Appendix B

Symbols and Abbreviations

B.1 Mathematical Symbols

$\hat{}$	denotes estimator
$\check{}$	denotes approximate estimator
$\tilde{}$	denotes reversal of a vector or matrix
$\bar{}$	denotes mean value
$*$	convolution
$\stackrel{\mathcal{FT}}{\longleftrightarrow}$	Fourier transform pair
$\lfloor\cdot\rfloor$	denotes integer part
$\lvert\cdot\rvert$	denotes absolute value
$\oint_C$	contour integral over C
$\arg\max\limits_x f(x)$	denotes the value of x that maximizes $f(x)$
$\mathbf{0}$	vector whose elements equal zero
$\mathbf{1}$	vector whose elements equal one
$\mathbf{1}_M$	vector whose M elements equal one
A	maximum amplitude of excitatory postsynaptic potentials
$\mathbf{A}_i$	$M \times M$ matrix describing temporal and spatial correlation
$A(z)$	denominator polynomial of an AR system transfer function
$A_p(z)$	denominator polynomial of a p^{th} order AR system transfer function
$A_p(e^{\jmath\omega})$	discrete-time Fourier transform of denominator polynomial
$A_x(\tau,\nu)$	Ambiguity function of signal $x(t)$
a	amplitude factor, average time delay in excitatory postsynaptic potentials
$\mathbf{a}$	signal amplitude vector
a_i	feedback coefficient of a linear, time-invariant system, amplitude factor
$\mathbf{a}_p$	vector of feedback coefficient of a linear, time-invariant system, signal amplitude vector

α	gain factor in various recursive algorithms,
	weight coefficient
B	continuous-time signal bandwidth
	maximum amplitude of inhibitory postsynaptic potentials,
$B(z)$	numerator polynomial of the transfer function
$B(e^{j\omega})$	discrete-time Fourier transform of numerator polynomial
b	multiplicative parameter,
	average time delay in inhibitory postsynaptic potentials
$b(n)$	signal envelope
b_i	feedforward coefficient of a linear, time-invariant system
β	waveform duration parameter,
	exponential weighting factor
β_l	discretized waveform duration parameter,
	l^{th} ECG measurement on the observed signal
$\tilde{\beta}_l$	l^{th} ECG measurement on the reconstructed signal
$\beta(n)$	time-varying waveform duration parameter
C_i	interaction between neuron subpopulations
$\mathbf{C}_x$	covariance matrix of $\mathbf{x}$
C_ψ	normalization factor in the inverse wavelet transform
$C_x(t,\Omega)$	general Cohen's class time–frequency distribution
c	constant
c_i	partial fraction expansion coefficient
$c_j(k)$	dyadic scaling expansion coefficient
$c_j^u(k)$	$c_j(k)$ with zeros inserted
c_w	constant
D	decimation/interpolation factor,
	signal duration,
	interval preceding EP stimulus
D_0, D_1	QRS detection threshold parameters (refractory period)
$D_{\text{HT}}(\Omega)$	Fourier transform of $d_{\text{HT}}(t)$
$D_{\text{HT}_k}(\Omega)$	Fourier transform of $d_{\text{HT}}(t)$ at $k/\overline{T}_0$
$D_{\text{E}}^u(\Omega)$	Fourier transform of $d_{\text{E}}^u(t)$
$D^u(\Omega)$	Fourier transform of $d^u(t)$
$D(\Omega)$	Fourier transform of $d(t)$
$\mathbf{d}$	vector of unevenly distributed signal samples at $d(t_i)$
$d^u(t)$	unevenly sampled signal (continuous-time)
$d^i(t)$	interpolated signal (continuous-time)
$d^e(t)$	evenly sampled signal (continuous-time)
$d_{\text{HT}}(t)$	heart timing signal
$d_{\text{IT}}(k)$	interval tachogram signal
$d_{\text{IIT}}(k)$	inverse interval tachogram signal

$d_{\mathrm{IF}}(t)$	interval function
$d_{\mathrm{IIF}}(t)$	inverse interval function
$d^{i}_{\mathrm{IIF}_s}(t)$	inverse interval function interpolated with RR interval shifting
$d_{\mathrm{E}}(t)$	event series signal
$d_{\mathrm{LE}}(t)$	lowpass filtered event series signal
$d_{\mathrm{SR}}(k)$	interval tachogram from sinus rhythm
$d(n)$	heart rate signal
d_i	pole of a linear, time-invariant system
$d_j(k)$	coefficient of the dyadic wavelet expansion
$d^u_j(k)$	$d_j(k)$ with zeros inserted
$\Delta(n)$	discrete-time segmentation function
$\Delta\mathbf{w}(n)$	time-varying weight error vector
$\Delta\mathbf{w}^b$	bias in the weight vector estimate of the LMS algorithm
$\Delta\omega_i$	3-dB bandwidth of i^{th} spectral component
Δ_t	continuous-time signal duration
Δ_Ω	continuous-time signal bandwidth
Δ_n	discrete-time signal duration
Δ_ω	discrete-time signal bandwidth
Δ_θ	tolerance interval time in QRS detector evaluation
$\Delta\hat{s}_a(n)$	difference signal between two subaverages
Δt_{i_j}	time distance from i^{th} beat to the $(i+j)^{\text{th}}$ beat
$\Delta\beta$	step size for discretized duration parameter β, normalized ECG measurement error
$\delta(t)$	continuous-time unit impulse function (Dirac function)
$\delta(n)$	discrete-time unit impulse function
$e(n)$	discrete-time error signal
$e_p(n)$	prediction error of a p^{th} order AR system
$e_r(n)$	prediction error within a reference window
$e_t(n)$	prediction error within a test window
$e^+(n)$	forward prediction error
$e^-(n)$	backward prediction error
$\mathbf{e}_i$	error signal vector in block LMS algorithm
$E[\cdot]$	expected value
E_s	energy of s(n)
$\mathcal{E}$	mean-square error
$\mathcal{E}(n)$	mean-square error function at time n
$\mathcal{E}_{ex}(n)$	excess mean-square error function at time n
$\mathcal{E}_{min}$	minimum mean-square error
$\mathcal{E}_{\mathbf{w}}$	mean-square error function of the weight vector $\mathbf{w}$
$\mathcal{E}_{\mathbf{w}}(n)$	mean-square error function of the weight vector $\mathbf{w}$ at time n
ϵ	fraction of a number

ε	tolerance in data compression
ϵ^2	least-squares error
F	continuous-time frequency
F_i	constant frequency values for series development
F_s	sampling rate
F_c	cut-off frequency
F_I	mean repetition frequency
$\mathbf{F}_m(n)$	matrix used in LMS filtering
f	normalized discrete-time frequency
f_c	normalized cut-off frequency
$f(\cdot)$	nonlinear function, e.g., a sigmoid
$\mathbf{\Phi}$	matrix defining a set of basis functions
$\Phi(\Omega)$	Fourier transform of $\varphi(t)$
ϕ	phase
$\boldsymbol{\varphi}$	vector basis function
$\varphi(n)$	discrete-time basis function
$\varphi(t)$	continuous-time phase function, scaling function in wavelet representation
$\varphi_{j,k}(t)$	dyadically sampled scaling function
$G(t,\Omega)$	two-dimensional Fourier transform of the kernel function $g(\tau,\nu)$
$g(\tau,\nu)$	two-dimensional continuous-time kernel function
$g(l,n)$	sum across basis functions of the product at samples l and n
g_M	update factor in recursive weighted averaging
$g(n)$	QRS detector threshold (refractory period)
$g(\cdot)$	interpolation function
$g(n,\varepsilon)$	slope function with tolerance ε for SAPA data compression
$\lvert\Gamma_{xy}(e^{j\omega})\rvert^2$	magnitude squared coherence of $x(n)$ and $y(n)$
$\Gamma_{\mathrm{SPI}}(n)$	spectral purity index
$\Gamma(\nu)$	Gamma function
γ	reflection coefficient of a lattice filter, trimming factor in the trimmed mean estimator
$\mathbf{H}$	matrix of sine/cosine basis functions
$H(e^{j\omega})$	frequency response of $h(n)$
$H(e^{j\omega},n)$	time-varying frequency response of $h(k,n)$
$H^c(e^{j\omega})$	clipped frequency response of $h(n)$
$H(z)$	transfer function of $h(n)$
$H_{ij}(z)$	cross-transfer function relating $x_i(n)$ and $x_j(n)$
$H_p(z)$	transfer function of order p AR system
$\mathcal{H}_i$	Hjorth descriptor
h	complex-valued parameter
$\mathbf{h}$	vector of discrete-time impulse response, vector of unevenly spaced samples

$h(k)$	impulse response of a discrete-time, linear, time-invariant filter
$h_\varphi(n)$	sequence of scaling coefficients
$h_\psi(n)$	sequence of wavelet coefficients
$h(k,n)$	impulse response of a discrete-time, linear, time-varying filter
$h(t)$	continuous-time impulse response
$h_e(t)$	impulse response (excitatory postsynaptical potentials)
$h_i(t)$	impulse response (inhibitory postsynaptical potentials)
$\mathbf{I}$	identity matrix
$\mathbf{i}$	column vector whose top element is one and the remaining zero
J	finest scale in wavelet decomposition, total number of leads
$\Im(\cdot)$	imaginary part
$\mathcal{J}(\cdot)$	mean-square error for continuous functions
$\jmath$	$\sqrt{-1}$
K	size of truncated set of basis functions, number of occurrence times, periodogram segmentation parameter, subset used in trimmed mean
K_ψ	number of vanishing moment for a certain wavelet
$K_{mc}(x)$	kurtosis of signal $x(n)$
$\kappa(t)$	continuous-valued indexing function
$\kappa_x(k_1,k_2)$	third-order moment (cumulant) of discrete-time process $x(n)$
L	filter length, segmented signal length
$L(\mathbf{x})$	likelihood ratio
$\mathcal{L}$	Lagrangian function
$\mathbf{\Lambda}$	diagonal matrix of eigenvalues
λ	eigenvalue, Lagrange multiplier
λ_r	average firing rate
M	number of channels/leads/events
$M(\Omega)$	Fourier transform of $m(t)$, number of realizations in an ensemble
$\mathcal{M}(p)$	function for model order determination
$m(t)$	continuous-time modulation function
m_i	constant amplitude factors in function series development
m_x	mean value of stationary process $x(n)$
$\mathbf{m}_x$	vector of mean values for $\mathbf{x}$
$m_x(n)$	mean value of the process $x(n)$
μ	adaptation parameter in the LMS algorithm, fraction of peak amplitude for QRS detection threshold selection
N	signal length,

	negative amplitude peak of an EP
N_{φ}	number of scaling coefficients $h_{\varphi}(n)$
N_{ψ}	number of wavelet coefficients $h_{\psi}(n)$
N_{FN}	number of false negative detections
N_{FP}	number of false positive detections
N_{TN}	number of true negative detections
N_{TP}	number of true positive detections
N_D	number of true detections
N_M	number of missed detections
N_F	number of false alarms
n	sequence index
∇_x	gradient with respect to the x
$\nabla_{\mathbf{w}}$	gradient with respect to the vector $\mathbf{w}$
η	threshold value
$\eta(n)$	influence function for robust, recursive averaging
$\eta_I(n)$	interval-dependent threshold
η_T	threshold parameter for wavelet denoising
Ω	continuous-time radian frequency
$\overline{\Omega}$	center of gravity of $X(\Omega)$
ω	discrete-time radian frequency
$o(k)$	binary variable
P	power,
	positive amplitude peak in evoked potentials
P_D	probability of detection
P_F	probability of false detection
P_M	probability of missed detection
P_i	power of the i^{th} component of a rational power spectrum
$P_x(\Omega)$	characteristic function of x
$\mathcal{P}_{\text{CR}}$	data compression ratio
$\mathcal{P}_{\text{PRD}}$	percentage root mean-square difference
$\mathcal{P}_{\text{RMS}}$	root mean-square parameter
$\mathcal{P}_{\text{WDD}}$	weighted diagnostic distortion
p	model order
$p_{1,2}$	pair of complex-conjugate poles
$p_x(x)$	probability density function of x
$p(\mathbf{x};\theta)$	probability density function of $\mathbf{x}$ with θ as a parameter
$p(\mathbf{x};\boldsymbol{\theta})$	probability density function of $\mathbf{x}$ with $\boldsymbol{\theta}$ as a parameter vector
$\Psi(\Omega)$	continuous-time Fourier transform of the mother wavelet
$\psi(\cdot)$	influence function for robust averaging
$\psi(t)$	mother wavelet
$\psi_{s,\tau}(t)$	continuous-time family of wavelet functions
$\psi_{j,k}(t)$	dyadic discretized family of wavelet functions

q	model order
$q(n)$	discrete-time signal
R	threshold value in the IPFM model
$\mathcal{R}$	eigenvalue-based performance index
r	radius in complex plane, steepness of a sigmoid function between the two levels
r_i	RR interval preceding the i^{th} beat
$\mathbf{r}_j$	autocorrelation vector of j lags of $x(n)$
$r(n)$	instantaneous RR interval estimate
$r_x(k)$	autocorrelation function of $x(n)$
$r_x(n_1, n_2)$	autocorrelation function of $x(n)$ between the samples n_1 and n_2
$r_x(k; n)$	time-varying autocorrelation function of $x(n)$
$r_x(\tau)$	autocorrelation function of $x(t)$
$r_{xy}(k)$	cross-correlation function of $x(n)$ and $y(n)$
ρ	cross-correlation coefficient, exponential damping factor
ρ_{ij}	cross-correlation coefficient between $x_i(n)$ and $x_j(n)$
$\rho_q(l)$	energy-normalized autocorrelation function of $q(n)$
$\mathbf{R}_x$	autocorrelation matrix of $\mathbf{x}$
$\mathbf{R}_v(n)$	spatial correlation matrix between different channels at time n
$\mathbf{R}_V$	$M \times M$ correlation matrix between the M different EPs in $\mathbf{V}$
$\mathbf{r}_{xy}$	cross-correlation vector of $x(n)$ and $\mathbf{y}(n)$
$\mathrm{Res}[\cdot, \star]$	residue of a complex-valued function $(\cdot)$ at pole $(\star)$
$\Re(\cdot)$	real part
$S_x(e^{j\omega})$	power spectrum of $x(n)$
$S_A(e^{j\omega})$	discrete-time Fourier transform of the analytic signal $s_A(n)$
$S_x^r(e^{j\omega})$	rhythmic activity of the EEG power spectrum
$S_x^a(e^{j\omega})$	unstructured activity of the EEG power spectrum
$S_x(z)$	complex power spectrum of $x(n)$
$S_x(\Omega)$	energy spectrum of $x(t)$ in time–frequency representations
$S_m(\Omega)$	power spectrum of $m(t)$
$S_d^u(\Omega)$	power density spectrum of $d^u(t)$
$S_x(t, \Omega)$	spectrogram of $x(t)$
$S_{mc}(x)$	skewness of $x(n)$
s	IPFM model parameter related to ectopic beats
$\mathbf{s}$	signal vector
$s(n)$	discrete-time signal
$\check{s}(n)$	Hilbert transform of $s(n)$
$s_A(n)$	the analytic signal of $s(n)$
$s(t)$	lowpass envelope signal
$s_c(t/\beta)$	lowpass envelope signal with varying duration β
$\hat{\mathbf{s}}_a$	vector ensemble average estimator

$\hat{\mathbf{s}}_{a_l}$	vector ensemble subaverage estimator
$\hat{\mathbf{s}}_{e,M}$	vector exponential average estimator based on M EPs
$\hat{\mathbf{s}}_{r,M}$	recursive, robust average estimator
$\hat{\mathbf{s}}_w$	vector weighted average estimator
$\hat{s}_a(n)$	ensemble average estimator
$\hat{s}_{a,M}(n)$	ensemble average estimator based on M EPs
$\hat{s}_{a_l}(n)$	ensemble subaverage estimator
$\hat{s}_{e,M}(n)$	exponential average estimator based on M EPs
$\hat{s}_{\text{med}}(n)$	ensemble median estimator
$\hat{s}_{\text{tri}}(n)$	trimmed mean estimator
ξ	signal energy
$\xi(n)$	running signal energy
σ_v^2	variance of $v(n)$
σ_v	standard deviation of $v(n)$
σ	width parameter,
	variance of a Gaussian PDF
T	sampling interval
$\mathbf{T}$	synthesis matrix for ECG lead transformation
T_I	mean interval length
t	continuous-time
$\bar{t}$	"center of gravity" of $x(t)$
t_i	occurrence time of the i^{th} beat
t_{k_e}	occurrence time of a sinus beat preceding an ectopic beat
$\hat{t}^f_{k_e+1}$	forward extended time (sinus rhythm replacing the ectopic)
$\hat{t}^b_{k_e}$	backward extended time (sinus rhythm preceding the ectopic)
t_e	occurrence time of an ectopic beat
τ	time delay/latency (continuous-time),
	convergence time for the LMS algorithm
$\boldsymbol{\theta}$	unknown vector parameter
θ	unknown scalar parameter
θ_i	discrete-time occurrence time
θ_i'	discrete-time segmentation onset of i^{th} PQRST complex
U	window signal power normalization factor
$u(t)$	continuous-time unit step function
$\mathbf{V}$	$N \times M$ matrix modeling the noise of M different EPs
$V(z)$	transfer function of the discrete-time noise $v(n)$
$V[\cdot]$	variance of a estimate
$\mathcal{V}_j$	space spanned by the translated scaling function at scale j
$\mathbf{v}$	vector of discrete-time noise samples
$\mathbf{v}_i$	vector of noise signal samples from i^{th} evoked potential
$v(t)$	continuous-time noise

$v(n)$	discrete-time noise
$\mathbf{v}(n)$	$M \times 1$ vector of noise samples at time n of M channels
ν	continuous-time radian frequency, conduction velocity
W	window length
$W_B(e^{j\omega})$	discrete-time Fourier transform of the Bartlett window
$W_x(t,\Omega)$	continuous-time Wigner–Ville distribution
$W_{x_1,x_2}(t,\Omega)$	cross Wigner–Ville distribution between $x_1(t)$ and $x_2(t)$
$\mathcal{W}_j$	space spanned by the translated wavelet function at scale j
$\mathbf{w}$	weight vector
$\mathbf{w}^o$	optimal weight vector
$\mathbf{w}(n)$	weight vector at time n
w_i	scalar weight
$w_{j,k}$	coefficient expansion of the discrete wavelet transform
$w(n)$	discrete-time window or weighting function
$w(s,\tau)$	continuous wavelet transform
$w_B(n)$	Bartlett window
$\overline{\omega}_i$	i^{th} order spectral moment
ω_i	peak frequency of the i^{th} spectral component
ω_c	normalized cut-off frequency
$\mathbf{X}$	data matrix of an ensemble of signal
$\mathbf{X}_M$	data matrix of an ensemble of M signals
$X(z)$	transfer function of $x(n)$
$X(e^{j\omega})$	discrete-time Fourier transform of $x(n)$
$X(\Omega)$	Fourier transform of $x(t)$
$X_A(\Omega)$	Fourier transform of the analytic signal $x_A(t)$
$X(t,\Omega)$	STFT of $x(t)$
$x(n)$	observed discrete-time signal
$x_{i,l}(n)$	observed discrete-time (i^{th} beat and l^{th} lead)
$\tilde{x}(n)$	reconstructed signal
$\hat{x}_p(n)$	p^{th} order linear prediction of $x(n)$
$x^{(i)}(n)$	i^{th} order discrete-time "derivative" of $x(n)$
$x(t)$	continuous-time signal
$x_c(t)$	continuous-time signal
$x_A(t)$	analytic signal of $x(t)$
$x(n)$	observed signal
$x_i(t)$	i^{th} continuous-time signal in an ensemble, wavelet approximation signal at scale i of $x(t)$
$\mathbf{x}$	vector of signal samples
$\tilde{\mathbf{x}}$	reconstructed/decompressed signal vector
$\mathbf{x}_i$	vector of signal samples from i^{th} evoked potential

$\mathbf{x}'_i$	piled lead vector of signals $\mathbf{x}_{i,j}$ from i^{th} beat
$\mathbf{x}_{i,j}$	vector of signal samples from i^{th} QRS complex at j^{th} lead
$\mathbf{x}_p$	vector of p signal samples preceding n
$\mathbf{x}(n)$	$M \times 1$ vector of samples at time n of M channels
$\boldsymbol{\mathcal{X}}$	space expanded by a set of basis functions
$y(n)$	discrete-time filter output
$y_\infty(t)$	continuous-time ECG baseline wander signal
$y(t)$	cubic spline approximation of baseline wander
$y_j(n)$	wavelet decomposition detail signal at scale j
$\mathbf{Z}$	complex variable matrix
z	complex variable
$\tilde{z}_{e,i}$	exponentially updated peak amplitude at i^{th} beat
$z_{1,2}$	pair of complex-conjugate zeros
$z(n)$	discrete-time signal
$z_d(n)$	discrete-time signal decimated from $z(n)$
$z_u(n)$	discrete-time signal with zeros inserted

B.2 Abbreviations

A/D	analog-to-digital (conversion)
AEP	auditory evoked potentials
AIC	Akaike information criterion
ANS	autonomic nervous system
AR	autoregressive
AR(p)	autoregressive process of order p
ARMA	autoregressive moving average
ARV	average rectified value
AV	atrioventricular
AZTEC	amplitude zone time epoch coding
BAEP	brainstem auditory evoked potentials
BCI	brain–computer interface
BPM	beats per minute
BLMS	block least mean-square
CCU	coronary care unit
CNS	central nervous system
CPU	central processing unit
CSA	compressed spectral array
CWD	Choi–Williams distribution
CWT	continuous wavelet transform
dB	decibel

DC	direct current
DFT	discrete Fourier transform
DSP	digital signal processor
DTFT	discrete-time Fourier transform
DWT	discrete wavelet transform
DWPT	discrete wavelet packet transform
ECG	electrocardiogram
ECoG	electrocorticogram
EEG	electroencephalogram
EG	electrogram
EGG	electrogastrogram
EMG	electromyogram
ENG	electroneurogram
EOG	electrooculogram
EP	evoked potential
ERG	electroretinogram
FT	Fourier transform (continuous-time)
FFT	Fast Fourier Transform (discrete-time)
FIR	finite impulse response
GAL	gradient adaptive lattice
HRV	heart rate variability
HT	heart timing
Hz	Hertz
IBIS	integrate body mind information system database
ICU	intensive care unit
IF	interval function
IIF	inverse interval function
IIR	infinite impulse response
IIT	inverse interval tachogram
IT	interval tachogram
IPFM	integral pulse frequency modulation
KL	Karhunen–Loeve
KLT	KL transform
kHz	kilohertz
LMS	least mean-square
LTST	long-term ST database
MA	moving average
MDL	minimum description length
MEG	magnetoencephalogram
ML	maximum likelihood
MSE	mean-square error
MMSE	minimum mean-square error

MRI	magnetic resonance imaging
MUAP	motor unit action potential
MVC	maximal voluntary contraction
NN	normal-to-normal RR interval
PC	personal computer
PDF	probability density function
PET	positron emission tomography
pNN50	pairs of NN RR intervals differing by more than 50 ms
PNS	peripheral nervous system
PRD	percentage root mean-square difference
PWVD	pseudo Wigner–Ville distribution
REM	rapid eye movement
RMS	root mean-square
Res	residue
rMSSD	root mean-square of successive differences
ROC	receiver operating characteristic
SA	sinoatrial (node)
SAPA	scan-along polygonal approximation
SDNN	standard deviation of NN intervals
SEM	spectral error measure
SEP	somatosensory evoked potentials
SNR	signal-to-noise ratio
SPA	spectral parameter analysis
SPECT	single photon emission computed tomography
SPI	spectral purity index
SQUID	superconducting quantum interference device
SSW	spikes and sharp waves
STFT	short-time Fourier transform
SVD	singular value decomposition
SVPB	supraventricular premature beat
TINN	triangular interpolation index
VCG	vectorcardiogram
VEP	visual evoked potentials
VPB	ventricular premature beat
VT	ventricular tachycardia
WCT	Wilson central terminal
WVD	Wigner–Ville distribution
WDD	weighted diagnostic distortion