Formelsamling i kretsteori, ellära och elektronik

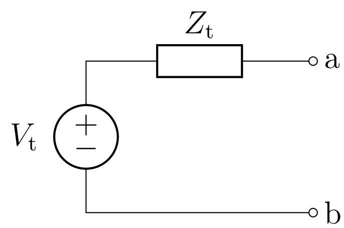
Elektro- och informationsteknik
Lunds tekniska högskola
Januari 2022

Kretsteori

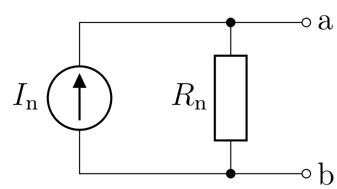
Komplexvärden

- Realdelskonvention: $v(t) = \mathrm{Re}\{V\mathrm{e}^{\mathrm{j}\omega t}\}$ och $i(t) = \mathrm{Re}\{I\mathrm{e}^{\mathrm{j}\omega t}\}$.
- Imaginärdelskonvention: $v(t) = \mathrm{Im}\{V\mathrm{e}^{\mathrm{j}\omega t}\}$ och $i(t) = \mathrm{Im}\{I\mathrm{e}^{\mathrm{j}\omega t}\}$.

Tvåpolsekvivalenter



Thévenin



Norton

Komplex effekt

$$
\begin{aligned}
S &= \frac{1}{2}VI^* = P + \mathrm{j}Q = |S|(\cos\varphi + \mathrm{j}\sin\varphi) \\
S &= \text{komplex effekt [VA]} \\
|S| &= \text{skenbar effekt [VA]} \\
P &= \mathrm{Re}\, S = \text{aktiv effekt (=tidsmedelvärdet av effektförbrukningen) [W]} \\
Q &= \mathrm{Im}\, S = \text{reaktiv effekt } [\mathrm{VA_r}]{=}[\mathrm{VAR}] \\
\cos\varphi &= \text{effektfaktor}
\end{aligned}
$$

Effektanpassningsregeln

$$
Z_\mathrm{L} = Z_\mathrm{i}^* \quad \text{och} \quad \max\{P_\mathrm{L}\} = \frac{|V|^2}{8R_\mathrm{i}}
$$

Ömsesidig induktans

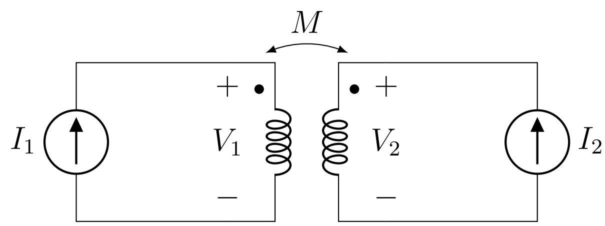
$$\begin{cases} V_1 = \mathrm{j}\omega L_1 I_1 + \mathrm{j}\omega M I_2 \\ V_2 = \mathrm{j}\omega L_2 I_2 + \mathrm{j}\omega M I_1 \end{cases}$$

$L_1, L_2 =$ självinduktanser

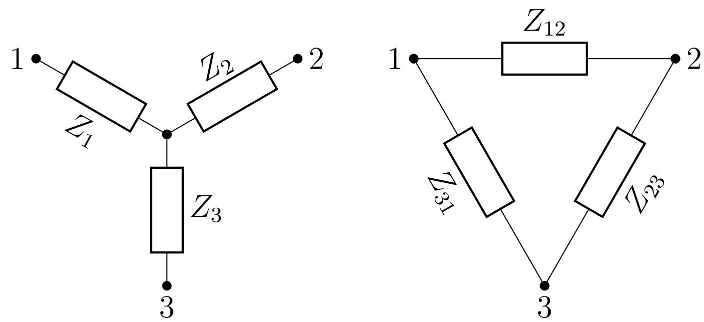
$M =$ ömsesidig induktans

$M = k\sqrt{L_1 L_2}$ där $0 \leq k \leq 1$

$k =$ kopplingsfaktorn



Nätverkstransformation



Y till Δ

$$Z_{12} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$
$$Z_{23} = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1}$$
$$Z_{31} = Z_3 + Z_1 + \frac{Z_3 Z_1}{Z_2}$$

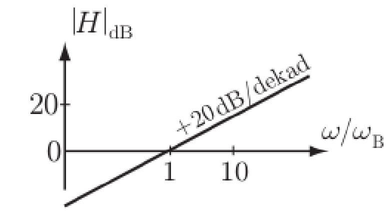
Δ till Y

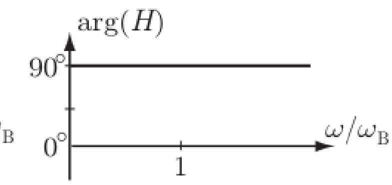
$$Z_1 = \frac{Z_{31} Z_{12}}{Z_{12} + Z_{23} + Z_{31}}$$
$$Z_2 = \frac{Z_{12} Z_{23}}{Z_{12} + Z_{23} + Z_{31}}$$
$$Z_3 = \frac{Z_{23} Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

Rätlinjeapproximationer av Bodediagram

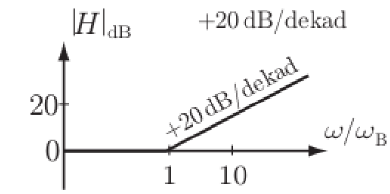
$H(s)$	Amplitud	Fas

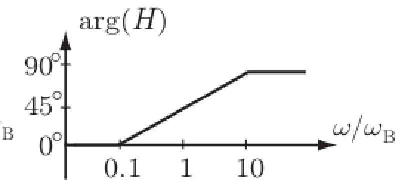
$$\frac{s}{\omega_\mathrm{B}}$$



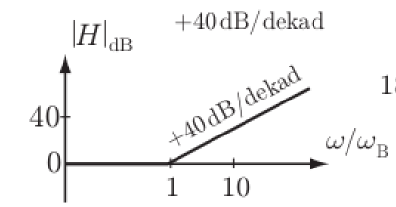


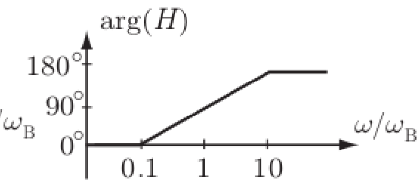
$$1+\frac{s}{\omega_\mathrm{B}}$$



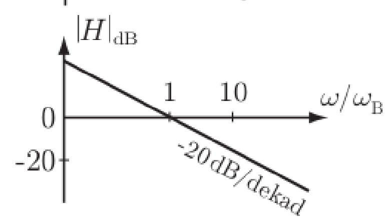


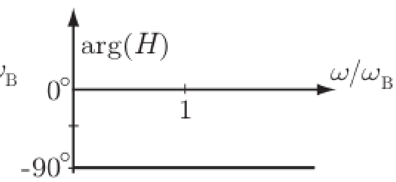
$$1+2\zeta\frac{s}{\omega_\mathrm{B}}+\frac{s^2}{\omega_\mathrm{B}^2}$$



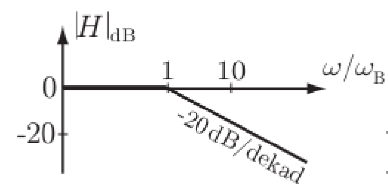


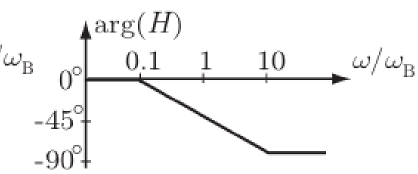
$$\left(\frac{s}{\omega_\mathrm{B}}\right)^{-1}$$



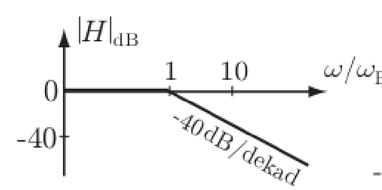


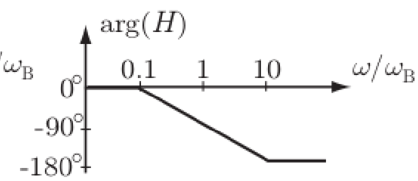
$$\left(1+\frac{s}{\omega_\mathrm{B}}\right)^{-1}$$





$$\left(1+2\zeta\frac{s}{\omega_\mathrm{B}}+\frac{s^2}{\omega_\mathrm{B}^2}\right)^{-1}$$



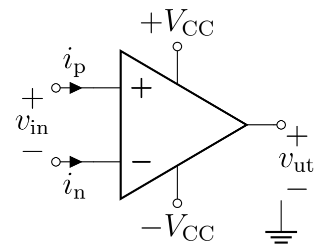


OBS! Det skall gälla att $|\zeta| \leq 1$.

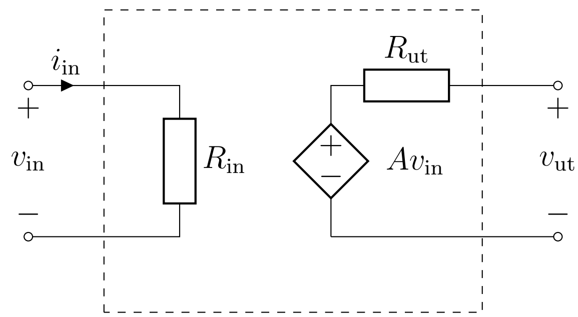
Elektronik

Ideal operationsförstärkare (OP)

För en ideal OP är $i_\mathrm{p} = i_\mathrm{n} = 0$. Vi använder vanligtvis negativ återkoppling där också $v_\mathrm{in} = 0$.



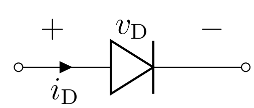
Kretsmodell av spänningsförstärkare



Dioder

Shockleyekvationen för en diod är

$$i_\mathrm{D} = I_\mathrm{s}\left(\mathrm{e}^{\frac{v_\mathrm{D}}{nV_T}} - 1\right)$$

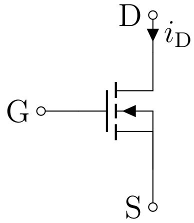
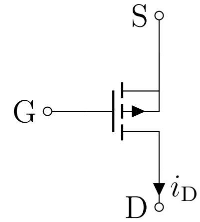


där $V_T = \frac{kT}{q}$, $q \approx 1.6 \cdot 10^{-19}\,\mathrm{C}$ och $k \approx 1.38 \cdot 10^{-23}\,\mathrm{J/K}$.

Småsignalresistansen är

$$r_\mathrm{d} = \frac{1}{\left.\frac{\mathrm{d}i_\mathrm{D}}{\mathrm{d}v_\mathrm{D}}\right|_\mathrm{arbetspunkt}}$$

MOSFET

	NMOS	PMOS
Kretssymbol		
$\mu \approx$	$675\,\mathrm{cm^2V^{-1}s^{-1}}$	$240\,\mathrm{cm^2V^{-1}s^{-1}}$
$\kappa \approx$	$115\,\mu\mathrm{AV^{-2}}$	$40\,\mu\mathrm{AV^{-2}}$
$V_\mathrm{t} \approx$	$+0.5\,\mathrm{V}$	$-0.6\,\mathrm{V}$
Subtröskel (strypt område)	$v_\mathrm{GS} \leq V_\mathrm{t}$, $v_\mathrm{DS} \geq 0$, $i_\mathrm{D} = 0$	$v_\mathrm{GS} \geq V_\mathrm{t}$, $v_\mathrm{DS} \leq 0$, $i_\mathrm{D} = 0$
Linjärt område	$v_\mathrm{GS} \geq V_\mathrm{t}$, $0 \leq v_\mathrm{DS} \leq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(2(v_\mathrm{GS} - V_\mathrm{t})v_\mathrm{DS} - v_\mathrm{DS}^2)$	$v_\mathrm{GS} \leq V_\mathrm{t}$, $0 \geq v_\mathrm{DS} \geq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(2(v_\mathrm{GS} - V_\mathrm{t})v_\mathrm{DS} - v_\mathrm{DS}^2)$
Mättnads-område	$v_\mathrm{GS} \geq V_\mathrm{t}$, $v_\mathrm{DS} \geq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(v_\mathrm{GS} - V_\mathrm{t})^2$	$v_\mathrm{GS} \leq V_\mathrm{t}$, $v_\mathrm{DS} \leq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(v_\mathrm{GS} - V_\mathrm{t})^2$
v_DS, v_GS	Vanligtvis positiva	Vanligtvis negativa

$$K = \frac{W}{L}\frac{\kappa}{2}$$

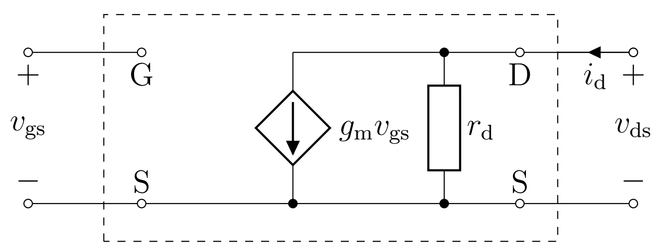
Småsignalmodell

Småsignalmodell för en FET, där

$$g_\mathrm{m} = \left.\frac{\partial i_\mathrm{D}}{\partial v_\mathrm{GS}}\right|_{\text{arbetspunkt}}$$

och

$$\frac{1}{r_\mathrm{d}} = \left.\frac{\partial i_\mathrm{D}}{\partial v_\mathrm{DS}}\right|_{\text{arbetspunkt}}$$



Ellära

Lorentzkraften

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Elektriskt fält och potential från punktladdning

$$\boldsymbol{E} = \frac{q}{4\pi\varepsilon_0 r^2}\boldsymbol{e}_r \qquad V = \frac{q}{4\pi\varepsilon_0 r}$$

Spänning

$$V = V_1 - V_2 = \int_{P_1}^{P_2} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r}$$

Elektrisk dipol

$$\boldsymbol{p} = p\boldsymbol{e}_z \qquad p = q\ell \qquad \boldsymbol{E} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\boldsymbol{e}_r + \sin\theta\boldsymbol{e}_\theta)$$

Polarisation $\boldsymbol{P}$ och elektrisk flödestäthet $\boldsymbol{D}$

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}$$
$$\boldsymbol{P} = \chi_\mathrm{e}\varepsilon_0\boldsymbol{E}$$
$$\varepsilon_\mathrm{r} = 1 + \chi_\mathrm{e}$$

Strömtäthet och resistans för rak ledare

$$\boldsymbol{J} = \frac{i}{A}\boldsymbol{e}_x \qquad R = \rho\frac{\ell}{A}$$

Plattkondensator

$$C = \varepsilon_\mathrm{r}\varepsilon_0\frac{A}{d}$$

RCL-beräkningar

R	C	L
$R = \dfrac{v_a - v_b}{i}$	$C = \dfrac{q}{v_a - v_b}$	$L = \dfrac{\phi}{i}$
$\int_S \boldsymbol{J} \cdot \boldsymbol{e}_n \,\mathrm{d}S = i$	$\oint_S \boldsymbol{D} \cdot \boldsymbol{e}_n \,\mathrm{d}S = q$	$\int_S \boldsymbol{B} \cdot \boldsymbol{e}_n \,\mathrm{d}S = \phi$
$\int_{P_a}^{P_b} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} = v_a - v_b$	$\int_{P_a}^{P_b} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} = v_a - v_b$	$\oint_C \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{r} = i$
$\boldsymbol{J} = \sigma \boldsymbol{E}$	$\boldsymbol{D} = \varepsilon_0 \varepsilon_\mathrm{r} \boldsymbol{E}$	$\boldsymbol{B} = \mu_0 \mu_\mathrm{r} \boldsymbol{H}$

Fälten uppfyller följande villkor:

$$\nabla \times \boldsymbol{E} = 0 \quad \Leftrightarrow \quad \boldsymbol{E} = -\nabla V \quad \Leftrightarrow \quad \oint_C \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} = 0$$

$$\nabla \cdot \boldsymbol{J} = 0 \quad \Leftrightarrow \quad \oint_S \boldsymbol{J} \cdot \boldsymbol{e}_n \,\mathrm{d}S = 0$$

$$\nabla \cdot \boldsymbol{B} = 0 \quad \Leftrightarrow \quad \oint_S \boldsymbol{B} \cdot \boldsymbol{e}_n \,\mathrm{d}S = 0$$

Kretsparametrarna i effekt- och energiuttryck:

	R	C	L
Krets	$p = Ri^2 = v^2/R$	$w_\mathrm{e} = \dfrac{1}{2}Cv^2 = \dfrac{1}{2}q^2/C$	$w_\mathrm{m} = \dfrac{1}{2}Li^2 = \dfrac{1}{2}\phi^2/L$
Fält	$p = \int \boldsymbol{E} \cdot \boldsymbol{J} \,\mathrm{d}V$	$w_\mathrm{e} = \dfrac{1}{2} \displaystyle\int \boldsymbol{D} \cdot \boldsymbol{E} \,\mathrm{d}V$	$w_\mathrm{m} = \dfrac{1}{2} \displaystyle\int \boldsymbol{B} \cdot \boldsymbol{H} \,\mathrm{d}V$

Transmissionsledningar

Ledningsekvationerna, förlustfri ledning

$$-\frac{\partial v}{\partial x} = L'\frac{\partial i}{\partial t}$$
$$-\frac{\partial i}{\partial x} = C'\frac{\partial v}{\partial t}$$

Allmän lösning, förlustfri ledning

$$v(x,t) = v^+(x - v_\mathrm{p}t) + v^-(x + v_\mathrm{p}t)$$
$$i(x,t) = \frac{1}{Z_0}\big(v^+(x - v_\mathrm{p}t) - v^-(x + v_\mathrm{p}t)\big)$$

$$v_\mathrm{p} = \frac{1}{\sqrt{L'C'}} \qquad Z_0 = \sqrt{\frac{L'}{C'}} \qquad L'C' = \mu_\mathrm{r}\mu_0\varepsilon_r\varepsilon_0$$

Ledningsekvationerna, sinusformigt tidsberoende

$$-\frac{\mathrm{d}V}{\mathrm{d}x} = R'I + \mathrm{j}\omega L'I$$
$$-\frac{\mathrm{d}I}{\mathrm{d}x} = G'V + \mathrm{j}\omega C'V$$

Allmän lösning, sinusformigt tidsberoende

$$V(x) = V_1\mathrm{e}^{-\gamma x} + V_2\mathrm{e}^{\gamma x}$$
$$I(x) = \frac{1}{Z_0}\big(V_1\mathrm{e}^{-\gamma x} - V_2\mathrm{e}^{\gamma x}\big)$$

Utbredningskonstant

$$\gamma = \sqrt{(R' + \mathrm{j}\omega L')(G' + \mathrm{j}\omega C')} = \alpha + \mathrm{j}\beta$$

Karakteristisk impedans

$$Z_0 = \sqrt{\frac{R' + \mathrm{j}\omega L'}{G' + \mathrm{j}\omega C'}}$$

Impedansen för en ledning med längden ℓ avslutad med Z_L

$$Z_\mathrm{in} = Z_0 \frac{Z_\mathrm{L}\cosh(\gamma\ell) + Z_0\sinh(\gamma\ell)}{Z_0\cosh(\gamma\ell) + Z_\mathrm{L}\sinh(\gamma\ell)} = Z_0 \frac{1+\Gamma \mathrm{e}^{-2\gamma\ell}}{1-\Gamma \mathrm{e}^{-2\gamma\ell}}$$

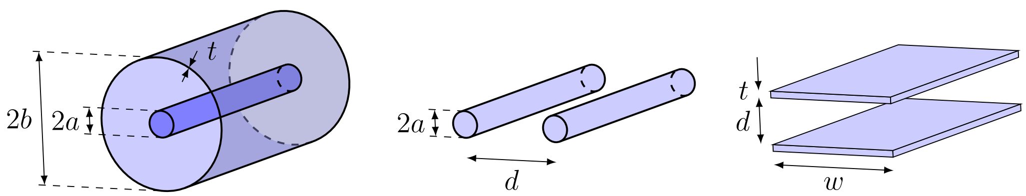
Impedansen för en förlustfri ledning med längden ℓ avslutad med Z_L

$$Z_\mathrm{in} = Z_0 \frac{Z_\mathrm{L}\cos(\beta\ell) + \mathrm{j}Z_0\sin(\beta\ell)}{Z_0\cos(\beta\ell) + \mathrm{j}Z_\mathrm{L}\sin(\beta\ell)} = Z_0 \frac{1+\Gamma \mathrm{e}^{-\mathrm{j}2\beta\ell}}{1-\Gamma \mathrm{e}^{-\mathrm{j}2\beta\ell}}$$

Reflektionsfaktorn för spänning vid belastningen

$$\Gamma = \frac{Z_\mathrm{L} - Z_0}{Z_\mathrm{L} + Z_0}$$

Kretsparametrar för transmissionsledningsgeometrier



Parameter	Koaxial	Tvåtråds	Platta
R' [Ω/m]	$\frac{1}{2\pi\delta\sigma_\mathrm{c}}\left[\frac{1}{a}+\frac{1}{b}\right]$ ($\delta \ll a, t$)	$\frac{1}{\pi a\delta\sigma_\mathrm{c}}$ ($\delta \ll a$)	$\frac{2}{w\delta\sigma_\mathrm{c}}$ ($\delta \ll t$)
L' [H/m]	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{2a}$	$\frac{\mu d}{w}$
G' [S/m]	$\frac{2\pi\sigma}{\ln\frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\sigma w}{d}$
C' [F/m]	$\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$	$\frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\varepsilon w}{d}$ ($w \gg d$)

Inträngningsdjupet i metallen är $\delta = 1/\sqrt{\pi f \mu_\mathrm{c}\sigma_\mathrm{c}}$.

Matematiska formler och samband

Trigonometriska formler

$$\sin\alpha = \cos(\alpha - \pi/2) \qquad \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos\alpha = \sin(\alpha + \pi/2) \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos^2\alpha + \sin^2\alpha = 1 \qquad 2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$
$$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha \qquad 2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$
$$2\sin\alpha\cos\alpha = \sin 2\alpha \qquad 2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$A\cos\alpha + B\sin\alpha = \sqrt{A^2+B^2}\cos(\alpha-\beta) \quad \text{där} \quad \cos\beta = \frac{A}{\sqrt{A^2+B^2}},\ \sin\beta = \frac{B}{\sqrt{A^2+B^2}}$$

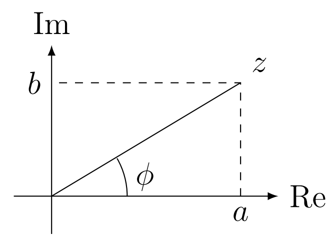
$$\mathrm{e}^{\mathrm{j}\alpha} = \cos\alpha + \mathrm{j}\sin\alpha \qquad \cos\alpha = \frac{\mathrm{e}^{\mathrm{j}\alpha} + \mathrm{e}^{-\mathrm{j}\alpha}}{2} \qquad \sin\alpha = \frac{\mathrm{e}^{\mathrm{j}\alpha} - \mathrm{e}^{-\mathrm{j}\alpha}}{2\mathrm{j}}$$

Komplexa tal

$$z = a + \mathrm{j}b = |z|\mathrm{e}^{\mathrm{j}\phi}$$

där

$$|z| = \sqrt{a^2+b^2} \quad \text{och om } a > 0 \text{ är} \quad \phi = \arctan\frac{b}{a}$$



Ekvationssystem (2×2)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$

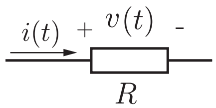
med lösning

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$

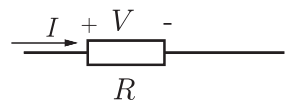
Laplacetransformen

$$\frac{\mathrm{d}}{\mathrm{d}t} \longrightarrow s$$

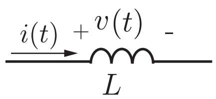
Tidsdomän $\longrightarrow$ Laplacedomän



$v(t) = Ri(t)$ $\xrightarrow{\mathcal{L}}$



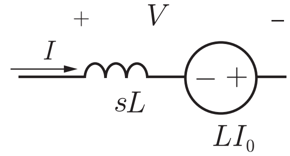
$V(s) = RI(s)$



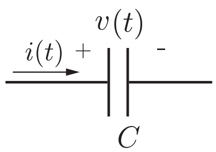
$v(t) = L\dfrac{\mathrm{d}i(t)}{\mathrm{d}t}$

$i(0-) = I_0$

$\xrightarrow{\mathcal{L}}$



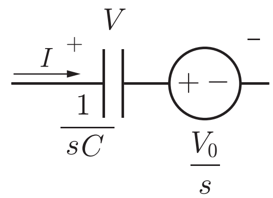
$V(s) = L(sI(s) - I_0)$



$C\dfrac{\mathrm{d}v(t)}{\mathrm{d}t} = i(t)$

$v(0-) = V_0$

$\xrightarrow{\mathcal{L}}$



$V(s) = \dfrac{I(s)}{sC} + \dfrac{V_0}{s}$

	$f(t)$	$F(s)$
1.	$\alpha f(t)$	$\alpha F(s)$
2.	$f_1(t) + f_2(t) + f_3(t) + \cdots$	$F_1(s) + F_2(s) + F_3(s) + \cdots$
3.	$\dfrac{\mathrm{d}f(t)}{\mathrm{d}t}$	$sF(s) - f(0^-)$
4.	$\displaystyle\int_0^t f(\tau)\,\mathrm{d}\tau$	$\dfrac{F(s)}{s}$
5.	$f(t-a)\,\mathrm{u}(t-a), \quad a > 0$	$\mathrm{e}^{-as}F(s)$
6.	$\mathrm{e}^{-at}f(t)$	$F(s+a)$
7.	$f(at), \quad a > 0$	$\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$

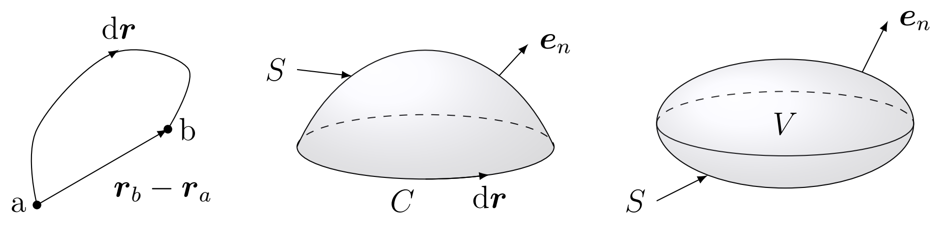
Begynnelsevärdessatsen $\quad \lim\limits_{t\to 0_+} f(t) = \lim\limits_{s\to\infty} sF(s)$

Slutvärdessatsen $\quad \lim\limits_{t\to\infty} f(t) = \lim\limits_{s\to 0} sF(s)$

	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$\dfrac{\mathrm{d}^n}{\mathrm{d}t^n}\,\delta(t)$	s^n
3.	$\mathrm{u}(t)$, enhetssteget	$\dfrac{1}{s}$
4.	$\dfrac{t^n}{n!}\;\mathrm{u}(t)$	$\dfrac{1}{s^{n+1}}$
5.	$\mathrm{e}^{-at}\;\mathrm{u}(t)$	$\dfrac{1}{s+a}$
6.	$\dfrac{t^n}{n!}\,\mathrm{e}^{-at}\;\mathrm{u}(t)$	$\dfrac{1}{(s+a)^{n+1}}$
7.	$\dfrac{\mathrm{e}^{-at}-\mathrm{e}^{-bt}}{b-a}\;\mathrm{u}(t)$	$\dfrac{1}{(s+a)(s+b)}$
8.	$\dfrac{a\mathrm{e}^{-at}-b\mathrm{e}^{-bt}}{a-b}\;\mathrm{u}(t)$	$\dfrac{s}{(s+a)(s+b)}$
9.	$\sin(\omega_0 t)\;\mathrm{u}(t)$	$\dfrac{\omega_0}{s^2+\omega_0^2}$
10.	$\cos(\omega_0 t)\;\mathrm{u}(t)$	$\dfrac{s}{s^2+\omega_0^2}$
11.	$\big(\sin(\omega_0 t)-\omega_0 t\,\cos(\omega_0 t)\big)\;\mathrm{u}(t)$	$\dfrac{2\omega_0^3}{(s^2+\omega_0^2)^2}$
12.	$\omega_0 t\,\sin(\omega_0 t)\;\mathrm{u}(t)$	$\dfrac{2\omega_0^2 s}{(s^2+\omega_0^2)^2}$
13.	$\mathrm{e}^{-at}\,\sin(\omega_0 t)\;\mathrm{u}(t)$	$\dfrac{\omega_0}{(s+a)^2+\omega_0^2}$
14.	$\mathrm{e}^{-at}\,\cos(\omega_0 t)\;\mathrm{u}(t)$	$\dfrac{s+a}{(s+a)^2+\omega_0^2}$

Vektoranalys och koordinatsystem

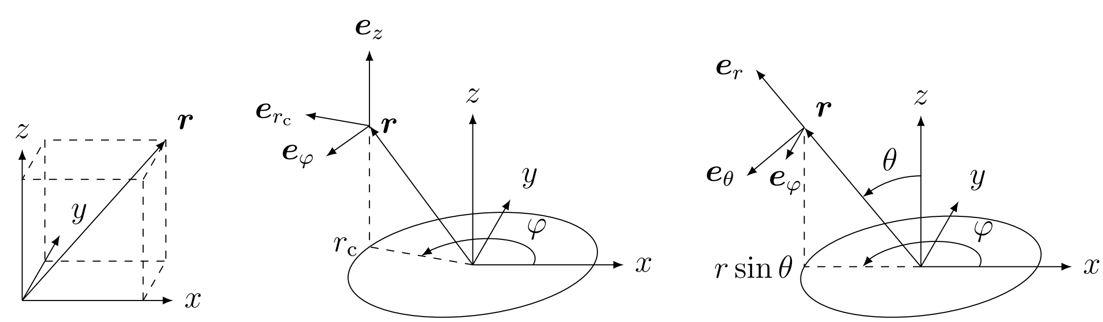
Integralsatser



Kurvintegral Ytintegral Volymintegral

Analysens huvudsats:	$\int_a^b f'(x)\,\mathrm{d}x = f(b) - f(a)$
Gradientsatsen:	$\int_C \nabla\psi(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{r} = \psi(\boldsymbol{r}_b) - \psi(\boldsymbol{r}_a)$
Stokes sats:	$\int_S (\nabla \times \boldsymbol{A}) \cdot \boldsymbol{e}_n\,\mathrm{d}S = \oint_C \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{r}$
Gauss sats:	$\int_V \nabla \cdot \boldsymbol{A}\,\mathrm{d}V = \oint_S \boldsymbol{A} \cdot \boldsymbol{e}_n\,\mathrm{d}S$

Koordinatsystem



Kartesiska Cylindriska Sfäriska

Kartesiska koordinater

$$
\begin{aligned}
\boldsymbol{r} &= \boldsymbol{e}_x x + \boldsymbol{e}_y y + \boldsymbol{e}_z z \\
\nabla\psi &= \boldsymbol{e}_x \frac{\partial\psi}{\partial x} + \boldsymbol{e}_y \frac{\partial\psi}{\partial y} + \boldsymbol{e}_z \frac{\partial\psi}{\partial z} \\
\nabla\cdot\boldsymbol{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\
\nabla\times\boldsymbol{A} &= \boldsymbol{e}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \boldsymbol{e}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \boldsymbol{e}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \\
\nabla^2\psi &= \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \\
\mathrm{d}\boldsymbol{r} &= \boldsymbol{e}_x \,\mathrm{d}x + \boldsymbol{e}_y \,\mathrm{d}y + \boldsymbol{e}_z \,\mathrm{d}z \\
\mathrm{d}S &= \begin{cases} \mathrm{d}y\,\mathrm{d}z & (\text{normalriktning } \boldsymbol{e}_x) \\ \mathrm{d}x\,\mathrm{d}z & (\text{normalriktning } \boldsymbol{e}_y) \\ \mathrm{d}x\,\mathrm{d}y & (\text{normalriktning } \boldsymbol{e}_z) \end{cases} \\
\mathrm{d}V &= \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z
\end{aligned}
$$

Cylindriska koordinater

$$
\begin{aligned}
x &= r_\mathrm{c}\cos\varphi & r_\mathrm{c} &= \sqrt{x^2+y^2} \\
y &= r_\mathrm{c}\sin\varphi & \varphi &= \begin{cases} \arccos\frac{x}{\sqrt{x^2+y^2}} & y \geq 0 \\ 2\pi - \arccos\frac{x}{\sqrt{x^2+y^2}} & y < 0 \end{cases} \\
z &= z & z &= z \\
\boldsymbol{e}_x &= \boldsymbol{e}_{r_\mathrm{c}}\cos\varphi - \boldsymbol{e}_\varphi\sin\varphi & \boldsymbol{e}_{r_\mathrm{c}} &= \boldsymbol{e}_x\cos\varphi + \boldsymbol{e}_y\sin\varphi \\
\boldsymbol{e}_y &= \boldsymbol{e}_{r_\mathrm{c}}\sin\varphi + \boldsymbol{e}_\varphi\cos\varphi & \boldsymbol{e}_\varphi &= -\boldsymbol{e}_x\sin\varphi + \boldsymbol{e}_y\cos\varphi \\
\boldsymbol{e}_z &= \boldsymbol{e}_z & \boldsymbol{e}_z &= \boldsymbol{e}_z
\end{aligned}
$$

Differentialoperatorer

$$
\begin{aligned}
\nabla\psi &= \boldsymbol{e}_{r_\mathrm{c}} \frac{\partial\psi}{\partial r_\mathrm{c}} + \boldsymbol{e}_\varphi \frac{1}{r_\mathrm{c}} \frac{\partial\psi}{\partial\varphi} + \boldsymbol{e}_z \frac{\partial\psi}{\partial z} \\
\nabla\cdot\boldsymbol{A} &= \frac{1}{r_\mathrm{c}} \frac{\partial(r_\mathrm{c} A_{r_\mathrm{c}})}{\partial r_\mathrm{c}} + \frac{1}{r_\mathrm{c}} \frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z} \\
\nabla\times\boldsymbol{A} &= \boldsymbol{e}_{r_\mathrm{c}} \left(\frac{1}{r_\mathrm{c}} \frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \boldsymbol{e}_\varphi \left(\frac{\partial A_{r_\mathrm{c}}}{\partial z} - \frac{\partial A_z}{\partial r_\mathrm{c}}\right) \\
&\quad + \boldsymbol{e}_z \frac{1}{r_\mathrm{c}} \left(\frac{\partial}{\partial r_\mathrm{c}}(r_\mathrm{c} A_\varphi) - \frac{\partial A_{r_\mathrm{c}}}{\partial\varphi}\right) \\
\nabla^2\psi &= \frac{1}{r_\mathrm{c}} \frac{\partial}{\partial r_\mathrm{c}} \left(r_\mathrm{c} \frac{\partial\psi}{\partial r_\mathrm{c}}\right) + \frac{1}{r_\mathrm{c}^2} \frac{\partial^2\psi}{\partial\varphi^2} + \frac{\partial^2\psi}{\partial z^2}
\end{aligned}
$$

Differentialer

$$
\begin{aligned}
\mathrm{d}\boldsymbol{r} &= \boldsymbol{e}_{r_\mathrm{c}}\,\mathrm{d}r_\mathrm{c} + \boldsymbol{e}_\varphi r_\mathrm{c}\,\mathrm{d}\varphi + \boldsymbol{e}_z\,\mathrm{d}z \\
\mathrm{d}S &= \begin{cases} r_\mathrm{c}\,\mathrm{d}\varphi\,\mathrm{d}z & (\text{normalriktning } \boldsymbol{e}_{r_\mathrm{c}}) \\ \mathrm{d}r_\mathrm{c}\,\mathrm{d}z & (\text{normalriktning } \boldsymbol{e}_\varphi) \\ r_\mathrm{c}\,\mathrm{d}r_\mathrm{c}\,\mathrm{d}\varphi & (\text{normalriktning } \boldsymbol{e}_z) \end{cases} \\
\mathrm{d}V &= r_\mathrm{c}\,\mathrm{d}r_\mathrm{c}\,\mathrm{d}\varphi\,\mathrm{d}z
\end{aligned}
$$

Sfäriska koordinater

$$
\begin{aligned}
x &= r\sin\theta\cos\varphi & r &= \sqrt{x^2+y^2+z^2} \\
y &= r\sin\theta\sin\varphi & \theta &= \arccos\frac{z}{\sqrt{x^2+y^2+z^2}} \\
z &= r\cos\theta & \varphi &= \begin{cases} \arccos\frac{x}{\sqrt{x^2+y^2}} & y \geq 0 \\ 2\pi - \arccos\frac{x}{\sqrt{x^2+y^2}} & y < 0 \end{cases} \\
\boldsymbol{e}_x &= \boldsymbol{e}_r\sin\theta\cos\varphi + \boldsymbol{e}_\theta\cos\theta\cos\varphi & \boldsymbol{e}_r &= \boldsymbol{e}_x\sin\theta\cos\varphi \\
&\quad - \boldsymbol{e}_\varphi\sin\theta\sin\varphi & &\quad + \boldsymbol{e}_y\sin\theta\sin\varphi + \boldsymbol{e}_z\cos\theta \\
\boldsymbol{e}_y &= \boldsymbol{e}_r\sin\theta\sin\varphi + \boldsymbol{e}_\theta\cos\theta\sin\varphi & \boldsymbol{e}_\theta &= \boldsymbol{e}_x\cos\theta\cos\varphi + \boldsymbol{e}_y\cos\theta\sin\varphi \\
&\quad + \boldsymbol{e}_\varphi\cos\varphi & &\quad - \boldsymbol{e}_z\sin\theta \\
\boldsymbol{e}_z &= \boldsymbol{e}_r\cos\theta - \boldsymbol{e}_\theta\sin\theta & \boldsymbol{e}_\varphi &= -\boldsymbol{e}_x\sin\varphi + \boldsymbol{e}_y\cos\varphi
\end{aligned}
$$

Differentialoperatorer

$$
\begin{aligned}
\nabla\psi &= \boldsymbol{e}_r\frac{\partial\psi}{\partial r} + \boldsymbol{e}_\theta\frac{1}{r}\frac{\partial\psi}{\partial\theta} + \boldsymbol{e}_\varphi\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\varphi} \\
\nabla\cdot\boldsymbol{A} &= \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi} \\
\nabla\times\boldsymbol{A} &= \boldsymbol{e}_r\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta A_\varphi) - \frac{\partial A_\theta}{\partial\varphi}\right) + \boldsymbol{e}_\theta\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right) \\
&\quad + \boldsymbol{e}_\varphi\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right) \\
\nabla^2\psi &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}
\end{aligned}
$$

Differentialer

$$
\begin{aligned}
\mathrm{d}\boldsymbol{r} &= \boldsymbol{e}_r\,\mathrm{d}r + \boldsymbol{e}_\theta\, r\,\mathrm{d}\theta + \boldsymbol{e}_\varphi\, r\sin\theta\,\mathrm{d}\varphi \\
\mathrm{d}S &= \begin{cases} r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi & (\text{normalriktning } \boldsymbol{e}_r) \\ r\sin\theta\,\mathrm{d}r\,\mathrm{d}\varphi & (\text{normalriktning } \boldsymbol{e}_\theta) \\ r\,\mathrm{d}r\,\mathrm{d}\theta & (\text{normalriktning } \boldsymbol{e}_\varphi) \end{cases} \\
\mathrm{d}V &= r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\varphi
\end{aligned}
$$