The Simplest Introduction To Linear Motion I Can Think Of

Mario I. Caicedo

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An important note One secret to master problem solving in general and science in particular is to examine each new situation with great care looking for similarities to some situations or problems we might have already found.

1 Kinematics

Kinematics is the branch of classical mechanics that describes the motion of points, objects and systems of groups of objects, without reference to the causes of motion (i.e., forces). The study of kinematics is often referred to as the "geometry of motion".

The physics of motion is quite an extensive subject. Beginners usually go no further than constant accelerated motion along a straight line, and some times in 2D. Here we will concentrate on the motion of objects along a straight line, a picture of such might be a car moving along a straight racing track.

2 Position and displacement

Have the above been said, think of the track and the starting point (the simplest mathematical model for this is a line where we mark a special point and call it origin -$\mathcal{O}$-).

Have $\mathcal{O}$ been chosen, we let x be the signed position of a point (the moving body) with respect to $\mathcal{O}$, let us for now take negative x for positions to the left of x and plus for positions to the right of the origin, this sign assignment is called orientation. In order to give a precise description of the motion we need a clock to measure time (t), the measurement has to be done in some units, may they be, seconds, hours, microseconds, etc. As we did with the track, the description of the motion requires us to think of some initial moment of time which we usually call t_0, it is quite customary to state $t_0 = 0$.

To be effective when doing physics we must learn the **dialect of physics**[1], physicists use regular words but endow them with very specific precise meanings. Imagine yourself walking along a $1\,mi$ long boulevard. let's say from $6\,pm$ to $9\,pm$. You begin your at some street corner, a position we call $x(6\,pm) = 0$, then you walk for a while and $10\,min$ later, you reach a library which is half a mile from the initial corner so $x(6:10\,pm) = +0.5\,mi$, where we have used our free will to choose the positive sing for x when yo walk from the the initial corner to the library. The displacement Δx between $t_0 = 6\,pm$ and $t_1 = 6:10\,pm$ is defined to be

$$\Delta x_{t_ot} \equiv x(t) - x(t_0) = x(6:10\,pm) - x(6:00\,pm) = 0.5\,mi\,, \tag{1}$$

You stay in the library for $40\,min$, go out and walk for $5\,min$ to an ice cream shop which is located $1/4\,mi$ of the initial corner in the direction to the library. In physicist dialect, your position at $t_2 = 55\,min$ is $x(55\,min) = 0.25\,mi$. At this point is where people tend to get

[1] I have intentionally used the mathematical notation $x(t)$ to refer to the position at time t. With it, I am making a non-trivial link with math. We want to think of the particle's positions as a function of time (why a function? Well, because a particle can't be in two different places at the same time, can it?).

confused because habits are deeply rooted in the mind, meaning that there is a psychological refusal to accept the new, more precise dialect. Just look at the following facts:

- **Positions**
  - $x(t_0) = x(6:00\,pm) = 0$, At six pm in the evening you are at the starting point of your walk (**motion**).
  - $x(t_1) = x(6:10\,pm) = +0.5\,mi$, 10 min later you are half a mile of the starting point in whatever you chose to be the positive direction.
  - $x(t_2) = x(6:55\,pm) = +0.25\,mi$. At $6:55\,pm$ you are a quarter mile from the starting point in the positive direction.
- **Displacements**
  - $\Delta_{t_0,t_1}x = x(t_1) - x(t_0) = x(6:10\,pm) - x(6:00\,pm) = +0.5\,mi$. Between $6:00\,pm$ and $6:10\,pm$ your displacement was half a mile in the positive direction.
  - $\Delta_{t_1,t_2}x = x(t_2) - x(t_1) = x(6:55\,pm) - x(6:10\,pm) = -0.25\,mi$. Between $6:10\,pm$ and $6:55\,pm$ your displacement was a quarter mile in the negative direction.

The funniest part comes in if you remain for quite a long time in the ice cream shop chatting with some friends, at $8:54\,pm$ you remember you are going to lose some tv show, say good bye, and rush back to the initial corner arriving there at exactly $9:00\,pm$, then, even though you logged a walk of $1.0\,mi$ your total displacement is

$$\Delta_{t_0,t_fin}x = x(t_fin) - x(t_0) = x(9:00\,pm) - x(6:00\,pm) = 0.00\,mi\,, \tag{2}$$

there is nothing wrong here, you indeed walked a mile, but the name of the total distance you walked is the **trajectory's length**, also known as the total arc length.

A simpler example would be running in an Olympic athletics track. Everyone knows the length of the track is a quarter mile, so, if you run four laps you will have logged a mile but your total displacement in the period of time it takes you to run such distance is definitively $0\,mi$

3 The notions of velocity and speed

Figure 1: Elaine Thompson-Herah's 10.61 s to win Tokyo gold adjusts to the fastest wind-legal time ever, 10.57 s.

Everyone understands the meaning of the words **fast** and **slow**. We are all aware that if three runners sprint in a race figure 1, the fastest one is that who reaches the finish line in the

Figure 2: Instrument Cluster Meter of an SUV with the speedometer at the center

least time, while the slowest is that who those in the longest time, the runner arriving in second place is neither the fastest nor the lowest, but is definitively slower than the fastest and faster than the slowest.

Any person who has ridden in a car has at least seen the speedometer (see figure 2). The numbers on the speedometer indicate the speed of the car in miles per hour (mph). For example, if the speedometer reads 30 mph, we know that it will take us about 18 minutes to travel 9 miles. Let's examine how we know it will take 18 minutes. Most of us agree that speed (what the speedometer measures) is the rate at which the car travels a distance in unit time. In our case, the car is moving at a speed of

$$30\,mph = 30\,miles/(60\,minutes) = 0.5\,miles/minute \tag{3}$$

In other words, at a speed of 30 mph, the car travels half a mile in one minute. So, traveling 9 miles, which is equal to 18 times half a mile, will take 18 times the time to travel half a mile, or 18 minutes.

The information we get from the speedometer is only about the rate of distance per unit time, we don't know if the motion is forward, backwards to the east or to the west. Given the above we conclude that if a car travels a distance of $50\,mi$ in $2\,h$, the speed of the car is

$$speed = \frac{50\,mi}{2\,h} = 25\,mi/h \tag{4}$$

4 Motion at constant velocity

Let us imagine an experiment, it consists on observing an object moving along a line and recording its position at certain times. The result of our *Gedankenexperiment* is presented in the form some data with two significant figures

t (s)	x (m)
0.0	1.0
1.0	0.8
2.0	0.6
3.0	0.4
4.0	0.2
5.0	0.0
6.0	−0.2
7.0	−0.4

Table 1: Experiment measuring the position of a toy car along a straight track

Some examination of the table shows that the toy began its motion 1.0 m to the left of the origin and moved to the left along the track in such a way that its last recorder position, 7.0 seconds after the motion began to be recorded is 40 cm to the left of the origin.

It is important to note that the reported data does not say anything about what was the state of motion just before $t = 0$, the object might have being somehow moving along the track or it might have been standing still at the initial position $x(0.0) = 1.0\ m$.

During the time of the experiment, and thinking at the different instants of time at which the measurements took place, the toy went through a series of changes in position called **displacements** (Δx) defined by

$$\Delta x(t) \equiv x(t + \Delta t) - x(t)\,, \tag{5}$$

The quantity Δt appearing in formula 5 is just the time interval between two successive measurements.

We might sketch a graphic of the data and would find a straight line with

$$\text{slope} \equiv v = \frac{\Delta x}{\Delta t} = -0.2\ m/s\,. \tag{6}$$

formula 6 is quite similar to formula 4 defining the speed. There are some subtle differences though.

In formula 6 v stands for *velocity*, which is to be understood as a signed quantity. This is related to the fact that the quantity that enters the definition of velocity is displacement which is a signed quantity while distance which is what appears in the definition of speed is not signed.

Speed is therefore the magnitude of the velocity and as we already knew, it just talks about the motion being fast or slow.

The sign in the velocity sign tells us whether the motion is rightwards or leftwards, that is, velocity tells us about direction of motion and speed.

In our experiment, v is negative meaning -as we already knew- that the motion is to the left.

To give a better notion of speed we might quote that NASCAR cars may reach a top speed of nearly 200 mph or 321 km/h. F1 cars are quite more impressive, Honda, who took their RA106 to the Bonneville Salt Flats in the US, a site famous for top-speed runs, to try and break 400 km/h. They were unsuccessful, but set a 397.36 km/h (246.9 mph) top speed, to claim the highest speed in an F1. In actual F1 races and due to the difficulties imposed by the circuits, F1 cars have typical speeds (magnitudes of v) close to 161 mph (260 Km/h) which is 5.6 times faster than the usual city limit of 30 mph.

Our experiment corresponds to the the simplest motion of all. Known as *Uniform Rectilinear Motion* (URM) is the motion of an object that travels along a straight line always in the same direction and at constant speed. If we call x_0 the position of the particle at the initial time $t = t_0$ and v_0 the velocity at t_0, then the position of the particle $t \geq t_0$ is given by the formula

$$\boxed{x(t) = v_0\,(t - t_0) + x_0} \tag{7}$$

5 Uniformly accelerated motion

We begin this section by introducing the simplest possible generalization of formula 7, namely

$$\boxed{\begin{aligned} v(t) &= a(t - t_0) + v_0 \\ x(t) &= \frac{a\,(t - t_0)^2}{2} + v_0\,(t - t_0) + x_0\,. \end{aligned}} \tag{8}$$

Where x_0, v_0 and a are constants with dimensions

$$\boxed{\begin{aligned} [x_0] &= \text{Length} = L \\ [v_0] &= \text{Length per unit time} = \frac{L}{T}\,, \quad \textit{and}, \\ [a] &= \text{Length per time per time} = \frac{L}{T^2} \end{aligned}} \tag{9}$$

For formulas 8 to make some sense, we need to give some physical (concrete, experimental) meaning to the symbols appearing in them.

We begin with $v(t) = a(t - t_0) + v_0$, this formula expresses a motion where the velocity in not constant, in fact it changes linearly with

$$\text{slope} = a \tag{10}$$

a the *acceleration* is a signed quantity having a quite more delicate meaning, a tells us about how velocity changes in time.

5.1 An useful formula

Before attacking the problems we will derive an useful formula that applies for uniform accelerated motion along a line.

We begin by recalling that for this condition

$$x(t) = \frac{a(t - t_0)^2}{2} + v_0(t - t_0) + x_0 \tag{11}$$

$$v(t) = a(t - t_0) + v_0 \tag{12}$$

From 12

$$t - t_0 = \frac{v(t) - v_0}{a} \tag{13}$$

We now substitute eq. 13 into eq. 13 to get

$$\begin{aligned} 2(x(t) - x_0) = a\left(\frac{v(t) - v_0}{a}\right)^2 + 2v_0\left(\frac{v(t) - v_0}{a}\right) = \\ = \frac{(v(t) - v_0)^2}{a} + 2v_0\left(\frac{v(t) - v_0}{a}\right) \end{aligned} \tag{14}$$

But

$$\begin{aligned}
&\left[\frac{(v(t)-v_0)^2}{a}\right] + 2v_0\left(\frac{v(t)-v_0}{a}\right) = \\
&= \frac{v(t)^2 - 2v(t)v_0 + v_0^2}{a} + \frac{2v_0v(t) - 2v_0^2}{a} = \\
&= \frac{v(t)^2 - 2v(t)v_0 + v_0^2 + 2v_0v(t) - 2v_0^2}{a} = \\
&= \frac{v(t)^2 - v_0^2}{a}
\end{aligned} \tag{15}$$

and form here we conclude that

$$2a(x(t)-x_0) = v(t)^2 - v_0^2 \tag{16}$$

This formula is usually written as (d stands for displacement)

$$\boxed{v_f^2 - v_{ini}^2 = 2ad} \tag{17}$$

6 Acceleration, what is it?

To develop an intuition about acceleration, imagine a drag racing car, certainly any video will show that those machines can go form zero to very high speeds in short periods of time, to be precise, a top fuel dragster accelerates from a standstill to 100 mph (160.9 km/h) in as little as 0.8 seconds. In terms of a that mean that the magnitude of a is 'big', with big meaning in comparison to a standard car which to go from 60 mph take nearly 5 seconds. In both examples, the magnitudes of the acceleration are

$$\boxed{\begin{aligned}
a_{dragster} &= \frac{160}{0.8} \text{ miles per second per second} = 200 \text{ miles per second per second} \\
a_{standard\ car} &= \frac{60}{5} \text{ miles per second per second} = 12 \text{ miles per second per second}
\end{aligned}} \tag{18}$$

said in words, a dragster acceleration is nearly 17 times bigger than a standard's car. For further comparison, the dragster acceleration is even bigger than that of a jet fighter plane during lift of from a carrier

Discussion Topic 1 *Sit with your friends or some faculty and try to go deep into the question: What exactly is acceleration?*

Discussion Topic 2 *What is the meaning of the constants v_0 and x_0 in equation 8.*

Discussion Topic 3 *What are the formulas for position and velocity for non accelerated motion?, How do you interpret the resulting formulas in terms of a familiar setting?*

Prob 1 *How much does it take to travel 200 miles at the 55 mph speed limit?*

Example 1 *As for today, the Olympic records for the 100 m track are 9.63 seconds, set by Usain Bolt in 2012, and 10.62 seconds, set by Florence Griffith-Joyner in 1988.*

1. *Find the speeds of said runners.*

$$v_{UB} = \frac{100\ m}{9.63\ s} = 10.39\ m/s \tag{19}$$

$$v_{GJ} = ? \tag{20}$$

2. *Transform the values to different units*

$$\begin{aligned} v_{UB} = 10.39\ m/s = 10.39 \frac{Km}{1000\,m} \frac{3600\,s}{h} = 37.38\ \ Km/h = \\ = 37.38 \frac{1\ mil}{1.609\ Km} = 23.23\ mph \end{aligned} \tag{21}$$

Prob 2 *Are the above the true speeds of the athletes?*

Figure 3: Skydiver in "free" fall

Example 2 *"Near" the surface of the Earth, an object in free fall in a vacuum will accelerate at approximately* $9.8\,,\, m/s^2$, **independently of its mass**. *With air resistance acting on an object that has been dropped, the object will eventually reach a terminal velocity, which is around 53 m/s (190 km/h or 118 mph) for a human skydiver.*

1. *How much time does it take to reach the terminal velocity? Since the skydiver falls with constant acceleration,*

$$v_{term} = a_{fall} t_{reach\ t.v.} \tag{22}$$

that means

$$t_{reach\ t.v.} = \frac{v_{term}}{a_{fall}} \tag{23}$$

putting the numbers together,

$$t_{reach\ t.v.} = \frac{53\ m/s}{9.8, m/s^2} = 5.4\ s \tag{24}$$

2. *How much distance does a skydiver fall till she reaches the terminal velocity?*

   To answer this question we must assume that the initial velocity of the skydiver is 0*, setting up the observer in the plane, the initial position is also 0, so the distance is simply*

$$y = \frac{a\,t^2}{2} \tag{25}$$

   To find the distance the skydiver falls until she reaches the terminal velocity, all that is needed is to substitute $t_{reach\ t.v.} = 5.4\ s$ *in this formula, once again, we put the numbers toghethr to get,*

$$y = \frac{9.8\ \ m/s \times\ (5.4\ s)^2}{2} = 142.8\ m \tag{26}$$

Figure 4: The Ferrari 812 Superfast

Example 3 *A flagship Ferrari should be fast, that's obvious, but with a name like Superfast, the 812 really had to put its money where its mouth is. Fortunately for Ferrari, the 6.5-litre V12 will happily launch it from 0-60 mph in just 2.9 seconds, onto a top speed of 211 mph.*

Remark *Before going any further, we must realize that this physical situation is very similar to that of example 2 (why?)*

1. *Find the accelerarion of the 812.*

   The acceleration is just the change in speed divided by the time it takes to reach the speed,

$$a = \frac{60\ mph}{2.9\ s} = \frac{60 \times 1609\ m}{3600\ s} \times \frac{1}{2.9\ s} = 9.24\, m/s^2 \tag{27}$$

2. *In how much time does the 812 reach its maximum speed? The maximum speed of the car is*

$$211\ \ mph = \frac{211 \times 1609\ m}{3600\ s} = 94\ \ m/s \tag{28}$$

   To reach that speed the 812 needs

$$t = \frac{94.3\ \ m/s}{9.24\ s} = 10.20\ \ s \tag{29}$$

Prob 3 *A car is travelling at* 36 Km/h*, the breaks are applied suddenly so the resulting* **acceleration** *is* 10 m/s^2*, how much time passes until the car stops?*

Prob 4 *On 14 October 2012 skydiver Felix Baumgartner did a freefall parachute jump from a height of* 38969.4 m*, smashing through eight world records and the sound barrier all in one go.*

Could we analyze this situation the same way we did in example 2

6.1 An important observation regarding acceleration

In all the examples given above, the above we have discusses motions in which the speed increases due to the acceleration, is this always so?. The answer is **definitely** , we already know that a signals a change in velocity, in one dimension a smooth change in velocity may

Figure 5: Felix Baumgartner begining his famous jump

acceleration and velocity	speed
$a = 0$	constant velocity
parallel	increase
antiparallel	decrease

mean an increase or decrease in speed. Consider figure 6, in it three objects are moving to the right so their velocity is positive, the car, accelerating with positive a goes faster and faster, the truck, instead, with $a < 0$ is reducing its sped, while the cyclist with $a = 0$ is riding smoothly at constant velocity.

In order to use the right notions we refer to these situations as acceleration being and velocity being parallel, antiparallel or non accelerated motion.

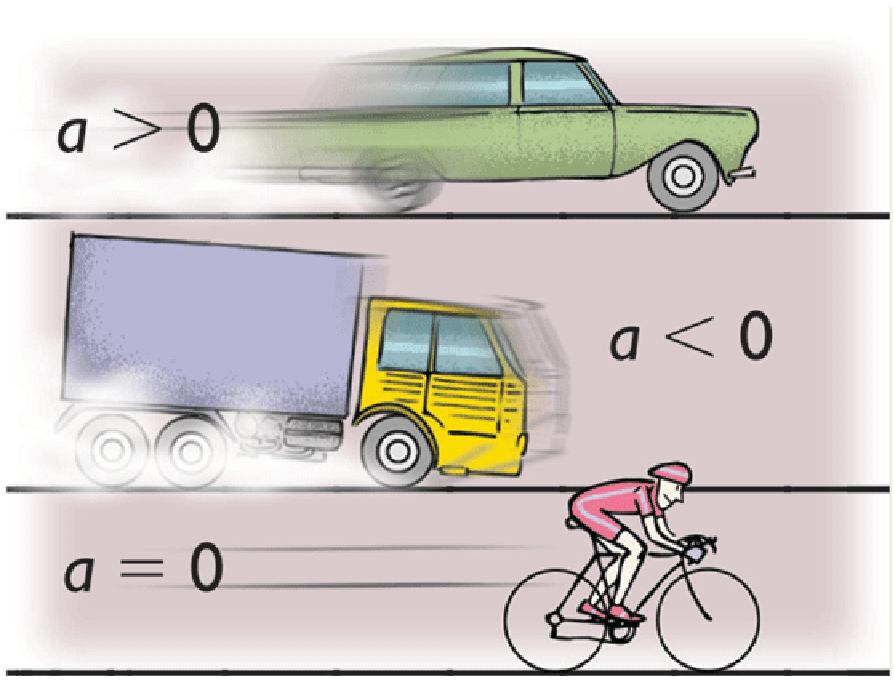

Figure 6: Changing in speed due to acceleration

7 Dynamics

We begin this section by stating (without any further explanations) Newton's first and second law

1. A body will reamain in constant velocity linear motion unless is acted upon by a forece.
2. When a force acts on a body of mass (inertia) M the body acquires an acceleration given by
$$\vec{F} = M\vec{a} \tag{30}$$

Before going any further it is important to explain that many people get confused with the concepts of **mass** and **weight**.

Mass is the inertia, i.e. the resistance to be accelerated.

Weight on the other hand is a completely different concept. The weight of an object is the force with which earth (or any astronomical object) attracts a body when it is close to the earth's surface.

The International Space Station orbits the Earth because of the gravitational pull that the earths exerts on the ISS, were not for gravity, the ISS would travel in a straight line for ever. The same happens with the moon and of course with earth which is pulled by the sun. But much more interesting (and therefore fun), to all stars in the Milky Way which are attracted to a Giant Black Hole lying at the very center of our Galaxy. According to the legend, young Newton realized that the very same pull that makes an apple fall form a branch of an apple tree is what makes the moon go around the earth, and boy, ¡he was right!

Figure 7: The apple and the moon are pulled towards earth. Gravity is a universal force, all objects are attracted towards each other

Example 4 *As happens with length or time, forces also have units, in fact, they have* **derived units** *which are defined by Newton' s second law. The force required to give a* 1 Kg *object an acceleration of* 1 m/s^2 *is called a Newton (symbol: N). Therefore,*

$$1\ N = 1\ Kg \times m/s^2 \tag{31}$$

Figure 8: Balance

Figure 9: Dynamometer

Figure 10: Balance

Prob 5 *The kerb weight of the ferrari 812 (example 3) is* 1744 kg *(3,845 lb), what force is needed for pushing it with it's acceleration.*

Prob 6 *Given that a free falling body on earth falls with an acceleration very close to* 9.8 m/s^2*, ¿how much does a* 100 Kg *object weight?*

8 g's a typical measure of acceleration

If you go to the movies and watch, let's say, **Top Gun: Maverick**, you will constantly hear something like "...Mav is trying a 5g maneuver." That means a maneuver in which the plane, and therefore the pilot, were subject to high (in terms of magnitude) accelerations. The effect of acceleration on animals, and of course humans, is very important for their lives. A human can withstand up to 18g's without dying, but typically passes out at 6 to 7g's. But...

Well, the simplest answer to the question we have just posed is this: 1 g is the acceleration of an object in free fall near the earth, which is close to 32 ft per second per second or

$$1\ g = 9.78\ m/s^2 \approx 10\ m/s^2. \tag{32}$$

This actually means that if we let a ball fall from the top of a building it will acquire a down falling speed of 32 feet per second in the first second of its motion and will be falling a at rate of 64 ft per second after two seconds of free fall.

An interesting fact is that what we call **Weight** is nothing more that the force with which the earth attracts an object. Near the surface of the earth, the weight is simply

$$Weight \; = Mass \times 1 \; g = Mass \times 9.78 \; m/s^2 \approx Mass \times 10 \; m/s^2 \,, \tag{33}$$

where we must learn that the mass of an object is the resistance it offers to be accelerated.

$$Weight_{Mario} = 100 \times 10 \; m/s^2 = 1000 \; N \tag{34}$$

$$Weight_{rollercoaster} = 12 Weight_{Mario} = 12000 \; N \tag{35}$$

Velocity when you fall 2 meters.

$$v = \sqrt{2\, g\, height} = \sqrt{2 \times 10 \times 2} = 6.32\, m/s \tag{36}$$

acceleration to complete stop

$$a = v/\text{time it takes to stop} = 6.32\, m/s/(0.1\, s) = 63.2\, m/s^2 \approx 6\, g \tag{37}$$

The force that the floor applies on my skull equals my mass times $6g$

9 Interesting Videos

1. 50 min 1D Motion, lecture by Walter Lewin
2. ISS acceleration
3. Effect of Acceleration John Paul Stapp

4. Carrier Catapult, force accelerates objects

5. The Hulk, a roller coaster that uses a catapult

6. Famous Physicist Brian Cox subjet to several G' s

7. G forces felt and explained What happens when you are in a room which is in free fall?