```
ClearAll[q, q1, q2, q3, q4, q5, q6, nTf, nTa, aTb, bTc, cTd, dTe, eTf,
  nTf, L1, L21, L22, L33, L31, L32, L4, L5, R, Jt , Jinv, Jmultinv, t];
```

```
(* 1) Initializing Transformation Matrices *)
```

$$q = \begin{pmatrix} q1[t] \\ q2[t] \\ q3[t] \\ q4[t] \\ q5[t] \\ q6[t] \end{pmatrix};$$

$$nTa = \begin{pmatrix} Cos[q1[t]] & -Sin[q1[t]] & 0 & 0 \\ Sin[q1[t]] & Cos[q1[t]] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$aTb = \begin{pmatrix} Cos[q2[t]] & 0 & -Sin[q2[t]] & 0 \\ 0 & 1 & 0 & 0 \\ Sin[q2[t]] & 0 & Cos[q2[t]] & L1 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$bTc = \begin{pmatrix} Cos[q3[t]] & 0 & -Sin[q3[t]] & 0 \\ 0 & 1 & 0 & L21 \\ Sin[q3[t]] & 0 & Cos[q3[t]] & L22 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$cTd = \begin{pmatrix} 1 & 0 & 0 & L33 \\ 0 & Cos[q4[t]] & -Sin[q4[t]] & -L31 \\ 0 & Sin[q4[t]] & Cos[q4[t]] & L32 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$dTe = \begin{pmatrix} Cos[q5[t]] & 0 & -Sin[q5[t]] & L4 \\ 0 & 1 & 0 & 0 \\ Sin[q5[t]] & 0 & Cos[q5[t]] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$eTf = \begin{pmatrix} 1 & 0 & 0 & L5 \\ 0 & Cos[q6[t]] & -Sin[q6[t]] & 0 \\ 0 & Sin[q6[t]] & Cos[q6[t]] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

```
nTf = nTa . aTb . bTc . cTd . dTe . eTf // Simplify ; (*This is the final
 Transformation matrix (between the newtonian frame & frame f) *)

R = Join[{Part [nTf [[1, 4]]]}, {Part [nTf [[2, 4]]]} , { Part [nTf [[3, 4]]]}];

xf = Part[R[[1]]];
yf = Part[R[[2]]];
zf = Part[R[[3]]];

J = D[{Part[R[[1]]], Part[R[[2]]], Part[R[[3]]]},
   {{q1[t] , q2[t], q3[t], q4[t], q5[t], q6[t]}} ];
```

(* Calculating the pseudoinverse of the Jacobian matrix as $J^{\dagger} = J^{T}.[J.J^{T}]^{-1}$ *)

```
Jt = Transpose[J] ;
```

```
Jmult = J.Jt ;

Jmultinv = Inverse[Jmult];

Jinv = Jt . Jmultinv ;

V = Dt [ R , t];
A = Dt[ V , t] ;
```

$$Rin = \begin{pmatrix} Xin[t] \\ Yin[t] \\ Zin[t] \end{pmatrix};$$

```
Vin = Dt [Rin, t];
Ain = Dt [Vin , t];
qdot = Jinv . Vin;
qdoubledot = Dt[ qdot , t] ;



L1 = 0.3;
L21 = 0.15;
L22 = 0.6;
L33 = 0.075;
L31 = 0.15;
L32 =  0.075;
L4 = 0.075;
L5 = 0.075;


Rta3weed = R - Rin /. {Xin[t] → 0, Yin[t] → 0.5, Zin[t] → 0.6} ;
```

$$q0 = \begin{pmatrix} 0.5 \\ 1 \\ 0.5 \\ 1 \\ 0.5 \\ 1 \end{pmatrix};$$

$$qi = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix};$$

$$zero = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix};$$

```
Flag = True;
For[i = 0, Flag && i < 100, i++, Jinv0 =
   Jinv /. {q1[t] → q0[[1]][[1]], q2[t] → q0[[2]][[1]], q3[t] → q0[[3]][[1]],
     q4[t] → q0[[4]][[1]], q5[t] → q0[[5]][[1]], q6[t] → q0[[6]][[1]] };
  Rta3weed2 = Rta3weed /. {q1[t] → q0[[1]][[1]], q2[t] → q0[[2]][[1]],
     q3[t] → q0[[3]][[1]], q4[t] → q0[[4]][[1]],
     q5[t] → q0[[5]][[1]], q6[t] → q0[[6]][[1]] };
  qi = q0 - Jinv0 . (Rta3weed2 );
  Print[qi // MatrixForm]; Flag = (qi - q0 ≠ zero); q0 = qi; ];
```

$$q0 = \begin{pmatrix} 0.5 \\ 1 \\ 0.5 \\ 1 \\ 0.5 \\ 1 \end{pmatrix};$$

```
Clear [L1, L21, L22, L33, L31, L32, L4, L5];

Print[
  "The Number of iterations needed for reaching approximately zero error is " ,
  i];
```

$$\begin{pmatrix} -0.308603 \\ 2.38048 \\ -6.43692 \\ 2.47746 \\ 0.0362737 \\ 1. \end{pmatrix}$$

$$\begin{pmatrix} -1.1545 \\ 2.41642 \\ -9.49114 \\ 2.50488 \\ 0.836339 \\ 1. \end{pmatrix}$$

$$\begin{pmatrix} -2.17418 \\ 1.25773 \\ -5.83567 \\ 2.17515 \\ 0.511958 \\ 1. \end{pmatrix}$$

$$\begin{pmatrix} -1.82894 \\ 1.56213 \\ -7.36808 \\ 2.44774 \\ 1.13665 \\ 1. \end{pmatrix}$$

$$\begin{pmatrix} -1.66514 \\ 1.32603 \\ -6.80429 \\ 2.28931 \\ 1.24934 \\ 1. \end{pmatrix}$$

$$\begin{pmatrix} -1.68135 \\ 1.32698 \\ -6.71107 \\ 2.2608 \\ 1.25008 \\ 1. \end{pmatrix}$$

$$\begin{pmatrix} -1.6808 \\ 1.32548 \\ -6.71111 \\ 2.26075 \\ 1.24991 \\ 1. \end{pmatrix}$$

$$\begin{pmatrix} -1.6808 \\ 1.32548 \\ -6.71111 \\ 2.26075 \\ 1.24991 \\ 1. \end{pmatrix}$$

$$\begin{pmatrix} -1.6808 \\ 1.32548 \\ -6.71111 \\ 2.26075 \\ 1.24991 \\ 1. \end{pmatrix}$$

$$\begin{pmatrix} -1.6808 \\ 1.32548 \\ -6.71111 \\ 2.26075 \\ 1.24991 \\ 1. \end{pmatrix}$$

The Number of iterations needed for reaching approximately zero error is 10

```
Print [ "Final Transformation Matrix = ", "nTf =" ,
  nTf /. { Cos → "c" , Sin → "s" , q1[t] → "q1", q2[t] → "q2", q3[t] → "q3",
     q4[t] → "q4", q5[t] → "q5", q6[t] → "q6"}  // MatrixForm ] ;
Print [ "The forward position equations are represented as ",
   ("x"
    "y"
    "z") // MatrixForm, "=" ,
  R /. { Cos → "c" , Sin → "s" , q1[t] → "q1", q2[t] → "q2", q3[t] → "q3",
     q4[t] → "q4", q5[t] → "q5", q6[t] → "q6"} // MatrixForm ] ;
```

```
Print["The forward velocity equations are represented as ",
  ("Vx"
   "Vy"
   "Vz") // MatrixForm, "=" ,
  V /. {Cos → "c" , Sin → "s" , q1[t] → "q₁", q2[t] → "q₂", q3[t] → "q₃",
     q4[t] → "q₄", q5[t] → "q₅", q6[t] → "q₆"} // MatrixForm] ;
Print["The forward acceleration equations are represented as ",
  ("Ax"
   "Ay"
   "Az") // MatrixForm, "=" ,
  A /. {Cos → "c" , Sin → "s" , q1[t] → "q₁", q2[t] → "q₂", q3[t] → "q₃",
     q4[t] → "q₄", q5[t] → "q₅", q6[t] → "q₆"} // MatrixForm] ;

Print [ "The Jacobian matrix is ",
  "J = ", J  /. {Cos → "c" , Sin → "s"} // MatrixForm] ;
Print [ "The pseudoinverse of the Jacobian matrix is ", "J† = ",
  Jinv  /. {Cos → "c" , Sin → "s" , q1[t] → "q₁", q2[t] → "q₂",
     q3[t] → "q₃", q4[t] → "q₄", q5[t] → "q₅", q6[t] → "q₆"} // MatrixForm];

Print [ "The inverse position angles are computed using the Newton-Raphson
    method, since this is a numerical method, therefore intial values
    for the angles were needed to be set, By substituting with q0 =",
  q0 // MatrixForm , " the value of the angles will be: ",
  qi  // MatrixForm,
  " After reaching the minimal amount of error in ",
  i , " iterations"] ;

Print["The inverse velocity equations are represented as q̇ =",

  ("q̇₁"
   "q̇₂"
   "q̇₃"
   "q̇₄"
   "q̇₅"
   "q̇₆") // MatrixForm, "=",

  qdot /. {Cos → "c" , Sin → "s" , q1[t] → "q₁", q2[t] → "q₂", q3[t] → "q₃",
     q4[t] → "q₄", q5[t] → "q₅", q6[t] → "q₆"} // MatrixForm] ;
Print["The inverse acceleration equations are represented as q̈ =",

  ("q̈₁"
   "q̈₂"
   "q̈₃"
   "q̈₄"
   "q̈₅"
   "q̈₆") // MatrixForm, "=",
```

```
qdoubledot /. {Cos → "c" , Sin → "s" , q1[t] → "q1", q2[t] → "q2",
   q3[t] → "q3", q4[t] → "q4", q5[t] → "q5", q6[t] → "q6"} // MatrixForm ] ;
```

Final Transformation Matrix = nTf = $\begin{pmatrix} s[q_1]\, s[q_4]\, s[q_5] + c[q_1]\, (c[q_2+q_3]\, c[q_5] - c[q_4 \\ c[q_2+q_3]\, c[q_5]\, s[q_1] - (c[q_3]\, c[q_4]\, s[q_1]\, s[q_2] + c[q_2]\, c[q_4] \\ c[q_5]\, s[q_2+q_3] + c[q_2+q_3]\, c[q_4]\, s \\ 0 \end{pmatrix}$

The forward position equations are represented as $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s[q_1]\, (-L21 + L31 + L5\, s \\ c[q_1]\, (L21 - L31 - L5\, s[q_4]\, s[q_5]) + s[q \\ L1 + L22\, c[c \end{pmatrix}$

The forward velocity equations are represented as $\begin{pmatrix} Vx \\ Vy \\ Vz \end{pmatrix} = \begin{pmatrix} -s[q_1]\, (L21 - L31 - L5\, s[q_4]\, s[q_5])\, q1' \end{pmatrix}$

The forward acceleration equations are represented as $\begin{pmatrix} Ax \\ Ay \\ Az \end{pmatrix} = \begin{pmatrix} -c[q_1]\, (L21 - L31 - L5\, s[q_4]\, s[q_5] \end{pmatrix}$

The Jacobian matrix is J = $\begin{pmatrix} c[q1[t]]\, (-L21 + L31 + L5\, s[q4[t]]\, s[q5[t]] \\ -s[q1[t]]\, (L21 - L31 - L5\, s[q4[t]]\, s[q5[t]]) + c[q1[t]]\, (c[q2[t] + q3 \end{pmatrix}$

The pseudoinverse of the Jacobian matrix is $J^{\dagger}$ = $\begin{pmatrix} \overline{-((L5\, c[q_2+q_3]\, c[q_4]\, c[q_5] - L5\, s[q_2+q_3]\, s[q_5])\, (-L5\, c[q_1]\, c[q_5]} \\ \overline{-((L5\, c[q_2+q_3]\, c[q_4]\, c[q_5] - L5\, s[q_2+q_3]\, s[q_5])\, (-L5\, c[q_1]\, c[q_5]} \\ \overline{-((L5\, c[q_2+q_3]\, c[q_4]\, c[q_5] - L5\, s[q_2+q_3]\, s[q_5])\, (-L5\, c[q_1]\, c[q_5]} \\ \overline{-((L5\, c[q_2+q_3]\, c[q_4]\, c[q_5] - L5\, s[q_2+q_3]\, s[q_5])\, (-L5\, c[q_1]\, c[q_5]} \end{pmatrix}$

The inverse position angles are computed using the Newton-Raphson method, since this is a numerical method, therefore intial values for the angles were needed to be set, By substituting with q0 =

$\begin{pmatrix} 0.5 \\ 1 \\ 0.5 \\ 1 \\ 0.5 \\ 1 \end{pmatrix}$ the value of the angles will be: $\begin{pmatrix} -1.6808 \\ 1.32548 \\ -6.71111 \\ 2.26075 \\ 1.24991 \\ 1. \end{pmatrix}$

After reaching the minimal amount of error in 10 iterations

The inverse velocity equations are represented as $\dot{q} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{pmatrix} =$

$$\left(\begin{array}{l} \left(\frac{}{-((L5\, c[q_2+q_3]\, c[q_4]\, c[q_5]-L5\, s[q_2+q_3]\, s[q_5])} \right. \\ \left(\frac{}{-((L5\, c[q_2+q_3]\, c[q_4]\, c[q_5]-L5\, s[q_2+q_3]\, s[q_5])} \right. \\ \left(\frac{}{-((L5\, c[q_2+q_3]\, c[q_4]\, c[q_5]-L5\, s[q_2+q_3]\, s[q_5])} \right. \\ \left(\frac{}{-((L5\, c[q_2+q_3]\, c[q_4]\, c[q_5]-L5\, s[q_2+q_3]\, s[q_5])} \right. \end{array} \right.$$

The inverse acceleration equations are represented as $\ddot{q} = \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_6 \end{pmatrix} =$

$$\left(\left(- \frac{}{-((L5\, c[q_2+q_3]\, c[q_4]\, c[q_5]-L5\, s[q_2+q} \right. \right.$$