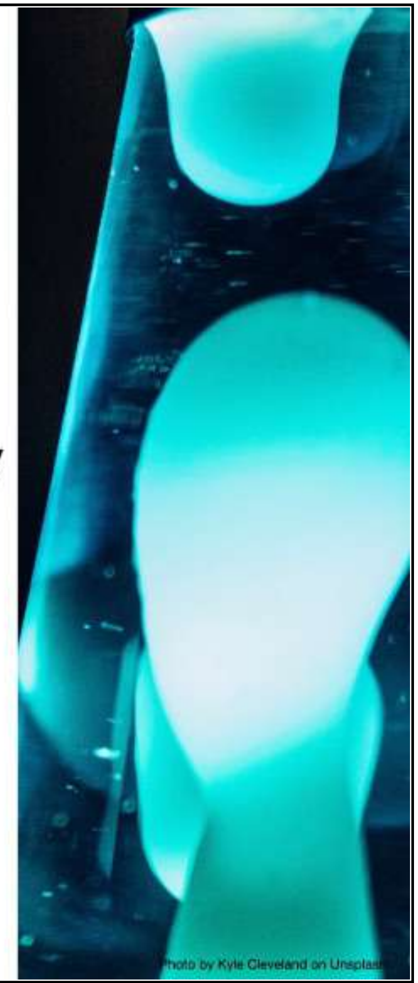
Randomness in Cryptography

Vanesa Daza



Fundamentals Principles of Modern Cryptography

1. Security depends on the resources of the attacker.
2. Kerckhoffs Principle
3. Security is impossible without randomness

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The One-Time Pad

Quick Review

(KeyGen, Enc, Dec)

$$\frac{\text{KeyGen:}}{\begin{array}{c} k \leftarrow \{0,1\}^\lambda \\ \text{return } k \end{array}} \qquad \frac{\text{Enc}_k(m)\text{:}}{\text{return } k \oplus m} \qquad \frac{\text{Dec}_k(c)\text{:}}{\text{return } k \oplus c}$$

- Perfectly Secure (unconditional security)
- Computationally Efficient
- Key too long and only used one time
- Malleable

crucial point: keys are uniformly selected at random, and true randomness is expensive.

How to make the OTP practical?

PRNG

basic idea : randomness → pseudo-randomness

Definition 2.2 *A* pseudorandom number generator (PRNG) *is a function*

$$G : \{0,1\}^{\ell} \to \{0,1\}^{h}$$

such that no efficient adversary can distinguish the output distribution of G from the uniform distribution on $\{0,1\}^h$. → for practical purposes, the output is considered uniformly random in $\{0,1\}^h$.

$s_0 = \text{seed}$

$s_i = G(s_{i-1})$, $i = 0,1,2,\dots$ or in general : $s_{i+1} = G(s_i, s_{i-1}, \dots, s_{i-t})$ for some t

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PRNG Example

Linear Congruential Generator

$$s_0 = \text{seed}$$
$$s_{i+1} \equiv a s_i + b \bmod m, \quad i = 0, 1, \ldots$$

integer constants (secret)

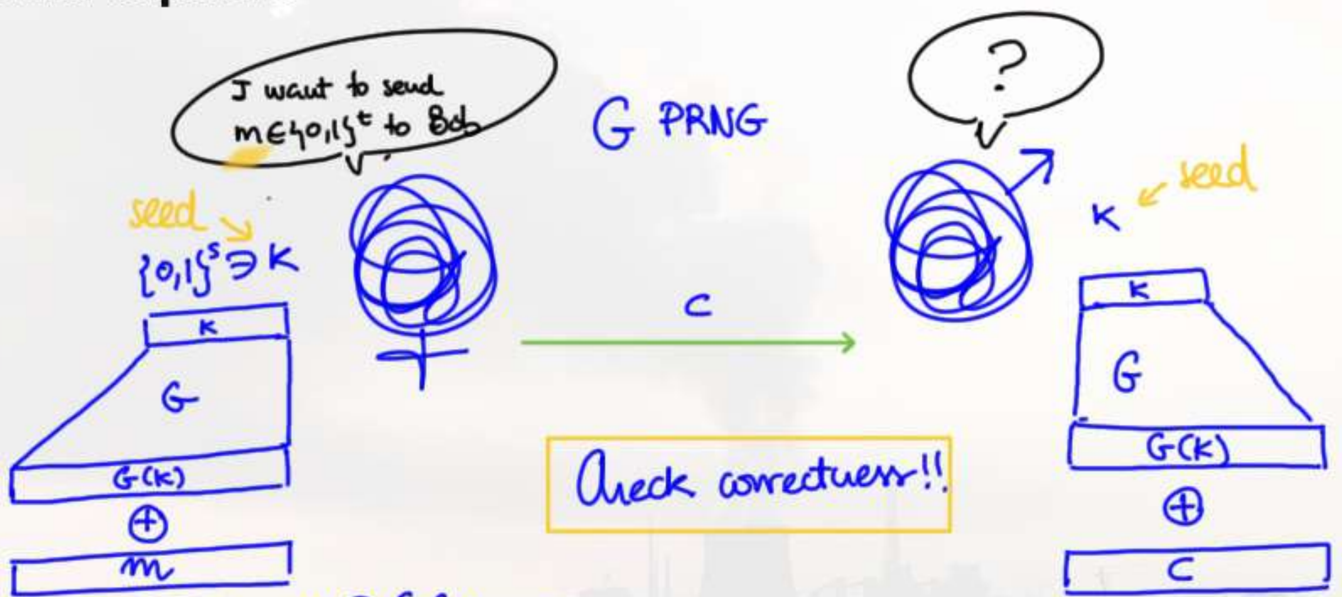
- produce a sequence of random looking integers between 0 and m-1
- choice of a, b and m is very important to guarantee good statistical properties

rand() function used in ANSI C

$$s_0 = 12345$$
$$s_{i+1} \equiv 1103515245 s_i + 12345 \bmod 2^{31}, \quad i = 0, 1, \ldots$$

How to make the OTP practical

Stream Ciphers



$Enc_K(m) = m \oplus G(K) = c$

$Dec_K(c) = c \oplus G(K)$

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<u>Example</u>: Linear feedback shift registers (LFSR)

$S_0 = K = k_1 \dots k_\ell$

$k_i = p_1 k_{i-1} \oplus \dots \oplus p_\ell \, k_{i-\ell}$

$p_i \in \{0,1\}$

$\ell = 3, \; p_1 = 1, \; p_2 = 0, \; p_3 = 1$

$K = 011$

$k_4 = 1 \cdot 1 + 0 \cdot 1 + 1 \cdot 0 = 1$

$k_5 = 1 \cdot 1 + 0 \cdot 1 + 1 \cdot 1 = 0$

$k_6 = 1 \cdot 0 + 0 \cdot 1 + 1 \cdot 1 = 1$

$\vdots$

But, the output of an LSFR of length ℓ repeats periodically, with a period of at most $2^\ell - 1$ $\longrightarrow$ not suitable for cryptography.

Still, a clever combination of some LFSR remains secure !! -

Stream Ciphers

Security

Length key <<< Length Plaintext → no perfect secrecy!!

We need to define what security means, and it will depend on PRNG

⊚ unpredictability

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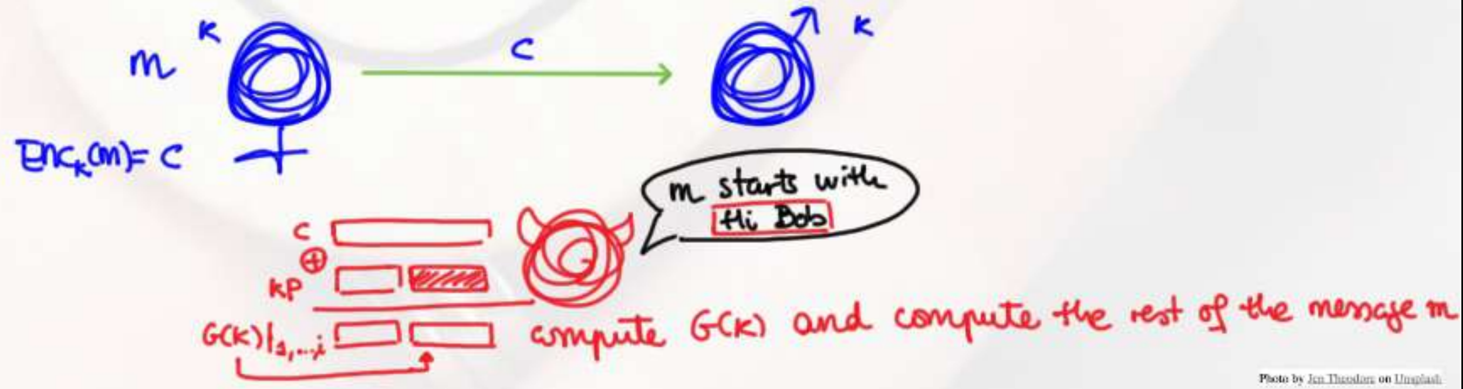
Predictability of PRNG

Informally, a PRNG is *predictable* if there exists an efficient way to predict a bit from previous computed bits.

$\exists i \quad G(k)|_{1,\dots,i} \xrightarrow{A} G(k)|_{i+1,\dots,n}$

Observe that if PRNG G is predictable, then the corresponding stream cipher is insecure. (known plaintext attack)



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Can we apply the 2-time attack to Stream Ciphers?

$$\left.\begin{array}{l} c_1 = m_1 \oplus G(k) \\ c_2 = m_2 \oplus G(k) \end{array}\right\} \longrightarrow m_1, m_2 ?$$

Observe that $c_1 \oplus c_2 = m_1 \oplus c_2$ and as it happened with the OTP, this could leak information on m_1 and m_2.

wired equivalent privacy

Attacking the WEP protocol (simplified version)

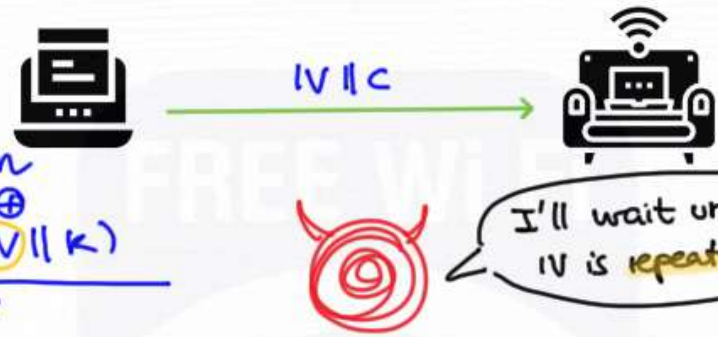
802.11b.WEP

G PRNG function called RC4

k $\qquad$ k

in $\{0,1\}^{24}$

$$\frac{m \oplus G(IV \| k)}{c}$$



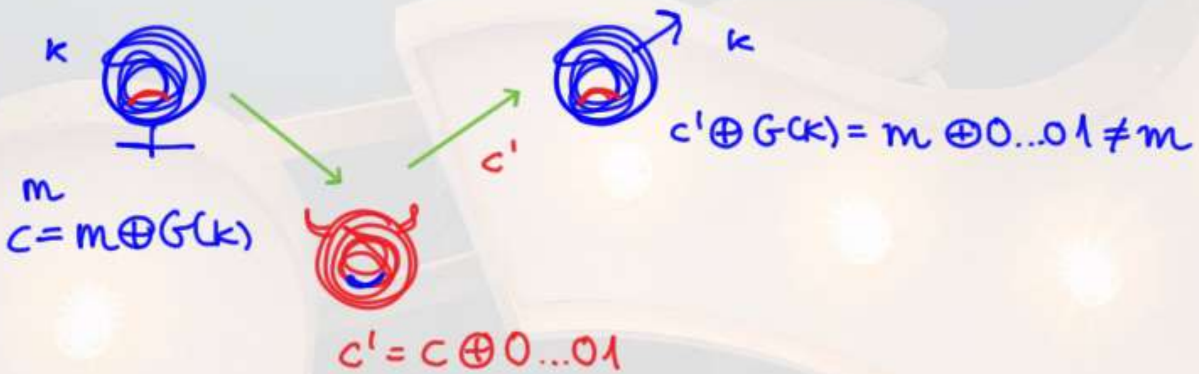
after 2^{24} frames reset to 0

$$\left.\begin{array}{l} IV \| c_1 \\ IV \| c_2 \end{array}\right\} \rightarrow c_1 \oplus c_2 = m_1 \oplus G(IV \| k) \oplus m_2 \oplus G(IV \| k) =$$
$$= m_1 \oplus m_2 \text{ as in a 2-time attack!!}$$

More efficient attacks were found because of the closeness of the keys: $k_1 = IV_1 \| k$, $k_2 = IV_2 \| k$ =!!

Are the Stream Ciphers malleable?

Similarly as it happens in the OTP, stream ciphers are malleable. Indeed just noticed that



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TOUR OF ACCOUNTING
OVER HERE WE HAVE OUR RANDOM NUMBER GENERATOR.
NINE NINE NINE NINE NINE NINE
10/25/01 © 2001 United Feature Syndicate, Inc.
ARE YOU SURE THAT'S RANDOM?
THAT'S THE PROBLEM WITH RANDOMNESS: YOU CAN NEVER BE SURE.

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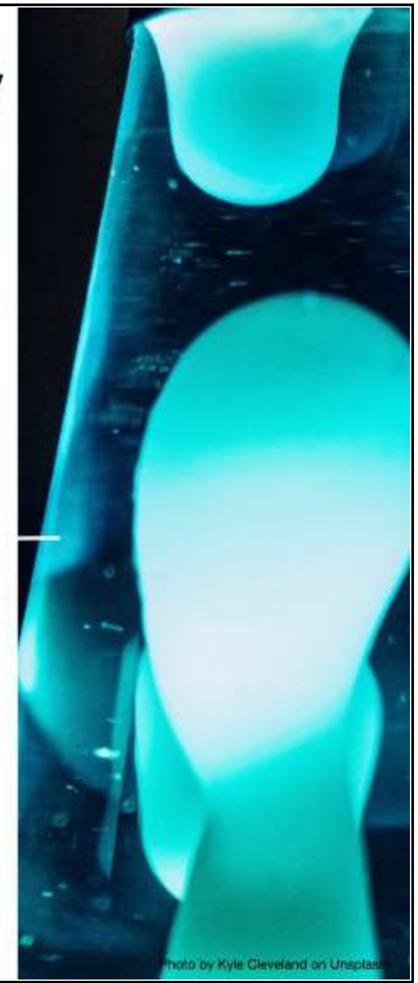

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