Electric Circuits I

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Electrical circuits are the invisible backbone of modern life.

- We rely on them constantly, from morning to night.
- They power our homes, offices, and vehicles, providing light, heat, and communication.
- From a simple light switch to complex industrial operations and healthcare, circuits are the conduits for energy.

Without them, our technologically advanced society would grind to a halt.

What is an electric circuit?

Definition

An electric circuit is an interconnected network of electrical components through which the electric current can flow in one or more closed loops.

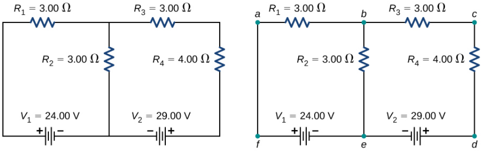


Figure: Left, scheme for a simple two loop circuit. Right, same circuit with nodes labeled

Key elements of a circuit

Power sources, conductors, and electrical loads.

Definition

A closed loop, or simply, a loop, is a continuous path for current to flow, starting from some point in the circuit, a voltage source, for example, passing through components, and returning to the original point. A circuit without a complete path for current to flow is considered open.

Components of a circuit include:

- Voltage sources: Provides the energy to drive the electric current (e.g., battery, generator).
- Loads: These are components that consume electrical energy and converts it into another form (e.g., light bulb, motor).
- Conductors: Material that allows electric current to flow easily (e.g., copper wire).
- Switches: These are devices that control the flow of electric current (e.g., on/off switch).

For a circuit to function, it must form a complete loop. If the circuit is broken, the flow of current stops.

Kirchhoff's Laws: The Foundation of Circuit Analysis

- Kirchhoff's Laws are fundamental principles for analyzing electrical circuits.
- They are derived directly from two foundational conservation principles in physics:
  - **Kirchhoff's Current Law (KCL)**: Based on the conservation of electric charge.
  - **Kirchhoff's Voltage Law (KVL)**: Based on the conservation of energy.
- These laws allow us to determine unknown currents and voltages in complex circuits.

Current Direction Convention

- For consistent circuit analysis, we must establish a convention for the direction of electric current.
- Electric current is the flow of charge. However, there are two types of charge: positive and negative.
- We will follow the historical convention established by **Benjamin Franklin**:
  - Current is defined as the direction of flow of **positive charges**.
  - This means current flows from a higher potential (positive terminal) to a lower potential (negative terminal) outside a voltage source.
- While electrons (negative charges) are usually the charge carriers in wires, this convention simplifies analysis without affecting the mathematical outcomes.
- This convention is standard in most circuit analysis texts, e.g., *Network Analysis* by M.E. Van Valkenburg.

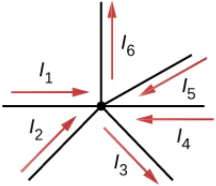
Kirchhoff's Current Law (KCL)

Also known as Kirchhoff's Junction Rule

- KCL is based on the fundamental principle of **conservation of electric charge**.
- This means that the algebraic sum of all currents entering and leaving any junction (or node) in a circuit must be zero.
- Equivalently, the total charge entering a junction per unit time must equal the total charge leaving it per unit time.

$$\boxed{\sum_{k=1}^{N} i_k = 0 \quad \text{or} \quad \sum_{\text{incoming}} i_{\text{in}} = \sum_{\text{outgoing}} i_{\text{out}}}$$

Convention: Currents entering a junction are typically assigned a positive sign, and currents leaving are assigned a negative sign for the $\sum i_k = 0$ formulation.



In the node of the figure there are four incoming and two outgoing currents

$$\begin{cases} i_1\ , i_2, i_4, i_5 & \textit{Incoming Currents} \\ i_3\ , i_6 & \textit{Outgoing currents} \end{cases}$$

Kirchhoff's first law applied to this case states

$$\boxed{i_1 + i_2 + i_4 + i_5 = i_3 + i_6}$$

Kirchhoff's Voltage Law (KVL): The Physics

Conservation of Energy and Potential

- KVL is fundamentally rooted in the **conservation of energy**.
- Recall that potential difference (voltage, V) is defined as the change in potential energy (U) per unit charge (q): $V = \Delta U / q$.
- As a charge traverses a closed loop in a circuit, it returns to its starting potential energy level.
- Therefore, the total energy supplied by voltage sources within the loop must be exactly equal to the total energy dissipated or stored by all other components in that same loop.
- This ensures that no energy is gained or lost as charge completes a full circuit.

Energy in = Energy out within any closed path.

Kirchhoff's Voltage Law (KVL): Engineering Application

Voltage Drops and Rises in a Loop

- For practical circuit analysis, KVL is applied by considering **voltage rises** and **voltage drops** around a closed loop.
- A **voltage rise** occurs when moving from a lower potential to a higher potential (e.g., across an ideal voltage source from negative to positive terminal).
- A **voltage drop** occurs when moving from a higher potential to a lower potential (e.g., across a resistor in the direction of current flow, or across an ideal voltage source from positive to negative terminal).

- KVL states that the algebraic sum of all voltage drops and rises around any closed loop must be zero:

$$\boxed{\sum_{\text{all elements in loop}} V_k = 0}$$

*Common convention: Voltage rises are positive, voltage drops are negative (or vice-versa, as long as consistent).

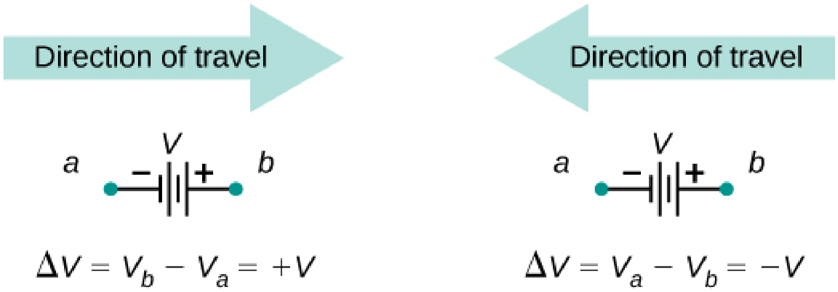


Figure: Sign conventions for voltage increase or drop across a voltage source

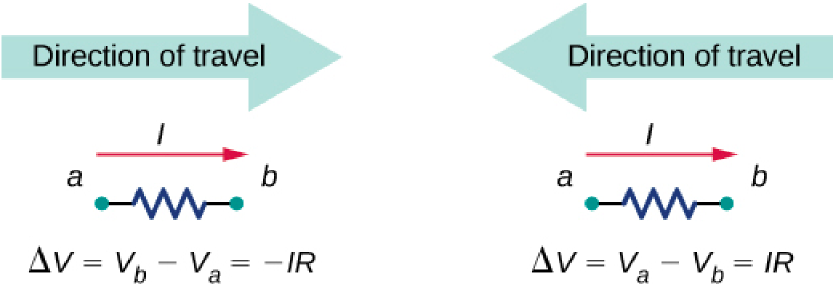
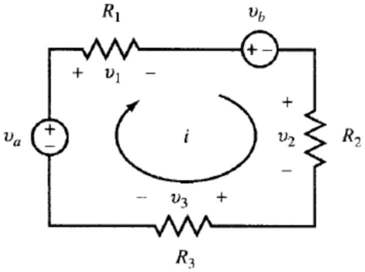


Figure: Sign conventions for voltage increase or drop across resistances

A One Loop Circuit



If we walk the circuit beginning in the negative pole of v_a, the increases and drops of voltage are given by

$$v_a - v_1 - v_b - v_2 - v_3 = 0$$

which in terms of Ohm's law are:

$$\boxed{v_a - R_1 i - v_b - R_2 i - R_3 i = 0}$$

Voltage Divisor [One Loop]

$$v = (R_1 + R_2 + R_3)\, i$$

the voltage drop at R_1

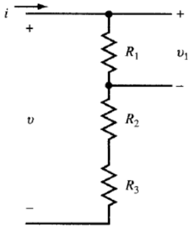
$$v_1 = R_1 i$$

and therefore,

$$v_1 = \frac{R_1}{R_1 + R_2 + R_3}\, v\,,$$

if all the resistances are equal.

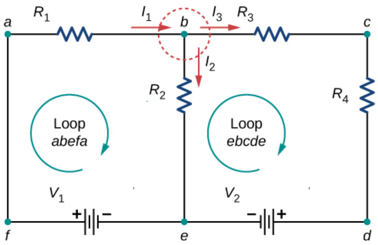
$$\boxed{v_1 = \frac{v}{3}\, v}$$



The circuit divides the source voltage in sections of one third of the source voltage.

What happens if we use N identical resistances?

Two Loop Circuit A



There are clearly three closed paths that can be followed in this circuit, namely (*abefa*), (*ebcde*) and (*facdf*). We name a path by segments that begin and end at the referred nodes. It may be argued that there are many more paths, but, they are, either the same but traversed in the opposite sense or paths where segments are traversed several times.

Two Loop Circuit B

The circuit shows two interesting nodes where the current divides, namely b and e. At node b i_1 arrives, splits into two pieces i_2 and i_3 so

$$\boxed{i_1 = i_2 + i_3},$$

while at node e i_3 and i_2 converge to leave the node as just one current, i_1, leading to

$$i_2 + i_3 = i_1 ,$$

so we have no new information form this node.

Two Loop Circuit C

Let us write the loop equations for the three loops

$$
\begin{aligned}
(abefa): \quad & -R_1 i_1 - R_2 i_2 + v_1 = 0 \\
(ebcde): \quad & R_2 i_2 - R_3 i_3 - R_4 i_3 - v_2 = 0 \\
(facdf): \quad & -R_1 i_1 - R_3 i_3 - R_4 i_3 - v_2 + v_1 = 0
\end{aligned}
\tag{1}
$$

There is something very interesting here, the equation for the big loop $(facdf)$ is the sum of the equations for the two small loops, meaning that the information coming form the big loop is superfluous, i.e. it is sufficient to think about the two small loops (or one small loop and the big one).

Two Loop Circuit C

The circuit is finally described by a set of three linear equations with three unknowns, namely

$$\begin{aligned} i_i - i_2 - i_3 &= 0 \\ -R_1 i_1 - R_2 i_2 + v_1 &= 0 \\ R_2 i_2 - R_3 i_3 - R_4 i_3 - v_2 &= 0 \end{aligned} \tag{2}$$

Our problem is no longer physics, but math, which teaches us appropriate techniques to deal with these systems

Capacitors in KVL: Applying the Loop Rule

Same Voltage Drop/Rise Logic

- When applying Kirchhoff's Voltage Law (KVL) to a loop containing a capacitor, the sign convention for the capacitor's voltage ($v_C(t)$) is exactly the same as for a resistor or any other component.
- Based on the assumed voltage polarity (from PSC) and your chosen direction of traversing the loop:
  - If you move from the **positive (+) to the negative (-)** terminal of $v_C(t)$, it's a **voltage drop** (subtract $v_C(t)$ in KVL).
  - If you move from the **negative (-) to the positive (+)** terminal of $v_C(t)$, it's a **voltage rise** (add $v_C(t)$ in KVL).

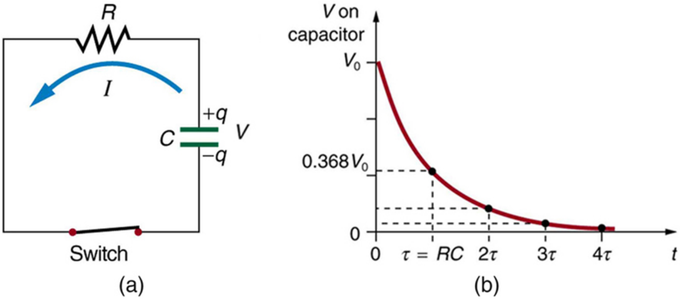
The capacitor voltage $v_C(t)$ will be an unknown variable that you solve for, which might be time-varying. Its actual polarity will be determined by the sign of your solution.

Beware

Treat capacitor voltages as you would any other voltage source or drop in the loop. You must exercise particular care with the signs of current and charge

Let us discuss the simplest capacitor discharge problem. According to the convention we are using and "walking" the loop as the arrow in the current, the loop equaqtion is

$$V - IR = 0\,, \quad V = \frac{q}{C}$$



Now and here is when we excersie care. The charge in the positive plate is being drained by the current I so

$$\frac{dq}{dt} = -I$$

so, at the end the equation for the loop takes the form,

$$\frac{dq}{dt} = -\frac{q}{RC} \tag{3}$$

The solution of equation 3 is equation with solution

$$q(t) = Q_0\, e^{-\frac{t}{\tau}} \,,$$

where

- Q_0 is the initial charge of the capacitor and
- $\tau = RC$ is universally known in engineering as the time constant of the circuit.