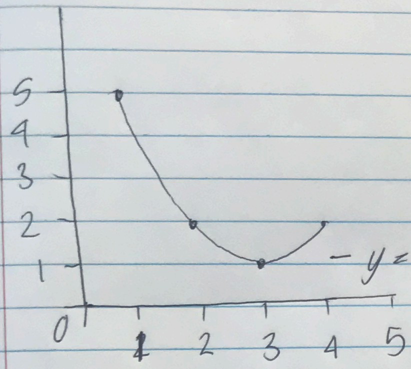
Question 1

$$f(x)\begin{cases} c(x^2-6x+10) & 1<x<4 \\ 0 & \text{otherwise} \end{cases}$$



Since $x^2-6x+10$ is non negative for all x
the function could be a pdf as it is non negative.

It should also have a total probability area of 1 as there is a constant c that should make this plausible if $c \geq 0$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{1} f(x)\,dx + \int_{1}^{4} c(x^2-6x+10)\,dx + \int_{4}^{\infty} 0\,dx = 1$$

$$0 + c\left[\frac{x^3}{3} - \frac{6}{2}x^2 + 10x\right]_1^4 + 0 = 1$$

$$c\left[\left(\frac{(4^3)}{3} - 3(4^2) + 10(4)\right) - \left(\frac{(1^3)}{3} - 3(1^2) + 10(1)\right)\right] = 1$$

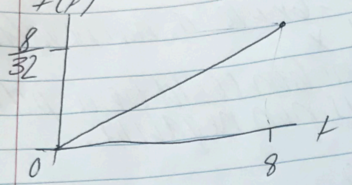
$$c[6] = 1$$

$$c = \frac{1}{6}$$

Question 2

$$f(t) \begin{cases} \frac{t}{32} & 0 \le t \le 8 \\ 0 & \text{otherwise} \end{cases}$$

a) $f(t)$



b) For $t<0$

$$F_T(t) = \int_{-\infty}^{\infty} f_T(u)\,du = \int_{-\infty}^{t} 0\,du = 0$$

For $0 \le t \le 8$

$$F_T(t) = \int_{-\infty}^{\infty} f_T(u)\,du = \int_{-\infty}^{0} 0\,du + \int_{0}^{t} \frac{u}{32}\,du$$

$$= 0 + \frac{1}{32}\left[\frac{u^2}{32}\right]_0^t$$

$$= \frac{t^2}{64}$$

For $t<8$

$$F_T(t) = \int_{-\infty}^{\infty} f_T(u)\,du = \int_{-\infty}^{0} 0\,du + \int_{0}^{8} \frac{u}{32}\,du + \int_{8}^{\infty} 0\,du$$

$$= 0 + \frac{1}{32}\left[\frac{u^2}{2}\right]_0^8 + 0$$

$$= 1$$

$$F_T(t) = \begin{cases} \frac{t^2}{64} & 0 \le t \le 8 \\ 1 & t > 8 \\ 0 & t < 0 \end{cases}$$

c) $P(T \geq 5) = 1 - P(T \leq 5)$

$= 1 - F_T(5)$

$= 1 - \frac{5^2}{64}$

$= 0.61$

d) $P(3 \leq T \leq 5) = P(T \leq 5) - P(T \leq 3)$

$= F_T(5) - F_T(3)$

$= \frac{5^2}{64} - \frac{3^2}{64}$

$= 0.25$

e $P(T \geq 5 \mid T \geq 3) = \frac{P((T \geq 5) \cap (T \geq 3))}{P(T \geq 3)}$

$= \frac{P(T \geq 5)}{P(T \geq 3)}$

$= \frac{1 - F_T(5)}{1 - F_T(3)}$

$= \frac{0.609375}{0.859375}$

$= 0.7091$

Question 3

$$f(x) = \begin{cases} Ax(3-x) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,dx = 1$

$0 + A\int_{0}^{2} 3x - x^2\,dx + 0 = 1$

$A\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$

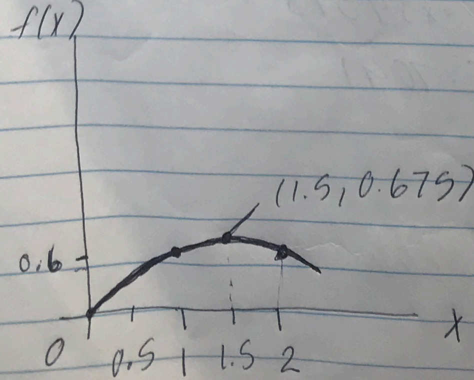
$A\left[\left(\frac{3(2)^2}{2} - \frac{(2)^3}{3}\right) - \left(\frac{3(0)^2}{2} - \frac{(0)^3}{3}\right)\right] = 1$

$A[3.3\dot{3}] = 1$

$A = \frac{1}{3.3\dot{3}}$

$A = 0.3$

b) $f(x) \begin{cases} 0.3x(3-x) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$



c) $E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$= \int_{-\infty}^{0} x f(x) dx + \int_0^2 x\ f(x) dx + \int_2^{\infty} x f(x) dx$

$= \int_{-\infty}^{0} 0 dx + \int_0^2 x \times 0.3x(3-x) dx + \int_2^{\infty} 0 (dx)$

$= 0 + 0.3 \int_0^2 x^2 (3-x) dx + 0$

$= 0.3 \left[\frac{3x^3}{3} - \frac{x^4}{4} \right]_0^2$

$= 0.3 \left[\left(2^3 - \frac{(2^4)}{4} \right) - 0 \right]$

$= 1.2$ (hunreds of dollars)

$= \$120 \Leftarrow 1.2 \times \$100$

The mean cost is \$120

d) $Var(x) = E(x^2) - E(x)$

$= \left(\int_{-\infty}^{0} x^2 f(x) dx + \int_0^2 x^2 f(x) dx + \int_2^{\infty} x^2 f(x) dx \right) - 1.2^2$

$= \left(\int_{-\infty}^{0} 0 dx + \int_0^2 x^2 \times 0.3 \times x(3-x) dx + \int_2^{\infty} 0 dx \right) - 1.2^2$

$= \left(0 + 0.3 \int_0^2 3x^3 - x^4 dx + 0 \right) - 1.2^2$

$= 0.3 \left[\frac{3(x^4)}{4} - \frac{(x^5)}{5} \right]_0^2 - 1.2^2$

$= 0.3 \left[\frac{3}{4}(2^4) - \frac{(2^5)}{5} \right] - 1.2^2$

$= 1.68 - 1.2^2$

$= 0.24$ (hunreds of dollars)

Variance cost is \$24 $\Leftarrow (0.24 \times 100)$

Question 4

$$f(x)\begin{cases} 0 & x<1 \\ -0.25x^3+1.5x^2-2.25x+1 & 1\le x\le 3 \\ 1 & x\ge 3 \end{cases}$$

a) $P(X\le 1.5)=F_X(1.5)$

$= -0.25(1.5^3)+1.5(1.5^2)-2.25(1.5)+1$

$= 0.15625$

b) $$f(x)=\frac{d}{dx}F(x)=\begin{cases} \frac{d}{dx}0 & x<1 \\ \frac{d}{dx}\left[-\frac{1}{4}x^3+\frac{3}{2}x^2-\frac{9}{4}x+1\right] & 1\le x\le 3 \\ \frac{d}{dx}1 & x<3 \end{cases}$$

$$f(x)=\begin{cases} -\frac{3}{4}x^2+3x-\frac{9}{4} & 1\le x\le 3 \\ 0 & \text{otherwise} \end{cases}$$

c) $\cancel{Var(x)=4.2-\left(\int_{-\infty}^{\infty} x f(x)\,dx\right)^2}$

$Var(x)=E(X^2)-(E(X)^2)$

$E(X)=\int_{-\infty}^{\infty} x f(x)\,dx$

$=\int_{-\infty}^{1} 0\,dx+\int_1^3 x\cdot\left(-\frac{3}{4}x^2+3x-\frac{9}{4}\right)dx+\int_3^{\infty} 0\,dx$

$=0+\int_1^3 -\frac{3}{4}x^3+3x^2-\frac{9}{4}x\,dx+0$

$=\left[-\frac{3}{16}x^4+x^3-\frac{9}{8}x^2\right]_1^3$

$=\frac{27}{16}+\frac{5}{16}$

$=2$

$Var(x)=4.2-2^2$

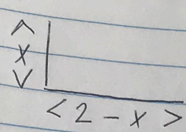
$=0.2$

Question 9

$X \sim U(0,2)$

$$f(x) = \begin{cases} \frac{1}{b-a} & (a \le x \le b) \\ 0 & \text{otherwise} \end{cases}$$

$$f_X(x) \begin{cases} \frac{1}{2} & 0 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$$

a) 

2m long stick broken into 2 peices at some distance x m the broken stick have lengths x and 2-x

Area = Lenght x widht
$= x(2-x)$
$= 2x - x^2$

b) $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^2 x\left(\frac{1}{2}\right)dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^2 = 1$

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^2 x^2\left(\frac{1}{2}\right)dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = \frac{4}{3}$

$E(A) = E(2X - X^2)$
$= E(2X) - E(X^2)$
$= 2E(X) - E(X^2)$
$= 2(1) - \left(\frac{4}{3}\right)$
$= \frac{2}{3}$

$Var(X) = E(X^2) - (E(X)^2)$
$= \frac{4}{3} - 1^2$
$= \frac{1}{3}$

$2Var(X) = \frac{1}{3} \times 2$
$= \frac{2}{3}$

$\therefore E(A) = 2Var(X)$