1a)
For $x < -2$
$F_X(x) = 0$

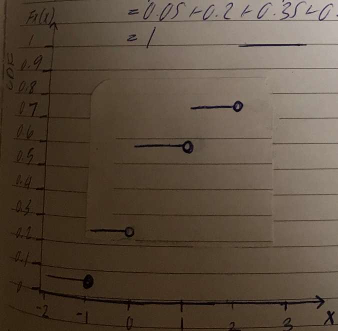
For $-2 \le x < -1$
$F_X(x) = P(x = -2)$
$= 0.05$

For $-1 \le x < 0$
$F_X(x) = P(x = -2) + P(x = -1)$
$= 0.05 + 0.2$
$= 0.25$

For $0 \le x < 1$
$F_X(x) = P(x = -2) + P(x = -1) + P(x = 0)$
$= 0.05 + 0.2 + 0.35$
$= 0.6$

For $1 \le x < 2$
$F_X(x) = P(x = -2) + P(x = -1) + P(x = 0) + P(x = 1)$
$= 0.05 + 0.2 + 0.35 + 0.15$
$= 0.75$

For $2 \le x < 3$
$F_X(x) = P(x = -2) + P(x = -1) + P(x = 0) + P(x = 1) + P(x = 2)$
$= 0.05 + 0.2 + 0.35 + 0.15 + 0.25$
$= 1$



1b)
$E(x) = X(s_1)p(s_1) + X(s_2)p(s_2) \ldots$
$E(x) = (-2 \times 0.05) + (-1 \times 0.2) + (0 \times 0.35) + (1 \times 0.15) + (2 \times 0.25)$
$E(x) = 0.35$

$Var(x) = E(x^2) - \mu_x^2$

$E(x^2) = ((-2^2) \times 0.05) + ((-1^2) \times 0.2) + ((0^2) \times 0.35) + (1^2 \times 0.15) +$
$(2^2 \times 0.25)$
$= 1.55$

$\mu_x = E(x) \therefore \mu_x^2 = E(x)^2$

$E(x)^2 = 0.35^2$
$= 0.1225$

$Var(x) = 1.55 - 0.1225$
$= 1.4275$

$\sigma x = \sqrt{Var(x)}$
$= \sqrt{1.4275}$
$= 1.19478\ldots$
$= 1.1948$

2a) $S = \{(1,1), (1,2), (1,3), (1,4)$
$(2,1), (2,2), (2,3), (2,4)$
$(3,1), (3,2), (3,3), (3,4)$
$(4,1), (4,2), (4,3), (4,4)\}$

2b) $S(x,y) = \{(1,0), (2,-1), (3,-2), (4,-3)$
$(2,1), (2,0), (3,-1), (4,-2)$
$(3,2), (3,1), (3,0), (4,-1)$
$(4,3), (4,2), (4,1), (4,0)\}$

2c)

x	1	2	3	4
x^2	1	4	9	16
$f_x(x)$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$

y	-3	-2	-1	0	1	2	3
y^2	9	4	1	0	1	4	9
$f_y(y)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

2d)
$\mu x = x(s1)p(s1) + x(s2)p(s2) \ldots$
$\mu x = (1 \times \frac{1}{16}) + (2 \times \frac{3}{16}) + (3 \times \frac{5}{16}) + (4 \times \frac{7}{16})$
$\mu x = 3.125$

$\mu y = y(s1)p(s1) + y(s2)p(s2) \ldots$
$\mu y = (-3 \times \frac{1}{16}) + (-2 \times \frac{2}{16}) + (-1 \times \frac{3}{16}) + (0 \times \frac{4}{16}) + (1 \times \frac{3}{16}) +$
$(2 \times \frac{2}{16}) + (3 \times \frac{1}{16})$
$\mu y = 0$

$\sigma x = \sqrt{var(x)}$
$var(x) = E(x^2) - \mu x^2$
$E(x^2) = (1^2 \times \frac{1}{16}) + (2^2 \times (\frac{3}{16})) + (3^2 \times \frac{5}{16}) + (4^2 \times \frac{7}{16})$
$= 10.625$
$var(x) = 10.625 - (3.125)^2$
$= 0.859375$

$\sigma x = \sqrt{0.859375}$
$\sigma x = 0.92702\ldots$
$\sigma x = 0.9270$

$var(y) = E(y^2) - \mu y^2$

$\sigma y = \sqrt{var(y)}$
$E(y^2) = (-3^2 \times \frac{1}{16}) + (-2^2 \times \frac{2}{16}) + (-1^2 \times \frac{3}{16}) + (0^2 \times \frac{4}{16})$
$+ (1^2 \times \frac{3}{16}) + (2^2 \times \frac{2}{16}) + (3^3 \times \frac{1}{16})$
$= 2.5$
$var(y) = 2.5 - (0)^2$
$= 2.5$

$\sigma y = \sqrt{2.5}$
$= 1.581138\ldots$
$= 1.5811$

e) $S_{xy} = \{(0), (-2), (-6), (-12)$
$(2), (0), (-3), (-8)$
$(6), (3), (0), (-4)$
$(12), (8), (4), (0)\}$

xy	-12	-8	-6	-4	-3	-2	0	2	3	4	6	8	12
$f_{xy}(xy)$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

$\sigma xy = E(XY) - \mu x \mu y$

$E(XY) = xy(s1)p(s1) + xy(s2)p(s2) \ldots$
$= (-12 \times \frac{1}{16}) + (-8 \times \frac{1}{16}) + (-6 \times \frac{1}{16}) + (-4 \times \frac{1}{16}) + (-3 \times \frac{1}{16})$
$+ (-2 \times \frac{1}{16}) + (0 \times \frac{4}{16}) + (2 \times \frac{1}{16}) + (3 \times \frac{1}{16}) + (4 \times \frac{1}{16})$
$+ (6 \times \frac{1}{16}) + (8 \times \frac{1}{16}) + (12 \times \frac{1}{16})$
$= 0$

$\sigma xy = 0 - (3.125 \times 0)$
$\sigma xy = 0$

$\sigma xy = 0$ suggest that X and Y are uncorrelated as

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}}$$

$$\rho = \frac{0}{\sqrt{Var(X)\,Var(Y)}}$$

$\rho = 0$

E) X and Y are independent if and only if for all i and j

$P(X=i \text{ and } Y=j) = P(X=i) \times P(Y=j)$

let $i=1$ $P(X=1 \text{ and } y=0) = \frac{1}{16}$

let $j=0$ $P(x=1) \times P(y=0) = \frac{1}{16} \times \frac{4}{16} = \frac{1}{64}$

$\frac{1}{16} \neq \frac{1}{64} \therefore P(x=i \text{ and } y=j) \neq P(x=i) \times P(y=j)$

This means that X and Y are not independent

3ai) The 4 conditions for a binomial ~~normal~~ Random variable

- fixed number of trials
- each trial has 2 possible outcomes
- each trial has ~~equal~~ fixed probability of success
- trials are independent

- N has a fixed number of trials $= n$
- N has 2 possible outcomes (+ or −) for each trial
- N has ~~equal~~ fixed probability of success for each trial, $p = 0.5$
- N trials are independent - outcome of one trial doesn't influence the outcome of any other trial

N meets the 4 conditions of a random binomial variable

ii) let n be the number of trials in N
let p be the probability of success

E(N) = ~~mean of N~~ $= n \times p = n \times 0.5 = \frac{n}{2}$

standard deviation of N $= \sqrt{n \times p \times (1-p)} = \sqrt{n \times 0.5 \times 0.5} = \sqrt{\left(\frac{n}{4}\right)}$

iii) X = no of uses $\quad X \sim B(5, 0.9)$

$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$= \left(\binom{5}{3} \times 0.5^3 \times 0.5^2\right) + \left(\binom{5}{4} \times 0.5^4 \times 0.5^1\right) + \left(\binom{5}{5} \times 0.5^5 \times 0.5^0\right)$

$= 0.5$

b) The 4 conditions for a Random binomial variable

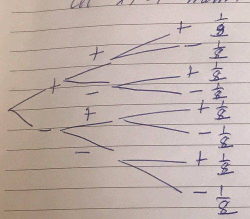
- fixed number of trials
- each trial has a fixed probability of success
- each trial has 2 possible outcome
- trials are independent

- Z has a fixed number of trials $= n$
- each trial has a fixed probability of success in Z
- a trial has 2 outcome in Z (assuming first and last day index is not counted - otherwise more than 2 outcomes)

• trials in Z are not independent as outcome of a trial depends on outcome of previous trial

Z does not meet the 4 conditions of a binomial random variable

ii) Let $x_i = 0$ mean there is not a trend
Let $x_i = 1$ mean that there is a trend



$P(x_i = 0) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
$= \frac{6}{8}$
$= \frac{3}{4}$

$P(x_i = 1) = \frac{1}{8} + \frac{1}{8}$
$= \frac{2}{8}$
$= \frac{1}{4}$

$Z = x_1 + x_2 + \dots + x_n$

x_i	0	1
$P(X = x_i)$	$3/4$	$1/4$

$E(X = x_i) = 0 \times (\frac{3}{4}) + 1 \times \frac{1}{4}$
$E(X = x_i) = \frac{1}{4}$

$E(Z) = E(\sum_1^n x_i)$
$= E(x_1 + x_2 + \dots + x_n)$
$= E(x_1) + E(x_2) + \dots + E(x_n)$
$= \frac{1}{4} + \frac{1}{4} + \dots + \frac{1}{4}$
$= n\frac{1}{4}$
$= \frac{n}{4}$

4a)
• "(same) peak hour each day" = occurance are counted in a fixed interval of time or space
• "emergency phone call" = a call occurs independently as a phone call doesn't influence the occurence of another phone call
• "emergency phone call" = a call can only occur one at a time (does not occur simultaneously)
• "the number of" = count the number of occurence

b) let n be number of hour
let s be number of calls on average which occurs within an interval (n=1)

$n = 1$
$s = 3.4$

$\mu_X = n \times s$
$\mu_X = 3.4$

$\sigma X = \sqrt{3.4}$
$\sigma X = 1.8439088\dots$
$\sigma X = 1.8439$

c) $p(x) = P(X = x) = \frac{\mu^x e^{-\mu}}{x!}$

$P(0) = \frac{3.4^0 \times e^{-3.4}}{0!}$

$P(0) = 0.03337\dots$
$= 0.0334$

d) $P(X \geq 10) = 1 - P(X \leq 9)$
$= 1 - (P(X=9) + P(X=8) + P(X=7) + P(X=6) + P(X=5) + P(X=4) + P(X=3) + P(X=2) + P(X=1) + P(X=0))$

$p(X = x) = \frac{\mu^x e^{-\mu}}{x!}$

$$= 1 - \left(\left(\frac{3.4^{9} \times e^{-3.4}}{9!}\right) + \left(\frac{3.4^{8} \times e^{-3.4}}{8!}\right) + \left(\frac{3.4^{7} \times e^{-3.4}}{7!}\right) + \left(\frac{3.4^{6} \times e^{-3.4}}{6!}\right) + \left(\frac{3.4^{5} \times e^{-3.4}}{5!}\right) + \left(\frac{3.4^{4} \times e^{-3.4}}{4!}\right) + \left(\frac{3.4^{3} \times e^{-3.4}}{3!}\right) + \left(\frac{3.4^{2} \times e^{-3.4}}{2!}\right) + \left(\frac{3.4^{1} \times e^{-3.4}}{1!}\right) + \left(\frac{3.4^{0} \times e^{-3.4}}{0!}\right)$$

$= 1 - 0.99729126724$

$= 0.002708 \ldots$

$P(X \geq 10) = 0.0027$

5a) $E(\hat{p}) = \frac{E(x)}{n}$

$E(X) = \frac{1}{330} \times (0 \times 17 + 1 \times 25 + 2 \times 150 + 3 \times 86 + 4 \times 37 + 5 \times 10 + 6 \times 3 + 7 \times 2)$

$E(x) = 2.46363\ldots$
$= 2.4636$

$E(\hat{p}) = \frac{2.4636}{7}$

$= 0.3519$

$q = p - 1$

$X \sim B(7, 0.3519)$

Estimated working $P(X=x) = \binom{n}{x} p^x q^{n-x}$

$P(X=0) = \binom{7}{0} \times 0.3519^0 \times 0.6481^7 = 0.0480$

$P(X=1) = \binom{7}{1} \times 0.3519^1 \times 0.6481^6 = 0.1825$

$P(X=2) = \binom{7}{2} \times 0.3519^2 \times 0.6481^5 = 0.2973$

$P(X=3) = \binom{7}{3} \times 0.3519^3 \times 0.6481^4 = 0.2680$

$P(X=4) = \binom{7}{4} \times 0.3519^4 \times 0.6481^5 = 0.1461$

$P(X \geq 5) = 1 - (P(X=4) + P(X=3) + P(X=2) + P(X=1) + P(X=0)) = 0.0571$

x	f_i	$e_i = P(X=x) \times 330$
0	17	15.84
1	25	60.225
2	150	98.109
3	86	88.77
4	37	48.213
5	10	18.843
6	3	
7	2	

$$X^2 = \sum_{i=0}^{7} \frac{(f_i - e_i)^2}{e_i}$$

$$X^2 = \frac{(17-15.84)^2}{15.84} + \frac{(25-60.225)^2}{60.225} + \ldots\ldots + \frac{(37-48.213)^2}{48.213} + \frac{(15-18.843)^2}{18.843}$$

$X^2 = 51.6119$

degree of freedom $= K - 1 -$ no. of estimated parameters
$V = 6 - 1 - 1$
$V = 4$

$\chi^{V}_{0.05} = \chi^{4}_{0.05} = 9.488$

From a chi square table we observe that for a chi square distribution with a d.o.f of 4, 5% of all values for the distribution is greater than 9.488
as $\chi^2 > \chi^4_{0.05}$ $(51.6115 > 9.488)$, this suggests that a binomial distribution is not a good fit

b) The 4 conditions for a random binomial variable
- fixed no of trials
- each trial has a fixed probability of success
- each trial has 2 outcome
- trial are independent

- there is a fixed number of trials ("sample of 330 visitors")
- trial adds to number of occurance so probability of success is not constant
- there is more than 2 outcomes ("0, 1, 2, 3, 4, 5, 6, 7")
- one visitor visiting an attraction does not effect another visitor visiting an attraction (unless traveling in a group or overcrowding issue)

The binomial distribution is not a good fit for the data as it does not fit the 4 requirements of a random binomial variable