Home Project 2

Convolution and Filtering

CS180 • Fall 2025

Back to Home ›

1A – Convolution Filters

This project studies linear filtering and multi-scale image representations through a sequence of controlled experiments. In Part 1, an implemention of a 2-D convolution from first principles is constructed and used to estimate spatial derivatives via finite differences, and replace naive differencing with derivative-of-Gaussian (DoG) filters to suppress noise. In Part 2, we use filtering as a tool for manipulating frequency content: unsharp masking increases the energy of high spatial frequencies to "sharpen" imagery; hybrid images juxtapose low-frequency content from one image with high-frequency content from another; and Gaussian/Laplacian stacks enable multi-resolution blending that hides seams across scales.

Dataset: Box Filter **k = 3** From scratch

SciPy

Box Filter — SciPy (k=3)

The pedagogical baseline, `conv2d_four_loops`, explicitly flips the kernel and accumulates the product over a zero-padded neighborhood using four for loops (two for spatial coordinates and two for kernel indices). The optimized variant, `conv2d_two_loops`, maintains identical semantics but reduces interpreter overhead by vectorizing the inner multiply–accumulate across complete row (or column) slices, leaving only two loops over output coordinates. For validation and runtime comparison I used `conv2d_scipy`, a thin wrapper around `scipy.signal.convolve2d` with *same* output size and *fill* (zero) boundary conditions. The box filter used in this section is created by `proj2/filters.box_filter(k)`, which returns a constant kernel whose entries sum to one, preserving DC gain. Visual and numerical comparisons confirm that both custom implementations match SciPy up to floating-point tolerance, while the two-loop version is substantially faster than four loops for moderate kernel sizes.

1B – Finite Difference Operator

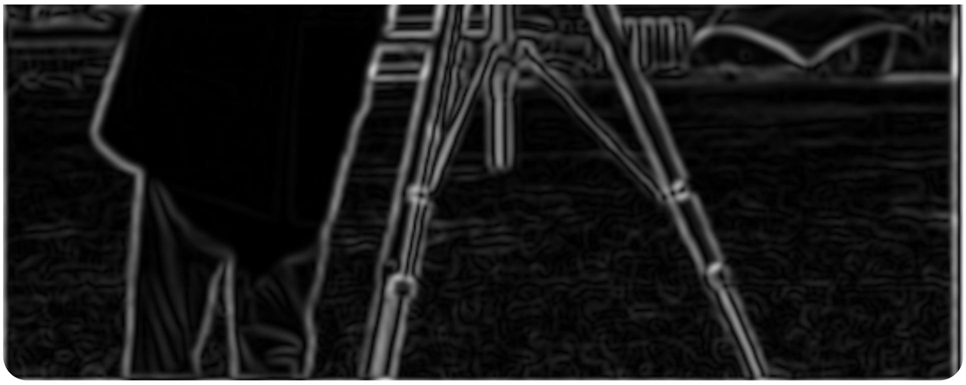
To estimate spatial derivatives, I convolve a grayscale image with the forward-difference stencils `Dx = [-1, 1]` and `Dy = Dx`^{T} using `conv2d_scipy`. The partials `Ix` and `Iy` are then combined into a gradient magnitude image `|∇I| = sqrt(Ix² + Iy²)` (implemented in `proj2/edges.py`). Binarized "edge maps" are obtained by thresholding `|∇I|` at a fraction of its maximum.

○ Ix ○ Iy ○ |∇I| ◉ Edges **t = 10**

Finite difference — edges (t=10)

1C – Derivative of Gaussian (DoG)

○ Smooth → Diff ◉ DoG (∂G ⊗ I) ○ Ix ○ Iy ◉ |∇I|



1C — DoG, |∇I|

I ended up replacing the raw differences with derivatives of a Gaussian. A normalized Gaussian kernel G_σ is constructed using $\mathrm{ksize} \approx 6\sigma + 1$ to capture sufficient support. Two pipelines are compared: (i) smoothing the image with G_σ and then applying D_x/D_y; and (ii) forming the DoG filters $\partial G/\partial x$, $\partial G/\partial y$ (by convolving G with the difference stencils) and applying a single convolution per axis. Both methods yield near-identical I_x, I_y, and $|\nabla I|$ when σ and support are matched, but the DoG route is computationally attractive and conceptually cleaner. For the bells-and-whistles visualization, I compute orientations $\theta = \mathrm{atan2}(I_y, I_x)$ and map them to hue in HSV, with value proportional to $|\nabla I|$; this exposes coherent edge directions across the scene.

2A – Unsharp Masking / Sharpening

Dataset: Taj ○ Source ◉ Blurred (low-pass) ○ High-pass

○ Sharpened

2A — Taj • Blurred (LPF)

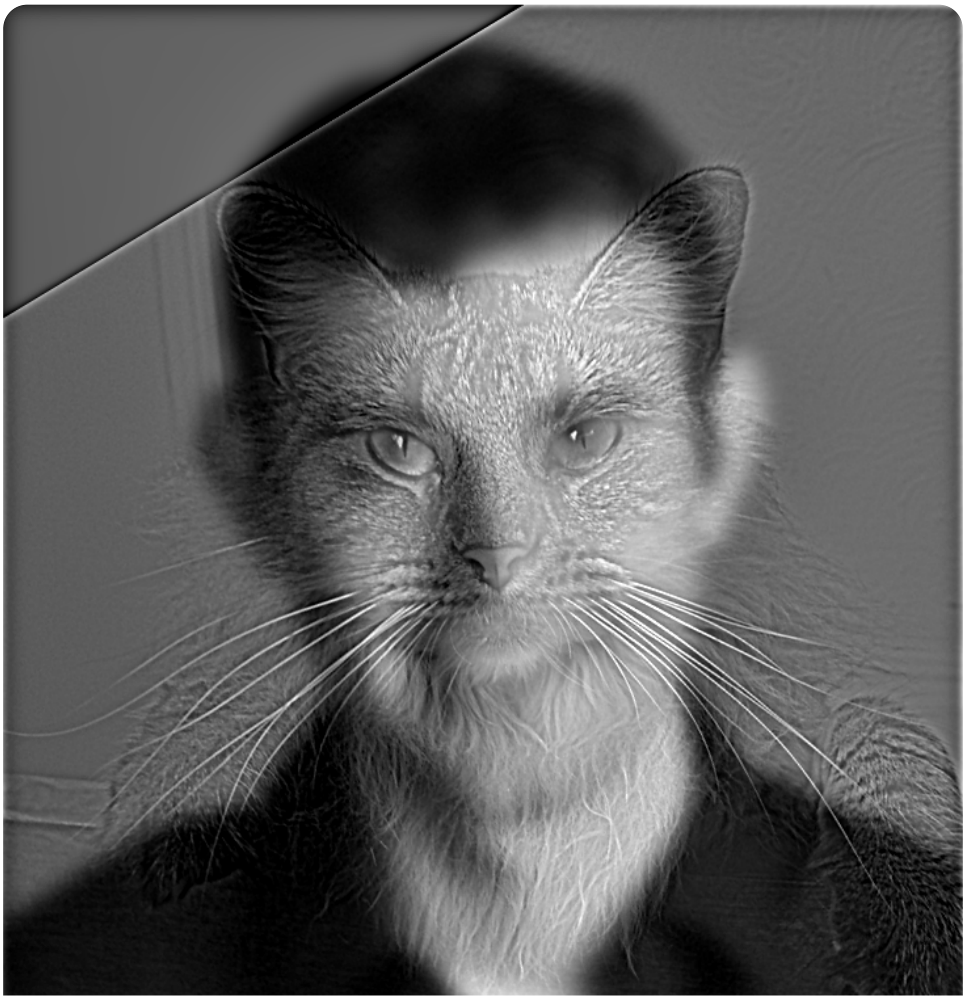
Unsharp masking is implemented in `proj2/sharpen.py`. A Gaussian blur `blur` $= G_\sigma \otimes I$ is subtracted from the input to isolate high frequencies `high = I − blur`; the sharpened output is $I' = \mathrm{clip}(I + \alpha \cdot \mathrm{high}, 0, 1)$. Parameters σ and α govern the spatial scale and strength of the enhancement: increasing σ shifts the emphasis to broader features, whereas large α may introduce halos near strong edges. I also demonstrate reversal by first blurring a sharp image (setting $\alpha = -1$ to visualize the low-pass) and then attempting to re-sharpen it; the comparison clarifies that sharpening restores local contrast but cannot recreate frequencies that have been eliminated by the blur.

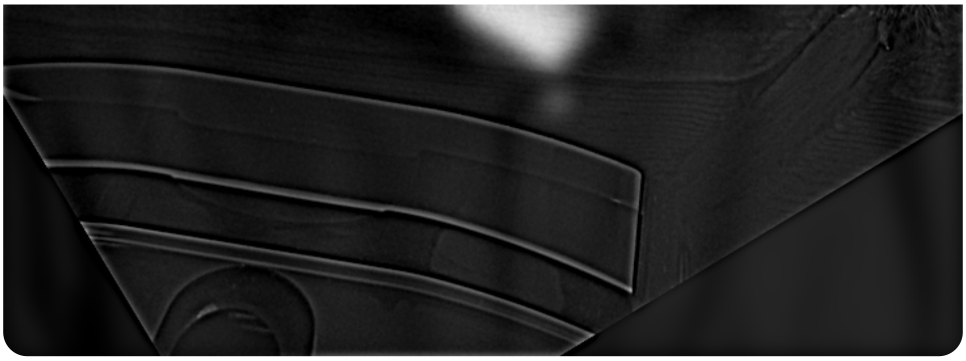
2B – Hybrid Images

Set: Example (A+B) ○ A (low-freq carrier) ○ B (high-freq detail) ○ Low-pass(A)

○ High-pass(B) ◉ Hybrid ○ FFT(A) ○ FFT(B) ○ FFT(Low) ○ FFT(High)

○ FFT(Hybrid)



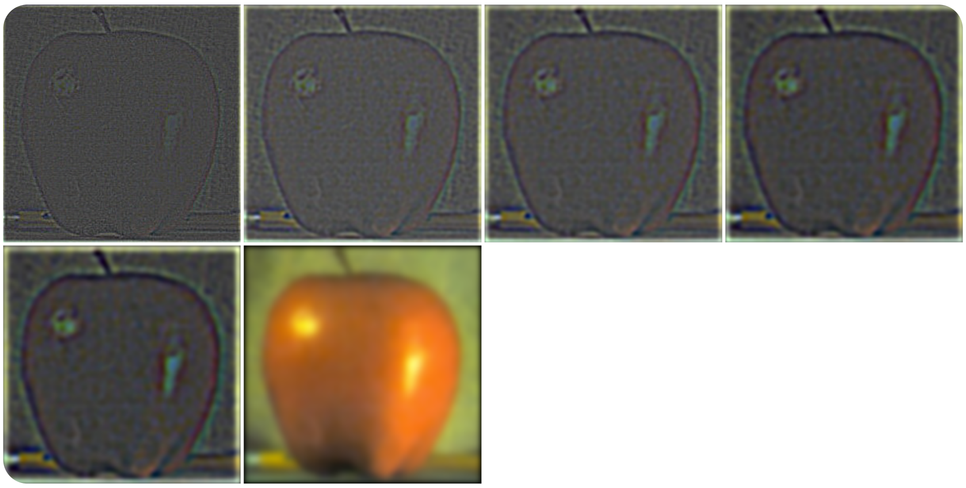


2B — Example • Hybrid

The hybrid construction in `proj2/hybrid.py` forms $H = LP_{\sigma L}(A) + HP_{\sigma H}(B)$, where $HP(B) = B - LP_{\sigma H}(B)$. The function `hybrid_image(A,B, low_ksize, σL, high_ksize, σH)` returns the low-pass of `A`, the high-pass of `B`, their clipped sum, and a log-magnitude FFT for analysis (`log_fft`). Images are pre-aligned and resized via `proj2/io_utils.match_size` so that semantic structures are co-located. Successful hybrids rely on three controls: accurate geometric alignment (to avoid double features), a sufficiently large `σL` that removes mid-frequencies from the carrier, and a moderate `σH` that preserves crisp detail without injecting noise. I tune cutoffs by inspecting the FFTs and by checking whether the interpretation of `H` flips with viewing distance as intended.

2C – Gaussian & Laplacian Stacks

Image: Apple ○ Gaussian Stack ● Laplacian Stack



2C — Laplacian stack • Apple

In stacks.py, `gaussian_stack(img, L, k, σ)` generates a stack of `L` images at full resolution by repeated blurring with the same σ; `laplacian_stack. Unlike pyramids, stacks avoid down-sampling and simplify subsequent per-pixel operations (e.g., stack-wise masking) at the` expense of memory. The Laplacian representation is a band-pass decomposition of the image and is a natural basis for frequency-aware blending.

2D — Multiresolution Blending

Mask: Vertical ◉ Result (Oraple) ○ Mask Stack ○ Laplacian Blended Stack

2D — Result (Oraple) • Vertical mask

We implement blending using the Burt–Adelson recipe using stacks. Given two color images A, B and a mask M∈[0,1], Laplacian stacks LA, LB and a Gaussian stack of the mask GM. At each level i, we compute $L_{blend}[i] = GM[i]\cdot LA[i] + (1-GM[i])\cdot LB[i]$, then collapse the blended Laplacian stack to reconstruct the output. A hard vertical step mask reproduces the classic “oraple” result; an irregular, feathered mask demonstrates that spatially varying, scale-appropriate smoothing of the seam yields perceptually (ALMOST) seamless composites even across textured regions.

JS: 2B viewer