Name: Marinna Ricketts-Uy

Section: 02 – Budhraja

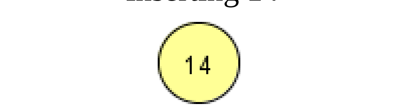
HW #: 4

Version: B

Username: pd12778

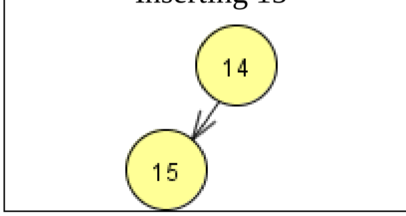
1. Inserting the following into a min heap: 14,15,9,3,4,6,8,12,10,13,7,11,1,2,5

Inserting 14



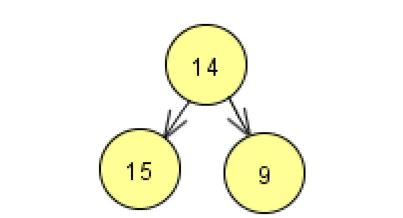
	14														
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 15

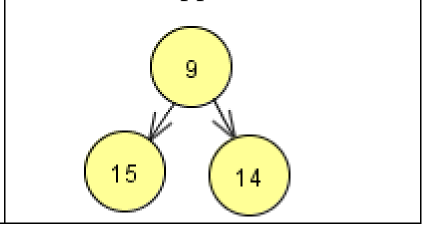


	14	15													
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 9

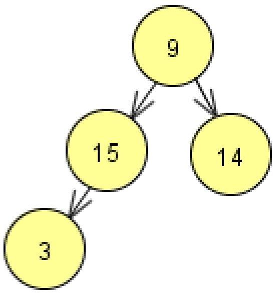


9 is swapped with 14

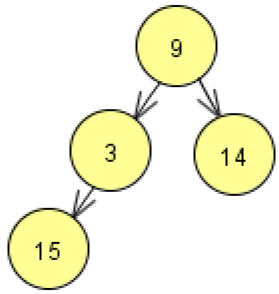


	9	15	14												
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

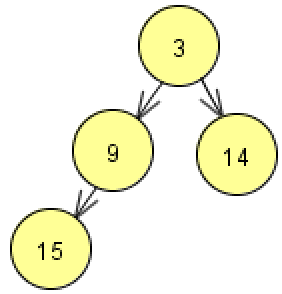
Inserting 3



3 is swapped with 15

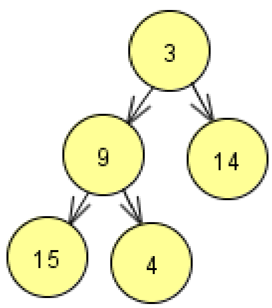


3 is swapped with 9

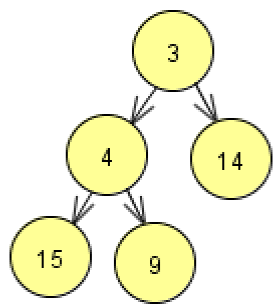


	3	9	14	15											
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 4

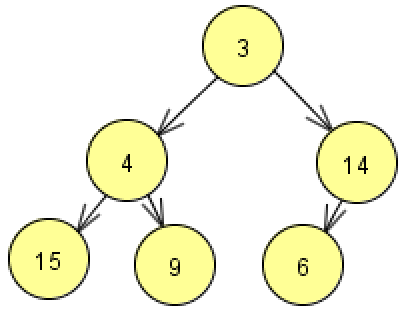


4 is swapped with 9

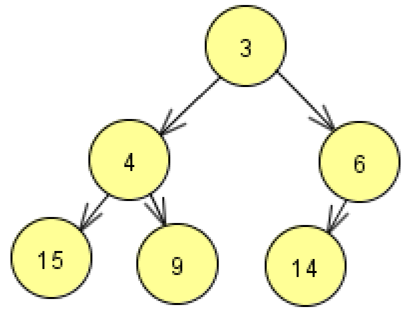


	3	4	14	15	9										
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 6

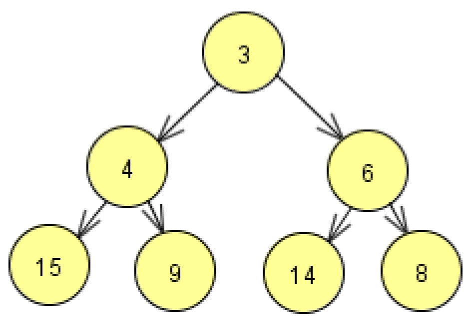


6 is swapped with 14



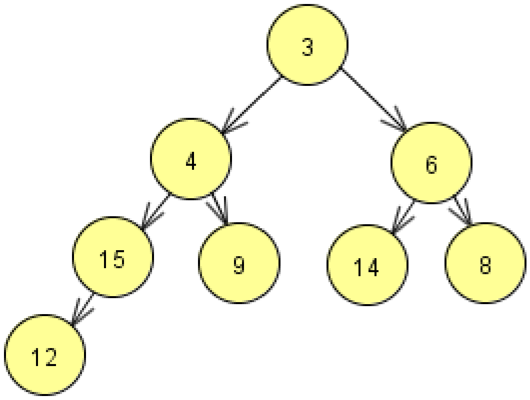
	3	4	6	15	9	14									
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 8

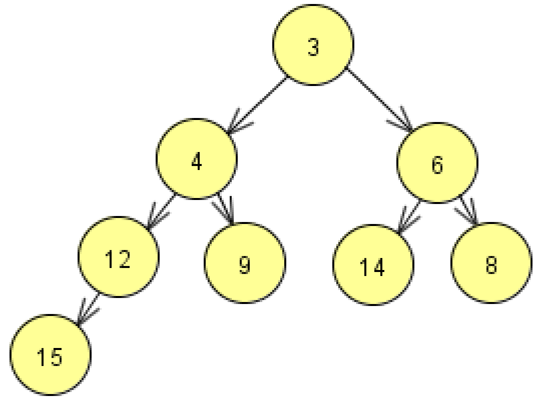


	3	4	6	15	9	14	8								
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 12

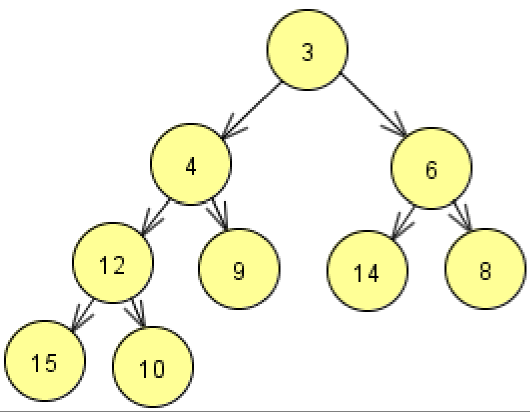


12 is swapped with 15

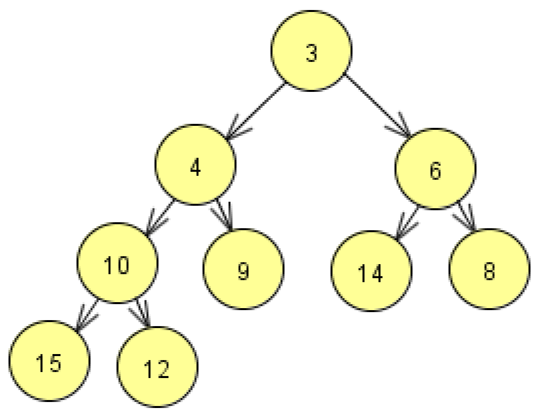


	3	4	6	12	9	14	8	15							
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 10

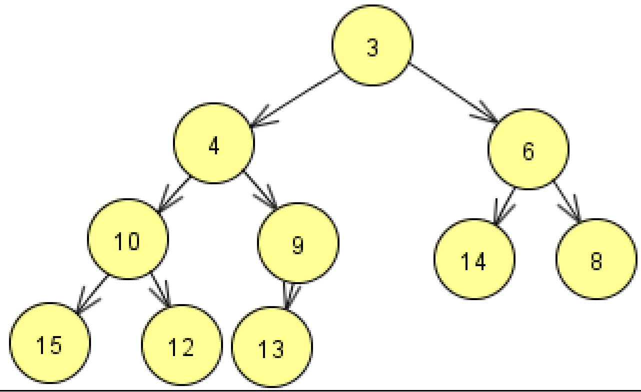


10 is swapped with 12



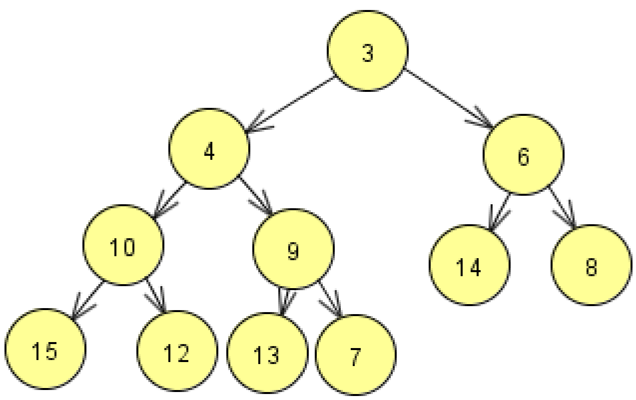
	3	4	6	10	9	14	8	15	12						
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 13

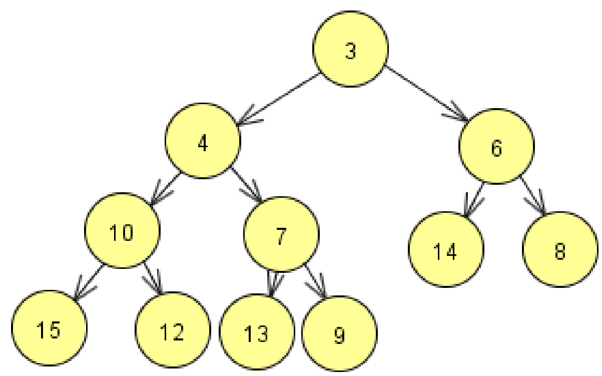


	3	4	6	10	9	14	8	15	12	13					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 7

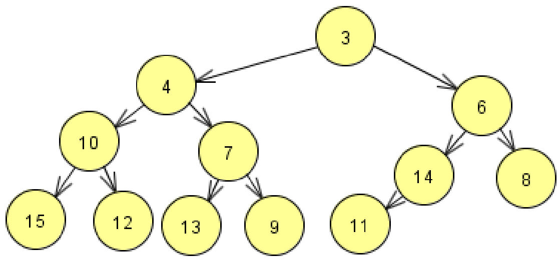


7 is swapped with 9

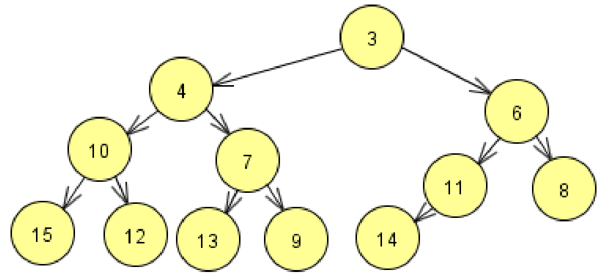


	3	4	6	10	7	14	8	15	12	13	9				
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 11

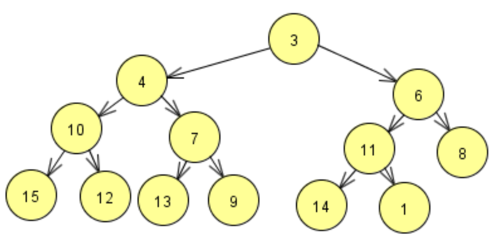


11 is swapped with 14

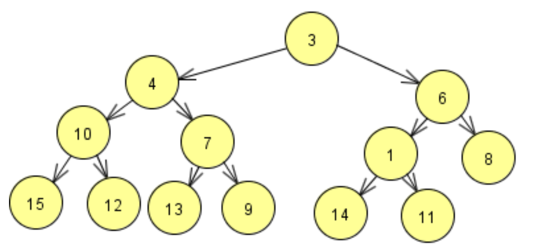


	3	4	6	10	7	11	8	15	12	13	9	14			
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

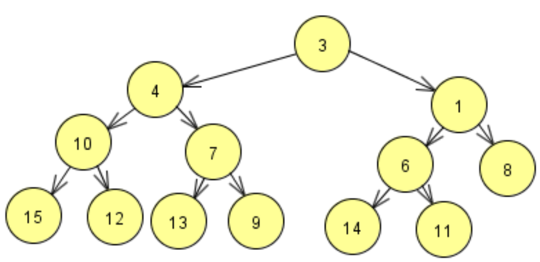
Inserting 1



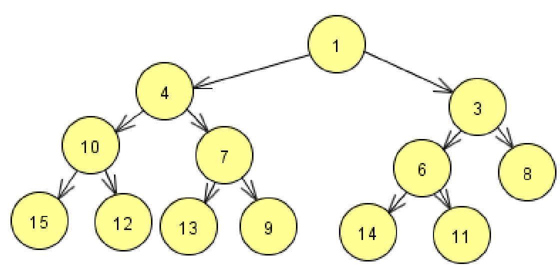
1 is swapped with 11



1 is swapped with 6

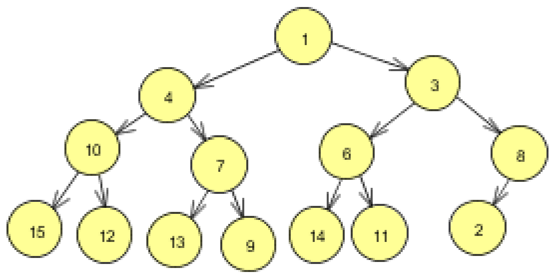


1 is swapped with 3

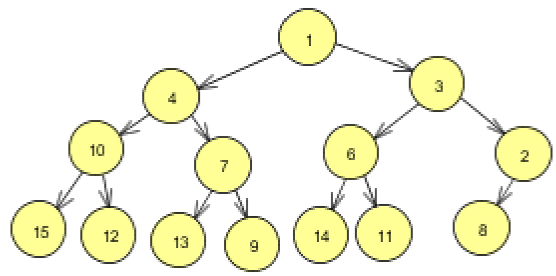


	1	4	3	10	7	6	8	15	12	13	9	14	11		
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

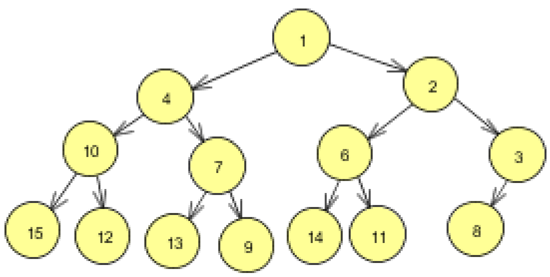
Inserting 2



2 is swapped with 8

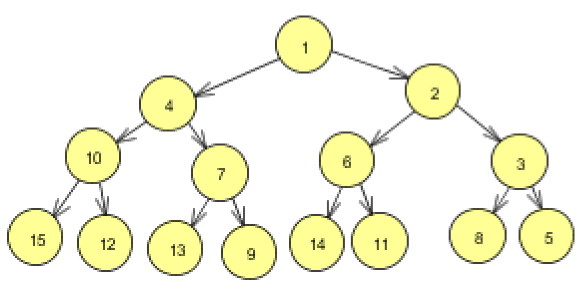


2 is swapped with 3



	1	4	2	10	7	6	3	15	12	13	9	14	11	8	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Inserting 5



	1	4	2	10	7	6	3	15	12	13	9	14	11	8	5
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

2.

a. Prove it must be at one of the leaves:

In order to prove that the maximum item in a min-heap must be one of the leaves, we can assume the opposite.

Proof by Contradiction:

We can assume that it is not a leaf. If this is the case, then the maximum item must have at least one child. If it has a child, and is the maximum item, then it must be greater than its child. However, then based on the property of a min-heap, this is not valid. The parent must be less than its children.

So, the maximum item must be one of the leaves in a min-heap.

b. Prove there are exactly ceiling(N/2) leaves:

The last leaf in a min-heap is at the N^{th} index. The parent is at index floor(N/2) and the leaves are indexed from floor(N/2) + 1 to N. So in order to calculate the number of leaves:

$$N - \text{floor}(N/2) = \text{ceiling}(N/2)$$

c. Prove every leaf must be examined to find it:

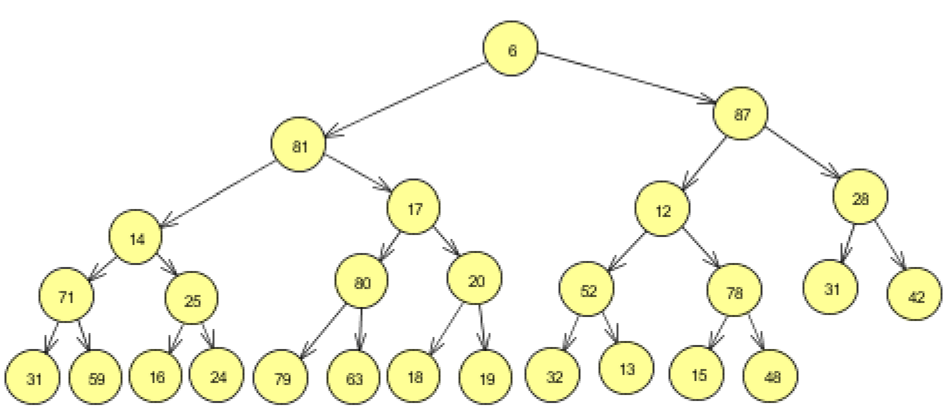
So let's assume the min-heap is a perfect tree of N nodes and the maximum item is M. The next step is to add N + 1 nodes that are all greater than M. The values added will be the leaves in the order in which each were inserted. Any of the values could be the maximum item in the min-heap.

So, the maximum item could be in any of the leaves, and every leaf must be examined in order to find which is the greatest amongst them all.

3.

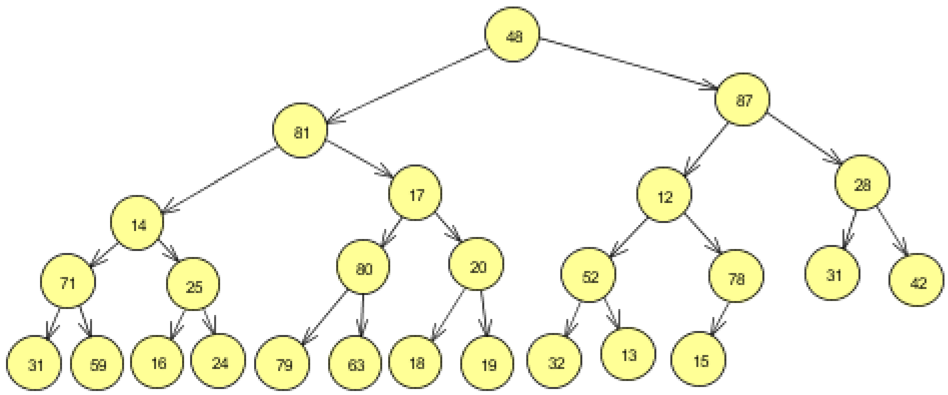
a. Figure 1 after two delete min operations:

Original:

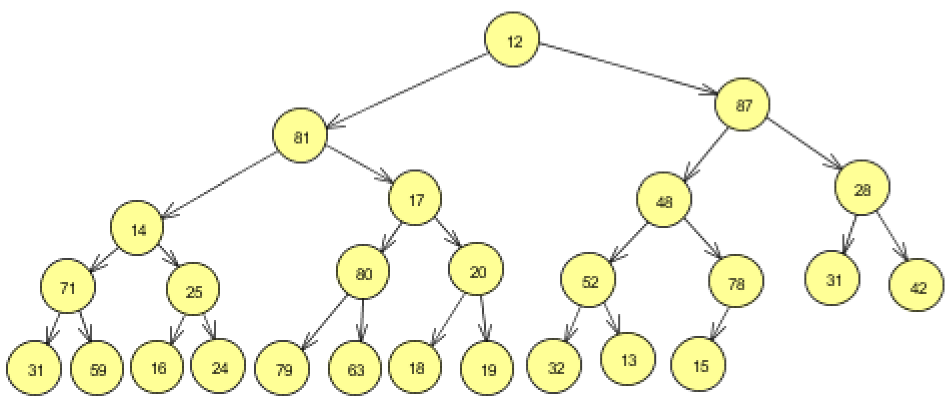


After one delete min operation:

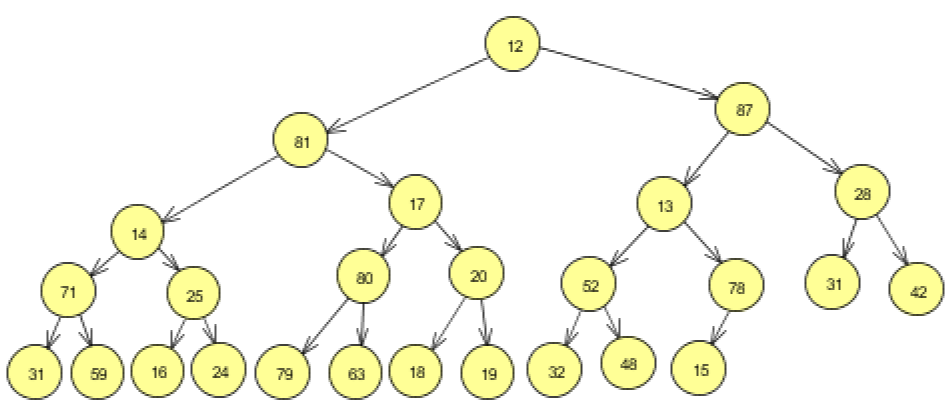
Step 1: Delete 6 and replace with last item in heap (48)



Step 2: Swap 48 with 12

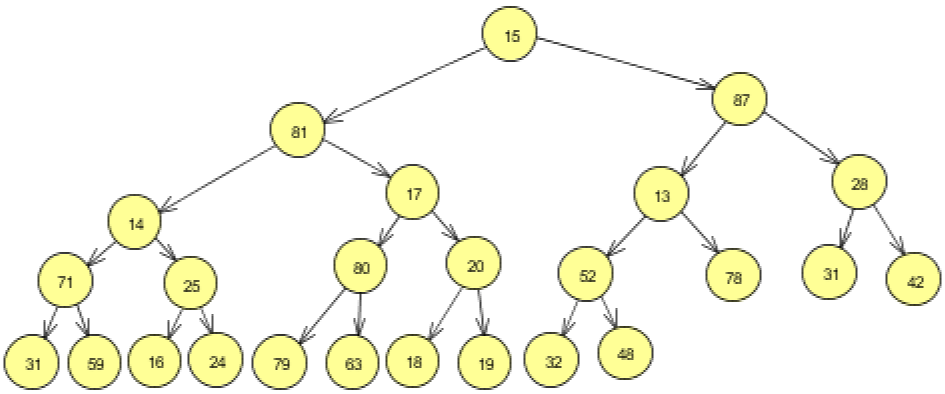


Step 3: Swap 48 with 13

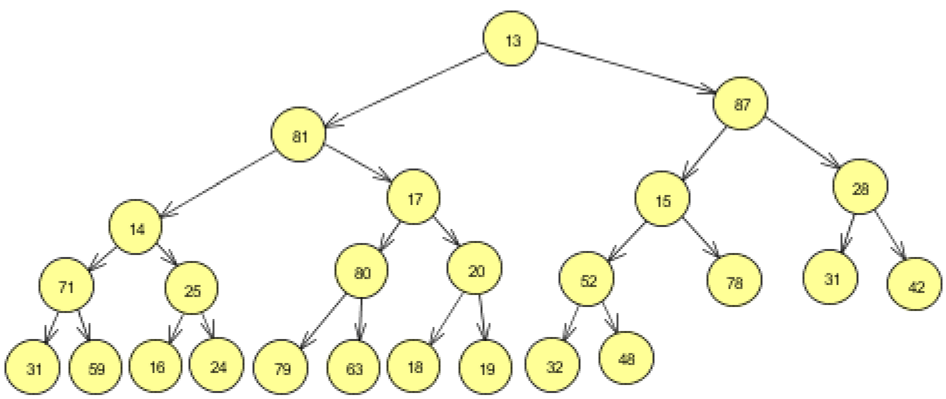


After another delete min operation:

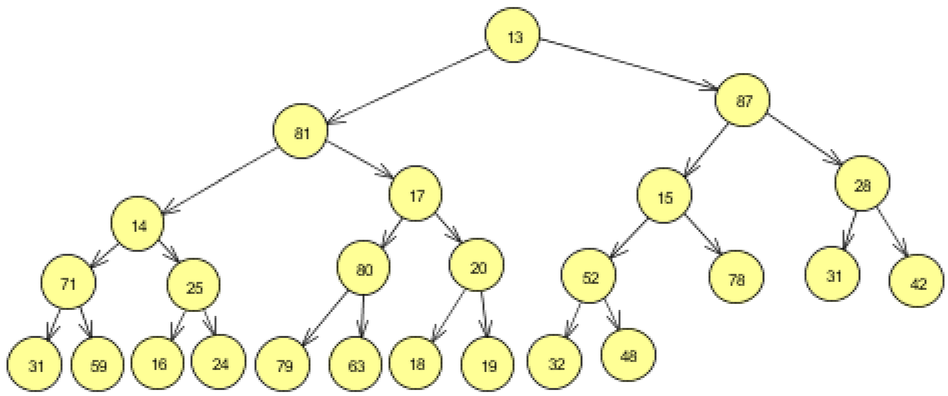
Step 1: Delete 12 and replace with last item in the heap (15)



Step 2: Swap 15 with 13

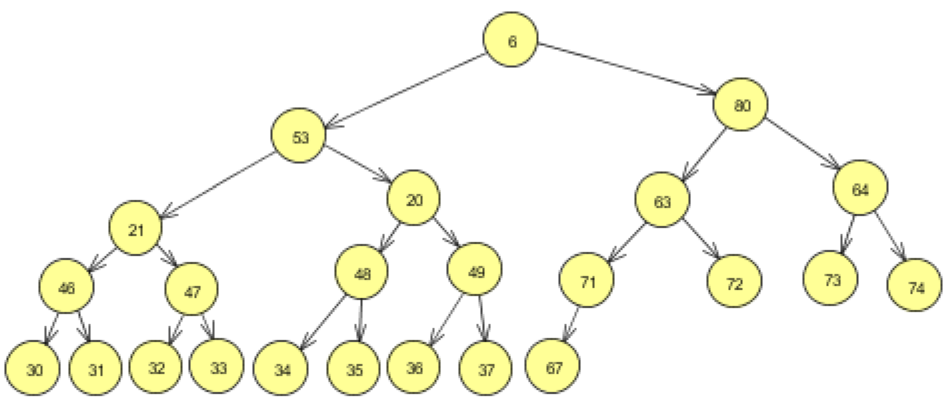


Final Answer (a):

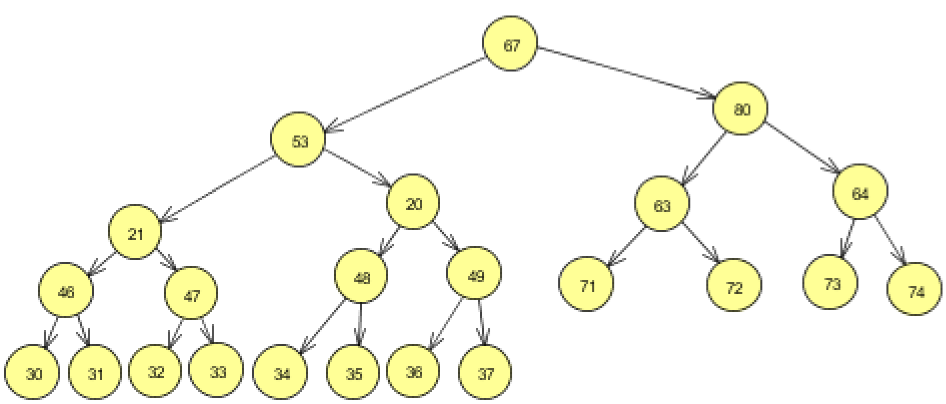


b. Figure 2 after one delete min operation:

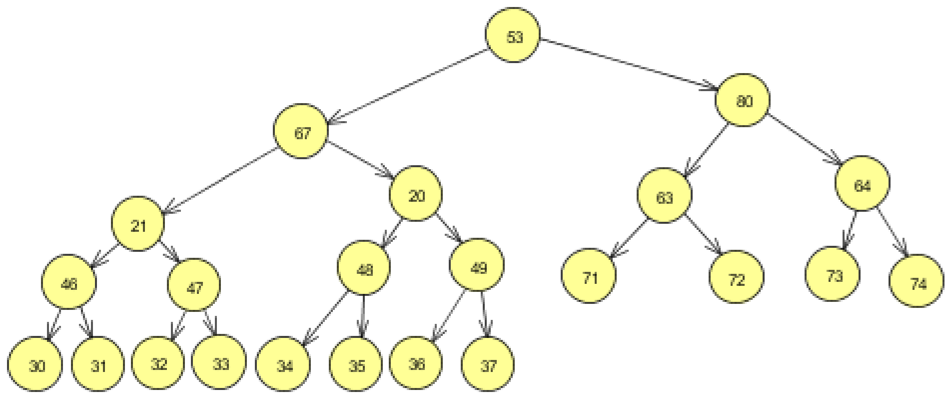
Original:



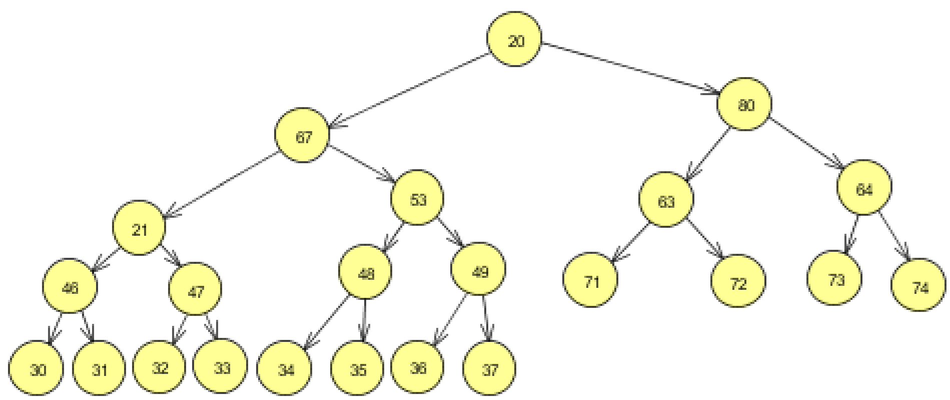
Step 1: Delete 6 and replace with last item in heap (67)



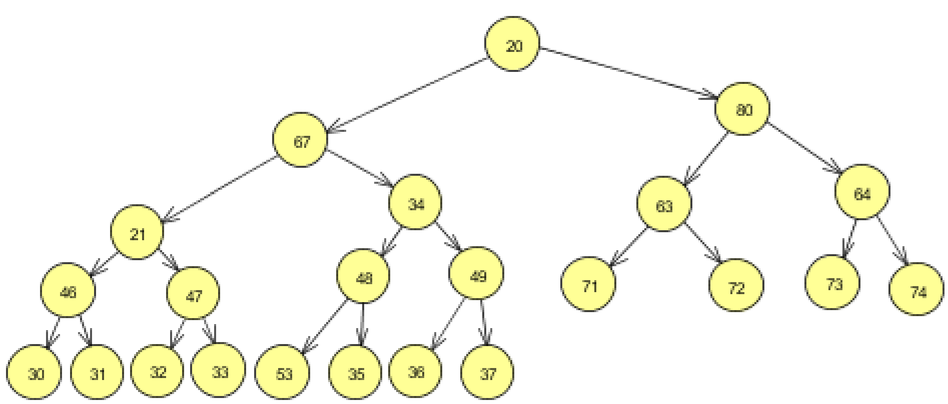
Step 2: Swap 67 with 53



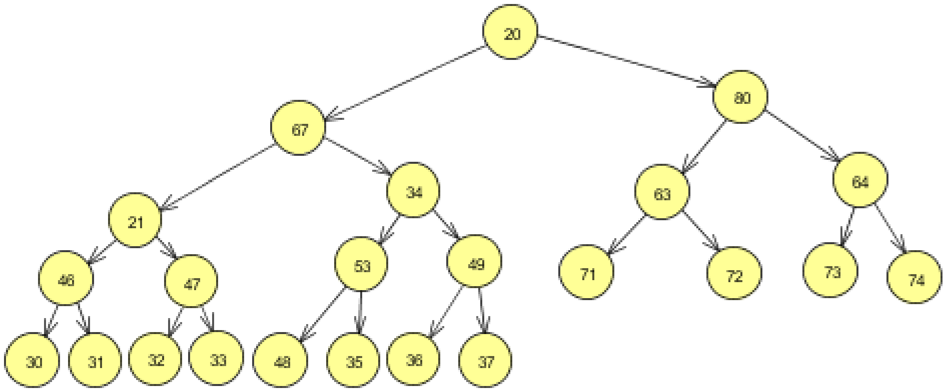
Step 3: Swap 53 with 20



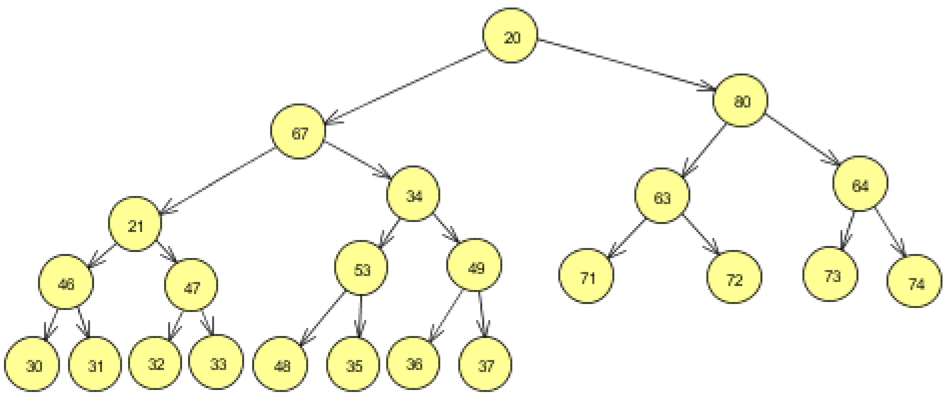
Step 4: Swap 53 with 34



Step 5: Swap 53 with 48

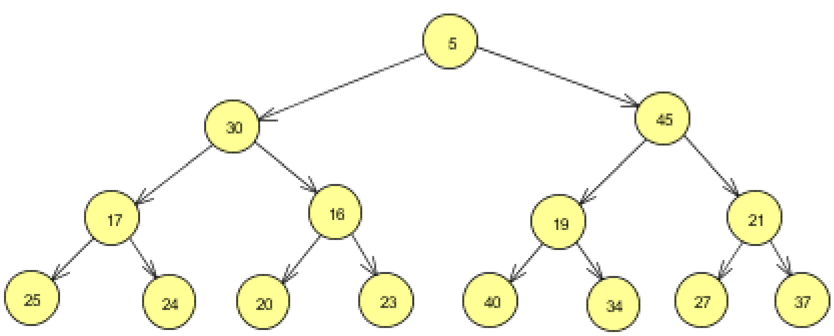


Final Answer (b):

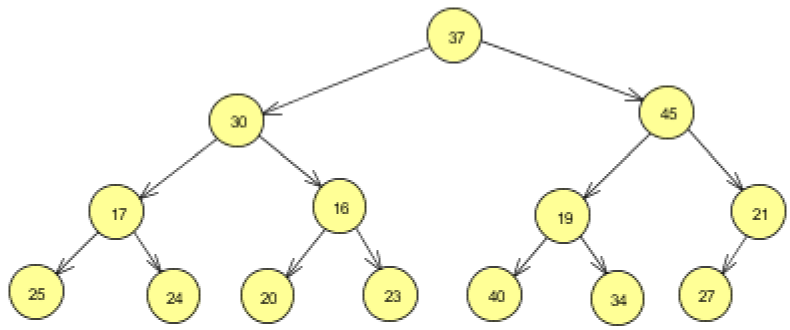


c. Figure 3 after one delete min operation:

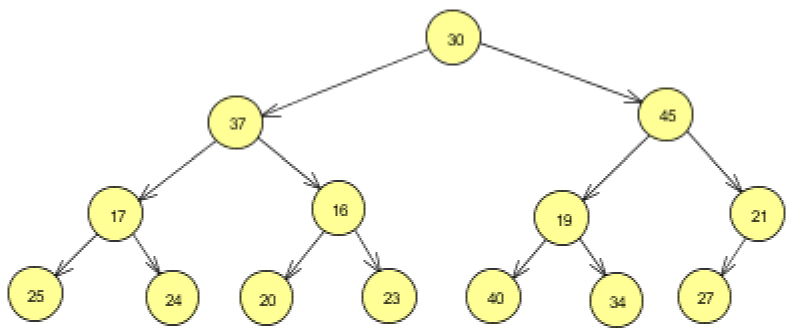
Original:



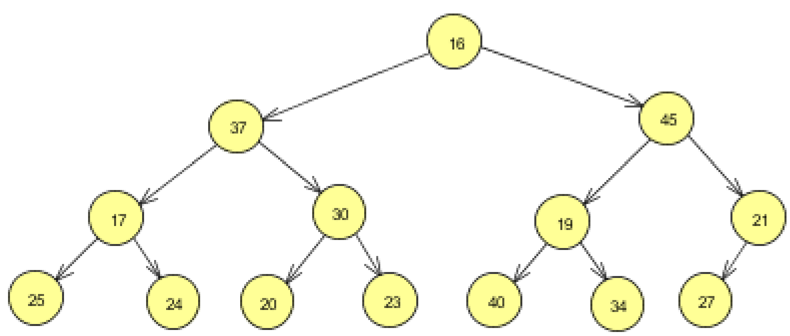
Step 1: Delete 5 and replace with last item in heap (37)



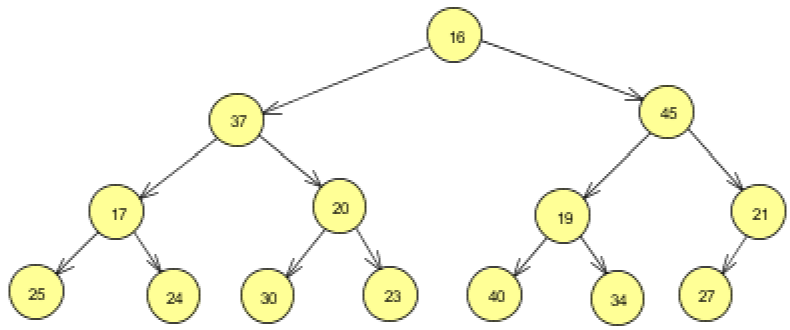
Step 2: Swap 37 with 30



Step 3: Swap 30 with 16



Step 4: Swap 30 with 20



Final Answer (c):