# A Framework for the Automated Verification of Algebraic Effects and Handlers (extended version)*

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**Abstract.** Algebraic effects and handlers are a powerful abstraction to build non-local control-flow mechanisms such as resumable exceptions, lightweight threads, co-routines, generators, and asynchronous I/O. All of such features have very evolved semantics, hence they pose very interesting challenges to deductive verification techniques. In fact, there are very few proposed techniques to deductively verify programs featuring these constructs, even fewer when it comes to automated proofs. In this paper, we present an extension to Cameleer, a deductive verification tool for OCaml code, that allows one to reason about algebraic effects and handlers. The proposed embeds the behavior of effects and handlers using exceptions and employs defunctionalization to deal with continuations exposed by effect handlers.

**Keywords:** OCaml· Algebraic Effects · Automated Proofs · Deductive Verification · Cameleer· GOSPEL

## 1 Introduction

The newest release of the OCaml compiler saw the introduction of a variety of new features that make parallel and concurrent code more efficient as well as easier to write. These include, amongst other, algebraic effects and handlers [22]. In short, algebraic effects are similar to exceptions: when an effect is performed, execution is halted and control is surrendered to the nearest enclosing handler. The main difference between exception and effect handlers is that the latter exposes a (delimited) continuation function that allows us to go back to the point in which the effect was performed and resume execution.

This addition to an industrial strength language means there is a large incentive for verification tools to add support for algebraic effects and handlers. Although some research has been done [5, 16] that presents reasoning rules for this kind of program, none of them explore automated proofs. Therefore, the research question we pose to ourselves is *what tools and methodologies must*

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*be developed in order to apply automated deductive proofs to OCaml programs, featuring effects and handlers?*

To answer this question, we present a framework that can automatically verify programs that employ algebraic effects. Such a framework is implemented as an extension to Cameleer [15], a deductive verification tool for OCaml code whose specification is written in GOSPEL [3] (Generic OCaml SPEcification Language) and then translated to WhyML, the specification and programming language of the Why3 verification framework [2]. Cameleer is also the only automated deductive verification platform (we are aware of) that targets OCaml programs, making it an ideal choice for this project.

To achieve this goal we must be able to represent the continuation function exposed by the handler in our proofs, which we will do using a translation of algebraic effects into Why3 that employs defunctionalization [21], a program transformation technique that turns higher-order programs into first-order ones. To the best of our knowledge, the use of defunctionalization in a proof scenario is novel.

The contributions of this work can be summarized as follows:

1. we extend GOSPEL to accommodate the needed logical elements to specify the behavior of algebraic effects and handlers;
2. we develop an embedding of OCaml handlers and effects in WhyML, employing defunctionalization;
3. we extend Cameleer to automatically translate GOSPEL annotated OCaml programs into this embedding, allowing us to conduct deductive verification inside the Why3 framework; and
4. we build a set of case studies verified using our proposed methodology.

*Notation.* OCaml and GOSPEL share many syntactic traits with WhyML. Since we show examples using these three languages throughout the paper, we put a label on the right-hand side of every listing to indicate the language in which such piece of code is written.

## 2 Algebraic Effects in a Nutshell

Before presenting how we plan to specify and prove algebraic effects, let us present a program that simulates a mutable reference, without actually using mutable state. For such purpose, we introduce the following two distinct algebraic effects:

```
effect Get : int                                    OCaml
effect Set : int -> unit
```

The role of `Get` and `Set` effects is to implement, respectively, the usual dereferencing and assignment operations.

One can read the above definitions as follows: when the `Get` effect is performed, the nearest enclosing handler calls the corresponding continuation by

passing as argument an `int` value, which stands for the current value of the reference. Conversely, the `Set` effect sends an `int`, the new value of the reference, to the handler. The handler will simply return `unit`. This means any future `Get` effect shall retrieve this same value.

Using these effects, we define the following function that increments the value of the reference by one:

```
let inc () : unit =                                    OCaml
  let current = perform Get in
  perform (Set (1 + current))
```

To call this function, we must define a handler that enforces the intended behavior. Abiding to our rule of not using mutable state, we create the following anonymous functions to represent the program state:

```
let create_env () : int -> int = match inc () with    OCaml
  | r ->                 fun _ -> r
  | effect Get k ->      fun x -> (continue k x) x
  | effect (Set n) k -> fun _ -> (continue k ()) n
```

Here, `create_env` returns another function which receives the initial value of the reference and handles each effect accordingly. In case no effect is performed, the handler falls into the first branch and it builds a function that always returns the result `r`.

When the `Get` effect is handled we return a function that calls the continuation Why3k with the argument `x`, that is, the current value of the reference. Important to note that when we call the continuation with the `continue` function, the handler remains installed, meaning it handles any remaining effects. Such kind of handlers is know as *deep handlers*, whereas *shallow handlers* can only handle at most one effect [7]. This also means that calling the continuation returns another function, seeing as it is the return type of the handler. After the call to `continue`, we resume the computation back to the point where the effect is performed, inside the `inc` function. As such, the `Set` effect is performed (and handled by the next branch of the same handler). This means `continue` returns a function, which we apply to `x` to ensure the value of the reference remains unchanged.

When the `Set` effect is handled, it returns a function that calls the continuation, which once again returns a new function. In turn, this function is applied to `n`, the value sent through the effect.

The use of `Get` and `Set` effects resemble the use of the state monad, for instance, in the Haskell language[1]. Indeed, algebraic effects have many similarities with monads, in the sense that both constructions provide abstraction over effectful computations. The upside of using effects is that performing effects is done in *direct style*, much like calling a function. Whereas with monads, one would need to encapsulate every operation inside the monad type. Moreover, effects provide a clear *abstraction barrier* between the action of performing an effect and its actual implementation [10].

[1] `https://wiki.haskell.org/State_Monad`

## 3 Specifying and Reasoning About Effects and Handlers

Proving the correctness of an handler leads to some specification and reasoning challenges, the greatest of which begin proving that every call to `continue` is sound. For the example of the previous Section, this would mean proving that for every `Get` effect the handler replies with the correct value. To do this, we must define some logical contract that the handler must respect when calling the continuation. Additionally, the function that performs the effect, must also respect a set of requirement to be able to perform an effect.

Throughout this paper, we refer to any function that calls `continue` as the *`Server`*, whereas any function calling `perform` is designated the *`Client`*. The rules that these two agents must ensure when performing or responding to an effect is described by *protocols* [5]. Protocols consist of preconditions (the conditions `Client` must respect when performing an effect), postconditions (the conditions that `Server` must respect when calling the continuation) and a `modifies` clause with all the variables `Server` is given write permissions to.

### 3.1 Client Specification

To present how we implemented protocols in Cameleer, we use a simple example: we create a program where `Client` sends some value to `Server`. The `Server` then takes that value and sets a global reference `p` to that value and returns `p`'s old value. To do this, we first define the reference as well as an effect, `XCHG`, that receives an integer value and returns another one. We then create a protocol with the rules we outlined, as follows:

*GOSPEL + OCaml*

```
let p : int ref = ref 0
effect XCHG : int -> int

(*@ protocol XCHG x :
    ensures !p = x && reply = old !p
    modifies p *)
```

Here, we define a GOSPEL protocol which ensures that, when `Server` returns control to `Client`, the reference holds `x`, the value passed by `Client`, and `Server` replies with `p`'s old value (hence, the use of the keyword `reply`). Additionally, we also state that `Server` is allowed to modify the contents of `p`. This protocol has no precondition since there is no restriction over what values `Client` may send. Every protocol is named after an effect, meaning it establishes the rules the two agents must obey when using such an effect to communicate.

Let us now move on to specifying a `Client` that uses this effect. This amounts to the following:

*GOSPEL + OCaml*

```
let xchg (n : int) : int =
  perform (XCHG n)
(*@ ensures !p = n && result = old !p
    performs XCHG *)
```

The `performs` clause lists the effects a certain function might perform. The postcondition of the `xchg` function is exactly the postcondition of the protocol, since it simply calls `perform` with effect `XCHG`.

### 3.2 Server Specification

Consider the following simple OCaml handler for the `XCHG` effect:

```
let server () =                                          GOSPEL + OCaml
  try xchg (xchg 42) with
  | effect (XCHG n) k ->
      let old_p = !p in
      p := n;
      continue k old_p
```

It is easy, at least informally, to observe that this handler respects the protocol defined previously. However, in the general case, this can only be established if we specify a condition whether the function exits through an *exceptional* branch or it ends normally. We refer to this piece of specification as the *handler invariant*, inspired from the fact we are dealing here with deep handlers, which provide a form of recursive treatment of effects. For the XGHG `Server`, we provide the following specification:

```
try xchg 42 with ...                                     GOSPEL + OCaml
(*@ try_ensures !p = old !p && result = 42
    returns int *)
```

The said invariant is provided via `try_ensures`, a new clause we have introduced in GOSPEL for the purpose of reasoning about handlers. For the above example, whenever the execution reaches the exceptional branch, we know the final instruction of the handler is a call to `continue`. Since the handler remains installed (again, due to it being a deep-handler), it keeps calling `continue` until no more effects are performed. In practice, this means the two effects produced by the double call to `xchg` are treated in this handler. After the second call to `continue`, function `server` halts its execution normally, hence the handler specification allows one to derive this function returns 42 and the value stored in `p` is unchanged (even if this reference is written twice). Finally, the `returns` keyword specifies the return type of this function...

## 4 Translation of Effects and Handlers into WhyML

Having presented our protocol-based approach to specify effects and handlers, we focus now on exploiting such specification for the purpose of deductive verification. We present in this Section a translation scheme from GOSPEL-annotated OCaml programs, with effects and handlers, into a WhyML program. Hence, the verification conditions generated by Why3 must ensure the original OCaml program adheres to its specification. This mainly amounts to prove that `Client` and `Server` respect the established protocols. To guide our presentation, we re-use the `xchg` example from Sec. 3.

### 4.1 Reasoning about `Client`

Each request `Client` makes, via an effect, must respect the corresponding protocol's precondition and that `Client`'s postcondition holds for any reply `Server` may send. In order to prove such properties, our translation pipeline introduces the following function, which simulates performing an `XCHG` effect:

```
val perform_XCHG (x : int) : int                                    WhyML
  ensures { let reply = result in
            reply = old !p && !p = x }
  writes  { p }
```

The specification of `perform_XCHG` is exactly that of the `XCHG` protocol. Using this function, the `Client` is straightforwardly translated into WhyML, as follows:

```
  let xchg (n : int) : int                                          WhyML
    ensures { !p = n && result = old !p }
  = perform_XCHG n
```

By using the `perform_XCHG` function, we can check if the request made satisfies the protocol's precondition and if the function's postcondition holds for any possible reply. The former is proved trivially: seeing as there is no precondition, `Client` can always make a request. As for the latter, the protocols imposes `Server` can only reply after setting the `p` reference to the value sent by `Client` and can only return the reference's old value. This is exactly what we put in `perform_XCHG` postcondition, hence the body of `xchg` also adheres to its specification.

### 4.2 Reasoning about `Server`

Using our translation approach, proving a `Client` is no different than proving functions that call other functions. Proving `Server`, on the other hand, poses some interesting challenges:

- How does one translate OCaml's effects into WhyML?
- The Why3 `xchg` function does not have any kind of exceptional return, it merely calls a function that simulates the handler's behavior. How do we embed this in Why3?
- How does one represent continuations, as well as OCaml's `continue` function, in Why3?
- What kind of specification should such continuations present?

To tackle the first issue, we simply choose to represent effects as WhyML *exceptions*. More specifically, we create an exception that receives the same arguments as the original effect. In the case of `XCHG`, it receives a single `int` value, resulting in the following encoding:

```
  exception XCHG int                                                WhyML
```

Using such representation for effects, OCaml effect handlers are translated into WhyML using exception handlers. The handler from Sec. 3.2 is translated as follows:

```
try xchg (xchg 42) with                                          WhyML
| XCHG n ->
    let old_p = !p in
    p := n;
    continue k old_p
```

To make this handler effective, the `xchg` function must be able to, indeed, throw an exception. Our translation scheme adds the following `raises` clause to the `perform_XCHG` function:

```
raises { XCHG }                                                  WhyML
```

In practice, we translate every `performs` clause from GOSPEL specification into a WhyML `raises` clause.

### 4.3 Representing Continuations

The attentive reader might notice that the continuation `k` is absent from the WhyML translation of handlers. Indeed, representing and dealing with continuations in Why3 is a challenge, since the framework has very limited support for higher-order computation.

Continuations are, in essence, functions that can only be called using the special `continue` function. Moreover, they can only be called once (if the same continuation is called multiple times, the OCaml runtime would throw an exception). We encode continuations in WhyML encoding via the following record type:

```
type continuation 'a 'b = abstract {                             WhyML
  mutable _valid : bool;
}
```

This declaration introduces a polymorphic type of continuations, with argument of type `'a` and result of type `'b`. The Boolean field `_valid` is used to check whether the continuation was *continue*d, ensuring the one-shot behavior. From a programming perspective, type `continuation` behaves as an *abstract* data type. One can only explore the value of `_valid` within specification or ghost code.

To use continuations as a value equipped with some logical contract, we adopt a technique [11, 20] that allows us to represent and manipulate the specification of continuations. This is done through the following pair of predicates:

```
type state                                                       WhyML

predicate pre  (continuation 'a 'b) 'a state
predicate post (continuation 'a 'b) 'a state state 'b
```

These predicates are not given a particular definition, but are axiomatized later, as needed. The `pre` predicate holds if the continuation's precondition holds. This predicate is applied to the continuation's argument and a representation of the state in which the continuation begins execution. On the other hand, `post` predicate holds if the continuation's postcondition holds. This predicate is applied to the continuation's argument, the initial state, the state after the continuation executed, and finally the value returned by the continuation. The `state` type is instantiated with the piece of mutable state that a particular program manipulates. For instance, for the `xchg` example, `state` would be refined as record type with a single mutable field `_p`, which represents the value stored in reference `p`.

Using this approach, we define the `continue` function in Why3, as follows:

```
val continue (k : continuation 'a 'b) (arg : 'a) : 'b      WhyML
  requires { k._valid }
  requires { pre k arg {_p = !p} }
  ensures  { post k arg (old {_p = !p}) {_p = !p} result }
  writes   { k._valid, p }
```

When this function is called, it will respect the contract of continuation `k`. Additionally, it requires the continuation to be valid (*i.e.*, not called yet). Once `continue` returns, the continuation is invalidated, since we state in the `writes` clause that the `k._valid` field is written, hence one can no longer suppose it is set to `true`.

This encoding of the `continuation` data type is in fact a form of defunctionalization [4], where `continue` resembles the Why3apply function that is generated during defunctionalization. A major difference in our approach is that we do not represent continuations as an algebraic data types. Instead, we fully capture of such values in their specification, as we explain in what follows.

### 4.4 Specifying continuations

Using all these different pieces, we can now conclude our WhyML proof by creating a value `k` of type `continuation` and specifying it using the `pre` and `post` predicates defined previously. For the example of the `XCHG` handler, this is done as follows:

```
try xchg (xchg 42) with                                    WhyML
| XCHG n ->
    val k : continuation int int in
    assume { ... };
    let old_p = !p in
    p := n;
    continue k old_p
```

The WhyML `val..in` construction allows one to introduce local abstract values, *i.e.*, without imposing a particular implementation. The conditions that go in the `assume` clause are essentially the specification for `k`. First, we state that the continuation is valid:

```
assume { k._valid }                                                    WhyML
```

Next, using the `pre` predicate, we state that the precondition for `k` is the protocol's *postcondition*. Although this might seem counterintuitive at first, the protocol's postcondition is the condition that `Server` must respect when it surrenders control to `Client` via the suspended continuation. Therefore, the `Client` must ensure the protocol's postcondition when calling the continuation. To recap, the protocol's postcondition is the following:

```
(*@ protocol XCHG n :                                          GOSPEL + OCaml
    ensures reply = old !p && !p = n *)
```

We must now translate the postcondition to WhyML and encode it with the `pre` predicate, as follows:

```
assume { let eff_p = !p in                                             WhyML
         forall reply state.
         pre k reply state <->
         reply = eff_p && state._p = n }
```

Within this clause, we state that the precondition of `k` is that the reply must be equal to `eff_p`, the value `p` takes when the continuation is created (in other words, when the effect is performed). Next, using the `state` type, we say that when `k` is called the value stored in `p` is equal to `n`, the value passed by `Client`.

Finally, using the `post` predicate, we state that the postcondition of `k` is the handler's postcondition. This is because calling `k` reinstalls the handler, any call to `k` terminates when it exists the handler. Once again, the postcondition of the handler is

```
try_ensures !p = old !p && result = 42                         GOSPEL + OCaml
```

To translate this to WhyML, we need to access the value `p` held before `xchg` was called. To do so, we create an auxiliary variable `init_p` and translate the handler's postcondition as follows:

```
let init_p = !p in                                                     WhyML
try xchg (xchg 42) with
| XCHG n ->
  ...
  assume { forall reply old_state state result.
           post k reply old_state state result <->
           state._p = 3 && result = init_p }
```

It is worth noting that we do not use the initial state in which the continuation is called nor the reply its its postcondition. This is because these values are only relevant for the continuation's precondition.

*The generated Verification Conditions.* We take a step back and observe what VCs Why3 generates for the resulting WhyML program, and what they mean

| Case study | # VCs | LOC / Spec. / Ghost | Proof time |
|---|---|---|---|
| Control inversion | 15 | 20 / 25 / 2 | 0.74 |
| Division interpreter | 19 | 18 / 16 / 2 | 0.74 |
| Koda-Ruskey algorithm | 142 | 26 / 20 / 0 | 17.60 |
| References | 36 | 37 / 22 / 3 | 0.91 |
| Xchg | 13 | 13 / 7 / 0 | 0.64 |
| Shallow Handlers | 13 | 21 / 7 / 0 | 0.54 |

Fig. 1: Summary of the case studies verified with extended Cameleer.

from the perspective of the original OCaml program. For `Client`, Why3 generates one VC checking if the postcondition of the `xchg` function holds. Seeing as this function has the same postcondition as the `perform_XCHG` function, this is proved trivially. As for `Server`, we generate four VCs. First, one must prove that the handler's invariant holds when the function terminates through the non-exceptional branch. Since the handler's invariant is the same as `xchg`'s postcondition, this is also proved. Next, the proof that the continuation has not been called yet (trivially, since the continuation is created valid) and that the continuation's precondition is respected. Given this handler respects the protocol, this VC is also dispatched. Finally the proof that the handler's invariant is maintained when we exit through the exceptional case. Seeing as the handler's last instruction is calling the continuation, whose postcondition is the handler's invariant, this is also proved.

We give in Appendix A a general presentation of our translation scheme from OCaml programs with effects and handlers to WhyML. We believe this approach is general enough, hence it could be adapted to any proof-aware language with exceptions.

## 5 Evaluation

To test our approach, we specify and verify a handful of illustrative case studies. Besides the `XCHG` example we have already presented in detail, we also verify the following case studies:

1. Turning a higher order iterator function into a generator. This process is commonly referred to as *control inversion*.
2. A division interpreter that performs an effect in case of a division by zero.
3. Implementing mutable state using algebraic effects and deep handlers, as shown in Sec. 2.
4. The Koda-Ruskey algorithm.
5. Implementing a deep handler using a shallow handler. Our support for shallow handlers is still experimental, as we are currently a significant number of bureaucratic verification conditions to deal with unhandled effects.

The reported case studies are all automatically verified, using a combination of Alt-Ergo 2.4.2, CVC4 1.8, and Z3 4.8.6 SMT solvers. Fig. 1 summarizes important metrics about our verified case studies: the number of generated verification

conditions for each example; the total lines of OCaml code, GOSPEL specification, and lines of ghost (these are also included in the number of OCaml LOC), respectively; and finally the time it takes (in seconds) to replay a proof. The runtimes were measured by averaging over ten runs on a Lenovo Thinkpad X1 Carbon 8th Generation, running Linux Mint 20.1, OCaml 5.0.0, and Why3 1.5.1.

All examples, including OCaml implementation, GOSPEL specification, and Why3 proof sessions files are publicly available[2]. It is worth noting that throughout this paper we use a friendly syntax for effect and handlers, which is not supported natively by the OCaml compiler (this was in fact the syntax proposed in an experimental-oriented switch of the compiler). However, all of our case studies are written using the effects and handlers library supported by OCaml 5.0.0.

*A more elaborate case study.* Most of the proofs enumerated, although not trivial, are relatively small scale examples. The outlier is the Koda-Ruskey algorithm [12] which enumerates the valid colorings of a forest of n-ary trees. The complete OCaml implementation and GOSPEL for this algorithm is depicted in Fig. 2. Very briefly put, the nodes of a tree can either be colored White or Black. If a node is colored White, all of its children must be White; if it is Black, there are no restrictions posed on the color of its children. Each time the algorithm reaches a new coloring, an effect is performed. The global array `bits` is used to represent each new coloring.

Not only is this a very challenging proof, it is the only implementation we are aware of for the Koda-Ruskey algorithm using algebraic effects. Although we are not going to go over the entire specification, the key part is that the protocol for this function is defined locally. This is because we want to state that, whenever an effect is performed, the forest has a valid coloring. Since the forest is passed as an argument, however, we must define the protocol within the scope of the function to refer to its argument. Our proof, namely auxiliary definitions and lemmas about forests and colorings, follows closely a previous Why3 for a first-order, stack-based version of the Koda-Ruskey algorithm [6].

## 6 Related Work

The most common type of static verifications programmers use for their programs is type-checking. A possible way of extending a type system's reach is by adding an effect system [1]. In normal type systems, we generally have type signatures such as $f : \alpha \to \beta$, where $f$ is a function that receives an argument of type $\alpha$ and returns a value of type $\beta$. When using an effect system, type signatures have an additional parameter: a row $\rho$. A row is simply a list of signatures indicating which effects this function performs. With this typing information, we can determine, at compile time, if a program will perform any unhandled effects, thereby removing yet another common source of errors. Effect systems can be used to track not only algebraic effects, but any kind of impure behavior

[2] `https://github.com/mrjazzybread/cameleer/tree/effects/tests`

```
effect Yield : unit                                                    OCaml

let koda_ruskey (f : forest) =
  let rec enum_eff (f : forest) ((baton : forest list)[@ghost])
  = match f with
    | E -> perform Yield
    | N (i, l, r) ->
       if bits.(i) = White then begin
         enum_eff r baton;
         bits.(i) <- Black;
         try enum_eff l (r :: baton) with
         | Yield -> enum_eff r baton; continue k ()
       end else begin
         begin try enum_eff l (r :: baton) with
         | effect Yield -> enum_eff r baton; continue k () end;
         bits.(i) <- White;
         enum_eff r baton end
  (*@ requires valid_nums_forest f (length bits)
      requires disjoint_stack f baton
      requires any_forest f bits.elts
      ensures forall i.
              not mem_forest i f && not mem_stack i baton ->
              bits[i] = old bits[i]
      ensures inverse (Cons f baton) (old bits.elts) bits.elts
      variant f
      protocol Yield {
        requires valid_coloring f bits.elts
        requires forall i.
                 not mem_stack i baton -> bits[i] = old bits[i]
        requires unchanged baton (old bits.elts) bits.elts ||
                 inverse baton (old bits.elts) bits.elts
        ensures forall i.
                not mem_stack i baton -> bits[i] = old bits[i]
        ensures inverse baton (old bits.elts) bits.elts
        modifies bits } *)
  in enum_eff f []
(*@ requires valid_nums_forest f (length bits)
    requires white_forest f bits.elts
    performs Yield *)
```

Fig. 2: The Koda-Ruskey algorithm implementation and specification.

such as mutable state and divergence [13]. This means we can imbue within the typing information whether or not a function is pure.

Although effect handlers are, still, relatively niche, there has been some work in modelling their behavior using equational theory. The vanguard of this approach was Plotkin and Power [17] where they reason over effects as equations of effectful computations. Some research has been done by applying this algebraic approach to effects and handlers. For example, Plotkin and Pretnar [18,19] developed a logic where the semantics of effects was captured using this approach and handlers were verified by determining if they satisfied a set of equalities. Additionally, some implementations have been developed that embed these reasoning rules into Coq, an interactive theorem prover [14, 24].

Another important line of research is reasoning about the concurrent behavior of effects and handlers into Separation Logic. For instance, Timany and Birkedal [23] developed a framework for the verification of full stack continuations in a language with *call/cc*. Additionally, Hinrichsen *et al.* [8] developed an extension of Coq [9] for reasoning over programs with message passing using *protocols* that define the rules that two agents must respect in their exchange. Drawing inspiration from this research, de Vilhena and Pottier [5] developed a simplified version of protocols for programs with algebraic effects. Their framework interprets effects and effect handlers as a communication between two agents: the one who raised the effect and its handler. To model this communication, they use protocols which describe the values that can be sent in either direction during this communication. The logic that governs these protocols is built on top of Iris, a higher order separation logic encoded in Coq. These protocols are, indeed, the main inspiration for the Cameleer extension we have developed.

## 7 Conclusions and Future Work

We have presented an extension to the Cameleer tool to specify and reason about algebraic effects and handlers. This lead us to introduce several extensions to the GOSPEL itself, namely the introduction of protocols at the level of specification, as well as the `performs` and `try_ensures` clauses. In order to explore such specification approach, we implemented inside Cameleer a translation scheme from OCaml into a WhyML embedding, which features exceptions and defunctionalization of continuations as a means to reason about effects and handlers. To the best of our knowledge, this extension to Cameleer is the first framework that proposes an automated verification path to algebraic effects.

As future work, we plan extend our tool to handle hidden state, seeing as we can only deal with mutable state defined at the top-level. Additionally, our tool can only prove handlers that remain installed when we call the generated continuation: we plan to add support for handlers where the continuation may produce an unhandled effect. Finally, our long term goal is to employ Cameleer in the effort of verifying larger OCaml libraries that make a significant use of effects

and handlers. A good candidate is the `eio`[3], a library that proposes direct-style concurrency primitives (implemented as effects and handlers) for the OCaml language.

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# A General Translation Scheme

$\mathcal{H}(e_{0t},\ \overline{E\bar{x}},\ \bar{e},\ \overline{S_{h ensures}}, term_{post},\ \tau,\ \Sigma,\ term_{state}) =$

*WhyML*

```
let handler () =
  ensures{S_{h ensures_1}}
  ensures{S_{h ensures_2}}
  ...
      let init_state = S_{x̄} in
      try e_{0t} with
      |...
      |E_n x̄ ->
          let eff_state = S_{x̄} in
          val gen_k unit : continuation (π2(Σ(E_n))) τ
          ensures{valid result}
          ensures{
              forall arg state.
              pre result arg state <->
              E_post arg eff_state state result
          }
          ensures{
              let f = result in
              forall arg irrelevant_old_state state result.
              post f arg irrelevant_old_state state result <->
                let state_old = init_state in
                term_post ∧ term_state
          } in let k = gen_k () in e_nt
       |...
  in handler ()
```

Fig. 3: Translation rule for effect handlers.

$$\mathcal{O}(\epsilon) = \texttt{true}$$
$$\mathcal{O}(x :: t) = \texttt{state}.x = \texttt{state\_old}.x \wedge \mathcal{O}(t)$$

Fig. 4: Unmodified State

$$\mathcal{T}\ (\tau_1 \to \tau_2 \to \tau_3) =$$
$$\quad let\ s = \mathcal{T}(\tau_2 \to \tau_3)\ in$$
$$\quad \tau_1 :: \pi_1(s), \pi_2(s)$$

$$\mathcal{T}\ (\tau_1 \to \tau_2 \to \tau_3) =$$
$$\quad (\tau_1 \to \tau_2) = \tau_1, \tau_2$$

$$\mathcal{T}\ (\tau_1 \to \tau_2 \to \tau_3) =$$
$$\mathcal{T}\ (\tau) = unit, \tau$$

Fig. 5: Effect Type Translation.

$\mathcal{D}\ (args = x_1 : \tau_1, x_2 : \tau_2, ..., x_n : \tau_n)\ \tau_r\ e_t\ \bar{S}\ term_{pre}\ term_{post} =$

*WhyML*

```
let f args S̄ = e_t in
val gen_f () : lambda (τ_1, τ_2, ..., τ_n) τ_r
ensures{
  forall arg state. pre result arg state <->
    let x_1, x_2 ... = arg in
    term_pre
}
ensures{
  let f = result in
  forall arg old_state state result.
  post f arg old_state state result <->
  let x_1, x_2 ... = arg in
  term_post
} in gen_f ()
```

Fig. 6: Defunctionalization Function.

$$\mathcal{S}(term :: xs) = h(term) \wedge \mathcal{S}(xs)$$
$$\mathcal{S}(\epsilon) = true$$

Fig. 7: Combination of terms

$\mathcal{P}(E,\ \bar{x},\ \Sigma,\ term_{pre},\ term_{post},\ mod) =$

*WhyML*

```
predicate pre_E (arg : π1(Σ(E))) (state : state) =
    let x̄ = arg in
    term_pre

predicate post_E (arg : π1(Σ(E))) (old_state : state)
                 (state : S_τ) (reply : π2(Σ(E))) =
    let x̄ = arg in
    term_post


val perform_E (arg: π1(Σ(E))) : π2(Σ(E))
requires{pre_E arg S_x̄}
ensures{post_E arg (old S_x̄) S_x̄ result}
writes{mod}
```

Fig. 8: Protocol Translation Function.

$$\frac{}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\texttt{effect } E \ : \ \tau]\!] \rightsquigarrow \texttt{exception } E \dashv \Sigma[E \to \mathcal{T}(\tau)]} \ (\textsc{TEffect})$$

$$\frac{}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\texttt{perform E}\bar{\texttt{x}}]\!] \rightsquigarrow \texttt{perform\_E } \bar{\texttt{x}} \dashv} \ (\textsc{TPerform})$$

$$\frac{\mathcal{S}(\overline{Sp}_{requires}) = term_{pre} \qquad \mathcal{S}(\overline{Sp}_{ensures}) = term_{post}}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\texttt{protocol } E\bar{x} : \overline{Sp}]\!] \rightsquigarrow \mathcal{P}(E,\ \bar{x},\ \Sigma,\ term_{pre},\ term_{post},\ Sp_{modifies}) \dashv} \ (\textsc{TProtocol})$$

$$\frac{\Sigma \circ \Delta \circ \nu \cup \overline{x} \circ \emptyset \vdash [\![e]\!] \rightsquigarrow e_t \dashv \mu' \qquad \mathcal{S}(\bar{S}_{requires}) = pre \qquad \mathcal{S}(\bar{S}_{ensures}) = post}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\texttt{fun } \bar{S}\ \overline{(x : \tau_{arg})} : \tau_{ret} \to e]\!] \rightsquigarrow \mathcal{D}\ \overline{(x : \tau_{arg})}\ \tau_{ret}\ e_t\ \bar{S}\ pre\ post \dashv} \ (\textsc{TFun})$$

$$\frac{}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\texttt{performs } E]\!] \rightsquigarrow \texttt{raises}\{E\ arg \to E\_pre\ arg\ S_{\bar{x}}\} \dashv} \ (\textsc{TPerformsClause})$$

$$\frac{\forall i\ \Sigma \circ \Delta \circ \nu \circ \mu_i \vdash [\![e_i]\!] \rightsquigarrow e_{it} \dashv \mu_{i+1} \qquad e_0 = x \implies x \in \nu}{\Sigma \circ \Delta \circ \nu \circ \mu_0 \vdash [\![e_0\ e_1\ ...\ e_n]\!] \rightsquigarrow \texttt{apply } e_0\ (e_2, e_3, ..., e_n) \dashv S_{vars}} \ (\textsc{TAppDefun})$$

$$\frac{\forall i\ \Sigma \circ \Delta \circ \nu \circ \mu_i \vdash [\![e_i]\!] \rightsquigarrow e_{it} \dashv \mu_{i+1} \qquad \neg x \in \nu}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![x\ e_0\ e_1\ ...\ e_n]\!] \rightsquigarrow x\ e_{0t}\ e_{1t}\ \ldots e_{nt} \dashv \Delta(x) \cup \mu_{n+1}} \ (\textsc{TApp})$$

$$\frac{\begin{array}{c}\forall i.0 < i \leqq n \implies \Sigma \circ \Delta \circ \nu \circ \mu_i \vdash [\![e_i]\!] \rightsquigarrow e_{it} \dashv \mu_{i+1} \qquad \mathcal{S}(\overline{Sh_{ensures}}) = term_{post} \\ Sh_{returns} = \tau \qquad e_{try} = try\ e_0\ with\ Sh\ \texttt{effect } E\bar{x}\ k \to e\end{array}}{\Sigma \circ \Delta \circ \nu \circ \emptyset \vdash [\![e_{try}]\!] \rightsquigarrow \mathcal{H}(\overline{E\bar{x}},\ \bar{e},\ \overline{Sh_{ensures}},\ term_{post},\ \tau,\ \Sigma,\ \mathcal{O}(S_{vars} \setminus \mu_{n+1})) \dashv \mu_n} \ (\textsc{TTry})$$

$$\frac{\begin{array}{c}\Sigma \circ \Delta \circ \nu \circ \emptyset \vdash [\![e]\!] \rightsquigarrow e_t \dashv \mu \\ e \neq fun\ ... \qquad \bar{x} \neq \epsilon \implies \Delta' = \Delta[f \to \mu] \qquad \bar{x} = \epsilon \implies \Delta' = \Delta\end{array}}{\Sigma \circ \Delta \circ \nu \circ \emptyset \vdash [\![\texttt{let } \bar{S}\ f\ \bar{x} = e]\!] \rightsquigarrow \texttt{let } \bar{S}\ f\ \bar{x} = e_t \dashv \Delta'} \ (\textsc{TLet})$$

$$\frac{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![e_1]\!] \rightsquigarrow e_{1t} \dashv \mu' \qquad \Sigma \circ \Delta \circ \nu \circ \mu' \vdash [\![e_2]\!] \rightsquigarrow e_{2t} \dashv \mu''}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\texttt{if } a \texttt{ then } e_1 \texttt{ else } e_2]\!] \rightsquigarrow \texttt{if } a \texttt{ then } e_{1t} \texttt{ else } e_{2t} \dashv \mu''} \ (\textsc{TIf})$$

$$\frac{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![e_1]\!] \rightsquigarrow e_{1t} \dashv \mu' \qquad \Sigma \circ \Delta \circ \nu \circ \mu' \vdash [\![e_2]\!] \rightsquigarrow e_{2t} \dashv \mu''}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![e_1\ ;\ e_2]\!] \rightsquigarrow e_1\ ;\ e_2 \dashv \mu''} \ (\textsc{TSeq})$$

$$\frac{\begin{array}{c}\bar{x} \neq \epsilon \implies \mu = \emptyset \\ \Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![e_1]\!] \rightsquigarrow e_{1t} \dashv \mu' \qquad e \neq fun\ ... \qquad \bar{x} \neq \epsilon \implies \Delta' = \Delta[f \to \mu'] \\ \bar{x} = \epsilon \implies \Delta' = \Delta \qquad \Sigma \circ \Delta' \circ \nu \circ \mu' \vdash [\![e_2]\!] \rightsquigarrow e_{2t} \dashv \mu''\end{array}}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\texttt{let } \bar{S}\ f\ \bar{x} = e_1 \texttt{ in } e_2]\!] \rightsquigarrow \texttt{let } \bar{S}\ f\ \bar{x} = e_{1t} \texttt{ in } e_{2t} \dashv \mu''} \ (\textsc{TLetIn})$$

$$\frac{\forall i.0 \leqq i < n \implies \Sigma \circ \Delta \circ \nu \circ \mu_i \vdash [\![e_i]\!] \rightsquigarrow e_{it} \dashv \Sigma \circ \Delta \circ \nu \circ \mu_{i+1}}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\texttt{match } a \texttt{ with } \overline{p \to e_i}]\!] \rightsquigarrow \texttt{match } a \texttt{ with } \overline{p \to e_{it}} \dashv \Sigma \circ \Delta \circ \nu \circ \mu_n} \ (\textsc{TMatch})$$

$$\frac{}{\Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\texttt{modifies } \bar{s}]\!] \rightsquigarrow \texttt{writes}\{s\} \dashv} \ (\textsc{TModifies}) \qquad \Sigma \circ \Delta \circ \nu \circ \mu \vdash [\![\epsilon]\!] \rightsquigarrow \epsilon \dashv \ (\textsc{TEmpty})$$

$$\frac{\Sigma \circ \Delta \circ \nu \circ \emptyset \vdash [\![d]\!] \rightsquigarrow d_t \dashv \Sigma' \circ \Delta' \qquad \Sigma' \circ \Delta' \circ \nu \circ \emptyset \vdash [\![\bar{d}]\!] \rightsquigarrow d_{tl} \dashv \Sigma'' \circ \Delta''}{\Sigma \circ \Delta \circ \nu \circ \emptyset \vdash [\![d :: \bar{d}]\!] \rightsquigarrow d_t :: d_{tl} \dashv \Sigma'' \circ \Delta''} \ (\textsc{TDecl})$$



Fig. 9: Inductive Translation Rules.