Magnetic Field

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Outline

A Tale of Physics

Imagine the following situation in a lab...

- **Leonard Hofstadter**, an experimental physicist, places a test charge q at rest. It remains at rest. (No electric force in play here!)
- Leonard then gives the charge a little push. As it moves, the charge begins to curve, but without gaining kinetic energy – meaning its speed does not change.
- Leonard claims: "Wow! There's definitely a force, but it's not an electric force!"
- Sheldon Cooper, the theoretician, adds: "This force must be perpendicular to the motion, that's why it does no work!"

- They repeat the experiment, giving the charge various speeds. They observe that as the speed increases, the radius of curvature of the path increases.
- Sheldon then proposes: "I postulate the existence of a new field, **B**, exerting a force on the charge. This force must be:"
    - Proportional to the charge q (because uncharged particles don't feel it).
    - Proportional to the speed v.
    - Perpendicular to both the velocity $\mathbf{v}$ and the field $\mathbf{B}$ itself.
- He then writes:

$$\boxed{\mathbf{F} = q(\mathbf{v} \times \mathbf{B})}$$

The Truth Behind the Tale: The Lorentz Force

- The "tale" of Leonard and Sheldon, while dramatized, captures the fundamental nature of a very real and crucial force in electromagnetism.
- This force, which acts on a moving charged particle in a magnetic field, is known as the **Lorentz Force**.
- It's named after the Dutch physicist **Hendrik Antoon Lorentz** (1853–1928), who provided its complete and formal mathematical formulation in the 1890s. Lorentz was a highly respected mentor and influential figure for Albert Einstein, providing crucial groundwork for Einstein's theory of relativity.

- The field **B** that exerts this force is most precisely called the Magnetic Flux Density or Magnetic Induction.
- While often colloquially referred to as the "Magnetic Field" (especially in contexts like vacuum or air where magnetization is negligible), **B** is **distinct** from the Magnetic Field Strength (or **H** field), which accounts for external currents and plays a vital role when magnetic materials are involved.

Units of Magnetic Flux Density (B)

- The SI unit for Magnetic Flux Density ($\mathbf{B}$) follows directly from the Lorentz force formula ($\mathbf{F} = q\mathbf{v} \times \mathbf{B}$), expressed as Force per (Charge $\times$ Velocity):

$$\text{Units of } \mathbf{B} = \frac{\text{Newton}}{\text{Coulomb} \cdot (\text{meter}/\text{second})} = \frac{\text{N}}{\text{C} \cdot \text{m}/\text{s}}$$

- For convenience, this derived unit is named the **tesla (T)**:

$$\boxed{1 \text{ Tesla (T)} = \frac{1 \text{ N}}{1 \text{ C} \cdot \text{m}/\text{s}}}$$

- Recalling that a Coulomb per second is an Ampere (1 A = 1 C/s), we can also write:

$$1 \text{ T} = \frac{1 \text{ N}}{1 \text{ A} \cdot \text{m}}$$

- An earlier (non-SI) unit for **B**, still in common use, is the **gauss (G)**:

$$1 \text{ Gauss (G)} = 10^{-4} \text{ Tesla (T)}$$

Approximate Magnetic Field Strengths

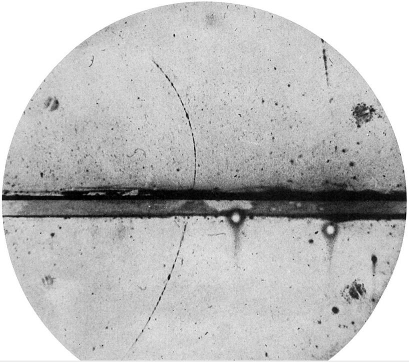
Magnetic fields vary enormously in strength depending on the source and location. Here are some approximate values for context:

Situation	**Magnetic Field (B)**
At surface of neutron star	10^{8} T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

- Note that Earth's magnetic field near the planet's surface is approximately 10^{-4} T, or 1 Gauss.

Charges Moving in Magnetic Fields

The POSITRON (yeah, antimatter!) was discovered on August 26, 1932 by Carl Anderson. The ion trail left on the photographic plate had a curvature matching the mass-to-charge ratio of an electron, but in a direction that showed its charge was positive



Let us briefly study the motion of a charged particle moving under the influence of a uniform magnetic induction.
Combining the Lorentz force formula with Newton' s 2^{nd} Law, splitting the velocity in its components parallel and perpendicular to the induction and using the properties of the vector product we find the acceleration

$$\mathbf{a} = -\left(\frac{q}{m}\mathbf{B}\right) \times \mathbf{v} = -\left(\frac{q}{m}\mathbf{B}\right) \times \left(\mathbf{v}_{\perp} + \mathbf{v}_{\parallel}\right) \tag{1}$$

$$\boxed{\mathbf{a} = -\left(\frac{q}{m}\mathbf{B}\right) \times \mathbf{v}_{\perp}} \tag{2}$$

This formula, in passing, clearly shows that the combination $q\mathbf{B}/m$ has units of reciprocal time

Since the magnetic induction is uniform we may choose the z axis to be oriented parallel to it so $\mathbf{B} = B\,\hat{\mathbf{e}}_z$ to write

$$\boxed{\begin{aligned} \frac{d\mathbf{v}_\perp}{dt} &= -\omega_c\,\hat{\mathbf{e}}_z \times \mathbf{v}_\perp \\ \omega_c &= \frac{q}{m} B \end{aligned}} \tag{3}$$

Here the quantity ω_c, is called **Cyclotron Frequency**

The Nature of the Motion: Why $\mathbf{v}_\perp$ Curves in a Circle

Let us carefully examine the consequences of the equation of motion

$$\frac{d\mathbf{v}_\perp}{dt} = \mathbf{a} = \omega_c(\mathbf{v}_\perp \times \hat{\mathbf{e}}_z)$$

- **1. Speed remains constant:**
  - The magnetic force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ is always perpendicular to the velocity vector $\mathbf{v}$.
  - When a force is perpendicular to motion, it does no work on the particle ($dW = \mathbf{F} \cdot d\mathbf{l} = 0$).
  - If no work is done, the particle's kinetic energy ($\frac{1}{2}mv^2$) does not change.
  - Implying that the magnitude of the velocity (the speed), $v_\perp = |\mathbf{v}_\perp|$, must remain constant.

The Nature of the Motion: Why $\mathbf{v}_\perp$ Curves in a Circle (Cont.)

- **2. Acceleration is purely centripetal (radial inward):**
  - e clearly established that the acceleration **a** is always perpendicular to $\mathbf{v}_\perp$
  - When an object moves with constant speed and its acceleration is always perpendicular to its velocity, the only type of motion possible is uniform circular motion.
  - In uniform circular motion, the velocity vector $\mathbf{v}_\perp$ is always tangential to the circular path.
  - The acceleration vector **a** always points towards the center of the circle (it's a centripetal acceleration). This means it has no component that would make the particle move further out or further in.

The Nature of the Motion: Why $\mathbf{v}_\perp$ Curves in a Circle (Cont.)

We have thus, rigurously proved that the velocity component $\mathbf{v}_\perp$ describes motion entirely in the plane perpendicular to $\mathbf{B}$, in a circle, with no radial component.

Just for the fun of it, let us calculate the radius of the circular motion of a charge that enters a magnetic field perpendicular to it. Since the motion is circular and there is only and one force only, we equate the force to the centripetal force

$$m\frac{v_\perp^2}{r} = qBv_\perp$$

yielding

$$\boxed{r = \frac{m\, v_\perp}{qB}}$$

A good exercise in physics is to discuss this result.

Representing Magnetic Fields: Field Lines

Just like electric fields, we can visualize magnetic fields using field lines (also known as magnetic flux lines).

Similar rules apply to magnetic field lines as to electric field lines:

- **1. Direction of B:** The tangent to a magnetic field line at any given point indicates the direction of the magnetic flux density (**B**) at that specific point.
- **2. Magnitude of B (Strength):** The spacing of the lines represents the magnitude of **B**.
  - Where the lines are closer together, the magnetic field is stronger.
  - Conversely, where they are further apart, the field is weaker.

Magnetic Field Lines of a Bar Magnet

The magnetic field lines near a bar magnet (a common permanent magnet) illustrate key properties:

- The lines all pass through the magnet itself.
- They all form closed loops. (This is a fundamental difference from electric field lines, which start and end on charges.)
- The external magnetic effects are strongest near its ends (the poles), where the field lines are most closely spaced.
- This is empirically demonstrated by how the magnetic field collects iron filings mainly near the two ends of the magnet (refer to Figure 28.1.4b).

Closed Loops and the Search for Magnetic Monopoles

A fundamental characteristic of magnetic field lines is that they always form closed loops. Unlike electric field lines, which originate from positive charges and terminate on negative charges, magnetic field lines have no beginning or end.
This observation has the following profound implication:

No Magnetic Monopoles:

The fact that magnetic field lines always close suggests that, unlike electric charges, there are **no isolated magnetic "charges" or "poles"** (magnetic monopoles).

If magnetic monopoles existed...

Magnetic field lines would originate from a "north monopole" and terminate on a "south monopole," similar to how electric field lines behave.

Decades-long Search

Despite extensive searches using highly sensitive instruments worldwide, physicists have never found definitive evidence of magnetic monopoles. Their existence remains hypothetical.

Magnetic Field Lines are Always Closed

The Mathematical Expression of The Absence of Monopoles

The **fundamental absence of magnetic monopoles** is expressed by the Gauss's Law for Magnetism:

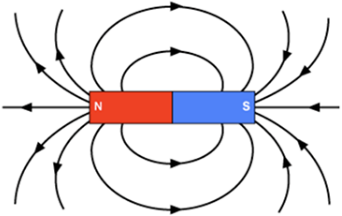
$$\boxed{\oint_S \mathbf{B} \cdot d\mathbf{A} = 0}$$

This integral states that the net magnetic flux through any closed surface (S) is always zero, meaning just as much magnetic field enters the surface as leaves it.

An Important Convention

Memorize

Magnetic field lines depart from north poles and converge in south poles



Electric Currents: The Source of Magnetic Fields

So far, we've discussed how magnetic fields exert forces on moving charges (the Lorentz force). But where do these magnetic fields come from?

- Historically, it was Hans Christian Ørsted who, in 1820, first observed that an electric current in a wire could deflect a compass needle. This groundbreaking discovery established the direct link between electricity and magnetism
- Lovely video from Kathy Loves Physics.

- Ørsted discovery means that moving electric charges (i.e., electric currents) are the fundamental source of magnetic fields.
- Just as stationary charges create electric fields, moving charges create both electric and magnetic fields.
- Our next goal is to quantify how a given current distribution produces a magnetic field $\mathbf{B}$ in the surrounding space.

Introducing the Biot-Savart Law: Conceptual Basis

The **Biot-Savart Law** is a fundamental equation in magnetostatics that allows us to calculate the magnetic field $\mathbf{B}$ generated by a steady electric current.

- It is analogous to Coulomb's Law in electrostatics, which calculates electric fields from charges.
- Instead of point charges, the Biot-Savart Law considers the magnetic field produced by a small segment of current, called a **current element**, $I\,d\mathbf{l}$.

The infinitesimal magnetic field $d\mathbf{B}$ produced by this current element at a point P:

- Is **proportional to the current (I)** and the **length ($d\mathbf{l}$)** of the current element.
- Is **inversely proportional to the square of the distance (r^2)** from the current element to point P.
- Depends on the direction of the current element relative to the position vector from the element to point P (given by a cross product).

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Biot-Savart Law

Analogous to Coulomb's Law in electrostatics, which calculates electric fields from charges.

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Biot-Savart Law

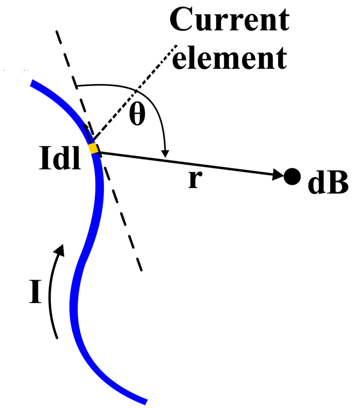
The Biot-Savart Law precisely describes the infinitesimal magnetic field $d\mathbf{B}$ produced at a point P by an infinitesimal current element $I\,d\mathbf{l}$:

$$\boxed{d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\, d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}} \quad \text{or} \quad \boxed{d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\, d\mathbf{l} \times \mathbf{r}}{r^3}} \tag{4}$$

Here:

- $d\mathbf{B}$:Is the infinitesimal magnetic field vector at point P.
- μ_0: The permeability of free space, is a fundamental constant. $\mu_0 = 4\pi \times 10^{-7}$ T $\cdot$ m/A (or N/A^2).
- I: The magnitude of the current in the wire and $d\mathbf{l}$ the infinitesimal vector length element along the direction of the current.
- $\mathbf{r}$ is the position vector from the current element to P, the point where the field is being calculated. The magnitude of $\mathbf{r}$ is r.
- The unit vector in the direction of $\mathbf{r}$ (i.e., $\mathbf{r}/r$) is denoted by $\hat{\mathbf{r}}$: .

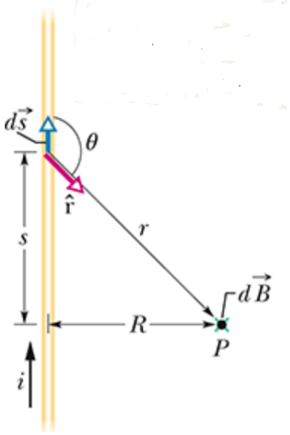
Biot Savart Law



$$\boxed{d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I \, d\mathbf{l} \times \mathbf{r}}{r^3}} \qquad (5)$$

To find the total magnetic field $\mathbf{B}$ from a current distribution, we integrate $d\mathbf{B}$ over the entire current path.

The Infinite Current Carrying Linear Cable



In the notation of the figure,

$$d\vec{s} = ds\,\hat{\mathbf{e}}_z \tag{6}$$

$$\mathbf{r} = R\,\hat{\mathbf{e}}_\rho - s\,\hat{\mathbf{e}}_z \tag{7}$$

$$r^3 = \left[R^2 + s^2\right]^{3/2} \tag{8}$$

Where $\hat{\mathbf{e}}_z$ is a unit vector oriented along the cable, and $\hat{\mathbf{e}}_\rho$ a radial unit vector perpendicular to the z axis and pointing outwards (in the direction of increasing R)

We now substitute the information of the previous slide into Biot and Savart law to get

$$d\mathbf{B} = \frac{\mu_o\, I}{4\pi} \frac{ds\,\hat{\mathbf{e}}_z \times [R\,\hat{\mathbf{e}}_\rho - s\,\hat{\mathbf{e}}_z]}{[R^2+s^2]^{3/2}} \tag{9}$$

now, $\hat{\mathbf{e}}_z \times \hat{\mathbf{e}}_z = 0$ and $\hat{\mathbf{e}}_z \times \hat{\mathbf{e}}_\rho = \hat{\mathbf{e}}_\phi$ where $\hat{\mathbf{e}}_\phi$ is a unit vector entering the slide perpendicularly to it.

In brief

$$\boxed{d\mathbf{B} = \frac{\mu_o\, I\, R}{4\pi}\,\hat{\mathbf{e}}_\phi\,\frac{ds}{[R^2+s^2]^{3/2}}} \tag{10}$$

To find **B** we must add all the contributions and this means integrating form $s = -\infty$ to $s = +\infty$

We are then interested in

$$\boxed{I = \int_{-\infty}^{\infty} \frac{ds}{[R^2 + s^2]^{3/2}}} \tag{11}$$

This integral is quite easy, make the substitution $s = R\,tan(\alpha)$ so $du = R\,sec^2\alpha\,d\alpha$ and $R^2 + s^2 = R^2(+tan^2\alpha) = R^2\,sec^2\alpha$ while the limits change to $\alpha = -\pi/2$ and $\alpha = \pi 2$

Upon substitution,

$$\boxed{I = \frac{1}{R^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cos\alpha\,d\alpha = \frac{1}{R^2} sin\alpha\Big|_{\frac{\pi}{2}}^{-\frac{\pi}{2}} = \frac{2}{R^2}} \tag{12}$$

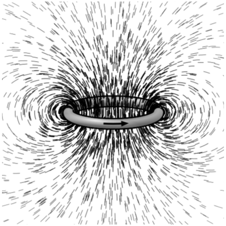
The magnetic field produced by an *infinite* cable carrying a current I is proportional to the current and varies as the reciprocal of the distance to the cable and its lines are circles in the plane perpendicular to the cable oriented according the right hand rule with respect to the current

$$\boxed{\mathbf{B} = \frac{\mu_o\, I}{2\pi R}\,\hat{\mathbf{e}}_\phi} \tag{13}$$

This result is extremely compact, elegant and simple. In addition, it has striking similarities with the electric field produced by a long line of charge but shows an important difference with such a field, which is radial while the magnetic induction is tangential

The Ring of Current

A ring or loop carrying a current I produces an extremely interesting magnetic field configuration. If the curent is high an the area of the loop is small the fields lines closely resemble the lines of an ideal dipole.



To avoid mathematical difficulties we will calculate the field along the axis of a current loop

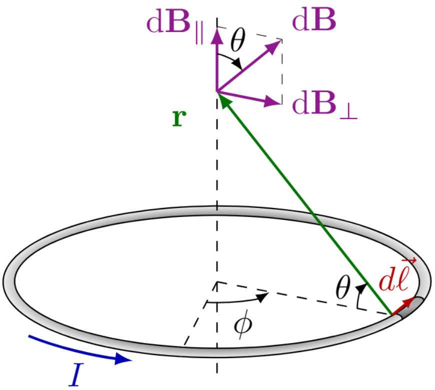
Using the figure,

$$d\vec{\ell} = Rd\phi\,\hat{\mathbf{e}}_\phi \tag{14}$$

$$\mathbf{r} = -R\,\hat{\mathbf{e}}_\rho + z\,\hat{\mathbf{e}}_z \tag{15}$$

$$r^3 = \left[R^2 + z^2\right]^{3/2} \tag{16}$$

Where $\hat{\mathbf{e}}_\phi$ is the unit vector oriented along the loop in the direction of increasing ϕ, and $\hat{\mathbf{e}}_\rho$ is the usual radial unit vector



The contribution to the induction caused by the infinitesimal piece of current is

$$\begin{aligned} d\mathbf{B} &= \frac{\mu_o I}{4\pi} \frac{R\, d\phi \hat{\mathbf{e}}_\phi \times (-R\,\hat{\mathbf{e}}_\rho + z\,\hat{\mathbf{e}}_z)}{(R^2+z^2)^{3/2}} = \\ &= \frac{\mu_o I}{4\pi} \left[\frac{R^2\, d\phi\, \hat{\mathbf{e}}_z}{(R^2+z^2)^{3/2}} + \frac{zR\, d\phi\, \hat{\mathbf{e}}_\rho}{(R^2+z^2)^{3/2}} \right] \end{aligned} \tag{17}$$

The vector in blue is what in the figure is referred to as $d\mathbf{B}_\perp$, which we know -by symmetry arguments-, will not contribute to $\mathbf{B}$.

The integration on ϕ from 0 to 2π renders the contribution parallel to the plane of the loop nil, the component along the z axis, on the other hand is

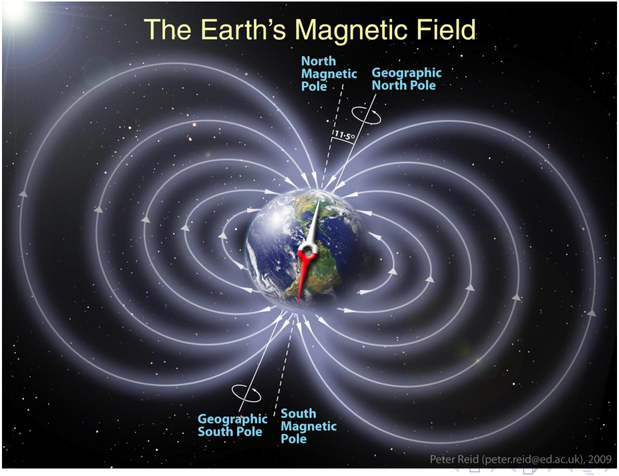
$$\mathbf{B} = \frac{\mu_o I}{2\pi} \frac{\pi R^2}{(R^2 + z^2)^{3/2}} \hat{\mathbf{e}}_z = \frac{\mu_o}{2\pi} \frac{IA}{(R^2 + z^2)^{3/2}} \hat{\mathbf{e}}_z \qquad (18)$$

Where $A = \pi R^2$ is the area enclosed by the ring. The magnetic moment of the loop is usually defined as $\mu = IA$, and so, for $R << z$ the magnetic field along the axis of the loop is

$$\mathbf{B} = \frac{\mu_o}{2\pi} \frac{\mu}{z^3} \hat{\mathbf{e}}_z \qquad (19)$$

These Calculations are hard, we need ...

Our planet has a magnetic field that ranges between 0.25 to 0.65 Gauss, but we must always recall that our North Magnetic Pole is Magnetically South!



This field protects us from harmful cosmic rays and solar winds that would otherwise make life on Earth close to impossible, and it helps us in many other ways.

- **Paleomagnetism and Geochronology:** Dating geological formations (like rocks, sediments, and archaeological artifacts) by analyzing their remanent magnetization, which records the Earth's ancient magnetic field at the time of their formation. This is crucial for understanding plate tectonics, continental drift, and past environmental changes.
- **Hydrocarbon and Mineral Exploration:**
  - Detailed mapping of large sedimentary basins to identify structural features (faults, folds) and basement topography, which are key in petroleum and natural gas exploration.
  - Direct and indirect detection of various mineral deposits (e.g., iron ore, nickel, copper, kimberlites for diamonds) by identifying their distinct magnetic signatures or the magnetic characteristics of associated host rocks.

- **Geophysical Surveying and Hazard Assessment:**
  - Mapping geological structures and subsurface features for engineering purposes, groundwater exploration, and environmental studies (e.g., locating buried utilities, unexploded ordnance, or landfill boundaries).
  - Monitoring volcanic activity and earthquake precursors, as changes in stress within the Earth's crust can sometimes induce detectable magnetic field variations.
- **Space Weather and Navigation:**
  - Understanding and forecasting space weather events (like solar flares and coronal mass ejections) that interact with the Earth's magnetic field, impacting satellites, power grids, and communication systems.
  - Enhancing navigation systems (e.g., compasses, GPS augmentation) by providing accurate models of the Earth's main magnetic field and its variations.

- **Archaeological Prospection:** Locating and mapping buried archaeological features (like kilns, hearths, walls, ditches) that have distinct magnetic properties due to heating or human activity.


LECTURE SUMMARY
- PHIESOPHENT LIKE YOU PHILSOMENS NEVET ARE PHEDANT
- PEGROBES YOUR SACKEST FLUNT
- COOR INY ANGHANDAY THE
- COPFRY
PHILESHPERT

- **Lorentz Force:** Describes the force on a moving charge (q) in a magnetic field ($\mathbf{B}$): $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. It's why charges curve without changing speed.
- **Biot-Savart Law:** Calculates the magnetic field ($\mathbf{B}$) generated by electric currents ($I\, d\mathbf{l}$). The fundamental source of magnetism.
- **Magnetic Field Lines:** Always form **closed loops**. This fundamental property implies the absence of magnetic monopoles (expressed by Gauss's Law for Magnetism).
- **Ampere's Law:** Connects electric currents to the circulation of magnetic fields.
- **Earth's Magnetic Field:** Vital protection from cosmic rays, and a valuable tool in geophysics.