Guarantees of Origin and Market Power in the Spot Electricity Market

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Abstract

Consumers, governments and corporations are becoming more aware of the origin of the energy that they consume, and the guarantees of origin (GO) market is increasing in Europe and worldwide. More than 25% of the electricity consumed in Europe is consumed by using GO markets. We work out the subgame perfect Nash equilibrium when the spot and the GO markets operate sequentially, and different market designs are implemented in the GO market. We find that the introduction of GO market could have a pro-competitive effect in the spot market. Moreover, the change on prices in the spot market induced by the introduction of a GO market could reverse the flow of electricity between nodes in the spot market.

1 Introduction

Consumers, governments and corporations are becoming more aware of the origin of the energy that they consume, and the guarantees of origin (GO) market is increasing in Europe and worldwide. More than 25% of the electricity consumed in Europe is consumed by using GO markets. However, our knowledge of the GO market and its interaction with the spot market is still very limited. By characterizing the subgame perfect Nash equilibrium when the competition in the spot and the GO markets is imperfect, and when those markets operate sequentially, we study if the introduction of a GO market has a pro-competitive effect on the spot market. We complete the analysis by studying the impact of different

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market designs and market structures on the equilibrium.

In the basic set up, two suppliers with identical production capacity compete in prices when the competition in the spot and the GO markets is imperfect. The demand in both markets is inelastic and it is located in two different nodes connected by a transmission line. The spot ant the GO markets operate sequentially. In the spot market, the supplier that sets the lower price is dispatched first and satisfies the electricity in its node and in the other node up to the transmission capacity. The supplier that sets the higher price is dispatched last and satisfies the residual demand in its node. The quantities dispatched in the spot market are suppliers' production capacities in the GO market.

We extend the basic set up in three directions. First, in the "GO, no-constraint" design, we assume that the transmission line is not taken into account to work out the equilibrium in the GO market, i.e, in the GO market, the suppliers can sell their entire production capacities even when the transmission line in that market is congested. This market design reflects the current market design of the GO market, where the suppliers can buy or sell GOs without taken into account the transmission constraints in that market. In the GO market, the supplier that sets the lower price is dispatched first and sells its entire production capacity. The supplier that sets the higher price is dispatched last and satisfies the residual demand.

Second, in the "GO, constraint" design we take into account the transmission constraint to work out the dispatch in the GO market, i.e, in the GO market, the suppliers face production or transmission constraints. Finally, in the "spot, green-grey technologies" case, we propose a different market structure in the spot market where the suppliers produce by using green and grey technologies. In the spot market the suppliers can serve their consumers by using both technologies, but in the GO market, they can only serve their consumers by using the green technology.

We find that the introduction of a GO market has a pro-competitive effect on the spot market only with the "GO, no-constraint" design. In that case, the suppliers compete fiercely in the spot market to be dispatched first in that market and to have more production capacity to sell in the GO market. In contrast, with the "GO, constraint" design, and when the transmission line is congested in the GO market, the equilibrium in that market is determined exclusively by the transmission constraint, and the suppliers do not compete fiercely in the spot market, since, due to the transmission constraint in the GO market, they cannot sell their entire production capacity in that market.

With the "GO, no-constraint" design, when the demand in the spot market is asymmetric, and the equilibrium is characterized exclusively in the spot market, the supplier located in the high-demand node faces higher residual demand, and it sets higher prices than the supplier located in the low-demand node. In contrast, when the spot and the GO markets operate sequentially, the supplier located in the high demand node sets lower prices in the spot market, since when it is dispatched first in that market, it sells more electricity, and has more production capacity to sell in the GO market. Hence, when the demand in the spot market is asymmetric, the introduction of a GO market, not only has a pro-competitive effect in the spot market, but also can reverse the flow of electricity between nodes.

The analysis of the "spot, green-grey technologies" case will be ready in the next version of the paper.

2 The model

The set-up of the model is different for the three cases that we study. The timing is the same, but suppliers' dispatch and profits are different for the three cases that we study. We present the set-up and the timing for the three cases: "GO, no-constraint," "GO, constraint," and "Spot, green-grey technologies."

2.1 Set-up of the model

"GO, no-constraint":

There are two nodes $i = 1, 2$ connected by a transmission line with capacity T. In the spot market, the demand in each node is inelastic (a_1^s, a_2^s). There are two suppliers, $i = 1, 2$, each with capacity $(k_1^s = k_2^s = k^s)$ located in nodes $1, 2$, where suppliers' production capacities satisfy two requirements. First, $T \leq k^s$, i.e., the suppliers cannot sell their entire production capacity into the other node. Therefore, the transmission constraint could be binding. Second, $k^s + T > max\{\overline{a}_1^s, \overline{a}_2^s\}$, i.e., the installed production capacity in one node plus the electricity that flows from the other node is enough to satisfy the peak demand in both nodes.

The spot electricity market is organized as a nodal price market, where the equilibrium price in nodes 1 and 2 is different when the transmission line is congested. When the transmission line is congested, it is profitable to buy electricity in the cheap node and to sell it in the expensive node. We assume that the congestion rents are captured by the transmission system operator.[1]

In the GO market, the demand in each node is inelastic (a_1^{go}, a_2^{go}). Suppliers' production capacities in the GO market coincide with suppliers' dispatch in the spot market.[2] As in the spot market, the two nodes are connected by a transmission line with capacity T. However,

[1]This assumption is in line with the current design of nodal markets where the transmission system operator captures the congestion rents.

[2]This assumption is in line with the current design of the market, where the suppliers need to be dispatched in the spot market to participate in the GO market.

in the “GO, no-constraint” design, we assume that the suppliers in the GO market can sell their entire production capacity in the other node. This assumption is in line with the current GO market design, where the transmission capacity is not taken into account to clear the GO market.

“GO, constraint”:

The set-up is as in the “GO no-constraint” design, but when the GO market is cleared, the suppliers cannot sell more electricity into the other node than the transmission capacity.

“Spot, green-grey technologies”:

In the “GO, no-constraint” and the “GO, constraint” designs, we have assumed that the suppliers can sell the production capacity dispatched in the spot market into the GO market. However, that is not necessarily true, since in the GO market it could be the case that there is no demand for that production capacity, since it is not “green” enough.[3] In the “Spot, green-grey technologies” design, we assume that the suppliers dispatched in the spot market can sell their production capacities in the GO market only when that production capacity is “green” enough.

2.2 Timing of the game

“GO, no-constraint”:

The suppliers observe the demand in both nodes (1 and 2) and both markets (spot and GO), and simultaneously and independently, set their prices in the spot market ($p^s \equiv (p_1^s; p_2^s)$). The transmission system operator collects the prices, and calls the suppliers into operation. Supplier 1's output (supplier 2's output is symmetric) in the spot market is defined by:

$$q_1^s(p^s) = \begin{cases} q_1^{s1} = min\{a_1^s + a_2^s, a_1^s + T, k^s\} & \text{if } p_1^s \leq p_2^s \\ q_1^{s2} = max\{0, a_1^s - T, a_1^s + a_2^s - k^s\} & \text{if } p_1^s > p_2^s \end{cases} \tag{1}$$

When supplier 1 sets the lower price in the spot market ($p_1^s \leq p_2^s$) (left-branch, figure 1), it is dispatched first in the auction. When the transmission line is not congested and supplier 1 has enough production capacity, it satisfies the demand in both nodes ($q_1^{s1} = a_1^s + a_2^s$);[4] when the transmission line is congested, supplier 1 satisfies the demand in its own node and the demand in the other node up to the transmission constraint ($q_1^{s1} = a_1^s + T$); finally,

[3] In the GO market, the demand for GOs is differentiated, and some consumers want to acquire electricity only from local wind or solar farms, or from new hydro power plants. Therefore, it is important to take into account GOs consumers' demand to clear the GO market.

[4] We use the superindex $s1$ to refer to the left-branch of the tree in figure 1, and the superindex $s2$ to refer to the right-branch of the tree in figure 1.

Figure 1: Spot electricity market

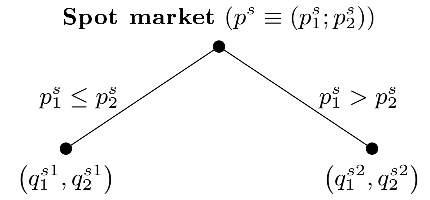
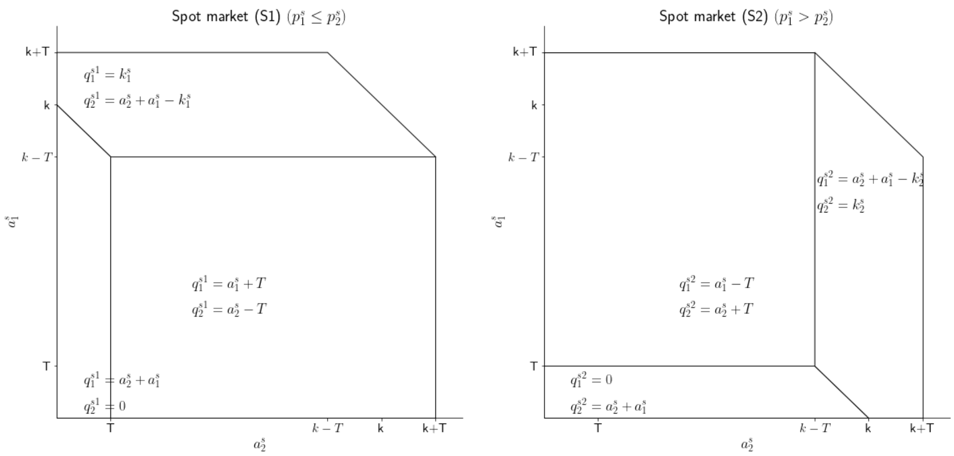


Figure 2: Dispatch spot market: $T = 2$, $k_1^s = k_2^s = k^s = 12$



supplier 1 cannot satisfy more demand that its production capacity ($q_1^{s1} = k^s$) (left-hand side, figure 2).[5]

When supplier 1 sets the higher price in the spot market ($p_1^s > p_2^s$) (right-branch, figure 1), it is dispatched last in the auction. When the transmission line is not congested and the other supplier has enough production capacity to satisfy the demand, supplier 1's residual demand is nil ($q_1^{s2} = 0$); when the transmission line is congested, supplier 1's residual demand is ($q_1^{s2} = a_1^s - T$); finally, when the demand is high enough, supplier 1's residual demand is ($q_1^{s2} = a_1^s + a_2^s - k^s$).

The quantities dispatched in the spot market will be suppliers' production capacities in

[5] In the models of price competition with capacity constraints, the tie-breaking rule is crucial determining the existence of the equilibrium (Dasgupta and Maskin, 1986). In the presence of production or transmission costs, the tie-breaking rule needs to be designed to minimize those costs. In the models in this paper, there are no production or transmission costs and different tie breaking rules could be implemented. The chosen tie-breaking rule gives priority in the dispatch to the supplier located in the high-demand node; when the demand in both nodes is equal, the suppliers satisfy the demand in their own nodes. This tie-reaking rule minimizes transmission losses.

the GO market, i.e., $k_i^{go1} = q_i^{s1}, k_i^{go2} = q_i^{s2}\ \forall\ i = 1,2$. Therefore, in the spot market, the suppliers are competing not only to satisfy the demand in that market, but also to have production capacity to compete in the GO market. After knowing their dispatch in the spot market, the suppliers, simultaneously and independently, set their prices in the GO market ($p^{go} \equiv (p_1^{go}; p_2^{go})$). The transmission system operator collects the prices, and calls the suppliers into operation. When in the spot market, $p_1^s > p_2^s$, the suppliers are in the right-branch, figure 3, $k_1^{go2} = q_1^{s2} = a_1^s - T$, $k_2^{go2} = q_2^{s2} = a_2^s + T$,[6] and supplier 1's output (supplier 2's output is symmetric) in the GO2 market (right-branch, figure 3) is defined by:

$$q_1^{go2}(p^{go2}) = \begin{cases} q_1^{go21} = min\{a_1^{go} + a_2^{go}, k_1^{go2}\} & \text{if } p_1^{go2} \leq p_2^{go2} \\ q_1^{go22} = max\{0, a_1^{go} + a_2^{go} - k_2^{go2}\} & \text{if } p_1^{go2} > p_2^{go2} \end{cases} \tag{2}$$

When supplier 1 sets the lower price in the GO2 market, it is dispatched first and satisfies the total demand ($q_1^{go21} = a_1^{go} + a_s^{go}$) up to supplier 1's production capacity in the GO2 market ($q_1^{go21} = k_1^{go2} = q_1^{s2}$) (right-left-branch GO2 market, figure 3; left-hand side, figure 4).[7]. When supplier 1 sets the higher price in the GO2 market, it is dispatched last and satisfies the residual demand. When the demand is low, supplier 1's residual demand is nil; when the demand is high enough, supplier 1's residual demand is ($q_1^{go22} = a_1^{go} + a_2^{go} - k_2^{go2}$) (right-right-branch GO1 market, figure 3; right-hand side, figure 4).[8]

In the GO1 market (left-branch, figure 3), supplier 1's output (supplier 2's output is symmetric) is as in the GO2 market, but taken into account that $k_1^{go1} = q_1^{s1} = a_1^s + T$, $k_2^{go1} = q_2^{s1} = a_2^s - T$.

After the suppliers are called into operation, the profits are worked out in the spot and in the GO market (GO1 or GO2 market, depending on the relation between p_1^s and p_2^s). The spot market is designed as a nodal market in which the transmission system operator captures the congestion rents. Therefore, suppliers' profits are obtained by multiplying suppliers' quantities (dispatch) by their own price, and supplier 1's profits in the spot market are defined by:

$$\pi_1^s(p^s) = p_1^s q_1^s \tag{3}$$

[6] From now on, we assume that in the spot market, the transmission line is always congested. Moreover, for the examples in the paper, we assume that the demand is within the small square area in figure 2. This assumption gives us the opportunity to study the effect of transmission constraints determining the equilibrium in the spot market. When the transmission line is not congested, the equilibrium is as in Fabra et al. (2006).

[7] We use the superindex $go11$ to refer to the left-left-branch of the tree in figure 3; the superindex $go12$ to refer to the left-right-branch of the tree in figure 3; the superindex $go21$ to refer to the right-left-branch of the tree in figure 3; and the superindex $go22$ to refer to the right-right-branch of the tree in figure 3

[8] The tie-breaking rule is as in the spot market.

Figure 3: Spot and GO markets

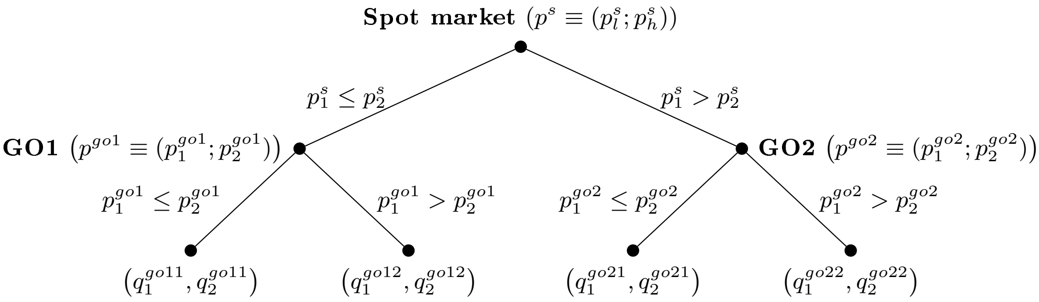
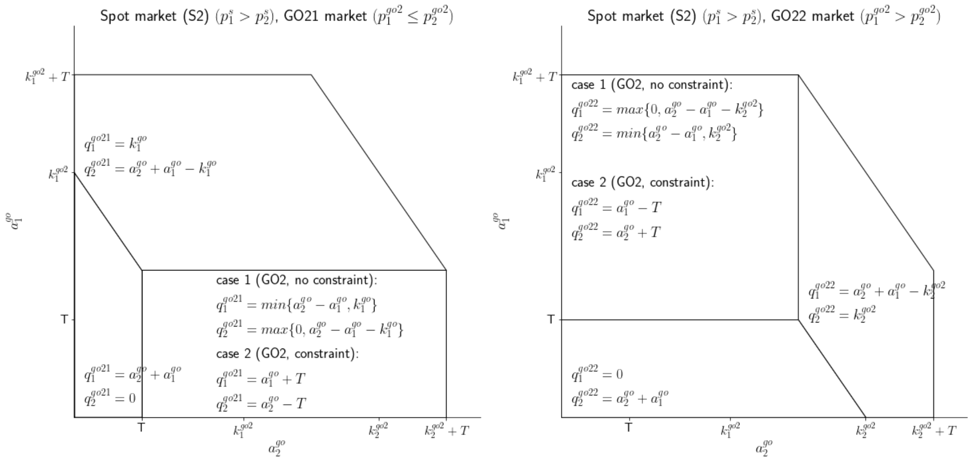


Figure 4: Dispatch GO2 market $(p^s_1 > p^s_2)$: $T = 2$, $k_1 = k_2 = k = 12$, $a^s_1 = a^s_2 = 7$, $k^{go2}_1 = q^{s2}_1 = a^s_1 - T = 5$, $k^{go2}_2 = q^{s2}_2 = a^s_2 + T = 9$



When supplier 1 sets the lower price in the spot market, it is dispatched first and its profits in that market are $(\pi^{s1}_1 = p^{s1}_1(a^s_1 + T))$. When supplier 1 sets the higher price in the spot market, it is dispatched last and its profits in that market are $(\pi^{s2}_1 = p^{s2}_1(a^s_1 - T))$.

Supplier 1's profits in the GO1 market are defined by (supplier 2's profits follow the same formula):

$$\pi^{go1}_1(p^{go1}) = p^{go1}_1 q^{go1}_1 \tag{4}$$

Supplier 1' profits in the GO2 market are defined by (supplier 2's profits follow the same formula):

$$\pi^{go2}_1(p^{go2}) = p^{go2}_1 q^{go2}_1 \tag{5}$$

By summing supplier 1's profits in the spot and in the GO markets, we obtain supplier 1's total profits (supplier 2's profits follow the same formula):

$$\pi_1(p^s) = \begin{cases} p_1^{s1}(a_1^s + T) + \pi_1^{go1}(p^{go1}) & \text{if } p_1^s \le p_2^s \\ p_1^{s2}(a_1^s - T) + \pi_1^{go2}(p^{go2}) & \text{if } p_1^s > p_2^s \end{cases} \tag{6}$$

<u>"GO, constraint"</u>:

When the transmission constraint is taken into account to work out the equilibrium in the GO market, supplier 1's output (supplier 2's output is symmetric) in the GO2 market (right-branch, figure 3) is defined by:

$$q_1^{go2}(p^{go2}) = \begin{cases} q_1^{go21} = min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} & \text{if } p_1^{go2} \le p_2^{go2} \\ q_1^{go22} = max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\} & \text{if } p_1^{go2} > p_2^{go2} \end{cases} \tag{7}$$

When the transmission line is not congested in the GO market, the equilibrium is as in equation 2. When the transmission line in the GO market is congested, and supplier 1 submits the lower bid in that market, it satisfies the demand in its own node and the demand in node 2 up to the transmission capacity ($q_1^{go21} = a_1^{go} + T$) (right-left-branch GO2 market, figure 3; left-hand side, figure 4). When supplier 1 submits the higher bid in the GO market, it satisfies the residual demand in its own node ($q_1^{go22} = a_1^{go} - T$) (right-right-branch GO2 market, figure 3; right-hand side, figure 4).

Suppliers' profits in the spot and the GO markets are as in equations 3, 4, 5, and 6.

<u>"Spot, green-grey technologies"</u>: To be defined.

3 Equilibrium

In this section, we characterize the subgame perfect Nash equilibrium. First, we work out the equilibrium in the GO1 and the GO2 markets. Second, based on the equilibrium in the GO1 and the GO2 markets, we characterize the equilibrium in the spot market. To characterize the equilibrium in the GO1, the GO2 and the spot markets, we proceed in three steps: First, we prove that a pure strategies equilibrium does not exist in any of those markets. Second, we find the support of the mixed strategies equilibrium. Third, we find the mixed strategies equilibrium.

We begin characterizing the equilibrium in the GO1 market (figure 3). First, we prove that a pure strategies Nash equilibrium does not exist.

Lemma 1. In the GO2 market, and when both suppliers face a positive residual demand, a pure strategies Nash equilibrium does not exist.

Proof: In the GO2 market both suppliers face a positive residual demand, and a pure strategies Nash equilibrium does not exist, since the suppliers stablish a price war having incentives to undercut each other to be dispatched first in the auction. In particular, a pair of prices ($p_1^{go2} = p_2^{go2} = 0$) is not a pure strategies Nash equilibrium, since at least one supplier has incentives to increase its price and satisfy the residual demand. A pair of prices ($p_1^{go2} = p_2^{go2} > 0$) is not a pure strategies Nash equilibrium, since both suppliers have incentives to reduce their price to be dispatched first in the auction. A pair of prices ($p_1^{go2} > p_2^{go2} > 0$) is not a pure strategies Nash equilibrium, since supplier 2 has incentives to increase its price, but still undercutting supplier 1 to be dispatched first at a higher bid. $\square$

Once that we prove that a pure strategies Nash equilibrium does not exist, we find the support of the mixed strategies equilibrium.

Lemma 2. In the mixed strategies equilibrium in the GO2 market, both suppliers randomize in the interval $p^{go2} \in \left[max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}, \overline{p}^{go}\right]$. Where $\underline{p}_1^{go2}$ solves $\underline{p}_1^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} = \overline{p}^{go} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}$, and $\underline{p}_2^{go2}$ solves $\underline{p}_2^{go2} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} = \overline{p}^{go} max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}$.

Proof: Each supplier can guarantee its own residual profit by setting the price cap in the GO2 market and satisfying the residual demand ($\overline{p}^{go} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}$), ($\overline{p}^{go} max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}$). Therefore, none of the suppliers will set price lower than ($\underline{p}_1^{go2}$) or ($\underline{p}_2^{go2}$), where $\underline{p}_1^{go2}$ solves $\underline{p}_1^{go2}$ solves $\underline{p}_1^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\} = \overline{p}^{go} max\{0, a_1^{go} - T, a_1^{go} + a_2^{go} - k_2^{go2}\}$, and $\underline{p}_2^{go2}$ solves $\underline{p}_2^{go2}$ solves $\underline{p}_2^{go2} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} = \overline{p}^{go} max\{0, a_2^{go} - T, a_1^{go} + a_2^{go} - k_1^{go2}\}$. Moreover, none of the suppliers sets a price $p^{go2} \in \left[min\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}, max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}\right]$, since the supplier for which $\underline{p}_i^{go2}$ is lower knows that the other supplier will never sets a price lower than $\underline{p}_j^{go2}$, and it can increase its bid, but still undercutting the other supplier and increase its profits. Therefore, the lower bound of the support of the mixed strategies equilibrium is $\left(max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}\right)$. The upper bound of the support is equal to the price cap ($\overline{p}^{go}$). Hence, both suppliers randomize in the interval $p^{go2} \in \left[max\left\{\underline{p}_1^{go2}, \underline{p}_2^{go2}\right\}, \overline{p}^{go}\right]$. $\square$

In lemma 2, we find the support of the mixed strategies equilibrium. In proposition 1, we characterize the equilibrium in the GO2 market.

Proposition 1. In the mixed strategies equilibrium in the GO2 market, the suppliers randomize by using the next cumulative distribution functions:

$$F_1(p^{go2}) = \begin{cases} 0 & \text{if } p^{go2} < \underline{p}^{go2} \\ \dfrac{min\{a_1^{go}+a_2^{go}, a_2^{go}+T, k_2^{go2}\}}{min\{a_1^{go}+a_2^{go}, a_2^{go}+T, k_2^{go2}\} - max\{0, a_2^{go}-T, a_1^{go}+a_2^{go}-k_1^{go2}\}} \dfrac{p^{go2}-\underline{p}^{go2}}{p^{go2}} & \text{if } p^{go2} \in \left(\underline{p}^{go2}, \overline{p}^{go}\right) \\ 1 & \text{if } p^{go2} = \overline{p}^{go} \end{cases} \quad (8)$$

$$F_2(p^{go1}) = \begin{cases} 0 & \text{if } p^{go2} < \underline{p}^{go2} \\ \dfrac{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}}{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}} \dfrac{p^{go2}-\underline{p}^{go2}}{p^{go2}} & \text{if } p^{go2} \in \left(\underline{p}^{go2}, \overline{p}^{go}\right) \\ 1 & \text{if } p^{go2} = \overline{p}^{go} \end{cases} \quad (9)$$

Proof: To work out the Cumulative Distribution Function (CDF), we follow Varian (1980) and Kreps and Scheinkman (1984). I work out supplier 2's CDF. The steps to work out supplier 1's CDF are identical.

The proof follows four steps. In the first step, the payoff function for any supplier is:

$$\begin{aligned} \pi_1(p^{go2}) &= F_2(p^{go2})\left[p^{go2}max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right] + \\ & (1-F_2(p^{go2}))\left[p^{go2}min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}\right] = \\ &= -p^{go2}\ F_2(p^{go2})\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right] + \\ & p^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} \end{aligned} \quad (10)$$

With probability $F_2(p^{go2})$, supplier 2 sets the lower price, and supplier 1 is dispatched last in the auction. In that case, supplier 1's profits are $p^{go2}max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}$. With probability $(1-F_2(p^{go2}))$, supplier 2 sets the higher price, and supplier 1 is dispatched first in the auction. In that case, supplier 1's profits are $p^{go2}min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}$.

In the second step, $\pi_1(p^{go2}) = \overline{\pi}_1^{go2} \forall p^{go2} \in S$, where S is the support of the mixed strategies worked out in lemma 2, and $\overline{\pi}_1^{go2}$ is the average profit, i.e., each strategy in the support generates the same expected payoff. Then,

$$\begin{aligned} \overline{\pi}_1^{go2} &= -p^{go2}\ F_2(p^{go2})\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right] + \\ & p^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} \Rightarrow \\ F_2(p^{go2}) &= \frac{p^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - \overline{\pi}_1^{go2}}{p^{go2}\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right]} \end{aligned} \quad (11)$$

The third step, at $\underline{p}^{go2}$, $F_2(\underline{p}^{go2}) = 0$. Then,

$$\overline{\pi}_1^{go2} = \underline{p}^{go2}min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} \quad (12)$$

In the fourth step, plugging 12 into 11, we obtain supplier 2's mixed strategies.

$$\begin{aligned} F_2(p^{go2}) &= \frac{p^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - \underline{p}^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}}{p^{go2}\left[min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}\right]} = \\ &= \frac{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}}{min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\} - max\{0, a_1^{go}-T, a_1^{go}+a_2^{go}-k_2^{go2}\}} \frac{p^{go2}-\underline{p}^{go2}}{p^{go2}} \quad \forall p^{go2} \in S \end{aligned} \quad (13)$$

Step fourth concludes the proof. $\square$

Once that we work out the equilibrium in the GO markets, we work out the equilibrium in the spot market. To characterize the equilibrium in that market, it is necessary to add the expected profits in the GO markets to the profits in the spot market, and then work out the support of the mixed strategies equilibrium, and suppliers' CDFs.

First, we work out the support of the mixed strategies equilibrium.

Lemma 3. In the mixed strategies equilibrium in the spot market, both suppliers randomize in the interval $p^s \in \left[max\left\{\underline{p}_1^s, \underline{p}_2^s\right\}, \overline{p}^s\right]$. Where, $\underline{p}_1^s$ solves $\underline{p}_1^s(a_1^s+T)+\underline{p}^{go1}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go1}\} = \overline{p}^s(a_1^s-T)+\underline{p}^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}$, and $\underline{p}_2^s$ solves $\underline{p}_2^s(a_2^s+T)+\underline{p}^{go2}\ min\{a_1^{go}+a_2^{go}, a_2^{go}+T, k_2^{go2}\} = \overline{p}^s(a_2^s-T)+\underline{p}^{go1}\ min\{a_1^{go}+a_2^{go}, a_2^{go}+T, k_2^{go1}\}$.

The proof of lemma 3 is as in lemma 2, but to work out $\underline{p}_1^s$ and $\underline{p}_2^s$ in the spot market, it is necessary to take into account suppliers' expected profits in the GO1 and GO2 markets. In particular to work out $\underline{p}_1^s$, it is necessary to solve $\underline{p}_1^s(a_1^s+T)+\underline{p}^{go1}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go1}\} = \overline{p}^s(a_1^s-T)+\underline{p}^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}$. In the right-hand side, $\overline{p}^s(a_1^s-T)$ represents supplier 1's profits in the spot market when it serves the residual demand and sets the price cap in that market, and ($\underline{p}^{go2}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go2}\}$) represents supplier 1's expected profits in the GO2 market. Supplier 1 can guarantee that profit by serving the residual demand and setting a price equal to the price cap in the spot market. Therefore, it will never set a price in the spot market for which its profits are lower than the profits in the right-hand side. The price that makes supplier 1 be indifferent between satisfy the residual demand in the spot market at the price cap plus the expected profits in the GO2 market is $\underline{p}_1^s$. That price equalize supplier 1's profits in the spot market when it is dispatched first in that market ($\underline{p}_1^s(a_1^s+T)$) plus supplier 1's expected profits in the GO1 market ($\underline{p}^{go1}\ min\{a_1^{go}+a_2^{go}, a_1^{go}+T, k_1^{go1}\}$).

We complete the characterization of the equilibrium by working out suppliers CDFs. As when we work out the lower bound of the support in lemma 3, to work out the CDFs, it is necessary to take into account the expected profits in the GO1 and GO2 markets.

Proposition 2. In the mixed strategies equilibrium in the spot market, the suppliers randomize by using the next cumulative distribution functions:

$$F_1(p^s) = \begin{cases} 0 & \text{if } p^s < \underline{p}^s \\ \dfrac{(p^s - \underline{p}^s)(a_2^s + T)}{p^s\left[(a_2^s + T)(a_2^s - T)\right] + \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go2}\} - \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_2^{go} + T, k_2^{go1}\}} & \text{if } p^s \in \left(\underline{p}^s, \overline{p}^s\right) \\ 1 & \text{if } p^s = \overline{p}^s \end{cases} \quad (14)$$

$$F_2(p^s) = \begin{cases} 0 & \text{if } p^s < \underline{p}^s \\ \dfrac{(p^s - \underline{p}^s)(a_1^s + T)}{p^s\left[(a_1^s + T)(a_1^s - T)\right] + \underline{p}^{go1} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go1}\} - \underline{p}^{go2} min\{a_1^{go} + a_2^{go}, a_1^{go} + T, k_1^{go2}\}} & \text{if } p^s \in \left(\underline{p}^s, \overline{p}^s\right) \\ 1 & \text{if } p^s = \overline{p}^s \end{cases} \quad (15)$$

The proof of proposition 2 is as in proposition 1, but taken into account suppliers' expected payoffs in the GO1 and the GO2 markets.

4 Competitive and welfare analysis

Once that we characterize the equilibrium, we conduct a static comparative analysis example in which we study the interaction between the GO and the spot market. In particular, we study if the introduction of a GO market has a pro-competitive effect in the spot market.

We organize our analysis by using three different examples. First, we assume that the demand in the spot and in the GO markets are fully symmetric ($a_1^s = a_2^s = 8$), ($a_1^{go} = a_2^{go} = 4$), and the transmission line is congested in both directions (square, figure 5; square, left-hand side, figure 6) ("Example1: Symmetric"). Second, we assume that the demand in the spot market is asymmetric ($a_1^s = 8, a_2^s = 7$), the demand in the GO market is low ($a_1^{go} = a_2^{go} = 3$), and the transmission line is congested in both directions (square, figure 5; square, right-hand side, figure 6) ("Example2: Asymmetric spot demand, low GO demand"). Finally, we assume that the demand in the spot market is asymmetric ($a_1^s = 8, a_2^s = 7$), the demand in the GO market is high ($a_1^{go} = 4, a_2^{go} = 3$), and the transmission line is congested in both directions (square, figure 5; square, right-hand side, figure 6) ("Example3: Asymmetric spot demand, high GO demand").

"Example1: Symmetric."

In example 1, by keeping everything symmetric (suppliers' production capacities, demands in the spot and demands in the GO markets), we study the competitive effect of the GO market. By choosing demands in the spot and the GO markets in which the transmission line is congested in both directions, we compare the cases "GO, no-constraint," "GO, constraint," i.e., we can study if the competitive effect of the GO market occurs when the transmission constraints are not taken into account to clear the GO market, or the competitive effect also occurs when the transmission constraints are taken into account to clear the GO market.

Figure 5: Spot market, equilibrium areas

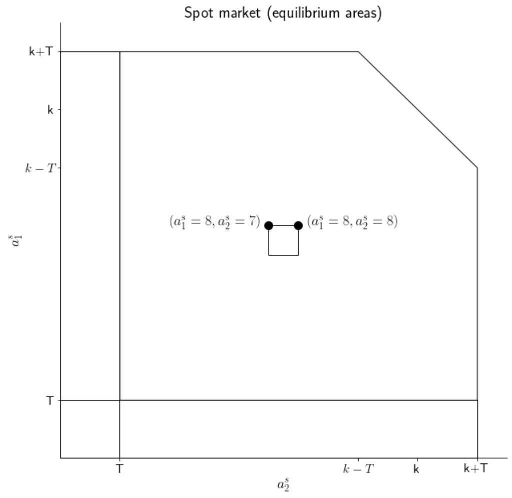


Figure 6: GO market, equilibrium areas

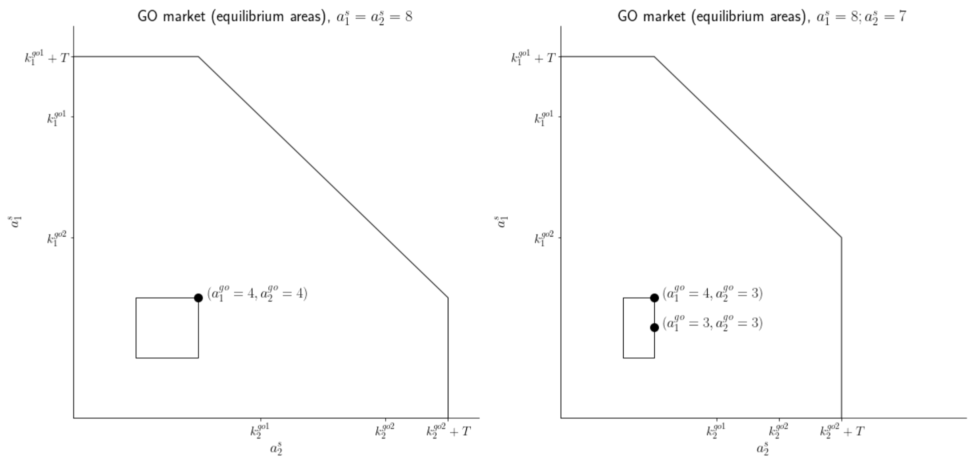
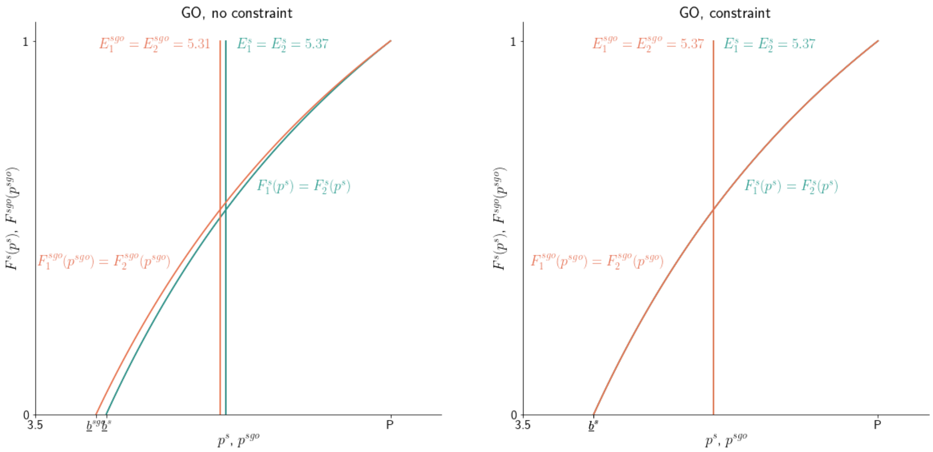


Figure 7: Spot-GO. “Example1: Symmetric” ($a_1^s = a_2^s = 8, a_1^{go} = a_2^{go} = 4$)



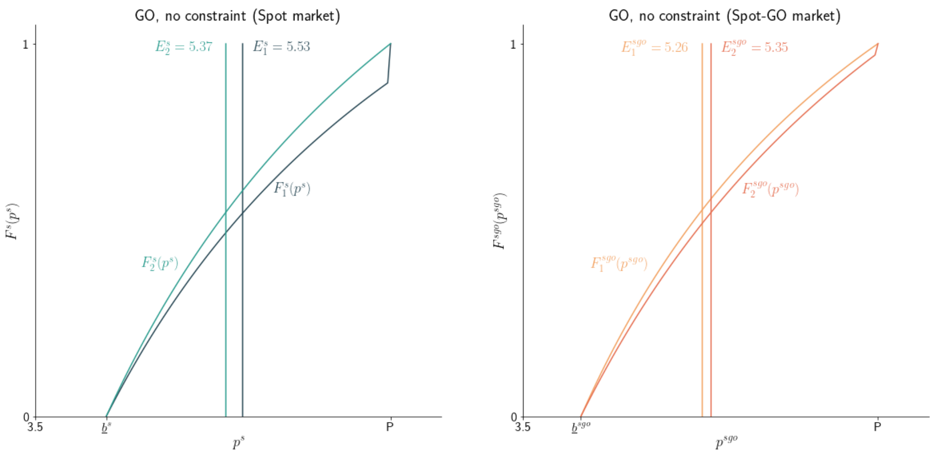
The introduction of the GO market has a pro-competitive effect in the spot market only in the “GO, no-constraint” design. The introduction of a GO market makes that the suppliers compete fiercely in the spot market to sell more quantity in that market, and therefore, to have more production capacity to compete in the GO market (left-hand side, figure 7). The suppliers prefers to lost part of their profits in the spot market, since by doing that, they will have more production capacity to compete in the GO market. However, this is true when the transmission constraints are not taken into account to clear the GO market. If those constraints are taken into account, “GO, constraint” design, the equilibrium in the GO market is determined exclusively by the transmission constraints, and the suppliers have no incentives to compete fiercely in the spot market, since they will not be able to sell that production capacity in the GO market (right-hand side, figure 7).

“Example2: Asymmetric spot demand, low GO demand.”

In example 1, we have shown that when in the GO market, the transmission line is congested in both directions, the introduction of a GO market does not change the equilibrium in the spot market. In examples 2 and 3, we focus on the “GO, no-constraint” design, since only in that case, the introduction of a GO market increases competition in the spot market.

In example 2, we introduce asymmetries in the demand in the spot market, and keep the demand in the GO market low. In the spot market, the supplier located in the high demand node (supplier 1) faces higher residual demand and it sets higher prices with higher probabilities ($E_1^s = 5.53 > E_2^s = 5.37$) (right-hand side, figure 8). The introduction of the GO market change the things substantially, since the supplier located in the high-demand node in the spot market also can sell more electricity in that market, and therefore, its production capacity to compete in the GO market is also larger. Therefore, the supplier

Figure 8: "Example2: Asymmetric spot demand, low GO demand" ($a_1^s = 8, a_2^s = 7, a_1^{go} = a_2^{go} = 3$)



located in the spot market has incentives to set lower prices, since by doing that it has more production capacity to sell in the GO market, and the flow of electricity changes ($E_1^{sgo} = 5.26 < E_2^{sgo} = 5.35$) (left-hand side, figure 8).

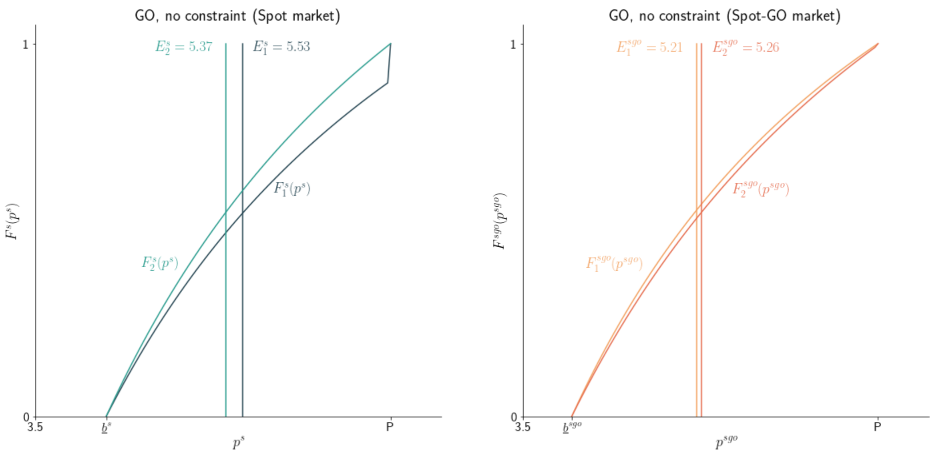
"Example2: Asymmetric spot demand, high GO demand."

In example 3, as in example 2, we introduce asymmetries in the demand in the spot market, but we study the case in which, the demand in the GO market is high. In the spot market, the equilibrium is as in example 2 ($E_1^s = 5.53 > E_2^s = 5.37$) (right-hand side, figure 9). After an increase in the demand in the GO market, the supplier located in the high-demand node in the spot market still has incentives to set lower prices, since by doing that it has more production capacity to sell in the GO market. However, the supplier located in the low-demand node in the spot market faces an increase in the demand in the GO market, and its incentives to set lower prices in the spot market to have access to the demand in the GO market increase. Therefore, the difference between suppliers' equilibrium prices shrink when the demand in the GO market increases ($E_1^{sgo} = 5.21 < E_2^{sgo} = 5.26$) (left-hand side, figure 9).

5 Conclusion

Consumers, governments and corporations are becoming more aware of the origin of the energy that they consume, and the guarantees of origin (GO) market is increasing in Europe and worldwide. More than 25% of the electricity consumed in Europe is consumed by using GO markets. We characterize the equilibrium when the spot and the GO markets operate

Figure 9: “Example3: Asymmetric spot demand, high GO demand” ($a_1^s = 8, a_2^s = 7, a_1^{go} = 4, a_2^{go} = 3$)



sequentially.

We propose three different market designs: “GO, no-constraint,” “GO, constraint,” and “Spot, green-grey technologies.” We show that the introduction of a GO market has a pro-competitive effect in the spot market only in the “GO, no-constraint.” Moreover, we find that when the demand in the spot market is asymmetric, the introduction of a GO market could reverse the flow of electricity between nodes.