



# Project #3: Matrix-free LSQR

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## Working Plan

**Phase 0:** Understanding of LSQR method

**Phase 1:** Programm and test of the LSQR method with a small matrix

**Phase 2:** Try and modify LSQR method for a matrix **A** that ~~can not fully stored in the memory~~ is really big

# What we have done

- cuBlas implementation
  - Only using cuBlas subroutines for the lsqr implementation
- cuSparse implementation
  - Using cuSparse subroutines for the lsqr implementation
  - If there is no subroutine implemented → using own kernels
    - Norm kernel
    - Addition and subtraction vector kernel
    - Multiply scalar with a vector
- Kernel implementation
  - Norm kernel
  - Addition and subtraction kernel
  - Multiply scalar to vector
  - Multiply sparse matrix with vector
- Matlab and Python code for Testing and creating sparse matrices and vectors

# Project - Read data and compile

- Read Data
  - First number has to be the row size
  - Second number has to be the column size
  - Then values for the matrix or vector
- How to start the program?

## Modes:

- 0: cuBlas
- 1: Kernels
- 2: cuSparse

Mode

```
:~/PMPP_Project/build$ ./lsqr ../Data/Matrix5000x5000p50.txt ../Data/Vector5000x1p50.txt 5000 0
```

Directory

Path of matrix and vector

Maximum number of iteration

# Example

A terminal window with a title bar that reads "dominik — ssh dmarino@gccg201.igd.fraunhofer.de — 111x29". The terminal content shows the command `./lsqr ../Data/Matrix5000x5000p50.txt ../Data/Vector5000x1p50.txt 5000 0` being entered at the prompt `dmarino@gccg201:~/PMPP_Project/build$`.

```
dominik — ssh dmarino@gccg201.igd.fraunhofer.de — 111x29
dmarino@gccg201:~/PMPP_Project/build$ ./lsqr ../Data/Matrix5000x5000p50.txt ../Data/Vector5000x1p50.txt 5000 0
```

# Sparse GPU Kernels for Deep Learning

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**Abstract**—Scientific workloads have traditionally exploited high levels of sparsity to accelerate computation and reduce memory requirements. While deep neural networks can be made sparse, achieving practical speedups on GPUs is difficult because these applications have relatively moderate levels of sparsity that are not sufficient for existing sparse kernels to outperform their dense counterparts. In this work, we study sparse matrices from deep learning applications and identify favorable properties that can be exploited to accelerate computation. Based on these insights, we develop high-performance GPU kernels for two sparse matrix operations widely applicable in neural networks: sparse matrix–dense matrix multiplication and sampled dense–dense matrix multiplication. Our kernels reach 27% of single-precision peak on Nvidia V100 GPUs. Using our kernels, we demonstrate sparse Transformer and MobileNet models that achieve 1.2–2.1 $\times$  speedups and up to 12.8 $\times$  memory savings without sacrificing accuracy.

**Index Terms**—Neural networks, sparse matrices, graphics processing units

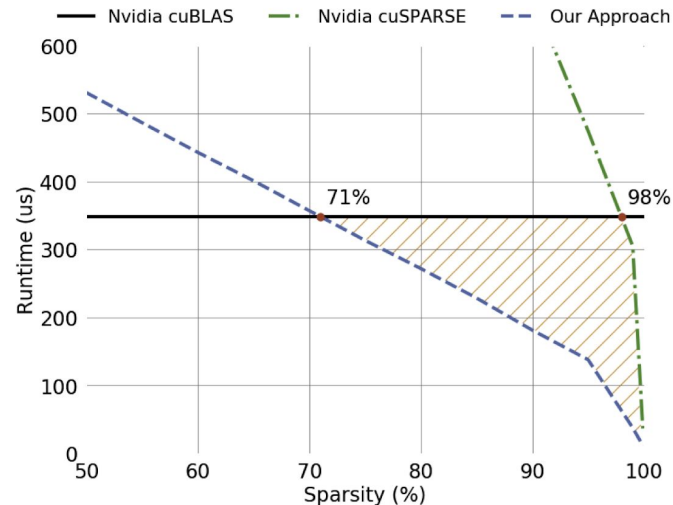
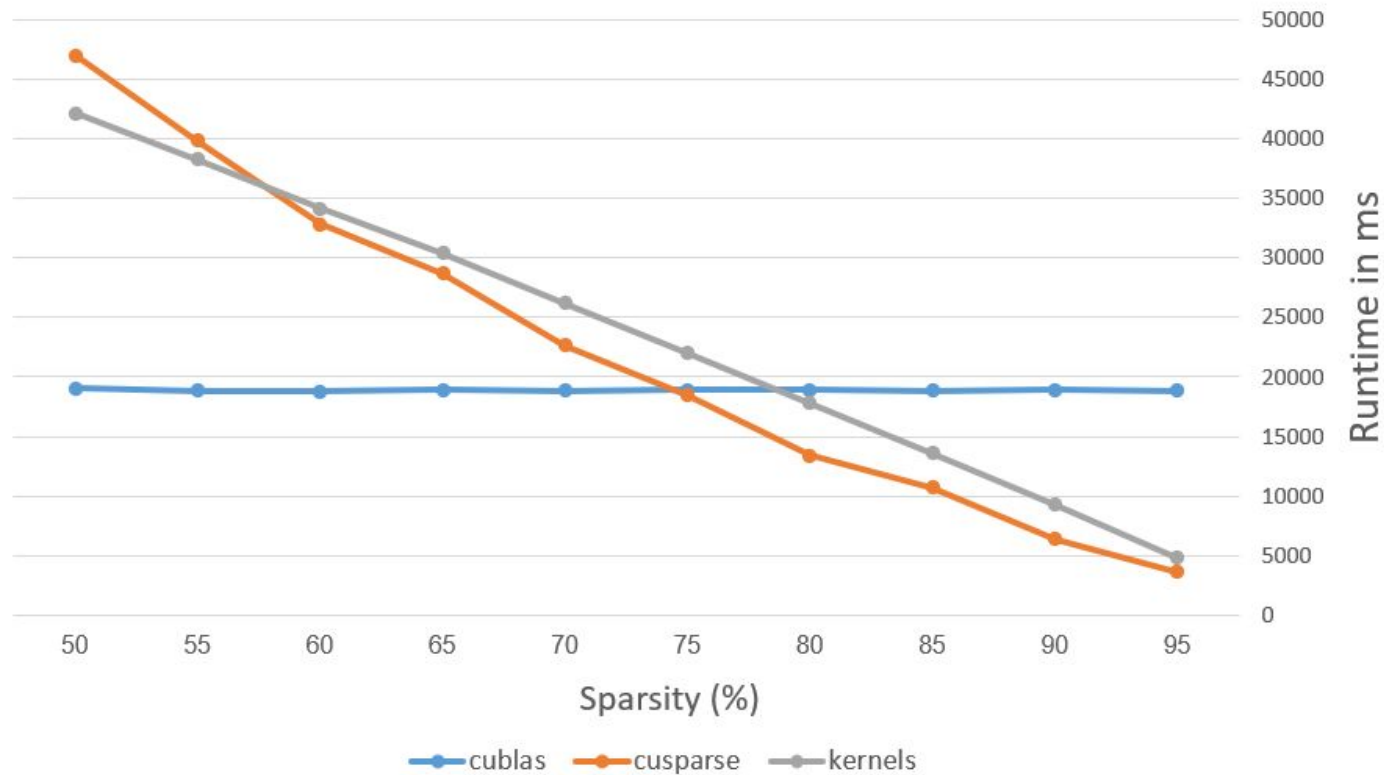


Fig. 1. Sparse matrix–matrix multiplication runtime for a weight-sparse

S.LG] 31 Aug 2020

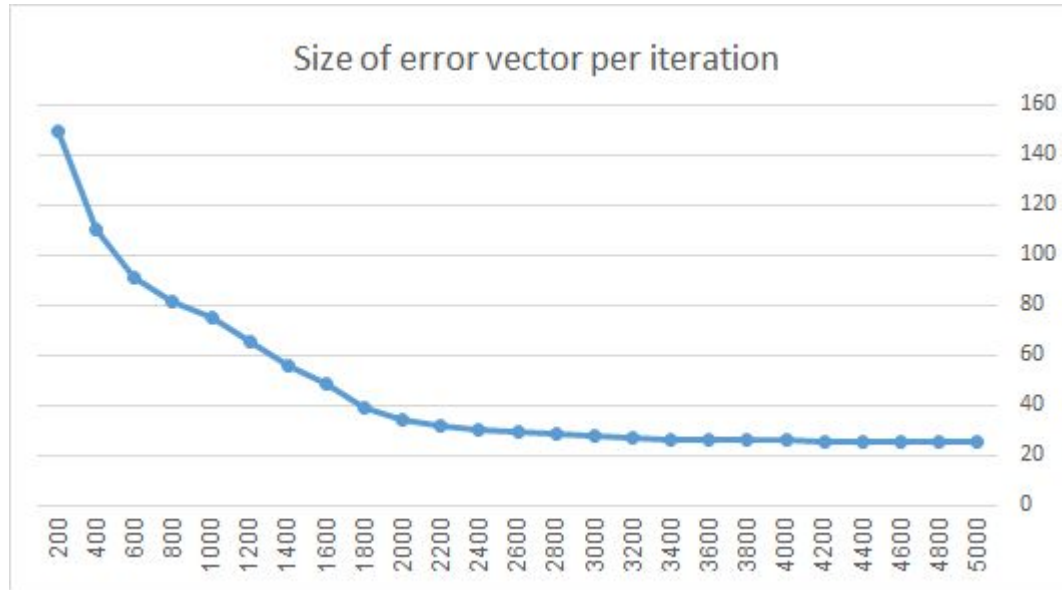
LSQR Runtime implementation





# Convergence of the algorithm

(square matrix, 5000\*5000, sparsity: 50%, values are integers in range 0-100)  $\rightarrow |Ax-b|$



# Troubles

- “Voodoo” Errors in kernels - when using more than one block of threads, variables had “left over” values in shared memory
- Errors with Linking/using old libraries of cublas/cusparse
- Uninvertible matrices might cause weird error values
- The algorithm is working for about 150 iterations and then goes completely wrong
- Problems with cudaMallocPitch
- Problems with transposing the matrix

```
if(tid < size){  
    sdata[tid] = in_data[i];           //load global data in sh_memory  
}else{  
    sdata[tid] = 0;  
}  
__syncthreads();
```

# What we have learned

- cuBlas und cuSparse are old-fashioned and hard to work with
- Checking cuda errors after every operation
- Cuda error codes are partial and confusing
- Shared memory needs to be initialized to 0
- Memory needs to be freed as soon as possible
- It's better to avoid allocating memory and copying it
- Using sparse libraries is worth it at about 80% sparsity

```
cusparseStatus_t  
cusparseCsr2cscEx2_bufferSize(cusparseHandle_t handle,  
    int m,  
    int n,  
    int nnz,  
    const void* csrVal,  
    const int* csrRowPtr,  
    const int* csrColInd,  
    void* cscVal,  
    int* cscColPtr,  
    int* cscRowInd,  
    cudaDataType valType,  
    cusparseAction_t copyValues,  
    cusparseIndexBase_t idxBase,  
    cusparseCsr2CscAlg_t alg,  
    size_t* bufferSize)
```

```
cusparseStatus_t  
cusparseCsr2cscEx2(cusparseHandle_t handle,  
    int m,  
    int n,  
    int nnz,  
    const void* csrVal,  
    const int* csrRowPtr,  
    const int* csrColInd,  
    void* cscVal,  
    int* cscColPtr,  
    int* cscRowInd,  
    cudaDataType valType,  
    cusparseAction_t copyValues,  
    cusparseIndexBase_t idxBase,  
    cusparseCsr2CscAlg_t alg,  
    void* buffer)
```

# Possible improvements

- Matrix multiplication with shared memory, warps to unroll
- Working on a better stopping criteria (taking into account improvement slow-down)
- Detect if given matrix **A** is invertible
- More general use of functions, that allow for example for transpose operation

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*Do you have any questions?*