PROBLEMA DE BODE RESUELTO:

Al analizar un circuito, se ha obtenido la función de transferencia

$$T(w) = \frac{2\cdot 10^4 + jw\,10^3}{1 + jw\,10^{-4}}$$

(a) Pinte el diagrama de Bode del módulo de la función de transferencia.

(b) Pinte el diagrama de Bode del argumento de la función de transferencia

SOLUCIÓN

Esta función de transferencia puede verse como el producto de tres funciones de transferencia:

$$T(\omega) = T_1(\omega) \cdot T_2(\omega) \cdot T_3(\omega) = \underbrace{2 \cdot 10^4}_{T_1(\omega)} \cdot \underbrace{\left(1 + j\frac{\omega}{2 \cdot 10}\right)}_{T_2(\omega)} \cdot \underbrace{\frac{1}{1 + j\omega 10^{-4}}}_{T_3(\omega)}$$

siendo:

$$T_1(\omega) = 2 \cdot 10^4$$

$$T_2(\omega) = 1 + j\frac{\omega}{\omega_{o2}} \quad \text{con} \quad \omega_{o2} = 20$$

$$T_3(\omega) = \frac{1}{1 + j\frac{\omega}{\omega_{o3}}} \quad \text{con} \quad \omega_{o3} = 10^4$$

con lo que cada una de ellas resulta:

- $T_1(\omega)$:

$$T_1(\omega) = 2 \cdot 10^4 \Rightarrow T_1(\omega) = |T_1(\omega)|\, e^{j \arg T_1(\omega)} =$$

$$= 2 \cdot 10^4 e^{j \cdot 0}$$

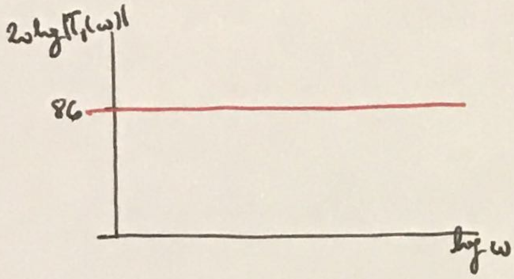
* DB Módulo:

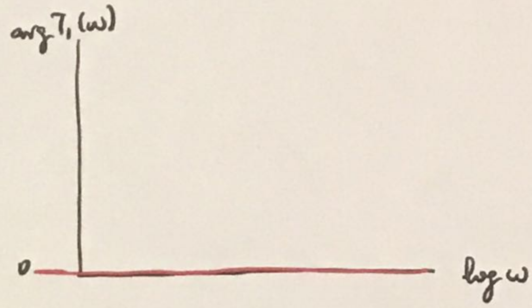
$$20 \log |T_1(\omega)| = 20 \cdot \log 2 \cdot 10^4 =$$

$$= 20 \log 2 + 80 = 86$$

* DB Argumento

$$\arg T_1(\omega) = 0$$





- $T_2(\omega)$:

$$T_2(\omega) = 1 + j\frac{w}{w_{02}} \Rightarrow T_2(\omega) = |T_2(\omega)|\, e^{j\arg T_2(\omega)}$$

con $w_{02} = 20$

$$|T_2(\omega)| = \sqrt{1 + \frac{w^2}{w_{02}^2}}$$

$$\arg T_2(\omega) = \arctan \frac{w}{w_{02}}$$

* DB Módulo:

$$20\log\sqrt{1+\frac{w^2}{w_{02}^2}}$$

$$w \gg w_0 \Rightarrow 20\log\frac{w}{w_{02}} =$$

$$= 20\log w - 20\log w_{02}$$

$$w \ll w_0 \Rightarrow 20\log 1 = 0$$

$$w = w_0 \Rightarrow 20\log\sqrt{2} = 20\log 2^{\frac{1}{2}} =$$

$$= 10\log 2 = 3$$

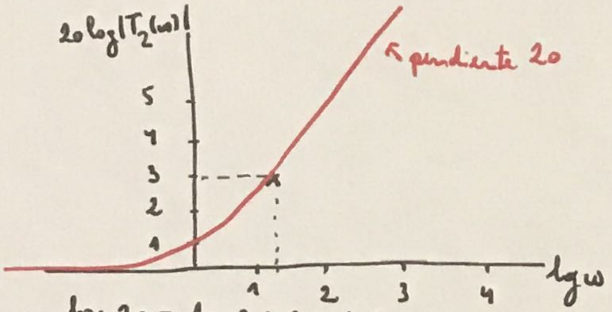
* DB Argumento

$$\arctan\frac{w}{w_{02}}$$

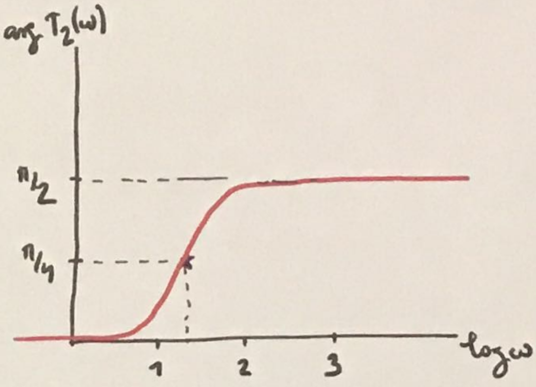
$$w \gg w_0 \rightarrow \arg T_2(\omega) = \frac{\pi}{2}$$

$$w \ll w_0 \rightarrow \arg T_2(\omega) = 0$$

$$w = w_0 \rightarrow \arg T_2(\omega) = \frac{\pi}{4}$$



$\log 20 = \log 2 + 1 = 1{,}3$



- **$T_3(\omega)$:** $T_3(\omega) = \dfrac{1}{1 + j\dfrac{\omega}{\omega_{03}}}$

con $\omega_{03} = 10^4$

$T_3(\omega) = |T_3(\omega)|\, e^{j \arg T_3(\omega)}$

$|T_3(\omega)| = \dfrac{1}{\sqrt{1 + \dfrac{\omega^2}{\omega_{03}^2}}}$

$\arg T_3(\omega) = e^{-j \operatorname{arctg} \frac{\omega}{\omega_{03}}}$

* DB Módulo:

$20\log|T_3(\omega)| = 20\log \dfrac{1}{\sqrt{1 + \dfrac{\omega^2}{\omega_{03}^2}}}$

$\omega \gg \omega_0 \Rightarrow 20\log \dfrac{\omega_{03}}{\omega} =$

$= 20\log \omega_{03} - 20\log \omega$

$\omega \ll \omega_0 \Rightarrow 20\log 1 = 0$

$\omega = \omega_{03} \Rightarrow 20\log 2^{-1/2} = -10\log 2 =$

$= -3$

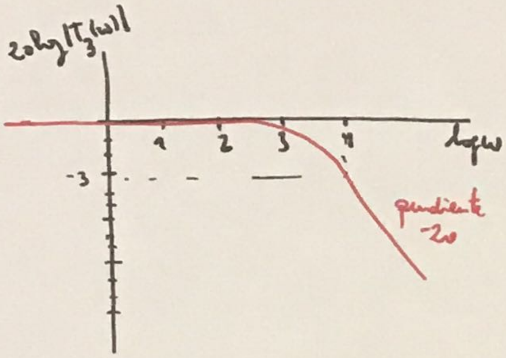
* DB Argumento

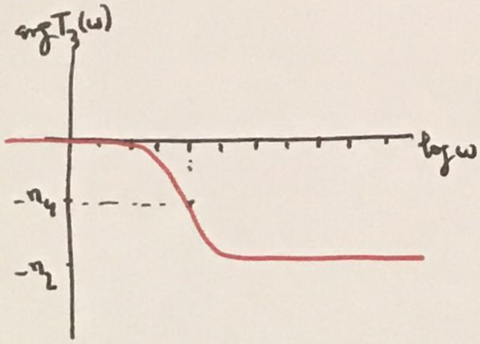
$-\operatorname{arctg} \dfrac{\omega}{\omega_{03}}$

$\omega \gg \omega_{03} \Rightarrow \arg T_3(\omega) = -\dfrac{\pi}{2}$

$\omega \ll \omega_{03} \Rightarrow \arg T_3(\omega) = 0$

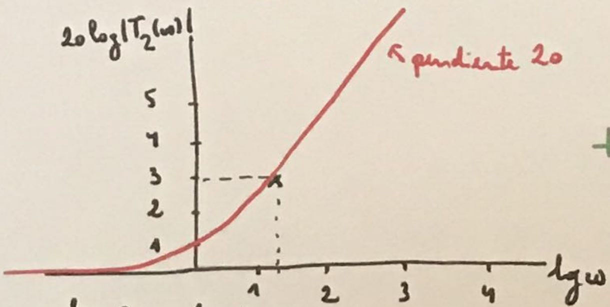
$\omega = \omega_{03} \Rightarrow \arg T_3(\omega) = -\dfrac{\pi}{4}$





De manera que el diagrama de Bode total en módulo será sumar los diagramas de Bode en módulo. Sumamos 2 y 3.

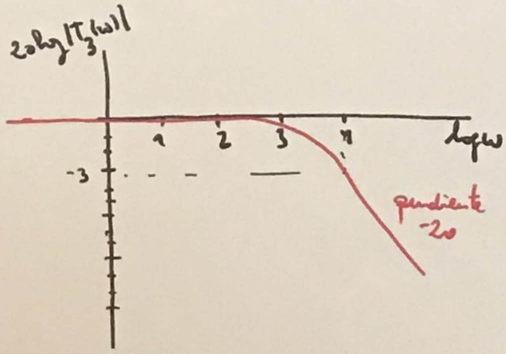
2



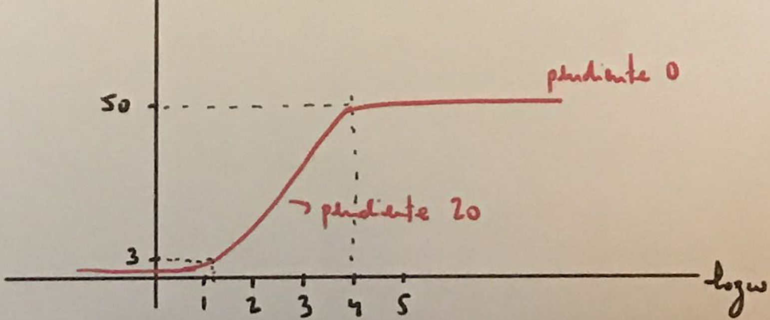
$\log 20 = \log 2 + 1 = 1{,}3$

+

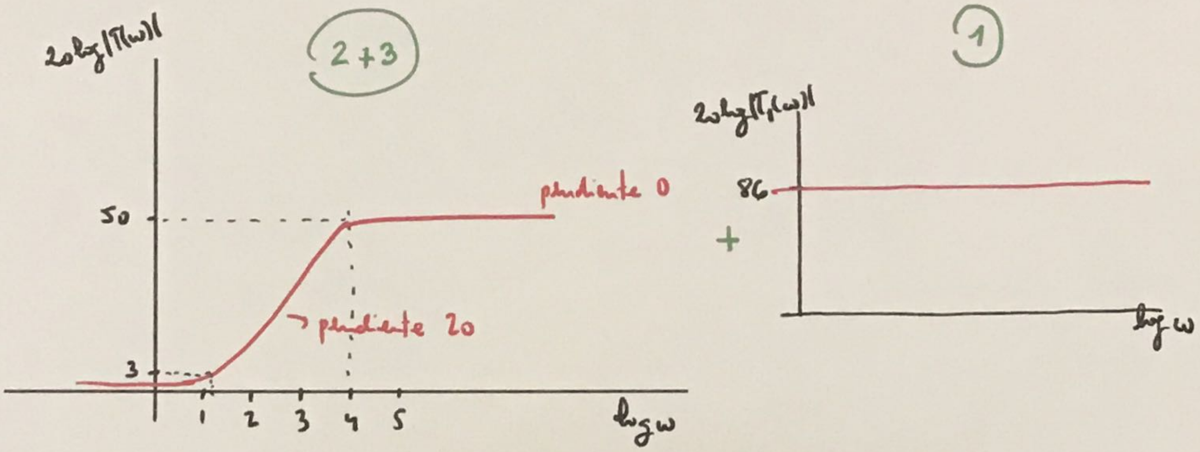
3

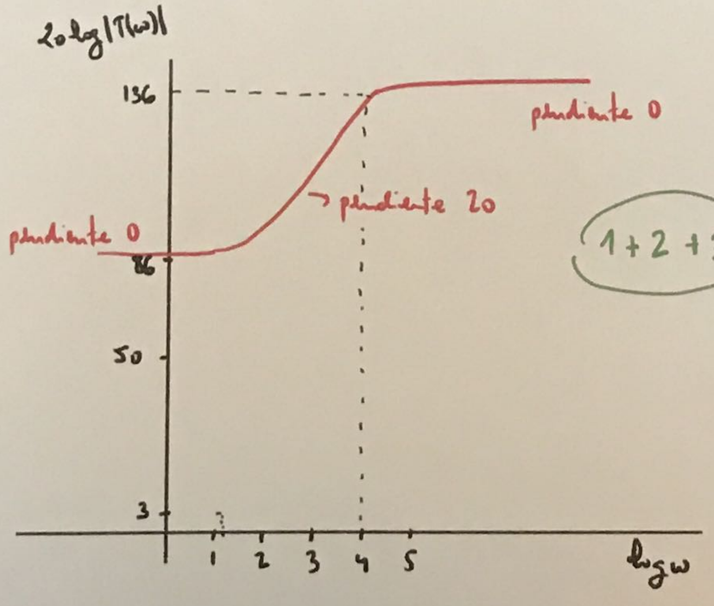


2+3

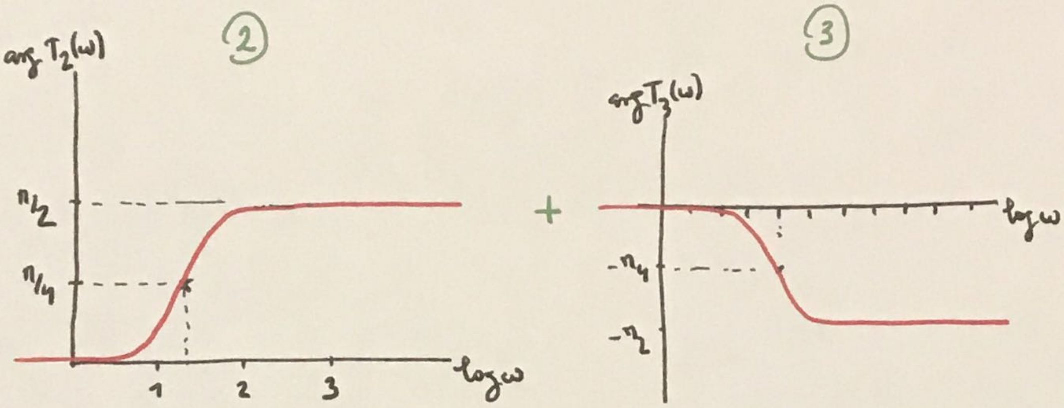


Y el resultado lo sumamos con 1

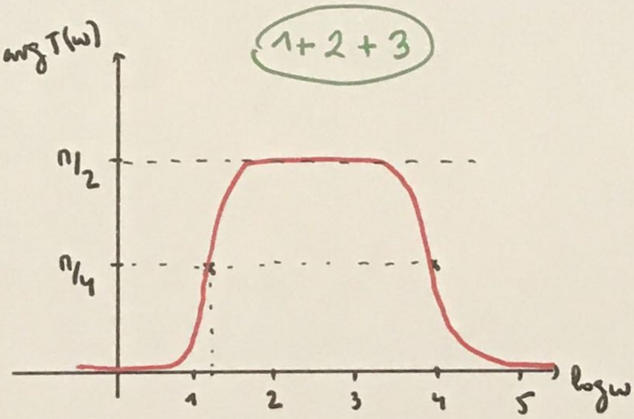




Y el diagrama de Bode total en argumento será sumar las gráficas de los diagramas de Bode en argumento. Como el de $T_1(w)$ es nulo, en realidad, basta con sumar el de $T_2(w)$ y el de $T_3(w)$



2
arg $T_2(\omega)$
$\pi/2$
$\pi/4$
1
2
3
$\log \omega$
+
3
arg $T_3(\omega)$
$-\pi/4$
$-\pi/2$
$\log \omega$



1+2+3
arg $T(\omega)$
$\pi/2$
$\pi/4$
1
2
3
4
5
$\log \omega$