Mario I. Caicedo

Mathematics Teaching Philosophy

My name is Mario Iván Caicedo Sandgren, and I am a Professor of Physics at Universidad Simón Bolívar (USB), a top-tier Venezuelan technological institution with high prestige in Latin America. My background in physics provides a unique perspective that enriches my approach to teaching mathematics, emphasizing its power, beauty, and applications. Over my 33-year academic career, thanks to the structure of USB, I have been fortunate to research and teach in four areas: theoretical and mathematical physics, mathematics, geosciences, and complexity science.

I have taught calculus-based courses for engineering and science students, as well as advanced undergraduate and graduate physics courses, including those on geometrical and topological methods. I have also supervised 38 undergraduate research projects, four Master's theses, and jointly guided one PhD student. In all these projects, I encouraged students to present their work at conferences, believing that public exposition strengthens understanding and confidence. Informally, I have taught pure mathematics to groups of students who have asked for help reinforcing their courses, covering subjects ranging from real and complex analysis and linear algebra to differential equations (both ordinary and partial), differential geometry, and Lie groups and Lie algebras.

Through all these years, I have learned that no one really teaches anything to anyone. Teachers act as catalysts for the learning process, facilitating exploration rather than delivering finished truths. Nonetheless, for convenience, I will continue to use the words *teach* and *teacher*. I have identified four different teaching scenarios: classroom lecturing, direct interaction in scheduled meetings, informal discussions in unplanned gatherings, and mentoring advanced students. I also think of students in three broad categories: general audiences, students in introductory courses, and advanced students. Each intersection of these categories calls for slightly different approaches, yet in all cases, empathy and respect are essential.

Students approach learning with different motivations and skills. General audiences are often curious, as when visiting a museum; introductory students seek concepts and techniques useful for future courses; advanced students are developing professional skills and require guided challenges to build confidence and independent thinking.

Even in mathematics, ideas presented in the classroom are seldom grasped fully. For instance, when asking students to expand $(a+b)^2$ and $(a+b)(a-b)$, they almost always get $a^2+2ab+b^2$ and a^2-b^2. Yet, when asked to compute 23×17 or 12^2, very few recognize shortcuts like $20^2-3^2 = 391$ or $100+2\times 2\times 10+2^2 = 144$. Similarly, in advanced courses, very few notice that

$$\sqrt{18} = \sqrt{16+2} = 4\sqrt{1+1/8} \approx 4(1+1/16) = 4.25,$$

which is remarkably close to the exact value—to four decimal places—of 4.2426. These examples illustrate the difference between procedural correctness and deeper understanding, a distinction I strive to highlight in all courses.

Students also come with preconceptions. A concrete example I use in class comes from topology: the definition of a manifold via a maximal atlas. Many students initially struggle to see why we need a maximal atlas, or how different charts interact. To make this tangible, I bring a physical atlas of the Earth to the classroom. I show that each chart (or map) covers a region, some maps overlap, and together they give a clear picture of the entire surface. This hands-on demonstration allows students to see the abstract definition come alive and understand the necessity of the maximal atlas. Demonstrations and short visual examples like this, often sketched on the board, are central to helping students internalize abstract mathematical concepts. Whenever possible, I also supplement lessons with short videos or computational illustrations that reinforce these ideas.

When presenting proofs, I habitually draw sketches or diagrams to provide intuition and visual guidance. I believe that connecting abstract concepts with geometric or graphical intuition helps students internalize reasoning. For instance, in linear algebra, I illustrate linear transformations by mapping vectors in a plane; in analysis, I often draw function graphs to support epsilon-delta arguments. Throughout, I integrate simple examples that are concrete and often amusing, helping to anchor abstract ideas in experience.

I also make a point of introducing the human side of mathematics. Whenever possible, I share brief stories of mathematicians and their historical context, placing discoveries within the flow of human culture—Rome before the Industrial Revolution, Babylon before Rome, Columbus before the American Revolution—to show that mathematics is a human endeavor embedded in history. I include art, literature, and other cultural references sparingly, always to illuminate the beauty and context of the concepts. The profound reward of this approach comes when a simple prompt—to find a relation between Salvador Dalí or Leonardo da Vinci and mathematics—leads students to return with huge grins, eagerly sharing their discoveries of Pacioli's *De Divina Proportione* or the influence of topologist René Thom. In these moments of shared joy, the boundary between science and art dissolves, and learning becomes a collaborative adventure.

In the classroom, I encourage deliberation, questioning, and discussion. I provide detailed notes in advance so that class time can be devoted to exploring ideas, resolving doubts, and connecting concepts. Problem sets are designed to illuminate theoretical results and foster independent thinking, while computational exercises allow students to compare analytical reasoning with numerical outcomes. I make myself as available as possible to students, always maintaining a respectful and approachable attitude.

Finally, to summarize my approach, I maintain what I call a *flying checklist*:

1. Treat students with kindness and respect at all times.
2. Encourage questions and curiosity; the only silly question is the one left unasked.
3. Help students manage frustration and anxiety, guiding them to find joy in discovery.
4. Be accessible and approachable.
5. Combine rigor with intuition: in proofs and examples, connect abstract ideas with visual or concrete illustrations.
6. Provide notes before lectures and emphasize key ideas during class.
7. Go into technical details while maintaining perspective on the larger conceptual picture.
8. Lead by example, maintaining high standards without arrogance.

9. Utilize technology and open-source tools to enhance understanding and verification.
10. Design assessments based on understanding and practice, not extreme time pressure.
11. Make learning an engaging and joyful experience.

In essence, my teaching philosophy is rooted in empathy, clarity, and the human dimension of mathematics. I aim to cultivate students' intuition, reasoning, and curiosity, while highlighting the beauty and cultural context of the discipline. My ultimate goal is to foster confident, thoughtful learners who can appreciate both the rigor and the elegance of mathematics.

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