Contents

1	Compilation (Chapter 1)	2
2	Lexical Analysis (Chapter 2)	4
3	Parsing (Chapter 3)	6
4	Type Checking (Chapter 4)	9
5	JVM Code Generation (Chapter 5)	11
6	Translating JVM Code to MIPS Code (Chapter 6)	14
7	Register Allocation (Chapter 7)	16

Compilation (Chapter 1) 1

PROBLEMS

Problem 1. What are the changes that you will need to make in the j-- code tree in order to support @ as the remainder operator on primitive integers? For example, 17 @ 5 = 2.

Problem 2. Write down the following class names in internal form:

- java.util.ArrayList
- jminusminus.Parser
- Employee

Problem 3. Write down the JVM type descriptor for each of the following field/constructor/method declarations:

```
private int N;
private String s;
• public static final double PI = 3.141592653589793;
• public Employee(String name) ...
• public Coordinates(float latitude, float longitude) ...
• public Object get(String key) ...
• public void put(String key, Object o) ...
• public static int[] sort(int[] n, boolean ascending) ...
```

• public int[][] transpose(int[][] matrix) ...

Problem 4. Write a program GenSquare.java that produces, using CLEmitter, a Square.class program, which accepts an integer n as command-line argument and prints the square of that number as output.

SOLUTIONS

Solution 1.

```
$j/j--/lexicalgrammar
REM ::= "@"
$j/j--/grammar
multiplicativeExpression ::= unaryExpression {(STAR | REM) unaryExpression}
$j/j--/src/jminusminus/TokenInfo.java
enum TokenKind {
    REM("0"),
$j/j--/src/jminusminus/Scanner.java
        case '@':
             nextCh();
             return new TokenInfo(REM, line);
```

\$j/j--/src/jminusminus/Parser.java

```
private JExpression multiplicativeExpression() {
   int line = scanner.token().line();
   boolean more = true;
   JExpression lhs = unaryExpression();
   while (more) {
      if (have(STAR)) {
         lhs = new JMultiplyOp(line, lhs, unaryExpression());
      } else if (have(REM)) {
         lhs = new JRemainderOp(line, lhs, unaryExpression());
      } else {
         more = false;
      }
   }
   return lhs;
}
```

\$j/j--/src/jminusminus/JBinaryExpression.java

```
class JRemainderOp extends JBinaryExpression {
   public JRemainderOp(int line, JExpression lhs, JExpression rhs) {
        super(line, "@", lhs, rhs);
   }

   public JExpression analyze(Context context) {
        lhs = (JExpression) lhs.analyze(context);
        rhs = (JExpression) rhs.analyze(context);
        lhs.type().mustMatchExpected(line(), Type.INT);
        rhs.type().mustMatchExpected(line(), Type.INT);
        type = Type.INT;
        return this;
   }

   public void codegen(CLEmitter output) {
        lhs.codegen(output);
        rhs.codegen(output);
        output.addNoArgInstruction(IREM);
   }
}
```

Solution 2.

- java/util/ArrayList
- jminusminus/Parser
- Employee

Solution 3.

- I
- Ljava/lang/String;
- D
- (Ljava/lang/String;)V
- (FF) V
- (Ljava/lang/String;)Ljava/lang/Object;
- (Ljava/lang/String;Ljava/lang/Object;)V
- ([IZ)[I
- I]](I]]) •

Solution 4.

```
import jminusminus.CLEmitter;
import static jminusminus.CLConstants.*;
import java.util.ArrayList;
public class GenSquare {
   public static void main(String[] args) {
       CLEmitter e = new CLEmitter(true);
       ArrayList < String > accessFlags = new ArrayList < String > ();
        accessFlags.add("public");
        e.addClass(accessFlags, "Square", "java/lang/Object", null, true);
       accessFlags.clear();
        accessFlags.add("public");
       accessFlags.add("static");
        e.addMethod(accessFlags, "main", "([Ljava/lang/String;)V", null, true);
       e.addNoArgInstruction(ALOAD_0);
       e.addNoArgInstruction(ICONST_0);
       e.addNoArgInstruction(AALOAD);
       e.addMemberAccessInstruction(INVOKESTATIC, "java/lang/Integer",
                "parseInt", "(Ljava/lang/String;)I");
       e.addNoArgInstruction(ISTORE_1);
        e.addMemberAccessInstruction(GETSTATIC, "java/lang/System", "out",
                "Ljava/io/PrintStream;");
       e.addNoArgInstruction(ILOAD_1);
       e.addNoArgInstruction(ILOAD_1);
       e.addNoArgInstruction(IMUL);
        e.addMemberAccessInstruction(INVOKEVIRTUAL, "java/io/PrintStream",
                "println", "(I)V");
        e.addNoArgInstruction(RETURN);
        e.write();
   }
```

2 Lexical Analysis (Chapter 2)

PROBLEMS

Problem 1. Consider the following j-- program:

```
import java.lang.System;
public class Greetings {
    public static void main(String[] args) {
        System.out.println("Hi " + args[0] + "!");
    }
}
```

List the tokens in the program, along with their line numbers and their images.

Problem 2. Consider a language over the alphabet $\{a,b\}$ that consists of strings ending in ab.

- a. Provide a regular expression for the language.
- b. Draw a state transition diagram that recognizes the language.

Problem 3. Consider the regular expression (a|b)* over the alphabet $\{a,b\}$.

- a. Describe the language implied by the regular expression.
- b. Use Thompson's construction to derive a non-deterministic finite state automaton (NFA) recognizing the same language.
- c. Use powerset construction to derive an equivalent deterministic finite state automaton (DFA).

d. Use the partitioning method to derive an equivalent minimal DFA.

Problem 4. Suppose we wish to add support for the do-while statement in j--.

What changes will you need to make in the hand-written and JavaCC lexical analyzers in the j-- code tree in order to support the do-while statement?

SOLUTIONS

Solution 1.

```
: import = import
          : <IDENTIFIER > = java
1
1
          : <IDENTIFIER > = lang
1
1
         : <IDENTIFIER> = System
1
1
         : public = public
3
3
         : class = class
3
         : <IDENTIFIER> = Greetings
         : { = {
         : public = public
         : static = static
          : void = void
         : <IDENTIFIER> = main
         : <IDENTIFIER> = String
         : [ = [
         : ] = ]
         : <IDENTIFIER> = args
         : ) = )
         : { = {
         : <IDENTIFIER> = System
5
         : <IDENTIFIER> = out
5
5
         : <IDENTIFIER> = println
5
         : ( = (
         : <STRING_LITERAL > = "Hi "
5
5
          : <IDENTIFIER> = args
5
5
          : [ = [
5
          : <INT_LITERAL> = 0
5
         : ] = ]
5
5
          : <STRING_LITERAL > = "!"
5
         : ) = )
5
          : } = }
6
          : \langle EOF \rangle = \langle EOF \rangle
```

Solution 2.

```
a. (a|b)*ab
```

b.

Solution 3.

a. The language consists of strings with any number of a's or b's.

b.

c.

d.

Solution 4.

```
$j/j--/lexicalgrammar
```

```
...
DO ::= "do"
...
```

\$j/j--/src/jminusminus/TokenInfo.java

```
enum TokenKind {
    ...
    DO("do"),
    ...
}
```

\$j/j--/src/jminusminus/Scanner.java

```
...
reserved.put(DO.image(), DO);
...
```

\$j/j--/src/jminusminus/j--.jj

```
TOKEN:
{
...
| < CHAR: "do" >
...
}
```

3 Parsing (Chapter 3)

Problems

Problem 1. Consider the following grammar:

$$S ::= (L) \mid \mathbf{a}$$

$$L ::= L \ S \mid \epsilon$$

- a. What language does this grammar describe?
- b. Show the parse tree for the string (a() (a(a))).
- c. Derive an equivalent LL(1) grammar.

Problem 2. Show that the following grammar is ambiguous:

$$S ::= \mathtt{a} \, S \, \mathtt{b} \, S \, \mid \, \mathtt{b} \, S \, \mathtt{a} \, S \, \mid \, \epsilon$$

Problem 3. Consider the following context-free grammar:

```
\begin{split} S &::= A \text{ a} \\ A &::= \text{b d } B \mid \text{e } B \\ B &::= \text{c } A \mid \text{d } B \mid \epsilon \end{split}
```

- a. Compute first and follow for S, A and B.
- b. Construct an LL(1) parsing table for this grammar.
- c. Show the steps in parsing b d c e a.

Problem 4. Consider the following grammar:

```
S ::= L = R
S ::= R
L ::= * R
L ::= i
R ::= L
```

- a. Construct the canonical LR(1) collection.
- b. Construct the Action and Goto tables.
- c. Show the steps in the parse for *i = i.

Problem 5. Suppose we wish to add support for the do-while statement in j--.

What changes will you need to make in the hand-written and JavaCC parsers in the j-- code tree in order to support the do-while statement?

SOLUTIONS

```
Solution 1.
```

Solution 2.

Solution 3.

Solution 4.

Solution 5.

\$j/j--/grammar

```
statement ::= block
| IF parExpression statement [ELSE statement]
| WHILE parExpression statement
| DO statement WHILE parExpression SEMI
| RETURN [expression] SEMI
| SEMI
| statementExpression SEMI
```

\$j/j--/src/jminusminus/Parser.java

```
private JStatement statement() {
    int line = scanner.token().line();
   if (see(LCURLY)) {
       return block();
   } else if (have(IF)) {
        JExpression test = parExpression();
        JStatement consequent = statement();
        JStatement alternate = have(ELSE) ? statement() : null;
        return new JIfStatement(line, test, consequent, alternate);
   } else if (have(WHILE)) {
        JExpression test = parExpression();
        JStatement statement = statement();
        return new JWhileStatement(line, test, statement);
   } else if (have(DO)) {
        JStatement statement = statement();
        mustBe(WHILE);
        JExpression test = parExpression();
        mustBe(SEMI);
        return new JDoWhileStatement(line, statement, test);
   } else if (have(RETURN)) {
        if (have(SEMI)) {
           return new JReturnStatement(line, null);
        } else {
            JExpression expr = expression();
            mustBe(SEMI);
            return new JReturnStatement(line, expr);
       }
   } else if (have(SEMI)) {
        return new JEmptyStatement(line);
   } else { // Must be a statementExpression
        JStatement statement = statementExpression();
        mustBe(SEMI):
        return statement;
   }
```

\$j/j--/src/jminusminus/JDoWhileStatement.java

```
package jminusminus;
import static jminusminus.CLConstants.*;
class JDoWhileStatement extends JStatement {
    private JStatement body;
    private JExpression condition;
    public JDoWhileStatement(int line, JStatement body, JExpression condition) {
        super(line);
        this.body = body;
        this.condition = condition;
    public JWhileStatement analyze(Context context) { return this; }
    public void codegen(CLEmitter output) { }
    public void writeToStdOut(PrettyPrinter p) {
       p.printf("<JDoWhileStatement line=\"\d\">\n", line());
        p.indentRight();
       p.printf("<Body>\n");
       p.indentRight();
        body.writeToStdOut(p);
       p.indentLeft();
       p.printf("</Body>\n");
        p.printf("<TestExpression>\n");
       p.indentRight();
        condition.writeToStdOut(p);
       p.indentLeft();
       p.printf("</TestExpression>\n");
```

```
p.indentLeft();
        p.printf("</JDoWhileStatement>\n");
$j/j--/src/jminusminus/j--.jj
private JStatement statement(): {
    int line = 0;
    JStatement statement = null;
    JExpression test = null;
    JStatement consequent = null;
    JStatement alternate = null;
    JStatement body = null;
    JExpression expr = null;
{
    try {
        statement = block() |
        <IF> { line = token.beginLine; }
        test = parExpression()
        consequent = statement()
        // Even without the lookahead below, which is added to
        // suppress JavaCC warnings, dangling if-else problem is
        // resolved by binding the alternate to the closest
        // consequent.
        [
            LOOKAHEAD ( <ELSE> )
            <ELSE> alternate = statement()
        { statement =
            new JIfStatement( line, test, consequent, alternate ); } |
        <WHILE> { line = token.beginLine; }
        test = parExpression()
        body = statement()
        { statement = new JWhileStatement( line, test, body ); } |
        <DO> { line = token.beginLine; }
        body = statement()
        <WHILE>
        test = parExpression()
        <SEMI>
        { statement = new JDoWhileStatement( line, body, test ); } |
        <RETURN> { line = token.beginLine; }
            expr = expression()
        <SEMI>
        { statement = new JReturnStatement( line, expr ); } |
        { statement = new JEmptyStatement( line ); } |
        // Must be a statementExpression
        statement = statementExpression()
        <SEMI>
    }
    catch ( ParseException e ) {
        recoverFromError( new int[] { SEMI, EOF }, e );
    { return statement; }
```

4 Type Checking (Chapter 4)

PROBLEMS

Problem 1. Consider the following j-- program:

```
package pass;
import java.lang.Integer;
import java.lang.System;
public class Sum {
    private static String MSG = "SUM = ";
   private int n;
    public Sum(int n) {
        this.n = n;
    public int compute() {
        int sum = 0, i = n;
        while (i > 0) {
            sum += i--;
        return sum;
    public static void main(String[] args) {
        int n = Integer.parseInt(args[0]);
       Sum sum = new Sum(n);
       System.out.println(MSG + sum.compute());
}
```

- a. How does pre-analysis (JCompilationUnit.preAnalyze()) of the program work?
- b. How does analysis (JCompilationUnit.analyze()) of the program work?
- c. How are the declarations of the local variables sum and i handled in the compute() method?
- d. How are offsets assigned to the parameters/variables in the program's constructor and the two methods?
- e. How is the simple variable n resolved in the main() method?
- f. How is the field selection MSG resolved in the main() method?
- g. How are the message expressions System.out.println(...) and sum.compute() resolved in the main() method?
- h. How is argument to System.out.println() analyzed in the main() method?

Problem 2. When can you cast an expression of type Type1 to another type Type2?

Problem 3. Consider the following j-- program:

```
public class Mystery {
    public int f(int x) {
        int y = x * x;
        return z;
    }
}
```

Is the program syntactically/semantically correct? If not, why and how does j-- figure it out?

Problem 4. How would you do semantic analysis for the do-while statement, ie, implement analyze() in JDoWhileStatement.java?

SOLUTIONS

Solution 1.

- a. See section 4.4 of our text.
- b. See section 4.5 of our text.

- c. See section 4.5.2 of our text.
- d. See section 4.5.2 of our text.
- e. See section 4.5.3 of our text.
- f. See section 4.5.4 of our text.
- g. See section 4.5.4 of our text.
- h. See section 4.5.5 of our text.

Solution 2. See section 4.5.6 of our text.

Solution 3. The program is syntactically correct, but semantically wrong since the variable z is not declared before use. During analysis of the return statement, the simple variable z is looked up in the chain of contexts, starting at the local context. The lookup is unsuccessful, and an error is reported that the variable has not been declared.

Solution 4.

```
public JDoWhileStatement analyze(Context context) {
   body = (JStatement) body.analyze(context);
   condition = condition.analyze(context);
   condition.type().mustMatchExpected(line(), Type.BOOLEAN);
   return this;
}
```

5 JVM Code Generation (Chapter 5)

Problems

Problem 1. Reconsider the j-- program sum from above:

```
package pass;
import java.lang.Integer;
import java.lang.System;
public class Sum {
   private static String MSG = "SUM = ";
   private int n;
    public Sum(int n) {
        this.n = n;
    public int compute() {
        int sum = 0, i = n;
        while (i > 0) {
            sum += i--;
        return sum;
    public static void main(String[] args) {
        int n = Integer.parseInt(args[0]);
        Sum sum = new Sum(n);
        System.out.println(MSG + sum.compute());
   }
```

How does JVM bytecode generation (JCompilationUnit.codegen()) for the program work?

Problem 2. Suppose 1hs and rhs are boolean expressions. How does j-- generate code for the following statements?

```
a. boolean x = lhs && rhs;
```

```
b. if (lhs && rhs) {
    then_statement
} else {
    else_statement
}

C. while (lhs && rhs) {
    statement
}
```

Problem 3. Suppose x is an object and y is an integer field within.

a. What is the JVM bytecode generated for the following statement? How does the runtime stack evolve as the instructions are executed?

```
++x.y;
```

b. If z is also an integer, what is the JVM bytecode generated for the following statement? How does the runtime stack evolve as the instructions are executed?

```
z = ++x.y;
```

Problem 4. How is code generated for the expression "The first perfect number is " + 6?

Problem 5. How is code generated for casts?

Problem 6. How would you generate JVM bytecode for the do-while statement, ie, implement codegen() in JDoWhileStatement.java?

SOLUTIONS

Solution 1. Consult sections 5.2 - 5.6 of our text.

Solution 2.

```
lhs code
a.
           branch to Target on false
           rhs code
           branch to Target on false
           push 1 on stack
           goto End
  Target: push 0 on stack
  End:
b.
           lhs code
           branch to Target on false
           rhs code
           branch to Target on false
           {\tt then\_statement} \ {\tt code}
           goto End
  Target: else_statement code
  End:
          . . .
           lhs code
C. Test:
           branch to Target on false
           rhs code
           branch to Target on false
           body code
           goto Test
  Target: ...
```

Solution 3. We use table on slide 26 from the JVM Code Generation chapter.

```
a. Bytecode:
    aload x'
dup
getfield y
iconst_1
iadd
putfield y

Runtime stack (right to left is top to bottom):

| x |
| x | x
| x | y
| x | y | 1
| x | y+1
|
```

```
b. Bytecode:
  aload x
  dup
  getfield y
  iconst_1
  iadd
  dup_x1
  putfield y
  Runtime stack (right to left is top to bottom):
  | x |
  | x | x
  | x | y
  | x | y | 1
  | x | y+1
  | y+1 | x | y+1
  | y+1
```

Solution 4. Since the left-hand-side expression of + is a string, the operation denotes string concatenation, and is represented in the AST as a JStringConcatenationOp object. The codegen() method therein does the following:

- 1. Creates an empty string buffer, ie, a StringBuffer object, and initializes it.
- 2. Appends the string "The first perfect number is " to the buffer using StringBuffer's append(String x) method.
- 3. Appends the integer value 6 to the buffer using StringBuffer's append(int x) method.
- 4. Invokes the toString() method on the buffer to produce a string on the runtime stack.

Solution 5. Analysis determines both the validity of a cast and the necessary converter, which encapsulates the code generated for the particular cast. Each Converter implements a method <code>codegen()</code>, which generates any code necessary to the cast. Code is first generated for the expression being cast, and then for the cast, using the appropriate converter.

Solution 6.

```
public void codegen(CLEmitter output) {
    String bodyStart = output.createLabel();
    output.addLabel(bodyStar);
    body.codegen(output);
    condition.codegen(output, bodyStart, true);
}
```

6 Translating JVM Code to MIPS Code (Chapter 6)

PROBLEMS

Problem 1. Consider the following j-- program SpimSum.java:

```
import spim.SPIM;

public class SpimSum {
    public static int compute(int n) {
        int sum = 0, i = n;
        while (i > 0) {
            sum += i--;
        }
        return sum;
    }

    public static void main(String[] args) {
        int result = SpimSum.compute(100);
        SPIM.printInt(result);
        SPIM.printChar('\n');
    }
}
```

The JVM bytecode for the SpimSum.compute() method are listed below, with linebreaks denoting basic blocks.

```
public static int compute(int);
  Code:
    stack=2, locals=3, args_size=1
       0: iconst_0
       1: istore_1
       2: iload_0
       3: istore_2
       4: iload_2
       5: iconst_0
       6: if_icmple
                         19
       9: iload_1
      10: iload_2
      11: iinc
                         2, -1
      14: iadd
      15: istore_1
      16: goto
      19: iload_1
      20: ireturn
```

The HIR instructions for the method are listed below.

```
BO succ: B1
Locals: IO
IO: LDLOC O
B1 [0, 3] dom: B0 pred: B0 succ: B2
Locals: IO I3 IO
I3: 0
B2 [LH] [4, 6] dom: B1 pred: B1 B3 succ: B3 B4
Locals: IO I5 I6
I5: [ I3 I11 ]
I6: [ I0 I10 ]
I7: 0
8: if I6 <= I7 then B4 else B3
B3 [LT] [9, 16] dom: B2 pred: B2 succ: B2
Locals: IO I11 I10
I9: -1
I10: I6 + I9
```

```
I11: I5 + I6
12: goto B2

B4 [19, 20] dom: B2 pred: B2
Locals: I0 I5 I6
I13: ireturn I5
```

- a. Draw the HIR flow graph for SpimSum.compute().
- b. Suppse that the HIR to LIR translation procedure assigns virtual registers v32, v33, v34, v37, and 38 to HIR instructions I3 I5, I6, I10, and I11 respectively. How are the Phi functions I5 and I6 in block B2 resolved?

Problem 2. What optimization techniques would you use to improve each of the following code snippets, and how?

```
a. static int f() {
       return 42;
  7
   static int g(int x) {
       return f() * x;
b. static int f() {
       int x = 28;
       int y = 42
       int z = x + y * 10;
       return z:
c. static int f(int x) {
      int y = x * x * x;
       return x * x * x;
  }
d. static int f(int[][] a) {
       int sum = 0;
       int i = 0;
       while (i <= a.length - 1) {</pre>
           int j = 0;
           while (j < a[0].length - 1) {</pre>
               sum += a[i][j];
               j = j + 1;
           }
           i = i + 1;
       }
       return sum;
  }
```

where a, b, and c have the same dimensions.

```
g. static int f(SomeObject o) {
    return o.x * o.y * o.z;
}
```

where x, y, and z are integer fields in o.

SOLUTIONS

Solution 1.

 \mathbf{a}

b. The Phi functions 15 and 16 in block B2 are resolved by adding move instructions at the end of B2's predecessors B1 and B3, as follows:

```
B1:
    move V32 V33
    move $a0 V34

B3:
    move V37 V33
    move V38 V34
```

Solution 2. a. Inlining

```
static int g(int x) {
   return 42 * x;
}
```

b. Constant folding and constant propagation

```
static int f() {
    return 448;
}
```

c. Common subexpression elimination

```
static int f(int x) {
    int y = x * x * x;
    return y;
}
```

d. Lifting loop invariant code

```
static int f(int[][] a) {
   int sum = 0;
   int i = 0;
   while (i <= a.length - 1) {
      int j = 0;
      int[] a_ = a[i];
      while (j < a_.length - 1) {
         sum += a_[j];
         j = j + 1;
      }
      i = i + 1;
   }
   return sum;
}</pre>
```

- e. Array bounds check elimination. Since a, b, and c have the same dimensions, perform the array bounds check, ie, check if the indices i and j are within bounds, just once instead of three times.
- f. Null check elimination. Perform null check on the object o just once instead of three times.

7 Register Allocation (Chapter 7)

PROBLEMS

Problem 1. The LIR instructions for the compute() method from the SpimSum program above are listed below.

```
B0
B1
0: LDC [0] [V32|I]
5: MOVE [V32|I] [V33|I]
10: MOVE $a0 [V34|I]
15: LDC [0] [V35|I]
20: BRANCH [LE] [V34|I] [V35|I] B4
В3
25: LDC [-1] [V36|I]
30: ADD [V34|I] [V36|I] [V37|I]
35: ADD [V33|I] [V34|I] [V38|I]
40: MOVE [V37|I] [V34|I]
45: MOVE [V38|I] [V33|I]
50: BRANCH B2
В4
55: MOVE [V33|I] $v0
60: RETURN $v0
```

- a. Compute the liveUse and liveDef sets (local liveness information) for each basic block in the method.
- b. Compute the liveIn and liveOut sets (global liveness information) for each basic block in the method.
- c. Compute the liveness interval for each virtual register in the method's LIR, with ranges and use positions.

Problem 2. Using the liveness intervals for the virtual registers in the LIR for the SpimSum.compute() method

- a. Build an interference graph G for the method.
- b. Represent G as an adjacency matrix.
- c. Represent G as an adjacency list.
- d. Is G 2-colorable? If so, give a register allocation using two physical registers r_1 and r_2 .
- e. Is G 3-colorable? If so, give a register allocation using three physical registers r_1 , r_2 , and r_3 .

SOLUTIONS

Solution 1.

```
a. B0
    liveUse:
    liveDef:

B1
    liveUse: $a0
    liveDef: V32, V33, V34

B2
    liveUse: $a0, V34
    liveDef: V35

B3
    liveUse: V33, V34
    liveDef: V33, V34, V36, V37, V38
B4
    liveUse: V33
    liveUse: $v0
```

```
b. B4
    liveIn: V33
    liveOut:

B3
    liveIn: V33, V34
    liveOut: V33

B2
    liveIn: $a0, V34
    liveOut: V33, V34

B1
    liveIn: $a0
    liveIn: liveOut: $a0, V34

C. v0: [55, 60]
    a0: [0, 10]
    V32: [0, 5]
    V33: [5, 35] [45, 55]
    V34: [10, 50]
    V35: [15, 20]
    V36: [25, 30]
    V37: [30, 40]
```

Solution 2.

V38: [35, 45]

a.

b.

c.

d.