Question 10

Question A

P(A|B) < P(A) show that P(B|A) < P(B)
P(A|B)=P(AnB)/P(B)
P(AnB)/P(B) < P(A)   <=substitution
P(AnB) < P(A) * P(B)   <=multiply each side by P(B)
P(AnB)/P(A) < P(B)   <= divide each side by P(A)
P(AnB)/P(A)=P(B|A)
P(B|A) < P(B)   <=substitution

Question B

i)

μx=E(x)= ∑xipi = x1p1+x2p2+…+xnpn
all xi
E(x)=(0*0.1)+(1*0.2)+(2*0.3)+(3*0.4)
E(x)=2

ii)

E(x^2)= ∑xi^2pi = x1^2*p1+x2^2*p2+…+xnpn
all xi
E(x^2)=(0.1*(0^2))+(0.2*(1^2))+(0.3*(2^2))+(0.4*(3^2))
E(x^2)=5

σ^2 =Var(x)=E[(X-μx)^2]=∑(xi-μx)^2*pi = E(X^2)-(E(x)^2)
all i
Var(x)=5-(2^2)
Var(x)=1

σx=√σ^2= √Var(x)
σx=√1
σx=1

iii)

-Let X be the number of computer sold
-The number of computers repurchased by the manufacturer is K=3-X
-S= 5000 (contribution margin 1000-500)
-O= -500 (original cost)
-R= 200 (repurchase returns)

h(X=0)=(X*S)+K(O-R)=(0*500)+3(-500+200)= - 900
h(X=1)=(X*S)+K(O-R)=(1*500)+2(-500+200)= -100
h(X=2)=(X*S)+K(O-R)=(2*500)+1(-500+200)= 700
h(X=3)=(X*S)+K(O-R)=(3*500)+0(-500+200)= 1500

μx=E(x)= ∑xipi = x1p1+x2p2+…+xnpn
all xi
E(h(X))=(- 900*0.1)+(-100*0.2)+(700*0.3)+(1500*0.4)

E(h(X))= 700
The expected profit is $700

Question C

i)

Poisson condition

1)occurrences/events are counted in a fixed interval of time or space: the experiment counts the number of error (event) on each page (interval)
2)events occur independently: the occurrence of an error on a page does not effect the probability of an occurrence of a future error
3)events occur one at a time: an error can only occur one at a time

<u>The experiment meets all 3 conditions of a Poisson random variable</u>

ii)

$\bar{x}= (1/n)* \sum_{i=1}^{n} X_i x_i = (1/n)*(X1* x1+X2* x2,\ldots,Xn*xn)$

$\bar{x}=((0*4)+(1*11)+(2*31)+(3*21)+(4*15)+(5*9)+(6*4)+(7*5))*(1/100)$

<u>$\bar{x}=3$</u>

$E(x^2)= \sum_{\text{all } x_i} x_i^2 p_i = x1^2*p1+x2^2*p2+\ldots+xnpn$

$E(x^2)=(((0^2)*4)+((1^2)*11)+((2^2)*31)+((3^2)*21)+((4^2)*15)+((5^2)*9)+((6^2)*4)+((7^2)*5))*(1/100)$

$E(x^2)=11.78$

$\sigma^2 =Var(x)=E[(X-\mu x)^2]=\sum_{\text{all } i}(xi-\mu x)^2*pi = E(X^2)-(\bar{x}^2)$

$Var(x)=11.78-(3^2)$

$Var(x)=2.78$

$\sigma x=\sqrt{\sigma^2}= \sqrt{Var(x)}$

$\sigma x=\sqrt{2.78}$

$\sigma x=1.66733320005$

<u>$\sigma x=1.6673$</u>

iii)

t=1 (interval 1 page)

$\mu=\bar{x}/t$

$\mu=3/1$

$\mu=3$ <= estimated parameter

<u>$X\sim Poi(\mu) = X\sim Poi(3)$</u>

iv)

$X\sim Poi(3)$

$\mu=3$

$P(x)=P(X=x)=(\mu^x*e^{(-\mu)})/x!)$

$P(x)=P(X=2)=(3^2*e^{(-3)})/2!)$

<u>$P(x)=P(X=2)=0.2240$ (4 d.p)</u>

v)

x	0	1	2	3	4	5	≥6	Total
(pi*total number of pages) = ei	4.98	14.94	22.4	22.4	16.8	10.08	8.39	99.99

(note: pi when x=7 < 5, therefore combined with x=6)

$X^2= \sum((fi^2)/ei) - n$

all i

$X^2=((4^2)/4.98)+((11^2)/14.94)+((31^2)/22.4)+((21^2)/22.4)+((15^2)/16.8)+((9^2)/10.08)+((9^2)/8.39)-100$

$X^2=104.984121884-100$

$X^2=4.9841$

$v=(k-1-\text{number of estimated parameters})$

$v=(7-1-1)$

$v=5$ df

$X^5(0.05)=11.07$ <=chi squared table

$4.9841<11.07$ therefore $X^2<X^5(0.05)$

Because X^2 is less than $X^5(0.05)$ this indicates that poisson is a good fit for the data. As a result it is plausible that the typist made random errors through the document as the errors have occur randomly due to fixed probability .