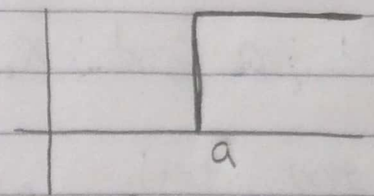
No: maths   Date: Sec. 4   Eng / Mona

" Series Solution of ODE "

$u(t-a)$

$$L\{u(t-a)\} = \frac{e^{-as}}{s}$$



$$L\{u(t-a)\, f(t-a)\} = e^{-as} L\{f(t)\}$$

Problems

Questions

① $L\{\cos t \; u(t-\pi)\}$

② $L\{e^{t} u(t-1)\}$

③ $L\{t \sin t \;\; u(t-\pi)\}$

④ $f(t) = \begin{cases} 2 & 0<t<2 \\ t & 2<t<3 \\ 3 & t>3 \end{cases}$ $F(s) = ?!$

Date:

Solution

① $L\{\cos(t-\pi+\pi)\ u(t-\pi)\}$

$= e^{-\pi s} L\{\cos(t+\pi)\}$ $\quad \cos\pi \cos t - \sin\pi \sin t$

$= -\cos t$

$= -e^{-\pi s} \frac{s}{s^2+1}$ #

② $L\{e^{(t-1+1)} u(t-1)\}$

$= e^{-s} L\{e^{t+1}\} = e^{-s} \cdot e \frac{1}{s-1}$ #

or

$L\{u(t-1)\} = \frac{e^{-s}}{s}$

$L\{e^{t} u(t-1)\} = \frac{e^{-(s-1)}}{s-1}$ #

③ $f(t) = t\sin t$

$L\{(t-\pi+\pi)\ \sin(t-\pi+\pi)\ u(t-\pi)\}$

$= e^{-\pi s} L\{(t+\pi)\ \sin(t+\pi)\}$ $\quad \sin t \cos\pi + \cos t \sin\pi$

$= -\sin t$

$= e^{-\pi s} L\{-t\sin t - \pi \sin t\}$

$= e^{-\pi s} \left[\frac{d}{ds} \frac{1}{s^2+1} - \pi \frac{1}{s^2+1}\right]$ #

YASSIN

(4) $f(t) = 2 + (t-2)\,u(t-2) - (3+t)\,u(t-3)$

$$F(s) = L\{f(t)\} = \frac{2}{s} + e^{-2s} \underbrace{L\{t\}}_{\frac{1}{s^2}} - e^{-3s} \underbrace{L\{t\}}_{\frac{1}{s^2}}$$

$$= \frac{2}{s} + e^{-2s}\frac{1}{s^2} - e^{-3s}\frac{1}{s^2} \quad //$$

This case used when $P(x) =$ رقم

$$P(x)\,y'' + q(x)\,y' + r(x)\,y = 0$$

If
$P(x) \neq 0 \longrightarrow x = x_0$ ordinary Point
$P(x) = 0 \longrightarrow$ Singular

معادلات الحل

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n\,a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1)\,a_n x^{n-2}$$

(1) $y'' + y = 0$

Sol

$P(x) = 1 \neq 0$

Let

، عوض بمعادلات الحل

$$\sum_{n=2}^{\infty} a_n\, n(n-1)\, x^{n-2} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$n = n + 2$

$$\sum_{0}^{\infty} a_{n+2}(n+2)(n+1)x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[(n+1)(n+2)a_{n+2} + a_n\right] x^n = 0$$

$$(n+1)(n+2)\,a_{n+2} + a_n = 0$$

$$a_{n+2} = -\frac{1}{(n+1)(n+2)}\,a_n \leftarrow \text{R.R}$$

when

$$\left.\begin{aligned}
n=0 \quad & a_2 = \frac{-1}{2}a_0 \\
n=1 \quad & a_3 = \frac{-1}{6}a_1 \\
n=2 \quad & a_4 = \frac{-1}{12}a_2 = \frac{1}{24}a_0 \\
n=3 \quad & a_5 = \frac{1}{42}a_3 = \frac{1}{42}\cdot\frac{-1}{6}a_1
\end{aligned}\right\}$$

لازم كلهم بدلالة a_0 و a_1

$$\therefore\; y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

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② $y'' - 2xy' + y = 0$

Sol

$$P(x) = 1 \neq 0$$

← عوض في معادلات الحل

$$\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} - 2\sum_{n=1}^{\infty} a_n n x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$n = n+2$

$$= \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n - 2\sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$= 2a_2 + a_0 + \sum_{n=1}^{\infty}\left[(n+2)(n+1)a_{n+2} - (2n-1)a_n\right]x^n = 0$$

$$2a_2 + a_0 = 0 \longrightarrow a_2 = \frac{-1}{2}a_0$$

$\rightarrow (n+2)(n+1)\, a_{n+2} = (2n-1)\, a_n$

$\rightarrow a_{n+2} = \dfrac{(2n-1)}{(n+2)(n+1)}\, a_n \leftarrow$ R.R

when R.R نعوض c

$n=1 \qquad a_3 = \frac{1}{2} a_1$

$n=2 \qquad a_4 = \frac{3}{12} a_2 = \frac{-1}{8} a_0$

$n=3$

$n=4$

continue ...

$\therefore \; y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

$$P(x)\, y'' + q(x)\, y' + r(x)\, y = 0$$

$P(x), q(x), r(x) \rightarrow$ are Polynomial

معادلات الحل

$$y = \sum_{n=0}^{\infty} a_n x^{n+s}$$

$$y' = \sum_{n=0}^{\infty} (n+s)\, a_n x^{n+s-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+s)(n+s-1)\, a_n x^{n+s-2}$$

different

S_1, S_2

diff

equal

$S_1 - S_2 = \text{fraction}$

$S_1 - S_2 = \text{int}$

$\therefore y_{G.S} = A y_1 + B y_2$

where $y_1 = y\big|_{s=s_1}$, $y_2 = y\big|_{s=s_2}$

③ $2xy'' + (x+1)y' + 3y = 0$

sol

عوض عن مشتقات الحل

$$2\sum_{n=0}^{\infty}(n+s)(n+s-1)a_n x^{n+s-1} + \sum_{n=0}^{\infty}(n+s)a_n x^{n+s}$$

$$+\sum_{n=0}^{\infty}(n+s)a_n x^{n+s-1} + 3\sum_{n=0}^{\infty}a_n x^{n+s} = 0$$

$$= \sum_{n=0}^{\infty}\left[2(n+s)(n+s-1)+(n+s)\right]a_n x^{n+s-1}$$

$$+\sum_{n=0}^{\infty}\left[(n+s)+3\right]a_n x^{n+s} = 0$$

$n+s-1 \Longrightarrow n+s$

$n-1 = n \Longrightarrow n = n+1$

$$= \sum_{n=-1}^{\infty}\left[2(n+s+1)(n+s)+(n+s+)\right]a_{n+1}x^{n+s}$$

$$+\sum_{n=0}^{\infty}\left[(n+s)+3\right]a_n x^{n+s} = 0$$

$$= \left[2s(s-1)+s\right]a_0 x^{s-1} + \sum_{n=0}^{\infty}\Big[(n+s+1)\left[2(n+s)+3\right]a_{n+1}$$

$$+\left[(n+s)+3\right]a_n\Big]x^{n+s}$$

$2s^2 - s = 0 \longrightarrow s(2s-1) = 0 \qquad s_1 = 0 \qquad s_2 = \frac{1}{2}$

$\therefore y = Ay_1 + By_2 \longrightarrow y_2 = y\big|_{s=s_2}$

$y_1 = y\big|_{s=s_1}$

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$$a_{n+1} = -\frac{(n+3)}{(n+1)(2n+)} a_n$$

at $S=0$

when $n=0$ $a_1 =$

$n=1$ $a_2 =$

Continue

$n=2$

$n=3$

$$\therefore y = \left[a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots\right] x^0$$

at $S = \frac{1}{2}$

when $n=0$

$n=1$ Continue ... $y_{G.S} = Ay_1 + By_2$

$n=2$

$n=3$

Reham maher