Projeto 2 – Solução de EDO

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Informações

- Os exercícios podem ser feitos ou individualmente os em duplas de 2 pessoas.

- O nome do arquivo deve ser: RA-Projeto 2.

- Os exercícios podem ser resolvidos em Python, VBA, Octave ou outra linguagem computacional.

Exercícios a serem feitos

1 - Exercício Proposto 6.5 do livro texto.

2 - Exercício Proposto 6.6 do livro texto.

3 - Resolva o problema da aleta apresentado a seguir.

4 – Resolva o problema do trocador de calor apresentado a seguir.

Exercício 1: Proposed Problem 6.5

6.5) This system is adapted from Incropera et al. (2006). A very long cylindrical metal bar with diameter D, length L, and thermal conductivity k has one end maintained at T_w by constant contact with a hot wall. The surface of this cylinder is exposed to ambient air at a constant temperature of T_∞ with a convection heat transfer coefficient of h. The system was left for a long time until it became completely stable.

a. Write ODEs that describes the temperature profile and define the two boundary conditions. Consider that there is no radial temperature profile inside the bar. Make assumptions if needed to simplify the mathematical solution of this problem.
b. Solve the mathematical model using VBA and RK4.

Determine and plot the temperature profiles along the bar length when it is manufactured from pure copper, aluminum, and stainless steel.

Consider the following numerical values:

$$\mathrm{D} = 5\ \mathrm{mm}$$
$$T_w = 100^\circ \mathrm{C}$$
$$T_\infty = 25^\circ \mathrm{C}$$
$$h = 100\mathrm{W}/(\mathrm{m}^2\mathrm{K})$$

Use qualquer linguagem de programação

Exercício 1: Proposed Problem 6.5 (cont)

$$\text{Copper:} \quad k = 398\text{W}/(\text{m K})$$
$$\text{Aluminum:} \quad k = 180\text{W}/(\text{m K})$$
$$\text{Stainless steel:} \quad k = 14\text{W}/(\text{m K})$$

c. Determine for each metallic material the minimum length that the bar must have for the bar temperature profile to reach a minimum plateau. Using the minimum lengths for each one of the metals, determine the heat loss for each material.

Hint: In order to numerically integrate second-order ODEs more easily, the following substitution can be very handy:

$$\frac{dT}{dx} = y = f(x)$$
$$\frac{d^2T}{dx^2} = \frac{dy}{dx} = g(x)$$

Instead of directly solving one single second-order differential equation, it is possible to break it into two first-order differential equations to be solved independently. Observe that, in this problem, one of the boundary conditions must suffer this change in variable too.

Exercício 2: Proposed Problem 6.6

6.6) Imagine the two concentric cylinders modeled in Example 4.8 and assume that the system has reached a steady state, so it can be represented by:

$$r\frac{d^2T}{dr^2}+\frac{dT}{dr}=0$$
$$\text{At } r=R_1, \quad T=T_0$$
$$\text{At } r=R_2, \quad \frac{dT}{dr}=-\frac{h}{k}(T-T_{env})$$

Assume the following numerical values: $R_1=0.5$ cm, $R_2=3$ cm, $T_0=100$ °C, $T_{env}=25$ °C, $k=180\mathrm{W/(mK)}$, and $h=100\ W/(m^2\mathrm{K})$. Use the same hint suggested in Proposed Problem 6.5 and solve this problem using VBA and RK4. Plot the radial profile of the temperature.

Use qualquer linguagem de programação

Resumo do problema 4.8

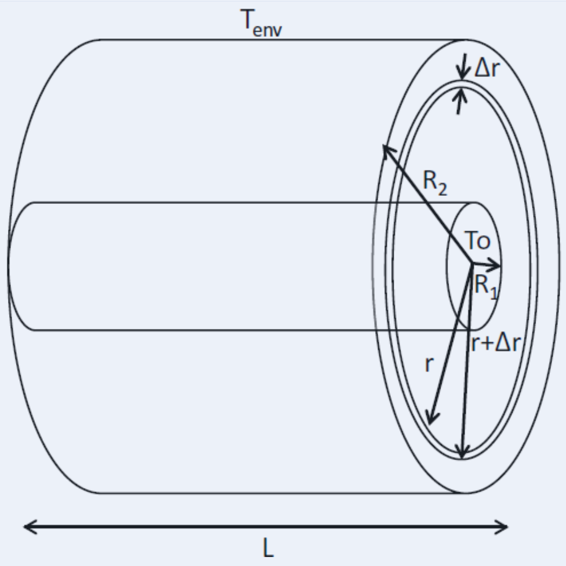


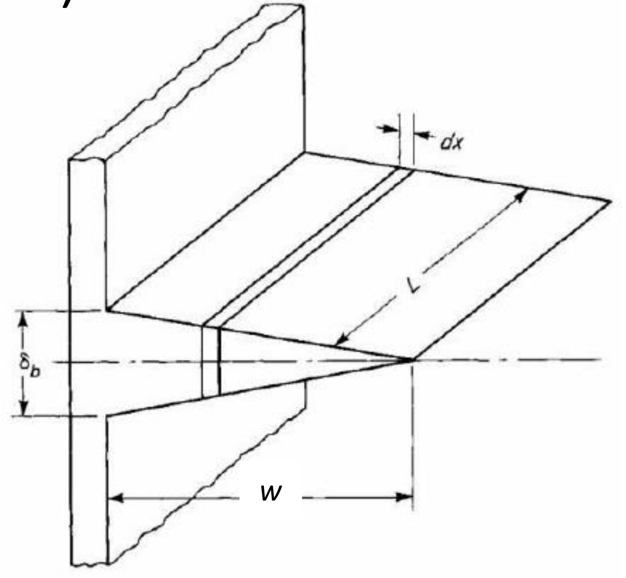
Fig. 4.15: Two concentric cylinders with radial heat conduction along the aluminum annulus

Modelagem em estado transiente

$$\frac{\partial^2T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{\rho c_p}{k}\frac{\partial T}{\partial t}$$

$$\text{At } t=0, \quad T=T_1, \quad \text{for } R_1\le r\le R_2$$
$$\text{At } r=R_1, \quad T=T_0, \quad \text{for } t>0$$
$$\text{At } r=R_2, \quad \frac{dT}{dr}=-\frac{h}{k}(T-T_{env}), \quad \text{for } t>0$$

Exercício 3 - Problema de aleta com área variável (em colaboração com o prof. Dirceu – EQ641)



Considere uma aleta de área de seção triangular com condutividade térmica k, conforme mostrada na figura ao lado. A aleta perde calor por convecção para um meio com temperatura de T_∞ com um coeficiente de transferência de calor h. A base da aleta é mantida em uma temperatura de T_b.

a) Realize um balanço de energia e mostre que a equação diferencial que descreve o perfil de temperatura na aleta é dada pela Eq. (1), onde $A_{st}(x)$ é a área da seção transversal e $P(x)$ é o perímetro da aleta. Apresente as principais considerações do problema.

$$\frac{d}{dx}\left(A_{st}(x)\frac{dT}{dx}\right) - \frac{hP(x)}{k}\left(T - T_\infty\right) = 0 \qquad (1)$$

b) Transforme a EDO de 2a ordem em 2 de primeira ordem e resolva por RK, a fim de determinar o perfil de temperatura ao longo do comprimento da aleta. Considere os parâmetros abaixo.

h (W/m^2 · °C)	150	δ_b (m)	0,040
k (W/m · °C)	125	L (m)	0,15
w (m)	1	T_b (°C)	300
T_∞ (°C)	25		

c) Compare os resultados numéricos com os analíticos obtidos na disciplina EQ641.

Exercício 4 - Problema de trocalor calor bitubular com variação das propriedades ao longo do comprimento

Tome como base o problema aprentado na aula 14 (esboçado a seguir), determine o perfil de temperature do benzeno e tolueno assumindo que o coeficiente de película varia com o comprimento.

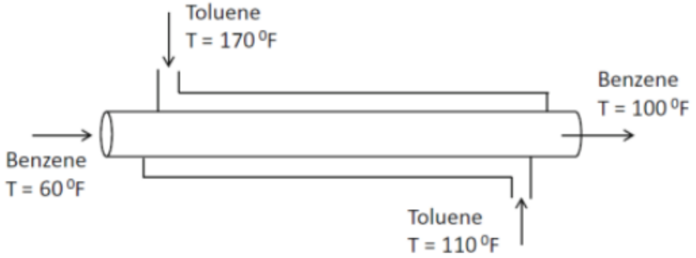
Sistemas de EDO – Ex: tocador calor bitubular

Balanço de Energia:

Benzeno: $f(x, Tb, Tt) = \frac{dTb}{dx} = Mb(Tt - Tb)$, condição inicial: em x = 0, T_{ben} = 60⁰F

Tolueno: $g(x, Tb, Tt) = \frac{dTt}{dx} = Mt\,(Tb - Tt)$, condição inicial: em x = 0, T_{tol} = 170⁰F, onde:

$Mb = \frac{U \times 1.25 \times \pi}{W_{ben} \times Cp_{ben}}$ = constante e $Mt = \frac{U \times 1.25 \times \pi}{W_{tol} \times Cp_{tol}}$ = constante



Parta dos dados de Cp e U abaixo, e depois inclua correlações no seu programa

Dados:

- Wben = 9820 lb/h
- Wtol = 6330 lb/h
- Cpben = 0.425 Btu/(lb)(°F)
- Cptol = 0.44 Btu/(ln)(°F)
- U = 0.80 Btu/(h)(in^2)(°F)
- h (passo) = 20 in

Exemplo de código visto em aula para propriedades constantes

```
from cmath import pi
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp

#Definições do problema
Wben = 9820   # lb/h
Wtol = 6330   # lb/h
Cpben = 0.425 # Btu/(lb)(F)
Cptol = 0.440 # Btu/(lb)(F)
U = 0.8       # Btu/(h)(in^2)(F)
Mb = 1.25*pi*U/(Wben*Cpben)
Mt = 1.25*pi*U/(Wtol*Cptol)
print("Valor de Mb:", Mb)
print("Valor de Mt:", Mb)

#### Sistema de EDOs
def temperaturefunction(x,y):
    Tb = y[0]
    Tt = y[1]
    dTbdx = Mb*(Tt-Tb)
    dTtdx = Mt*(Tb-Tt)
    return np.array([dTbdx,dTtdx])
##############

x_intervalo = np.array([0, 1600]) #intervalo do x para resolver as EDOs
x_passo = np.linspace(x_intervalo[0], x_intervalo[1],100) #passo de integração, com base no x inicial (x_span[0]), no x final (x_span[1]) e na quantidade de intervalos

y0 = np.array([60,170]) #Temperaturas do Tb e Tt em x = 0 (condição inicial)

perfiltemperatura = solve_ivp(temperaturefunction,x_intervalo,y0,t_eval=x_passo,method='RK45') #resolver a função

Tb = perfiltemperatura.y[0]
Tt = perfiltemperatura.y[1]

plt.plot(x_passo, Tb, 'b')
plt.plot(x_passo, Tt, 'r')
plt.legend(["Tben", "Ttol"])
plt.title("Perfil de Temperatura (°F) ao Longo do Eixo x (in)")
plt.xlabel("Comprimento do Eixo x do Trocado de Calor (in)")
plt.ylabel("Temperatura (°F)")
plt.show()
####
```

$\frac{dTb}{dx} = Mb(Tt - Tb)$, condição inicial: em x = 0, $T_{ben} = 60^0F$

$\frac{dTt}{dx} = Mt\,(Tb - Tt)$, condição inicial: em x = 0, $T_{tol} = 170^0F$,

Considerando propriedades variando com a temperatura

Propriedades que variam: Cp e U (coeficiente película)

Como calcular U?

$$\boxed{Nu_D = 0.023\, Re_D^{4/5}\, Pr^n}$$

Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$

Correlacionando Nu e h:

$$\boxed{h = \frac{k}{D} Nu_D}$$

- Calcular Nu para o tubo e anel,
- Calcular h para o tubo e anel
- Para anel, considere diâmetro equivalente)
- k = condutividade do líquido
- D = diâmetro tubo ou diâmetro equivalente

Table 8.4 Summary of convection correlations for flow in a circular tube[a,b,e]

Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform q_s''
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \dfrac{0.0668\, Gz_D}{1 + 0.04\, Gz_D^{2/3}}$	(8.57)	Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform T_s, $Gz_D = (D/x)\, Re_D\, Pr$
$\overline{Nu}_D = \dfrac{\dfrac{3.66}{\tanh[2.264\, Gz_D^{-1/3} + 1.7\, Gz_D^{-2/3}]} + 0.0499\, Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432\, Pr^{1/6}\, Gz_D^{-1/6})}$	(8.58)	Laminar, combined entry, $Pr \gtrsim 0.1$, uniform T_s, $Gz_D = (D/x)\, Re_D\, Pr$
$\dfrac{1}{\sqrt{f}} = -2.0 \log\left[\dfrac{e/D}{3.7} + \dfrac{2.51}{Re_D \sqrt{f}}\right]$	(8.20)[c]	Turbulent, fully developed
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21)[c]	Turbulent, fully developed, smooth walls, $3000 \lesssim Re_D \lesssim 5 \times 10^6$
$Nu_D = 0.023\, Re_D^{4/5}\, Pr^n$	(8.60)[d]	Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
$Nu_D = 0.027\, Re_D^{4/5}\, Pr^{1/3} \left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.61)[d]	Turbulent, fully developed, $0.7 \lesssim Pr \lesssim 16{,}700$, $Re_D \gtrsim 10{,}000$, $L/D \gtrsim 10$
$Nu_D = \dfrac{(f/8)(Re_D - 1000)\, Pr}{1 + 12.7 (f/8)^{1/2} (Pr^{2/3} - 1)}$	(8.62)[d]	Turbulent, fully developed, $0.5 \lesssim Pr \lesssim 2000$, $3000 \lesssim Re_D \lesssim 5 \times 10^6$, $(L/D) \gtrsim 10$
$Nu_D = 4.82 + 0.0185 (Re_D\, Pr)^{0.827}$	(8.64)	Liquid metals, turbulent, fully developed, uniform q_s'', $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $3 \times 10^{-3} \lesssim Pr \lesssim 5 \times 10^{-2}$, $10^2 \lesssim Re_D\, Pr \lesssim 10^4$
$Nu_D = 5.0 + 0.025 (Re_D\, Pr)^{0.8}$	(8.65)	Liquid metals, turbulent, fully developed, uniform T_s, $Re_D\, Pr \gtrsim 100$

[a] The mass transfer correlations may be obtained by replacing Nu_D and Pr by Sh_D and Sc, respectively.

[b] Properties in Equations 8.53, 8.55, 8.60, 8.61, 8.62, 8.64, and 8.65 are based on T_m; properties in Equations 8.19, 8.20, and 8.21 are based on $T_f = (T_s + T_m)/2$; properties in Equations 8.57 and 8.58 are based on $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.

[c] Equation 8.20 pertains to smooth or rough tubes. Equation 8.21 pertains to smooth tubes.

[d] As a first approximation, Equations 8.60, 8.61, or 8.62 may be used to evaluate the average Nusselt number $\overline{Nu}_D$ over the entire tube length, if $(L/D) \gtrsim 10$. The properties should then be evaluated at the average of the mean temperature, $\overline{\overline{T}}_m = (T_{m,i} + T_{m,o})/2$.

[e] For tubes of noncircular cross section, $Re_D \equiv D_h u_m/\nu$, $D_h \equiv 4A_c/P$, and $u_m = \dot{m}/\rho A_c$. Results for fully developed laminar flow are provided in Table 8.1. For turbulent flow, Equation 8.60 may be used as a first approximation.

Calculando Nu (vide capítulo 8 do Incropera)

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^n$$

$$Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

$$Re_D \equiv \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$$

Considerar variação das seguintes propriedades ao longo do comprimento do tubo:

- Viscosidade
- Condutividade térmica
- Calor específico
- Densidade

Alguns dados do Perry

Cp	MM	C0	C1	C2	C3	C4	Tmin (K)	Tmax (K)	Tmin (C)	Tmax (C)	Cp(min)	Cp(max)	Correlação
Benzene	78.114	1.29E+05	-1.70E+02	6.48E-01	0.00E+00	0.00E+00	278.68	353.24	5.53	80.09	1.3251E+05	1.504E+05	$c_{p\ liq} = C_1 + C_2 \times T + C_3 \times T^2 + C_4 \times T^3 + C_5 \times T^4$
Toluene	92.141	1.40E+05	-1.52E+02	6.95E-01	0.00E+00	0.00E+00	178.18	5.00E+02	-94.97	226.85	1.3507E+05	2.3774E+05	
Water	18.015	2.76E+05	-2.09E+03	8.13E+00	-1.41E-02	9.37E-06	273.16	533.15	0.01	260	7.6150E+04	8.9394E+04	
Hydrazine	32.045	7.98E+04	5.09E+01	4.34E-02	0.00E+00	0.00E+00	274.69	653.15	1.54	380	9.7078E+04	1.3158E+05	

ρ	MM	C0	C1	C2	C3	Tmin (K)	Tmax (K)	Tmin (C)	Tmax (C)	ρ(min)	ρ(max)	Correlação
Benzene	78.114	1.0162	0.2655	562.16	0.28212	278.68	562.16	5.53	289.01	11.421	3.827	$\rho_{liq} = \frac{C_1}{C_2^{1+(1-\frac{T}{C_3})^{C_4}}}$
Toluene	92.141	0.8488	0.26655	591.8	0.2878	178.18	591.8	-94.97	318.65	10.495	3.184	
Water	18.015	5.459	0.30542	647.13	0.081	273.16	333.15	0.01	60	55.583	54.703	
Water	18.015	4.9669	0.27788	647.13	0.1874	333.15	403.15	60	130	54.687	51.935	
Water	18.015	4.391	0.2487	647.13	0.2534	403.15	647.13	130	373.98	52.344	17.656	
Hydrazine	32.045	1.0516	0.16613	653.15	0.1898	274.69	653.15	1.54	380	31.934	6.330	

Outras Referências para obtenção propriedades:

- Livro Gases and Liquid Properties, Reid et al.
- Tabelas Incropera (ajustar curva)