COFLASY-M technical notes

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These notes describe the main features of `COsmological evolution of flavor ASYmmetries - Mathematica (COFLASY-M)`, a Mathematica code accompanying [1]. The code numerically solves the quantum kinetic equations (QKEs) governing the evolution of primordial lepton asymmetries from the early Universe to BBN time. We refer the reader to [1] for a complete derivation of the equations and discussion of the dynamics of the system.

The code consists of the following files:

- `BasicModule.m`: Contains all the definitions, equations and functions used to solve the system of QKEs.
- `COFLASY-M.nb`: Acts as the main front end notebook which loads `BasicModule.m` and contains a series of examples that serve as a tutorial for the user. These examples refer to some of the evolution plots in [1].

To use the code open the `COFLASY-M.nb` notebook and evaluate the "Basic module" section. Then you can go through the "Examples" section of the notebook, which serves as a tutorial for the user to get familiar with the code.

In the following, we describe the sections contained in `BasicModule.m`, and at the end of this document we describe the output generated by the "Examples" section in the `COFLASY-M.nb` notebook.

I. PRELIMINARIES

In this section we define the following global constants: the reduced Planck mass M_P, the Fermi constant G_F, the W boson mass M_w, the squared sine of the weak mixing angle $\sin^2\theta_W$, the electron and muon masses $m_{e,\mu}$. We work in natural units and in MeV energy units by setting MeV $= 1$ for all dimensionful quantities. We fix the values of the neutrino parameters $(\Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{13}, \theta_{23})$ as in Appendix A of [1] and $\delta_{CP} = 0$. All these parameters may be modified freely by the user. Inverted neutrino mass hierarchy can be considered by switching the sign of Δm^2_{31}.

We also define the Gell-Mann matrices λ_i, with $i = 1, ..., 8$, along with the structure constants of $SU(3)$

$$f^{ijk} = \frac{1}{4i}\mathrm{Tr}(\lambda_i[\lambda_j, \lambda_k])\,, \tag{1}$$

where $i, j, k = 1, ..., 8$ and repeated indices are summed. Finally, we define the function `gellMannComponents[X]` which for any given 3×3 complex matrix returns its components in the basis formed by $(\mathbb{1}, \lambda_i)$ as a nine-dimensional vector, see Sec. IIC of [1] for more details.

II. EQUATIONS

The main goal of the code is to compute the evolution of the 3×3 neutrino and anti-neutrino density matrices in the presence of primordial asymmetries among neutrinos and anti-neutrinos, from the early Universe down to BBN times. We will work in the limit of neglecting the energy transfer among neutrinos and the cosmic plasma, which is achieved by imposing $z_\nu = z_\gamma = 1$ in the equations derived in [1]. This limit is found to deliver accurate results for the evolution and asymptotic values of the neutrino flavor asymmetries, but does not allow for an accurate calculation of the effective number of relativistic degrees of freedom $N_{\rm eff}$, as this crucially depends on the energy trasfer rates that we are neglecting here. We refer the reader to [1] for further discussion.

The neutrino and anti-neutrino density matrices $\rho(T, k)$ and $\bar{\rho}(T, k)$ are in general functions of both temperature T and momentum k. The crucial approximation of [1] is that we assume that the momentum dependence may be factored

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out from the temperature dependence as a Fermi-Dirac (FD) distribution without chemical potential following the temperature of the thermal plasma, i.e. our ansatz for the full density matrices in the limit $z_\nu = z_\gamma = 1$ is

$$\rho(x,y) = r(x)\, f_{\rm fd}(y,0)\,, \qquad \bar\rho(x,y) = \bar r(x)\, f_{\rm fd}(y,0)\,, \tag{2}$$

where we introduced the dimensionless variables $x = T_{\rm ref}/T$ and $y = k/T$. $T_{\rm ref}$ is a reference temperature that may be fixed to any value, but a common convention is $T_{\rm ref} = m_e$. Given this ansatz, we can directly integrate the kinetic equations over momentum and solve for the temperature-dependent components $r(x)$ and $\bar r(x)$. In particular, the QKEs take the form

$$\begin{aligned} Hxr' &= -i\left[\langle\mathcal{H}_0\rangle + \langle V_c\rangle + \langle V_s\rangle\,(r-\bar r), r\right] + \langle\mathcal{I}\rangle\,, \\ Hx\bar r' &= +i\left[\langle\mathcal{H}_0\rangle + \langle V_c\rangle - \langle V_s\rangle\,(r-\bar r), \bar r\right] + \langle\bar{\mathcal{I}}\rangle\,, \end{aligned} \tag{3}$$

where we denote $' = \partial_x$. The Hubble parameter H is given by

$$H = \sqrt{g_*(x)}\frac{\pi}{\sqrt{90}M_P}\left(\frac{T_{\rm ref}}{x}\right)^2. \tag{4}$$

In the limit $z_\nu = z_\gamma = 1$, the effective number of degrees of freedom $g_*(x)$ under the approximation (2) is estimated as

$$g_*(x) = g_*^\gamma + g_*^{e^\pm} + g_*^\nu = 2 + \frac{7}{8}\cdot 4 + \frac{7}{8}\cdot {\rm Tr}[r+\bar r]\,. \tag{5}$$

In all cases, $\langle ...\rangle$ denotes momentum average and we work in the flavor basis. We refer the reader to Sec. II of [1] for the derivation of the equations and for a complete discussion and description of each term. In the following we write each contribution in the limit $z_\nu = z_\gamma = 1$. The vacuum Hamiltonian contribution is given by

$$\langle\mathcal{H}_0\rangle = \left(\frac{x}{T_{\rm ref}}\right)\frac{\pi^2}{36\zeta(3)}UM^2U^\dagger\,, \tag{6}$$

where $M^2 = {\rm diag}(0, \Delta m^2_{21}, \Delta m^2_{31})$ and U is the PMNS matrix, where we follow the PDG convention. The matter potentials contribute as

$$\langle V_c\rangle = -2\sqrt{2}\frac{7\pi^4}{180\zeta(3)}\frac{G_F}{M_W^2}\left(\frac{T_{\rm ref}}{x}\right)^5(\mathbb{E}+\mathbb{P})\,, \tag{7}$$

$$\langle V_s\rangle = \frac{3}{\sqrt{2}}G_F\frac{\zeta(3)}{\pi^2}\left(\frac{T_{\rm ref}}{x}\right)^3, \tag{8}$$

where $(\mathbb{E}+\mathbb{P})$ is a diagonal matrix in flavor space encoding the dimensionless energy and pressure densities of charged leptons and anti-leptons, and it is well approximated by

$$(\mathbb{E}+\mathbb{P})_{ee} \approx \frac{7\pi^2}{45} - \frac{1}{6}m_e^2\left(\frac{x}{T_{\rm ref}}\right)^2, \tag{9}$$

$$(\mathbb{E}+\mathbb{P})_{\mu\mu} \approx \frac{2}{\pi^2}m_\mu^3\left(\frac{x}{T_{\rm ref}}\right)^3 K_3\left(m_\mu\frac{x}{T_{\rm ref}}\right), \tag{10}$$

where K_i denotes the modified Bessel function of second kind. For the τ component we can safely assume $(\mathbb{E}+\mathbb{P})_\tau = 0$.

Collision terms

The effect of neutrino interactions with the plasma is encoded in the collision terms $\langle\mathcal{I}\rangle$ and $\langle\bar{\mathcal{I}}\rangle$. In the context of large primordial neutrino asymmetries, it has been recently discovered and emphasized in [2] that considering the exact collision terms is important. Given our ansatz for the density matrices (2), a first principles calculation of the three main contributions to the collision terms was performed in [1]: those arising from $\nu e \to \nu e$ scatterings,

$\nu\bar{\nu} \to e^+e^-$ annihilation, and those from neutrino self-interactions, $\nu\nu \to \nu\nu$ and $\nu\bar{\nu} \to \nu\bar{\nu}$. Assuming Fermi-Dirac statistics, in the fully relativistic limit[1] and imposing $z_\nu = z_\gamma = 1$, we can write them in the following compact form that is implemented in the code,

$$\langle\mathcal{I}\rangle^{\nu e\leftrightarrow\nu e} = 4F_{\rm FD}\frac{8G_F^2T^5}{\pi^3}(1-\epsilon)\left\{[G^L r G^L(\mathbb{1}-\epsilon\, r)+(\mathbb{1}-\epsilon\, r)G^L r G^L]-[rG^L(\mathbb{1}-\epsilon\, r)G^L+G^L(\mathbb{1}-\epsilon\, r)G^L r]\right\} \tag{11}$$

$$\begin{aligned}\langle\mathcal{I}\rangle^{\nu\bar{\nu}\leftrightarrow e^+e^-} &= F_{\rm FD}\frac{8G_F^2T^5}{\pi^3}\left\{[G^L(\mathbb{1}-\epsilon\,\bar{r})G^L(\mathbb{1}-\epsilon\, r)+(\mathbb{1}-\epsilon\, r)G^L(\mathbb{1}-\epsilon\,\bar{r})G^L]-(1-\epsilon)^2[rG^L\bar{r}G^L+G^L\bar{r}G^L r]\right\}\\ &+ F_{\rm FD}\frac{8G_F^2T^5}{\pi^3}\left\{[G^R(\mathbb{1}-\epsilon\,\bar{r})G^R(\mathbb{1}-\epsilon\, r)+(\mathbb{1}-\epsilon\, r)G^R(\mathbb{1}-\epsilon\,\bar{r})G^R]-(1-\epsilon)^2[rG^R\bar{r}G^R+G^R\bar{r}G^R r]\right\}\end{aligned} \tag{12}$$

$$\langle\mathcal{I}\rangle^{\nu\bar{\nu}\leftrightarrow\nu\bar{\nu}} = \frac{1}{4}F_{\rm FD}\frac{8G_F^2T^5}{\pi^3}\left\{2{\rm Tr}[r\bar{r}][\mathbb{1}-\epsilon\,(r+\bar{r})]-(r\bar{r}+\bar{r}r)({\rm Tr}[\mathbb{1}]-\epsilon\,{\rm Tr}[r+\bar{r}])\right\}\,, \tag{13}$$

while the collision term for $\nu\nu \leftrightarrow \nu\nu$ scatterings vanishes within our momentum average ansatz (2). Here $\mathbb{1} = {\rm diag}(1,1,1)$, $G^L = {\rm diag}(g_L, \tilde{g}_L, \tilde{g}_L)$, $G^R = {\rm diag}(g_R, g_R, g_R)$ with $g_L = \sin^2\theta_W + 1/2$, $\tilde{g}_L = \sin^2\theta_W - 1/2$, and $g_R = \sin^2\theta_W$. In the fully relativistic limit where $m_e \to 0$, the collision terms for anti-neutrinos are written simply by doing the replacement $r \leftrightarrow \bar{r}$.

The collision terms have been written in terms of expansion parameters that take into account that spin statistics represent only a small correction to the interaction rates. We numerically find $F_{\rm FD} \simeq 0.995$ and $\epsilon \simeq 0.049$ for Fermi-Dirac statistics, while for $\epsilon \to 0$ and $F_{\rm FD} \to 1$ one obtains the collision terms found using Maxwell-Boltzmann statistics for the momentum dependence of the density matrix.

As shown before, the collision terms are non-linear and they cannot be expressed as simple temperature dependent damping coefficients multiplying the respective density matrix elements, $\langle\mathcal{I}\rangle_{\alpha\beta} \simeq -d_{\alpha\beta}r_{\alpha\beta}$. However, collisions terms of the damping form are significantly simpler to implement when solving the QKEs and it is thus useful to derive approximate results of this form. In this context, we provide in [1] a damping approximation taking into account all the collision terms based on linearizing the full results above. To this end we consider the density matrices r and $\bar{r}$ to have 1's in the diagonal and then include the leading order off-diagonal components from either r or $\bar{r}$. Considering only the $e\mu$ and $e\tau$ components we can write:

$$\langle\mathcal{I}\rangle^{\rm damping\ approx} = -D\begin{pmatrix}0 & r_{12} & r_{13}\\ r_{12}^* & 0 & 0\\ r_{13}^* & 0 & 0\end{pmatrix}\,, \qquad \text{with} \qquad D = F_{\rm FD}\frac{16}{\pi^3}G_F^2T_\gamma^5(1-\epsilon)(3+2\sin^2\theta_W)\,, \tag{14}$$

and similarly for anti-neutrinos by replacing $r \leftrightarrow \bar{r}$ everywhere. This results in a damping rate which is a factor 1.55 larger than the one from $\nu e \to \nu e$ scatterings only.

SU(3) decomposition

We find that it is numerically convenient to expand the QKEs in terms of $SU(3)$ adjoint basis matrices λ_i, the Gell-Mann matrices. For the neutrino density matrix, this expansion is written as

$$r = r_0\mathbb{1} + r_i\lambda_i = r_0\mathbb{1} + \vec{r}\cdot\vec{\lambda}\,, = \begin{pmatrix} r_0 + r_3 + \frac{r_8}{\sqrt{3}} & r_1 - ir_2 & r_4 - ir_5\\ r_1 + ir_2 & r_0 - r_3 + \frac{r_8}{\sqrt{3}} & r_6 - ir_7\\ r_4 + ir_5 & r_6 + ir_7 & r_0 - \frac{2r_8}{\sqrt{3}}\end{pmatrix}\,, \tag{15}$$

where r_0 and r_i are real coefficients, and similarly for anti-neutrinos by replacing $r \leftrightarrow \bar{r}$ everywhere. Note that the vectorial notation corresponds to eight-dimensional vectors. In this manner, we may write the QKEs in Eq. (3) as differential equations over the real coefficients,

$$\begin{aligned} r^{0\prime} &= c^0\,, \quad r^{i\prime} = 2f^{ijk}r^k\left(h_0^j + v_c^j - v_s\bar{r}^j\right) + c^i\\ \bar{r}^{0\prime} &= \bar{c}^0\,, \quad \bar{r}^{i\prime} = -2f^{ijk}\bar{r}^k\left(h_0^j + v_c^j - v_s r^j\right) + \bar{c}^i\,.\end{aligned} \tag{16}$$

[1] This is a good approximation for the majority of the parameter space, but limitations are discussed in [1].

where we have expanded the various contributions to the QKEs in Eq. (3) in the basis of Gell-Mann matrices and written them in units of Hx, i.e. $\langle \mathcal{H}_0 \rangle / (Hx) \to h_0^a$, $\langle V_c \rangle / (Hx) \to v_c^a$, $\langle \mathcal{I} \rangle / (Hx) \to c^a$ and $\langle \bar{\mathcal{I}} \rangle / (Hx) \to \bar{c}^a$, while $v_s = \langle V_s \rangle / (Hx)$ with $\langle V_s \rangle$ given by Eq. (8). These vectors are obtained in the code by applying the function `gellMannComponents[X]` to the various contributions, following the same convention as in Eq. (15). Note that terms proportional to the identity matrix cancel in the commutator. The reader can also find the explicit expressions for h_0^a and v_c^a in [1].

We implement the compact form (16) of the QKEs in the code. We store separate forms of the equations depending on whether the full collision term or the damping approximation is considered. We also allow the user to choose to evolve the QKEs in the adiabatic limit, corresponding to $v_s = 0$. For large amplitudes of initial asymmetries this is an excellent approximation, except for some specific flavor directions. This is discussed in detail in [1].

III. INITIAL CONDITIONS

All off-diagonal elements of the neutrino density matrices are set to zero initially, $r_{\alpha\beta} = \bar{r}_{\alpha\beta} = 0$, with $\alpha \neq \beta$. Motivated by averaging over a Fermi-Dirac distribution with chemical potential and comparing to our momentum averaged ansatz, we choose as initial conditions for the diagonal components

$$r_{\alpha\alpha}^{\rm ini} = r_{\alpha\alpha}^{\rm fd} + \epsilon_{\rm db}\,, \quad \bar{r}_{\alpha\alpha}^{\rm ini} = \bar{r}_{\alpha\alpha}^{\rm fd} + \epsilon_{\rm db}\,, \tag{17}$$

where

$$r_{\alpha\alpha}^{\rm fd} = -\frac{4}{3\zeta(3)}{\rm Li}_3\left(-{\rm e}^{\xi_\alpha}\right)\,, \quad \bar{r}_{\alpha\alpha}^{\rm fd} = -\frac{4}{3\zeta(3)}{\rm Li}_3\left(-{\rm e}^{-\xi_\alpha}\right)\,, \tag{18}$$

with ${\rm Li}_s(x)$ being the polylogarithm and $\xi \equiv \mu/T$ and μ referring to the chemical potential. $\epsilon_{\rm db}$ is a numerically small correction term in order to ensure that detailed balance is fulfilled within our momentum averaged approximation[2] and is given as

$$\epsilon_{{\rm db},\alpha} = \frac{-\sqrt{4\epsilon^2\left((\bar{r}_{\alpha\alpha}^{\rm fd} - r_{\alpha\alpha}^{\rm fd})^2 + 1\right) - 4\epsilon\left((\bar{r}_{\alpha\alpha}^{\rm fd} - r_{\alpha\alpha}^{\rm fd})^2 + 2\right) + (\bar{r}_{\alpha\alpha}^{\rm fd} - r_{\alpha\alpha}^{\rm fd})^2 + 4} - 2\epsilon(\bar{r}_{\alpha\alpha}^{\rm fd} + r_{\alpha\alpha}^{\rm fd} - 1) + \bar{r}_{\alpha\alpha}^{\rm fd} + r_{\alpha\alpha}^{\rm fd}}{4\epsilon - 2}\,. \tag{19}$$

The functions to solve the QKEs receive (ξ_e,ξ_μ,ξ_τ), but we include functions to transform from $\xi_\alpha + \xi_\alpha^3/\pi^2$ and from the spherical coordinates parametrization $(A, \theta, \phi,)$ defined as

$$A = \sqrt{\sum_\alpha (\xi_\alpha + \xi_\alpha^3/\pi^2)^2}\,, \quad \xi_e + \xi_e^3/\pi^2 = A\sin\theta\cos\phi\,, \quad \xi_\mu + \xi_\mu^3/\pi^2 = A\sin\theta\sin\phi\,, \quad \xi_\tau + \xi_\tau^3/\pi^2 = A\cos_\theta\,, \tag{20}$$

where in the case of total lepton number conservation $\theta = \arctan(-\cos\phi - \sin\phi, 1)$, where $\arctan(x, y)$ represents the principal value of the arc tangent of y/x.

IV. FUNCTIONS TO SOLVE THE SYSTEM

The code includes two functions to solve the system of QKEs, generate plots showing the evolution of neutrino asymmetries and export the results to a .dat file:

- `solveFD[`$\xi_{e\_}$`, `$\xi_{\mu\_}$`, `$\xi_{\tau\_}$`, Filename_:"output", Tini_:20 MeV, Tave_:1.5 MeV, Tfinal_:1 MeV, Accval_:8, Precval_:8, SolverMethod_:"BDF", Adiabatic_:False, TimeoutAdiabatic_:300]`.
- `solveDamping[`$\xi_{e\_}$`, `$\xi_{\mu\_}$`, `$\xi_{\tau\_}$`, Filename_:"output", Tini_:20 MeV, Tave_:1.5 MeV, Tfinal_:1 MeV, Accval_:8, Precval_:8, SolverMethod_:"BDF", Adiabatic_:False, TimeoutAdiabatic_:300]`.

The only difference between the two functions above is that `solveFD` considers the collision terms of Eqs. (11), (12) and (13), while `solveDamping` considers the damping approximation terms of Eq. (14). Both functions receive the same arguments:

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[2] Detailed balance in the momentum average approximation is discussed in Appendix B.5.b of [1].

- ξ_e: Numerical value for the initial reduced chemical potential for the electron flavor.
- ξ_μ: Numerical value for the initial reduced chemical potential for the muon flavor.
- ξ_τ: Numerical value for the initial reduced chemical potential for the tau flavor.
- `Filename`: String containing the name of the .dat file where the results will be stored.
- `Tini`: Temperature in MeV at which the reduced chemical potentials receive the values introduced above.
- `Tave`: Temperature in MeV at which the solver finishes the evolution and the oscillations are averaged.
- `Tfinal`: The results at `Tave` are extended asymptotically until this final temperature, only relevant for plotting and for the output .dat file.
- `Accval`: Accuracy setting for `NDSolve`.
- `Precval`: Precision setting for `NDSolve`.
- `SolverMethod`: String containing the solving method for `NDSolve`.
- `Adiabatic`: Boolean variable which is `True` in order to evolve the system in the adiabatic approximation from the beginning, and `False` in order to evolve the full system.
- `TimeoutAdiabatic`: Time in seconds after which the solver switches from the full system to the adiabatic approximation. Only applies when `Adiabatic=True`.

Note also that several of the arguments are optional and will receive default values if not specified, with the only mandatory arguments being (ξ_e,ξ_μ,ξ_τ). A consistent input for temperatures should preserve `Tini` $<$ `Tave` $\leq$ `Tfinal`. The functions first check that this condition is satisfied, otherwise the default values are used. In any case, `Tini` should be significantly larger than the temperature at which neutrino oscillations in the thermal plasma become efficient in order to fully capture the dynamics of the system and be consistent with our initial conditions $r_{\alpha\beta} = \bar{r}_{\alpha\beta} = 0, (\alpha \neq \beta)$. `Tave` should be sufficiently small so that all MSW transitions have terminated and collisions are inefficient to keep neutrinos in equilibrium with the thermal plasma. Therefore, ideally one should consider `Tini` $\geq 15\,\mathrm{MeV}$ and `Tave` $\leq 2\,\mathrm{MeV}$.

The functions use `NDSolve` to evolve the system from `Tini` to `Tave`, and during the evolution the current value of temperature and a progress bar are displayed via `Monitor`. The accuracy and precision settings of `NDSolve` are controlled by `Accval` and `Precval`. Typically `Accval` = `Precval` = 8 gives good results. One may choose smaller values but we found that the physical evolution of the system may be affected already for values smaller than 6. `SolverMethod` dictates the solving method used by `NDSolve`. Typically both `"BDF"` and `"StiffnessSwitching"` work well, `"BDF"` is usually faster but `"StiffnessSwitching"` may be able in some cases to reach smaller values of `Tave` without numerical issues.

The functions will evolve the full system or consider the adiabatic approximation depending on the user input for the boolean variable `Adiabatic`. When `NDSolve` is evolving the full system, `TimeoutAdiabatic` contains a timeout in seconds after which `NDSolve` switches to the adiabatic approximation via the use of `WhenEvent`, which is able to set $v_s = 0$ mid-evolution. This is extremely useful in some cases where it is numerically very expensive to evolve the full system down to low values of `Tave`.

At `Tave`, the dynamics are dominated by fast oscillations arising from the vacuum flavor transitions. However, any observable which is integrated over some time larger than the fast oscillation frequency will only be sensitive to the average spectrum. Therefore, at `Tave` we estimate the average of these fast oscillations to extract the asymptotic value of the difference in neutrino and anti-neutrino density matrices as

$$\Delta r_\alpha = r_{\alpha\alpha} - \bar{r}_{\alpha\alpha} = \left(U \langle U^\dagger \Delta r_\alpha^{T_{\rm ave}} U \rangle_{\rm ave} U^\dagger\right)_{\alpha\alpha} , \tag{21}$$

where $\Delta r_\alpha^{T_{\rm ave}}$ is evaluated at $T =$ `Tave` obtained from `NDSolve` and $\langle .. \rangle_{\rm ave}$ means that we only keep the diagonal elements. This asymptotic value remains constant until `Tfinal`.

Finally, we generate a plot showing the neutrino flavor asymmetry obtained as (in the limit $z_\nu = z_\gamma = 1$)

$$\Delta n_\alpha = \left(\frac{n - \bar{n}}{T^3}\right)_{\alpha\alpha} = \frac{3}{4}\frac{\zeta(3)}{\pi^2}(r_{\alpha\alpha} - \bar{r}_{\alpha\alpha}) = \frac{1}{6}\left(\xi_\alpha + \frac{\xi_\alpha^3}{\pi^2}\right) \tag{22}$$

from `Tini` down to `Tfinal`, where the final equality assumes a Fermi-Dirac distribution for neutrinos. The user can change the overall normalization of asymmetries at any point in the notebook by manually modifying the global

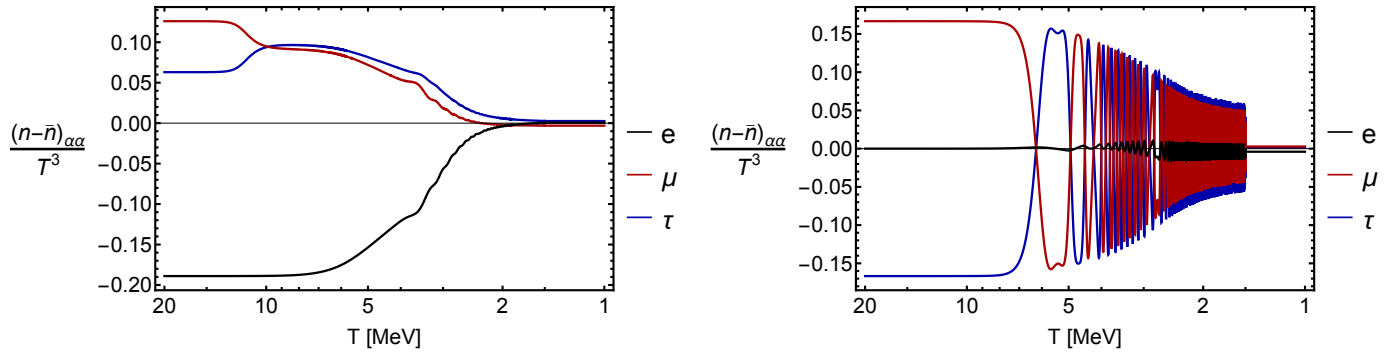

FIG. 1: Examples of time evolution plots generated by the function `solveFD` in the code. In the left (right) panel we specify $A = \sqrt{2}, \phi = \arctan(-3, 2) \simeq 2.55$ ($A = \sqrt{2}, \phi = \pi/2$).

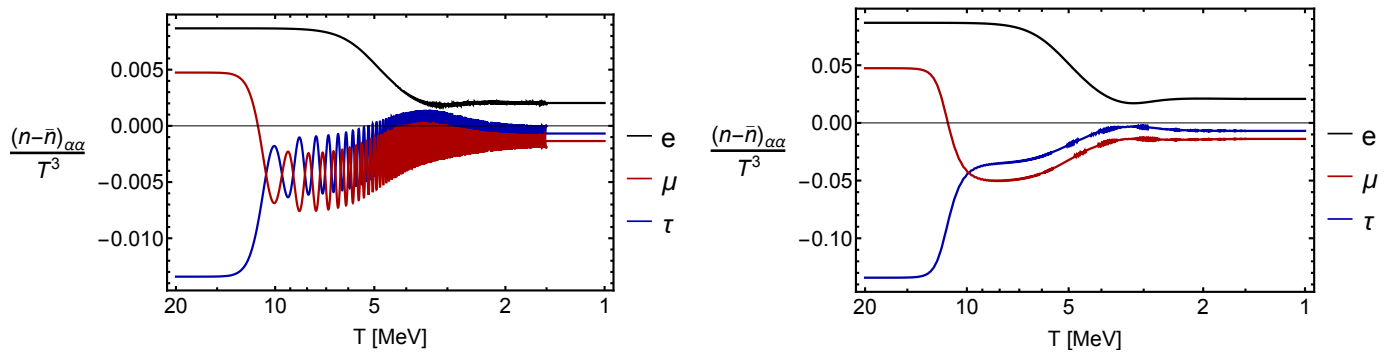

FIG. 2: Examples of time evolution plots generated by the function `solveDamping` in the code. We specify $\phi = 0.5$ and $A = 0.1\,(1)$ for the left (right) panel.

variable `FacΔn`. The normalisation used in [1] is `FacΔn` $= 1$, but e.g. some figures in [2] use `FacΔn` $= 100 \times 6$. We also export these results to a .dat file with the name given by `Filename`. This file is stored in a folder called "output" within the same folder as the `COFLASY-M` code files. The output file contains 1000 values of logarithmically spaced temperatures in MeV, along with their corresponding values of neutrino flavor asymmetries, i.e. the output file contains three columns with the following format: $(T\,[\text{MeV}], \Delta n_e, \Delta n_\mu, \Delta n_\tau)$. We also `Print` the final value of ξ_e which is obtained from inverting Eq. (22). The evolution of r_i and $\bar{r}_i$ is stored in the global variable `sol` that can also be accessed by the user after the output is generated. Similarly, the asymptotic flavor asymmetries are stored in the global variables Δnefinal, Δnμfinal and Δnτfinal, while the final ξ_α are stored in `ξαfinal`.

V. OUTPUT AND EXAMPLES

In order to use the code, you need to evaluate the "Basic module" section of the `COFLASY-M.nb` notebook. Then the "Examples" section of the `COFLASY-M.nb` notebook serves as a tutorial for the user to get started with the code. It contains the benchmarks of Figures 3, 5 and 6 (left) from [1] as examples with FD collisions, along with the benchmarks of Figure 12 from [1] as examples with damping collision terms. The benchmark of Figure 5 (right) is also shown with damping collision terms to illustrate differences with respect to the FD collision case.

In the following we show a minimal example Mathematica input code which calls the function `solveFD` to solve the system of QKEs:

```
(*We provide initial conditions over (A,\[Phi])*)
Aval = Sqrt[2];
\[Phi]val = ArcTan[-3, 2];
(*We translate from (A,\[Phi]) to (\[Xi]e,\[Xi]\[Mu],\[Xi]\[Tau]), \
alternatively the user may provide conditions over \
(\[Xi]e,\[Xi]\[Mu],\[Xi]\[Tau]) directly*)
```

```
\[Xi]eini = \[Xi]tildeto\[Xi][\[Xi]tile[Aval, \[Phi]val]];
\[Xi]\[Mu]ini = \[Xi]tildeto\[Xi][\[Xi]til\[Mu][Aval, \[Phi]val]];
\[Xi]\[Tau]ini = \[Xi]tildeto\[Xi][\[Xi]til\[Tau][Aval, \[Phi]val]];

(*This is the minimum input required to run the solving functions, \
with all the optional arguments using default values*)
(*We run the system with FD collisions*)
Fig3bFD = solveFD[\[Xi]eini, \[Xi]\[Mu]ini, \[Xi]\[Tau]ini]
```

In the minimal example above, the only input are the initial reduced chemical (ξ_e,ξ_μ,ξ_τ), with all the arguments of the `solveFD` function taking default values. In particular, the example above solves the full system for $A = \sqrt{2}$ and $\phi = \pi/2$ with vanishing total lepton number. A solution with the damping approximation can be obtained by replacing `solveFD` → `solveDamping`. In contrast, the user may provide specific values for all the optional arguments as in the following example:

```
(*We provide initial conditions for (A,\[Phi])*)
Aval = Sqrt[2];
\[Phi]val = \[Pi]/2;

(*We translate from (A,\[Phi]) to (\[Xi]e,\[Xi]\[Mu],\[Xi]\[Tau])*)

\[Xi]eini = \[Xi]tildeto\[Xi][\[Xi]tile[Aval, \[Phi]val]];
\[Xi]\[Mu]ini = \[Xi]tildeto\[Xi][\[Xi]til\[Mu][Aval, \[Phi]val]];
\[Xi]\[Tau]ini = \[Xi]tildeto\[Xi][\[Xi]til\[Tau][Aval, \[Phi]val]];

(*We provide initial temperature (Tini), temperature at which oscillation
average is taken (Tave), and final temperature (Tfinal)*)
(*Note we must provide Tini < Tave < Tfinal*)

Tini = 20 MeV;
Tave = 1.5 MeV;
Tfinal = 1 MeV;

(*We provide accuracy and precision settings for NDSolve*)

Accval = 8;
Precval = 8;

(*Adiabatic=False runs the full system, while Adiabatic=True runs the system
in the adiabatic approximationw where Vs=0*)

Adiabatic=False;

(*Execution time in seconds after which the solver switches to the adiabatic
approximation (only when Adiabatic=False)*)

TimeoutAdiabatic = 300;

(*We call the function that solves the system, export results and generates a
plot*)

Fig5bFD =
 solveFD["Fig5b", \[Xi]eini, \[Xi]\[Mu]ini, \[Xi]\[Tau]ini, Tini,
  Tave, Tfinal, Accval, Precval, "BDF", False, TimeoutAdiabatic]
```

The solving functions display messages indicating that the evolution has started, and during the evolution the current temperature for every step is displayed along with a progress bar. When the evolution finishes, the final value of ξ_e is displayed, and the evolution of neutrino flavor asymmetries Δn_α is saved to an output .dat file. This file is stored in "output/`Filename`.dat" within the same folder where the Mathematica code is stored. Simultaneously, an evolution plot is generated in the notebook. Two illustrative examples are shown in Fig. 1, which correspond

to the benchmarks of Figures 3 (right) and 5 (right) in [1]. Similarly, two illustrative examples using the damping approximation are shown in Fig. 2, which correspond to the benchmarks of Figures 12 (left and right) in [1].

[1] V. Domcke, M. Escudero, M. Fernández Navarro, and S. Sandner, Lepton Flavor Asymmetries: from the very early Universe to BBN, arXiv:2501.XXXXX.

[2] J. Froustey and C. Pitrou, Primordial neutrino asymmetry evolution with full mean-field effects and collisions, JCAP **03** (03), 065, arXiv:2110.11889 [hep-ph].