1) **<u>Call me . . . if you can</u>**
<u>A box contains 200 cell phones; 5 are defective and 195 are fine.</u>

(a) Two cell phones are selected at random with replacement by a quality control engineer.

(i) Calculate the probability that neither cell phone is defective.
(195/200)*(195/200)= 0.950625

(ii) Calculate the probability that exactly one cell phone is defective.
(195/200)*(5/200) + (5/200)*(195/200) = 0.04875

(iii) Calculate the probability that both cell phones are defective.
(5/200)*(5/200)= 0.000625

(iv) Explain whether or not the three probabilities you have just calculated should add to unity.

Let W = "Cellphone works"
Let W'= "Cellphone defective"

Sample space of (a) = {(W∩W),(W∩W'),(W'nW),(W'nW')}
P(S) = 1

Probabilities calculated:
P(W∩W)=0.950625
P((W∩W')∪(W'nW))=0.04875
P(W'nW')=0.000625

The 3 probabilities calculated make up the sample space of (a)
Hence their sum should add to unity (1)
P(W∩W)+P((W∩W')∪(W'nW))+P(W'nW') = {(W∩W)+(W∩W')+(W'nW)+(W'nW')}
0.950625+0.04875+0.000625=1

(b) Another quality control engineer now selects two cell phones at random without replacement.

(i) Calculate the probability that neither cell phone is defective.
(195/200)*(194/199)= 0.95050251256

(ii) Calculate the probability that exactly one cell phone is defective.
(195/200)*(5/199) + (5/200)*(195/199) = 0.04899497487

(iii) Calculate the probability that both cell phones are defective.
(5/200)*(4/199)= 0.00050251256

(iv) Explain whether or not the three probabilities you have just calculated should add to unity.

Let W = "Cellphone works"
Let W'= "Cellphone defective"

Sample space of (b) = $\{(W \cap W),(W \cap W'),(W' n W),(W' n W')\}$
$P(S)) = 1$

Probabilities calculated:
$P(W \cap W)=0.95050251256$
$P((W \cap W') \cup (W' n W))=0.04899497487$
$P(W' n W')= 0.00050251256$

The 3 probabilities calculated make up the sample space of (b)
Hence their sum should add to unity (1)
$P(W \cap W)+P((W \cap W') \cup (W' n W))+P(W' n W') = \{(W \cap W)+(W \cap W')+(W' n W)+(W' n W')\}$
$0.95050251256+0.04899497487+ 0.00050251256 =1$

(c) Explain whether the defectiveness of the second sampled cell phone is independent of the defectiveness of the first sampled cell phone in each of parts (a) and (b).

Two events (e.g A and B) are independent if $P(AnB)=P(A)*P(B)$

For part (a)
$P(W' n W')= 0.000625$
$P(W')*P(W')= (5/200)*(5/200)$
$P(W')*P(W')= 0.000625$
$P(W' n W')=P(W')*P(W')$
In part (a) the defectiveness of the second sampled cell phone is independent of the defectiveness of the first sampled cell phone

For part (b)
$P(W' n W')= 0.00050251256$
$P(W')*P(W')= (5/200)*(5/200)$
$P(W')*P(W')= 0.000625$
$P(W' n W') \neq P(W')*P(W')$
In part (b) the defectiveness of the second sampled cell phone is not independent of the defectiveness of the first sampled cell phone

2) A committee of size 4 is to be chosen from a group of 6 men and 8 women. The 4 names are to be chosen randomly "out of a hat".

(a) Describe the sample space for this situation (but don't try and list all the sample points, just describe it in terms of combinations or permutations with or without replacement) and calculate the probability of selecting any given committee.

There are (6+8)14 people in total
A committee consist of 4 people
(No replacement and if order chosen is not important then)

$${}^{n}C_{k} : \frac{n!}{k!(n-k)!}$$

n=14
k=4

^nC_k : =14!/4!(14-4)!

14C4=14!/4!(10)!
14C4=14*13*11*10!/4!(10)!
14C4=14*13*12*11/4! = 1001

There are 1001 ways a committee of 4 can be chosen from the 16 people

(b) What is the probability that a committee will have a balance of two men and two women?
Let B be "committee will have a balance of two men and two women?"

$^nC_k : \frac{n!}{k!(n-k)!}$

n(B)=(6C2)*(8C2)
n(B)=(6!/2!(6-2)!)*(8!/2!(8-2)!)
n(B)=(6!/2!4!)*(8!/2!6!)
n(B)=8!/2!4!2!
n(B)=8*7*6*5*4!/2!4!2!
n(B)=8*7*6*5/2!2!
n(B)=8*7*6*5/2*1*2*1
n(B)=420
P(B)=420/1001

P(B)=0.41958..

(c) What is the probability that at least one woman will be selected?
Let W be "at least one woman will be selected"

$^nC_k : \frac{n!}{k!(n-k)!}$

P(W) = 1-P(W')
P(W) = 1-(6C4/14C4)
P(W) = 1-((6!/4!(6-4)!)/1001)
P(W) = 1-((6!/4!(2)!)/1001)
P(W)= 1-((6*5*4!/4!(2)!)/1001)
P(W)= 1-((6*5/(2)!)/1001)
P(W)= 1-(15/1001)
P(W)=0.9850…

3) Beethoven wrote 9 symphonies, Mozart wrote 27 piano concertos and Schubert composed 15 string quartets. A university radio station announcer wishes to play 3 different compositions from these collections in a day.
(a) How many ways can the announcer choose to play if the order of compositions matters?
Total songs= 9+27+15
Total songs= 51
Number of sets=3

$^nP_k = \frac{n!}{(n-k)!}$

n=51
k=3
Let K be total number of sets
n(K)=51P3
n(K)=51!/(51-3)!
n(K)=51!/(48)!
n(K)=51*50*49*48!/(48)!
n(K)=51*50*49
n(K)=124950 number of sets can be played

(b) If the announcer wishes to play a Beethoven symphony first, then a Mozart piano concerto but the last piece is not restricted, in how many ways can this be done?

^nC_k : $\frac{n!}{k!(n-k)!}$

Let T be number of sets if a beethhoven symphony is first, mozart concerto is second and a random piece third

‘3 different compositions’ - hence no repetition (51-2=49)
n(T)=9C1*27C1*49C1
n(T)=(9!/1!8!)*(27/1!26!)*(49!/1!48!)
n(T)=(9!/8!)*(27!/26!)*(49!/48!)
n(T)=(9*8!/8!)*(27*26!/26!)*(49*48!/48!)
n(T)=9*27*49
n(T)=11907 number of sets with beethhoven symphony is first, mozart concerto is second and a random piece third can be played

(c) If it does not matter in what order the three selected compositions are played, what is the probability that they are all from Mozart?
Let M be all three songs are from Mozart
Let A be all possible sets from all musician

^nC_k : $\frac{n!}{k!(n-k)!}$

n(M)=27C3
n(M)=27!/3!(27-3)!
n(M)=27!/3!(27-3)!
n(M)=27!/3!24!
n(M)=27*26*25*24!/3!24!
n(M)=27*26*25/3*2*1
n(M)=2925

n(A)=51C3
n(A)=51!/3!48!
n(A)=51*50*49*48!/3!48!
n(A)=(51*50*49)/(3*2*1)
n(A)=20825

P(M)=n(M)/n(A)
P(M)=2925/20825
P(M)=0.140456..

(d) The station manager decides that on each successive day, a Beethoven symphony will be played, followed by a Mozart piano concerto, followed by a Schubert string quartet. For roughly how many years could this policy be continued before exactly the same program would have to be repeated?

$^{n}C_{k}$: $\frac{n!}{k!(n-k)!}$

Let U be Beethoven symphony will be played, followed by a Mozart piano concerto, followed by a Schubert string quartet

n(U)=9C1*27C1*15C1
n(U)=(9!/1!8!)*(27!/1!26!)*(15!/1!14!)
n(U)=(9*8!/1!8!)*(27*26!/1!26!)*(15*14!/1!14!)
n(U)=9*27*15
n(U)=3645 number of ways

Number of years = 3645/365
Number of years = 9.98
Number of years = 10 years

4 The blood group distribution in a certain country is A: 41%, B: 9%, AB: 4%, O: 46%. For a randomly selected individual from that country, calculate:
(a) the odds on having blood group A,

O(A)=P(A)/P(A')
O(A)=0.41/1-0.41
O(A)=0.41/0.59
O(A)= 41:59

(b) the odds on having blood group A or blood group B,
O(AuB)=P(AuB)/1-P(AuB)
O(AuB)=0.5/1-0.5
O(AuB)=0.5/0.5
O(AuB)=0.5:0.5 = 1:1

(c) the odds on having blood group A, given that the individual does not have blood group O.
O(A|O')=P(A|O')/P(A'|O')

P(A|O')=P(AnO')/P(O')
Since the events are independent
P(A|O')=P(A)/P(O')
P(A|O')=0.41/1-0.46
P(A|O')=0.41/0.54
P(A|O')=0.75925925925

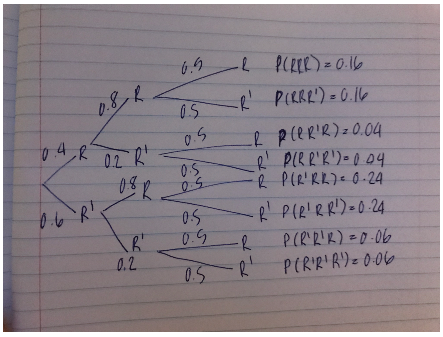
P(A'|O')=P(A'nO')/P(O')

P(A|O’)=P(AB)+P(B)/P(O’)
P(A|O’)=0.13/1-0.46
P(A|O’)=0.13/0.54
P(A|O’)=0.24074074074

O(A|O’)=P(A|O’)/P(A’|O’)
O(A|O’)=(0.41/0.54)/(0.13/0.54)
O(A|O’)=(0.41/0.54)*(0.54/0.13)
O(A|O’)=0.41/0.13
O(A|O’)=41:13

5 On the way to work a person passes through three intersections monitored by traffic lights. The location and operation of these traffic lights is such that, to all intents and purposes, they appear to operate independently to a person travelling from one to another. The probability of a red light is 0.4, 0.8, and 0.5 respectively for each of the traffic lights.

Let R be the event of a red light



(a) Find the probability function of X, the number of red lights the person encounters in a single trip.
P(X=0)=P(R’R’R’)=0.06
P(X=1)=P(RR’R’)+P(R’R’R)+P(R’RR’)=0.04+0.06+0.24=0.34
P(X=2)=P(RRR’)+P(RR’R)+P(R’RR)=0.16+0.04+0.24=0.44
P(X=3)=P(RRR)=0.16

(b) Compute the mean of X.
E(x)=(0*0.06)+(1*0.34)+(2*0.44)+(3*0.16)
E(x)=0+0.34+0.88+0.46
E(x)=1.7

(c) Assume that the waiting time for each red light is two minutes. What is the mean waiting time in one trip?
Mean waiting time in one trip=1.7*2
Mean waiting time in one trip=3.4 minutes