Faraday' s Law of Induction

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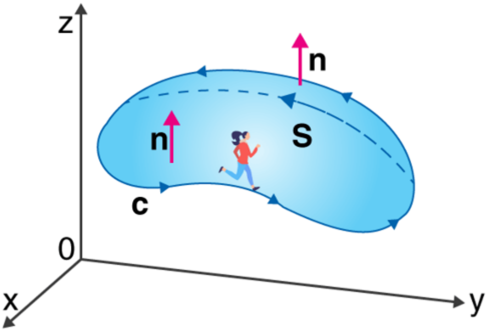
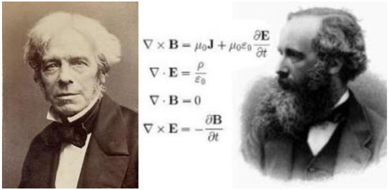
Outline

Curves and Surface Boundaries

Given a closed curve $\mathcal{C}$ and any open surface $(\mathcal{S})$ based on that curve, the surface is said to be oriented by $\mathcal{C}$ if the normals to $\mathcal{S}$ are oriented by $\mathcal{C}$

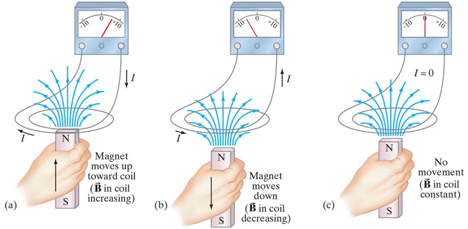




The phenomenon of induced currents was carefully investigated and demonstrated by Michael Faraday in his experiments conducted at the Royal Institution in London during the 1830s. The mathematical expression of Faraday's findings became a celebrated part of the work of James Clerk Maxwell, developed in the mid-19th century (notably, his equations published in the 1860s).

Experimental Results

A simple experiment demonstrates that when a bar magnet is moved into or out of a solenoid, a current is induced in the solenoid. Further examination reveals that the appearance of this current is due to the change in magnetic flux through the surface enclosed by the solenoid. This current is commonly referred to as an **induced current**.



Faraday's law of induction

The circulation of the electric field along a closed curve $\mathcal{C}$ [electromotive force or $\mathcal{E}$] equals the time rate of change of the magnetic flux through any surface $\mathcal{S}$ having $\mathcal{C}$ as its boundary.

The circulation of the electric field around a closed loop $\mathcal{C}$ is equal to the negative of the time rate of change of the magnetic flux through any surface $\mathcal{S}$ bounded by $\mathcal{C}$.

$$\mathcal{E} = \oint_{\mathcal{C}} \mathbf{E} \cdot d\vec{\ell} = -\frac{d\Phi_{\mathcal{S}}(\mathbf{B})}{dt} \tag{1}$$

A plane circuit composed of N turns, each of area $\mathcal{S}$, is placed in the $x - y$ perpendicular to an alternating uniform magnetic field that varies with time according to

$$\mathbf{B} = \mathcal{B}\, sin(\omega t)\, \hat{\mathbf{e}}_z$$

If we take the normal to the circuit to be $\hat{\mathbf{e}}_z$, then the flux of the induction field over one turn of the circuit is

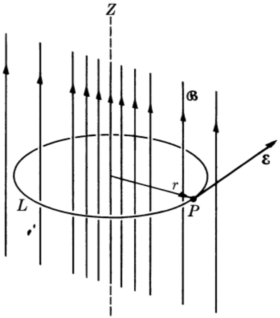
$$\Phi_{\mathcal{S}}(\mathbf{B}) = \mathcal{B}\mathcal{S}\, sin(\omega t)$$

Farady's law then implies that the FEM induced in the whole circuit is

$$\mathcal{E} = -\frac{\Phi_{\mathcal{S}}(\mathbf{B})}{dt} = -N\mathcal{B}\mathcal{S}\,\omega\, cos(\omega t)$$

The induced emf is oscillatory or alternating with the same frequency as the magnetic field!.

In a region of space there is a magnetic field which is parallel to the z-axis and which has axial (cylindrical) symmetry; that is, its magnitude at each point depends on the distance r to the z-axis only. The magnitude of the induction field also varies with time. Can the the electric field $\mathbf{E}$ at each point of space be calculated?.



The symmetry suggests that the electric field must be of the form $\mathbf{E} = E(r)\,\hat{\mathbf{e}}_\phi$, the fem (circulation of the electric field) along a circle of radius r is easily calculated to be

$$\mathcal{E} = 2\pi r\, E(r) = -\frac{d}{dt}\Phi_{\mathcal{S}}(\mathbf{B}(r,t))$$

This in turn implies that the farther we can go without any further knowledge about the magnetic induction field is that

$$\mathbf{E} = -\frac{1}{2\pi r}\frac{d}{dt}\left[\Phi_{\mathcal{S}}(\mathbf{B}(r,t))\right]\,\hat{\mathbf{e}}_\phi$$

The law of electromagnetic induction, as expressed in

$$\oint_{\mathcal{S}} \mathbf{E} \cdot d\vec{l} = -\frac{d}{dt}\Phi_{Int(\mathcal{S})}(\mathbf{B}),$$

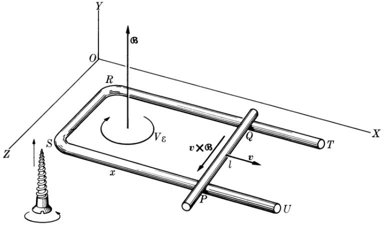
implies the existence of a local electric field whenever the magnetic field at that point is changing with time. As expressed in

$$\mathcal{E} = -\frac{d}{dt}\Phi(\mathbf{B}$$

it implies the existence of an emf when the magnetic flux through the circuit changes with time. It is important to discover whether the same results occur when the change in flux is due to a motion or deformation of the path $\mathcal{S}$ without $\mathbf{B}$ necessarily changing with time.

Moving Conductor

In order to investigate on the issue raised in the previous slide, let us consider the figure, where the conductor PQ can move parallel to itself with velocity **v** while maintaining contact with the U shaped conductor. The system $PQRS$ forms a closed circuit. Additionally, suppose that there is a uniform magnetic field **B** perpendicular to the plane of the system.



1. Each charge q in the moving conductor PQ is subject to a force $q\mathbf{v} \times \mathbf{B}$ acting along QP.
2. The same force on the charge could be assumed to be due to an "equivalent" electric field $\mathbf{E}_{eq}$ given by $q\mathbf{E}_{eq} = q\mathbf{v} \times \mathbf{B}$ or $\mathbf{E}_{eq} = \mathbf{v} \times \mathbf{B}$.
3. Since $\mathbf{v}$ and $\mathbf{B}$ are perpendicular, the relation among the magnitudes is $E_{eq} = vB$
4. If $PQ = \ell$, there is a voltage difference between P and Q given by $V = E_{eq}\ell = vB\ell$.
5. No forces are exerted on the sections QR, RS, and SP, since they are stationary.

6. Therefore the circulation of E_{eq} (or the emf) along circuit $PQRS$ is just $V_E = V$ in the direction of $\mathbf{v} \times \mathbf{B}$; that is,

$$\boxed{V_E = Bv\ell} \tag{2}$$

7. On the other hand, if we call the lenght SP x, the area of $PQRS$ is ℓx and therefore, the magnetic flux through $PQRS$ is $\Phi(\mathbf{B}) = B\ell x$.
8. The change of flux per unit time is then

$$\frac{d}{dt}\Phi(\mathbf{B}) = B\ell\frac{dx}{dt} = B\ell v$$

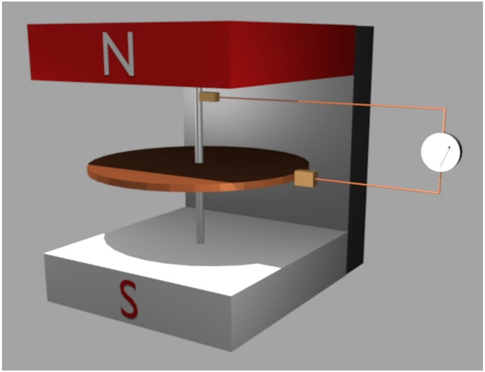
9. Comparing with equation 2:

$$\frac{d\Phi(\mathbf{B})}{dt} = V_{\mathbf{E}}$$

The fem is due to the moving parts of the circuit shape change!

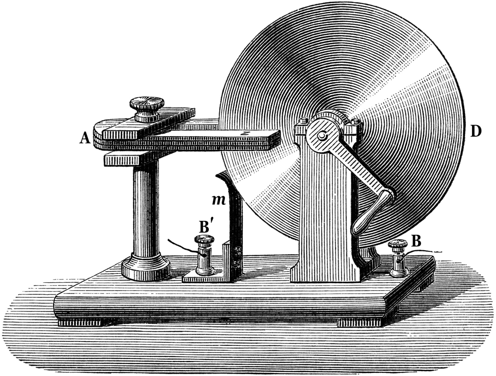
The minus sign is not included because we are considering only the relation between the magnitudes. However, Faraday' induction law still holds in sign, since the flux Φ is increasing and the sign of Vs is that of $\mathbf{v} \times \mathbf{B}$, so that it agrees with the Figure.

Homoploar Generator (Faraday Disk)



A homopolar generator is a DC electrical generator comprising an electrically conductive disc or cylinder rotating in a plane perpendicular to a uniform static magnetic field.

In this device is also known as a unipolar generator, acyclic generator, disk dynamo, or Faraday disc. A voltage difference is created between the center of the disc and the rim (or ends of the cylinder) with an electrical polarity that depends on the direction of rotation and the orientation of the field.



As happended with the bar moving on the U shaped conductor, in a rotating disc within a uniform magnetic field, an electromotive force ($\mathcal{E}$) is generated between the center and the edge of the disk.

As the conducting disc rotates, the free charge carriers within it (electrons) experience a magnetic force due to their movement in the magnetic field. This is nothing but the Lorentz force ($\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$), where q, $\mathbf{v}$ and $\mathbf{B}$ are the charge and the velocity of a charge carrier and the magnetic field respectively.

In the case of a rotating disc, the electrons (charge charriers) move tangentially and the magnetic field is typically applied parallel to the axis of rotation. This leads to a radial Lorentz force which pushes the electrons towards the periphery of the disc, while the positive ions in the lattice remain relatively stationary. This separation of charges creates a radial electric field pointing outwards from the center.

A steady state is reached when the electric force due to the charge separation balances the magnetic Lorentz force. This balance of forces is described by the equation

$$e\mathbf{E} = e(\mathbf{v} \times \mathbf{B})\,.$$

The radial electric field generates a voltage between the center and the edge of the disc. This voltage is the induced EMF.

$$\begin{aligned}\mathcal{E} &= \int \mathbf{E} \cdot d\vec{\ell} = r\,\omega\hat{\mathbf{e}}_\phi \times B\hat{\mathbf{e}}_z \cdot dr\hat{\mathbf{e}}_r = \\ &= \int \mathbf{E} \cdot d\vec{\ell} = qrB\,\omega B\hat{\mathbf{e}}_r \cdot dr\hat{\mathbf{e}}_r = \\ &= \frac{qr^2B}{2}\end{aligned} \tag{3}$$

Videos

1. Niel Degrasee Tyson [about 35 seconds of history]
2. TAM Farady's Induction Demo [almost 1 min]
3. Short Faraday's Biography [20 min]
4. Magnetic, Electric Fields EM Waves: History and Physics [30 min]