Algebra Based Physics: Electromagnetism

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December 30, 2024

Contents

An important note One secret to master problem solving in general and science in particular is to examine each new situation with great care looking for similarities to some situations or problems we might have already found.

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Chapter 1

Circuits

This chapter is heavily based on the following excellent web resource LibreTexts Physics

Electrical circuits are the invisible backbone of modern life. From the moment we wake up to the time we sleep, we rely on countless circuits. They power our homes, offices, and vehicles, bringing light, heat, and enabling communication. From the simple act of turning on a light switch to complex operations in industries and healthcare, electrical circuits are the conduits for energy that drive our world. Without them, our technologically advanced society would grind to a halt.

Electrical circuits are the fundamental building blocks of our technological world. While simple circuits power our homes and appliances, their integration and miniaturization have led to revolutionary advancements. Electronic circuits, composed of tiny components like transistors, are the heart of computers, smartphones, and countless other devices. These circuits process information at incredible speeds, allowing for complex calculations, communication, and entertainment. The ability to integrate billions of transistors onto a single chip has fueled the digital age, transforming how we live, work, and interact. From the supercomputers that power scientific research to the microcontrollers in our cars, electrical circuits are the unseen force driving innovation and progress. But, what is an electric circuit? For the purposes of

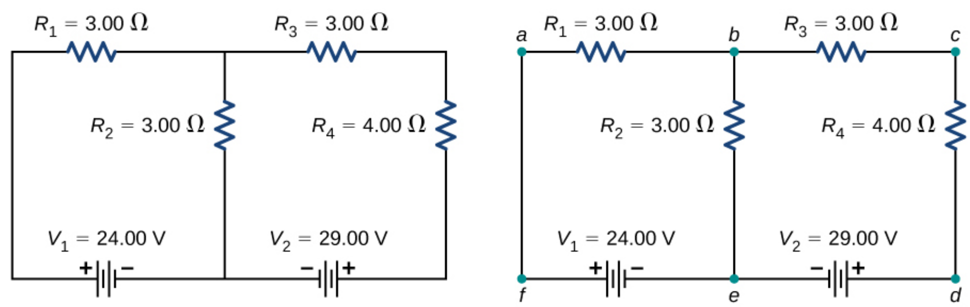

Figure 1.1: Left, scheme for a simple two loop circuit. Right, same circuit with labelled nodes

these notes, what follows constitutes a close enough definition, an electric circuit is an interconnected network of electrical components through which electric current can flow in one or more closed loops. This definition encompasses both simple and complex circuits, including those with multiple loops. Key elements of a circuit typically include a power source, conductors, and electrical loads.

A closed loop, or simply, a loop, is a continuous path for current to flow, starting from some point in the circuit, a voltage source, for example, passing through components, and returning to the original point. A circuit without a complete path for current to flow is considered open.

Components of a circuit include:

- Voltage sources: Provides the energy to drive the electric current (e.g., battery, generator).
- Loads: These are components that consume electrical energy and converts it into another form (e.g., light bulb, motor).
- Conductors: Material that allows electric current to flow easily (e.g., copper wire).
- Switches: THese are devices that control the flow of electric current (e.g., on/off switch).

For a circuit to function, it must form a complete loop. If the circuit is broken, the flow of current stops.

Figure 1.2 shows a two loop circuit, the right side figure ha the nodes labelled.

To analyze and calculate the voltages and currents within intricate electrical circuits, we rely on a fundamental set of principles known as Kirchhoff's rules. These rules, derived from the conservation of charge and energy, provide a systematic approach to solving circuit problems. Complex circuits, characterized by multiple interconnected components and loops, require these rules to determine the behavior of the circuit and its constituent elements.

1.1 Kirchoff's Rules

Kirchhoff's first rule (the junction rule) applies to the charge entering and leaving a junction. A junction, or node, is a connection of three or more wires. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out.

$$\boxed{\sum_{incoming\ currents} i_{in} = \sum_{outgoing\ currents} i_{out}}$$

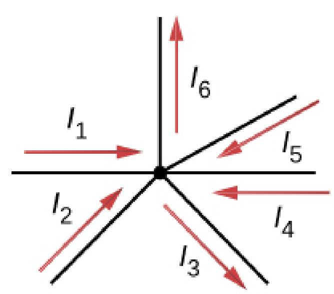

Figure 1.2: In this node there are four incoming and two outgoing currents, KIrchff law for nodes states, $i_1 + i_2 + i_4 + i_5 = i_3 + i_6$

$$\begin{cases} in \;\; i_1\,, i_3 \\ out \;\; i_2\,, i_4\,, i_5 \end{cases}$$

$$i_1 + i_3 = i_2 + i_4 + i_5$$

Kirchhoff's second rule (the loop rule) applies to potential differences. The loop rule is stated in terms of potential V rather than potential energy, but the two are related since $U = qV$. In a closed loop, whatever energy is supplied by a voltage source, the energy must be transferred into other forms by the devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit.

Kirchhoff's loop rule states that the algebraic sum of potential differences, including voltage supplied by the voltage sources and resistive elements, in any loop must be equal to zero.

1.1.1 Conventions

In order to apply Kirchoff's rules a sign convention must be established.

Citing Van Valkenburg[1], *A voltage source causes current to flow within the source in the direction from the negative to the positive or out terminal and into the negative terminal. This particular convention follows a decision made by Benjamin Franklin in 1752. Franklin's choice was made before electricity was identified with the electron, before the electron or the nature of charge were known. Actually electrons flow from the negative terminal to the positive terminal, which is in the opposite direction to that established by Franklin.*

To distinguish the two conventions, the flow of electrons is termed electron current and current assumed positive in the direction of Franklin's convention is called conventional current (or simply current, since this is the current we will use).

[1] NETWORK ANALYSIS, M. E. VAN VALKENBURG

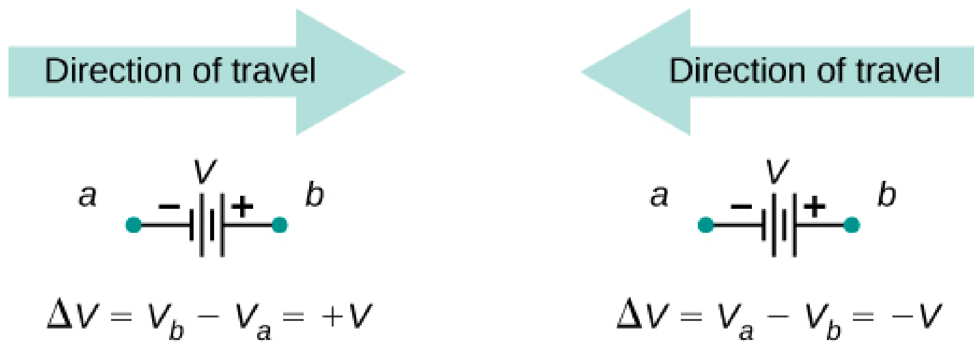

Figure 1.3: Sign conventions for voltage increase or drop across a voltage source

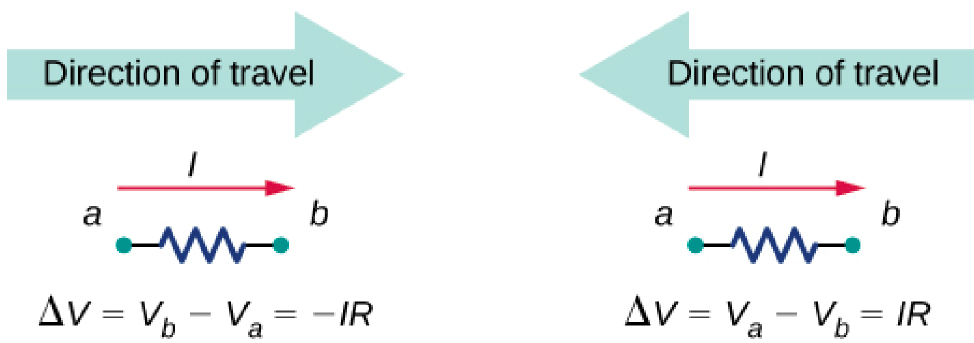

Figure 1.4: Sign conventions for voltage increase or drop when going through a resistance

If the negative terminal is used as a reference in measuring the potential of the positive terminal of a potential source, that voltage is considered positive and is spoken of as a voltage rise. Conversely, if the positive terminal is considered to be the reference in measuring the potential of the negative terminal of the voltage source, the voltage is considered negative and is spoken of as a voltage drop.

1.2 Examples

Example 1 *Let us begin by an extremely simple example. Figure 1.5 shows a one loop circuit with two voltage sources and three resistors. To make things easier each voltage increase or drop has been given a name and the direction of the current is clearly shown.*

If we walk the circuit beginning in the negative pole of v_a, the increases and drops of voltage are given by

$$v_a - v_1 - v_b - v_2 - v_3 = 0$$

which in terms of Ohm's law are:

$$\boxed{v_a - R_1 i - v_b - R_2 i - R_3 i = 0}$$

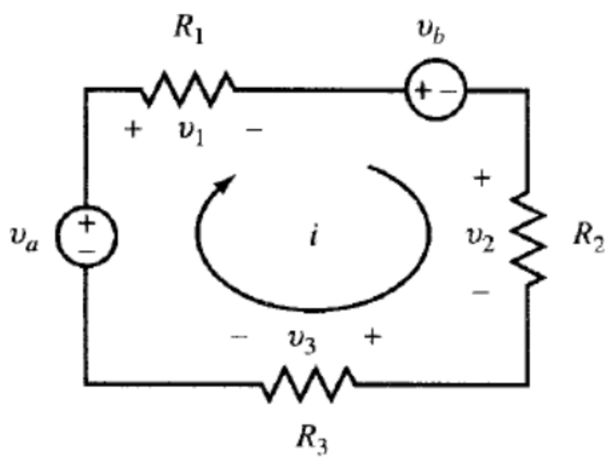

Figure 1.5: A simple one loop circuit with 5 circuit elements

Example 2 *Our second example, known as* **Voltage Divisor**, *shown in figure 1.6 is also extremely simple, but has many practical applications.*

$$v - R_1 i - R_2 i - R_3 i = 0$$

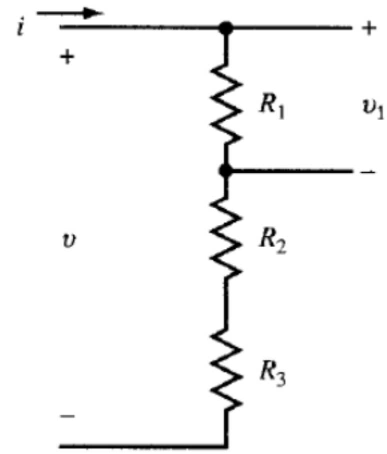

Figure 1.6: Voltage Divisor

The circuit has just one loop that goes through a voltage source and 3 resistances. The loop law implies

$$v = (R_1 + R_2 + R_3)\, i$$

the voltage drop at R_1 is simlpy

$$v_1 = R_1 i$$

and therefore,

$$v_1 = \frac{R_1}{R_1 + R_2 + R_3}\, v\,,$$

if all the resistances are equal.

$$v_1 = \frac{v}{3}\, v\,,$$

i.e. the circuit divides the source voltage in sections of one third of the source voltage.

What happens if we use N identical resistances?

Example 3 *Our next example, shown in figure 1.8is a two loop circuit.*

We first note that there are clearly three closed paths that can be followed in this circuit, namely $(abefa)$, $(ebcde)$ and $(facdf)$, where the notation is clear, we name a path by segments

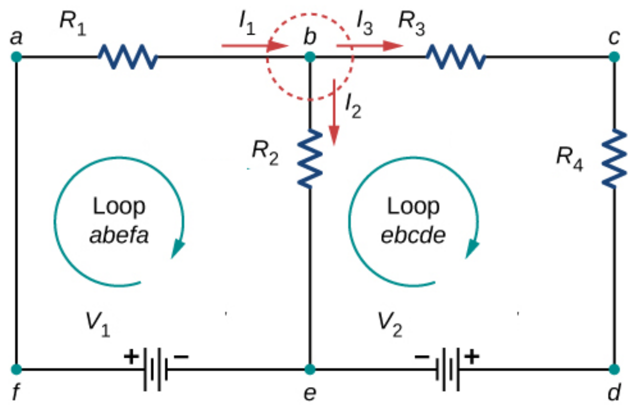

Figure 1.7: A two loop circuit with one voltage source

that begin and end at the referred nodes. It may be argued that there are many more paths, but, they are, either the same but traversed in the opposite sense or oaths where segments are traversed several times.

The circuit shows two interesting nodes where the current divides, namely b and e. At node b i_1 arrives, splits into two pieces i_2 and i_3 so

$$i_1 = 1_2 + i_3 \,,$$

while at node e i_3 and i_2 converge to leave the node as just one current, i_1, leading to

$$i_2 + i_3 = i_1 \,,$$

so we have no new information form this node.

Let us write the loop equations for the three loops

$$\begin{aligned} (abefa): &\qquad -R_1 i_1 - R_2 i_2 + v_1 = 0 \\ (ebcde): &\qquad R_2 i_2 - R_3 i_3 - R_4 i_3 - v_2 = 0 \\ (facdf): &\qquad -R_1 i_1 - R_3 i_3 - R_4 i_3 - v_2 + v_1 = 0 \end{aligned} \tag{1.1}$$

There is something very interesting here, the equation for the big loop ($facdf$) is the sum of the equations for the two small loops, meaning that the information coming form the big loop is superfluous, i.e. it is sufficient to think about the two small loops (or one small loop and the big one).

The circuit is finally described by a set of three linear equations with three unknowns, namely

$$\begin{aligned} i_i - i_2 - i_3 &= 0 \\ -R_1 i_1 - R_2 i_2 + v_1 &= 0 \\ R_2 i_2 - R_3 i_3 - R_4 i_3 - v_2 &= 0 \end{aligned} \tag{1.2}$$

Our problem is no longer physics, but math, which teaches us appropriate techniques to deal with these systems

Discussion Topic 1 *The theory of electric circuits or networks has been thoroughly studied and one of its main results has to do with the number of equations that are needed to completely describe a circuit. This is an elementary course and we will not pursuit this important topic, sufficient is to say that for a two loop circuit we need two loop equations and a constraint given by Kirchoff's node law.*

Example 4 **Current Divisor**

The current divisor is a two loop circuit parallel connected to a voltage source.

At node p there are 3 current, i which is incoming, i_1 that goes out through R_1 and say i_4 that goes to node q

At node q we also have 3 currents, i_4 that arrives, i_2 that goes to R_2 and i_3 that goes to R_3.

According to Kirchhoff's laws

$$i = i_1 + i_4\,, \qquad i_4 = i_2 + i_3$$

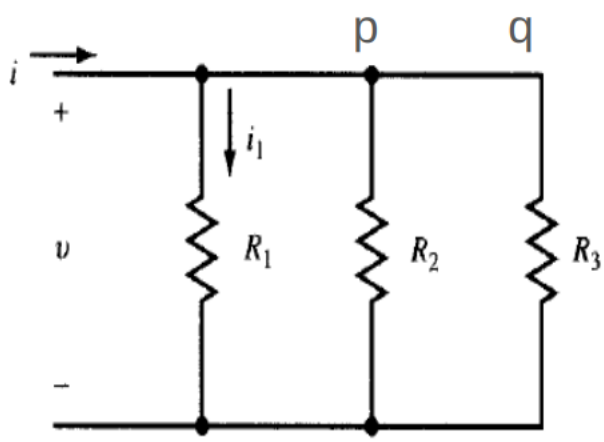

Figure 1.8: The current divisor

therefore

$$i = i_1 + i_2 + i_3\,,$$

but:

$$i_1 = \frac{v}{R_1}\,, i_2 = \frac{v}{R_2} \;\; and \;\; i_3 = \frac{v}{R_3}$$

so

$$\frac{i_1}{i} = \frac{v/R_1}{v(1/R_1 + 1/R_2 + 1/R_3)} = v\frac{1/R_1}{v(1/R_1 + 1/R_2 + 1/R_3)}$$

$$\boxed{\frac{i_1}{i} = \frac{R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}}$$

implying that if the resistances are all equal,

$$i_1 = \frac{i}{3}$$

Chapter 2

AC Current

Unlike direct current (DC), where the flow of electric charge is constant in magnitude and direction, alternating current (AC) is characterized by its periodic reversal in direction. This oscillatory behavior is crucial for the efficient transmission of electrical energy over long distances.

AC power sources, such as those found in homes and businesses, generate voltage that varies sinusoidally with time. This alternating voltage drives an alternating current through electrical circuits. The frequency of this alternation, typically 50 or 60 Hertz (Hz), determines the number of complete cycles per second. The key parameters describing AC waveforms include peak voltage, peak current, root mean square (RMS) voltage and current, and phase angle.

The widespread use of AC power is attributed to its advantages in power generation, transmission, and distribution. Transformers, devices that efficiently increase or decrease AC voltages, are essential components in the electrical power grid.

A typical AC voltage source can be represented by the following equation:

$$V(t) = V_m \, sin(\omega t + \phi)$$

Where:

V(t) is the instantaneous voltage at time t V_m is the peak voltage (amplitude) ω is the angular frequency (in radians per second), t is time and ϕ is the phase angle (in radians) This equation describes a sinusoidal waveform with a peak value of V_m, oscillating at a frequency

$$f = \frac{\omega}{2\pi}\, cycles/s\,,$$

and with an initial phase shift of ϕ radians.

An important concept in alternating current is that of **RMS Voltage**, *this is the effective value of AC voltage. RMS (Root Mean Square) voltage expresses the effectiveness of an alternating current (AC) voltage in delivering power to a load. It's essentially the equivalent DC voltage that would produce the same amount of heat in a resistor.*

For a sinusoidal AC voltage waveform, the RMS voltage (V_{rms}) is related to the peak voltage (V_m) by the following formula:

$$V_{rms} = \frac{Vp}{\sqrt{2}}$$

Example: If the peak voltage of an AC waveform is 100 volts, the RMS voltage would be approximately 70.7 volts ($\frac{\sqrt{2}}{2} 100\, V$).

RMS voltage is used in power calculations for AC circuits. Most AC voltmeters display RMS values. Understanding RMS voltage is crucial for working with AC power systems and electronics. By converting AC voltages to their RMS equivalents, we can apply DC power calculation formulas and obtain meaningful results for AC circuits.

2.1 Probs

1. *What power (in kW) is supplied to the starter motor of a large truck that draws 225 A of current from a 23.5 V battery hookup?*

We can use the formula for electrical power:

$$Power = Voltage \times Current$$

Substituting the given values:

$$P = 23.5\,V \times 225\,A = 5287.5\,Watts$$

To convert Watts to kilowatts, we divide by 1000, so

$$Power(kW) = 5287.5Watts/1000 = 5.2875\,kW$$

2. *A hair dryer has a switch that allows it to be used in the US (where the voltage is 120 V) and in Europe (where the voltage is 230 V). It accomplishes this by having two different resistance settings. When the switch is in the "120 V" position, the hair dryer uses a resistance value that allows it to operate at its rated power of 1520 W when connected to 120 V, and when the switch is in the "230 V" setting, the hair dryer uses a resistance that allows it to still operate at its rated power of 1520 W when connected to 230 V.*

   A tourist takes their 1520 W dual voltage hair dryer to Europe but forgets to set the switch to the "230 V" position. When they plug it into the 230 V receptacle and turn it on, it begins to smoke. What power does the hair dryer consume as a result of this mistake?

   Since power can be calculated by the formula

$$Power(P) = V^2/R\,,$$

   the resistance required for the operation under the desired conditions is

$$R = \frac{(120\,V)^2}{1520\,W} = 9.47\,\Omega$$

The power consumption at 230 V with the resistance just found is

$$P = \frac{(230\,V)^2}{9.47\,\Omega} = 5588.6\,Watts$$

or nearly $5.59\,kW$ *when incorrectly used in Europe.*

This significantly exceeds the rated power of the hair dryer, explaining why it would smoke due to overheating.

Note: This calculation assumes that the resistance of the hair dryer remains constant, which might not be entirely accurate due to factors like temperature dependence. However, it provides a good approximation of the increased power consumption.

3. *Two lightbulbs are each connected to a voltage of 120 V. One has a power of 25 W, the other 100 W.*

   Which bulb has the higher resistance?

   Which bulb carries more current? 25 W bulb 100 W bulb

   Which bulb has the higher resistance? The 25 W bulb has the higher resistance.

   We know that

$$Power = \frac{Voltage^2}{Resistance}\,.$$

   Since both bulbs have the same voltage (120V), the bulb with lower power must have higher resistance to compensate for the lower power output.

   The 100 W bulb carries more current. Indeed,

$$Power = Voltage \times Current\,.$$

   Since both bulbs have the same voltage, the bulb with higher power must have higher current to produce that higher power output.

4. *A battery of voltage V delivers power P to a resistor of resistance R connected to it. By what factor will the power delivered to the resistor change if the following changes are made? Give your answers as a factor of the original power P.*

(a) *The resistance is changed to 2.60R.*

We use the power formula: $P = V^2/R$

When the Resistance is changed to $2.60R$ *the power is changed to*

$$P' = \frac{V^2}{(2.60R)} = \frac{P}{2.60},$$

therefore, the power changes by a factor

$$F = \frac{P'}{P} = \frac{1}{2.60},$$

that is a power decreases by a factor of 1/2.60.

(b) *The voltage of the battery is now 2.60V, but the resistance is R.*

Voltage changed to 2.60V, resistance unchanged

$$New\ power = P' = (2.60V)^2/R = 6.76\,P$$

$$Factorchange = P'/P = 6.76,$$

Power increases by a factor of 6.76.

(c) *The resistance is 2.60R and voltage is 2.60V.*

$$New\ power = P' = (2.60V)^2/(2.60R) = 2.60\ P$$

$$Factorchange = P'/P = 2.60,$$

For a power increase of 2.60.

(d) Resistance changed to 2.60R and voltage changed to V/2.60

$$New\ power = P' = (V/2.60)^2/(2.60R) = P/(2.60)^3$$

$$Factor\ change = P'/P = 1/(2.60)^3$$

For a power decrease of $1/(2.60)^3$.

Summary of Results:

(a) Power decreases by a factor of 1/2.60.

(b) Power increases by a factor of 6.76.

(c) Power increases by a factor of 2.60.

(d) Power decreases by a factor of $1/(2.60)^3$.

5. *When an AC source is connected across a 16.0Ω resistor, the output voltage is given by*

$$V = 120\,Volt\,sin(80\pi t).$$

Determine the following quantities.

The delivered information consists on the resistance ($R = 16.0\,,\Omega$) *and the voltage source formula. From there we can read*

(a) Maximum Voltage (Vm), or amplitude is:

Vm = 120 V

(b) RMS Voltage (Vrms)

For the RMS voltage we just multiply the amplitude by the appropriate factor

$$Vrms = \frac{\sqrt{2}}{2}Vm \approx 84.85\,,V$$

(c) RMS Current (Irms)

This is trivially determined by using Ohms law on the rms voltage, i.e.

$$Irms = Vrms/R$$

$$Irms = 84.85V/16.0\Omega \approx 5.30A$$

(d) Peak Current (Ip).

Since the resistor is a linear component, the current and voltage waveforms will have the same shape. Therefore, the peak current is related to the RMS current by the same factor as the peak voltage to RMS voltage:

$$Ip = \sqrt{2}\, Irms$$

$$Ip = 5.30A\sqrt{2} \approx \ = 7.50\ A$$

(e) Current at t = 0.0045 s

First, we need to find the instantaneous voltage at t = 0.0045 s:

$$V = (120\, V)sin(80\pi 0.0045) \approx 97.98V$$

Then, using Ohm's law: $I = V/R$*,*

$$I = \frac{97.98\, V}{16.0\, \Omega} \approx 6.12A$$

Therefore, the current at t = 0.0045 s is approximately

6.12 A.

6. *An AC voltage source is connected to a resistor ($R = 1.70E2\,\Omega$). The resistor is connected to an AC voltage source whose output is given by the expression*

$$(1.00E2\,V) sin 2\pi f\,t.$$

(a) *What is the rms voltage across the resistor?*

Since the resistor is connected to the source and no more components are present, the voltage across the resitor is eqaul to the voltage of the source, therefore the RMS Voltage is calculated as:

$$V_{rms} = V_m/\sqrt{2}\,.$$

From the given voltage expression, we can read the peak voltage, $V_m = 170\,,V$.

Therefore,

$$V_{rms} = 170./\sqrt{2}\,V = 120.2\,V.$$

(b) *What is the rms current flowing through the resistor?*

7. *An AC voltage source and a resistor are connected in series to make up a simple AC circuit. If the source voltage is given by*

$$V(t) = V_0\,sin(2\pi f t)$$

and the source frequency is 15.7 Hz, at what time t will the current flowing in this circuit be 71.0% of the peak current?

The angular frequency of the source is

$$\omega = 2\pi f = 2\pi \times 15.7\,Hz \approx 98.96 rad/s$$

Since the circuit contains only a resistor, the current and voltage are in phase, this means that the current as a function of time is given by:

$$I(t) = I_{peak}\, sin(\omega t)$$

We want to find the time when

$$I(t) = 0.71 I_{peak}\,,$$

meaning that we want to solve the elementary trigonometric equation

$$0.71\, I_{peak} = I_{peak}\, sin(\omega t)$$

with solution

$$\omega t = arcsin(0.71)$$

wich in turn yields

$$t = \frac{arcsin(0.71)}{98.96\, rad/s} \approx 0.0072 s$$

so, the current will be 71% *of its peak value at approximately 7.2 milliseconds.*

Note: There are multiple solutions to the equation $sin(\omega t) = 0.71$ *within one period of the waveform. However, we've found the first instance where the current reaches* 71% *of its peak value.*

Appendix A

Description of the Course

LEARNING OUTCOMES

1. *Upon successful completion of this course, students should be able to:*
2. *Analyze the behavior of electric fields, electric potential and electric forces associated with electric charges.*
3. *Determine the current, voltage and power of AC and DC circuits that include power sources, resistances, capacitors, inductors and transformers.*
4. *Describe magnetic fields from currents and permanent magnets and determine the magnetic forces on moving charges and currents and determine the magnetic torque on a coil.*
5. *Apply Faraday's and Lenz's Laws to calculate induced currents and voltages.*
6. *Describe the properties of light waves and the geometric behavior of light rays associated with reflection, refraction, dispersion and total internal refraction.*

7. *Describe quantum mechanical phenomena including blackbody radiation, the photoelectric effect, the wave nature of matter and the uncertainty principle.*
8. *Determine emission and absorption spectra associated with atomic physics.*

COURSE TOPICS Planned physics topics for this course include:

1. *Electric Charge*
2. *Insulators, Conductors & Polarization*
3. *Electric Force*
4. *Electric Fields*
5. *Electric Field Lines*
6. *Conductors in Static Equilibrium*
7. *Uniformly Charged Spheres*
8. *Electric Potential*
9. *Equipotential Lines*
10. *Capacitors and Capacitance*
11. *Capacitors in Series & Parallel*
12. *Solving Capacitor Circuits*
13. *Electric Current*
14. *Resistance*

15. *Ohm's Law*

16. *Electric Power*

17. *AC vs DC Circuits*

18. *Resistances in Series/Parallel*

19. *Home Wiring and Electrical Hazards*

20. *Battery emf & Internal Resistance*

21. *Kirchhoff's Circuit Rules*

22. *Metering*

23. *RC Circuits*

24. *Magnetism*

25. *Magnetic Fields & Field Lines*

26. *Magnetic Force on a Charge*

27. *Motion of a Charge in a B-field*

28. *Magnetic Force on a Wire*

29. *Torque on a Current Loop & Motors*

30. *Magnetic Fields from Current*

31. *Magnetic Force Between Parallel Wires*

32. *Induced emf & Magnetic Flux*

33. Faraday's Law

34. Motional emf

35. Generators

36. Transformers

37. Inductance & Inductors

38. RL Circuits

39. Maxwell's Equations & EM Spectrum

40. Sources and Properties of EM Waves

41. Power and Intensity of EM Waves

42. Doppler Effect

43. Geometric Optics & Reflection

44. Index of Refraction & Snell's Law

45. Total Internal Reflection

46. Dispersion

47. Images by Thin Lenses

48. Images by Mirrors

COURSE MATERIALS