Electric Current

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August 14, 2025

From Static Charges to Flowing Electricity

So far, we've focused on the fascinating world of **electrostatics**:

- Charges are at rest or moving at constant velocity.
- Electric fields are static.
- Potential energy is stored in configurations (like capacitors).

But how do we use electricity? How does a light bulb turn on, or a phone charge? The answer isn't in static charges, but in their movement.

To truly harness electricity and build the technology that surrounds us, we must understand how electric charge is transported through materials. This shift from static configurations to the continuous flow of charge is our next big step!

Electric Current: The Flow of Charge

While electric current describes the flow of charge, we first consider the current density vector **J**, which provides a more fundamental, microscopic description of this flow.

- **J** describes how charges move. It allows **us** to find the amount of charge flowing per unit time through a unit cross-sectional area, perpendicular to the flow.
- It is a **vector quantity**, and its direction is defined as the direction of flow of positive charge.
- **Units:** Amperes per square meter (A/m^2).

Current Density and Charge Carrier Motion

Think of a material where some charges (called charge carriers) are moving with a velocity known as drift velocity. Given the standard notation

- n: Number of charge carriers per unit volume (charge carrier density, m^{-3})
- q: Charge of a single carrier (Coulombs, C)
- $\mathbf{v}_d$: Average drift velocity of the charge carriers (meters per second, m/s). This is the average velocity component of the charged particles in the direction of the electric force acting on them.

The current density $\mathbf{J}$ the charge carriers within the material is

$$\boxed{\mathbf{J} = nq\mathbf{v}_d}$$

To gain a clearer understanding, imagine a material where the charge carrier density (n) and drift velocity (v_d) are uniform. The flux of $\mathbf{J}$ through a cross-sectional area A perpendicular to $\mathbf{J}$ is:

$$\Phi_A(\mathbf{J}) = \int_A \mathbf{J} \cdot \hat{\mathbf{n}} dA = nq\, v_d A$$

Giving this formula some thought, we will realize it is just the charge per unit time that crosses the surface. This is what people usually call electric current I.

NOTE

For a current to exist, there must be charge carriers free to move (like electrons in a metal or ions in an electrolyte) and a force acting on them.

The electric current (I) is the total rate at which charge flows through a given cross-sectional area. It's the macroscopic manifestation of current density.

- For any surface A through which charge flows, the flux of $\mathbf{J}$ is the current that passes through a surface

$$\boxed{I = \int \mathbf{J} \cdot d\mathbf{A}}.$$

If the current density $\mathbf{J}$ is uniform and perpendicular to the cross-sectional area A, then:

$$I = JA$$

Units:

The SI unit for electric current is the **Ampere (A)**, where 1 Ampere = 1 Coulomb per second (1 A = 1 C/s).

Drift Velocity vs. Signal Speed

It's important to distinguish between the very slow drift velocity of individual charge carriers and the nearly instantaneous speed at which an electrical signal propagates.

- Drift Velocity (v_d): This is typically very slow, on the order of millimeters per second in metals. Individual electrons literally "drift" through the conductor.
- Speed of Electric Signal: When you flip a light switch, the light turns on almost instantly. This is because the electric field, which pushes the electrons, propagates through the wires at a speed close to the speed of light (c).

Analogy:

Think of a water pipe already full of water. When you open the tap, water immediately comes out the other end, even though any single water molecule takes a long time to travel through the pipe. The pressure wave travels quickly.

What Drives Current? The Electric Field

In electrostatics, we learned that the electric field inside a conductor in equilibrium is zero. However, for a current to flow, there must be an electric field present within the conductor.

- An external electric field ($\mathbf{E}$) exerts a force on the free charge carriers within the conductor.
- This force causes the charge carriers to accelerate, leading to their drift.
- Due to collisions with the atoms of the conductor, this acceleration is not continuous, and they reach an average drift velocity $\mathbf{v}_d$.
- Therefore, a sustained electric current implies a sustained electric field within the conductor.

Contrast with Electrostatics

Remember: in electrostatic equilibrium, charges arrange themselves to cancel out any internal electric field. With current, charges are constantly moving, implying a continuous electric field driving them.

Ohm's Law: Microscopic Form

The relationship between the electric field that drives the current and the resulting current density is fundamental.

For many materials (especially metals) at a constant temperature, the current density $\mathbf{J}$ is directly proportional to the applied electric field $\mathbf{E}$ [Ohm's Law]:

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

Where σ (sigma) is the electrical conductivity of the material.

The reciprocal of conductivity is resistivity (ρ): $\rho = 1/\sigma$.

So, Ohm's Law can also be written as:

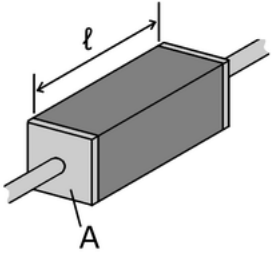
$$\boxed{\mathbf{E} = \rho \mathbf{J}}$$

UNITS:

- Conductivity (σ) Siemens per meter (S/m) or $(Omh.m)^{-1}$
- Resisitivity (ρ) $Omh.m$

Resistance (R) - The Macroscopic Property

While resistivity (ρ) is an intrinsic property of a material, resistance (R) is a macroscopic property of a specific object (like a wire) that depends on the material it is made of, its shape, and size.
Think of a conductor of uniform cross-sectional area A and length ℓ, carrying a uniform current density $\mathbf{J}$ driven by a uniform electric field $\mathbf{E}$:



Under the described circumstances:

- The potential difference across the conductor is $V = E\ell$.
- The current through the conductor is $I = JA$.

Using Ohm's Law in microscopic form ($\mathbf{E} = \rho\mathbf{J}$), so

$$E\ell = \rho JL$$

therefore

$$V = \rho\frac{I}{A}\ell$$

which yields

$$\boxed{V = \left(\frac{\rho\,\ell}{A}\right) I}$$

Ohm's Law: Macroscopic Form

The previous derivation, naturally suggests defining the resistance (R) of the conductor as:

$$\boxed{R = \frac{\rho L}{A}}$$

Clearly, the SI unit for resistance is the Ohm (Ω).
THe $V - -I$ relation across the conductor can now be cast as

$$\boxed{V = IR},$$

whic is the well known form of Ohm's Law:

Power Dissipation in Resistors

When current flows through a resistor, electrical potential energy is continuously converted into other forms of energy, typically thermal energy. This is known as Joule heating. The rate at which energy is dissipated (or converted) is the power (P)

- Consider a tiny charge dQ moving through a potential difference V in a short time dt.
- The work done on the charge by the electric field is $dU = VdQ$
- The power is $P = dU/dt = V(dQ/dt)$.
- Since $I = dQ/dt$:

$$\boxed{P = IV}$$

- Using Ohm's Law ($V = IR$ or $I = V/R$), we can express power in alternative forms:

$$\boxed{P = I^2R \quad \text{or} \quad P = \frac{V^2}{R}}$$

- A little reminder, the SI unit for power is the Watt (W)= 1 Joule per second.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Summary of Electric Current

We've explored the fundamental aspects of electric current:

- Current density vector: $\mathbf{J} = nq\mathbf{v}_d$, naturally defined from microscopic charge flow.
- Electric Current, flux of the current vector through some surface ($I = \int \mathbf{J} \cdot \hat{\mathbf{n}} dA$.
- The crucial role of the Electric Field ($\mathbf{E}$) in driving current within conductors.
- Introduced conductivity and resistivity and Ohm's Law as a relation between fields ($\mathbf{J} = \sigma \mathbf{E}$ or $\mathbf{E} = \rho \mathbf{J}$)

- Concept of Resistance ($R = \rho L/A$).
- The familiar expression of Ohm's Law ($V = IR$) as a current voltage relation.
- Finally, Power Dissipation ($P = IV = I^2R = V^2/R$) in resistances (circuits).

This lays the groundwork for analyzing complete circuits involving current, voltage, and resistance!