Electrostatic Energy

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Energy in the Electric Field: Where Does it Reside?

We know that electric fields exert forces on charges. These forces accelerate the charges, and therefore, we expect the kinetic energy of the charges to be changed -in general-.
A change in the kinetic energy of a particle means that work was done on it; consequently, we may safely conclude that an electric field does work on a charge which in turn implies that energy is transferred to the charge.

The above paragraph naturally rises a question: where is the energy transferred to the charge by the electric field transferred from?

- This energy doesn't just appear or disappear; it must be stored somewhere in the region where the interaction is occurring.
- The crucial insight is that this energy is stored, not *in* the charges themselves, but **in the electric field** that these charges create and that permeates space.
- We can think of the electric field not just as a mediator of forces, but as an active carrier and reservoir of energy.
- This energy is distributed throughout space wherever the field exists, and we can define its concentration through **energy density**.

Energy of a System of Charges: Work of Assembly

Electric potential energy (U) is associated with the configuration of a system of charges. This energy represents the work done to assemble the charge distribution from infinitely separated components.

- If we bring a first charge into empty space, no work is done ($U = 0$).
- To bring a second charge near the first, work is done against (or by) the electric field created by the first charge. This work becomes the potential energy of the two-charge system.
- For multiple charges, the total potential energy of the system is the sum of the work required to bring each charge from infinity to its final position, in the presence of all previously placed charges.

Defining Electric Energy Density (u_E)

If energy is stored in the electric field, then we can quantify how much energy is stored per unit volume of space. This is called the **electric energy density** (u_E).

- u_E represents the amount of electric potential energy stored in a small volume of space where an electric field exists.
- The electric energy density (u_E) in a vacuum (or air) is given by the formula:

$$\boxed{u_E = \frac{1}{2}\epsilon_0 E^2}$$

- **Units:** Since energy (U) is in Joules (J) and volume (V) is in cubic meters (m^3), the unit for energy density is Joules per cubic meter (J/m^3).

Total Electric Field Energy

If the energy density $u_E = \frac{1}{2}\epsilon_0 E^2$ is known at every point in space, we can find the total electric potential energy (U_{total}) of a charge configuration by integrating the energy density over all space:

$$U_{\text{total}} = \int_{\text{all space}} u_E \, dV = \int_{\text{all space}} \frac{1}{2}\epsilon_0 E^2 \, dV$$

- This integral sums up the energy contributions from every tiny volume element (dV) where an electric field exists.
- This approach to calculating electrostatic energy is equivalent to the work of assembly method ($U = \frac{1}{2}\sum_i q_i V_i$), but offers a different, field-centric perspective.
- It emphasizes that energy is a property of the field itself, regardless of whether there are charges in that specific volume element.

Consistency Check: Energy Density in a Parallel-Plate Capacitor

Consider a parallel plate capacitor of cross section A and interplate distance d carrying charge Q.

The magnitude of the electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}\,,$$

the general expression for the electrostatic energy density is

$$u_E = \frac{1}{2}\epsilon_0 E^2\,.$$

For our problem, it turns out to be

$$u_E = \frac{1}{2}\epsilon_0 \left(\frac{Q}{\epsilon_0 A}\right)^2 = \frac{1}{2}\frac{Q^2}{\epsilon_0 A^2}$$

The total energy is calculated integrating u_E in the whole volume (vol) of the capacitor, that is,

$$E = \int_{vol} u_E \, dv \,,$$

substitution yields

$$U = \int_{vole} \frac{1}{2} \frac{Q^2}{\epsilon_0 A^2} dv = \frac{1}{2} \frac{Q^2}{\epsilon_0 A^2} \int_{vol} dv = \frac{1}{2} \frac{Q^2}{\epsilon_0 A^2} \times Ad = \frac{1}{2} Q^2 \frac{d}{\epsilon_0 A}$$

We have just concluded that the rigorous treatment of the electric energy density implies that

Energy Stored in the Capacitor

$$\boxed{U = \frac{Q^2}{2C}}$$

Which is the expression we found when treated energy as work done by moving charges between the plates of the capacitor


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In every region of space where an electrostatic field (E) is found, there is energy distributed by the density

$$u_E = \frac{1}{2}\epsilon_0\, E^2\,,$$

the total energy stored in a volume *vol* is found as

$$U = \int_{vol} u_e\, dv$$