Solution to exercise 1.19

Viscosities side by side - solution to exercise 1.19

In the Lab 1 exercise you have noted that the relative flow rates of two laminated liquids will affect the widths of the two streams. Additionally some of you have noted that the differences in viscosity of two laminated liquids can affect the relative widths of the two streams. To exemplify, even if the two flow rates are set to the same value the stream widths are not the same unless the viscosity is the same.

In this exercise you will try to show how this can be explained. Try to work with variables in the expressions. Here is the setting:

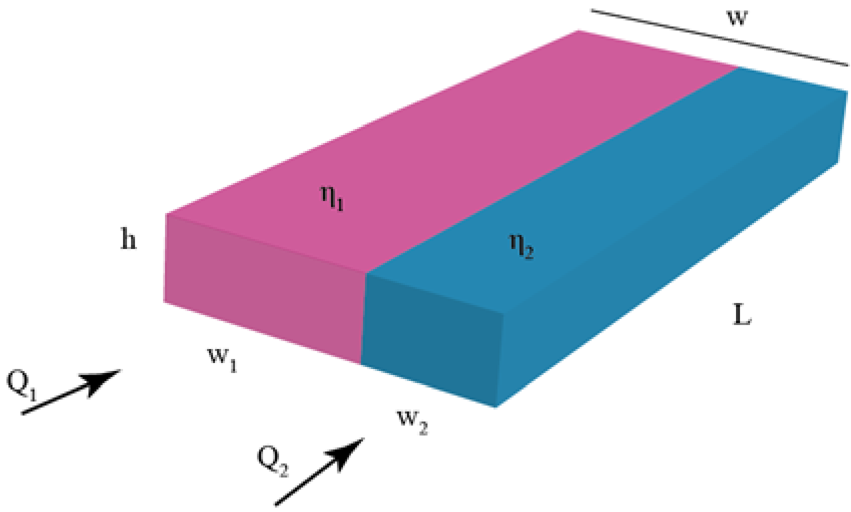
Two liquids of different viscosities η_1 and η_2 flows side by side in a low aspect ratio channel ($h \ll w$) at flow rates Q_1 and Q_2. How broad (w_i) will the respective streams be? We assume here for simplicity that the side walls as well as the interface between the two liquids have no effect on the flow so that the resistance to flow can be described by the formula for a parallel plate geometry, $R = 12\eta L/(h^3 w)$. Since the liquids have different viscosities we will need to treat them as two individual parallel channels of a combined width $w = w_1 + w_2$.

A) Try to derive an expression that indicates the position of the interface between the two liquids e.g. w_1/w_2. Remember that the pressure drop $p = p_1 = p_2$ along the channel must be the same for the two liquids because there is no rigid wall separating them. What factors determine the widths of the streams?

We can express the pressure *p* at the inlet of each channel as $p_i = Q_i R_i = Q_i 12\eta_i L/(h^3 w_i)$. Even if we would not really care what the pressure is at the inlet of the channel we can at least say that the pressure for the two flows must be the same at all points along the channel, because there is no rigid wall separating them. By setting $p_1 = p_2$ a lot of things cancel out and we arrive at the result that $Q_1\eta_1/w_1 = Q_2\eta_2/w_2$. A higher viscosity leads to a broader stream.

B) Try to figure out a way to derive the pressure at the inlet as a function of the two flow rates, the channel dimensions and the two viscosities.

If we do care about knowing the pressure at the inlet we can use that $w = w_1 + w_2$ is known and that $w_2 = w_1 Q_2\eta_2/(Q_1\eta_1)$. Solving for w_1 leads to $w_1 = w / (1 + Q_2\eta_2/(Q_1\eta_1))$ and inserting this into Ohm's law of laminar flows for one of the flow streams leads to $p = Q_1 R_1 = 12L(Q_1\eta_1 + Q_2\eta_2)/(h^3 w)$.



w
η_1
h
η_2
L
w_1
Q_1
w_2
Q_2