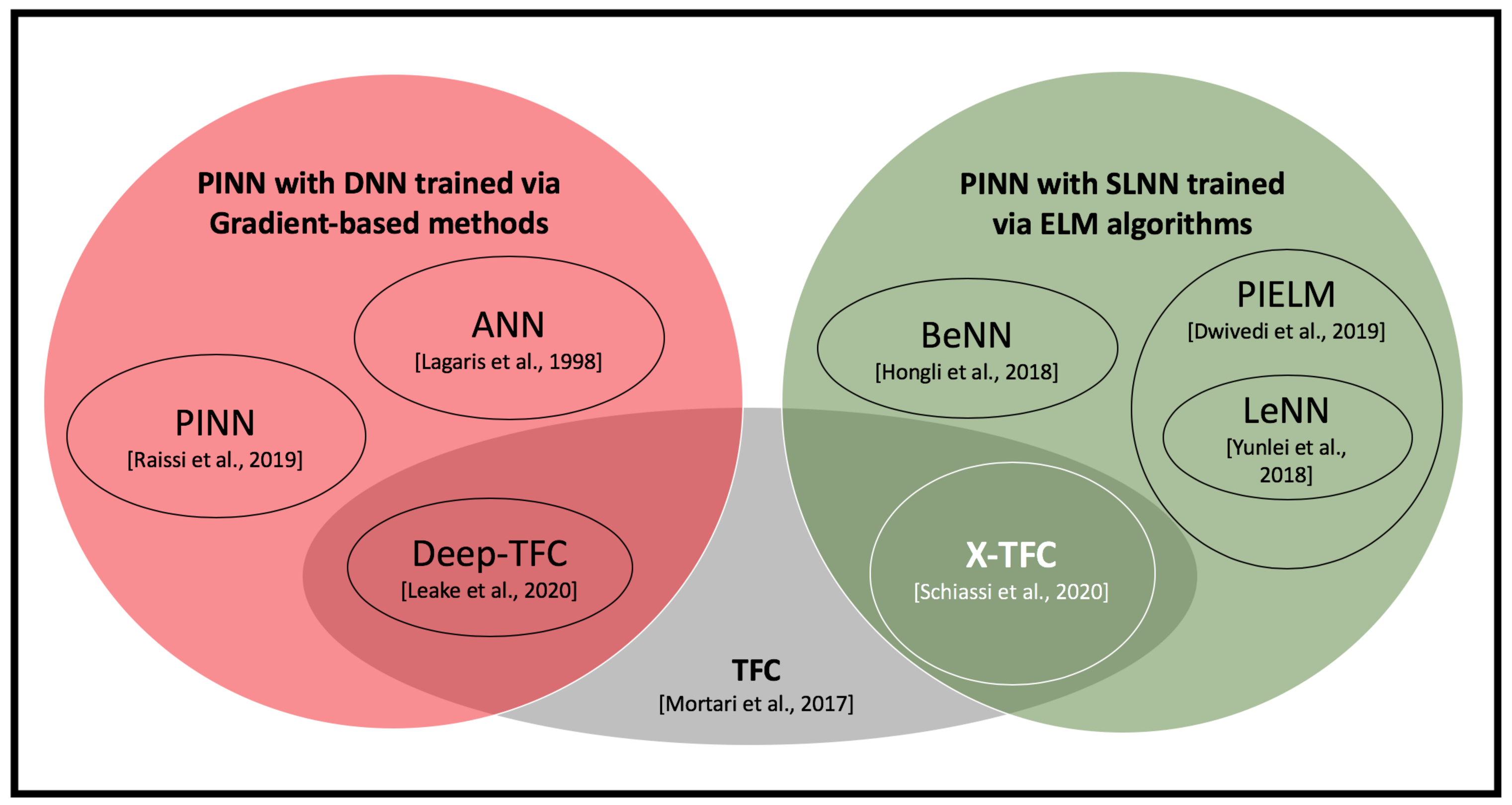
PHYSICS-INFORMED SOLUTIONS OF RADIATIVE TRANSFER PROBLEMS

Mario De Florio (PhD Student), Roberto Furfaro (Advisor)
Systems & Industrial Engineering, The University of Arizona

(1) INTRODUCTION

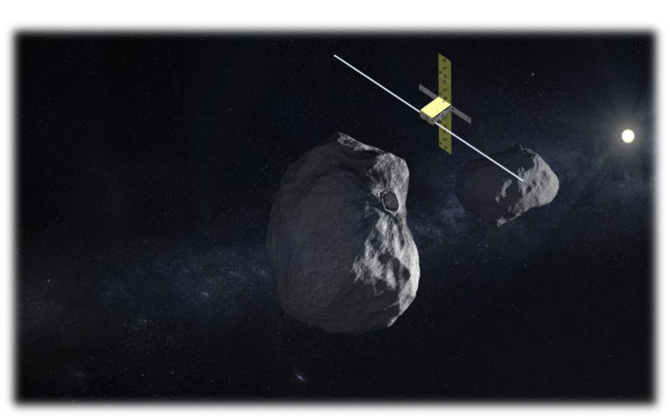
Overview: The Extreme Theory of Functional Connections (X-TFC) is a physics-informed Neural Network (PINN) method: synergy of the TFC, introduced by Mortari, and the PINN, introduce by Raissi et al.

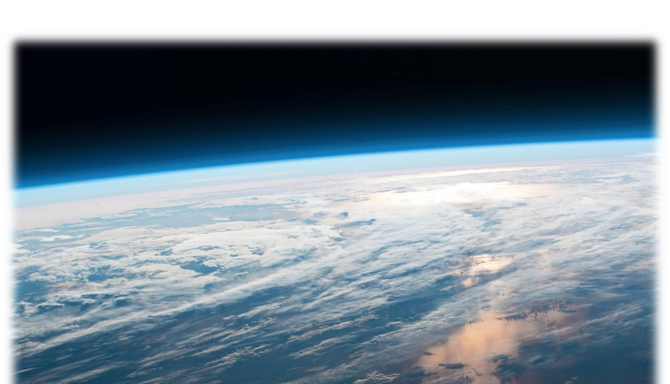


Goal: to develop an accurate and robust **physics-informed** framework to solve the Radiative Transfer Problem

(2) TRANSPORT THEORY FOR REMOTE SENSING

The **(Photon) Transport Theory** represents the theoretical underpinning of remote sensing.

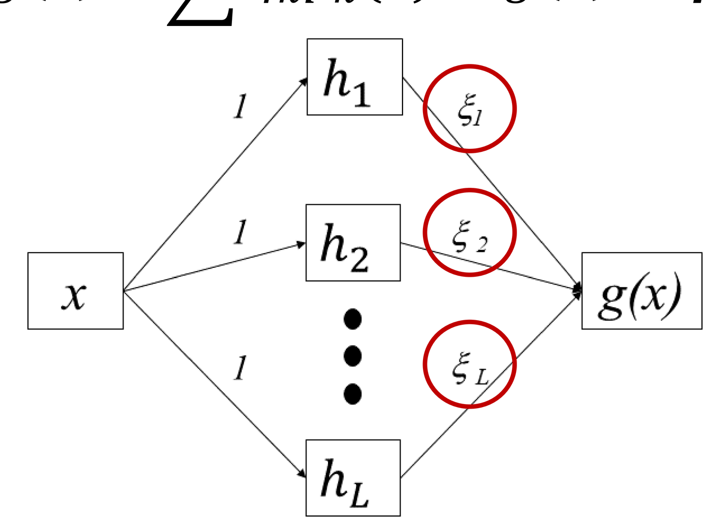




(3) THEORY OF FUNCTIONAL CONNECTION (TFC)

- **Methodology:** X-TFC uses the constrained expressions that are a sum of a free-chosen function *g(x)* and a term which analytically satisfies the boundary conditions.
- For the problems treated here, X-TFC employs a single layer ChNN to expand the free chosen function.
  - The ChNN is trained via ELM.

$$y(x) = g(x) + \sum^{n} \eta_k p_k(x) = g(x) + \boldsymbol{\eta}^T \boldsymbol{p}(x)$$

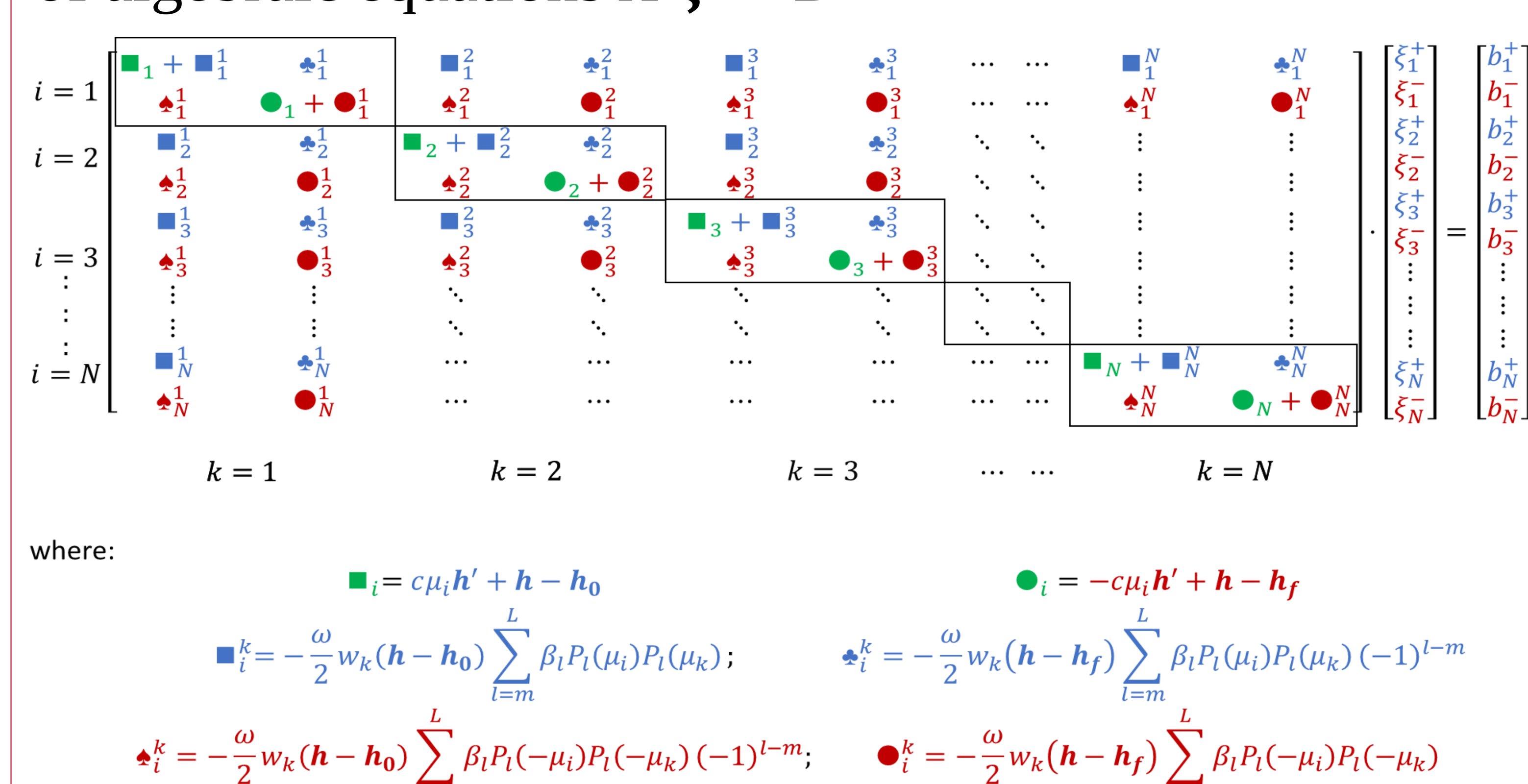


(4) RADIATIVE TRANSFER PROBLEM

The Radiative Transfer Equation according to Chandrasekhar is:

$$\mu \frac{\partial}{\partial t} I(\tau, \mu, \phi) + I(\tau, \mu, \phi) = \frac{\omega}{4\pi} \int_{-1}^{1} \int_{0}^{2\pi} p(\cos\Theta)\, I(\tau, \mu', \phi') d\phi' d\mu'$$

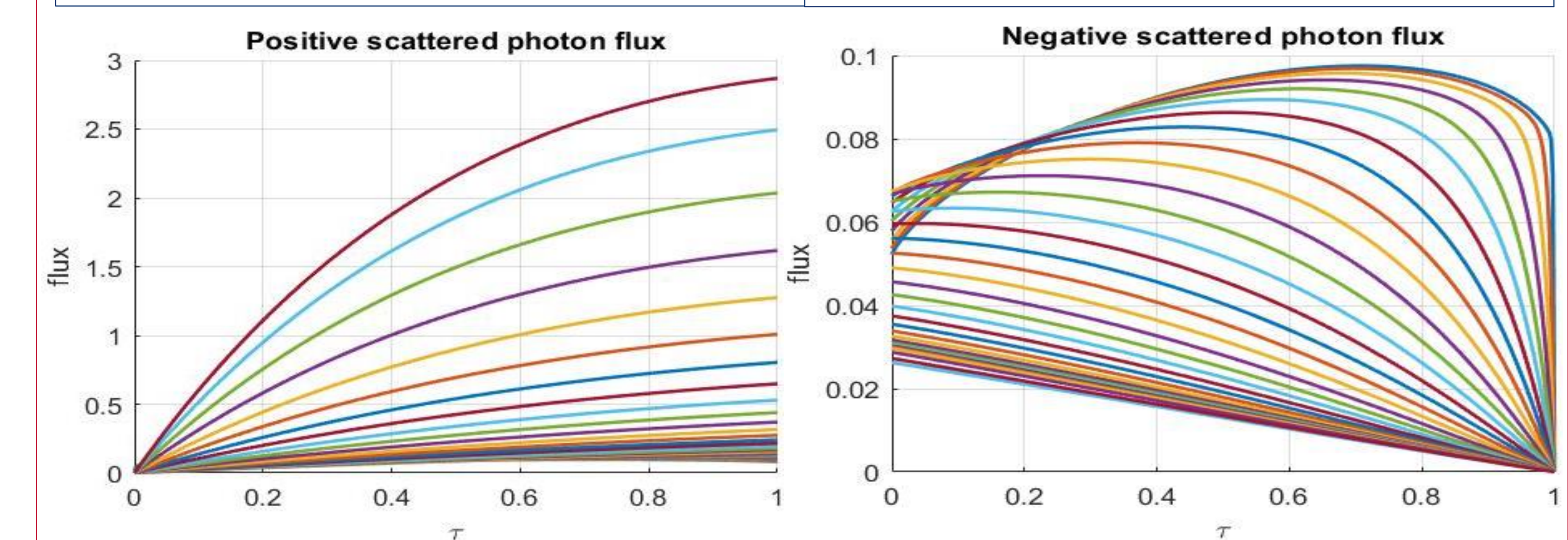
As the nature of the equation is linear, by applying TFC the problem reduces to the solution of the linear system of algebraic equations $\boldsymbol{A}\, \boldsymbol{\xi} = \boldsymbol{B}$



(5) RESULTS AND DISCUSSIONS

Haze L Problem ($\omega = 0.9$; $\mu_0 = 1$; $\tau_0 = 1$)

TFC N = 35	vs.	*ADO* N = 150	CPU time for the Least-Squares ≅ 5 s



7 digits Benchmark [Barry D. Ganapol]

μ	$\tau = 0$	$\tau = 0.05\Delta$	$\tau = 0.1\Delta$	$\tau = 0.2\Delta$	$\tau = 0.5\Delta$	$\tau = 0.75\Delta$	$\tau = \Delta$
-1.0	2.7971665e-2	2.6583431e-2	2.5179489e-2	2.2342144e-2	1.3751918e-2	6.7043863e-3	0
-0.9	3.0180197e-2	2.8742728e-2	2.7276328e-2	2.4282402e-2	1.5036989e-2	7.3279307e-3	0
-0.8	3.1447755e-2	3.0070750e-2	2.8641094e-2	2.5662054e-2	1.6096218e-2	7.8550379e-3	0
-0.7	3.4383906e-2	3.3055747e-2	3.1640694e-2	2.8605980e-2	1.8314210e-2	9.0026264e-3	0
-0.6	3.9130810e-2	3.7890987e-2	3.6513506e-2	3.3427829e-2	2.2097749e-2	1.1061916e-2	0
-0.5	4.5637920e-2	4.4617111e-2	4.3383966e-2	4.0403191e-2	2.8008565e-2	1.4514343e-2	0
-0.4	5.3511337e-2	5.2985449e-2	5.2140980e-2	4.9686294e-2	3.6854018e-2	2.0230769e-2	0
-0.3	6.1542012e-2	6.1991418e-2	6.1978635e-2	6.0886294e-2	4.9616521e-2	2.9827680e-2	0
-0.2	6.6956243e-2	6.9056402e-2	7.0499635e-2	7.2037240e-2	6.6696988e-2	4.6300639e-2	0
-0.1	6.5529583e-2	7.0004105e-2	7.3432378e-2	7.8590386e-2	8.4462845e-2	7.3611021e-2	0
-0.0	5.1748534e-2	6.1709609e-2	6.8016336e-2	7.7466538e-2	9.3997859e-2	9.7483668e-2	0
0.0	0	6.1709609e-2	6.8016336e-2	7.7466538e-2	9.3997859e-2	9.7483668e-2	7.9312594e-2
0.1	0	2.2494931e-2	3.9518122e-2	6.2652973e-2	9.6254337e-2	1.0871841e-1	1.0818930e-1
0.2	0	1.3100677e-2	2.5340076e-2	4.6772905e-2	9.1591615e-2	1.1381468e-1	1.2421167e-1
0.3	0	1.0194331e-2	2.0270341e-2	3.9472225e-2	8.7467481e-2	1.1662007e-1	1.3571209e-1
0.4	0	9.5290644e-3	1.9067650e-2	3.7770256e-2	8.8332315e-2	1.2250259e-1	1.4826767e-1
0.5	0	1.0263750e-2	2.0502330e-2	4.0649220e-2	9.6418345e-2	1.3581653e-1	1.6751584e-1
0.6	0	1.2529327e-2	2.4909477e-2	4.9065634e-2	1.1533634e-1	1.6223048e-1	2.0070062e-1
0.7	0	1.7417124e-2	3.4415206e-2	6.7081148e-2	1.5398186e-1	2.1356335e-1	2.6167192e-1
0.8	0	2.8562211e-2	5.6020429e-2	1.0769702e-2	2.3848565e-1	3.2254956e-1	3.8692070e-1
0.9	0	6.1633112e-2	1.1976311e-1	2.2612375e-1	4.7610276e-1	6.1970346e-1	7.1774509e-1
1.0	0	3.2812354e-1	6.2906510e-1	1.1563161	2.2483946	2.7414726	2.9776602

(6) CONCLUSIONS

- We developed a robust **physics-informed** framework to solve the Radiative Transfer Problem with high accuracy.
- Currently we are working on the application of our method to Tranport Theory problems modeled via PDEs (photon and neutron transport)

(7) ACKNOWLEDGEMENT

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Space Systems Engineering Laboratory

Mario De Florio, email: mariodf@email.arizona.edu