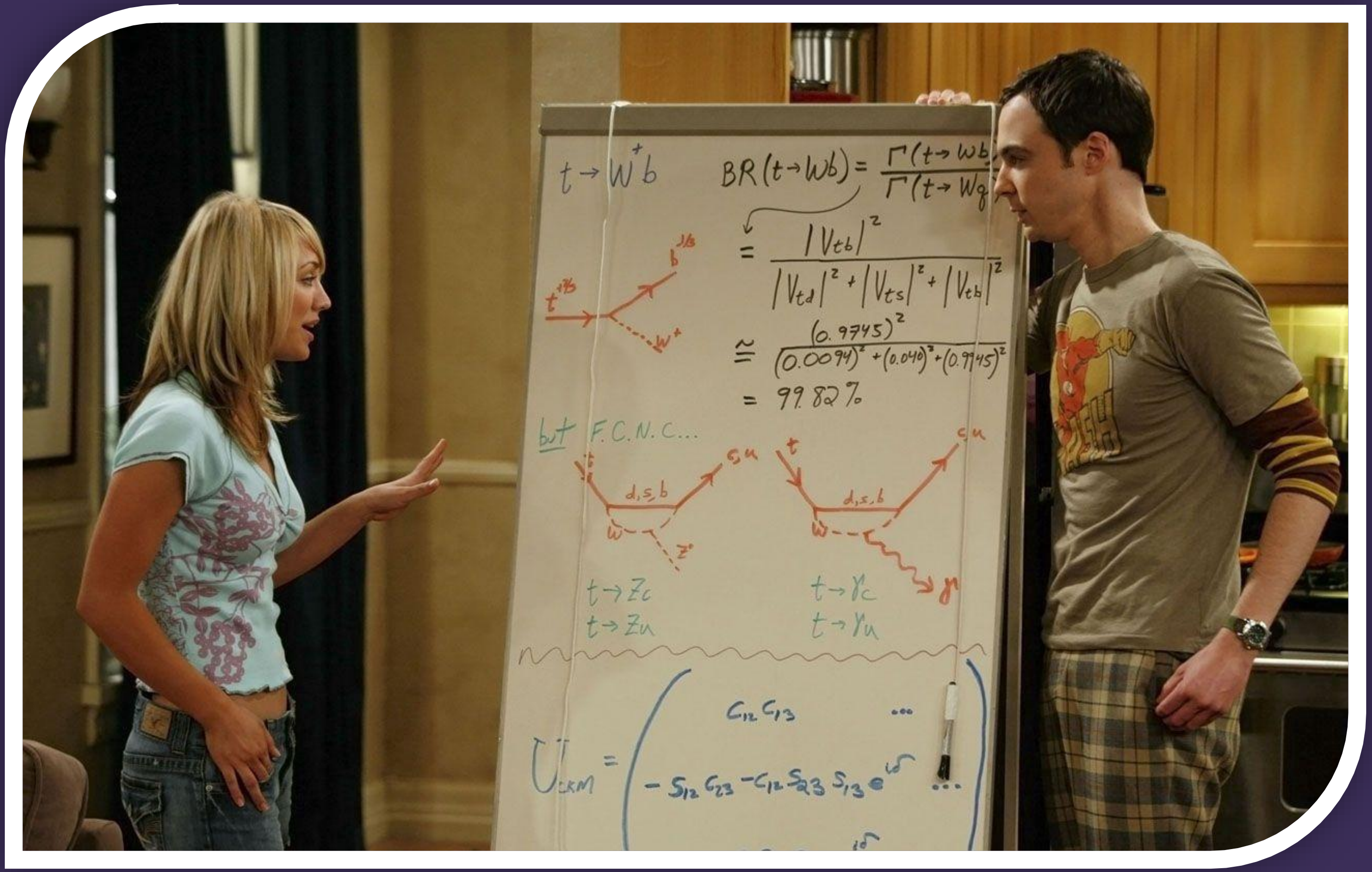
Microsoft

Introduction to Quantum Computing

some useful operators and protocols



Mario Cuomo

- Cloud Solution Architect @ Microsoft
- Bachelor's in Computer Science @ Roma Tre University
- Attending Master’s in Computer Science @ Roma Tre University
- Ex Unity and Microsoft Learn Student Ambassador
- Enthusiast for Artificial Intelligence, cryptography, Microsoft culture in Inclusion&Diversity
- mariocuomo.github.io

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Why?

Why?

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Agenda

- teleportation protocol

Agenda

- bit vs qubit
- teleportation protocol

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- superposition
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- decay of superposition
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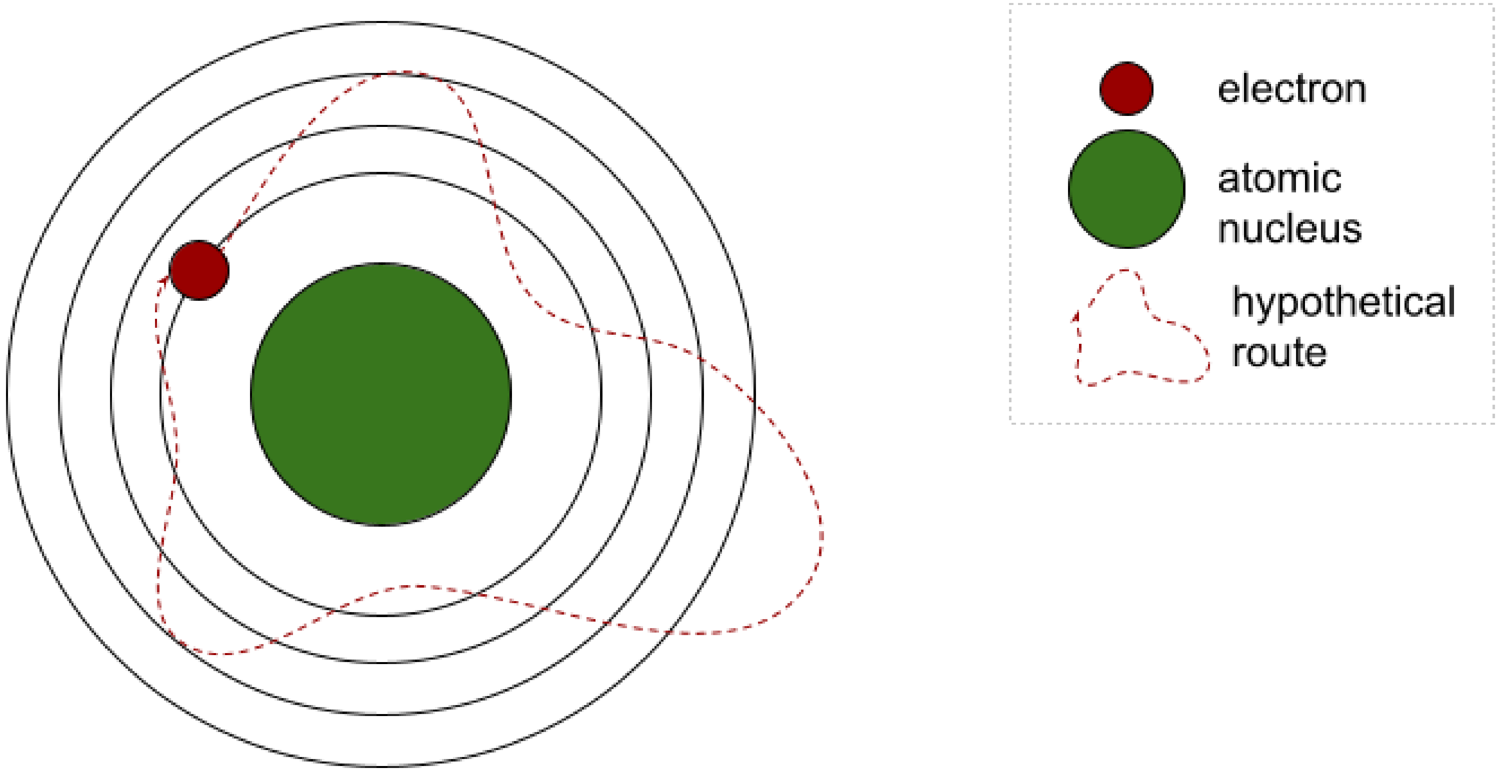
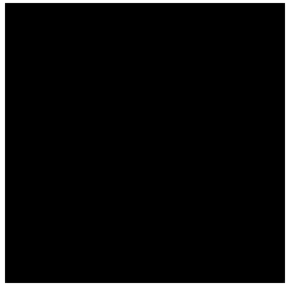
- entanglement
- operators on two qubit
- no cloning theorem
- teleportation protocol
- useful resources

prerequisite



bit vs qubit



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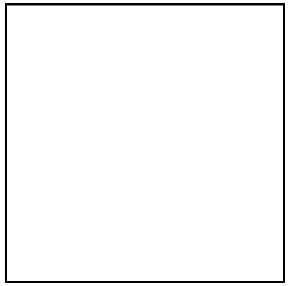

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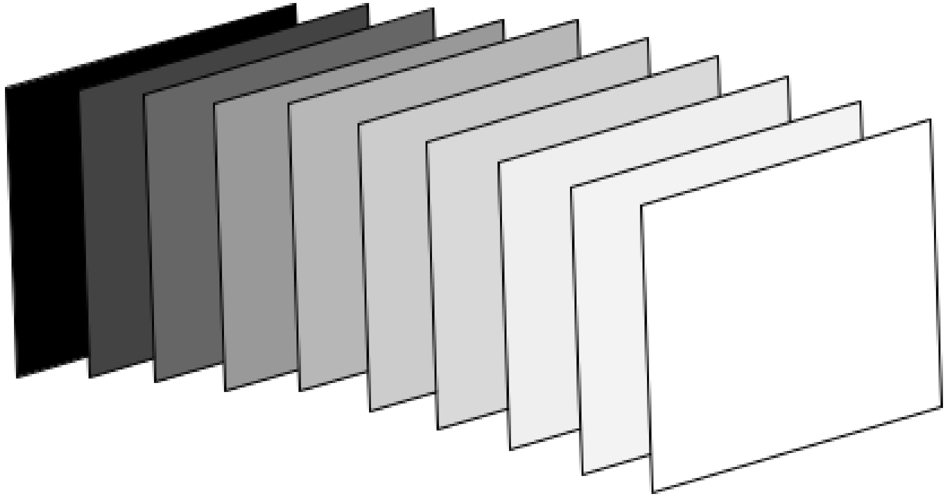
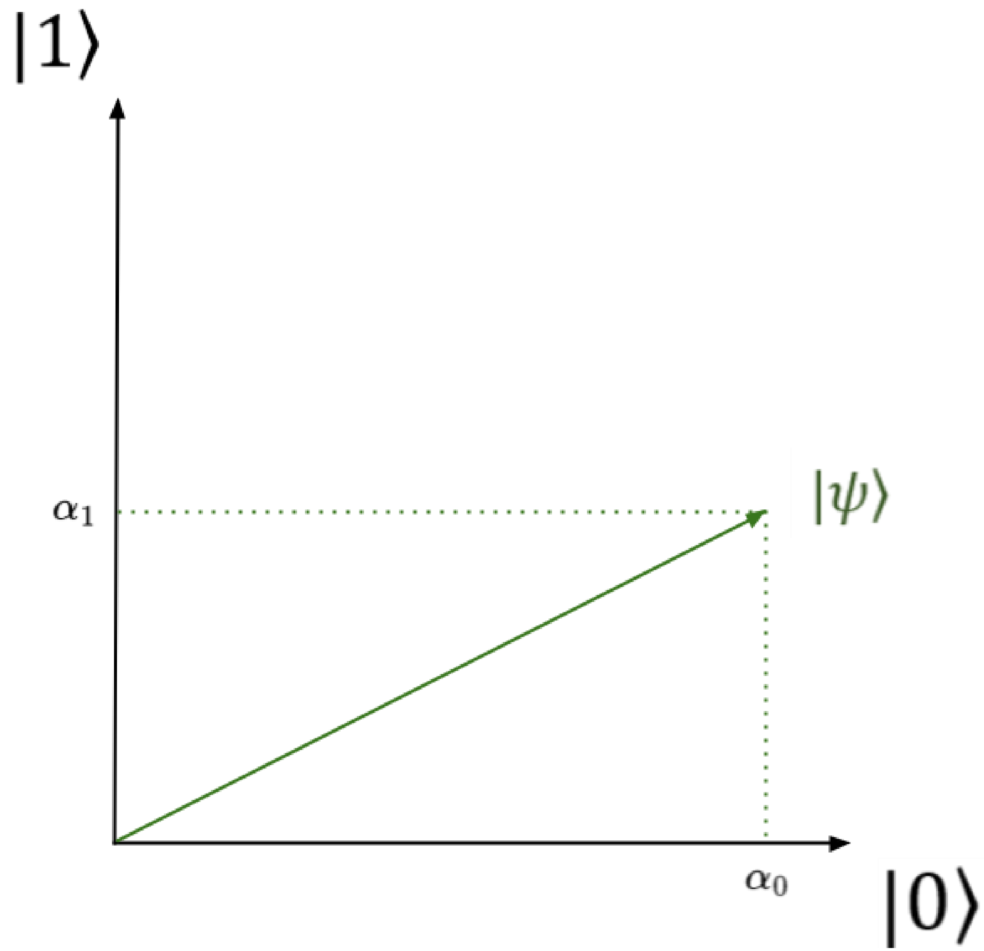
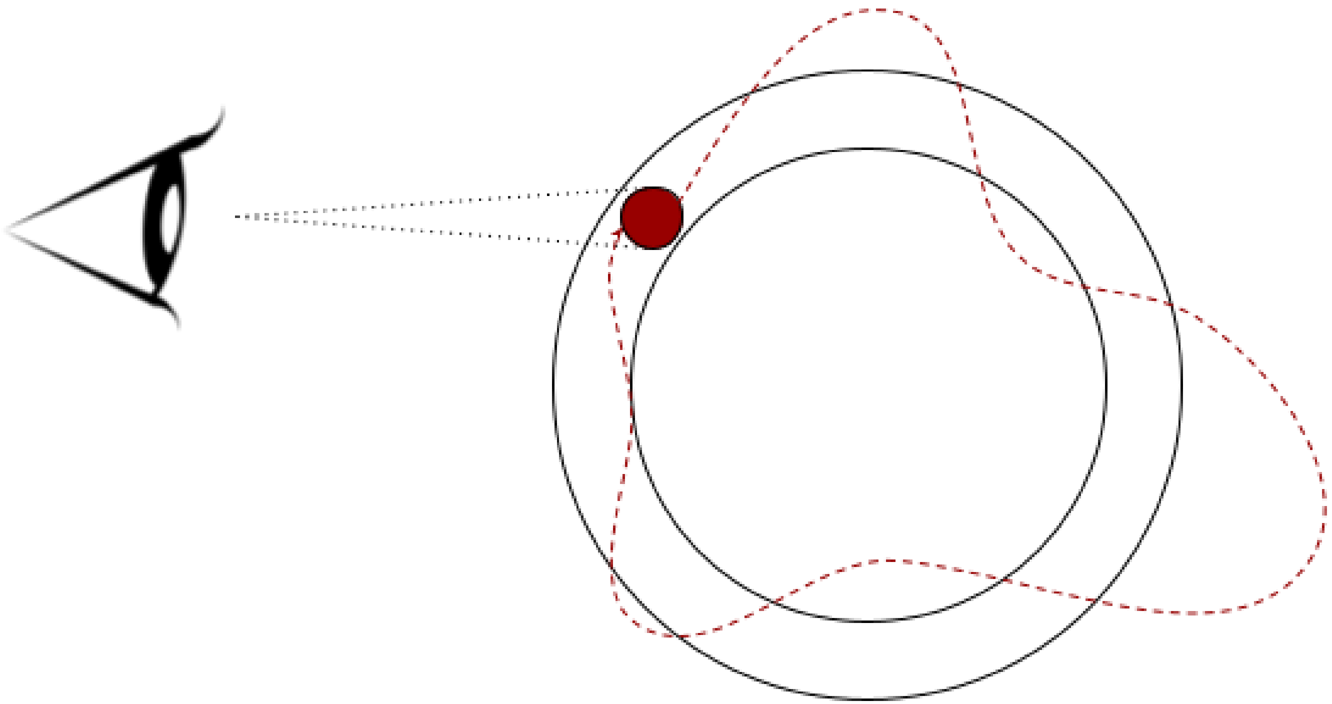


superposition

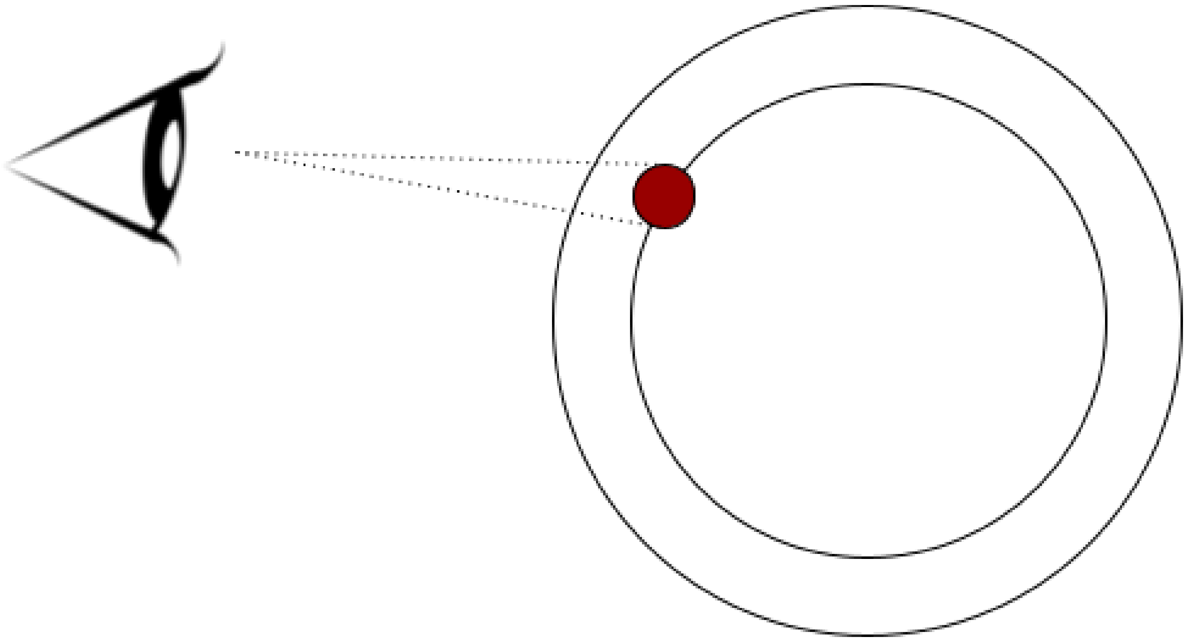


$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle$$

decay of superposition



decay of superposition



example

example

$|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$ has the following properties

example

$|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$ has the following properties

- $\left|\frac{\sqrt{3}}{2}\right|^2 = \frac{3}{4} = 0.75 = 75\%$ to be in $|0\rangle$ state

example

$|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$ has the following properties

- $\left|\frac{\sqrt{3}}{2}\right|^2 = \frac{3}{4} = 0.75 = 75\%$ to be in $|0\rangle$ state
- $\left|\frac{1}{2}\right|^2 = \frac{1}{4} = 0.25 = 25\%$ to be in $|1\rangle$ state

example

$|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$ has the following properties

- $\left|\frac{\sqrt{3}}{2}\right|^2 = \frac{3}{4} = 0.75 = 75\%$ to be in $|0\rangle$ state

- $\left|\frac{1}{2}\right|^2 = \frac{1}{4} = 0.25 = 25\%$ to be in $|1\rangle$ state

NOTE
$|a_0|^2 + |a_1|^2 = 1$

it’s your time!

it’s your time!

If $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, what is probability to be in $|0\rangle$ state?

it's your time!

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$A.\ \ 0\%$

it's your time!

If $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, what is probability to be in $|0\rangle$ state?

$A.\ \ 0\%$

B. 100%

it's your time!

If $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, what is probability to be in $|0\rangle$ state?

$A.$ 0%

B. 100%

C. 50%

Pauli matrices

Pauli matrices

- No error I

Pauli matrices

- No error I
- Bit flip X

Pauli matrices

- No error I
- Bit flip X
- Phase flip Z

Pauli matrices

- No error I
- Bit flip X
- Phase flip Z
- Bit & Phase flip Y

Pauli matrices

- No error I
- Bit flip X
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NOTE

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle = \binom{a_0}{a_1}$$

no error I

no error I

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

no error I

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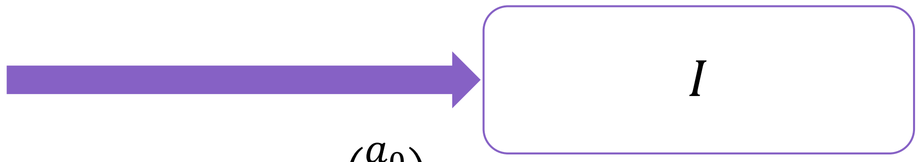
$\boxed{I}$

no error I

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

no error I

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\quad} \boxed{\;I\;} \xrightarrow{\quad}$$

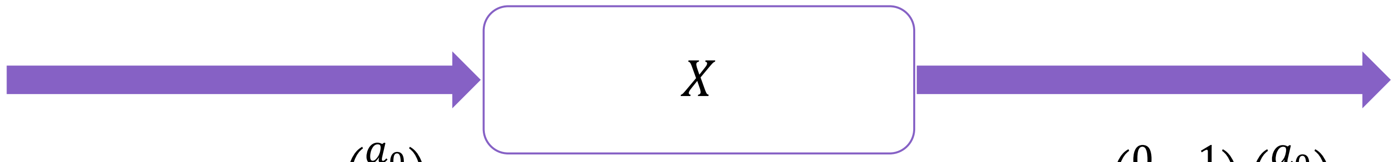
$|\psi\rangle = a_0|0\rangle + a_1|1\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$

$I|\psi\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = |\psi\rangle$

bit flip X

bit flip X

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



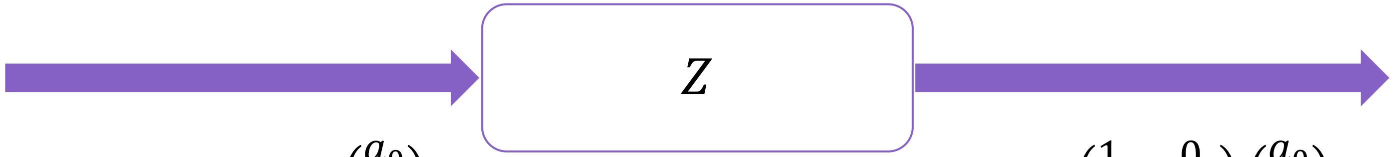
$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$X|\psi\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_0 \end{pmatrix} = a_1|0\rangle + a_0|1\rangle$$

phase flip Z

phase flip Z

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$Z|\psi\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} a_0 \\ -a_1 \end{pmatrix} = a_0|0\rangle - a_1|1\rangle$$

bit & phase flip Y

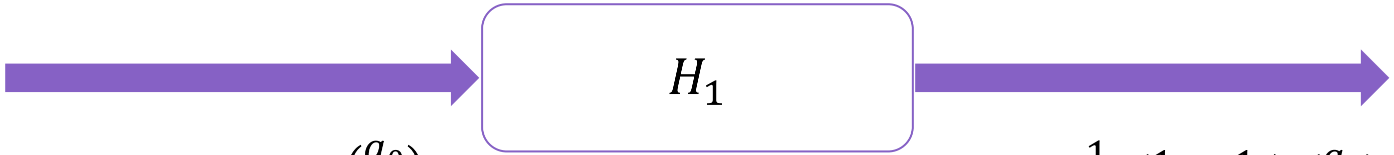
bit & phase flip Y

$$Y = XZ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \longrightarrow \boxed{Y} \longrightarrow Y|\psi\rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} a_1 \\ -a_0 \end{pmatrix} = a_1|0\rangle - a_0|1\rangle$$

Hadamard H_1

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

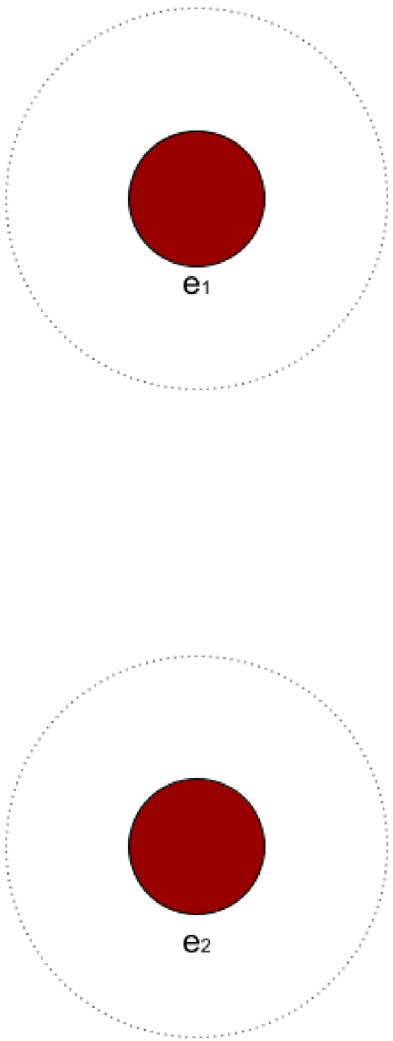


$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

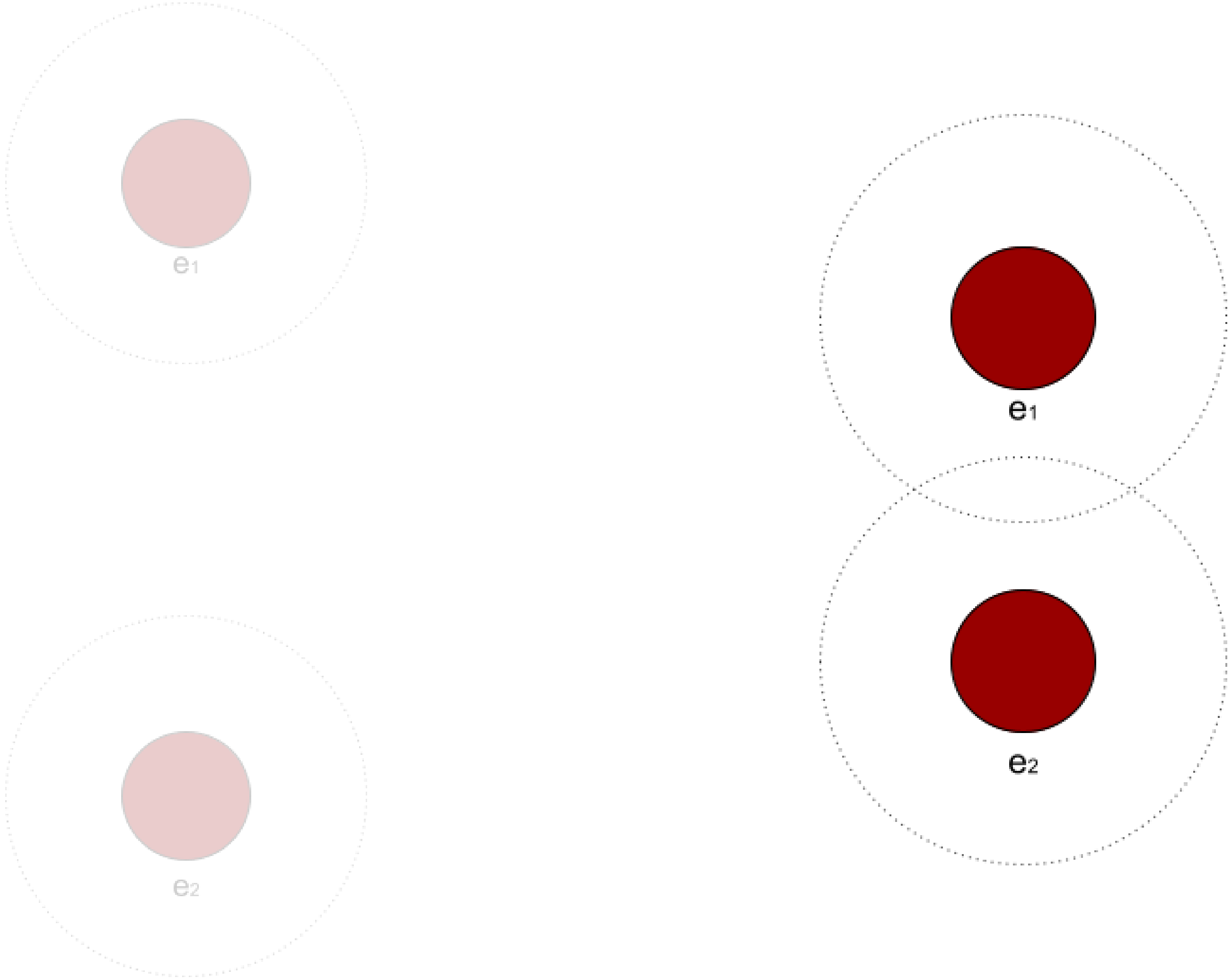
$$H_1|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} a_0 + a_1 \\ a_0 - a_1 \end{pmatrix}$$

entanglement

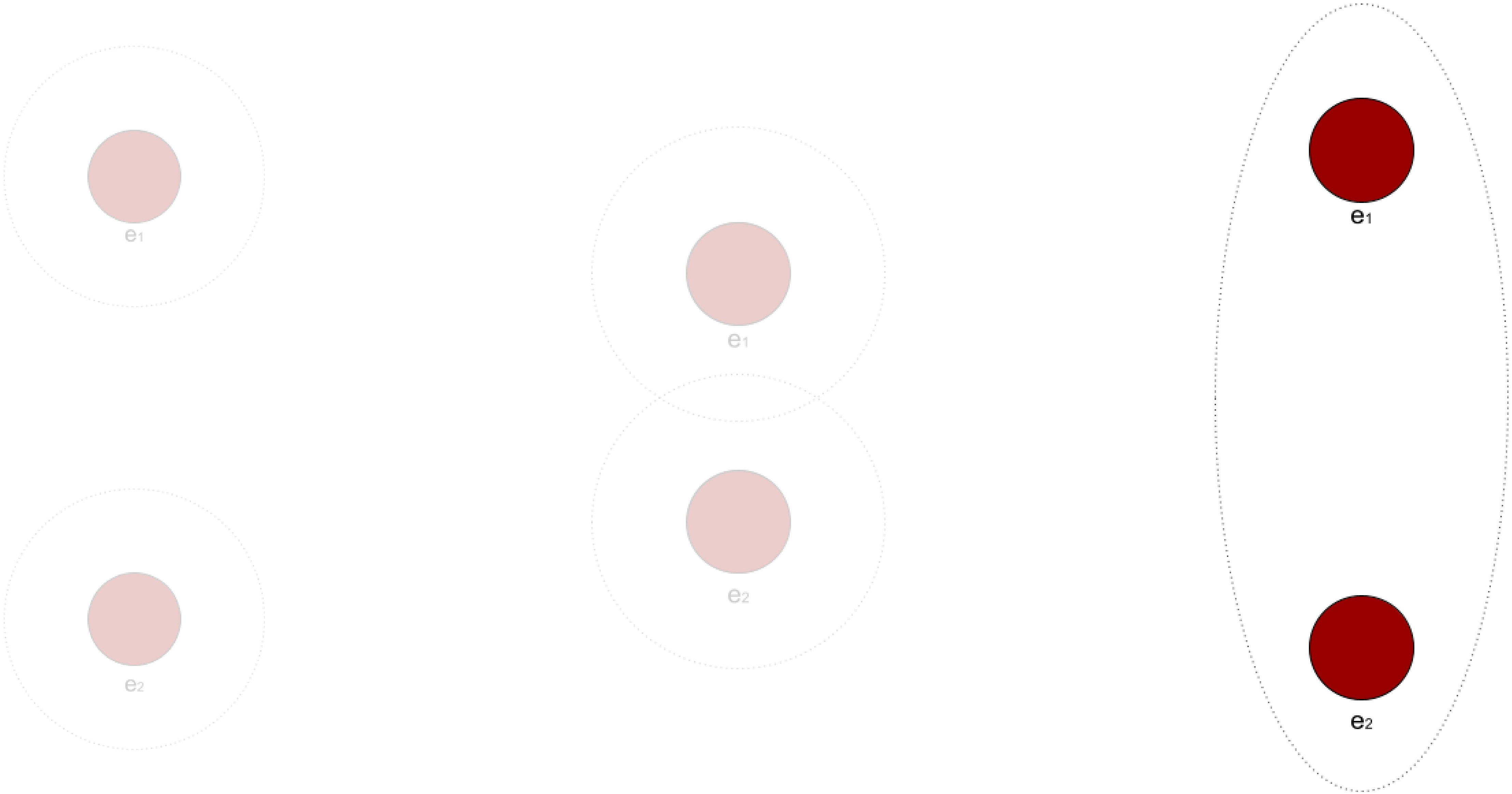
entanglement



entanglement



entanglement



example

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$$|\psi_a\rangle = a_0|0\rangle + a_1|1\rangle$$

$$|\psi_b\rangle = b_0|0\rangle + b_1|1\rangle$$

example

$$|\psi_a\rangle = a_0|0\rangle + a_1|1\rangle$$

$$|\psi_b\rangle = b_0|0\rangle + b_1|1\rangle$$

$$\longrightarrow$$

$$|\psi_a\psi_b\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$$

example

$$|\psi_a\rangle = a_0|0\rangle + a_1|1\rangle$$

$$|\psi_b\rangle = b_0|0\rangle + b_1|1\rangle$$

$$\longrightarrow$$

$$|\psi_a\psi_b\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

controlledNot

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

controlledNot

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

control	target	$CNOT$
0	0	$\lvert 00\rangle$
0	1	$\lvert 01\rangle$
1	0	$\lvert 11\rangle$
1	1	$\lvert 10\rangle$

create Bell state

create Bell state

$|\psi_a\rangle = |0\rangle$

$|\psi_b\rangle = |0\rangle$

create Bell state

$$|\psi_a\rangle = |0\rangle$$

$$|\psi_b\rangle = |0\rangle$$

$$H_1|\psi_a\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

create Bell state

$|\psi_a\rangle = |0\rangle$

$|\psi_b\rangle = |0\rangle$

$$H_1|\psi_a\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$cNot(H_1|\psi_a\rangle, |\psi_b\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\\ 1\\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Agenda

- bit vs qubit
- superposition
- decay of superposition
- operators on one qubit
- entanglement
- operators on two qubit
- no cloning theorem
- teleportation protocol
- useful resources

tensor product

tensor product

$$|\psi_a\rangle = a_0|0\rangle + a_1|1\rangle$$

$$|\psi_b\rangle = b_0|0\rangle + b_1|1\rangle$$

$$\longrightarrow$$

$$|\psi_a\psi_b\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$$

tensor product

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$$\longrightarrow \quad |\psi_a\psi_b\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$$

$$|\psi_a\psi_b\rangle$$

tensor product

$$|\psi_a\rangle = a_0|0\rangle + a_1|1\rangle$$

$$|\psi_b\rangle = b_0|0\rangle + b_1|1\rangle$$

$$|\psi_a\psi_b\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$$

$$|\psi_a\psi_b\rangle = |\psi_a\rangle\otimes|\psi_b\rangle$$

tensor product

$$|\psi_a\rangle = a_0|0\rangle + a_1|1\rangle$$

$$|\psi_b\rangle = b_0|0\rangle + b_1|1\rangle$$

$$\Rightarrow \quad |\psi_a\psi_b\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$$

$$|\psi_a\psi_b\rangle = |\psi_a\rangle \otimes |\psi_b\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} \otimes \begin{pmatrix} b_0 \\ b_1 \end{pmatrix}$$

tensor product

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$$|\psi_a\rangle = a_0|0\rangle + a_1|1\rangle$$

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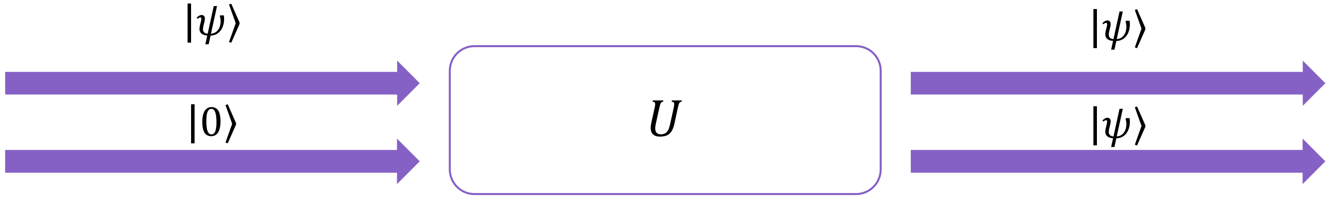
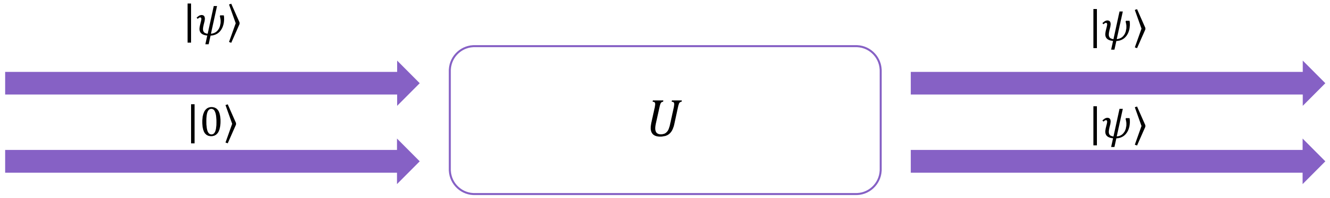
no cloning theorem

no cloning theorem

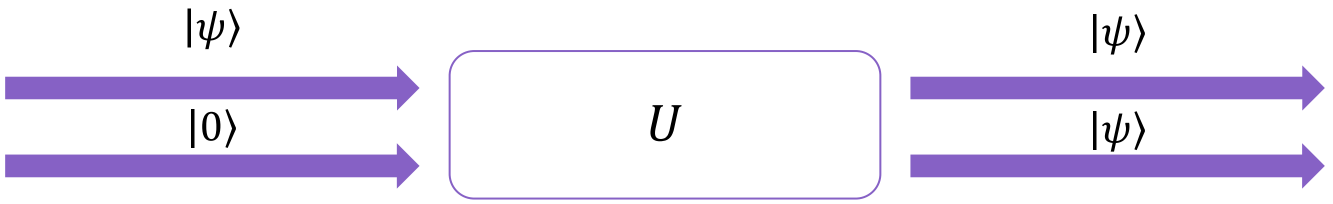


no cloning theorem



$$U(|0\rangle \otimes |0\rangle) = |0\rangle \otimes |0\rangle$$
$$U(|1\rangle \otimes |0\rangle) = |1\rangle \otimes |1\rangle$$

no cloning theorem



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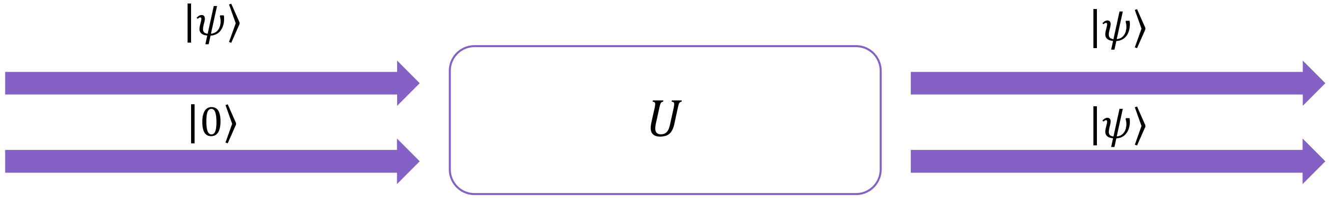
no cloning theorem



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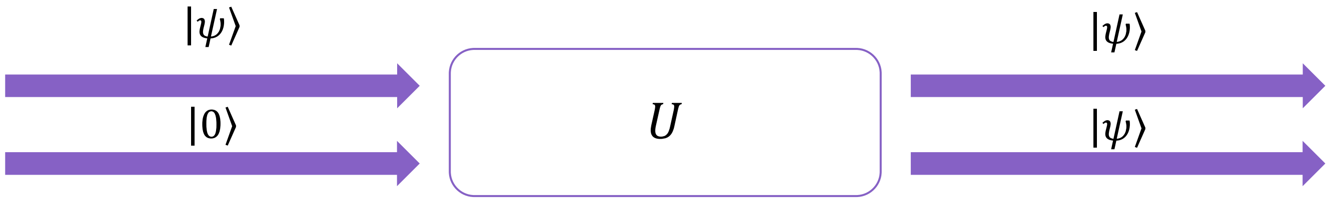
$$U(|\psi\rangle \otimes |0\rangle)$$

no cloning theorem



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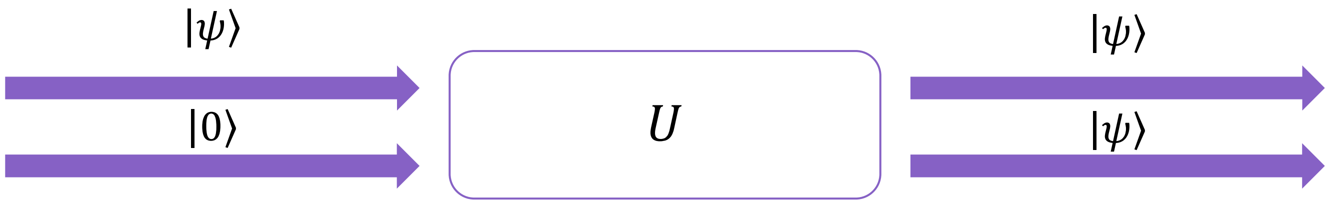
$$U(|\psi\rangle \otimes |0\rangle) = U\left(\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

no cloning theorem



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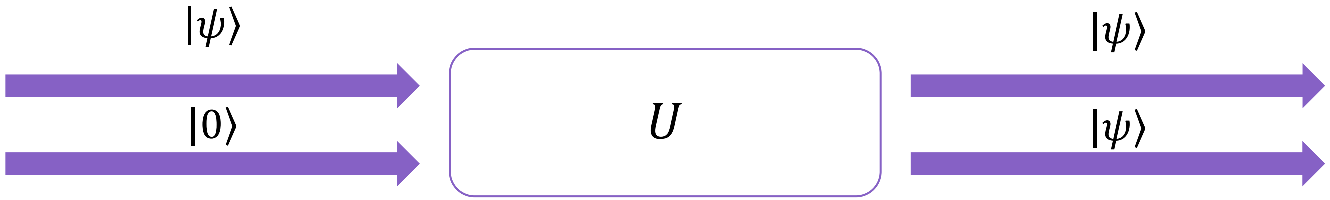
no cloning theorem



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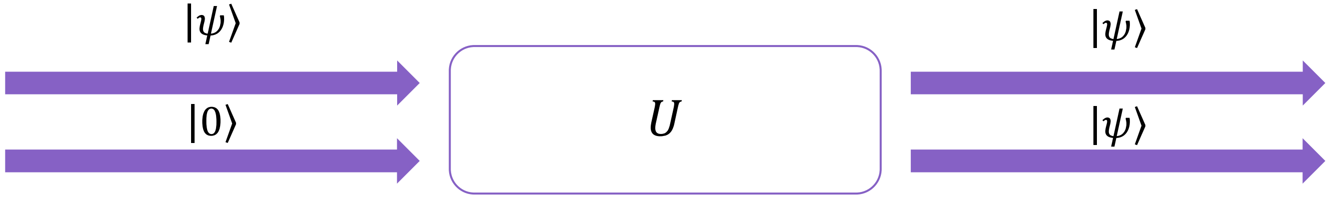
no cloning theorem



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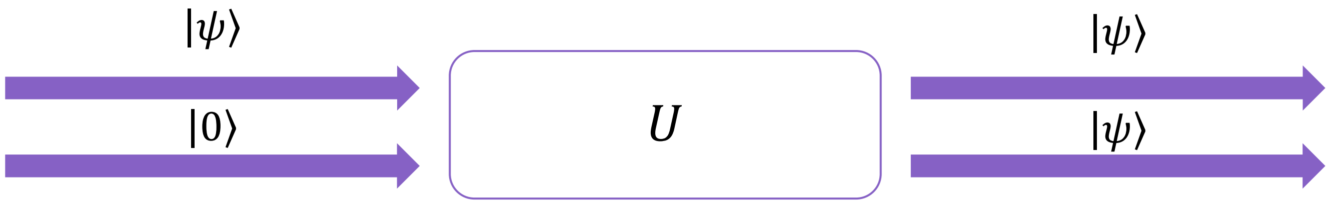
no cloning theorem



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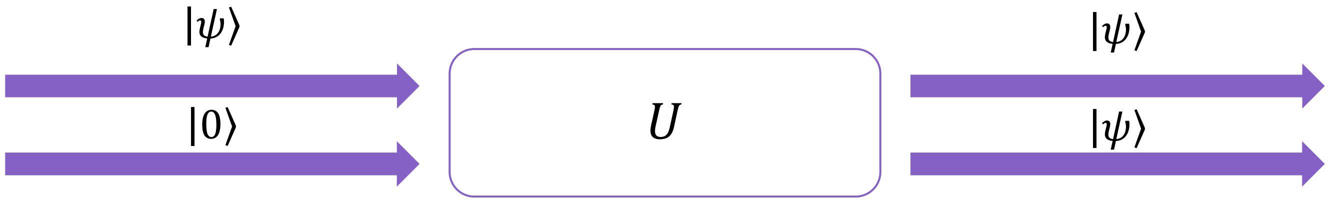
no cloning theorem



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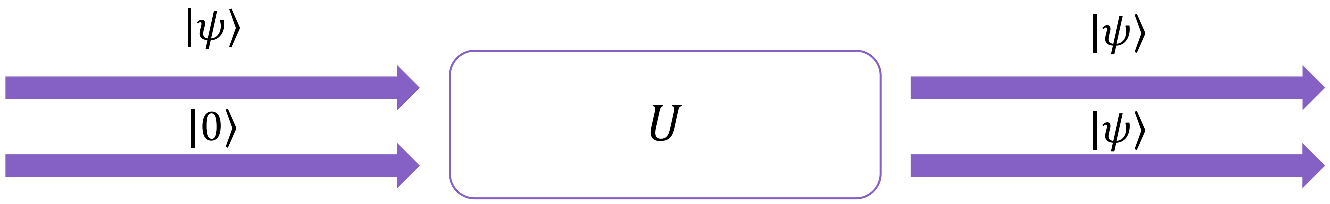

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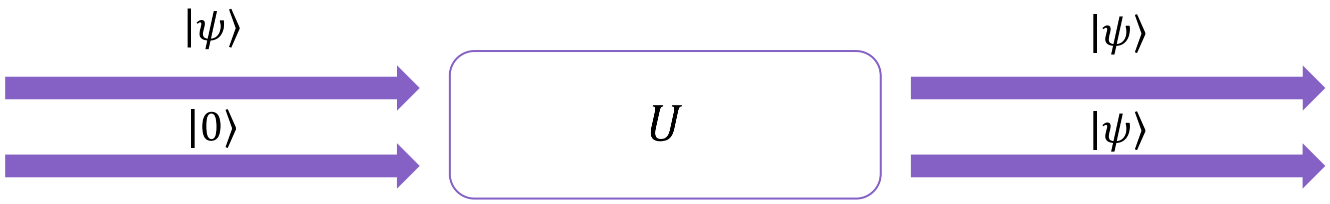

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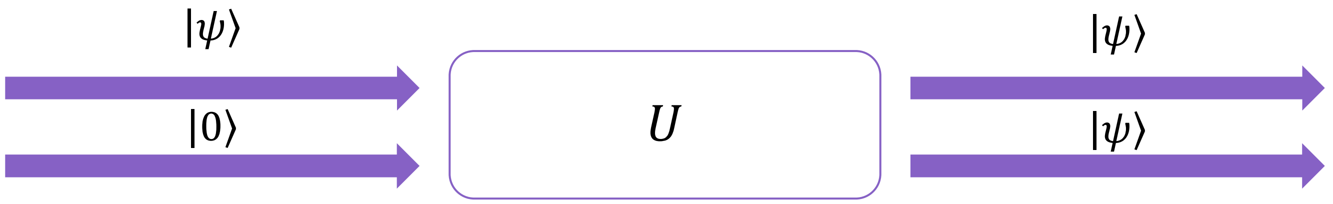

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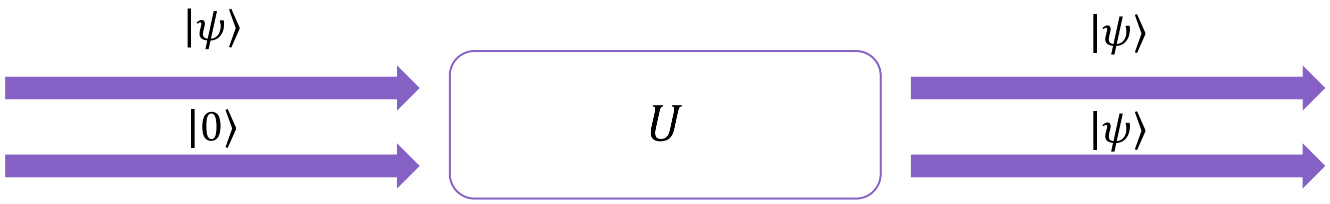

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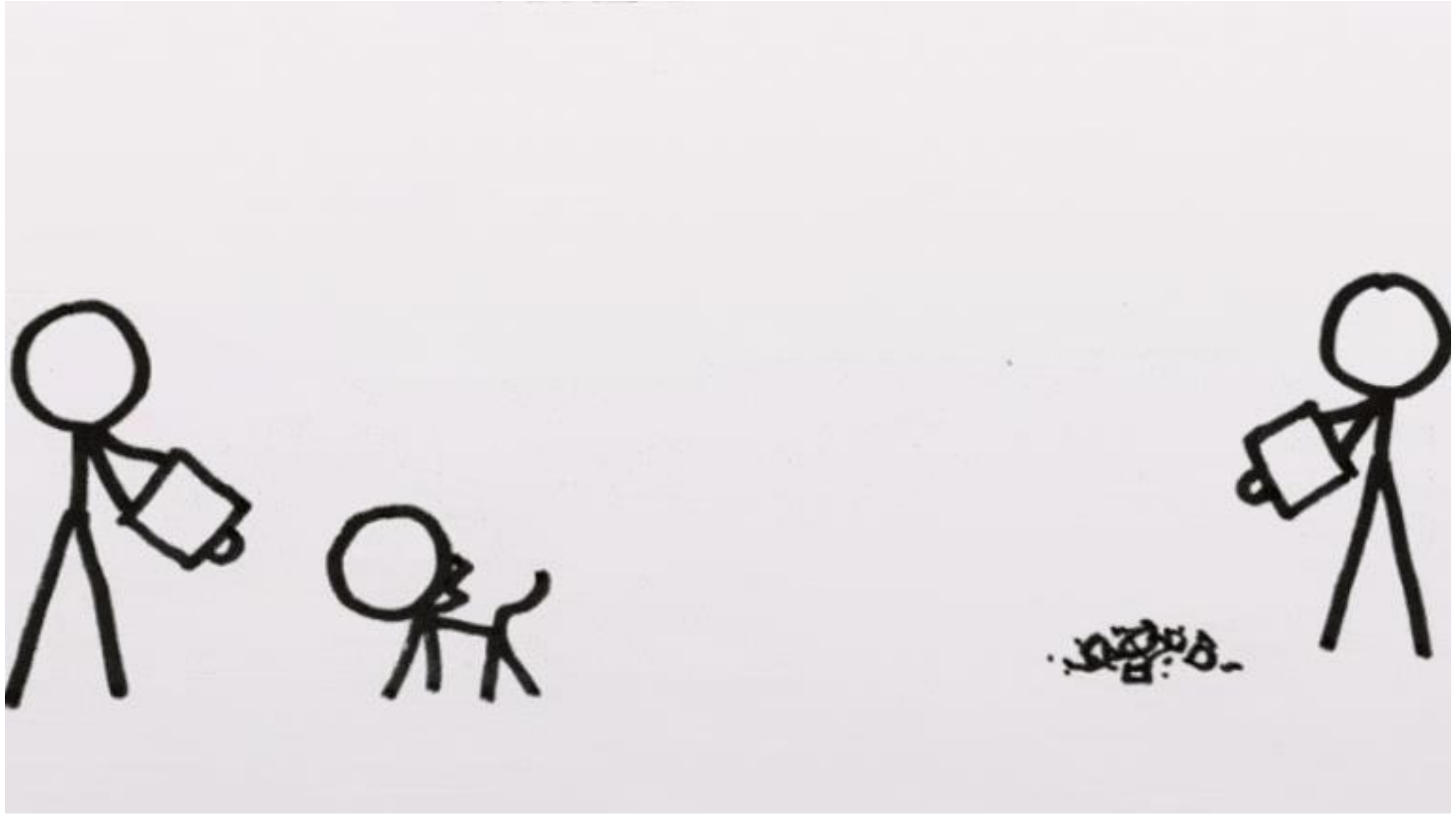
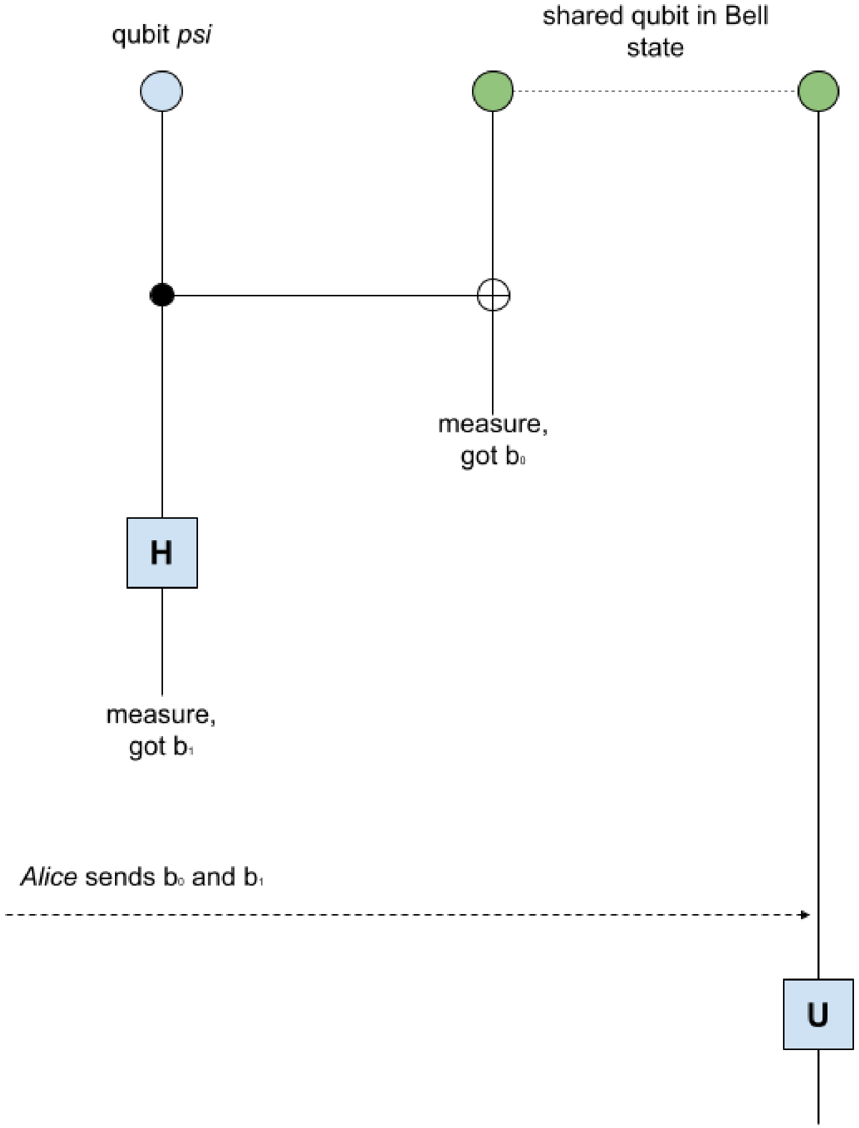
teleportation protocol

teleportation protocol



teleportation protocol



b_0	b_1	operator U
0	0	**I**
1	0	**X**
0	1	**Z**
1	1	**Y**

demo

resources

- What is Quantum Computing | Microsoft Azure
- Q# Holiday Calendar 2022 - Q# Blog (microsoft.com)
- Superdense coding — due bit al prezzo di un qubit. | by mariocuomo | Oct, 2022 | Medium
- mariocuomo@microsoft.com ✉

Microsoft

Thank you for your attention