Oscillating Circuits and Alternating Currents

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Outline

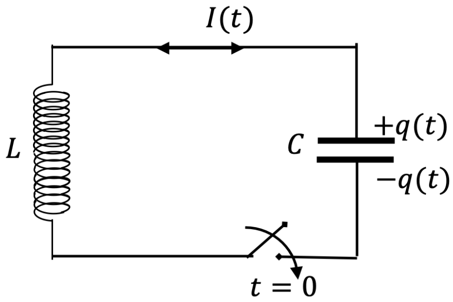
1. The Tank Circuit
2. Solving the equation
3. Physical Interpretation

Why Study LC (Tank) Circuits?

- **Non-Trivial Behavior:** Unlike simple resistive circuits, LC circuits introduce dynamic, time-varying responses that are fundamental to understanding AC circuits and transient phenomena.
- **Fundamental Concept:** LC circuits are the simplest form of resonant circuits, illustrating the core principles of energy oscillation.
- **Natural Oscillators:** They possess the inherent ability to oscillate, converting energy between electric and magnetic fields, forming the basis for signal generation.
- **Frequency Selectivity:** They are essential for filtering and tuning applications, allowing specific frequencies to be selected or rejected (e.g., in radio receivers and transmitters).

- **Signal Generation:** Beyond being natural oscillators, they are the heart of many practical oscillators used in clocks, communication systems, and radio frequency (RF) devices.
- **Energy Storage & Transfer:** They demonstrate how energy can be efficiently stored and transferred within a circuit at specific resonant frequencies, crucial for power electronics and wireless power transfer.
- **Ubiquitous Applications:** Found everywhere from simple AM/FM radios to complex mobile phones, RFID systems, MRI machines, and high-frequency power converters.

The LC or tank circuit is a circuit composed of at most the three elements shown in the figure



After closing the switch, the loop law describes the behavior of the circuit as

$$\frac{q}{C} + L\frac{dI}{dt} = 0 \tag{1}$$

Taking the time derivative, equation 1 can be cast as

$$\frac{d^2q(t)}{dt^2}+\frac{q(t)}{LC}=0\,, \tag{2}$$

This is a second order ordinary differential equation, in passing, just by noting the position of the values of the circuit elements in the equation, we can deduce that $\omega_0=1/\sqrt{LC}$ -called the natural angular frequency- has units of $time^{-1}$.
Every text will show that the general solution of eq 2

$$q(t)=q_0\,sin(\omega_0 t+\delta)\,, \tag{3}$$

where q_0 and δ are constants to be found by imposing initial conditions to the circuit.

A little bit of math teaches us that

- The current (dq/dt) flowing through the circuit is $i = q_0\omega_o\, cos(\omega_0 t + \delta) = i_{max}\; cos(\omega_0 t + \delta)$
- The electric and magnetic energies are
  - $E_E = \frac{q_0^2\, sin^2(\omega_0 t+\delta)}{2C}$
  - $E_B = \frac{L}{2}\, q_0^2\omega_o^2\, cos^2(\omega_0 t + \delta) = \frac{q_0^2\, cos^2(\omega_0 t+\delta)}{2C}$
- These formulas show that the storing of energy is periodically changing from being mostly electrical to mostly magnetic and so on, with the total energy (the sum of both) being constant since there are no sources or dissipators in this highly ideal circuit

For the sake of simplicity let us take $\delta = -\pi/2$ so the charge and the current time dependencies are

$$q(t) = q_0\, cos(\omega_0 t)\,, \quad i(t) = -i_{max}\, sin(\omega_0 t) \tag{4}$$

this formulas imply that both physical quantities change periodically with a period

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{LC}\,. \tag{5}$$

Besides, carefully looking the figure, one can realize that the current peaks at a time $T/4$ later than the charge, in engineering dialect it is said that engineers say that the current lags the charge

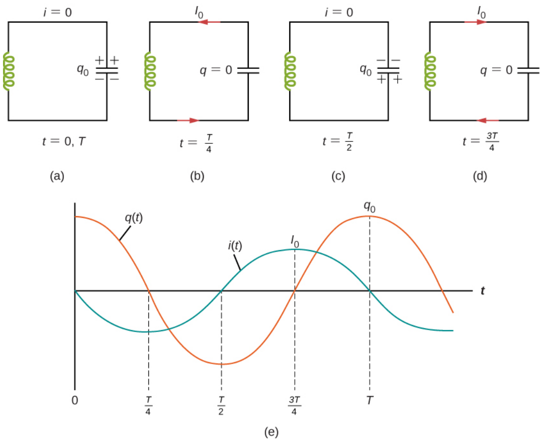


FIGURE: Charge and curretn in anLC circuit. Note the inversions in the direction of the current

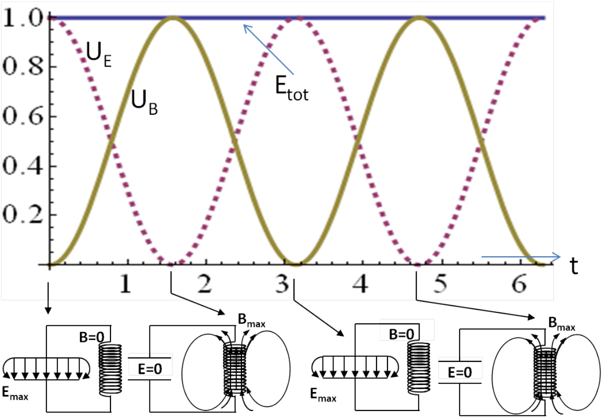


Figure: Energy moves periodically between the capacitor and the inductor.