Elementary Electrostatics Coulomb' s Law

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July 23, 2025

A Long Journey to Understanding

- Early human observations of natural phenomena:
  - Lightning.
  - Electric fish.
- The first systematic *scientific* observations are attributed to the Ancient Greeks.
- **Thales of Miletus** (c.626/623- c.548/545 BC. 6th century BCE):
  - Noticed that when **amber** (fossilized tree resin) was rubbed with fur or cloth, it gained the ability to attract light objects like feathers or straw.
  - This was the earliest recorded investigation into what we now call static electricity.

Thales, First Philosopher/Scientist

Urania, the Muse of Astronomy Reveals to Thales the Secrets of the Skies. Antonio Canova 1798-1799

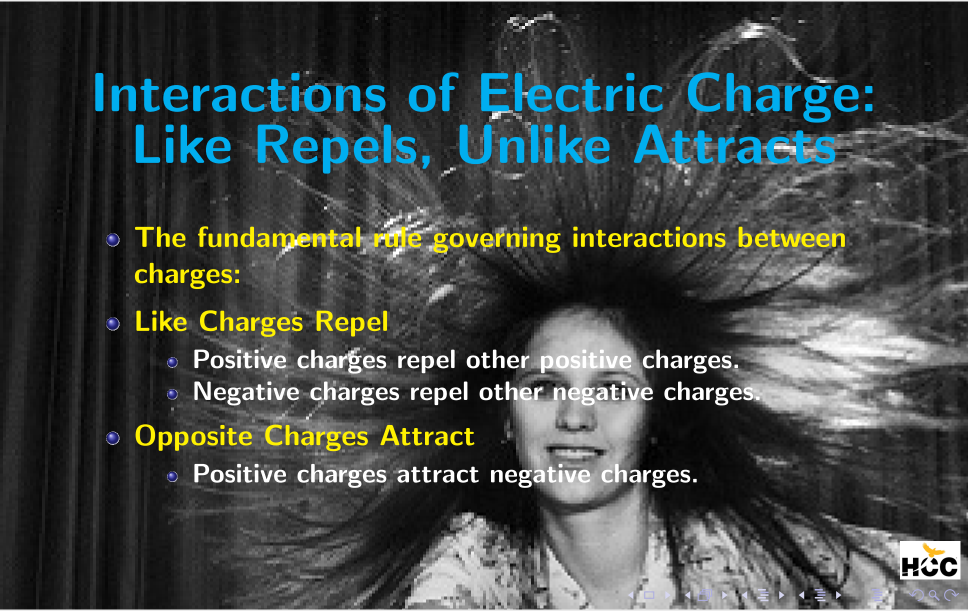
- Proposed natural explanations for phenomena, moving beyond mythological ones.
- Credited with predicting a solar eclipse (likely the one of 585 BCE).
- Used constellations (like Ursa Minor) for navigation.
- Credited with several fundamental geometric theorems, including: Thales's Theorem (Intercept Theorem): If two intersecting lines are cut by two parallel lines, then the corresponding segments on the intersecting lines are proportional. (Used to measure height of pyramids).

The Etymology of “Electricity”

- The ancient Greek word for amber is $\eta\lambda\epsilon\kappa\tau\rho\text{ø}\nu$ (*elektron).*
- *Centuries later, William Gilbert (17th century) used the Latin term "electricus" (meaning "like amber") to describe this attractive property.*
- *This gave rise to our modern terms: "electricity" and "electron".*

The Fundamental Property: Electric Charge

- An intrinsic fundamental property of matter.
- Responsible for electric and magnetic phenomena.
- Exists in two types:
  - Positive (+)
  - Negative (-)
- The SI unit of electric charge is the Coulomb (C).



Interactions of Electric Charge: Like Repels, Unlike Attracts

- The fundamental rule governing interactions between charges:
- Like Charges Repel
  - Positive charges repel other positive charges.
  - Negative charges repel other negative charges.
- Opposite Charges Attract
  - Positive charges attract negative charges.

Key Concept

These attractive and repulsive forces are described (quantified) by Coulomb's Law.

Key Concept

These forces are fundamental for the existence of Ordinary Matter (Atoms and Molecules).
Chemical bonding, for instance, is a direct consequence of electromagnetic interactions between the charged constituents (nuclei and electrons) of the atoms, with a primary role played by the valence electrons in the outermost shells.

The Discrete Nature of Charge: Quantization

- Electric charge is **quantized**.
  - It exists only in discrete, indivisible units.
  - It does not exist in arbitrary fractional amounts.
- The smallest unit of charge is the **elementary charge**, denoted by e.
  - $e \approx 1.602 \times 10^{-19}$ Coulombs (C).
- All observable charges are integer multiples of e.
  - For any charged object, its total charge $Q = \pm Ne$, where N is an integer.
- **Charge Carriers:**
  - Electron: Charge of $-e$
  - Proton: Charge of $+e$

Beyond Elementary: The Curious Case of Quarks

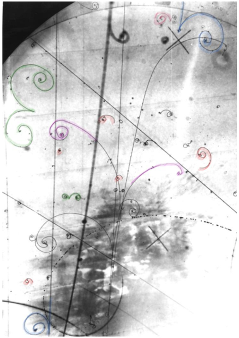
- While the elementary charge e is the smallest observed free unit of charge, there's a fascinating exception:
- **Quarks:** Fundamental particles that make up protons, neutrons, and other hadrons.
- Quarks carry **fractional electric charges**:
  - Up (u), Charm (c), Top (t) quarks have charge $+2/3e$.
  - Down (d), Strange (s), Bottom (b) quarks have charge $-1/3e$.

More on Quarks

Why this doesn't contradict quantization for observable charges:

- Quarks are never found in isolation; they are always **confined** within composite particles (e.g., protons are uud, neutrons are udd).
  - The total charge of any such composite particle (and any free particle) is always an integer multiple of e.
  - Example: Proton charge $(uud) = (+2/3e) + (+2/3e) + (-1/3e) = +e$.
  - Example: Neutron charge $(udd) = (+2/3e) + (-1/3e) + (-1/3e) = 0$.

Understanding Electric Interactions



This bubble chamber picture shows some electromagnetic events such as pair creation or materialization of high energy photon into an electron-positron pair (green tracks), the Compton effect (red tracks), the emission of electromagnetic radiation by accelerating charges (violet tracks) (bremsstrahlung) and the knock-on electrons or delta ray (blue tracks)

Early Explorations into Electric Phenomena

- Ancient observations of "static electricity" (e.g., rubbing amber).
- Recognition of attractive and repulsive forces between charged objects.
  - Like charges repel.
  - Unlike charges attract.
- Early qualitative understanding, but lacking a precise quantitative law.

Charles-Augustin de Coulomb and His Torsion Balance

- French physicist Charles-Augustin de Coulomb (1736-1806).
- Dedicated to quantifying the force between electric charges.
- The challenge: Measuring very weak forces accurately.
- His ingenious solution: The torsion balance.

Charles-Augustin de Coulomb.

The Torsion Balance: Precision in Electrostatics

- A device used to measure very weak forces.
- Principle: Force causes a known twist in a thin wire, which can be precisely measured.
- Coulomb used it to measure the force between small charged spheres.
- Allowed for the first quantitative investigation of electrostatic forces.

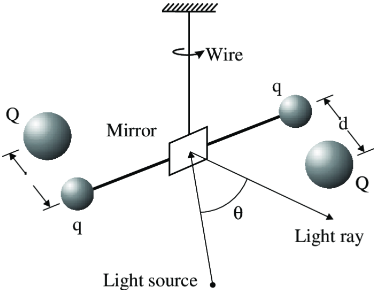


Diagram of Coulomb's torsion balance.

A Parallel Pursuit: Henry Cavendish and Gravitation

- British scientist Henry Cavendish (1731-1810).
- Famous for the "Cavendish Experiment" (1798).
- Used a very similar torsion balance apparatus.
- Purpose: To measure the gravitational constant (G) and the Earth's density.
- **Parallel:** Both Coulomb and Cavendish leveraged the precision of the torsion balance for fundamental force measurements.
- Cavendish also performed electrostatic experiments but didn't publish as formally or widely.

Quantifying Electric Force: Coulomb's Law

- Based on his precise measurements, Coulomb established:
- The force (F) between two point charges:
  - is directly proportional to the product of the magnitudes of the charges ($q_1 q_2$).
  - is inversely proportional to the square of the distance (r^2) between them.
  - Is directed along the line joining the two particles

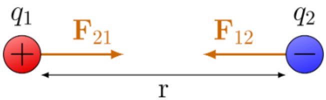


Figure: Forces between two opposite charges

Coulomb's force that charge q_1 exerts on q_2

In the international system of units

$$\begin{aligned} \mathbf{F}_{12} &= \kappa \frac{q_1 q_2}{r^2}\, \hat{\mathbf{u}}_{12} \\ \kappa &= \frac{1}{4\pi\epsilon_0}\,, \epsilon_0 : \text{ permitivity of empty space} \end{aligned} \tag{1}$$

- $\kappa \approx 8.987 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
- q_1, q_2: electric charges
- r: Distance between the charges
- $\hat{\mathbf{u}}_{12}$ unit vector pointing from q_1 to q_2

The Principle of Superposition: Definition

- When multiple electric charges are present, the net electrostatic force exerted on any one charge is the **vector sum** of the individual electrostatic forces exerted on it by all the other charges.
- **Key Insight:** The force between any pair of charges is entirely independent of the presence of any other charges in the system.
    - This means charges do not "screen" or interfere with the force between other charges.
- This principle simplifies the calculation of forces in complex multi-charge systems.

Formal Statement

If there are n charges $q_2, q_3, \ldots, q_n$ exerting forces $\vec{F}_{21}, \vec{F}_{31}, \ldots, \vec{F}_{n1}$ on a charge q_1, then the net force on q_1 is:

$$\vec{F}_{\text{net},1} = \vec{F}_{21} + \vec{F}_{31} + \ldots + \vec{F}_{n1}$$

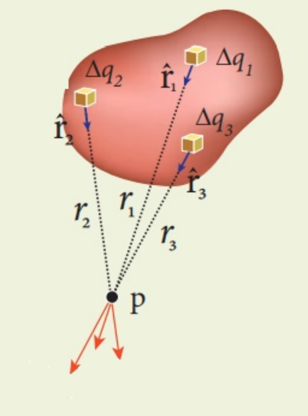
$$\vec{F}_{\text{net},1} = \sum_{i=2}^{n} \vec{F}_{i1} \tag{2}$$

(where $\vec{F}_{i1}$ is the force on q_1 due to q_i)

Remember!

Force is a vector quantity. Direction is crucial for vector addition.

Distributed Charges



- Charges may be continuously distributed (density)
- When that happens, the sum in thew superposition principle (formula 2 becomes an integral.

$$\mathbf{F}(P) = \frac{q_P}{4\pi\epsilon_0} \int \frac{dq\,\hat{\mathbf{r}}_{dq,P}}{r^2_{dq,P}}$$

Where, dq is the charge element, and $\mathbf{r}_{dq,P}$ the vector that joins the charge element with the charge at point P

Fundamental Principle

"In any isolated system, the net electric charge remains constant."

- This means that electric charge can neither be created nor destroyed.
- It can only be transferred from one object or location to another.
- The total amount of positive charge minus the total amount of negative charge in a closed system never changes.
- Analogous to the conservation of energy or momentum.

Conservation of Charge: Examples

Examples of Charge Transfer:

- **Rubbing a balloon on hair:** Electrons are transferred from hair to balloon, making the balloon negatively charged and the hair positively charged. The total charge of the hair+balloon system remains zero.
- **Pair Production/Annihilation:** In nuclear physics, an electron and a positron (anti-electron) can be created from energy, or annihilate back into energy. In both processes, the net charge is conserved (0 for both sides).

Conservation of Charge: Significance

- Essential for understanding electrical circuits, static electricity, and all electromagnetic interactions.
- Explains why current flows in a closed loop.
- A cornerstone of classical and quantum electrodynamics.

  NO experiment has ever shown a violation of charge conservation.

How Strong Is The Electromagnetic Interaction

Observation

In almost every general audience science books or tv documentaries dealing with modern physics one hears that the gravitational interaction is much weaker than the electromagnetic one.

Exercise: To understand the observation above, compare the gravitational force between two electrons with the corresponding Coulomb force. Expressed in equations, calculate the ratio

$$\frac{F_e}{F_g}$$

Quantifying the Force Difference: Electrons as a Case Study

- We will consider the two electrons as separated by an arbitrary distance r.

1. Electromagnetic Force (F_e)

For two electrons:

$$F_e = \kappa \frac{e^2}{r^2}$$

2. Gravitational Force (F_g)

For two electrons:

$$F_g = G \frac{m_e^2}{r^2}$$

- **Ratio of Forces:**

$$\frac{F_e}{F_g} = \frac{\kappa \frac{e^2}{r^2}}{G \frac{m_e^2}{r^2}} = \frac{\kappa}{G} \frac{e^2}{m_e^2}$$

Notice that the distance r cancels out, demonstrating that this ratio is fundamental and independent of separation!

Remember the Constants (in SI)

- Elementary charge, $e \approx 1.602 \times 10^{-19}$ C
- Electron mass, $m_e \approx 9.109 \times 10^{-31}$ kg
- Coulomb's constant, $k \approx 8.987 \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$
- Gravitational constant, $G \approx 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

- Next Step Plug in the values for the constants:

$$
\begin{aligned}
\frac{F_e}{F_g} &= \frac{8.987\times 10^{9}\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2}{6.674\times 10^{-11}\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2}\times\frac{(1.609\times 10^{-19}\mathrm{C})^2}{(9.109\times 10^{-31}\ \mathrm{kg})^2} = \\
&= \frac{(8.987\times 10^{9})\times(2.566\times 10^{-38})}{(6.674\times 10^{-11})\times(8.297\times 10^{-61})} = \\
&= \frac{2.306\times 10^{-28}}{5.537\times 10^{-71}}
\end{aligned}
$$

$$
\boxed{\frac{F_e}{F_g}\approx 4.165\times 10^{42}}
$$

The Astonishing Result

The electromagnetic force between two electrons is approximately 4.165×10^{42} times stronger than the gravitational force between them.

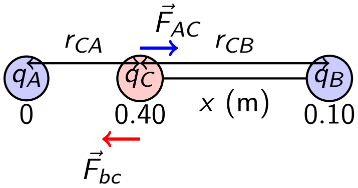
- This colossal difference highlights why gravity is only significant for objects with immense mass, whereas electromagnetism dominates interactions at the atomic and molecular scales.

Forces on a Charge Between Two Others

Two point charges, $q_A = +5.0 \times 10^{-6}$ C ($+5.0\,\mu$C) and $q_B = -3.0 \times 10^{-6}$ C ($-3.0\,\mu$C), are placed on the x-axis. q_A is at $x = 0$ and q_B is at $x = 0.40$ m. A third charge, $q_C = +6.0 \times 10^{-6}$ C ($+6.0\,\mu$C), is placed at $x = 0.10$ m.

What is the net electrostatic force on charge q_C?

The situation is depicted in the figure below



Our first step is finding the relevant distances

$$r_{CA} = 0.10 \text{ m} - 0 \text{ m} = 0.10 \text{ m}$$
$$r_{CB} = 0.40 \text{ m} - 0.10 \text{ m} = 0.30 \text{ m}$$

The forces that charges q_A and q_B exert on charge q_C are

$$\mathbf{F}_{AC} = k\frac{q_A q_C}{r_{CA}^2}\hat{\mathbf{e}}_x = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times$$
$$\frac{(+5.0 \times 10^{-6})(+6.0 \times 10^{-6})\ C^2}{(0.10)^2\ m^2}\,\hat{\mathbf{e}}_x =$$
$$= (8.99 \times 10^9)\frac{30.0 \times 10^{-12}}{0.01}\ \text{N}\ \hat{\mathbf{e}}_x = 26.97\ \text{N}\ \hat{\mathbf{e}}_x$$

Where $\hat{\mathbf{e}}_x$ is the unit vector directed from charge q_A to charge q_C ($\mathbf{F}_{AC}$ is repulsive)

Similarly

$$\mathbf{F}_{CB} = k\frac{q_B q_C}{r_{CB}^2}\hat{\mathbf{e}}_x = (8.99 \times 10^9 \ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)\times$$
$$\frac{(-3.0 \times 10^{-6})(6.0 \times 10^{-6})\ C^2}{(0.30)^2\ m^2}\hat{\mathbf{e}}_x =$$
$$= -(8.99 \times 10^9)\frac{18.0 \times 10^{-12}}{0.09}\ \mathrm{N}\,\hat{\mathbf{e}}_x = -1.798\ \ \mathrm{N}\,\hat{\mathbf{e}}_x$$

$\mathbf{F}_{BC}$ is antiparallel to $\hat{\mathbf{e}}_x$ and is therefore, attractive

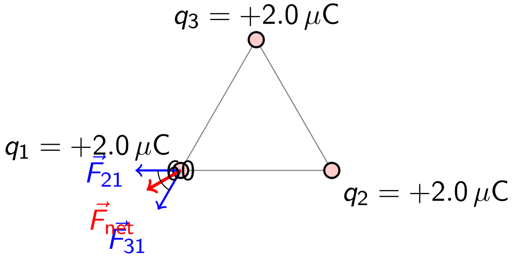
The net force is simply [Superposition Principle], the vector addition of the two forces, i.e.

$$\mathbf{F}_{\mathrm{net}} = \mathbf{F}_{AC} + \mathbf{F}_{BC}$$
$$\mathbf{F}_{\mathrm{net}} = (26.97\ \mathrm{N})\,\hat{\mathbf{e}}_x + (-1.798\ \mathrm{N}\,\hat{\mathbf{e}}_x) =$$
$$= (26.97 - 1.798)\,\hat{\mathbf{e}}_x = (25.172\ \mathrm{N})\,\hat{\mathbf{e}}_x$$
$$\boxed{\mathbf{F}_{\mathrm{net}} = 25.172\ \mathrm{N}\,\hat{\mathbf{e}}_x}$$

Charges on a Triangle

Three equal positive point charges, $q_1 = q_2 = q_3 = +2.0 \times 10^{-6}$ C ($+2.0\,\mu$C), are located at the vertices of an equilateral triangle with sides of length $a = 0.50$ m. What is the net electrostatic force on charge q_1?

This figure shows the situation



We begin by choosing the basis vector $\hat{\mathbf{e}}_x$ to be a vector directed from charge q_1 to charge q_2 and $\hat{\mathbf{e}}_y$ to be perpendicular to $\hat{\mathbf{e}}_x$ and oriented to the upper part of the figure.
Accordingly,

$$\begin{aligned} \mathbf{F}_{21} &= k\frac{q_1 q_2}{a^2}\left[-\hat{\mathbf{e}}_x\right] \\ &= -(8.99\times 10^9)\frac{(2.0\times 10^{-6})^2}{(0.50)^2}\,\hat{\mathbf{e}}_x \\ &= -(8.99\times 10^9)\frac{4.0\times 10^{-12}}{0.25}\,\hat{\mathbf{e}}_x = -0.144\ \mathrm{N} \end{aligned} \tag{3}$$

where the sign comes from the fact that we are calculating the force exerted by q_2 on q_1 which by definition is antiparallel to $\hat{\mathbf{e}}_x$

As for $\mathbf{F}_{31}$, it has the same magnitude $\mathbf{F}_{21}$, and written as a linear combination of the basis vectors, it is

$$\mathbf{F}_{31} = k\frac{q_3 q_1}{a^2} = F_{21} = -0.144 \text{ N} \left[cos\theta\, \hat{\mathbf{e}}_x + sin\theta\, \hat{\mathbf{e}}_y\right]$$

Where F_{21} is the magnitude of $\mathbf{F}_{21}$, the sign comes from the orientation of the force and$\theta = 60^\circ$ or $(\pi/3\, rad)$
substitution yields

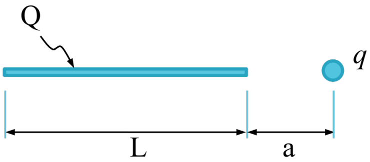
$$\mathbf{F}_{31} = -0.072 \text{ N}\, \hat{\mathbf{e}}_x - 0.125 \text{ N}, \hat{\mathbf{e}}_y \tag{4}$$

We might use eqns 3 and 3 to find the net force bay a straightforward algebraic sum. Instead, and just for the fun of it, we note that the two forces $\vec{F}_{21}$ and $\vec{F}_{31}$ make a 60 angle with each other. Therefore, the parallelogram rule implies that the magnitude of the net force is

$$F_{\text{net}} = 2F_{21}\cos(30) \tag{5}$$
$$= 2 \times 0.144 \times \frac{\sqrt{3}}{2} \tag{6}$$
$$= 0.144\sqrt{3} = 0.249 \text{ N} \tag{7}$$

The final answer to the problem may be expressed by stating: $\vec{F}_{\text{net}} = 0.25$ N, directed away from the triangle's center

Distributed Charges



The test charge q is located along the line of a thin uniformly charged bar that carries a total charge Q. What is the force the bar exerts on q

We first set $mat\hat{h}bfe_x$ to be the unit vector along the line containing the bar and directed to the right.
THe infinitesimal force that an element of charge (dQ) located at some point along the bar exerts on q is clearly

$$d\mathbf{F} = \frac{q}{4\pi\epsilon_0}\,\frac{dQ}{x^2}\,\hat{\mathbf{e}}_x\,, \tag{8}$$

where, $dQ = \lambda\, dx$, $\lambda = Q/L$ being the linear charge density with SI units C/m.
Using the supeposition principle, the total force exerted on q is the sum of the forces produced by each element of charge with turns out to be the integral:

$$\mathbf{F} = \frac{q}{4\pi\epsilon_0}\int_a^{L+a}\frac{dQ}{x^2}\,\hat{\mathbf{e}}_x \tag{9}$$

The integration is elementary,

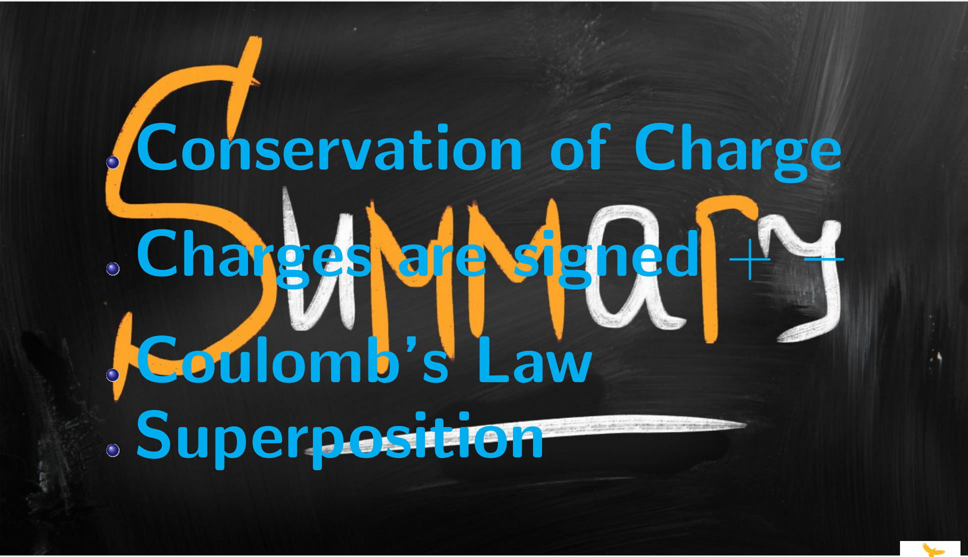
$$\mathbf{F} = \frac{q}{4\pi\epsilon_0} \int_a^{L+a} \frac{dQ}{x^2}\,\hat{\mathbf{e}}_x = -\frac{q\lambda}{4\pi\epsilon_0}\,\frac{1}{x}\Big|_a^{L+a}\,\hat{\mathbf{e}}_x = \\ = \frac{q\lambda}{4\pi\epsilon_0}\,[\frac{1}{a} - \frac{1}{L+a}]\,\hat{\mathbf{e}}_x \tag{10}$$

A little bit of algebra yields

$$\mathbf{F} = \frac{q\lambda}{4\pi\epsilon_0}\,[\frac{1}{a} - \frac{1}{L+a}]\,\hat{\mathbf{e}}_x = \frac{q\lambda\,L}{4\pi\epsilon_0}\,\frac{1}{a(L+a)}\,\hat{\mathbf{e}}_x \tag{11}$$

which reduces to

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\,\frac{qQ}{a(L+a)}\,\hat{\mathbf{e}}_x \tag{12}$$



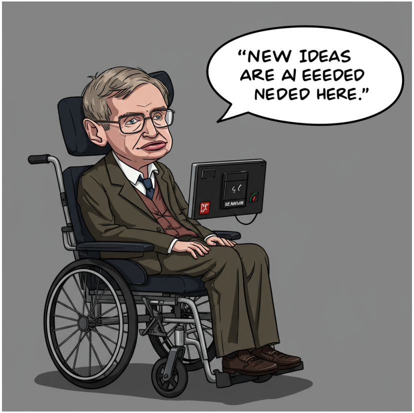
- Conservation of Charge
- Charges are signed $+ \; -$
- Coulomb's Law
- Superposition

Action at Distance an Unpleasant Idea

Troublesome

It seems as if particles detected themselves and interacted instantaneously

Violation of Causality
If actions at a distance are instantaneous, it can lead to problems with the concept of cause and effect, as it's difficult to determine which event came first in a chain of interactions.
Conceptual Challenges The concept of instantaneous action can be counterintuitive and lead to philosophical questions about the nature of reality. In essence, the problem isn't with the concept of action at a distance itself, but rather with the implications of its potential clash with our understanding of causality and the limits of the speed of light.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