Guarantees of Origin and Market Power in the Spot Electricity Market*

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Abstract

Consumers, governments and corporations are becoming more aware of the origin of the energy that they consume, and the guarantees of origin (GO) market is increasing in Europe and worldwide. More than 25% of the electricity consumed in Europe is consumed by using GO markets. We work out the sub-game perfect Nash equilibrium when the spot and the GO markets operate sequentially. We study the impact on the equilibrium of changes on the GO market design and changes on the structural parameters (GO demand, and green production capacity installed in the spot market). We find that under one of the market designs analyzed in the paper, the introduction of a GO market has a pro-competitive effect in the spot market. We also find that an increase in demand in the GO market and a decrease in the green production capacity installed in the spot market have a pro-competitive impact in the spot market.

KEYWORDS: electricity auctions, guarantees of origin, spot electricity market, transmission constraints.

1 Introduction

Consumers, governments and corporations are becoming more aware of the origin of the energy that they consume, and the guarantees of origin (GO) market is increasing in Eu-

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rope and worldwide. More than 25% of the electricity consumed in Europe is consumed by using GO markets. However, our knowledge of the GO market and its interaction with the spot market is still very limited. By characterizing the sub-game perfect Nash equilibrium when the competition in the spot and the GO markets is imperfect, and when those markets operate sequentially, we study if the introduction of a GO market has a pro-competitive effect on the spot market. We complete the analysis by studying the impact of different market designs and market structures on the equilibrium.

The spot and the GO markets are different markets. In the spot market, the suppliers sell electricity and the consumers buy electricity. In the GO market, the sellers sell guarantees of origin that guarantee that the electricity sold come from a renewable source, and the consumers buy those guarantees of origin.

In the set up of the model, two suppliers with identical production capacity compete in prices when the competition in the spot and the GO markets is imperfect. The demand in both markets is inelastic and it is located in two different nodes connected by a transmission line. The suppliers production capacity in the spot market could be larger than the transmission capacity, i.e., the transmission line could be congested. The spot and the GO markets operate sequentially. In the spot market, the supplier that sets the lower price is dispatched first and satisfies the electricity in its node and in the other node up to the transmission capacity. The supplier that sets the higher price is dispatched last and satisfies the residual demand in its own node. The quantities dispatched in the spot market are suppliers' production capacities in the GO market. As in the spot market, in the GO market, the supplier that sets the lower price is dispatched first and the supplier that sets the higher price is dispatched last and satisfies the residual demand. This timing is in line with the current market design where to operate in the GO market, the suppliers need to be dispatched in the spot market.

We work out the sub-game perfect Nash equilibrium when the spot and the GO markets operate sequentially and when two market designs are introduced by the auctioneer. First, in the "GO, no-constraint" design, we assume that the transmission line is not taken into account to work out the equilibrium in the GO market, i.e, in the GO market, the suppliers can sell their entire production capacities even when the transmission line in that market is congested. This market design reflects the current market design of the GO market, where the suppliers can buy or sell GOs without taken into account the transmission constraints in that market. In contrast, in the "GO, constraint" design, the transmission line is taken into account to clear the GO market. This market design is in line with the current GO redesign proposals.

We characterize the sub-game perfect Nash equilibrium for any set of parameters. However, to prove formally the impact that changes in the market design and the structural parameters have on the equilibrium, we need two introduce two more assumptions. First, we assume that the demand in the spot market is symmetric. Second, we assume that the demand in the GO market is such, that under the "GO, no-constraint" design, the equilibrium in that market is exclusively determined by the production capacity constraint in the GO market; but under the "GO, constraint" design, the equilibrium in that market is exclusively determined by the transmission capacity constraint in the GO market. These two assumptions allow us to isolate the impact that a change on the market design has on the equilibrium. Otherwise, the equilibrium in the GO market is determined by the production and the transmission capacities simultaneously and it is not possible to prove formally the impact that a change on the market design and a change on the structural parameters of the model have on the equilibrium prices in the spot electricity market. These assumptions are in line with the standard approach used in industrial organization, where it is necessary to keep the model as simple as possible to isolate the impact that changes on market design or structural parameters have on the equilibrium.

The introduction of a GO market has a pro-competitive effect on the spot electricity market only under the current market design ("GO, no-constraint" design). In that case, the suppliers compete fiercely in the spot market to sell as much production capacity as possible, since by doing that, they will have a lot of production capacity to sell in the GO market. In contrast, under the "GO, constraint" design, those incentives disappear since the suppliers cannot sell their entire production capacity in the GO market due to the transmission constraint in that market. Therefore, they compete less fiercely in the spot market, and the prices in that market are higher.

An increase in demand in the GO market has a pro-competitive effect on the spot electricity market, since in that case, the suppliers have incentives to compete fiercely in that market to sell as much production capacity as possible in the GO market because the demand (and the profits associated to that demand) are higher. A reduction on suppliers' green production capacity[1] has a pro-competitive effect on the GO market, since in that case, it is very risky for the suppliers to be dispatched last in the spot market because they will have very little production capacity to sell in the GO market. Therefore, they compete fiercely in the spot market to sell as much production capacity in that market, and that increase competition in the spot market.

Our findings have import policy implications. First, the current GO market design has a pro-competitive impact on the spot electricity market increasing consumers welfare. However, the fact that the transmission line is not taken into account to clear the GO market could distort long-term investment decisions. Second, any policy that makes the consumers become more aware of the origin of their electricity and increases the demand that come from green production capacity has a pro-competitive impact on the spot electricity prices.

[1] In the GO market, the demand for GOs is differentiated, and some consumers want to acquire electricity only from local wind or solar farms, or from new hydro power plants. Therefore, it is important to take into account GOs consumers' demand to clear the GO market.

Finally, the pro-competitive impact of a GO market is more important when the green production capacity installed is low.

The models of price competition with capacity constraints have been broadly used in the industrial organization literature. In particular, in the models that endogenize the emergence of a price leader in duopoly models (Deneckere and Kovenock, 1992; Canoy, 1996; Osborne and Pitchik, 1986). They have been also used to endogenize production capacity decisions. Kreps and Scheinkman (1984) characterize the sub-game perfect Nash equilibrium when the suppliers, first invest in production capacity, and then compete in prices.[2] Our paper extend that literature by characterizing the equilibrium when the suppliers compete in prices with capacity constraints in two markets that operate sequentially. To some extend, our model is similar to Kreps and Scheinkman (1984), since the suppliers, first compete to be dispatched in the spot market (and that dispatch will be the "production capacity" in the GO market), and then compete in prices again in the GO market. However, in our model, the suppliers do not invest in "physical production capacity".

The sequential markets literature characterizes the equilibrium when two markets operate sequentially. First, the suppliers compete in a forward market, and then, they compete in a spot market. That literature studies if the introduction of a forward market has a pro-competitive impact on the spot electricity market. Allaz and Vila (1993) work out the sub-game equilibrium when the forward and the spot markets operate sequentially and the suppliers compete in quantities, and they find that the introduction of a forward market has a pro-competitive impact on the spot electricity market. By using a similar set-up, but when the suppliers compete in prices, Mahenc and Salanié (2004) find that the introduction of a forward market has an anti-competitive impact on the spot electricity market. Koichiro and Reguant (2016) analyze data of sequential electricity markets with imperfect competition and restricted entry in arbitrage. They find that the suppliers with higher production capacity withhold capacity in the spot market to increase the prices in that market. In contrast, the suppliers with lower production capacity oversell electricity in the spot market and buy capacity in the intraday market at a lower price. Borenstein, Bushnell, Knittel and Wolfram (2008), and Saravia (2003) extends Koichiro and Reguant (2016) by introducing traders (arbitragers), and they find that the introduction of traders have a pro-competitive impact in the spot electricity market.

In the models that study sequential markets, the suppliers trade the same product several times. In contrast, we characterize the equilibrium in sequential markets when the suppliers sell different products. In the spot electricity market, the suppliers sell electricity, and in the GO market, the suppliers sell guarantees of origin.

In section 2, we present the set-up and the timing of the model. In section 3, we

[2] Crampes and Creti (2005), Le Coq (2002), and Moreno and Ubeda (2006) extend that model by introducing a uniform price auction in the second stage of the game.

characterize the equilibrium. In section 4, we study the impact that changes in the market design and changes in the structural parameters have on the equilibrium. In section 5, we conclude the paper. All the proofs are in the appendix.

2 The model

In this section, we present the set-up and the timing of the model for the two market designs analyzed in the paper: "GO, no-constraint" and "GO, constraint."

2.1 Set-up of the model

There are two nodes $i = 1, 2$ connected by a transmission line with capacity T. In the spot market, the demand in each node is inelastic (a_1^s, a_2^s). There are two suppliers, $i = 1, 2$, each with capacity $(k_1^s = k_2^s = k^s)$ located in nodes $1, 2$, where suppliers' production capacities satisfy two requirements. First, $T \leq k^s$, i.e., the suppliers cannot sell their entire production capacity into the other node. Therefore, the transmission constraint could be binding. Second, $k^s + T > max\,\{\overline{a}_1^s, \overline{a}_2^s\}$, i.e., the installed production capacity in one node plus the electricity that flows from the other node is enough to satisfy the peak demand in both nodes.

The spot electricity market is organized as a nodal price market, where the equilibrium price in nodes 1 and 2 is different when the transmission line is congested. When the transmission line is congested, it is profitable to buy electricity in the cheap node and to sell it in the expensive node. We assume that the congestion rents are captured by the transmission system operator.[3]

In the GO market, the demand in each node is inelastic (a_1^{go}, a_2^{go}). Suppliers' production capacities in the GO market coincide with suppliers' dispatch in the spot market, i.e., the suppliers carry their green production capacity dispatched in the spot market to compete in the GO market. This assumption is in line with the current design of the market, where the suppliers need to be dispatched in the spot market to participate in the GO market. As in the spot market, the two nodes are connected by a transmission line with capacity T. However, in the "GO, no-constraint" design, we assume that the suppliers in the GO market can sell their entire production capacity in the other node. This assumption is in line with the current GO market design, where the transmission capacity is not taken into account to clear the GO market. In contrast, in the "GO, constraint" design, we assume that the transmission capacity is taken into account to clear the GO market. This allows us to study the impact of both market designs on the competition in the spot market.

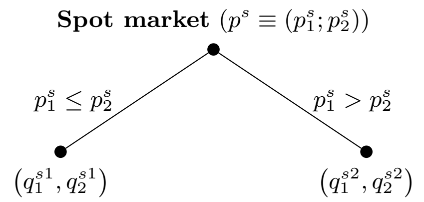
[3]This assumption is in line with the current design of nodal markets where the transmission system operator captures the congestion rents.

2.2 Timing of the game

The suppliers observe the demand in both nodes (1 and 2) and both markets (spot and GO), and simultaneously and independently, set their prices in the spot market ($p^s \equiv (p_1^s; p_2^s)$). The transmission system operator collects the prices, and calls the suppliers into operation. Supplier 1's output (supplier 2's output is symmetric) in the spot market is defined by:

$$q_1^s(p^s) = \begin{cases} q_1^{s1} = min\{a_1^s + a_2^s, a_1^s + T, k^s\} & \text{if } p_1^s \leq p_2^s \\ q_1^{s2} = max\{0, a_1^s - T, a_1^s + a_2^s - k^s\} & \text{if } p_1^s > p_2^s \end{cases} \tag{1}$$

Figure 1: Spot electricity market



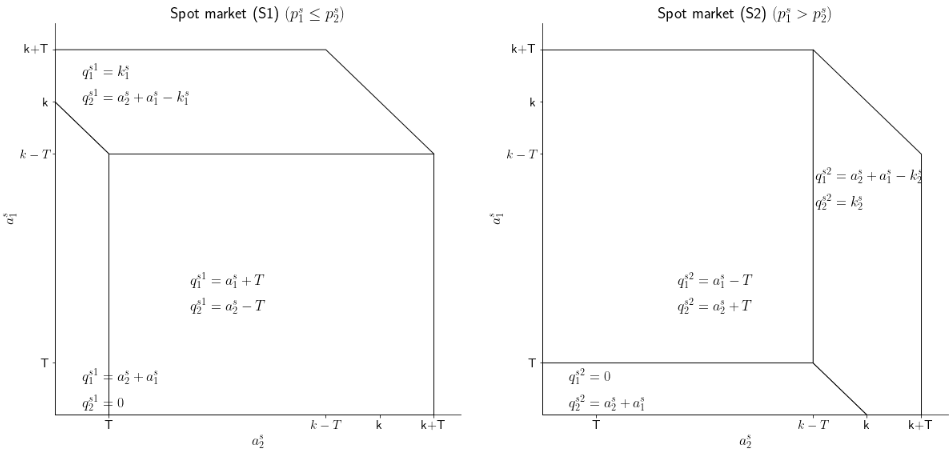
When supplier 1 sets the lower price in the spot market ($p_1^s \leq p_2^s$) (left-branch, figure 1), it is dispatched first in the auction. When the transmission line is not congested and supplier 1 has enough production capacity, it satisfies the demand in both nodes ($q_1^{s1} = a_1^s + a_2^s$);[4] when the transmission line is congested, supplier 1 satisfies the demand in its own node and the demand in the other node up to the transmission constraint ($q_1^{s1} = a_1^s + T$); finally, supplier 1 cannot satisfy more demand that its production capacity ($q_1^{s1} = k_1^s$) (left-hand side, figure 2).[5]

When supplier 1 sets the higher price in the spot market ($p_1^s > p_2^s$) (right-branch, figure 1), it is dispatched last in the auction. When the transmission line is not congested and the other supplier has enough production capacity to satisfy the demand, supplier 1's residual demand is nil ($q_1^{s2} = 0$); when the transmission line is congested, supplier 1's residual

[4]We use the superindex $s1$ to refer to the left-branch of the tree in figure 1, and the superindex $s2$ to refer to the right-branch of the tree in figure 1.

[5]In the models of price competition with capacity constraints, the tie-breaking rule is crucial determining the existence of the equilibrium (Dasgupta and Maskin, 1986). In the presence of production or transmission costs, the tie-breaking rule needs to be designed to minimize those costs. In the models in this paper, there are no production or transmission costs and different tie breaking rules could be implemented. The chosen tie-breaking rule gives priority in the dispatch to the supplier located in the high-demand node; when the demand in both nodes is equal, the suppliers satisfy the demand in their own nodes. This tie-reaking rule minimizes transmission losses.
For a complete characterization of the equilibrium in models of price competition with capacity constraints and positive costs see Blázquez (2018), Deneckere and Kovenock (1996), Fabra, von der Fehr and Harbord (2006), Osborne and Pitchik (1986).

Figure 2: Dispatch spot market: $T = 2$, $k_1^s = k_2^s = k^s = 12$



demand is ($q_1^{s2} = a_1^s - T$); finally, when the demand is high enough, supplier 1's residual demand is ($q_1^{s2} = a_1^s + a_2^s - k_2^s$).

The quantities dispatched in the spot market will be suppliers' production capacities in the GO market, i.e., $k_i^{go1} = \alpha_1 q_i^{s1}, k_i^{go2} = \alpha_2 q_i^{s2}\ \forall\ i = 1, 2$, where $\alpha_1, \alpha_2 \in [0, 1]$. Therefore, in the spot market, the suppliers are competing not only to satisfy the demand in that market, but also to have production capacity to compete in the GO market. In the previous expressions, the parameter α represents the proportion of production capacity dispatched in the spot market considered as green by the consumers. Therefore, when $\alpha = 1$, the consumers consider that the entire production capacity of the suppliers dispatched in the spot market is green, and when $\alpha = 0$, the consumers consider that none of that production capacity is green. This allows us to study the impact of technological differences between suppliers on the competition in the spot market.

After knowing their dispatch in the spot market, the suppliers, simultaneously and independently, set their prices in the GO market ($p^{go} \equiv (p_1^{go}; p_2^{go})$). The transmission system operator collects the prices, and calls the suppliers into operation. The timing in the GO market is the same for both market designs, but the dispatch differs depending on the market design. First, we compute the dispatch in the GO market under the "GO, no-constraint" design. Then, we compute the dispatch in the GO market under the "GO, constraint" design.

First, we compute the dispatch in the GO market under the "GO, no-constraint" design. When in the spot market, $p_1^s > p_2^s$, the suppliers are in the right-branch, figure 3, $k_1^{go2} = \alpha_1 q_1^{s2} = \alpha_1(a_1^s - T)$, $k_2^{go2} = \alpha_2 q_2^{s2} = \alpha_2(a_2^s + T)$,[6] and supplier 1's output (supplier 2's output

[6]From now on, we assume that in the spot market, the transmission line is always congested. This assumption gives us the opportunity to study the effect of transmission constraints determining the equilibrium

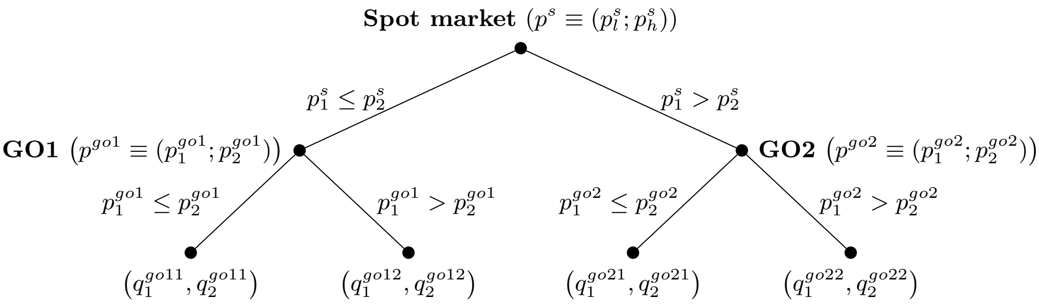
is symmetric) in the GO2 market (right-branch, figure 3) is defined by:

$$q_1^{go2}(p^{go2}) = \begin{cases} q_1^{go21} = min\{a_1^{go} + a_2^{go}, k_1^{go2}\} & \text{if } p_1^{go2} \leq p_2^{go2} \\ q_1^{go22} = max\{0, a_1^{go} + a_2^{go} - k_2^{go2}\} & \text{if } p_1^{go2} > p_2^{go2} \end{cases} \tag{2}$$

When supplier 1 sets the lower price in the GO2 market, it is dispatched first and satisfies the total demand ($q_1^{go21} = a_1^{go} + a_2^{go}$) up to supplier 1's production capacity in the GO2 market ($q_1^{go21} = k_1^{go2} = q_1^{s2}$) (right-left-branch GO2 market, figure 3; left-hand side, figure 4).[7]. When supplier 1 sets the higher price in the GO2 market, it is dispatched last and satisfies the residual demand. When the demand is low, supplier 1's residual demand is nil; when the demand is high enough, supplier 1's residual demand is ($q_1^{go22} = a_1^{go} + a_2^{go} - k_2^{go2}$) (right-right-branch GO1 market, figure 3; right-hand side, figure 4).[8]

In the GO1 market (left-branch, figure 3), supplier 1's output (supplier 2's output is symmetric) is as in the GO2 market, but taken into account that $k_1^{go1} = \alpha_1 q_1^{s1} = \alpha_1(a_1^s + T)$, $k_2^{go1} = \alpha_2 q_2^{s1} = \alpha_2(a_2^s - T)$.

Figure 3: Spot and GO markets



Under the "GO, constraint" design, the transmission constraint is taken into account to compute the dispatch in the GO market. In that case, supplier 1's output (supplier 2's output is symmetric) in the GO2 market (right-branch, figure 3) is defined by:

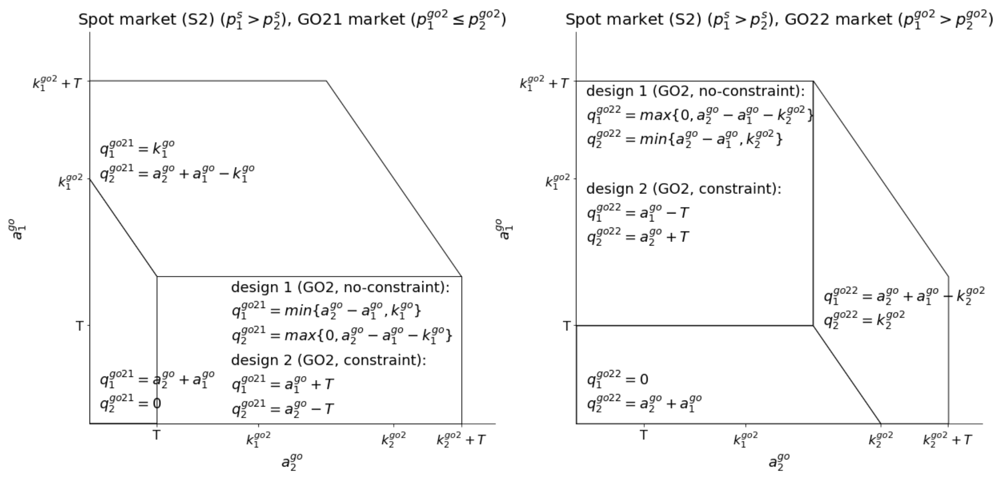
$$q_1^{go2}(p^{go2}) = \begin{cases} q_1^{go21} = min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} & \text{if } p_1^{go2} \leq p_2^{go2} \\ q_1^{go22} = max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\} & \text{if } p_1^{go2} > p_2^{go2} \end{cases} \tag{3}$$

in the spot market. When the transmission line is not congested, the equilibrium is as in Fabra et al. (2006).

[7] We use the superindex $go11$ to refer to the left-left-branch of the tree in figure 3; the superindex $go12$ to refer to the left-right-branch of the tree in figure 3; the superindex $go21$ to refer to the right-left-branch of the tree in figure 3; and the superindex $go22$ to refer to the right-right-branch of the tree in figure 3

[8] The tie-breaking rule is as in the spot market.

Figure 4: Dispatch GO2 market ($p_1^s > p_2^s$): $T = 2$, $k_1 = k_2 = k = 12$, $a_1^s = a_2^s = 7$, $k_1^{go2} = \alpha_1 q_1^{s2} = \alpha_1(a_1^s - T) = 5$, $k_2^{go2} = \alpha_2 q_2^{s2} = \alpha_2(a_2^s + T) = 9$, $\alpha_1 = \alpha_2 = 1$.



When the transmission line is not congested in the GO market, the dispatch is as in equation 2. When the transmission line in the GO market is congested, and supplier 1 submits the lower bid in that market, it satisfies the demand in its own node and the demand in node 2 up to the transmission capacity ($q_1^{go21} = a_1^{go} + T$) (right-left-branch GO2 market, figure 3; left-hand side, figure 4). When supplier 1 submits the higher bid in the GO market, it satisfies the residual demand in its own node ($q_1^{go22} = a_1^{go} - T$) (right-right-branch GO2 market, figure 3; right-hand side, figure 4).

After the suppliers are called into operation, the profits are worked out in the spot and in the GO market (GO1 or GO2 market, depending on the relation between p_1^s and p_2^s). The spot market is designed as a nodal market in which the transmission system operator captures the congestion rents. Therefore, suppliers' profits are obtained by multiplying suppliers' quantities (dispatch) by their own price, and supplier 1's profits in the spot market are defined by:

$$\pi_1^s(p^s) = p_1^s q_1^s \tag{4}$$

When supplier 1 sets the lower price in the spot market, it is dispatched first and its profits in that market are ($\pi_1^{s1} = p_1^{s1}(a_1^s + T)$). When supplier 1 sets the higher price in the spot market, it is dispatched last and its profits in that market are ($\pi_1^{s2} = p_1^{s2}(a_1^s - T)$).

Supplier 1's profits in the GO1 market are defined by (supplier 2's profits follow the same formula):

$$\pi_1^{go1}(p^{go1}) = p_1^{go1} q_1^{go1} \tag{5}$$

Supplier 1' profits in the GO2 market are defined by (supplier 2's profits follow the same formula):

$$\pi_1^{go2}(p^{go2}) = p_1^{go2} q_1^{go2} \tag{6}$$

By summing supplier 1's profits in the spot and in the GO markets, we obtain supplier 1's total profits (supplier 2's profits follow the same formula):

$$\pi_1(p^s) = \begin{cases} p_1^{s1}(a_1^s + T) + \pi_1^{go1}(p^{go1}) & \text{if } p_1^s \leq p_2^s \\ p_1^{s2}(a_1^s - T) + \pi_1^{go2}(p^{go2}) & \text{if } p_1^s > p_2^s \end{cases} \tag{7}$$

3 Equilibrium

In this section, we characterize the subgame perfect Nash equilibrium. We work out the equilibrium for the "GO, constraint" design, since the "GO, no-constraint" design is a particular case of the first one.[9] First, we work out the equilibrium in the GO1 and the GO2 markets. Second, based on the equilibrium in the GO1 and the GO2 markets, we characterize the equilibrium in the spot market. To characterize the equilibrium in the GO1, the GO2 and the spot markets, we proceed in three steps: First, we prove that a pure strategies equilibrium does not exist in any of those markets. Second, we find the support of the mixed strategies equilibrium. Third, we find the mixed strategies equilibrium.

We begin characterizing the equilibrium in the GO2 market (figure 3). The equilibrium in the GO1 market is symmetric. First, we prove that a pure strategies Nash equilibrium does not exist.

Lemma 1. In the GO2 market, and when at least one supplier faces a positive residual demand, a pure strategies Nash equilibrium does not exist.

Once that we prove that a pure strategies Nash equilibrium does not exist, we find the support of the mixed strategies equilibrium.

Lemma 2. In the mixed strategies equilibrium in the GO2 market, both suppliers randomize in the interval $p^{go2} \in \left[max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}, \overline{P}^{go}\right]$. Where $\underline{p}_1^{go2}$ solves $\underline{p}_1^{go2} min\{a_1^{go} +$

[9] In the "GO, no-constraint" design (equation 2), the transmission constraint in the GO market is not taken into account to compute the dispatch in that market. In contrast, in the "GO, constraint" design, the transmission constraint in the GO market is taken into account to compute the dispatch in that market (equation 3). Therefore, equation 2 is a particular case of equation 3.

$a_2^{go}, a_1^{go} + T, k_1^{go2}\} = \overline{P}^{go} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}$, and $\underline{p}_2^{go2}$ solves $\underline{p}_2^{go2} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} = \overline{P}^{go} max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}$.

In lemma 2, we find the support of the mixed strategies equilibrium. In proposition 1, we characterize the equilibrium in the GO2 market. To make the formulas more readable, we follow the notation used in equation 3, and in figures 3 and 4. In particular:

$$
\begin{aligned}
q_1^{go21} &= min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} \\
q_1^{go22} &= max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\} \\
q_2^{go22} &= min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} \\
q_2^{go21} &= max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}
\end{aligned}
$$

Proposition 1. When the spot and the GO markets operate sequentially, in the mixed strategies equilibrium in the GO2 market, the suppliers randomize by using the next cumulative distribution functions:

$$
F_1(p^{go2}) = \begin{cases} 0 & \text{if } p^{go2} < \underline{p}^{go2} \\ \dfrac{q_2^{go22}}{q_2^{go22} - q_2^{go21}} \dfrac{p^{go2} - \underline{p}^{go2}}{p^{go2}} & \text{if } p^{go2} \in \left(\underline{p}^{go2}, \overline{P}^{go}\right) \\ 1 & \text{if } p^{go2} = \overline{P}^{go} \end{cases} \tag{8}
$$

$$
F_2(p^{go2}) = \begin{cases} 0 & \text{if } p^{go2} < \underline{p}^{go2} \\ \dfrac{q_1^{go21}}{q_1^{go21} - q_1^{go22}} \dfrac{p^{go2} - \underline{p}^{go2}}{p^{go2}} & \text{if } p^{go2} \in \left(\underline{p}^{go2}, \overline{P}^{go}\right) \\ 1 & \text{if } p^{go2} = \overline{P}^{go} \end{cases} \tag{9}
$$

Once that we work out the equilibrium in the GO markets, we work out the equilibrium in the spot market. To characterize the equilibrium in that market, it is necessary to add the expected profits in the GO markets to the profits in the spot market, and then work out the support of the mixed strategies equilibrium, and suppliers' CDFs.

First, we work out the support of the mixed strategies equilibrium.

Lemma 3. In the mixed strategies equilibrium in the spot market, both suppliers randomize in the interval $p^{sgo} \in \left[max\left\{\underline{p}_1^{sgo}, \underline{p}_2^{sgo}\right\}, \overline{P}^{sgo}\right]$. Where, $\underline{p}_1^{sgo}$ solves $\underline{p}_1^{sgo}(a_1^s + T) + \underline{p}^{go1}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} = \overline{P}^{sgo}(a_1^s - T) + \underline{p}^{go2}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}$, and $\underline{p}_2^{sgo}$ solves $\underline{p}_2^{sgo}(a_2^s + T) + \underline{p}^{go2}\ min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} = \overline{P}^{sgo}(a_2^s - T) + \underline{p}^{go1}\ min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go1}\}$.

We complete the characterization of the equilibrium by working out suppliers CDFs in the spot electricity market. It is very important to take into account, that suppliers prices in the spot market determine their dispatch in that market, and therefore, their production capacities in the GO market, i.e., it is necessary to work out the sub-game perfect Nash equilibrium by backward induction. Hence, as when we work out the lower bound of the support in lemma 3, to work out the CDFs, it is necessary to take into account the expected profits in the GO1 and GO2 markets. To make the formulas more readable and clear, we follow the notation used in equations 1 and 3, and in figures 3 and 4. In particular:[10]

$$
\begin{aligned}
q_1^{s1} &= (a_1^s + T)\\
q_1^{s2} &= (a_1^s - T)\\
q_1^{go21} &= min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}\\
q_1^{go11} &= min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\}\\
q_2^{s1} &= (a_2^s - T)\\
q_2^{s2} &= (a_2^s + T)\\
q_2^{go22} &= min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\}\\
q_2^{go12} &= min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go1}\}
\end{aligned}
$$

Proposition 2. When the spot and the GO markets operate sequentially, in the mixed strategies equilibrium in the spot market, the suppliers randomize by using the next cumulative distribution functions:

$$
F_1(p^{sgo}) = \begin{cases} 0 & \text{if } p^{sgo} < \underline{p}^{sgo} \\ \dfrac{(p^{sgo} - \underline{p}^{sgo})q_2^{s2}}{p^{sgo}\left[q_2^{s2} - q_2^{s1}\right] + \underline{p}^{go2}q_2^{go22} - \underline{p}^{go1}q_2^{go12}} & \text{if } p^{sgo} \in \left(\underline{p}^{sgo}, \overline{P}^{sgo}\right) \\ 1 & \text{if } p^{sgo} = \overline{P}^{sgo} \end{cases} \tag{10}
$$

$$
F_2(p^{sgo}) = \begin{cases} 0 & \text{if } p^{sgo} < \underline{p}^{sgo} \\ \dfrac{(p^{sgo} - \underline{p}^{sgo})q_1^{s1}}{p^{sgo}\left[q_1^{s1} - q_1^{s2}\right] + \underline{p}^{go1}q_1^{go11} - \underline{p}^{go2}q_1^{go21}} & \text{if } p^{sgo} \in \left(\underline{p}^{sgo}, \overline{P}^{sgo}\right) \\ 1 & \text{if } p^{sgo} = \overline{P}^{sgo} \end{cases} \tag{11}
$$

To conclude the characterization of the equilibrium, we also present the CDFs when the spot market operates independently. This will help us to set a benchmark to compare it

[10]We are assuming that the transmission line in the spot market is congested in both directions. We focus on that case, because the objective of the paper is to understand the relation between the spot and the GO market, when the transmission line is congested. Proposition 2 can be easily modified to reflect all the cases in equation 1.

which the cases in which the spot and the guarantees of origin markets operate sequentially, and different market designs are implemented in the GO market.

Proposition 3. In absence of a GO market, in the mixed strategies equilibrium in the spot market, the suppliers randomize by using the next cumulative distribution functions:

$$F_1(p^s) = \begin{cases} 0 & \text{if } p^s < \underline{p}^s \\ \dfrac{(p^s - \underline{p}^s) q_2^{s2}}{p^s \left[q_2^{s2} - q_2^{s1} \right]} & \text{if } p^s \in \left(\underline{p}^s, \overline{P}^s \right) \\ 1 & \text{if } p^s = \overline{P}^s \end{cases} \tag{12}$$

$$F_2(p^s) = \begin{cases} 0 & \text{if } p^s < \underline{p}^s \\ \dfrac{(p^s - \underline{p}^s) q_1^{s1}}{p^s \left[q_1^{s1} - q_1^{s2} \right]} & \text{if } p^s \in \left(\underline{p}^s, \overline{P}^s \right) \\ 1 & \text{if } p^s = \overline{P}^s \end{cases} \tag{13}$$

Equations 12 and 13 seem to be a particular case of equations 10 and 11. However, it is important to notice that $\underline{p}^s$ and $\underline{p}^{sgo}$ do not need to coincide. Therefore, the comparison between the CDFs in the spot market in the absence of a GO market, and when the spot and the GO markets operate sequentially is more elaborated. We study this in the next section.

4 Analysis

In this section we study if the introduction of a GO market fosters the competition in the spot electricity market. We also study if changes in the structural parameters of the model (demand in the GO market, and green production capacity dispatched in the spot market) boost the competition in the spot electricity market.

When the spot and the GO markets operate sequentially, the dispatch in the spot market is the green production capacity that can be sold in the GO market. Therefore, for each pair of demands in the spot market (a_1^s, a_2^s) will be different production capacities in the GO market $(k_1^{go1}, k_2^{go1}; k_1^{go2}, k_2^{go2})$ (equations 2 and 3). Moreover, the fact that not all the production dispatched in the spot market is considered green by the consumers in the GO market complicates even more the analysis since in that case, the production capacities in the GO market are $(\alpha_1 k_1^{go1}, \alpha_2 k_2^{go1}; \alpha_1 k_1^{go2}, \alpha_2 k_2^{go2})$, with $\alpha \in [0, 1]$. Therefore, to isolate the impact of a change in the market design or in the structural parameters, it is crucial to choose the right demands in both markets. Otherwise, the equilibrium could be determined by the combination of many factors, and it could be impossible to isolate the impact of a change on the market design on the equilibrium.

In particular, to make the comparison between both market designs possible, and to isolate the impact of a change on the market design, we need to introduce two assumptions on the parameters. First, we keep the demand in the spot market symmetric ($a_1^s = a_2^s$) because we are interested in isolating the impact of a change in the market design, and to do that, we keep the demand in the spot market symmetric changing only the market design. Second, we also need to assume $k_1^{go1} = a_1^s + T > a_1^{go} + a_2^{go} > a_1^{go} + T > a_2^s - T = k_2^{go1}$ (and $k_2^{go2} = a_2^s + T > a_1^{go} + a_2^{go} > a_2^{go} + T > a_1^s - T = k_1^{go2}$, symmetric case). The last assumption is the crucial one, since it determines that under the "GO, no-constraint" design, the equilibrium in the GO market is determined by the production capacity constraint on the green energy sold in that market. In contrast, under the "GO, constraint" design, the equilibrium in the GO market is determined by the transmission capacity constraint in that market, i.e., we can isolate the impact that a change in the market design has on the equilibrium. For a different set of parameters, and under the "GO, constraint" design, the equilibrium in the GO market could be determined by the production and the transmission capacity constraints simultaneously, and it is not possible to isolate the effect and determine the impact that a change in the market design has on the equilibrium.

In the annex, we explain in detail how those two assumptions affect suppliers' dispatch, and therefore, suppliers' strategies in propositions 4-6. The results in those propositions apply for a broader set of demands in the spot and in the GO markets, but in those cases, the equilibrium is determined by different economic forces that could reinforce or mitigate each other. In the web page that complement this paper, the reader can work out the equilibrium for all the parameters covered in propositions 4-6, and also for a broader set of parameters.

We start our analysis comparing the effect that a change on the GO market design has on the equilibrium prices in the spot electricity market.

Proposition 4. The introduction of a guarantees of origin market has a pro-competitive effect on the spot electricity market only with the "GO, no-constraint" design.

The introduction of a GO market has a pro-competitive effect on the spot market only under the "GO, no-constraint" design. With that market design, the transmission constraint is not taken into account to work out the equilibrium in the GO market. Therefore, in the spot market, the suppliers compete fiercely trying to sell as much electricity as possible in that market, since by doing that, they will have more production capacity to sell in the GO market.

In the annex, we prove formally proposition 4. In particular, we prove that $\underline{p}^{sgo} \leq \underline{p}^s$, and that $F^{sgo} \geq F^s \; \forall p^{sgo} = p^s \in [\underline{p}^s, \overline{P}^s]$. The two previous relations imply that, under the "GO, no-constraint" design, the equilibrium price when the guarantees of origin and the spot markets operate sequentially are lower than when the spot market operates independently. Therefore, under the "GO, no-constraint" design, the introduction of a guarantees of origin

Figure 5: Proposition 4 ($a_1^s = a_2^s = 7, a_1^{go} = a_2^{go} = 3, \alpha_1 = \alpha_2 = 1$)

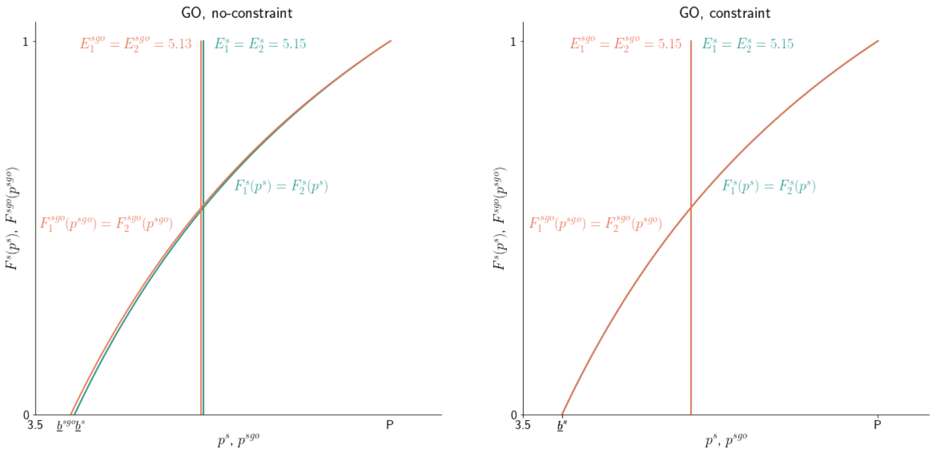


Table 1: Summary propositions

Design	Spot market						Integrated Spot and GO markets (design 1)					
Variables	p_1^s	p_2^s	CS^s	π_1^s	π_2^s	W^s	p_1^{sgo}	p_2^{sgo}	CS^{sgo}	π_1^{sgo}	π_2^{sgo}	W^{sgo}
Prop 4	5.159	5.159	25.78	35	35	95.78	5.137	5.137	26.08	34.67	34.67	95.42
Prop 5	5.159	5.159	25.78	35	35	95.78	5.159	5.159	25.78	35	35	95.78
Prop 6	5.159	5.159	25.78	35	35	95.78	5.157	5.157	25.8	34.98	34.98	95.75

Integrated Spot and GO markets (design 1 - "GO, no-constraint" design): The transmission constraint is not taken into consideration to work out the equilibrium in the GO market.

Prop 4: $a_1^s = a_2^s = 7$; $a_1^{go} = a_2^{go} = 3$; $\alpha_1^s = \alpha_2^s = 1$; $P^s = 7$; $P^{go} = 2$.

Prop 5: $a_1^s = a_2^s = 7$; $a_1^{go} = a_2^{go} = 2.5$; $\alpha_1^s = \alpha_2^s = 1$; $P^s = 7$; $P^{go} = 2$.

Prop 6: $a_1^s = a_2^s = 7$; $a_1^{go} = a_2^{go} = 2.5$; $\alpha_1^s = \alpha_2^s = 0.95$; $P^s = 7$; $P^{go} = 2$.

market has a pro-competitive effect on the spot market (row Prop 4, columns p_1^s, p_2^s, p_1^{sgo}, p_1^{sgo}, table 1). Statistically, the relations, $\underline{p}^{sgo} \le \underline{p}^s$, and $F^{sgo} \ge F^s$ $\forall p^{sgo} = p^s \in [\underline{p}^s, \overline{P}^s]$, imply that the suppliers' cumulative distribution functions when the guarantees of origin and the spot markets operate sequentially stochastic dominate the suppliers' cumulative distribution functions when the spot market operates independently (left-hand side panel, figure 5).

As a consequence of the decrease in the equilibrium prices in the spot market, the consumers surplus increase (row Prop 4, columns CS^s, CS^{sgo}, table 1), and suppliers' profits decrease (row Prop 4, columns π_1^s, π_2^s, π_1^{sgo}, π_2^{sgo}, table 1). The overall effect is a decrease in welfare (row Prop 4, columns WS^s, WS^{sgo}, table 1).

In the rest of this section, we study the impact that a change in the structural parameters of the model (demand in the GO market (a_1^{go}, a_2^{go}), and green production capacity that can be sold in the spot market (α_1, α_2)) has on the competition in the spot market. We focus our analysis on the "GO, no-constraint" design, since as we prove in proposition 4, that is the unique market design that has an impact on spot equilibrium prices.

Proposition 5. A decrease on the demand in the guarantees of origin market decreases the pro-competitive effect in the spot market induced by the introduction of a guarantees of origin market.

A decrease in the demand in the GO market decreases the incentives to compete fiercely in the spot electricity market. When the demand in the GO market is low, the potential suppliers' profits in that market are lower. Therefore, the incentives to sell more electricity in the spot market to have more production capacity to sell in the GO market decrease, and the pro-competitive impact of the introduction of a GO market can even disappear. That is the case of the example in table 1. When the demand in the GO market is low $a_1^{go} = a_2^{go} = 2.5$, both suppliers have enough green production in the GO to satisfy the entire demand in that market, and the introduction of a GO market has no impact on the spot electricity market, i.e., the equilibrium prices when the spot and the GO market operate sequentially are identical to the prices when the spot market operates independently (row Prop 5, table 1).

Proposition 6. A decrease on the green installed capacity that can be sold in the guarantees of origin market increases the pro-competitive effect induced by the introduction of a guarantees of origin market.

A reduction in the green production capacity that can be sold in the GO market has a pro-competitive effect in the spot electricity market. When the suppliers have less green production capacity to sell in the GO market, they compete fiercely to be dispatched in the spot market, since if they are dispatched last in that market, they will have very little production capacity to sell in the GO market. Consequently, a decrease in the green production capacity installed in the spot market makes suppliers compete fiercely increasing competition in the spot market.

In proposition 5, we prove that a decrease in demand in the GO market could even cancel the pro-competitive impact of the introduction of a GO market. In particular, when the demand in the GO market is low $a_1^{go} = a_2^{go} = 2.5$, the pro-competitive effect disappears. However, that is not true when the green production capacity in the spot market decreases. In particular, for that demand in the GO market, and when $\alpha = 0.95$, the pro-competitive effect emerges again, since in that case, the suppliers that compete in the GO market does not have enough production capacity to satisfy the total demand in that market, and the profits in that market are positive. Therefore, the suppliers have incentives to compete fiercely in the spot market to sell as much capacity as possible to have access to the profits in the GO market. In that case, the equilibrium prices in the spot market are lower, and the consumers surplus higher (row Prop 6, columns p_1^s, p_2^s, p_1^{sgo}, p_1^{sgo}, CS^s, CS^{sgo}, table 1). As in proposition 4, the decrease in prices has a negative impact on suppliers' profits (row Prop 6, columns π_1^s, π_2^s, π_1^{sgo}, π_2^{sgo}, table 1). The overall effect is a decrease in welfare (row Prop 6, columns WS^s, WS^{sgo}, table 1).

5 Conclusion

Consumers, governments and corporations are becoming more aware of the origin of the energy that they consume, and the guarantees of origin (GO) market is increasing in Europe and worldwide. More than 25% of the electricity consumed in Europe is consumed by using GO markets. We characterize the equilibrium when the spot and the GO markets operate sequentially. We study if the introduction of a GO market has a pro-competitive impact on the spot electricity market. We also analyze if changes on the structural parameters of the model (GO demand, and green production capacity that can be sold in the GO market) have a pro-competitive effect on the spot electricity market.

We work out the equilibrium when two market designs are introduced in the GO market. In the first market design, the transmission constraint between two different nodes in the GO market is neglected when the GO market is cleared ("GO, no-constraint"). That is the current GO market design. In the second market design, the transmission constraint between two different electricity nodes in the GO market is taken into account when the GO market is cleared ("GO, constraint"). That market design follows the same approach that the design in the spot market where the transmission constraint is taken into account to clear the market. Moreover, the "GO, constraint" design is in line with the current debate to redesign the GO market, where the market design proposals are in favor of taken into account the congestion in the transmission line when the GO market is cleared. That market design aims to avoid investment distortions induced by the artificial neglecting of the transmission constraint in the GO market.

We find that the introduction of a GO market has a pro-competitive impact on the spot electricity market only under the "GO, no-constraint" design. With that market design, the suppliers compete fiercely in the spot market to sell as much production capacity as possible, since by doing that, they will have more production capacity to sell in the GO market. In contrast, under the "GO, constraint" design those incentives disappear, since the suppliers cannot sell their entire production capacity in the GO market due to the transmission constraint in that market.

We also study the impact of a change on the structural parameters of the model on the equilibrium (GO demand, and green production capacity that can be sold in the GO market). A decrease on the demand in the GO market reduces the pro-competitive impact of the introduction of a GO market. A reduction of demand in the GO market makes that the suppliers have less incentives to compete fiercely in the spot market, since they will have access to less demand in the GO market. Therefore, they do not have incentives to sell as much production capacity as possible in the spot market, since they will not be able to sell all that production capacity in the GO market. A reduction on the green production capacity that can be sold in the GO market has also a pro-competitive impact on the spot market. In that case, it is very risky for the suppliers to be dispatched last in the spot market, since they will have very little green production capacity to sell in the GO market. Therefore, they compete fiercely to sell as much green production capacity as possible in the spot market to have more production capacity to sell in the GO market.

Our findings have import policy implications. First, the current GO market design has a pro-competitive impact on the spot electricity market. This contrast with the proposed redesign of the GO market that will have no impact on the competition in the spot electricity market. However, it is important to notice that the current GO market design does not have into consideration the transmission constraints to clear that market and that could induce long-term investment distortions. Therefore, it is not clear that the current GO market design is the correct one in the long-term. Second, an increase in the demand in the GO market has a pro-competitive impact on the spot electricity market. Therefore, any policy that increases the awareness of consumers about the origin og their electricity consumption will have a positive impact on consumers welfare. Third, a low penetration of green production capacity has a pro-competitive impact on the spot electricity market. Therefore, the pro-competitive effect will disappear when the penetration of green production capacity increases.

The objective of this paper is to characterize the equilibrium when the spot and the GO market operate sequentially. We also study the impact that changes in the market design and the structural parameters of the model have on the competition in the spot electricity market. To achieve that objective, we follow the standard industrial approach where we keep the model as simple as possible and change the market design and the structural parameters of the model to isolate the impact that those changes have on equilibrium. In our case, we assume symmetric demand in the spot market, and a set of parameters in the GO market that give us the opportunity to isolate the impact of a change on the market design. Despite those assumptions, we characterize the equilibrium for any combination of demands in the spot and the GO market. Therefore, our paper fully characterize the equilibrium for any set of parameters, and it useful not only to prove formally the results in this paper, but also to address other relevant questions in the interaction between the spot and the GO market, e.g., asymmetries in the demand in the spot and the GO market, asymmetries in the green capacity productions, long-term investment incentives in the spot electricity market, etcetera. He leave that analysis for future research.

Appendix

Lemma 1. In the proof, we assume that both suppliers face a positive residual demand.[11] In the GO2 market both suppliers face a positive residual demand, and a pure strategies Nash equilibrium does not exist, since the suppliers stablish a price war having incentives to undercut each other to be dispatched first in the auction. In particular, a pair of prices ($p_1^{go2} = p_2^{go2} = 0$) is not a pure strategies Nash equilibrium, since at least one supplier has incentives to increase its price and satisfy the residual demand. A pair of prices ($p_1^{go2} = p_2^{go2} > 0$) is not a pure strategies Nash equilibrium, since both suppliers have incentives to reduce their price to be dispatched first in the auction. A pair of prices ($p_1^{go2} > p_2^{go2} > 0$) is not a pure strategies Nash equilibrium, since supplier 2 has incentives to increase its price, but still undercutting supplier 1 to be dispatched first at a higher bid. □

Lemma 2. In the proof, we assume that both suppliers face a positive residual demand.[12] Each supplier can guarantee its own residual profit by setting the price cap in the GO2 market and satisfying the residual demand ($\overline{P}^{go} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}$), ($\overline{P}^{go} max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}$). Therefore, none of the suppliers will set a price lower than ($\underline{p}_1^{go2}$) or ($\underline{p}_2^{go2}$), where $\underline{p}_1^{go2}$ solves $\underline{p}_1^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} = \overline{P}^{go} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}$, and $\underline{p}_2^{go2}$ solves $\underline{p}_2^{go2} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} = \overline{P}^{go} max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}$.

In addition, none of the suppliers sets a price $p^{go2} \in \left[min\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}, max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}\right]$, since the supplier for which $\underline{p}_i^{go2}$ is lower knows that the other supplier will never sets a price lower than $\underline{p}_j^{go2}$, and it can increase its bid, but still undercutting the other supplier and increase its profits. Therefore, the lower bound of the support of the mixed strategies equilibrium is $\left(max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}\right)$. The upper bound of the support is equal to the price cap $\left(\overline{P}^{go}\right)$. Hence, both suppliers randomize in the interval $p^{go2} \in \left[max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}, \overline{P}^{go}\right]$. □

Proposition 1. To work out the Cumulative Distribution Function (CDF), we follow Varian (1980) and Kreps and Scheinkman (1984). I work out supplier 2's CDF. The steps to work out supplier 1's CDF are identical.

The proof follows four steps. In the first step, the payoff function for any supplier is:

$$\begin{aligned}\pi_1(p^{go2}) &= F_2(p^{go2})\left[p^{go2} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}\right] + \\ &\quad (1 - F_2(p^{go2}))\left[p^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}\right] = \\ &= -p^{go2}\ F_2(p^{go2})\left[min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} - max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}\right] + \\ &\quad p^{go2}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} \qquad (14)\end{aligned}$$

[11] The proof follows the same principles when only one supplier faces a positive residual demand.

[12] The proof follows the same principles when only one supplier faces a positive residual demand.

With probability $F_2(p^{go2})$, supplier 2 sets the lower price, and supplier 1 is dispatched last in the auction. In that case, supplier 1's profits are $p^{go2}max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}$. With probability $(1-F_2(p^{go2}))$, supplier 2 sets the higher price, and supplier 1 is dispatched first in the auction. In that case, supplier 1's profits are $p^{go2}min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}$.

In the second step, $\pi_1(p^{go2}) = \overline{\pi}_1^{go2} \forall p^{go2} \in S$, where S is the support of the mixed strategies worked out in lemma 2, and $\overline{\pi}_1^{go2}$ is the average profit, i.e., each strategy in the support generates the same expected payoff. Then,

$$\begin{aligned}
\overline{\pi}_1^{go2} &= -p^{go2}\ F_2(p^{go2})\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right] + \\
&\quad p^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} \Rightarrow \\
F_2(p^{go2}) &= \frac{p^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - \overline{\pi}_1^{go2}}{p^{go2}\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right]} \qquad (15)
\end{aligned}$$

The third step, at $\underline{p}^{go2}$, $F_2(\underline{p}^{go2}) = 0$. Then,

$$\overline{\pi}_1^{go2} = \underline{p}^{go2} min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} \qquad (16)$$

In the fourth step, plugging 16 into 15, we obtain supplier 2's mixed strategies.

$$\begin{aligned}
F_2(p^{go2}) &= \frac{p^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - \underline{p}^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}}{p^{go2}\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right]} = \qquad (17) \\
&= \frac{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}}{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}} \frac{p^{go2}-\underline{p}^{go2}}{p^{go2}} \quad \forall p^{go2} \in S
\end{aligned}$$

In the fifth step, by using lemma 2, we plug the value of the lower bound of the support in market GO2 ($\underline{p}^{go2}$) into equation 17, and it is straight forward to show that $F_2(p^{go2} = \overline{P}^{go}) = 1$.

Step fifth concludes the proof. □

Lemma 3. The proof of lemma 3 is as in lemma 2, but to work out $\underline{p}_1^{sgo}$ and $\underline{p}_2^{sgo}$ in the spot market, it is necessary to take into account suppliers' expected profits in the GO1 and GO2 markets. In particular to work out $\underline{p}_1^{sgo}$, it is necessary to solve $\underline{p}_1^{sgo}(a_1^s+T)+\underline{p}^{go1}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go1}\} = \overline{P}^{sgo}(a_1^s-T)+\underline{p}^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}$. In the right-hand side, $\overline{P}^{sgo}(a_1^s-T)$ represents supplier 1's profits in the spot market when it serves the residual demand and sets the price cap in that market, and $(\underline{p}^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\})$ represents supplier 1's expected profits in the GO2 market. Supplier 1 can guarantee that profit by serving the residual demand and setting a price equal to the price cap in the spot market. Therefore, it will never set a price in the spot market for which its profits are lower than the profits in the right-hand side. The price that makes supplier 1 be indifferent between satisfy the residual demand in the spot market at the price cap plus the expected profits in the GO2 market is $\underline{p}_1^{sgo}$. That price equalize supplier 1's profits in the spot market when it is dispatched first in that market ($\underline{p}_1^{sgo}(a_1^s + T)$) plus supplier 1's expected profits in the GO1 market ($\underline{p}^{go1}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\}$).

Proposition 2. We follow the same logic as in proposition 1. We work out supplier 2's CDF. The steps to work out supplier 1's CDF are identical.

The proof follows four steps. In the first step, the payoff function for any supplier is:

$$\begin{aligned}
\pi_1(p^{sgo}) &= F_2(p^{sgo})\left[p^{sgo}(a_1^s - T) + \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}\right] + \\
&\quad (1 - F_2(p^{sgo}))\left[p^{sgo}(a_1^s + T) + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\}\right] = \\
&= -p^{sgo}\ F_2(p^{sgo}) \\
&\quad \left[p^{sgo}(a_1^s + T) + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} - p^{sgo}(a_1^s - T) - \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\right] + \\
&\quad p^{sgo}(a_1^s + T) + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} \qquad (18)
\end{aligned}$$

With probability $F_2(p^{sgo})$, supplier 2 sets the lower price, and supplier 1 is dispatched last in the auction. In that case, supplier 1's profits in the spot market are $p^{sgo}(a_1^s - T)$, and supplier 1's expected profit in market GO2 are $\underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\}$. With probability $(1 - F_2(p^{sgo}))$, supplier 2 sets the higher price, and supplier 1 is dispatched first in the auction. In that case, supplier 1's profits in the spot market are $p^{sgo}(a_1^s + T)$, and supplier 1's expected profits in market GO1 are $\underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\}$.

In the second step, $\pi_1(p^{sgo}) = \overline{\pi}_1^{sgo} \forall p^{sgo} \in S$, where S is the support of the mixed strategies worked out in lemma 3, and $\overline{\pi}_1^{sgo}$ is the average profit, i.e., each strategy in the support generates the same expected payoff. Then,

$$\begin{aligned}
\overline{\pi}_1^{sgo} &= -p^{sgo}\ F_2(p^{sgo}) \\
&\quad \left[p^{sgo}(a_1^s + T) + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} - p^{sgo}(a_1^s - T) - \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\right] + \\
&\quad p^{sgo}(a_1^s + T) + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} \Rightarrow \\
F_2(p^{sgo}) &= \frac{p^{sgo}(a_1^s + T) + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} - \overline{\pi}_1^{go2}}{p^{sgo}(a_1^s + T) + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} - p^{sgo}(a_1^s - T) - \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}} \\
&= \frac{p^{sgo}(a_1^s + T) + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} - \overline{\pi}_1^{go2}}{p^{sgo}\left[(a_1^s + T) - (a_1^s - T)\right] + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} - \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}} \qquad (19)
\end{aligned}$$

The third step, at $\underline{p}^{sgo}$, $F_2(\underline{p}^{sgo}) = 0$. Then,

$$\overline{\pi}_1^{sgo} = \underline{p}^{sgo}(a_1^s + T) + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} \qquad (20)$$

In the fourth step, plugging 20 into 19, we obtain supplier 2's mixed strategies.

$$\begin{aligned}
F_2(p^{go2}) &= \frac{(p^{sgo} - \underline{p}^{sgo})(a_1^s + T)}{p^{sgo}\left[(a_1^s + T) - (a_1^s - T)\right] + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} - \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}} \\
&\quad \forall p^{go2} \in S \qquad (21)
\end{aligned}$$

In the fifth step, by using lemma 3, we plug the value of the lower bound of the support ($\underline{p}^{sgo}$) into equation 21, and it is straight forward to show that $F_2(p^{sgo} = \overline{P}^s) = 1$.

Step fifth concludes the proof. $\square$

Proposition 3. The proof follows the same steps that propositions 1 and 2.

Proposition 4. In the "GO, no-constraint" design, we prove that $\underline{p}^{sgo} \leq \underline{p}^s$, and that $F^{sgo} \geq F^s \; \forall p^{sgo} = p^s \in [\underline{p}^s, \overline{P}^s]$. The two previous relations imply that, under the "GO, no-constraint" design, the equilibrium price when the guarantees of origin and the spot markets operate sequentially are lower than when the spot market operates independently, i.e., under the "GO, no-constraint" design, the introduction of a guarantees of origin market has a pro-competitive effect on the spot market. In the "GO, constraint" design, we prove that $\underline{p}^{sgo} = \underline{p}^s$, and that $F^{sgo} = F^s \; \forall p^{sgo} = p^s \in [\underline{p}^s, \overline{P}^s]$. The two previous relations imply that, under the "GO, constraint" design, the equilibrium price when the guarantees of origin and the spot markets operate sequentially are equal than when the spot market operates independently.

To make the comparison between both market designs possible, and to isolate the impact of a change on the market design, we need to introduce two assumptions on the parameters. First, we keep the demand in the spot market symmetric ($a_1^s = a_2^s$) because we are interested in isolating the impact of a change in the market design, and to do that, we keep the demand in the spot market symmetric changing only the market design. We also need to assume $a_1^s + T > a_1^{go} + a_2^{go} > a_2^s - T$ (and $a_2^s + T > a_1^{go} + a_2^{go} > a_1^s - T$, symmetric case). The last assumption is the crucial one, since it determines that under the "GO, no-constraint" design, the equilibrium in the GO market is determined by the production capacity constraint on the green energy sold in that market. In contrast, under the "GO, constraint" design, the equilibrium in the GO market is determined by the transmission capacity constraint, i.e., we can isolate the impact that a change in the market design has on the equilibrium. For a different set of parameters, and under the "GO, constraint" design, the equilibrium in the GO market could be determined by the production and the transmission capacity constraints simultaneously, and it is not possible to isolate the effect and determine the effect that a change in the market design has on the equilibrium.

In the "GO, no-constraint" design, we prove that $\underline{p}^{sgo} \leq \underline{p}^s$, and that $F^{sgo} \geq F^s \; \forall p^{sgo} = p^s \in [\underline{p}^s, \overline{P}^s]$.

We prove that $\underline{p}^{sgo} \leq \underline{p}^s$. We proceed in two steps.

First, we prove that $\underline{p}_1^{go} = \underline{p}_2^{go}$. By lemma 2, we know that:

$$
\begin{aligned}
\underline{p}^{go1} &= min\left\{\frac{\overline{P}^{go}(a_1^{go}+a_2^{go}-k_2^{go1})}{a_1^{go}+a_2^{go}}, \frac{\overline{P}^{go}(a_1^{go}+a_2^{go}-k_1^{go1})}{k_2^{go1}}\right\} \\
&= min\left\{\frac{\overline{P}^{go}(a_1^{go}+a_2^{go}-(a_2^s-T))}{a_1^{go}+a_2^{go}}, \frac{\overline{P}^{go}(a_1^{go}+a_2^{go}-(a_1^s+T))}{a_2^s-T}\right\} \\
\underline{p}^{go2} &= min\left\{\frac{\overline{P}^{go}(a_1^{go}+a_2^{go}-k_2^{go2})}{k_1^{go2}}, \frac{\overline{P}^{go}(a_1^{go}+a_2^{go}-k_1^{go2})}{a_1^{go}+a_2^{go}}\right\} \\
&= min\left\{\frac{\overline{P}^{go}(a_1^{go}+a_2^{go}-(a_2^s+T))}{a_1^s-T}, \frac{\overline{P}^{go}(a_1^{go}+a_2^{go}-(a_1^s-T))}{a_1^{go}+a_2^{go}}\right\}
\end{aligned} \tag{22}
$$

Since $a_1^s = a_2^s$, it is straight forward to check that $\underline{p}^{go1} = \underline{p}^{go2}$.

Second, by using the previous result, we show that $\underline{p}^{sgo} \leq \underline{p}^s$. By using lemma 3, we know that,

$$
\begin{aligned}
\underline{p}^{sgo} &= min\left\{\underline{p}_1^{sgo}, \underline{p}_2^{sgo}\right\} \\
&= min\left\{\frac{\overline{P}^s(a_1^s-T)+\underline{p}^{go}k_1^{go2}-\underline{p}^{go}(a_1^{go}+a_2^{go})}{k_1^{go1}}, \frac{\overline{P}^s(a_2^s-T)+\underline{p}^{go}k_2^{go1}-\underline{p}^{go}(a_1^{go}+a_2^{go})}{k_2^{go2}}\right\} \\
&= min\left\{\frac{\overline{P}^s(a_1^s-T)+\underline{p}^{go}(a_1^s-T)-\underline{p}^{go}(a_1^{go}+a_2^{go})}{a_1^s+T}, \frac{\overline{P}^s(a_2^s-T)+\underline{p}^{go}(a_2^s-T)-\underline{p}^{go}(a_1^{go}+a_2^{go})}{a_2^s+T}\right\}
\end{aligned} \tag{23}
$$

Given that $a_1^s = a_2^s$, $\underline{p}_1^{sgo} = \underline{p}_2^{sgo}$. By using this result, we obtain that,

$$
\begin{aligned}
\underline{p}^{sgo} \leq \underline{p}^s &\Leftrightarrow \frac{\overline{P}^s(a_1^s-T)+\underline{p}^{go}(a_1^s-T)-\underline{p}^{go}(a_1^{go}+a_2^{go})}{a_1^s+T} \leq \frac{\overline{P}^s(a_1^s-T)}{a_1^s+T} \\
&\Leftrightarrow \underline{p}^{go}((a_1^s-T)-(a_1^{go}+a_2^{go})) \leq 0
\end{aligned} \tag{24}
$$

The inequality in 24 holds since, as we have explained in the assumptions in this proposition, we have assumed that $(a_1^{go}+a_2^{go}) \geq (a_1^s - T)$.

Second, we prove that $F^{sgo} \geq F^s \ \forall p^{sgo} = p^s \in [\underline{p}^s, \overline{P}^s]$. First, we define F_2^{sgo} and F_2^s. The proof for F_1^{sgo} and F_1^s follows the same steps.

$$
\begin{aligned}
F_2^s &= \frac{(p^s-\underline{p}^s)(a_1^s+T)}{p^s[(a_1^s+T)-(a_1^s-T)]} \\
&= \frac{[p^s(a_1^s+T)-\overline{P}^s(a_1^s-T)](a_1^s+T)}{p^s[(a_1^s+T)-(a_1^s-T)]} = \frac{A(a_1^s+T)}{B} \\
F_2^{sgo} &= \frac{(p^s-\underline{p}^{sgo})(a_1^s+T)}{p^s[(a_1^s+T)-(a_1^s-T)]+\underline{p}^{go}(a_1^{go}+a_2^{go})-\underline{p}^{go}(a_1^s-T)} \\
&= \frac{[p^s(a_1^s+T)-\overline{P}^s(a_1^s-T)-\underline{p}^{go}(a_1^s-T)+\underline{p}^{go}(a_1^{go}+a_2^{go})](a_1^s+T)}{p^s[(a_1^s+T)-(a_1^s-T)]+\underline{p}^{go}(a_1^{go}+a_2^{go})-\underline{p}^{go}(a_1^s-T)} = \frac{(A+a)(a_1^s+T)}{B+a}
\end{aligned} \tag{25}
$$

Where, $A = [p^s(a_1^s+T) - \overline{P}^s(a_1^s-T)]$, $B = p^s[(a_1^s+T)-(a_1^s-T)]$, $a = \underline{p}^{go}(a_1^{go}+a_2^{go}) - \underline{p}^{go}(a_1^s-T)$. It is important to notice that $A \leq B\ \forall p^s \in [\underline{p}^s, \overline{P}^s]$, and $A = B$ when $p^s = \overline{P}^s$. It also important to remark that $a > 0$, since by the assumptions, $(a_1^{go}+a_2^{go}) > (a_1^s - T)$.

By using equation 25, it is easy to prove that $F^{sgo} \geq F^s\ \forall p^{sgo} = p^s \in [\underline{p}^s, \overline{P}^s]$:

$$\begin{aligned} F_2^s \leq F_2^{sgo} \quad &\Leftrightarrow \quad \frac{A}{B} \leq \frac{A+a}{B+a} \Leftrightarrow \frac{B-\epsilon}{B} \leq \frac{(B-\epsilon)+a}{B+a} \\ &\Leftrightarrow \quad (B-\epsilon)(B+a) \leq ((B-\epsilon)+a)B \Leftrightarrow B^2+Ba-\epsilon B-\epsilon a \leq B^2-\epsilon B+aB \\ &\Leftrightarrow \quad -\epsilon a \leq 0 \end{aligned} \tag{26}$$

where inequality 26 holds because $\epsilon > 0$ and $a > 0$.

It is straight forward to show that $F_2^s(\overline{P}^s) = F_2^{sgo}(\overline{P}^s) = 1$.

In the "GO, constraint" design, we prove that $\underline{p}^{sgo} = \underline{p}^s$, and that $F^{sgo} = F^s\ \forall p^{sgo} = p^s \in [\underline{p}^s, \overline{P}^s]$.

We prove that $\underline{p}^{sgo} = \underline{p}^s$. We proceed in two steps.

First, we prove that $\underline{p}_1^{go} = \underline{p}_2^{go}$. By lemma 2, we know that:

$$\begin{aligned} \underline{p}^{go1} &= min\left\{\frac{\overline{P}^{go}(a_1^{go}-T)}{a_1^{go}+T}, \frac{\overline{P}^{go}(a_2^{go}-T)}{a_2^{go}+T}\right\} \\ \underline{p}^{go2} &= min\left\{\frac{\overline{P}^{go}(a_1^{go}-T)}{a_1^{go}+T}, \frac{\overline{P}^{go}(a_2^{go}-T)}{a_2^{go}+T}\right\} \end{aligned} \tag{27}$$

It is important to notice that in equation 27, the lower bound of the support is determined exclusively by the transmission constraints. This contrast with 22, where the lower bound is determined exclusively by the green production constraints. In that sense, the assumptions introduced to prove proposition 4 are crucial, since they give us the opportunity to isolate the impact of the market design determining the lower bound of the support. Based on those assumptions, it is straight forward to check that $\underline{p}^{go1} = \underline{p}^{go2}$.

Second, by using the previous result, we show that $\underline{p}^{sgo} = \underline{p}^s$. By using lemma 3, we know that,

$$\begin{aligned} \underline{p}^{sgo} &= min\left\{\underline{p}_1^{sgo}, \underline{p}_2^{sgo}\right\} \qquad (28) \\ &= min\left\{\frac{\overline{P}^s(a_1^s-T)+\underline{p}^{go}(a_1^{go}+T)-\underline{p}^{go}(a_1^{go}+T)}{a_1^s+T}, \frac{\overline{P}^s(a_2^s-T)+\underline{p}^{go}(a_2^s+T)-\underline{p}^{go}(a_2^{go}+T)}{a_2^s+T}\right\} \end{aligned}$$

Given that $a_1^s = a_2^s$, $\underline{p}_1^{sgo} = \underline{p}_2^{sgo}$. By using this result, we obtain that,

$$\underline{p}^{sgo} = \underline{p}^{s} \quad \Leftrightarrow \quad \frac{\overline{P}^{s}(a_1^s - T) + \underline{p}^{go}(a_1^{go} + T) - \underline{p}^{go}(a_1^{go} + T)}{a_1^s + T} = \frac{\overline{P}^{s}(a_1^s - T)}{a_1^s + T} \tag{29}$$

Second, we prove that $F^{sgo} \geq F^{s}\ \forall p^{sgo} = p^{s} \in [\underline{p}^{s}, \overline{P}^{s}]$. First, we define F_2^{sgo} and F_2^{s}. The proof for F_1^{sgo} and F_1^{s} follows the same steps.

$$\begin{aligned} F_2^{s} &= \frac{(p^{s} - \underline{p}^{s})(a_1^s + T)}{p^{s}[(a_1^s + T) - (a_1^s - T)]} \\ F_2^{sgo} &= \frac{(p^{s} - \underline{p}^{sgo})(a_1^s + T)}{p^{s}[(a_1^s + T) - (a_1^s - T)] + \underline{p}^{go}(a_1^{go} + T) - \underline{p}^{go}(a_1^{go} + T)} \end{aligned} \tag{30}$$

It is easy to check that in equation 30, $F_2^{sgo} = F_2^{s}\ \forall p^{sgo} = p^{s} \in [\underline{p}^{s}, \overline{P}^{s}]$. Moreover, it is straight forward to show that $F_2^{s}(p^{s}) = F_2^{sgo}(p^{s}) \leq 1\ \forall p^{sgo} = p^{s} \in [\underline{p}^{s}, \overline{P}^{s}]$, with strict equality when $p^{s} = \overline{P}^{s}$.

Proposition 5. In the "GO, no-constraint" design, we prove that $\underline{p}^{sgo}(\hat{a}_1^{go} + \hat{a}_2^{go}) \leq \underline{p}^{sgo}(a_1^{go} + a_2^{go})$, and that $F^{sgo}(\hat{a}_1^{go} + \hat{a}_2^{go}) \geq F^{sgo}(a_1^{go} + a_2^{go})\ \forall p^{sgo} \in [\underline{p}^{sgo}, \overline{P}^{s}]$. Where, $(\hat{a}_1^{go} + \hat{a}_2^{go}) > (a_1^{go} + a_2^{go})$. The previous relations imply that, under the "GO, no-constraint" design, an increase in demand in the guarantees of origin market decreases the equilibrium price when the guarantees of origin and the spot markets operate sequentially, i.e., under the "GO, no-constraint" design, an increase in demand in the guarantees of origin market has a pro-competitive effect on the spot market. We focus our attention in the "GO, no-constraint" design, since, as we proved in proposition 4, in the "GO, constraint" design $\underline{p}^{sgo} = \underline{p}^{s}$, and $F^{sgo} = F^{s}\ \forall p^{sgo} \in [\underline{p}^{sgo} = p^{s}, \overline{P}^{s}]$.

First, we prove that $\underline{p}^{sgo}(\hat{a}_1^{go} + \hat{a}_2^{go}) \leq \underline{p}^{sgo}(a_1^{go} + a_2^{go})$.

In proposition 3, we prove that $\underline{p}^{go1} = \underline{p}^{go2} = \underline{p}^{go}$. Therefore,

$$\underline{p}_1^{sgo} = \frac{\overline{P}^{s}(a_1^s - T) + \underline{p}^{go}(a_1^s - T) - \underline{p}^{go}(a_1^{go} + a_2^{go})}{a_1^s + T} \tag{31}$$

Taken the derivative of $\underline{p}_1^{sgo}$ respect $(a_1^{go} + a_2^{go})$, we obtain that[13],

$$\frac{\partial \underline{p}_1^{sgo}}{\partial (a_1^{go} + a_2^{go})} = -\frac{\underline{p}^{go}}{a_1^s + T} < 0 \tag{32}$$

[13]We can take the derivative of $\underline{p}_1^{sgo}$ respect (a_1^{go}) or respect $\underline{p}_1^{sgo}$ respect (a_2^{go}), and we obtain a similar result.

We prove that $F^{sgo}(\hat{a}_1^{go}+\hat{a}_2^{go}) \geq F^{sgo}(a_1^{go}+a_2^{go})\ \forall p^{sgo} \in [\underline{p}^{sgo}, \overline{P}^s]$.

From equation 25, we know that,

$$
\begin{aligned}
F_2^{sgo}(a_1^{go}+a_2^{go}) &= \frac{(p^s - \underline{p}^{sgo})(a_1^s+T)}{p^s[(a_1^s+T)-(a_1^s-T)] + \underline{p}^{go}(a_1^{go}+a_2^{go}) - \underline{p}^{go}(a_1^s-T)} \\
&= \frac{[p^s(a_1^s+T) - \overline{P}^s(a_1^s-T) - \underline{p}^{go}(a_1^s-T) + \underline{p}^{go}(a_1^{go}+a_2^{go})](a_1^s+T)}{p^s[(a_1^s+T)-(a_1^s-T)] + \underline{p}^{go}(a_1^{go}+a_2^{go}) - \underline{p}^{go}(a_1^s-T)} = \frac{(A+a)(a_1^s+T)}{B+a} \\
F_2^{sgo}(\hat{a}_1^{go}+\hat{a}_2^{go}) &= \frac{(p^s - \underline{p}^{sgo})(a_1^s+T)}{p^s[(a_1^s+T)-(a_1^s-T)] + \underline{p}^{go}(\hat{a}_1^{go}+\hat{a}_2^{go}) - \underline{p}^{go}(a_1^s-T)} \\
&= \frac{[p^s(a_1^s+T) - \overline{P}^s(a_1^s-T) - \underline{p}^{go}(a_1^s-T) + \underline{p}^{go}(\hat{a}_1^{go}+\hat{a}_2^{go})](a_1^s+T)}{p^s[(a_1^s+T)-(a_1^s-T)] + \underline{p}^{go}(\hat{a}_1^{go}+\hat{a}_2^{go}) - \underline{p}^{go}(a_1^s-T)} = \frac{(A+\hat{a})(a_1^s+T)}{B+\hat{a}}
\end{aligned} \tag{33}
$$

Where, $A = [p^s(a_1^s+T) - \overline{P}^s(a_1^s-T)]$, $B = p^s[(a_1^s+T)-(a_1^s-T)]$, $a = \underline{p}^{go}(a_1^{go}+a_2^{go}) - \underline{p}^{go}(a_1^s-T)$, $\hat{a} = \underline{p}^{go}(\hat{a}_1^{go}+\hat{a}_2^{go}) - \underline{p}^{go}(a_1^s-T)$. It is important to notice that $A \leq B\ \forall p^s \in [\underline{p}^s, \overline{P}^s]$, and $A = B$ when $p^s = \overline{P}^s$. It also important to remark that $a > 0$, $\hat{a} > 0$, since by the assumptions, $(a_1^{go}+a_2^{go}) > (a_1^s - T)$, $(\hat{a}_1^{go}+\hat{a}_2^{go}) > (a_1^s-T)$.

Second, by using equation 33, it is easy to prove that $F^{sgo}(\hat{a}_1^{go}+\hat{a}_2^{go}) \geq F^{sgo}(a_1^{go}+a_2^{go})\ \forall p^{sgo} \in [\underline{p}^{sgo}, \overline{P}^s]$:

$$
\begin{aligned}
F_2^{sgo}(\hat{a}_1^{go}+\hat{a}_2^{go}) \geq F_2^{sgo}(a_1^{go}+a_2^{go}) &\Leftrightarrow \frac{A+\hat{a}}{B+\hat{a}} \geq \frac{A+a}{B+a} \Leftrightarrow \frac{(B-\epsilon)+(a+\delta)}{B+(a+\delta)} \geq \frac{(B-\epsilon)+a}{B+a} \\
&\Leftrightarrow [(B-\epsilon)+(a+\delta)]\,(B+a) \geq [(B-\epsilon)+a]\,[B+(a+\delta)] \\
&\Leftrightarrow (B-\epsilon)B + (B-\epsilon)a + (a+\delta)B + (a+\delta)a \geq \\
&\ (B-\epsilon)B + (B-\epsilon)(a+\delta) + aB + a(a+\delta) \\
&\Leftrightarrow (B-\epsilon)a + aB + \delta B \geq (B-\epsilon)a + (B-\epsilon)\delta + aB \\
&\Leftrightarrow \delta B \geq \delta B - \epsilon\delta \Leftrightarrow 0 \geq -\epsilon\delta
\end{aligned} \tag{34}
$$

Where inequality 34 holds because $\epsilon > 0$ and $\delta > 0$.

It is straight forward to show that $F_2^{sgo}(\overline{P}^s) = 1$.

Proposition 6. In the "GO, no-constraint" design, we prove that $\underline{p}^{sgo}(\alpha < 1) \leq \underline{p}^{sgo}(\alpha = 1)$, and that $F^{sgo}(\alpha < 1) \geq F^{sgo}(\alpha = 1)\ \forall p^{sgo} \in [\underline{p}^{sgo}(\alpha = 1), \overline{P}^s]$. The previous relations imply that, under the "GO, no-constraint" design, a decrease in the green production capacity to be sold in the guarantees of origin market decreases the equilibrium price when the guarantees of origin and the spot markets operate sequentially, i.e., under the "GO, no-constraint" design, a decrease in the green production capacity to be sold in the guarantees of origin market has a pro-competitive effect on the spot market. As in proposition 5, we focus our attention in the "GO, no-constraint" design, since, as we proved in proposition 4, in the "GO, constraint" design $\underline{p}^{sgo} = \underline{p}^{s}$, and $F^{sgo} = F^{s}$ $\forall p^{sgo} \in [\underline{p}^{sgo} = p^{s}, \overline{P}^{s}]$.

First, we prove that $\underline{p}^{sgo}(\alpha < 1) \leq \underline{p}^{sgo}(\alpha = 1)$.

By using lemma 3, we know that,

$$\begin{aligned}
\underline{p}_1^{sgo} &= \frac{\overline{P}^s(a_1^s - T) + \dfrac{\overline{P}^{go}(a_1^{go} + a_2^{go} - \alpha(a_2^s - T))}{a_1^{go} + a_2^{go}}\alpha(a_1^s - T) - \dfrac{\overline{P}^{go}(a_1^{go} + a_2^{go} - \alpha(a_2^s - T))}{a_1^{go} + a_2^{go}}(a_1^{go} + a_2^{go})}{a_1^s + T} \\
&= \frac{\overline{P}^s(a_1^s - T) + \dfrac{\overline{P}^{go}(a_1^{go} + a_2^{go} - \alpha(a_2^s - T))}{a_1^{go} + a_2^{go}}\left[\alpha(a_1^s - T) - (a_1^{go} + a_2^{go})\right]}{a_1^s + T} \qquad (35)
\end{aligned}$$

By using equation 35, we take the derivative of $\underline{p}_1^{sgo}$ respect α:

$$\begin{aligned}
\frac{\partial \underline{p}_1^{sgo}}{\partial \alpha} &= -\frac{\overline{P}^{go}(a_2^s - T)}{a_1^{go} + a_2^{go}}\left[\alpha(a_1^s - T) - (a_1^{go} + a_2^{go})\right] + \frac{\overline{P}^{go}(a_1^{go} + a_2^{go} - \alpha(a_2^s - T))}{a_1^{go} + a_2^{go}}(a_2^s - T) \\
&= -\overline{P}^{go}(a_2^s - T)\alpha(a_1^s - T) + \overline{P}^{go}(a_2^s - T)(a_1^{go} + a_2^{go}) + \overline{P}^{go}(a_1^{go} + a_2^{go})(a_1^s - T) - \overline{P}^{go}\alpha(a_2^s - T)(a_1^s - T) \\
&= \overline{P}^{go}(a_1^s - T)\left[2(a_1^{go} + a_2^{go}) - 2\alpha(a_2^s - T)\right] > 0 \qquad (36)
\end{aligned}$$

Where the result in 36 holds by the assumptions in propositions 4-6, $(a_1^{go} + a_2^{go}) > \alpha(a_2^s - T)$ $\forall \alpha \in [0, 1]$.

Second, we prove that $F^{sgo}(\alpha < 1) \geq F^{sgo}(\alpha = 1)$ $\forall p^{sgo} \in [\underline{p}^{sgo}(\alpha = 1), \overline{P}^{s}]$.

From equation 25, we know that,

$$\begin{aligned}
F_2^{sgo}(\alpha) &= \frac{[p^s(a_1^s + T) - \overline{P}^s(a_1^s - T) - \alpha \underline{p}_1^{go}(\alpha)(a_1^s - T) + \underline{p}_1^{go}(\alpha)(a_1^{go} + a_2^{go})](a_1^s + T)}{p^s[(a_1^s + T) - (a_1^s - T)] + \underline{p}_1^{go}(\alpha)(a_1^{go} + a_2^{go}) - \alpha \underline{p}_1^{go}(\alpha)(a_1^s - T)} = \\
&= \frac{(A + a)(a_1^s + T)}{B + a} \qquad (37)
\end{aligned}$$

Where, $A = [p^s(a_1^s + T) - \overline{P}^s(a_1^s - T)]$, $B = p^s[(a_1^s + T) - (a_1^s - T)]$. It is important to notice that $A \leq B$ $\forall p^s \in [\underline{p}^s, \overline{P}^s]$, and $A = B$ when $p^s = \overline{P}^s$. and where,

$$\begin{aligned}
a(\alpha) &= \underline{p}_1^{go}(\alpha)\left[(a_1^{go} + a_2^{go}) - \alpha(a_1^s - T)\right] \\
&= \frac{\overline{P}^{go}(a_1^{go} + a_2^{go}) - \alpha(a_1^s - T)}{a_1^{go} + a_2^{go}}\left[(a_1^{go} + a_2^{go}) - \alpha(a_1^s - T)\right] \\
&= \frac{\overline{P}^{go}}{a_1^{go} + a_2^{go}}\left[(a_1^{go} + a_2^{go})^2 + \alpha^2(a_1^s - T)^2 - 2\alpha(a_1^{go} + a_2^{go})(a_1^s - T)\right] \qquad (38)
\end{aligned}$$

By using equation 38, we take the derivative of a respect α,

$$\begin{aligned}\frac{\partial a(\alpha)}{\partial \alpha} &= 2\alpha(a_1^s - T)^2 - 2(a_1^s - T)(a_1^{go} + a_2^{go}) \\ &= 2\alpha(a_1^s - T)\left[(a_1^s - T) - (a_1^{go} + a_2^{go})\right] < 0 \end{aligned} \tag{39}$$

The inequality in 39 holds by the assumptions in propositions 4-6, $(a_1^{go} + a_2^{go}) > \alpha(a_1^s - T)\ \forall \alpha \in [0, 1]$.

To prove that $F^{sgo}(\alpha < 1) \geq F^{sgo}(\alpha = 1)\ \forall p^{sgo} \in [\underline{p}^{sgo}(\alpha = 1), \overline{P}^s]$, we have to prove that,

$$\frac{(A + a(\alpha = 1))(a_1^s + T)}{B + a(\alpha = 1)} \leq \frac{(A + \hat{a}(\alpha < 1))(a_1^s + T)}{B + \hat{a}(\alpha < 1)} \tag{40}$$

Where, by using equation 39, we know that $\hat{a}(\alpha < 1) > a(\alpha = 1)$. Therefore,

$$\begin{aligned}\frac{(A + a)(a_1^s + T)}{B + a} \leq \frac{(A + \hat{a})(a_1^s + T)}{B + \hat{a}} \quad &\Leftrightarrow \quad \frac{((B - \epsilon) + a)}{B + a} \leq \frac{((B - \epsilon) + (a + \delta))}{B + (a + \delta)} \\ &\Leftrightarrow \quad (B - \epsilon)B + (B - \epsilon)(a + \delta) + aB + a(a + \delta) < \\ & \quad (B - \epsilon)B + (B - \epsilon)a + (a + \delta)B + (a + \delta)a \\ &\Leftrightarrow \quad (B - \epsilon)a + (B - \epsilon)\delta + aB < (B - \epsilon)a + (a + \delta)B \\ &\Leftrightarrow \quad (a + \delta)B - \epsilon\delta < (a + \delta)B \Leftrightarrow -\epsilon\delta < 0 \end{aligned} \tag{41}$$

Where inequality 34 holds because $\epsilon > 0$ and $\delta > 0$.

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