Grundläggande samband

Trigonometriska formler

$$\begin{array}{ll}
\sin\alpha = \cos(\alpha - \pi/2) & \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \\
\cos\alpha = \sin(\alpha + \pi/2) & \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\cos^2\alpha + \sin^2\alpha = 1 & 2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta) \\
\cos^2\alpha - \sin^2\alpha = \cos 2\alpha & 2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta) \\
2\sin\alpha\cos\alpha = \sin 2\alpha & 2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta) \\
\sin(-\alpha) = -\sin\alpha & \sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\cos(-\alpha) = \cos\alpha & \cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha) &
\end{array}$$

$$\cos\alpha = \tfrac{1}{2}\,(e^{j\alpha} + e^{-j\alpha}), \quad \sin\alpha = \tfrac{1}{2j}\,(e^{j\alpha} - e^{-j\alpha}), \quad e^{j\alpha} = \cos\alpha + j\sin\alpha$$

$$A\cos\alpha + B\sin\alpha = \sqrt{A^2+B^2}\cos(\alpha-\beta)$$

där $\cos\beta = \frac{A}{\sqrt{A^2+B^2}}$, $\sin\beta = \frac{B}{\sqrt{A^2+B^2}}$

$$\text{och } \beta = \begin{cases} \arctan\frac{B}{A} & \text{om } A \geq 0 \\ \arctan\frac{B}{A} + \pi & \text{om } A < 0 \end{cases}$$

$$A\cos\alpha + B\sin\alpha = \sqrt{A^2+B^2}\sin(\alpha+\beta)$$

där $\cos\beta = \frac{B}{\sqrt{A^2+B^2}}$, $\sin\beta = \frac{A}{\sqrt{A^2+B^2}}$

$$\text{och } \beta = \begin{cases} \arctan\frac{A}{B} & \text{om } B \geq 0 \\ \arctan\frac{A}{B} + \pi & \text{om } B < 0 \end{cases}$$

Grader	Rad	sin	cos	tan	cot
0	0	0	1	0	$\pm\infty$
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
90	$\frac{\pi}{2}$	1	0	$\pm\infty$	0

Z-transformen

Z-transform av kausala signaler

1.	$\mathcal{X}(z) = Z[x(n)] = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$	Transform
2.	$x(n) = Z^{-1}[\mathcal{X}(z)] = \frac{1}{2\pi j}\int_\Gamma \mathcal{X}(z)z^{n-1}dz$	Inverstransform
3.	$\sum_\nu a_\nu x_\nu(n) \longleftrightarrow \sum_\nu a_\nu \mathcal{X}_\nu(z)$	Linjäritet
4.	$x(n-n_0) \longleftrightarrow z^{-n_0}\mathcal{X}(z)$	Skift (n_0 positivt eller negativt heltal)
5.	$nx(n) \longleftrightarrow -z\,\frac{d}{dz}\,\mathcal{X}(z)$	Multiplikation med n
6.	$a^n x(n) \longleftrightarrow \mathcal{X}\left(\frac{z}{a}\right)$	Skalning
7.	$x(-n) \longleftrightarrow \mathcal{X}\left(\frac{1}{z}\right)$	Spegling av tidsföljden
8.	$\left[\sum_{\ell=-\infty}^{n} x(\ell)\right] \longleftrightarrow \frac{z}{z-1}\,\mathcal{X}(z)$	Summering
9.	$x * y \longleftrightarrow \mathcal{X}(z)\cdot\mathcal{Y}(z)$	Faltning
10.	$x(n)\cdot y(n) \longleftrightarrow \frac{1}{2\pi j}\int_\Gamma \mathcal{Y}(\xi)\mathcal{X}\left(\frac{z}{\xi}\right)\xi^{-1}d\xi$	Produkt
11.	$x(0) = \lim_{z\to\infty}\mathcal{X}(z)$ (om gränsvärdet existerar)	Begynnelsevärdesteoremet
12.	$\lim_{n\to\infty} x(n) = \lim_{z\to 1}(z-1)\mathcal{X}(z)$ (om ROC inkluderar enhetscirkeln)	Slutvärdesteoremet
13.	$\sum_{\ell=-\infty}^{\infty} x(\ell)y(\ell) = \frac{1}{2\pi j}\int_\Gamma \mathcal{X}(z)\mathcal{Y}\left(\frac{1}{z}\right)z^{-1}dz$	Parsevals teorem för reellvärda tidsföljder
14.	$\sum_{\ell=-\infty}^{\infty} x^2(\ell) = \frac{1}{2\pi j}\int_\Gamma \mathcal{X}(z)\mathcal{X}(z^{-1})z^{-1}dz$	– --

	Talföljd		Transform
	$x(n)$	$\longleftrightarrow$	$\mathcal{X}(z)$
15.	$\delta(n)$	$\longleftrightarrow$	1
16.	$u(n)$	$\longleftrightarrow$	$\frac{1}{1-z^{-1}}$
17.	$nu(n)$	$\longleftrightarrow$	$\frac{z^{-1}}{(1-z^{-1})^2}$
18.	$\alpha^n u(n)$	$\longleftrightarrow$	$\frac{1}{1-\alpha z^{-1}}$
19.	$(n+1)\alpha^n u(n)$	$\longleftrightarrow$	$\frac{1}{(1-\alpha z^{-1})^2}$
20.	$\frac{(n+1)(n+2)\ldots(n+r-1)}{(r-1)!}\alpha^n u(n)$	$\longleftrightarrow$	$\frac{1}{(1-\alpha z^{-1})^r}$
21.	$\alpha^n \cos\beta n\ u(n)$	$\longleftrightarrow$	$\frac{1-z^{-1}\alpha\cos\beta}{1-z^{-1}2\alpha\cos\beta+\alpha^2 z^{-2}}$
22.	$\alpha^n \sin\beta n\ u(n)$	$\longleftrightarrow$	$\frac{z^{-1}\alpha\sin\beta}{1-z^{-1}2\alpha\cos\beta+\alpha^2 z^{-2}}$
23.	$\mathbf{F}^n u(n)$	$\longleftrightarrow$	$(\mathbf{I}-z^{-1}\mathbf{F})^{-1}$

Enkelsidig Z-transform av icke kausala signaler

Beteckning

$\mathcal{X}^+(z)=\sum_{n=0}^{\infty}x(n)z^{-n}$	Enkelsidig Z-transform, $x(n)$ ej nödvändigtvis kausal
$\mathcal{X}(z)=\mathcal{X}^+(z)$	För kausala signaler

Vid skift av $x(n)$ erhålles:
i) skift ett steg

$$\begin{aligned} x(n-1) &\longleftrightarrow z^{-1}\mathcal{X}^+(z)+x(-1) \\ x(n+1) &\longleftrightarrow z\mathcal{X}^+(z)-x(0)\cdot z \end{aligned}$$

ii) skift n_0 steg ($n_0 \geq 0$)

$$\begin{aligned} x(n-n_0) &\longleftrightarrow z^{-n_0}\mathcal{X}^+(z)+x(-1)z^{-n_0+1}+ \\ &\qquad +x(-2)z^{-n_0+2}+\ldots+x(-n_0) \\ x(n+n_0) &\longleftrightarrow z^{n_0}\mathcal{X}^+(z)-x(0)z^{n_0}-x(1)z^{n_0-1}-\ldots-x(n_0-1)z \end{aligned}$$