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Name: Marinna Ricketts-uy
Section: 02
HW #: 2
Version: A
Username: pd12778

#1: Prove that $2^n > n^2$ for every positive integer n when $n \geq 5$

Base Case:
$n = 5$: $2^5 > 5^2$
$32 > 25$, which is true.

Assume true for $2^k > k^2$

Prove true for $2^{k+1} > (k+1)^2$:
Consider:

$$2^{k+1} - (k+1)^2 = 2 \cdot 2^k - (k^2 + 2k + 1)$$
$$= 2^k + 2^k - (k^2 + 2k + 1)$$
$$2^{k+1} - (k+1)^2 > 2^k + k^2 - (k^2 + 2k + 1)$$
$$> 2^k - (2k+1)$$
$$2^{k+1} - (k+1)^2 > 2^k - (2k+1) > 0$$
$$2^{k+1} - (k+1)^2 > 0$$
$$2^{k+1} > (k+1)^2$$

$\therefore$ Statement is true for $n \geq 5$
$\therefore$ By PMI, $2^n > n^2$ for $n \geq 5$

#2: Prove that $4^n - 1 = 3k$
(k is legit and not "n". Results will always be divisible by 3)

Base case:
$n = 1: \quad 4^{(1)} - 1 = 3k \rightarrow 3 = 3k \quad k = 1$
$n = 2: \quad 4^{(2)} - 1 = 3k \rightarrow 15 = 3k \quad k = 5$

Assume true for $4^t - 1 = 3k$

Prove true for $4^{t+1} - 1 = 3k$:

$$4^{t+1} - 1 = 3k$$
$$4 \cdot 4^t - 1 = 3k$$
$$(3+1)4^t - 1 = 3k$$
$$3 \cdot 4^t + 4^t - 1 = 3k$$

$\therefore$ $3 \cdot 4^t$ is divisible by 3 and $4^t - 1$ was assumed

#3: Prove that $\frac{1}{1\cdot 2}+\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

Base case:

$$n=1: \quad \frac{1}{1(1+1)}=\frac{1}{1+1}$$

$$\frac{1}{1\cdot 2}=\frac{1}{2} \checkmark$$

Assume for $\frac{1}{1\cdot 2}+\dots+\frac{1}{k(k+1)}=\frac{k}{k+1}$

Prove for $\frac{1}{1\cdot 2}+\dots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)} \stackrel{?}{=} \frac{k+1}{k+2}$

$$\frac{k}{k+1}+\frac{1}{(k+1)(k+2)} \stackrel{?}{=} \frac{k+1}{k+2}$$

$$\frac{(k+2)\,k}{(k+2)\,k+1}+\frac{1}{(k+1)(k+2)} \stackrel{?}{=} \frac{k+1}{k+2}$$

$$\frac{k^2+2k+1}{(k+1)(k+2)} \stackrel{?}{=} \frac{k+1}{k+2}$$

$$\frac{(k+1)(k+1)}{(k+1)(k+2)} \stackrel{?}{=} \frac{k+1}{k+2}$$

$$\frac{k+1}{k+2}=\frac{k+1}{k+2}$$

$\therefore$ By P.M.I., $\frac{1}{1\cdot 2}+\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

Code Complexity:

#4:

```
sum = 0;                          (1)
for (i=0; i<n; i++)               (2n+2)
  for(j=0; j<n; j++)              ((2n+2) x n)
    ++sum;                        (n^2)
```

$\text{Total} = 1 + 2n + 2 + 2n^2 + 2n + n^2$
$= \boxed{3n^2 + 4n + 3}$

Big Oh speed : $\boxed{O(n^2)}$

#5:

```
sum = 0;                          (1)
for (i=0; i<n; i+=2)              (2n+2)
  for(j=0; j<n; j++)              (2n+2) x n
    ++sum;                        (n^2)
```

$\text{Total} = 1 + 2n + 2 + 2n^2 + 2n + n^2$
$= \boxed{3n^2 + 4n + 3}$

Big Oh speed : $\boxed{O(n^2)}$

#6:

```
sum=0;                          (1)
for (i=1; i<n; i*=2)            (1 + log_2(n) + 1 + log_2(n))
  for (i=0; j<n; j++)           (2n+2) x log_2(n)
    ++sum;                      (n) x log_2(n)
```

Total $= 1 + 2 + 2\log_2(n) + 2n\log_2(n) + 2\log_2(n) + n\log_2 n$
$= \boxed{3n\log_2(n) + 4\log_2(n) + 3}$

Big Oh speed: $\boxed{O(n\log n)}$

#7:

```
    sum = 0;                        (1)
n   for (i=0; i<n; i++)             (2n+2)
n^2   for (j=0; j<i*i; j++)         (1+n^2+1+n^2) x n
n^2     for (k=0; k<j; k++)         (1+n^2+1+n^2) x n^2 x n
          ++sum;                    (n^5)
```

Total $= 1 + 2n + 2 + n + n^3 + n + n^3 + n^3 + n^5 + n^3 + n^5 + n^5$
$= \boxed{3n^5 + 4n^3 + 4n + 3}$

Big Oh speed: $\boxed{O(n^5)}$