Guarantees of Origin and Market Power in the Spot Electricity Market

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Abstract

Consumers, governments and corporations are becoming more aware of the origin of the energy that they consume, and the guarantees of origin (GO) market is increasing in Europe and worldwide. More than 25% of the electricity consumed in Europe is consumed by using GO markets. We work out the subgame perfect Nash equilibrium when the spot and the GO markets operate sequentially, and different market designs are implemented in the GO market. We find that the introduction of GO market could have a pro-competitive effect in the spot market. Moreover, the change on prices in the spot market induced by the introduction of a GO market could reverse the flow of electricity between nodes in the spot market.

1 Introduction

Consumers, governments and corporations are becoming more aware of the origin of the energy that they consume, and the guarantees of origin (GO) market is increasing in Europe and worldwide. More than 25% of the electricity consumed in Europe is consumed by using GO markets. However, our knowledge of the GO market and its interaction with the spot market is still very limited. By characterizing the subgame perfect Nash equilibrium when the competition in the spot and the GO markets is imperfect, and when those markets operate sequentially, we study if the introduction of a GO market has a pro-competitive effect on the spot market. We complete the analysis by studying the impact of different

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market designs and market structures on the equilibrium.

In the basic set up, two suppliers with identical production capacity compete in prices when the competition in the spot and the GO markets is imperfect. The demand in both markets is inelastic and it is located in two different nodes connected by a transmission line. The spot ant the GO markets operate sequentially. In the spot market, the supplier that sets the lower price is dispatched first and satisfies the electricity in its node and in the other node up to the transmission capacity. The supplier that sets the higher price is dispatched last and satisfies the residual demand in its node. The quantities dispatched in the spot market are suppliers' production capacities in the GO market.

We extend the basic set up in three directions. First, in the "GO, no-constraint" design, we assume that the transmission line is not taken into account to work out the equilibrium in the GO market, i.e, in the GO market, the suppliers can sell their entire production capacities even when the transmission line in that market is congested. This market design reflects the current market design of the GO market, where the suppliers can buy or sell GOs without taken into account the transmission constraints in that market. In the GO market, the supplier that sets the lower price is dispatched first and sells its entire production capacity. The supplier that sets the higher price is dispatched last and satisfies the residual demand.

Second, in the "GO, constraint" design we take into account the transmission constraint to work out the dispatch in the GO market, i.e, in the GO market, the suppliers face production or transmission constraints. Finally, in the "spot, green-grey technologies" case, we propose a different market structure in the spot market where the suppliers produce by using green and grey technologies. In the spot market the suppliers can serve their consumers by using both technologies, but in the GO market, they can only serve their consumers by using the green technology.

We find that the introduction of a GO market has a pro-competitive effect on the spot market only with the "GO, no-constraint" design. In that case, the suppliers compete fiercely in the spot market to be dispatched first in that market and to have more production capacity to sell in the GO market. In contrast, with the "GO, constraint" design, and when the transmission line is congested in the GO market, the equilibrium in that market is determined exclusively by the transmission constraint, and the suppliers do not compete fiercely in the spot market, since, due to the transmission constraint in the GO market, they cannot sell their entire production capacity in that market.

With the "GO, no-constraint" design, when the demand in the spot market is asymmetric, and the equilibrium is characterized exclusively in the spot market, the supplier located in the high-demand node faces higher residual demand, and it sets higher prices than the supplier located in the low-demand node. In contrast, when the spot and the GO markets operate sequentially, the supplier located in the high demand node sets lower prices in the spot market, since when it is dispatched first in that market, it sells more electricity, and has more production capacity to sell in the GO market. Hence, when the demand in the spot market is asymmetric, the introduction of a GO market, not only has a pro-competitive effect in the spot market, but also can reverse the flow of electricity between nodes.

The analysis of the "spot, green-grey technologies" case will be ready in the next version of the paper.

2 The model

The set-up of the model is different for the three cases that we study. The timing is the same, but suppliers' dispatch and profits are different for the three cases that we study. We present the set-up and the timing for the three cases: "GO, no-constraint," "GO, constraint," and "Spot, green-grey technologies."

2.1 Set-up of the model

"GO, no-constraint":

There are two nodes $i = 1, 2$ connected by a transmission line with capacity T. In the spot market, the demand in each node is inelastic (a_1^s, a_2^s). There are two suppliers, $i = 1, 2$, each with capacity $(k_1^s = k_2^s = k^s)$ located in nodes $1, 2$, where suppliers' production capacities satisfy two requirements. First, $T \leq k^s$, i.e., the suppliers cannot sell their entire production capacity into the other node. Therefore, the transmission constraint could be binding. Second, $k^s + T > max\{\overline{a}_1^s, \overline{a}_2^s\}$, i.e., the installed production capacity in one node plus the electricity that flows from the other node is enough to satisfy the peak demand in both nodes.

The spot electricity market is organized as a nodal price market, where the equilibrium price in nodes 1 and 2 is different when the transmission line is congested. When the transmission line is congested, it is profitable to buy electricity in the cheap node and to sell it in the expensive node. We assume that the congestion rents are captured by the transmission system operator.[1]

In the GO market, the demand in each node is inelastic (a_1^{go}, a_2^{go}). Suppliers' production capacities in the GO market coincide with suppliers' dispatch in the spot market.[2] As in the spot market, the two nodes are connected by a transmission line with capacity T. However,

[1] This assumption is in line with the current design of nodal markets where the transmission system operator captures the congestion rents.

[2] This assumption is in line with the current design of the market, where the suppliers need to be dispatched in the spot market to participate in the GO market.

in the "GO, no-constraint" design, we assume that the suppliers in the GO market can sell their entire production capacity in the other node. This assumption is in line with the current GO market design, where the transmission capacity is not taken into account to clear the GO market.

"GO, constraint":

The set-up is as in the "GO no-constraint" design, but when the GO market is cleared, the suppliers cannot sell more electricity into the other node than the transmission capacity.

"Spot, green-grey technologies":

In the "GO, no-constraint" and the "GO, constraint" designs, we have assumed that the suppliers can sell the production capacity dispatched in the spot market into the GO market. However, that is not necessarily true, since in the GO market it could be the case that there is no demand for that production capacity, since it is not "green" enough.[3] In the "Spot, green-grey technologies" design, we assume that the suppliers dispatched in the spot market can sell their production capacities in the GO market only when that production capacity is "green" enough.

2.2 Timing of the game

"GO, no-constraint":

The suppliers observe the demand in both nodes (1 and 2) and both markets (spot and GO), and simultaneously and independently, set their prices in the spot market ($p^s \equiv (p_1^s; p_2^s)$). The transmission system operator collects the prices, and calls the suppliers into operation. Supplier 1's output (supplier 2's output is symmetric) in the spot market is defined by:

$$q_1^s(p^s) = \begin{cases} q_1^{s1} = min\{a_1^s + a_2^s, a_1^s + T, k^s\} & \text{if } p_1^s \leq p_2^s \\ q_1^{s2} = max\{0, a_1^s - T, a_1^s + a_2^s - k^s\} & \text{if } p_1^s > p_2^s \end{cases} \tag{1}$$

When supplier 1 sets the lower price in the spot market ($p_1^s \leq p_2^s$) (left-branch, figure 1), it is dispatched first in the auction. When the transmission line is not congested and supplier 1 has enough production capacity, it satisfies the demand in both nodes ($q_1^{s1} = a_1^s + a_2^s$);[4] when the transmission line is congested, supplier 1 satisfies the demand in its own node and the demand in the other node up to the transmission constraint ($q_1^{s1} = a_1^s + T$); finally,

[3] In the GO market, the demand for GOs is differentiated, and some consumers want to acquire electricity only from local wind or solar farms, or from new hydro power plants. Therefore, it is important to take into account GOs consumers' demand to clear the GO market.

[4] We use the superindex $s1$ to refer to the left-branch of the tree in figure 1, and the superindex $s2$ to refer to the right-branch of the tree in figure 1.

Figure 1: Spot electricity market

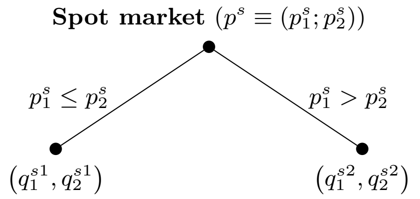
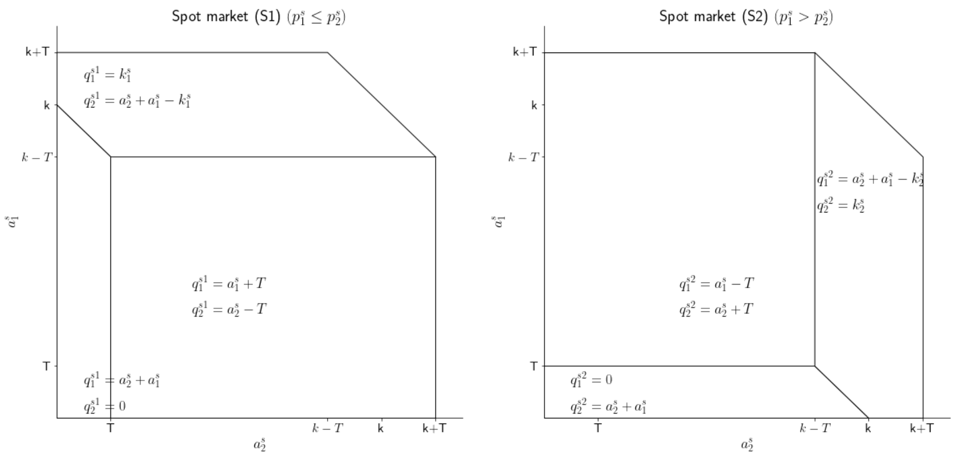


Figure 2: Dispatch spot market: $T = 2$, $k_1^s = k_2^s = k^s = 12$



supplier 1 cannot satisfy more demand that its production capacity ($q_1^{s1} = k^s$) (left-hand side, figure 2).[5]

When supplier 1 sets the higher price in the spot market ($p_1^s > p_2^s$) (right-branch, figure 1), it is dispatched last in the auction. When the transmission line is not congested and the other supplier has enough production capacity to satisfy the demand, supplier 1's residual demand is nil ($q_1^{s2} = 0$); when the transmission line is congested, supplier 1's residual demand is ($q_1^{s2} = a_1^s - T$); finally, when the demand is high enough, supplier 1's residual demand is ($q_1^{s2} = a_1^s + a_2^s - k^s$).

The quantities dispatched in the spot market will be suppliers' production capacities in

[5]In the models of price competition with capacity constraints, the tie-breaking rule is crucial determining the existence of the equilibrium (Dasgupta and Maskin, 1986). In the presence of production or transmission costs, the tie-breaking rule needs to be designed to minimize those costs. In the models in this paper, there are no production or transmission costs and different tie breaking rules could be implemented. The chosen tie-breaking rule gives priority in the dispatch to the supplier located in the high-demand node; when the demand in both nodes is equal, the suppliers satisfy the demand in their own nodes. This tie-reaking rule minimizes transmission losses.

the GO market, i.e., $k_i^{go1} = q_i^{s1}, k_i^{go2} = q_i^{s2}\ \forall\ i = 1, 2$. Therefore, in the spot market, the suppliers are competing not only to satisfy the demand in that market, but also to have production capacity to compete in the GO market. After knowing their dispatch in the spot market, the suppliers, simultaneously and independently, set their prices in the GO market ($p^{go} \equiv (p_1^{go}; p_2^{go})$). The transmission system operator collects the prices, and calls the suppliers into operation. When in the spot market, $p_1^s > p_2^s$, the suppliers are in the right-branch, figure 3, $k_1^{go2} = q_1^{s2} = a_1^s - T$, $k_2^{go2} = q_2^{s2} = a_2^s + T$,[6] and supplier 1's output (supplier 2's output is symmetric) in the GO2 market (right-branch, figure 3) is defined by:

$$q_1^{go2}(p^{go2}) = \begin{cases} q_1^{go21} = min\{a_1^{go} + a_2^{go}, k_1^{go2}\} & \text{if } p_1^{go2} \leq p_2^{go2} \\ q_1^{go22} = max\{0, a_1^{go} + a_2^{go} - k_2^{go2}\} & \text{if } p_1^{go2} > p_2^{go2} \end{cases} \tag{2}$$

When supplier 1 sets the lower price in the GO2 market, it is dispatched first and satisfies the total demand ($q_1^{go21} = a_1^{go} + a_s^{go}$) up to supplier 1's production capacity in the GO2 market ($q_1^{go21} = k_1^{go2} = q_1^{s2}$) (right-left-branch GO2 market, figure 3; left-hand side, figure 4).[7]. When supplier 1 sets the higher price in the GO2 market, it is dispatched last and satisfies the residual demand. When the demand is low, supplier 1's residual demand is nil; when the demand is high enough, supplier 1's residual demand is ($q_1^{go22} = a_1^{go} + a_2^{go} - k_2^{go2}$) (right-right-branch GO1 market, figure 3; right-hand side, figure 4).[8]

In the GO1 market (left-branch, figure 3), supplier 1's output (supplier 2's output is symmetric) is as in the GO2 market, but taken into account that $k_1^{go1} = q_1^{s1} = a_1^s + T$, $k_2^{go1} = q_2^{s1} = a_2^s - T$.

After the suppliers are called into operation, the profits are worked out in the spot and in the GO market (GO1 or GO2 market, depending on the relation between p_1^s and p_2^s). The spot market is designed as a nodal market in which the transmission system operator captures the congestion rents. Therefore, suppliers' profits are obtained by multiplying suppliers' quantities (dispatch) by their own price, and supplier 1's profits in the spot market are defined by:

$$\pi_1^s(p^s) = p_1^s q_1^s \tag{3}$$

[6]From now on, we assume that in the spot market, the transmission line is always congested. Moreover, for the examples in the paper, we assume that the demand is within the small square area in figure 2. This assumption gives us the opportunity to study the effect of transmission constraints determining the equilibrium in the spot market. When the transmission line is not congested, the equilibrium is as in Fabra et al. (2006).

[7]We use the superindex $go11$ to refer to the left-left-branch of the tree in figure 3; the superindex $go12$ to refer to the left-right-branch of the tree in figure 3; the superindex $go21$ to refer to the right-left-branch of the tree in figure 3; and the superindex $go22$ to refer to the right-right-branch of the tree in figure 3

[8]The tie-breaking rule is as in the spot market.

Figure 3: Spot and GO markets

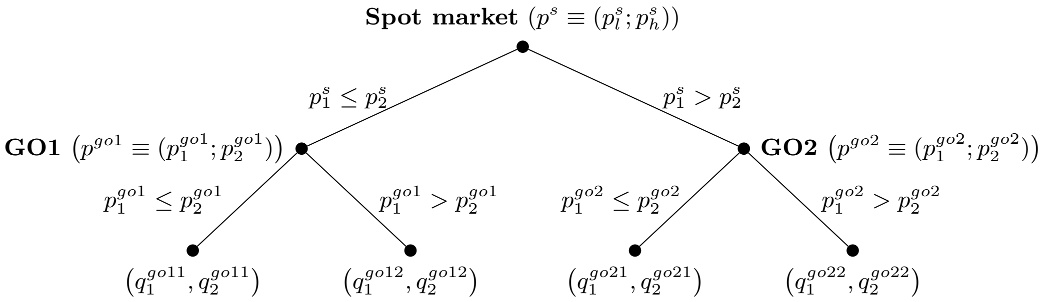
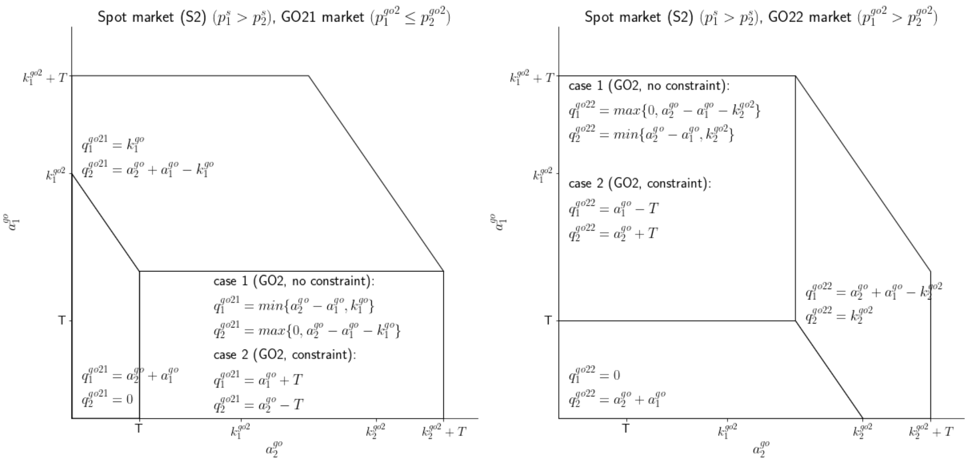


Figure 4: Dispatch GO2 market ($p_1^s > p_2^s$): $T = 2$, $k_1 = k_2 = k = 12$, $a_1^s = a_2^s = 7$, $k_1^{go2} = q_1^{s2} = a_1^s - T = 5$, $k_2^{go2} = q_2^{s2} = a_2^s + T = 9$



When supplier 1 sets the lower price in the spot market, it is dispatched first and its profits in that market are ($\pi_1^{s1} = p_1^{s1}(a_1^s + T)$). When supplier 1 sets the higher price in the spot market, it is dispatched last and its profits in that market are ($\pi_1^{s2} = p_1^{s2}(a_1^s - T)$).

Supplier 1's profits in the GO1 market are defined by (supplier 2's profits follow the same formula):

$$\pi_1^{go1}(p^{go1}) = p_1^{go1} q_1^{go1} \tag{4}$$

Supplier 1' profits in the GO2 market are defined by (supplier 2's profits follow the same formula):

$$\pi_1^{go2}(p^{go2}) = p_1^{go2} q_1^{go2} \tag{5}$$

By summing supplier 1's profits in the spot and in the GO markets, we obtain supplier 1's total profits (supplier 2's profits follow the same formula):

$$\pi_1(p^s) = \begin{cases} p_1^{s1}(a_1^s + T) + \pi_1^{go1}(p^{go1}) & \text{if } p_1^s \leq p_2^s \\ p_1^{s2}(a_1^s - T) + \pi_1^{go2}(p^{go2}) & \text{if } p_1^s > p_2^s \end{cases} \tag{6}$$

"GO, constraint":

When the transmission constraint is taken into account to work out the equilibrium in the GO market, supplier 1's output (supplier 2's output is symmetric) in the GO2 market (right-branch, figure 3) is defined by:

$$q_1^{go2}(p^{go2}) = \begin{cases} q_1^{go21} = min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} & \text{if } p_1^{go2} \leq p_2^{go2} \\ q_1^{go22} = max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\} & \text{if } p_1^{go2} > p_2^{go2} \end{cases} \tag{7}$$

When the transmission line is not congested in the GO market, the equilibrium is as in equation 2. When the transmission line in the GO market is congested, and supplier 1 submits the lower bid in that market, it satisfies the demand in its own node and the demand in node 2 up to the transmission capacity ($q_1^{go21} = a_1^{go} + T$) (right-left-branch GO2 market, figure 3; left-hand side, figure 4). When supplier 1 submits the higher bid in the GO market, it satisfies the residual demand in its own node ($q_1^{go22} = a_1^{go} - T$) (right-right-branch GO2 market, figure 3; right-hand side, figure 4).

Suppliers' profits in the spot and the GO markets are as in equations 3, 4, 5, and 6.

"Spot, green-grey technologies": To be defined.

3 Equilibrium

In this section, we characterize the subgame perfect Nash equilibrium. First, we work out the equilibrium in the GO1 and the GO2 markets. Second, based on the equilibrium in the GO1 and the GO2 markets, we characterize the equilibrium in the spot market. To characterize the equilibrium in the GO1, the GO2 and the spot markets, we proceed in three steps: First, we prove that a pure strategies equilibrium does not exist in any of those markets. Second, we find the support of the mixed strategies equilibrium. Third, we find the mixed strategies equilibrium.

We begin characterizing the equilibrium in the GO1 market (figure 3). First, we prove that a pure strategies Nash equilibrium does not exist.

Lemma 1. In the GO2 market, and when both suppliers face a positive residual demand, a pure strategies Nash equilibrium does not exist.

Proof: In the GO2 market both suppliers face a positive residual demand, and a pure strategies Nash equilibrium does not exist, since the suppliers stablish a price war having incentives to undercut each other to be dispatched first in the auction. In particular, a pair of prices ($p_1^{go2} = p_2^{go2} = 0$) is not a pure strategies Nash equilibrium, since at least one supplier has incentives to increase its price and satisfy the residual demand. A pair of prices ($p_1^{go2} = p_2^{go2} > 0$) is not a pure strategies Nash equilibrium, since both suppliers have incentives to reduce their price to be dispatched first in the auction. A pair of prices ($p_1^{go2} > p_2^{go2} > 0$) is not a pure strategies Nash equilibrium, since supplier 2 has incentives to increase its price, but still undercutting supplier 1 to be dispatched first at a higher bid. □

Once that we prove that a pure strategies Nash equilibrium does not exist, we find the support of the mixed strategies equilibrium.

Lemma 2. In the mixed strategies equilibrium in the GO2 market, both suppliers randomize in the interval $p^{go2} \in \left[max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}, \overline{p}^{go}\right]$. Where $\underline{p}_1^{go2}$ solves $\underline{p}_1^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} = \overline{p}^{go} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}$, and $\underline{p}_2^{go2}$ solves $\underline{p}_2^{go2} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} = \overline{p}^{go} max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}$.

Proof: Each supplier can guarantee its own residual profit by setting the price cap in the GO2 market and satisfying the residual demand ($\overline{p}^{go} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}$), ($\overline{p}^{go} max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}$). Therefore, none of the suppliers will set price lower than ($\underline{p}_1^{go2}$) or ($\underline{p}_2^{go2}$), where $\underline{p}_1^{go2}$ solves $\underline{p}_1^{go2}$ solves $\underline{p}_1^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} = \overline{p}^{go} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}$, and $\underline{p}_2^{go2}$ solves $\underline{p}_2^{go2}$ solves $\underline{p}_2^{go2} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} = \overline{p}^{go} max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}$. Moreover, none of the suppliers sets a price $p^{go2} \in \left[min\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}, max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}\right]$, since the supplier for which $\underline{p}_i^{go2}$ is lower knows that the other supplier will never sets a price lower than $\underline{p}_j^{go2}$, and it can increase its bid, but still undercutting the other supplier and increase its profits. Therefore, the lower bound of the support of the mixed strategies equilibrium is $\left(max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}\right)$. The upper bound of the support is equal to the price cap ($\overline{p}^{go}$). Hence, both suppliers randomize in the interval $p^{go2} \in \left[max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}, \overline{p}^{go}\right]$. □

In lemma 2, we find the support of the mixed strategies equilibrium. In proposition 1, we characterize the equilibrium in the GO2 market.

Proposition 1. In the mixed strategies equilibrium in the GO2 market, the suppliers randomize by using the next cumulative distribution functions:

$$F_1(p^{go2}) = \begin{cases} 0 & \text{if } p^{go2} < \underline{p}^{go2} \\ \dfrac{min\{a_1^{go}+a_2^{go}, a_2^{go}+T, k_2^{go2}\}}{min\{a_1^{go}+a_2^{go}, a_2^{go}+T, k_2^{go2}\} - max\{0, a_2^{go}-T, a_1^{go}+a_2^{go}-k_1^{go2}\}} \dfrac{p^{go2}-\underline{p}^{go2}}{p^{go2}} & \text{if } p^{go2} \in \left(\underline{p}^{go2}, \overline{p}^{go}\right) \\ 1 & \text{if } p^{go2} = \overline{p}^{go} \end{cases} \quad (8)$$

$$F_2(p^{go1}) = \begin{cases} 0 & \text{if } p^{go2} < \underline{p}^{go2} \\ \dfrac{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}}{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}} \dfrac{p^{go2}-\underline{p}^{go2}}{p^{go2}} & \text{if } p^{go2} \in \left(\underline{p}^{go2}, \overline{p}^{go}\right) \\ 1 & \text{if } p^{go2} = \overline{p}^{go} \end{cases} \quad (9)$$

Proof: To work out the Cumulative Distribution Function (CDF), we follow Varian (1980) and Kreps and Scheinkman (1984). I work out supplier 2's CDF. The steps to work out supplier 1's CDF are identical.

The proof follows four steps. In the first step, the payoff function for any supplier is:

$$\begin{aligned} \pi_1(p^{go2}) &= F_2(p^{go2})\left[p^{go2} max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right] + \\ &\quad (1-F_2(p^{go2}))\left[p^{go2} min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}\right] = \\ &= -p^{go2}\, F_2(p^{go2})\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right] + \\ &\quad p^{go2}\, min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} \end{aligned} \quad (10)$$

With probability $F_2(p^{go2})$, supplier 2 sets the lower price, and supplier 1 is dispatched last in the auction. In that case, supplier 1's profits are $p^{go2} max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}$. With probability $(1-F_2(p^{go2}))$, supplier 2 sets the higher price, and supplier 1 is dispatched first in the auction. In that case, supplier 1's profits are $p^{go2} min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}$.

In the second step, $\pi_1(p^{go2}) = \overline{\pi}_1^{go2} \forall p^{go2} \in S$, where S is the support of the mixed strategies worked out in lemma 2, and $\overline{\pi}_1^{go2}$ is the average profit, i.e., each strategy in the support generates the same expected payoff. Then,

$$\begin{aligned} \overline{\pi}_1^{go2} &= -p^{go2}\, F_2(p^{go2})\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right] + \\ &\quad p^{go2}\, min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} \Rightarrow \\ F_2(p^{go2}) &= \frac{p^{go2}\, min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - \overline{\pi}_1^{go2}}{p^{go2}\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right]} \end{aligned} \quad (11)$$

The third step, at $\underline{p}^{go2}$, $F_2(\underline{p}^{go2}) = 0$. Then,

$$\overline{\pi}_1^{go2} = \underline{p}^{go2} min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} \quad (12)$$

In the fourth step, plugging 12 into 11, we obtain supplier 2's mixed strategies.

$$\begin{aligned} F_2(p^{go2}) &= \frac{p^{go2}\, min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - \underline{p}^{go2}\, min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}}{p^{go2}\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right]} = \quad (13) \\ &= \frac{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}}{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}} \frac{p^{go2}-\underline{p}^{go2}}{p^{go2}} \quad \forall p^{go2} \in S \end{aligned}$$

Step fourth concludes the proof. $\square$

Once that we work out the equilibrium in the GO markets, we work out the equilibrium in the spot market. To characterize the equilibrium in that market, it is necessary to add the expected profits in the GO markets to the profits in the spot market, and then work out the support of the mixed strategies equilibrium, and suppliers' CDFs.

First, we work out the support of the mixed strategies equilibrium.

Lemma 3. In the mixed strategies equilibrium in the spot market, both suppliers randomize in the interval $p^s \in \left[max\left\{\underline{p}_1^s, \underline{p}_2^s\right\}, \overline{p}^s\right]$. Where, $\underline{p}_1^s$ solves $\underline{p}_1^s(a_1^s + T) + \underline{p}^{go1}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} = \overline{p}^s(a_1^s - T) + \underline{p}^{go2}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}$, and $\underline{p}_2^s$ solves $\underline{p}_2^s(a_2^s + T) + \underline{p}^{go2}\ min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} = \overline{p}^s(a_2^s - T) + \underline{p}^{go1}\ min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go1}\}$.

The proof of lemma 3 is as in lemma 2, but to work out $\underline{p}_1^s$ and $\underline{p}_2^s$ in the spot market, it is necessary to take into account suppliers' expected profits in the GO1 and GO2 markets. In particular to work out $\underline{p}_1^s$, it is necessary to solve $\underline{p}_1^s(a_1^s + T) + \underline{p}^{go1}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} = \overline{p}^s(a_1^s - T) + \underline{p}^{go2}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}$. In the right-hand side, $\overline{p}^s(a_1^s - T)$ represents supplier 1's profits in the spot market when it serves the residual demand and sets the price cap in that market, and $(\underline{p}^{go2}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\})$ represents supplier 1's expected profits in the GO2 market. Supplier 1 can guarantee that profit by serving the residual demand and setting a price equal to the price cap in the spot market. Therefore, it will never set a price in the spot market for which its profits are lower than the profits in the right-hand side. The price that makes supplier 1 be indifferent between satisfy the residual demand in the spot market at the price cap plus the expected profits in the GO2 market is $\underline{p}_1^s$. That price equalize supplier 1's profits in the spot market when it is dispatched first in that market $(\underline{p}_1^s(a_1^s + T))$ plus supplier 1's expected profits in the GO1 market $(\underline{p}^{go1}\ min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\})$.

We complete the characterization of the equilibrium by working out suppliers CDFs. As when we work out the lower bound of the support in lemma 3, to work out the CDFs, it is necessary to take into account the expected profits in the GO1 and GO2 markets.

Proposition 2. In the mixed strategies equilibrium in the spot market, the suppliers randomize by using the next cumulative distribution functions:

$$F_1(p^s) = \begin{cases} 0 & \text{if } p^s < \underline{p}^s \\ \dfrac{(p^s - \underline{p}^s)(a_2^s + T)}{p^s\left[(a_2^s + T)(a_2^s - T)\right] + \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} - \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go1}\}} & \text{if } p^s \in \left(\underline{p}^s, \overline{p}^s\right) \\ 1 & \text{if } p^s = \overline{p}^s \end{cases} \quad (14)$$

$$F_2(p^s) = \begin{cases} 0 & \text{if } p^s < \underline{p}^s \\ \dfrac{(p^s - \underline{p}^s)(a_1^s + T)}{p^s\left[(a_1^s + T)(a_1^s - T)\right] + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} - \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}} & \text{if } p^s \in \left(\underline{p}^s, \overline{p}^s\right) \\ 1 & \text{if } p^s = \overline{p}^s \end{cases} \quad (15)$$

The proof of proposition 2 is as in proposition 1, but taken into account suppliers' expected payoffs in the GO1 and the GO2 markets.

4 Welfare analysis

Once that we characterize the equilibrium, we conduct a welfare analysis in which we study the interaction between the GO and the spot market. In particular, we study if the introduction of a GO market has a pro-competitive effect in the spot market focusing on different symmetric and asymmetric cases. All the simulations of this section can be conducted in the webpage of the paper (link).

We study the symmetric and the asymmetric cases by using three different examples in each case. Those examples summarize the main results of our paper. The parameters used in the examples allow us to isolate the impact of the two market designs proposed in our paper: "GO, no-constraint" and "GO, constraint." For the chosen parameters, in the "GO, no-constraint" design, the equilibrium in the GO market is determined by the production capacity constraints. In contrast, in the "GO, constraint" design, the equilibrium in the GO market is determined by the transmission capacity constraints. This allows us to identify in a clear and clean way the impact of different market designs on the equilibrium.

In the symmetric case, in the first example ("Symmetric-Example 1: Baseline"), we assume that the demand in the spot and in the GO markets are fully symmetric ($a_1^s = a_2^s = 7$), ($a_1^{go} = a_2^{go} = 3$), the suppliers production capacities are green, so the the capacity dispatched in the spot market can be sold in the GO market ($\alpha_1 = \alpha_2 = 1$), and the transmission line is congested in both directions in the spot and in the GO market. We modify the "Symmetric-Example 1: Baseline" in two directions. First, we reduce the symmetrically the demand in the GO market ("Symmetric-Example 2: Low GO demand") ($a_1^s = a_2^s = 7$), ($a_1^{go} = a_2^{go} = 2.5$), ($\alpha_1 = \alpha_2 = 1$). Second, we reduce symmetrically the green capacity that can be sold in the GO market ("Symmetric-Example 3: Low green capacity") ($a_1^s = 7, a_2^s = 7, a_1^{go} = a_2^{go} = 3, \alpha_1 = \alpha_2 = 0.9$).

In the asymmetric case, we extend the "Symmetric-Example 1: Baseline" in three directions. First, we introduce asymmetries in the demand in the spot market ("Asymmetric.

Example 4: Asymmetric demand in the spot market") ($a_1^s = 7, a_2^s = 7.8$), ($a_1^{go} = a_2^{go} = 3$), ($\alpha_1 = \alpha_2 = 1$). Second, we introduce asymmetries in the green capacity that cam be sold om the GO market ("Asymmetric-Example 5 : Asymmetric green capacity") ($a_1^s = a_2^s = 7$), ($a_1^{go} = a_2^{go} = 3$), ($\alpha_1 = 1, \alpha_2 = 0.9$). Finally, we combine the two previous asymmetries ("Asymmetric-Example 6: Asymmetric demand and green capacity") ($a_1^s = 7, a_2^s = 7.8$), ($a_1^{go} = a_2^{go} = 3$), ($\alpha_1 = 1, \alpha_2 = 0.9$). Says something about the asymmetric demand in the GO market.

4.1 Symmetric case

We describe the three main results of this subsection by using three different examples.

"Symmetric-Example 1: Baseline."

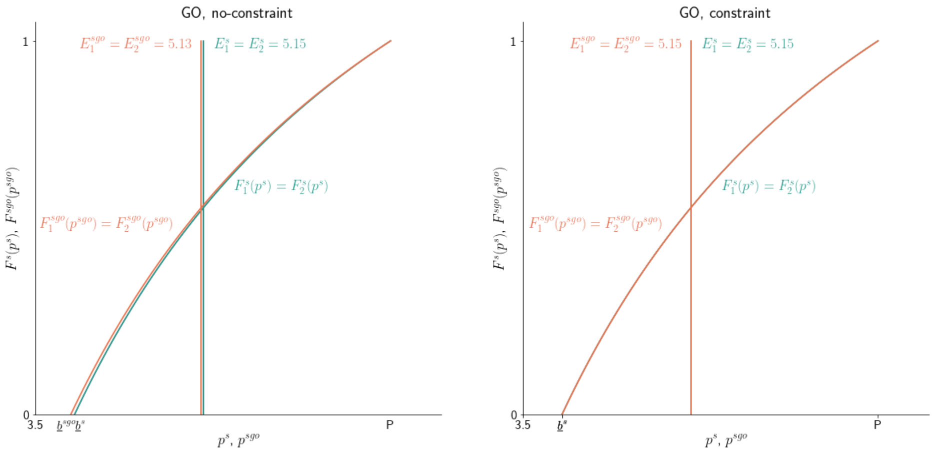
In example 1, by keeping everything symmetric (suppliers' production capacities, demands in the spot and demands in the GO markets), we study the if the introduction of the GO market has a pro-competitive effect of the spot market. By choosing demands in the spot and the GO markets in which the transmission line is congested in both directions, we compare the cases "GO, no-constraint," "GO, constraint," i.e., we can study if the competitive effect of the GO market occurs when the transmission constraints are not taken into account to clear the GO market, or the competitive effect also occurs when the transmission constraints are taken into account to clear the GO market.

The introduction of the GO market has a pro-competitive effect in the spot market only in the "GO, no-constraint" design. The introduction of a GO market makes that the suppliers compete fiercely in the spot market to sell more quantity in that market, and therefore, to have more production capacity to compete in the GO market (left-hand side, figure 5). The suppliers prefers to lose part of their profits in the spot market, since by doing that, they will have more production capacity to compete in the GO market. This result only holds when the transmission constraints are not taken into account to clear the GO market. If those constraints are taken into account, "GO, constraint" design, the equilibrium in the GO market is determined exclusively by the transmission constraints, and the suppliers have no incentives to compete fiercely in the spot market, since due to the transmission constraint in the GO market, they will not be able to sell that production capacity in that market (right-hand side, figure 5).

"Symmetric-Example 2: Low GO demand."

In the "Symmetric-Example 2: Low GO demand" case, we keep the same parameters that in the "Symmetric-Example 1: Baseline" case, but decreasing the demand in the GO market ($a_1^{go} = a_2^{go} = 2.5$). When the demand in the GO market decreases, the pro-competitive effect in the spot market that appears in the "GO, no-constraint" design is less intense. In that case, the suppliers have less incentives to compete fiercely in the spot market, since the

Figure 5: "Symmetric-Example 1: Baseline" ($a_1^s = a_2^s = 7, a_1^{go} = a_2^{go} = 3, \alpha_1 = \alpha_2 = 1$)



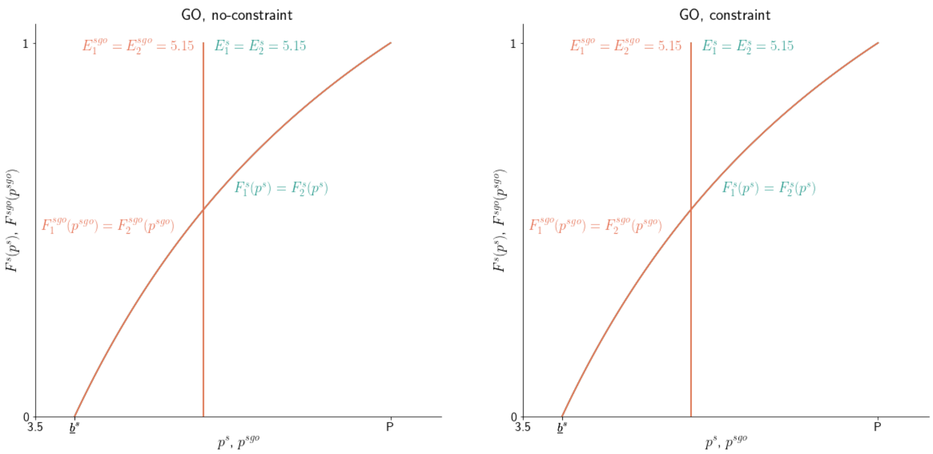
production capacity dispatched in that market is less demanded in the GO market. The pro-competitive effect in the spot market that appears in the "GO, no-constraint" design can even disappear when the demand in the GO market is low enough, since in that case, the residual demand in the GO market is nil, and the equilibrium price in that market are equal to suppliers' marginal costs. In that case, the equilibrium prices in the "GO, no-constraint" design and the "GO, constraint" design are the same (right-hand side, left-hand side, figure 6).

"Symmetric-Example 3: Low green capacity."

In the "Symmetric-Example 3: Low green capacity" case, we keep the same parameters that in the "Symmetric-Example 1: Baseline" case, but decreasing the production capacity dispatched that can be sold in the GO market ($\alpha_1 = \alpha_2 = 0.9$). The pro-competitive effect in the spot market that appears in the "GO, no-constraint" design is more intense when the green capacity that the suppliers can sell in the GO market decreases. In that case, the suppliers compete fiercely to be dispatched first in the spot market, since, if they are dispatched last, they will have very little production capacity to sell in the GO market. Therefore, the equilibrium prices in the spot market in the "Symmetric-Example 1: Baseline" case when the spot and the GO market operates sequentially are $E_1^{sgo} = E_2^{sgo} = 5.13$ (left-hand side, figure 5), and the equilibrium prices in the spot market in the "Symmetric-Example 3: Low green capacity" are $E_1^{sgo} = E_2^{sgo} = 5.10$ (left-hand side, figure 7).

It is important to notice that the pro-competitive effect in the spot market also appears in the "GO, constraint" design, since when $\alpha 1 = \alpha 2 = 0.9$, the equilibrium in the GO market is not determined by the transmission constraint, but by the production constraint. In that case, the equilibrium prices in the spot market in the "Symmetric-Example 1: Baseline" case

Figure 6: “Symmetric-Example 2: Low GO demand” ($a_1^s = 7, a_2^s = 7, a_1^{go} = a_2^{go} = 2.5, \alpha_1 = \alpha_2 = 1$)



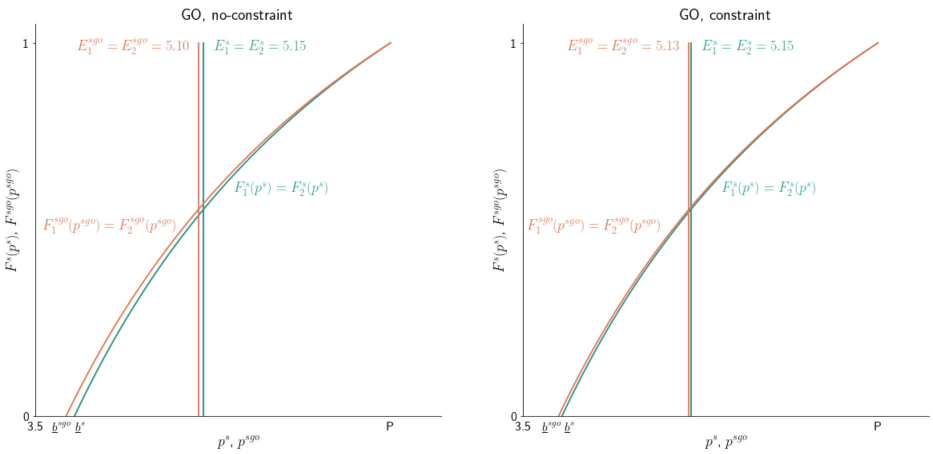
when the spot and the GO market operates sequentially are $E_1^{sgo} = E_2^{sgo} = 5.15$ (right-hand side, figure 5), and the equilibrium prices in the spot market in the “Symmetric-Example 3: Low green capacity” are $E_1^{sgo} = E_2^{sgo} = 5.13$ (right-hand side, figure 7). However, when $\alpha 1 = \alpha 2 = 0.9$, if $a_1^{go} = a_2^{go} = 2.5$, the pro-competitive effect in the “GO, constraint” design disappears, since for that demand in the GO market, the equilibrium is determined exclusively by the transmission constraint.

It is useful to connect “Symmetric-Example 2: Low GO demand” and “Symmetric-Example 3: Low green capacity.” In the “Symmetric-Example 2: Low GO demand” case, we show that when $a_1^s = a_2^s = 3$, $a_1^{go} = a_2^{go} = 2.5$ and $\alpha 1 = \alpha 2 = 1$, the pro-competitive effect in the “GO, no-constraint” design disappears, since the residual demand in the GO market is nil (left-hand side, figure 6). However, when $\alpha 1 = \alpha 2 = 0.9$ “Symmetric-Example 3: Low green capacity,” the pro-competitive effect in the spot market in the “GO, no-constraint” design appears again, since in that case, the suppliers face a positive residual demand in the GO market. The simulation and figures for this case can be studied by using the webpage of the paper (link).

So far, we focused our analysis in the symmetric case, and we find three main results. Result 1, the introduction of a GO market has a pro-competitive effect only in the “GO, no-constraint” design. Result 2, the pro-competitive effect decreases when the demand in the GO market decreases. Result 3, the pro-competitive effect increases when the suppliers cannot sell the production capacity dispatched in the spot market in the GO market.

After conclude our analysis of the symmetric case, we introduce asymmetries in the parameters of the model to study if the results obtained in the symmetric case hold.

Figure 7: "Symmetric-Example 3: Low green capacity" ($a_1^s = 7, a_2^s = 7, a_1^{go} = a_2^{go} = 3, \alpha_1 = \alpha_2 = 0.9$)



4.2 Asymmetric case

In the symmetric case (examples 1, 2 and 3), we have shown that when in the GO market, the transmission line is taken into account to work out the equilibrium ("GO, constraint" design), the introduction of a GO market does not change the equilibrium in the spot market. In the asymmetric case (examples 4, 5 and 6), we focus on the "GO, no-constraint" design, since only in that case, the introduction of a GO market increases competition in the spot market.

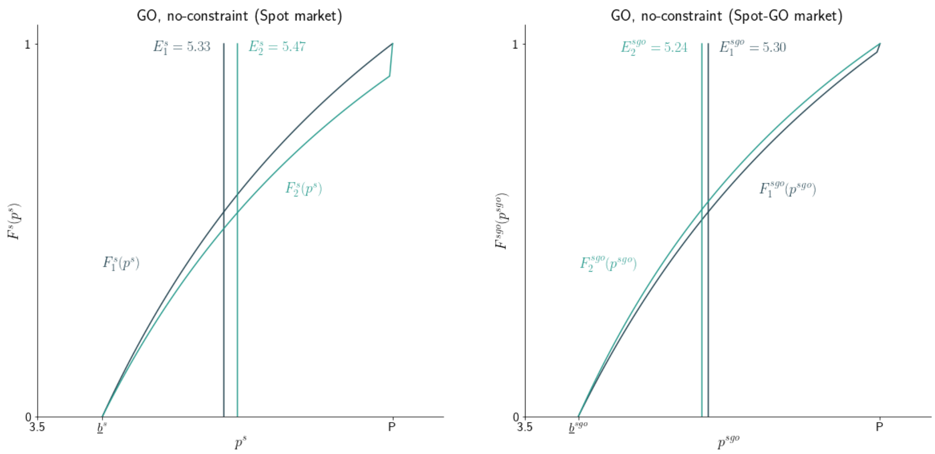
"Asymmetric-Example 4: Asymmetric demand in the spot market."

In "Asymmetric-Example 4: Asymmetric demand in the spot market," we extend the "Symmetric-Example 1: Baseline" by introducing asymmetries in the demand in the spot market ($a_1^s = 7, a_2^s = 7.8$).

When the spot market and the GO market do not operate sequentially, in the spot market, the supplier located in the high demand node (supplier 2) faces higher residual demand and the expected equilibrium price in that market is higher ($E_1^s = 5.33 < E_2^s = 5.47$) (right-hand side, figure 8). The introduction of the GO market changes the things substantially, since the supplier located in the high-demand node in the spot market also can sell more electricity in that market, and therefore, its production capacity to compete in the GO market is also larger. Therefore, the supplier located in the spot market has incentives to set lower prices, since by doing that, it has more production capacity to sell in the GO market. Therefore, the flow of electricity changes, and the prices are lower in both nodes ($E_1^{sgo} = 5.24 > E_2^{sgo} = 5.30$) (left-hand side, figure 8).

It is important to notice that the introduction of asymmetric demand in the GO market

Figure 8: "Asymmetric-Example 4: Asymmetric demand in the spot market" ($a_1^s = 7, a_2^s = 7.8, a_1^{go} = a_2^{go} = 3, \alpha_1 = \alpha_2 = 1$)



does not change the equilibrium, since in the "GO, no-constraint" design, the equilibrium is determined by aggregate demand in the GO market. Moreover, in the "GO, constraint" design, the asymmetries in demand in the GO market do not change the equilibrium, since when $p_i^{sgo} \leq p_j^{sgo}$, then $p_i^{sgo} = a_i^{sgo} + T$, and when $p_i^{sgo} > p_j^{sgo}$, then $p_i^{sgo} = a_i^{sgo} - T$. Given, that equation holds for both suppliers, the equilibrium does not change. We have to review this paragraph.

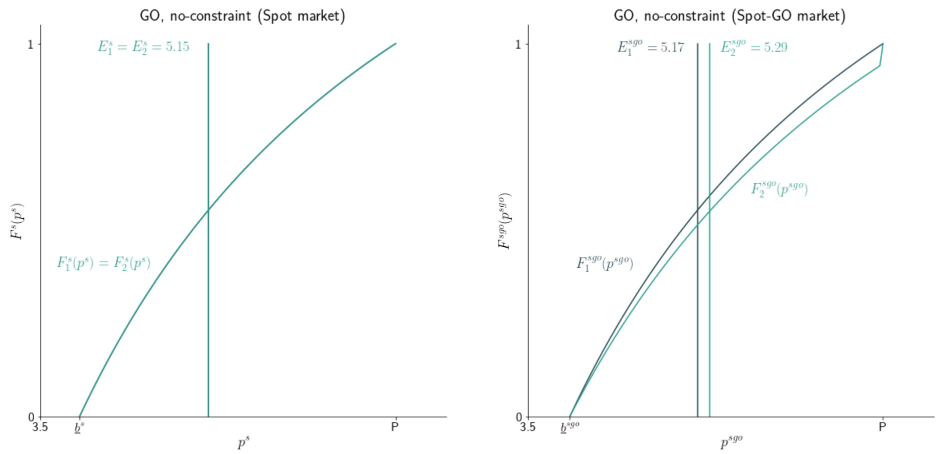
"Asymmetric-Example 5: Asymmetric green capacity."

In "Asymmetric-Example 5: Asymmetric green capacity," we extend the "Symmetric-Example 1: Baseline" by introducing asymmetries in the green capacity that cam be sold om the GO market ($\alpha_1 = 1, \alpha_2 = 0.9$).

The introduction of an asymmetry in the green capacity that can be sold in the GO market has an anti-competitive impact in the spot market, and the expected equilibrium price in the spot market when the spot and the GO market operate sequentially increases ($E_2^{sgo} = 5.29 > E_1^{sgo} = 5.17 > E_1^s = E_2^s = 5.15$). Moreover, when the spot and the GO market operate sequentially, the expected price in the node where the supplier has less green capacity to sell in the GO market is higher than in the node where the supplier has more production capacity to sell in the GO market ($E_2^{sgo} = 5.29 > E_1^{sgo} = 5.17$).

These two results contrast with the "Symmetric-Example 3: Low green capacity," where a symmetric decrease in the low green capacity has a pro-competitive effect in the spot market. When the green capacity decreases asymmetrically, the supplier with low green capacity anticipates that it will always have less production capacity in the GO market, and

Figure 9: "Asymmetric-Example 5: Asymmetric green capacity" ($a_1^s = 7, a_2^s = 7, a_1^{go} = a_2^{go} = 3, \alpha_1 = 1, \alpha_2 = 0.9$)



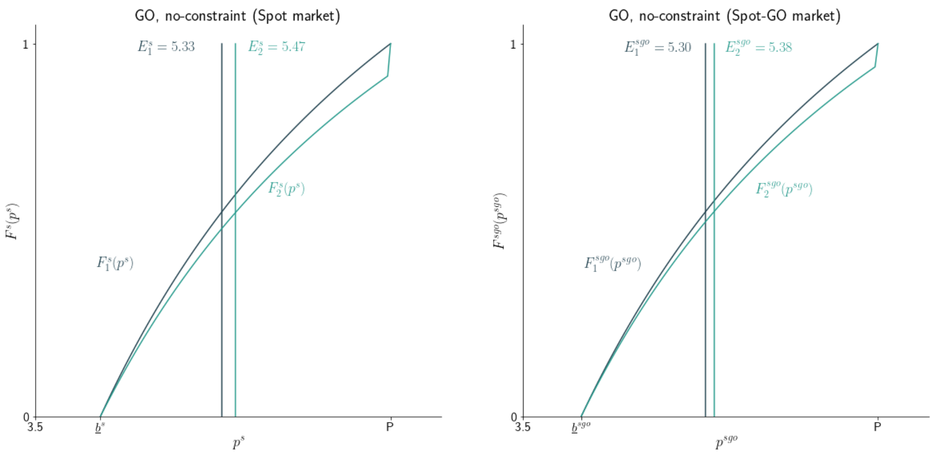
it finds more profitable to rise its price in the spot market and satisfy the demand in that market for a higher price. This result is in line with the models of price competition with capacity constraints, where an asymmetry in production capacity has an anti-competitive effect (Fabra et al. (2006)).

"Asymmetric-Example 6: Asymmetric demand and green capacity."

In the "Asymmetric-Example 4: Asymmetric demand in the spot market," the asymmetries in demand in the spot market have a pro-competitive effect when the spot and the GO markets operates sequentially. In contrast, in the "Asymmetric-Example 5: Asymmetric green capacity," the introduction of asymmetries in the green capacity have an anti-competitive effect. In the "Asymmetric-Example 6: Asymmetric demand and green capacity," we combine the asymmetries in demand and green capacity and study which effect dominates.

As can be observed in figure 10, the pro-competitive effect dominates, and the equilibrium prices in the spot market when the spot and the GO markets operate sequentially are lower than when the spot market operates independently ($E_1^{sgo} = 5.30 < E_1^s = 5.33$), ($E_2^{sgo} = 5.38 < E_2^s = 5.47$). Despite that the pro-competitive effect dominates, the electricity always flows from node 1 to node 2 ($E_1^s = 5.33 < E_2^s = 5.47$), ($E_1^{sgo} = 5.30 < E_2^{sgo} = 5.38$). This contrast with the "Asymmetric-Example 4: Asymmetric demand in the spot market," where the introduction of asymmetries in the demand in the spot market has a pro-competitive effect, and reverses the flow of electricity.

Figure 10: "Asymmetric-Example 6: Asymmetric demand and green capacity" ($a_1^s = 7, a_2^s = 7.8, a_1^{go} = a_2^{go} = 3, \alpha_1 = 1, \alpha_2 = 0.9$)



5 Conclusion

Consumers, governments and corporations are becoming more aware of the origin of the energy that they consume, and the guarantees of origin (GO) market is increasing in Europe and worldwide. More than 25% of the electricity consumed in Europe is consumed by using GO markets. We characterize the equilibrium when the spot and the GO markets operate sequentially.

We propose three different market designs: "GO, no-constraint," "GO, constraint," and "Spot, green-grey technologies." We show that the introduction of a GO market has a pro-competitive effect in the spot market only in the "GO, no-constraint." Moreover, we find that when the demand in the spot market is asymmetric, the introduction of a GO market could reverse the flow of electricity between nodes.