Answers to exercises

(PDF) (https://canvas.education.lu.se/courses/30286/files/5322790?wrap=1)

Please note: Explanations of concepts are in this document very brief or just hints and are in many cases not be considered satisfactory on an exam.

Definitions:

\rho = mass density

\eta = dynamic viscosity

v = mean flow velocity

L = some characteristic length e.g. smallest dimension

L = the length of a channel

W = the width of a channel or a flow stream

H = the height of a channel

a = radius of a sphere

r = radius of a shere, diameter of a tube or a drop

d = diameter of a tube

Q = volume flow rate

A = an area

P = the perimeter

p = pressure

1.1) L = 6.7 nm, Lead: The volume of a liquid can be related to the mass through the density.

1.2) inertial force and viscous force, in other terms: momentum and friction

1.3) (\rho v d) / \eta = (4 \rho Q) / (\pi \eta d)

1.4) Re < 2000

1.5.a) Re = 21

1.5.b) Re = 2546, I drink 1 dL in 10 seconds and I have a 5 mm diameter straw.

1.5.c) Re = 1400, (I use \eta_blood ~ 3 mPas)

1.6.a-e) I am not drawing this

1.6.f) The most narrow dimension.

1.7) Good for relating flow rate and pressure, The R in Ohm's law of fluid dynamics

1.8.a) See formula sheet "circle" (From Bruus, Theoretical Microfluidics)

1.8.c) see formula sheet "two plates" (From Bruus, Theoretical Microfluidics)

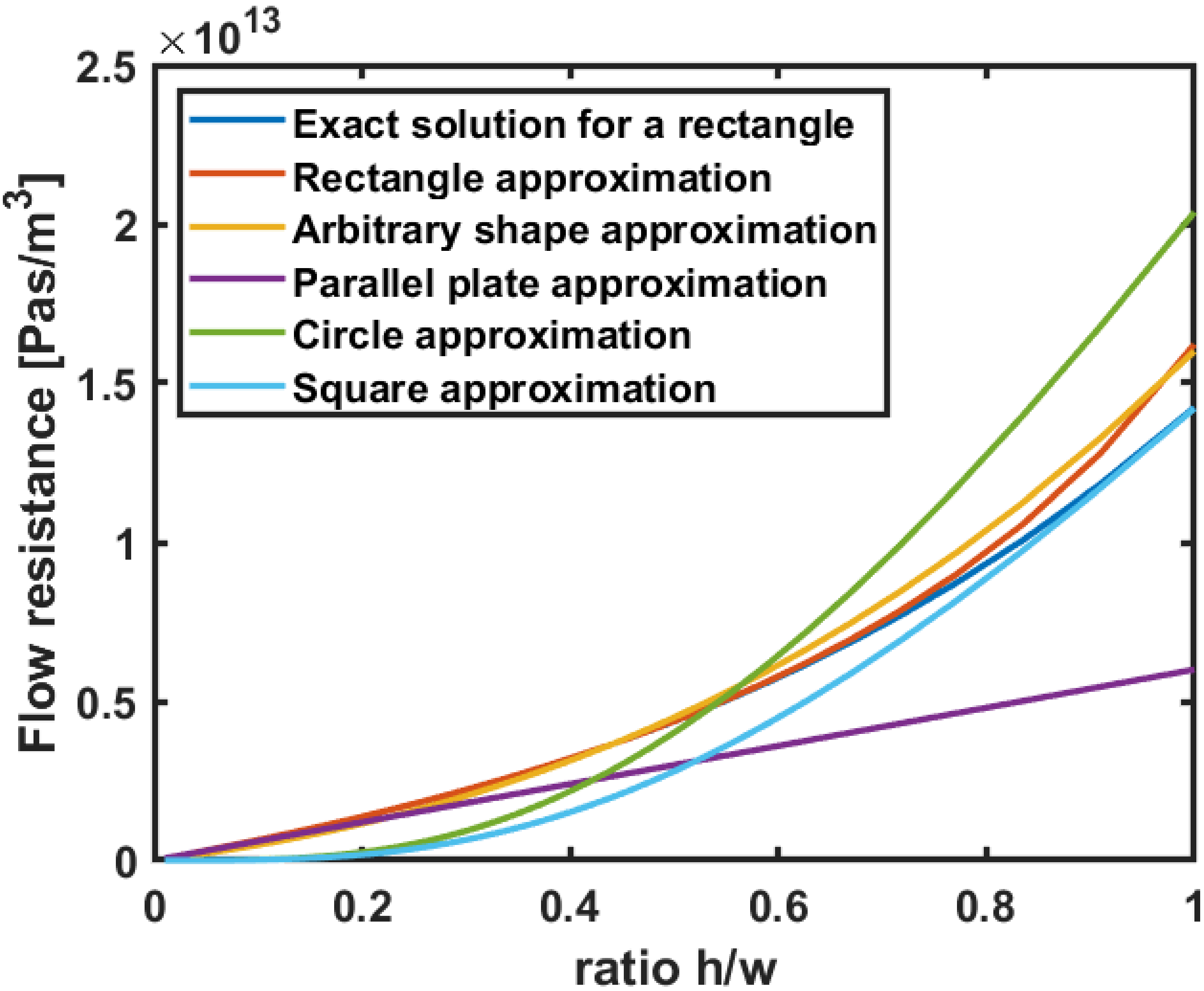
1.9.b) $R_{h\_arb} = 32\eta L/(h^4)$ = 32e13 [Pa s / m^3], $R_{h\_ded} = 28.4\eta L/(h^4)$ = 28.4e13 [Pas/m^3]

1.9.d) $R_{h\_arb} = 9\eta L/(h^4)$ = 9e13 [Pa s / m^3], $R_{h\_ded} = 8.76\eta L/(h^4)$ = 8.76e13 [Pas/m^3]

1.9.e) $R_{h\_arbitrary} = 6.71\eta L/(h^4)$ = 6.71e13 [Pa s / m^3]

1.10) $R = 2\eta LP^2/A^3$ where $P = 2\pi a$ and $A = \pi a^2$ => $R = 8\eta L/(\pi a^4)$ Q.E.D.

1.11. The final plot of the flow resistance should look something like this:



1.12.a) R_hyd = 5.1e12 [Pas/m^3], p = 85 [kPa]

1.12.b) R_hyd = 3.2e11 [Pas/m^3], p = 5.3 [kPa]

1.12.c) R_hyd = 5.4e12 [Pas/m^3], p = 90 [kPa]

1.12.d) R_hyd = 3.0e11 [Pas/m^3], p = 5.0 [kPa]

1.12.e) Q_1 = 59 [µL/min], Q_2 = 941 [µL/min]

1.13.a) R_chan = 2.54e11 [Pas/m^3]

1.13.b) Ask me when we meet

1.13.c) p1 = p2 = 18 [kPa]

1.14) To be shown in class or at the seminar

1.15) Hints: Find an expression for the flow velocity profile vs the pressure gradient (Folch page 103). Integrate the velocity over the cross section to get an expression for Q. Solve for the pressure gradient and insert in the expression for the velocity profile. Find max. For the rectangular channel, use a parallel plate geometry.

1.16) To be shown in class or at the seminar

1.17) 1 mPas

1.18) F = 10 [N]

1.19.a) W1/W2 = Q1\eta1/(Q2\eta2) (download full derivation in Canvas)

1.19.b) $p = 12L(Q_1\eta_1 + Q_2\eta_2)/(h^3w)$ (download full derivation in Canvas)

2.1) To be shown in class or at the seminar

2.2) diffusion length is x = w/2, assuming D = 10-9 m^2/s => t = 125 [s]

2.3.a) t = 125 [s]

2.3.b) Protein: x = 35 [μm], Neurotransmitter: t = 31 [ms]

2.4.a) a = 0.5 [μm], D = 4.4e-13 [m^2/s]

2.4.b) E-coli, cell volume of 0.6–0.7 μm^3, approximating as a sphere of a = 0.5 [μm], D = 4.4e-13 [m^2/s]

2.4.c) a = 5 [μm], D = 4.4e-14 [m^2/s]

2.4.d) a = 2.5 [nm], D = 8.8e-11 [m^2/s]

2.4.e) a = 0.5 [nm], D = 4.4e-10 [m^2/s]

3.1) Because molecules in a liquid want to stay in the bulk to minimize energy by maximizing the number of neighbors.

3.2) Because the surface tension compresses the droplet

3.3) \delta p = (2 \gamma) / r

3.4) \delta p = (2 \gamma cos[\Theta]) / r

3.5) h = (2 \gamma cos(\Theta)) / (\rho g r)

3.6) At the end of the capillary

3.7) Lead: friction

3.8) v = sqrt(r \gamma cos[\Theta] / [8 \eta t])

3.9) It will stop and the interface will become flat to minimize energy.

3.10) It cannot creep around a 90 degree corner (e.g. contact angle is ~25 degrees for water)

3.11.a) r = a

3.11.b) r can grow to infinity, \Delta p approaches 0

3.11.c) I know from the Bond number and capillary length that h ~ 2r ~ 3 mm.

4.1) F_g = \Delta\rho 4 \pi r^3 g / 3

4.2) F_d = F_g

4.3) F_d = 6 \pi \eta a u

4.4) \eta = 2 \Delta\rho r^2 g / (9 u)

4.5) \Delta\rho = 50 kg/m^3, t = 36 [s]

4.6) L = 3 [cm]

4.7.a) f = 2 [MHz]

4.7.b) t = 0.34 [s]

4.7.c) F_rad = 52.4 [pN]

4.7.d) F_g = 0.26 [pN] -> Q_g = F_g/F_rad*Q_rad = 0.98 [µL/min]

5.1) f = 8 / (\pi) ~= 2.55 rpm

5.2) d = 2 [8 η Q / (π ρ g)]^(1/4) = 287 µm

5.3) $p_{out} = \omega^2 \rho (R_{out}^2 - R_{in}^2) / 2$

Lead: p = F/A, F = ma, divide the tube in infinitesimally thin discs of thickness dr. Derive an expression for the dF as a function of r in a disc considering that a is a function of r due to the rotation. Then integrate the total force from Rin to Rout.

5.4) f = 12 Hz