Conductors and Insulators

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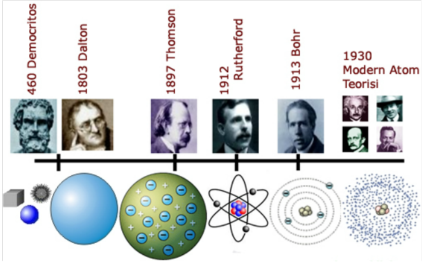
The Building Blocks of -ordinary- Matter

Atoms are composed of positively charged protons and electrically neutral neutrons, both tightly bound in a central nucleus, and negatively charged electrons "orbiting" the nucleus.
Protons and electrons carry charges of equal magnitude but opposite signs. Therefore, an atom with an equal number of protons and electrons is electrically neutral.



One Role of The electrons

Electrons are held close to the nucleus by the attractive electrical force between their opposite charges—a fundamental interaction essential for the existence of atoms and, consequently, everything around us.

The electrical properties of materials like conductors and insulators stem from the atomic structure.

Indeed, in "conductors" like copper, some of the outermost electrons are loosely bound and become conduction electrons, free to move throughout the material. This leaves behind positively charged atoms (ions). In contrast, insulators have very few, if any, such free electrons.

Classification of Materials

Conductors are materials through which charge can move rather freely; examples include metals (such as copper in common lamp wire), the human body, and tap water.

Non-conductors, also called insulators, are materials through which charge cannot move freely; examples include rubber (such as the insulation on common lamp wire), plastic, glass, and chemically pure water.

Semiconductors are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips.

Superconductors are materials that are perfect conductors, allowing charge to move without any hindrance. In these chapters we discuss only conductors and insulators.

Fundamental Fact

Since conductors have mobile charges (e.g., free electrons in metals), these charges redistribute in response to any net electric field.

- When a conductor is placed in an external electric field, mobile charges (electrons in metals) move under the field's influence.
- Redistribution continues until the internal electric field cancels the external field, resulting in zero net force on charges.
- In this state, called **electrostatic equilibrium**:
  - Charges are **static** (no macroscopic motion).
  - The net electric field inside the conductor is **zero**.
  - The excess charge resides entirely on the **surface**.

Visualizing Charge Redistribution

Imagine a dance floor crowded with people (our “free electrons” within a conductor).

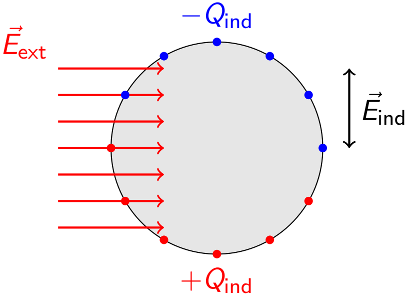
- Like dancers avoiding collisions, free electrons repel each other and move to **minimize mutual repulsion**.
- They redistribute until charges stop moving — reaching **electrostatic equilibrium**.
- **Key difference**: Dancers might stop due to fatigue, but electrons stop only when the net force on them is zero!

- The dance floor is obviously **finite**—dancers can't leave the floor, only move to its edges.
- Similarly, free charges are **trapped inside the conductor's boundaries**. They can't escape, so they maximize distance by moving to the surface.
- **Key Physics**:
  - Charges repel each other but are **confined by the conductor's geometry**.
  - The surface is the **only stable arrangement**:
    - Any charge inside would feel a net force pushing it outward.
    - On the surface, forces from other charges balance **tangentially** (only a normal component remains).

- External field $\vec{E}_{\text{ext}}$ forces charges to move.
- Redistribution of charges creates an opposing field $\vec{E}_{\text{ind}}$ until:

$$\vec{E}_{\text{net}} = \vec{E}_{\text{ext}} + \vec{E}_{\text{ind}} = 0 \quad \text{(inside conductor)}$$

- In **electrostatic equilibrium**:
  - Excess charge on **surface**.



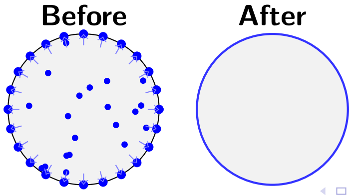
- Free electrons, like dancers in a crowded room, repel each other and flee to the edges.
- The conductor's boundaries act like walls: charges **cannot escape**, only spread out.

Before Equilibrium:

- Charges feel net force from external field.
- They accelerate, redistributing.

After Equilibrium:

- Charges stop moving when forces balance.
- $\vec{E}_{\text{net}} = 0$ inside.



- It will take sometime for this reorganization to occur, this is called **The "relaxation time" on our dance floor**.
- The relaxation time would then measure be how quickly everyone finds their new positions at the edges once the pushing starts.
- For very good "dancers" (like electrons in a conductor), this "settling" happens almost instantly (very short relaxation times)!

Based on the movement of free charges to equilibrium:

- **The electric field is zero everywhere inside a conductor.**
- If there were any electric field inside, it would exert a force on the free charges, causing them to move.
- Since the charges are in equilibrium (not moving), the net force on them must be zero, which means the electric field must be zero.
- This applies whether the conductor is solid or hollow.

Further consequences of charge redistribution in conductors:

- **Any excess charge on an isolated conductor resides entirely on its surface.**
- If there were excess charge in the interior, it would create an electric field there, violating the condition that $\mathbf{E} = 0$ inside.
- Mutual repulsion between charges forces them to the outermost surface.
- **The electric field just outside the surface of a charged conductor is always perpendicular to the surface.**
- If there were a component of $\mathbf{E}$ parallel to the surface, it would exert a force on surface charges, causing them to move along the surface, violating electrostatic equilibrium.

Electrostatics of Insulators

In stark contrast to conductors, insulators (often called **dielectrics** in this context) behave very differently in electric fields:

- **No Free Charges:** Insulators have virtually no free electrons. All electrons are tightly bound to their individual atoms or molecules.
- **Response to Electric Fields: Polarization**
  - When an external electric field is applied, these bound charges cannot flow through the material.
  - Instead, the atoms and molecules themselves become **polarized**.
    - Their electron clouds subtly shift, creating *induced electric dipoles*.
    - Or, for molecules with existing permanent dipoles, they tend to *realign* with the external field.

- **Internal Electric Field:** This polarization creates an internal electric field that *opposes* the external field.
    - The net electric field inside an insulator is therefore **reduced** compared to the external field, but it is generally **not zero**.
- **Charge Localization:** Any excess charge placed on the surface or within an insulator will remain localized exactly where it was placed, as there are no free charges to redistribute it.

While we often say conductors reach electrostatic equilibrium "instantaneously," there's a specific, extremely short time scale involved: the charge relaxation time (τ).

- This is the characteristic time it takes for excess charge within a conductor to redistribute itself, causing the electric field inside to become zero.
- It's defined as:

$$\tau = \frac{\epsilon}{\sigma}$$

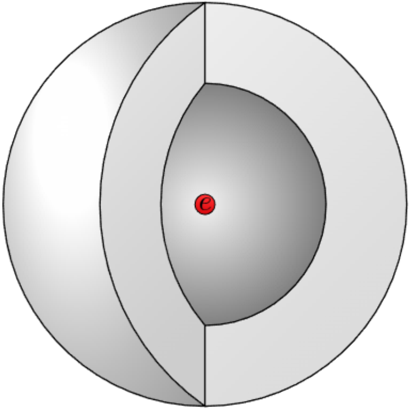
  where:
  - ϵ is the **permittivity** of the material (how it interacts with electric fields).
  - σ is the **electrical conductivity** of the material (how easily charge flows).

- For good conductors (like copper), σ is very large. This results in an incredibly small τ:
    - For copper, $\tau \approx 1.5 \times 10^{-19}$ seconds (or 0.15 attoseconds!).
- This means that for all practical purposes in electrostatics, charge redistribution and the establishment of $\mathbf{E} = 0$ inside conductors are indeed **effectively instantaneous**.
- In contrast, for insulators, σ is extremely small, leading to very long relaxation times, which is why charge can remain localized for extended periods.

To illustrate the fascinating physics of conductors, let's tackle a classic problem.

A point charge of magnitude e is placed at the center of a hollow, isolated conducting sphere, with an inner radius R_1 and an outer radius R_2.

How would the electric field and charges be distributed throughout all space?



To solve this problem, we'll rely on three powerful tools, two of which we have already used.

- **Gauss's Law:** Relates the electric flux through a closed surface to the enclosed charge.
- **Symmetry:** The spherical symmetry of the problem allows us to simplify calculations.
- **Properties of Conductors:** Specifically, how charges behave and where electric fields exist within and around them.

Let's define r as the radial distance from the center of the hollow sphere.

- **Inside the cavity ($r < R_1$):** If we draw any Gaussian surface within the hollow space, it encloses only the central point charge e. By Gauss's Law, the electric flux through this surface will be e/ϵ_0.
- **Within the conductor ($R_1 < r < R_2$):** In electrostatic equilibrium, the electric field is **zero** everywhere inside the conductor. Therefore, there is no electric flux through any Gaussian surface placed entirely within the conductor's material.

- **Outside the sphere ($r > R_2$):** When we consider a Gaussian surface outside the entire sphere, it encloses both the central charge e and any induced charges on the conductor. The total charge enclosed will determine the electric flux, which we'll find to be e/ϵ_0

To turn words into actual calculations, think of a spherical Gaussian surfaces concentric with the hollow sphere, their elements of surface are given by:$d\mathbf{A} = \hat{\mathbf{e}}_r\, dA$ where $\hat{\mathbf{e}}_r$ unit radial vector oriented in the direction of increasing radius.
Spherical symmetry tells us that the electric field must have the form

$$\mathbf{E} = E(r)\,\hat{\mathbf{e}}_r$$

- For any spherical gaussian surface of radius r centered in the charge, we will have

$$d\Phi_{Sphere}(\mathbf{E}) = E(r)\,\hat{\mathbf{e}}_r \cdot \hat{\mathbf{e}}_r\, dA = E(r)\, dA$$

- After integration:

$$\Phi_{Sphere}(\mathbf{E}) = \oint_{Sphere} E(r)\, dA = E(r) \oint_{Sphere} dA = E(r)\,(4\pi r^2)$$

Which, by virtue of Gauss' s law yield:

$$4\pi r^2\, E(r) = \begin{cases} \frac{e}{\epsilon_0} & r < R_1 \\ 0 = \frac{Q_{enclosed}}{\epsilon_0} & R_1 < r < R_2 \\ \frac{e}{\epsilon_0} & r > R_2 \end{cases} \tag{1}$$

We have therefore found the electric field:

$$\boxed{\mathbf{E}(\mathbf{r}) = \begin{cases} \frac{e}{4\pi\epsilon_0}\frac{\hat{\mathbf{e}}_r}{r^2} & r < R_1 \\ 0 & R_1 < r < R_2 \\ \frac{e}{4\pi\epsilon_0}\frac{\hat{\mathbf{e}}_r}{r^2} & r > R_2 \end{cases}} \tag{2}$$

We still have to deal with the charge distribution on the conducting hollow sphere.
Addressing this last problem is just a question of recalling that:
Any excess charge on an isolated conductor resides entirely on its surface.

Electric Field and Surface Charge

Result: Electric Field

The derived electric field is:

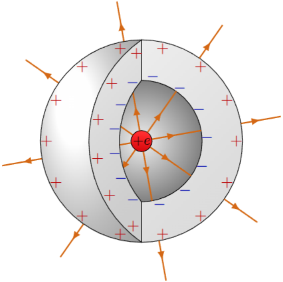
$$\boxed{\mathbf{E}(\mathbf{r}) = \begin{cases} \dfrac{e}{4\pi\epsilon_0}\dfrac{\hat{\mathbf{r}}}{r^2} & r < R_1 \\ 0 & R_1 < r < R_2 \\ \dfrac{e}{4\pi\epsilon_0}\dfrac{\hat{\mathbf{r}}}{r^2} & r > R_2 \end{cases}} \tag{3}$$

Resolving the Charge Distribution

To find the charge on the hollow sphere, recall:

- Excess charge on an isolated conductor resides entirely on its surface.

When the charge is positive ($+e$) the free charges redistribute as follows, negative charges go to the inner surface while positive charges go the outer surface



Surface Charge on the Inner Boundary

Bulk of the Conductor ($R_1 < r < R_2$)

- The region $R_1 < r < R_2$ is bounded by two surfaces:
  - Inner sphere (radius R_1)
  - Outer sphere (radius R_2)
- Any excess charge must reside on these surfaces (no bulk charge).

Gauss's Law Application

The enclosed charge Q_{enclosed} in 1 includes:

- Point charge $+e$ at the center
- Induced charge Q_{R_1} on the inner surface

$$Q_{\text{enclosed}} = e + Q_{R_1} \xrightarrow{\vec{E}=0 \text{ in conductor's bulk}} e + Q_{R_1} = 0$$

Result: Surface Charge Density

- Induced charge on R_1: $\boxed{Q_{R_1} = -e}$ (uniformly distributed)
- Surface charge density: $\sigma_1 = -\dfrac{e}{Area_1}$
- $Area_1 = 4\pi R_1^2$

$$\boxed{\sigma_1 = -\frac{e}{4\pi R_1^2}}$$

Charge Distribution on the Outer Surface

Key Assumption

The hollow conductor is **initially neutral**:

$$Q_{\text{conductor}} = Q_{R_1} + Q_{R_2} = 0$$

Charge Conservation

From previous results:

- Inner surface charge: $Q_{R_1} = -e$ (induced)
- Therefore, outer surface charge must be: $\boxed{Q_{R_2} = +e}$ (to preserve neutrality)

Gauss's Law Verification

For $r > R_2$:

$$Q_{\text{enclosed}} = \underbrace{e}_{\text{central}} + \underbrace{Q_{R_1} + Q_{R_2}}_{\text{conductor}} = e + (-e) + e = e$$

- Matches the field solution for $r > R_2$: $\mathbf{E} = \frac{e}{4\pi\epsilon_0}\frac{\hat{\mathbf{r}}}{r^2}$

Surface Charge Density

Outer surface charge distribution:

$$\boxed{\sigma_2 = \frac{Q_{R_2}}{4\pi R_2^2} = \frac{e}{4\pi R_2^2}} \quad \text{(uniform)}$$

In brief, the use of Gauss's Law, symmetry, and properties of conductors have allowed us to determine the electric field at any point:

$$\boxed{\mathbf{E}(\mathbf{r}) = \begin{cases} \frac{e}{4\pi\epsilon_0}\frac{\hat{\mathbf{e}}_r}{r^2} & r < R_1 \\ 0 & R_1 < r < R_2 \\ \frac{e}{4\pi\epsilon_0}\frac{\hat{\mathbf{e}}_r}{r^2} & r > R_2 \end{cases}}$$

And the charge distribution on the hollow sphere, which, being conducting, has to be distributed on its surface:

$$\boxed{\sigma = \begin{cases} -\frac{e}{4\pi R_1^2} & r = R_1 \\ \frac{e}{4\pi R_2^2} & r = R_2 \end{cases}} \tag{4}$$

The Power of Methodical Problem-Solving

Key Takeaways from This Solution

- No “magic” required - just systematic application of fundamental principles
- Complex problems become tractable when broken into logical steps

Our Problem-Solving Toolkit

- **Gauss’s Law**
  Related flux to enclosed charge
- **Symmetry**
  Guided surface choice and field direction
- **Conductor Properties**
  Explained charge redistribution
- **Charge Conservation**
  Ensured self-consistent solution

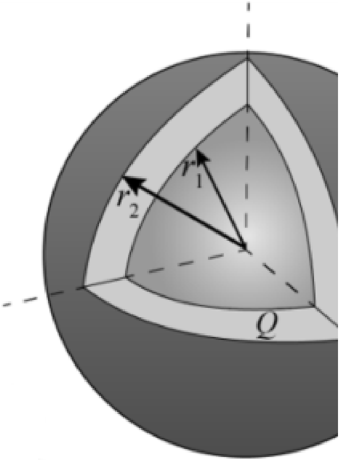
The Big Picture

This disciplined approach - applying core concepts methodically - works for:

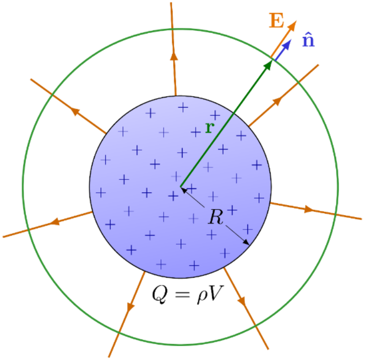
- This electrostatic problem
- More advanced EM scenarios
- Physics problem-solving in general

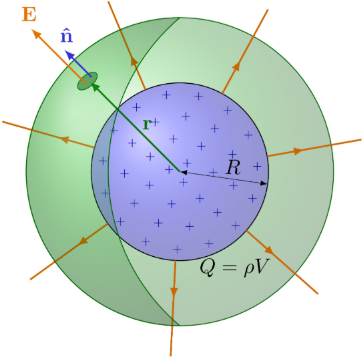
A hollow conducting sphere has an inner radius of $r_l = 1.5\ cm$ and an outer radius of $r_2 = 3.2\ cm$. The sphere has a net charge $Q = 1.9\ nC$

- What is the magnitude of the electric field at the center of the sphere?
  Answ:$0.0\ N/C$
- What is the magnitude of the electric field in the bulk of the conductor ($r_1 < r < r_2$)
  Answ:$0.0\ N/C$
- What is the magnitude of the electric field 5.9 m away from the center of the sphere?
  Answ:$1.49\ N/C$
- How is the charge distributed?



We have already studied this physical situation: the electric field produced by a uniform charge density (ρ). What we did not comment at that time is that this is physics of insulators, since only them can support a volumetric charge

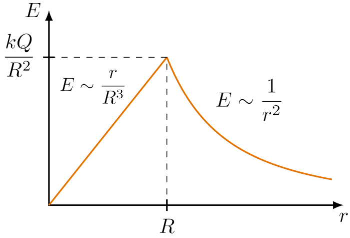




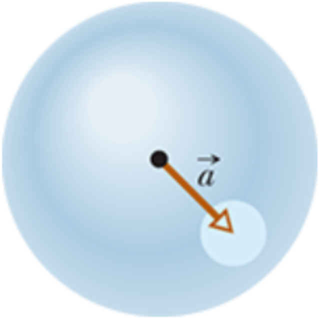
The field due to this charge distribution is

$$\mathbf{E}(r) = \begin{cases} \frac{Q}{4\pi\epsilon_0} \frac{r}{R^3}\,\hat{\mathbf{e}}_r & r < R \\ \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{\mathbf{e}}_r & r > R \end{cases} \tag{5}$$

where $\hat{\mathbf{e}}_r$ is the unit radial vector pointing outwards



A nonconducting solid sphere has a uniform volume charge density ρ. A spherical cavity is hollowed out of the sphere as shown in the figure. Find the electric field within the cavity.



The problem at hand has a underlying spherical symmetry that we are going to exploit. Let $\mathbf{E}_S$ the electric field of a completely solid sphere with charge density ρ, $\mathbf{E}_C$ the electric field within the volume of the cavity if it were filled with matter with the same charge density and $\mathbf{E}_A$ the electric field of the actual charge configuration.

The crucial observation is that the superposition principle guarantees that at any point P,

$$\mathbf{E}_S(P) = \mathbf{E}_C(P) + \mathbf{E}_A(P) \tag{6}$$

Let now $\mathbf{R}(P)$ and $\mathbf{r}(P)$ be the position vectors of a point P relative to the center of the big sphere and the small sphere (cavity) respectively.
Certainly:

$$\mathbf{R}(P) = \mathbf{r}(P) + \mathbf{a} \tag{7}$$

where $\mathbf{a}$ is the vector from the center of the large sphere to the center of the cavity. We have previously found that, the electric field inside a solid charged sphere is given by

$$\mathbf{E} = \frac{\rho}{3\epsilon_0}\mathbf{r}$$

where $\mathbf{r}$ is the position of the field point.

Using formula 6 we may write -for any point within the cavity-:

$$\frac{\rho}{3\epsilon_0}\mathbf{R}(P) = \frac{\rho}{3\epsilon_0}\mathbf{r}(P) + \mathbf{E}_A(P) \tag{8}$$

so

$$\mathbf{E}_A(P) = \frac{\rho}{3\epsilon_0}\left[\mathbf{R}(P) - \mathbf{r}(P)\right] , \tag{9}$$

which by virtue of formula 7 gives the following formula for the field at any point P in the interior of the cavity [a uniform field!!]

$$\boxed{\mathbf{E}_A(P) = \frac{\rho}{3\epsilon_0}\,\mathbf{a}} \tag{10}$$


LECTURE SUMMARY
- PHIESOPHENT LIKE YOU PHILSOMENS NEVET ARE PHEDANT
- PEGROBES YOUR SACKEST FLUNT
- COOR INY ANGHANDAY THE
- COPFRY
PHILESHPERT

- **Gauss's Law**

$$\boxed{\oint \mathbf{E}\cdot\hat{\mathbf{n}}\,dA = \frac{Q_{enclosed}}{\epsilon_0}}$$

- **Symmetry:** simplifies calculations. Exploit it
- **Properties of materials**
  - Insulators support volume charge density
  - Conductors: Charges move freely, $\mathbf{E} = 0$ in the bulk of a conductor. Charges acumulate in their boundaries (surface charge densitiy)