# Useful Formulas

## Set Operations

$$A \cup B, \quad A \cap B, \quad A^C$$
$$(A \cup B)^C = A^C \cap B^C,$$
$$(A \cap B)^C = A^C \cup B^C$$

## Basic Probability

$$P\Big(\bigcup_i A_i\Big) = \sum_i P(A_i) \text{ (disjoint } A_i\text{)}$$

$$P(A) + P(A^C) = 1$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Law of Total Probability :

$$P(A) = \sum_i P(A|B_i)P(B_i)$$

Bayes' Theorem :

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)}$$

## Counting

$$P_{N,k} = \frac{N!}{(N-k)!}, \quad C_{N,k} = \binom{N}{k} = \frac{N!}{k!(N-k)!}$$

## Expected Value & Variance

$$E(Y) = \sum_y yp(y)$$

$$V(Y) = \sum_y (y-\mu)^2 p(y) = E[Y^2] - \mu^2$$

$$\sigma = \sqrt{V(Y)}$$

## Binomial Distribution

$$Y \sim B(n,p)$$
$$P(Y=y) = \binom{n}{y} p^y (1-p)^{n-y}, \ y = 0, 1, \ldots, n$$
$$E[Y] = np, \quad V(Y) = np(1-p)$$

## Poisson Distribution

$$Y \sim \text{Poisson}(\lambda)$$
$$P(Y=y) = \frac{\lambda^y}{y!} e^{-\lambda}, \ y = 0, 1, 2, \ldots$$
$$E[Y] = \lambda, \quad V(Y) = \lambda$$

Approximation : $B(n,p) \approx \text{Poisson}(\lambda = np)$ for $n$ large, $p$ small.

## Tchebysheff's Theorem

$$P(|Y-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}, \quad k > 1$$

$$P(|Y-\mu| \geq k\sigma) \leq \frac{1}{k^2}$$