Formula sheet

FORMULA SHEET (pdf (https://canvas.education.lu.se/courses/24599/files/4000982?wrap=1))

Constants (assume these unless otherwise stated)

Water speed of sound: c = 1500 m/s

Water density: ρ = 1000 kg/m^3

Water dynamic viscosity: $\eta = 10^{-3}$ Pa·s = 10^{-3} kg/(m·s)

Water surface tension: $\gamma = 72 \cdot 10^{-3}$ N/m = $72 \cdot 10^{-3}$ kg/s^2

Water on glass contact angle: $\Phi = 25^\circ$

Boltzmann constant: $k_B \approx 1.38 \cdot 10^{-23}$ m^2·kg·s^{-2}·K^{-1}

Room temperature: T = 300 K

Standard acceleration due to gravity: g = 9.81 m/s^2

Standard atmospheric pressure: 101 kPa

Equations

The volume of a sphere with radius r:

$$V = 4\pi r^3/3$$

The area of a circle with radius r:

$$A = \pi r^2$$

The force acting on an accelerating body (Newton's 2nd law):

$$F = m \cdot a$$

The pressure inside a droplet with radius r_d:

$$p_d = 2 \cdot \gamma / r_d$$

The pressure at the liquid/air interface in a capillary of radius r_c:

$$p_{liquid} = p_{air} - 2 \cdot \gamma \cdot \cos(\Phi) / r_c$$

Washburn's equation for a capillary of radius r_c:

$$L = \sqrt{(r_c \cdot \gamma \cdot t \cdot \cos\theta)/(2 \cdot \eta)}$$

Hydrostatic pressure - the pressure at the bottom of a liquid column of height *h*:

$$p_h = \rho \cdot g \cdot h$$

Stokes' drag force on a particle with radius r:

$$F_d = 6 \cdot \pi \cdot r \cdot \eta \cdot u$$

Acoustic radiation force on a sphere of radius *r* in a standing acoustic wave of energy density E_{ac}:

$$F_r = 4 \cdot \pi \cdot r^3 \cdot \Phi \cdot k \cdot E_{ac} \cdot \sin(2ky)$$

Acoustic contrast for a particle (p) in a medium (m), compressibility (κ) and density (ρ):

$$\Phi = (1 - \kappa_p/\kappa_m) / 3 + (\rho_p/\rho_m - 1) / (2 \cdot \rho_p/\rho_m + 1)$$

Wave vector for a wave of wavelength λ:

$$k = 2 \cdot \pi / \lambda$$

1D planar characteristic diffusion, time (*t*), diffusion constant *D* and distance (*l*):

$$t = l^2 / (2D)$$

Diffusion constant of a sphere with hydrodynamic radius r_h at low Reynolds number:

$$D = k_B \cdot T/(6 \cdot \pi \cdot r_h \cdot \eta)$$

Reynolds number, density *ρ,* dimension *d*, velocity *u*, viscosity η:

$$Re = \rho \cdot d \cdot u/\eta$$

Pressure drop in a channel with flow resistance R and flow rate Q:

$$\Delta p = R \cdot Q$$

Fluidic resistances in series:

$$R_{tot} = R_1 + R_2$$

Fluidic resistances in parallel:

$$R_{tot} = R_1 \cdot R_2 / (R_1 + R_2)$$

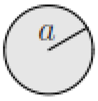
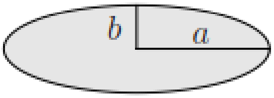
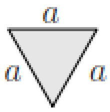
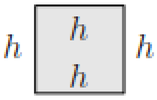
Speed of sound as a function of wavelength λ and frequency f:

$$c = \lambda \cdot f$$

Centrifugal acceleration a as a function of radial position r and rotation frequency f:

$$a = \omega^2 r = (2\pi f)^2 r$$

Table 1: *A list over the hydraulic resistance for straight channels with different cross-sectional shapes. The numerical values are calculated using the following parameters: η = 1 mPa s (water), L = 1 mm, a = 100 μm, b = 33 μm, h = 100 μm, and w = 300 μm. From Bruus, Theoretical Microfluidics, Oxford University Press (p. 75).*

shape		R_{hyd} expression	R_{hyd} $[10^{11}\ \frac{\text{Pa s}}{\text{m}^3}]$	reference
circle		$\frac{8}{\pi}\,\eta L\,\frac{1}{a^4}$	0.25	Eq. (3.39b)
ellipse		$\frac{4}{\pi}\,\eta L\,\frac{1+(b/a)^2}{(b/a)^3}\,\frac{1}{a^4}$	3.93	Eq. (3.38)
triangle		$\frac{320}{\sqrt{3}}\,\eta L\,\frac{1}{a^4}$	18.5	Eq. (3.46)
two plates		$12\,\eta L\,\frac{1}{h^3 w}$	0.40	Eq. (3.30)
rectangle		$\frac{12\,\eta L}{1-0.63(h/w)}\,\frac{1}{h^3 w}$	0.51	Eq. (3.58)
square		$28.4\,\eta L\,\frac{1}{h^4}$	2.84	Exercise 4.4
parabola		$\frac{105}{4}\,\eta L\,\frac{1}{h^3 w}$	0.88	Eq. (3.80)
arbitrary		$\approx 2\,\eta L\,\frac{\mathcal{P}^2}{\mathcal{A}^3}$	–	Eq. (3.27a)