Grundläggande samband

Trigonometriska formler

$$\sin\alpha = \cos(\alpha - \pi/2) \qquad \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos\alpha = \sin(\alpha + \pi/2) \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos^2\alpha + \sin^2\alpha = 1 \qquad 2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$
$$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha \qquad 2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$
$$2\sin\alpha\cos\alpha = \sin 2\alpha \qquad 2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$
$$\sin(-\alpha) = -\sin\alpha \qquad \sin\alpha + \sin\beta = 2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos(-\alpha) = \cos\alpha \qquad \cos\alpha + \cos\beta = 2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$
$$\cos^2\alpha = \tfrac{1}{2}(1+\cos 2\alpha)$$

$$\cos\alpha = \tfrac{1}{2}\left(e^{j\alpha} + e^{-j\alpha}\right), \quad \sin\alpha = \tfrac{1}{2j}\left(e^{j\alpha} - e^{-j\alpha}\right), \quad e^{j\alpha} = \cos\alpha + j\sin\alpha$$

Summa av geometrisk serie

$$\sum_{n=0}^{N-1} a^n = \begin{cases} N & \text{om } a = 1 \\ \frac{1-a^N}{1-a} & \text{om } a \neq 1 \end{cases}$$

Z-transformen

Z-transform av kausala signaler

1.	$\mathcal{X}(z) = Z[x(n)] = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$	Transform
2.	$x(n) = Z^{-1}[\mathcal{X}(z)] = \frac{1}{2\pi j}\int_{\Gamma} \mathcal{X}(z)z^{n-1}dz$	Inverstransform
3.	$\sum_{\nu} a_{\nu}x_{\nu}(n) \longleftrightarrow \sum_{\nu} a_{\nu}\mathcal{X}_{\nu}(z)$	Linjäritet
4.	$x(n-n_0) \longleftrightarrow z^{-n_0}\mathcal{X}(z)$	Skift (n_0 positivt eller negativt heltal)
5.	$nx(n) \longleftrightarrow -z\,\frac{d}{dz}\,\mathcal{X}(z)$	Multiplikation med n
6.	$a^n x(n) \longleftrightarrow \mathcal{X}\left(\frac{z}{a}\right)$	Skalning
7.	$x(-n) \longleftrightarrow \mathcal{X}\left(\frac{1}{z}\right)$	Spegling av tidsföljden
8.	$\left[\sum_{\ell=-\infty}^{n} x(\ell)\right] \longleftrightarrow \frac{z}{z-1}\,\mathcal{X}(z)$	Summering
9.	$x * y \longleftrightarrow \mathcal{X}(z)\cdot\mathcal{Y}(z)$	Faltning
10.	$x(n)\cdot y(n) \longleftrightarrow \frac{1}{2\pi j}\int_{\Gamma}\mathcal{Y}(\xi)\mathcal{X}\left(\frac{z}{\xi}\right)\xi^{-1}d\xi$	Produkt
11.	$x(0) = \lim_{z\to\infty}\mathcal{X}(z)$ (om gränsvärdet existerar)	Begynnelsevärdesteoremet
12.	$\lim_{n\to\infty} x(n) = \lim_{z\to 1}(z-1)\mathcal{X}(z)$ (om ROC inkluderar enhetscirkeln)	Slutvärdesteoremet
13.	$\sum_{\ell=-\infty}^{\infty} x(\ell)y(\ell) = \frac{1}{2\pi j}\int_{\Gamma}\mathcal{X}(z)\mathcal{Y}\left(\frac{1}{z}\right)z^{-1}dz$	Parsevals teorem för reellvärda tidsföljder
14.	$\sum_{\ell=-\infty}^{\infty} x^2(\ell) = \frac{1}{2\pi j}\int_{\Gamma}\mathcal{X}(z)\mathcal{X}(z^{-1})z^{-1}dz$	– –

	Talföljd		Transform
	$x(n)$	$\longleftrightarrow$	$\mathcal{X}(z)$
15.	$\delta(n)$	$\longleftrightarrow$	1
16.	$u(n)$	$\longleftrightarrow$	$\frac{1}{1-z^{-1}}$
17.	$nu(n)$	$\longleftrightarrow$	$\frac{z^{-1}}{(1-z^{-1})^2}$
18.	$\alpha^n u(n)$	$\longleftrightarrow$	$\frac{1}{1-\alpha z^{-1}}$
19.	$(n+1)\alpha^n u(n)$	$\longleftrightarrow$	$\frac{1}{(1-\alpha z^{-1})^2}$
20.	$\frac{(n+1)(n+2)\ldots(n+r-1)}{(r-1)!}\ \alpha^n u(n)$	$\longleftrightarrow$	$\frac{1}{(1-\alpha z^{-1})^r}$
21.	$\alpha^n \cos\beta n\ u(n)$	$\longleftrightarrow$	$\frac{1-z^{-1}\alpha\cos\beta}{1-z^{-1}2\alpha\cos\beta+\alpha^2 z^{-2}}$
22.	$\alpha^n \sin\beta n\ u(n)$	$\longleftrightarrow$	$\frac{z^{-1}\alpha\sin\beta}{1-z^{-1}2\alpha\cos\beta+\alpha^2 z^{-2}}$
23.	$\mathbf{F}^n u(n)$	$\longleftrightarrow$	$(\mathbf{I}-z^{-1}\mathbf{F})^{-1}$