Interaction effects between continuous variables (Optional)

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This is a very brief overview of this somewhat difficult topic. The course readings provide much more detail, and you should go over these carefully if you feel these kinds of interaction terms will be useful in your work.

Interactions between two continuous variables. We have focused on interactions between categorical and continuous variables. However, there can also be interactions between two continuous variables. For example, suppose that "Intentions" and "Actual Behavior" are both measured as continuous variables. Suppose further it is believed that the effect of intentions on behavior (i.e. the correspondence between what one wants to do and what one actually does) is greater at higher levels of income; that is, the higher one's income is, the more consistently one behaves. This model would be written as

$$\begin{aligned} Behavior &= \alpha + \beta_1 Intentions + \beta_2 Income + \beta_3 (Intentions * Income) + \varepsilon \\ &= \alpha + (\beta_1 + \beta_3 * Income) Intentions + \beta_2 Income \\ &= \alpha + \beta_1 Intentions + (\beta_2 + \beta_3 * Intentions) * Income \end{aligned}$$

A positive value for the effect of the interaction term would imply that the higher the income, the greater (more positive) the effect of intentions on behavior was. Similarly, the higher the intentions, the greater (more positive) the effect of income on behavior.

EXAMPLES.

- The greater the resources available, the stronger the effect of intentions on behavior. Those who are most able to get what they want are most likely to get it.

- The more religious someone is, the stronger the effect of moral values on behavior. The more religious someone is, the more compelled they will be to act on their moral values.

Interpreting Interactions between two continuous variables. As Jaccard, Turrisi and Wan (Interaction effects in multiple regression) and Aiken and West (Multiple regression: Testing and interpreting interactions) note, there are a number of difficulties in interpreting such interactions. There are also various problems that can arise. Both books note with regret that such interaction terms are not used more widely in the social sciences. Those who feel that such interaction terms may be theoretically justified in their analyses should consult these works for additional details. A few highlights from their discussions may be helpful:

- In general, models with interaction effects should also include the main effects of the variables that were used to compute the interaction terms, even if these main effects are not significant. Otherwise, main effects and interaction effects can get confounded. Further, it can be shown that, if main effects are not included, arbitrary changes in the zero point of the original variables can result in important changes in the apparent effects of the interaction terms.

- In models with multiplicative terms, the regression coefficients for X1 and X2 reflect *conditional* relationships. B1 is the effect of X1 on Y when X2 = 0. Similarly, B2 is the effect of X2 on Y when X1 = 0. For example, when X2 = 0, we get

$$\begin{aligned} Y &= \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 (X_1 * X_2) + \varepsilon \\ &= \alpha + \beta_1 X_1 + \beta_2 0 + \beta_3 (X_1 * 0) + \varepsilon \\ &= \alpha + \beta_1 X_1 + \varepsilon \end{aligned}$$

So, we can say that, for a person who has a score of 0 on X2, a 1 unit increase in X1 will produce, on average, a B1 increase in Y.

However, suppose someone has a score of 3 on X2. The effect of X1 is then

$$\begin{aligned} Y &= \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 (X_1 * X_2) + \varepsilon \\ &= \alpha + \beta_1 X_1 + \beta_2 * 3 + \beta_3 (X_1 * 3) + \varepsilon \\ &= \alpha + \beta_1 X_1 + 3\beta_2 + 3\beta_3 X_1 + \varepsilon \\ &= \alpha + 3\beta_2 + (\beta_1 + 3\beta_3) X_1 + \varepsilon \end{aligned}$$

So, when X2 = 3, a one unit increase in X1 will produce a (B1 + 3B3) unit increase in Y.

In short, if we want to ask, “What is the effect of X1 on Y”, the answer is “It depends on what X2 equals.”

- Of course, given the way X1 and X2 are scaled, 0 may or may not be a particularly meaningful value, e.g. no adult is 0 years old, nobody has an IQ or a height or a weight of 0. Hence, the main (i.e. non-interaction) effects in a model with interaction terms may have little meaning and may even be misleading.

- Effects can therefore often be made more interpretable by *centering* variables first. When we center a variable, we subtract the mean from each case, and then compute the interaction terms. When variables are centered, B1 is the effect of X1 on Y for the person who is “average” on X2.

- Alternatively, rather than centering around the mean, we might center around some other meaningful value. For example, we might recenter education so that a score of 0 corresponds to 12 years of high school. Then, the coefficient for the other X would correspond to the effect that variable has for high school graduates.

- We can also simply do the above by hand, e.g. take high, medium and low values for X2 and see what the effect of X1 is in each case. For example, suppose it conveniently is the case that the intercept is 0 and the betas are all 1, i.e.

$$y = X_1 + X_2 + X_1 X_2 + \varepsilon$$

Suppose further that 0, 5, and 10 are low, medium and high values of X2. We then get

	X2 = 0	X2 = 5	X2 = 10
Effect of X1 on Y	1	6	11

Hence, the effect of X1 on Y is 11 times greater for high values of X2 than it is for low values of X2.

More on Centering Continuous Variables. Centering can be useful when both variables are continuous. Again, what a model ultimately says and predicts does not depend on whether or not you center, but centering may make the results a little more meaningful and easy to interpret. For example, suppose a model includes the IVs education, income, and income*education, where income and education have both been centered about their means. Hence, the overall model is

$$\begin{aligned} E(Y) &= \alpha + \beta_1 Educ + \beta_2 Inc + \beta_3 Educ * Inc \\ &= \alpha + (\beta_1 + \beta_3 Inc) * Educ + \beta_2 Inc \\ &= \alpha + \beta_1 Educ + (\beta_2 + \beta_3 Educ) * Inc \end{aligned}$$

As the algebraically equivalent statements show, the effect of education on Y depends on the level of income, and the effect of income on Y depends on the level of education. Note that, for a person of average income (i.e. has a score of 0 on the centered Income variable) the model simplifies to

$$E(Y) = \alpha + \beta_1 Educ$$

- Hence, <u>when the variables are centered, the main effect of education is the effect of education on a person who has average income</u>.

Similarly, for a person with an average level of education (centered education = 0), the model simplifies to

$$E(Y) = \alpha + \beta_2 Inc$$

- That is, <u>when variables are centered, the main effect of income is the effect of income on a person who has an average level of education</u>.

Finally, for a person who has average education and average income,

$$E(Y) = \alpha$$

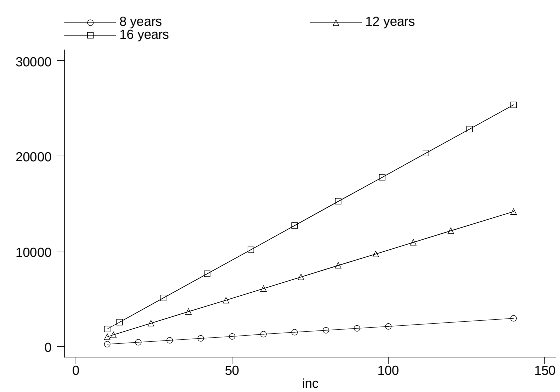
- i.e. <u>when variables are centered, the intercept is the predicted Y score for an average person</u>.

If you <u>didn't center</u>,

- the main effect of education would be the effect of education on a person who had 0 income
- the main effect of income would correspond to the effect of income on a person with 0 years of education
- the intercept would be the predicted Y score for a person who has 0 years of education and 0 income.
- Such numbers may or may not be terribly interesting. It depends on whether or not 0 happens to be an interesting value for your variables.

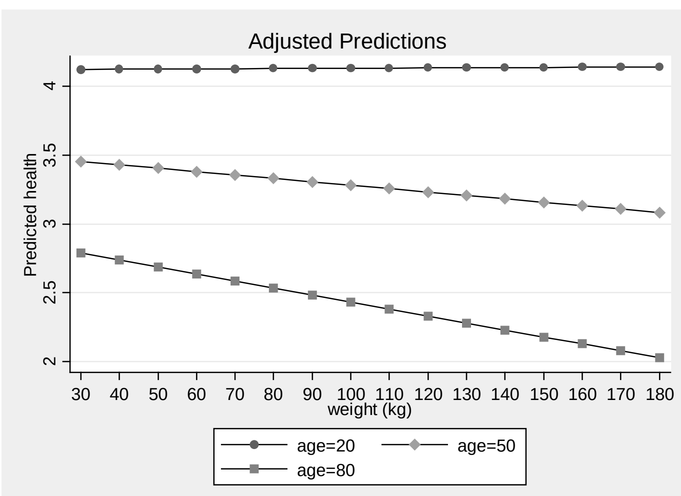
Again, what the model ultimately says and what it predicts are the same whether you center or not. But, looking at the effect of income on a person with average education may be more substantively interesting that focusing on the effect of income on a person who has no education. Whether or not you center won't change the estimated effect of the interaction (in this case B3) but it will change the estimated main effects. This is because the main effects mean different things depending on whether or not you have centered.

Graphing interactions between continuous variables. Graphing can be tricky for interactions involving two or more continuous variables but can still be useful. One approach is to plug in substantively interesting values for one of the IVs and then plot the other IV against the DV. For example, you could plot INCOME versus Y for high, medium and low levels of education (i.e. just like we had separate lines for men and women, we could have separate lines for each of the selected values of education.) If you did this for every possible value of education you'd either get a very messy graph or a 3-dimensional one, but if you choose a few interesting values you can give a feel for how the effect of income differs by education. For example, in the following hypothetical graph, we plot the relationship between Income and Y for 8, 12, and 16 years of education. The graph shows that, as education increases, the effect of income on Y gets greater and greater:



In Stata, we could do something like

```
. webuse nhanes2f, clear
. sum health age weight

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      health |     10335    3.413836    1.206196          1          5
         age |     10337     47.5637    17.21678         20         74
      weight |     10337    71.90088    15.35515      30.84     175.88

. reg health age weight c.age#c.weight

      Source |       SS       df       MS              Number of obs =   10335
-------------+------------------------------           F(  3, 10331) =  546.46
       Model |  2059.09026     3  686.36342           Prob > F      =  0.0000
    Residual |  12975.9311 10331  1.25601889           R-squared     =  0.1370
-------------+------------------------------           Adj R-squared =  0.1367
       Total |  15035.0214 10334   1.4549082           Root MSE      =  1.1207

--------------------------------------------------------------------------------
        health |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+----------------------------------------------------------------
           age |  -.0196621   .0030909    -6.36   0.000    -.0257208   -.0136034
        weight |   .0018507   .0021221     0.87   0.383    -.0023089    .0060104
               |
c.age#c.weight |  -.0000865    .000043    -2.01   0.044    -.0001708   -2.24e-06
               |
         _cons |   4.512782   .1522368    29.64   0.000     4.214368    4.811196
--------------------------------------------------------------------------------

. quietly margins, at(weight=(30(10)180) age = (20, 50, 80))
. marginsplot, noci scheme(sj) ytitle("Predicted health")
```



This shows us that the effect of weight on self-reported health is almost 0 for 20 year olds. But, as people get older and older, the effect of weight on self-reported health becomes more and more negative.