# Chapter 5: Public Goods

## Ch 36 in H. Varian $8^{th}$ Ed.

Slides by Mariona Segú, CYU Cergy Paris Université

Inspired by Michael D. Robinson, Mount Holyoke College

and by Stefanie Stantcheva, Harvard

# Introduction

So far…

→ Externalities can be solved if property right are properly assigned

→ But… not all externalities are solved like that, especially when there are more than 2 agents.

- Smoke example with 2 nonsmokers: Now, agents need to coordinate on how much clean air to sell.
- This can lead to coordination problems… (pollution in a whole country)

→ Public goods are one example of consumption externality

- Everyone must consume the same amount of the good
- Even if they have different preferences about it
- What is the optimal amount of public good?
- How can we socially decide about public goods?

# Public Goods – Definition

→ A good is **purely public** if it is both **nonexcludable** and **nonrival** in consumption.

- Nonexcludable: all consumers can consume the good. Individuals cannot deny each other the opportunity to consume a good.
- Nonrival: each consumer can consume all of the good. One individual's consumption of a good does not affect another's opportunity to consume the good

→ **Impure public goods**: Goods that satisfy the two public good conditions (non-rival in consumption and non-excludable) to some extent, but **not fully**.

# Public Goods – Examples

- Broadcast radio and TV programs.
- National defense.
- Public highways.
- Reductions in air pollution.
- National parks.

# Reservation Prices

- A consumer's reservation price for a unit of a good is his maximum willingness-to-pay for it.
- Consumer's wealth is $w$
- Utility of not having the good is $\mathrm{U}(w, 0)$
- Utility of paying p for the good is $\mathrm{U}(w-p, 1)$
- Reservation price is r such that $\mathrm{U}(w, 0)=\mathrm{U}(w-r, 1)$
  - Meaning, the price r such that consumer is indifferent between paying r and having the good and not having the good

# Reservation Prices: An Example

→ Consumer utility is $\mathrm{U}(x_1, x_2) = \mathrm{x}_1(x_2 + 1)$

→ Utility of not buying a unit of good 2 is

$$\mathrm{V}(w, 0\ ) = \frac{w}{p_1}\ (0 + 1) = \frac{w}{p_1}$$

→ Utility of buying one unit of good 2 at price p is

$$\mathrm{V}(w - p, 1\ ) = \frac{w-p}{p_1}\ (1 + 1) = \frac{2(w-p)}{p_1}$$

# Reservation Prices: An Example

- Reservation price r is such that $V(w, 0) = V(w - r, 1)$
- This is

$$\frac{w}{p_1} = \frac{2(w-r)}{p_1} \qquad \text{hence} \qquad r = \frac{w}{2}$$

- Reservation price of each person will depend on that person's wealth:
  - the maximum amount that an individual will be willing to pay will depend to some degree on how much that individual is able to pay.

# Outline

1. When Should a Public Good Be Provided?
2. Free-Riding
3. Variable Public Good Quantities
4. Quasilinear Preferences and Public Goods
5. Free-Riding Revisited
6. Collective decision mechanisms
7. Lindahl prices

# 1. When Should a Public Good Be Provided?

Example: two roommates A and B decide whether or not to purchase a TV. TV will be in the living room and enjoyed by both of them

→ The TV is a public good that costs c.

→ Individual wealth is $w_A$ and $w_B$

→ Individual payments for providing the public good are $g_A$ and $g_B$.

→ Each person's money left to spend on private consumption is $x_A$ and $x_B$

→ Budget constraint is then $x_A + g_A = w_A$ and $x_B + g_B = w_B$

→ The TV is purchased if $g_A + g_B \geq c$

# When Should a Public Good Be Provided?

- → Utilities depend on private good and on public good

$$U_A(x_A, G) \text{ and } U_B(x_B, G)$$

- → 2 possible allocations:
  - No TV: $(w_A, w_B, 0)$
  - TV is bought: $(x_A, x_B, 1)$ with $x_i = w_i - g_i$ for i=A,B

- → TV should be bought when both people are better off having the TV and paying their share than not having the TV

- → This is: payments must be individually rational, i.e.

$$U_A(w_A, 0) < U_A(w_A - g_A, 1)$$
$$U_B(w_B, 0) < U_B(w_B - g_B, 1)$$

# When Should a Public Good Be Provided?

→ Using the reservation wage definition…

$$U_A(w_A, 0) = U_A\,(w_A - r_A, 1) < U_A(w_A - g_A, 1)$$

$$U_B(w_B, 0) = U_B(w_B - r_B, 1) < U_B(w_B - g_B, 1)$$

→ Hence, $w_A - r_A < w_A - g_A$ and $w_B - r_B < w_B - g_B$

→ Which implies $r_A > g_A$ and $r_B > g_B$

1. The contribution to the public good needs to be smaller than the reservation price → **necessary condition**

2. The sum of willingnesses to pay must be greater than the cost of the TV → **sufficient condition**

$$r_A + r_B > g_B + g_A = c$$

# When Should a Public Good Be Provided?

If $r_A + r_B > g_B + g_A = c$, then it is Pareto-efficient to provide the public good.

→ Provision of public good only depends on individual's willingness to pay and on the total cost of the good

→ Provision of public good generally depends on each individuals' wealth (since $r_i$ depends generally on wealth)

- One exception is with quasilinear preferences

# Private Provision of a Public Good?

- Suppose $r_A > c$ and $r_B < c$
- Then A would supply the good even if B made no contribution.
- B then enjoys the good for free; **free-riding**.

# Private Provision of a Public Good?

- Suppose $r_A < c$ and $r_B < c$
- Then neither A nor B will supply the good alone.
- Yet, if $r_A + r_B > c$, then it is Pareto-improving for the good to be supplied.
- Still, A and B may try to free-ride on each other, causing no good to be supplied.

# 2. Free-Riding

- Suppose A and B each have just two actions – individually supply a public good, or not.
- Cost of supply c = $100.
- Reservation price to A from the good = $80.
- Reservation price to B from the good = $65.
- $80 + $65 > $100, so supplying the good is Pareto-improving.
- Payoffs for each consumer are
  - $P_i = r_i - c$ if $i$ buys TV
  - $P_i = r_i$ if the other buys TV
  - $P_i = 0$ if no one buys TV

# Free-Riding

Suppose only one player can buy the good…

| | | Player B | |
|---|---|---|---|
| | | Buy | Don’t Buy |
| Player A | Buy | | |
| | Don’t Buy | | |

# Free-Riding

Suppose only one player can buy the good…

| | | Player B | |
|---|---|---|---|
| | | Buy | Don't Buy |
| Player A | Buy | -\$20, -\$35 | -\$20, \$65 |
| | Don't Buy | \$80, -\$35 | \$0, \$0 |

## Free-Riding

| | | Player B | |
|---|---|---|---|
| | | Buy | Don’t Buy |
| Player A | Buy | -$20, -$35 | -$20, $65 |
| | Don’t Buy | $80, -$35 | $0, $0 |

**(Don’t’ Buy, Don’t Buy) is the unique NE.**

## Free-Riding

| | | Player B | |
|---|---|---|---|
| | | Buy | Don't Buy |
| Player A | Buy | -$20, -$35 | -$20, $65 |
| | Don't Buy | $80, -$35 | $0, $0 |

**But (Don't' Buy, Don't Buy) is inefficient.**

# Free-Riding

- Now allow A and B to make contributions to supplying the good.
- E.g. A contributes \$60 and B contributes \$40.

- Payoffs for each consumer are
  - $P_i = r_i - c_i$ if both contribute to the TV
  - $P_i = -c_i$ if only $i$ contributes to the TV
  - $P_i = 0$ if no one contributes to the TV

# Free-Riding

| | | Player B | |
|---|---|---|---|
| | | Contribute | Don't Contribute |
| Player A | Contribute | | |
| | Don't Contribute | | |

## Free-Riding

| | | Player B | |
|---|---|---|---|
| | | Contribute | Don't Contribute |
| Player A | Contribute | \$20, \$25 | -\$60, \$0 |
| | Don't Contribute | \$0, -\$40 | \$0, \$0 |

## Free-Riding

| | | Player B | |
|---|---|---|---|
| | | Contribute | Don't Contribute |
| Player A | Contribute | $20, $25 | -$60, $0 |
| | Don't Contribute | $0, -$40 | $0, $0 |

**Two NE: (Contribute, Contribute) and (Don't Contribute, Don't Contribute).**

# Free-Riding

- So allowing contributions makes possible supply of a public good when no individual will supply the good alone.
- But what contribution scheme is best?
- And free-riding can persist even with contributions.
  - Ex: incentive to lie about willingness to pay

# 3. Variable Public Good Quantities

- → E.g. how many broadcast TV programs, or how much land to include into a national park.
- → c(G) is the production cost of G units of public good.
- → Two individuals, A and B that ***collectively*** decide on the amount of public and private goods.
- → Private consumptions are $x_A$, $x_B$.

# Variable Public Good Quantities

→ Budget allocations must satisfy

$$x_A + x_B + c(G) = w_A + \ w_B$$

→ For consumer A, the maximization problem is:

$$\max_{x_A, x_B, G} U_A(x_A, G)$$

Subject to $$x_A + x_B + c(G) = w_A + \ w_B$$

And fixing $U_B$ to $\overline{u_B}$ , we add that $U_B(x_B, G) = \overline{u_B}$

The solution to this problem is such that

$$\left|MRS_{G,x_A}\right| + \left|MRS_{G,x_B}\right| = |MRS_A| + |MRS_B| = MC(G)$$

This is the **Samuelson rule**

# Variable Public Good Quantities

$$|MRS_A| + |MRS_B| = \frac{MU_G}{MU_{x_A}} + \frac{MU_G}{MU_{x_B}} = MC(G)$$

→ $MRS_A$ & $MRS_B$ are A & B's marg. rates of substitution between the private and public goods.

→ MC(G) is the marginal cost of providing an extra unit of G

→ This is the Pareto efficiency condition for public good supply

→ Remember: the public good is nonrival in consumption, so 1 extra unit of public good is fully consumed by both A and B.

→ Why?

# Variable Public Good Quantities

- To understand why, let's suppose $|MRS_A| + |MRS_B| < MC(G)$
- Ex: $|MRS_A| = \frac{1}{4}$ $|MRS_B| = \frac{1}{2}$ and $MC = 1$
- If the public good is reduced by 1 unit, A needs to be compensated with ¼ units of private good and B by ½. Hence, there is still ¼ of a dollar left over.
- Making 1 less public good unit releases more private good than the compensation payment requires $\Rightarrow$ Pareto-improvement from reduced G.
- Meaning that both consumers will be better off if G is reduced

# Variable Public Good Quantities

- Now, let's suppose $|MRS_A| + |MRS_B| > MC(G)$
- Ex: $|MRS_A| = \frac{2}{3}$ $|MRS_B| = \frac{1}{2}$ and $MC = 1$
- A would give up 2/3 units of private good to get 1 unit of G and B would give up ½. If they give up 2/3 and ½ we have more than enough to produce 1 more G and we could give back some of the extra money.
- Making 1 more public good unit is Pareto-improving
- Meaning that both consumers will be better off if G is increased

# Variable Public Good Quantities

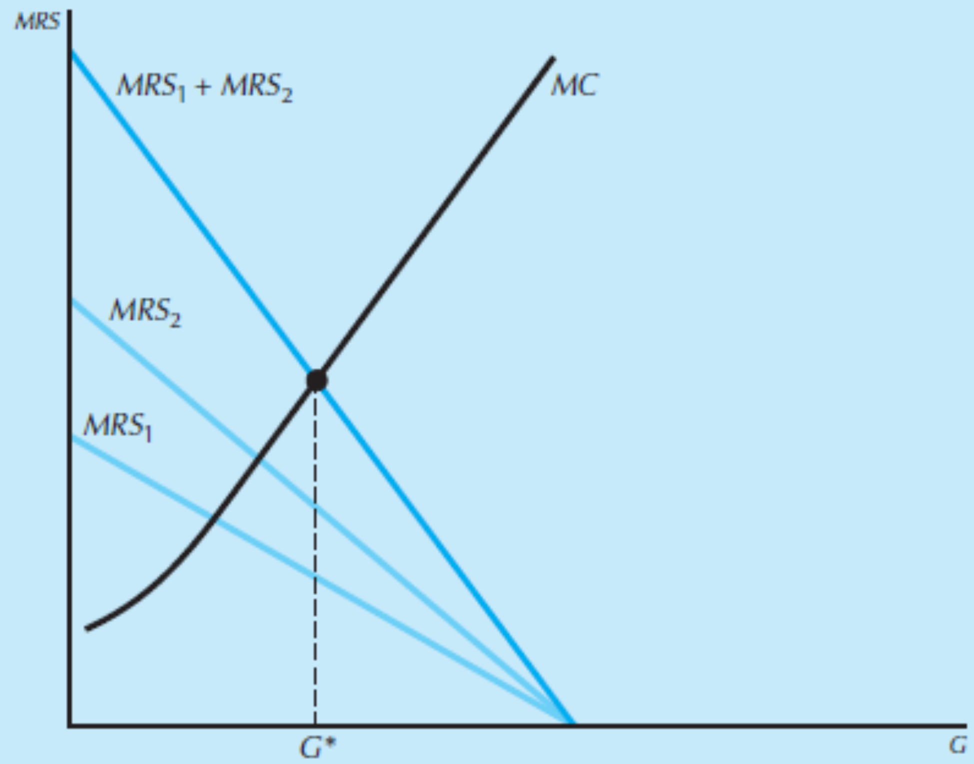
→ Hence, necessarily, efficient public good production requires

$$|MRS_A| + |MRS_B| = MC(G)$$

→ Suppose there are n consumers; i = 1,…,n. Then efficient public good production requires

$$\sum_{i=1}^{n} |MRS_i| = MC(G)$$

# Variable Public Good Quantities




**Determining the efficient amount of a public good.** The sum of the marginal rates of substitution must equal the marginal cost.

# 4. Quasilinear Preferences and Public Goods

→ Two consumers, A and B.

$U_i(x_i, G) = x_i + f_i(G)$ for i=A,B

→ Marginal utility of the private good is always 1

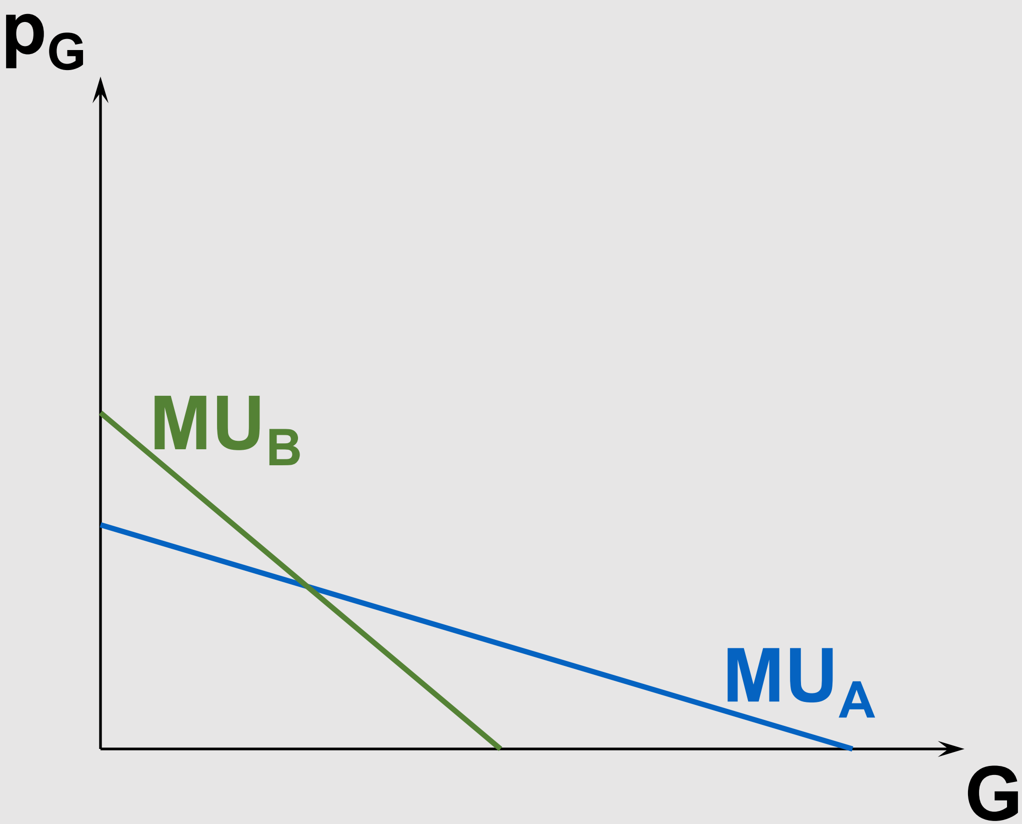
→ The $MRS_i$ are:

$$|MRS_i| = \frac{\frac{\Delta U_i(x_1, G)}{\Delta G}}{\frac{\Delta U_i}{\Delta x_1}} = \frac{\Delta U_i(x_1, G)}{\Delta G} = f'_i(G)$$

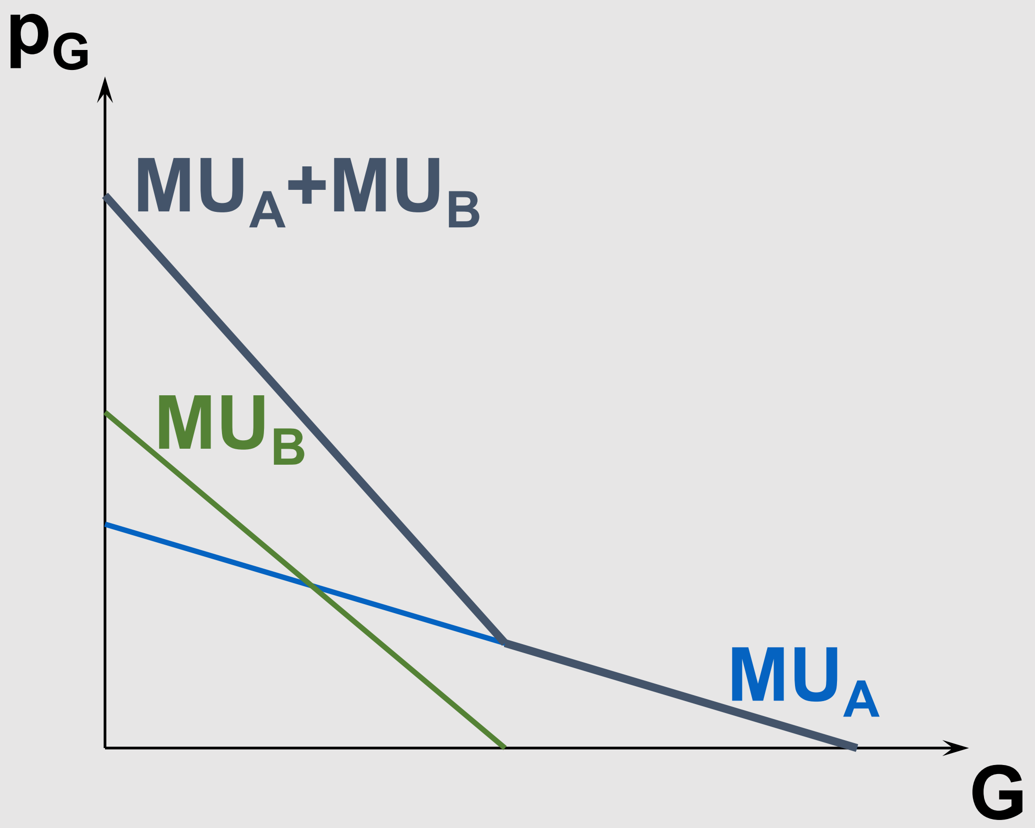
Since $|MRS_A| + |MRS_B| = MC(G)$

Then $f'_A(G) + f'_B(G) = MC(G)$

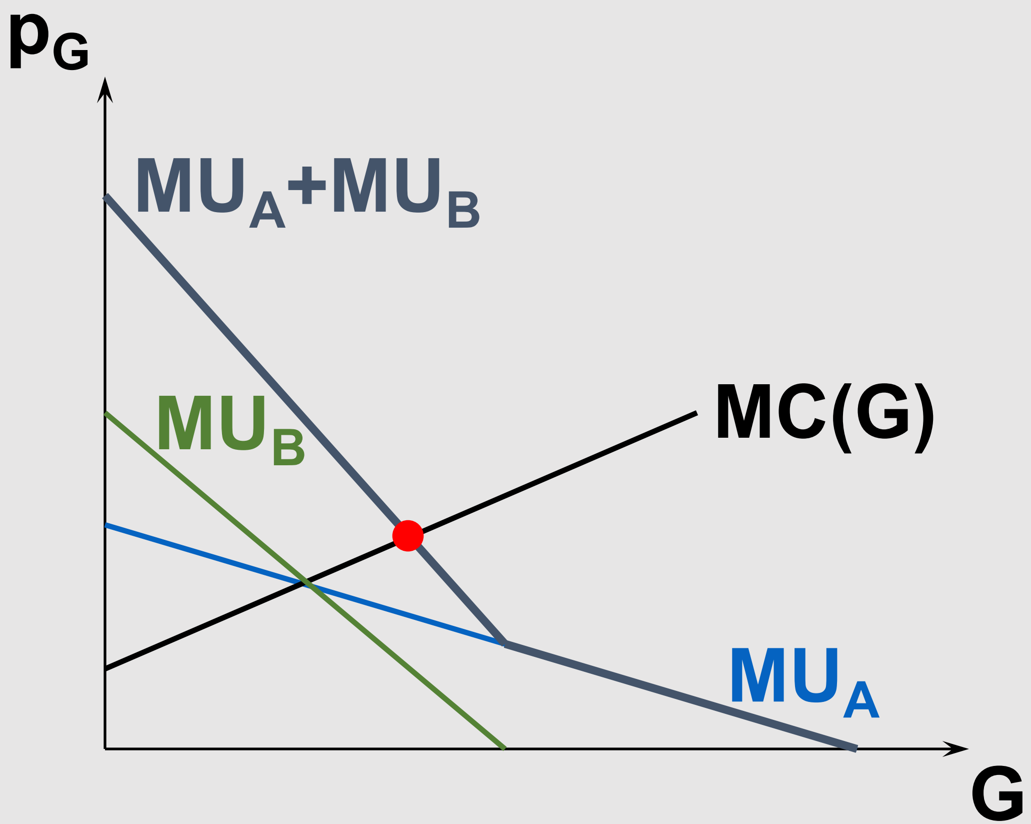
# Quasilinear Preferences and Public Goods



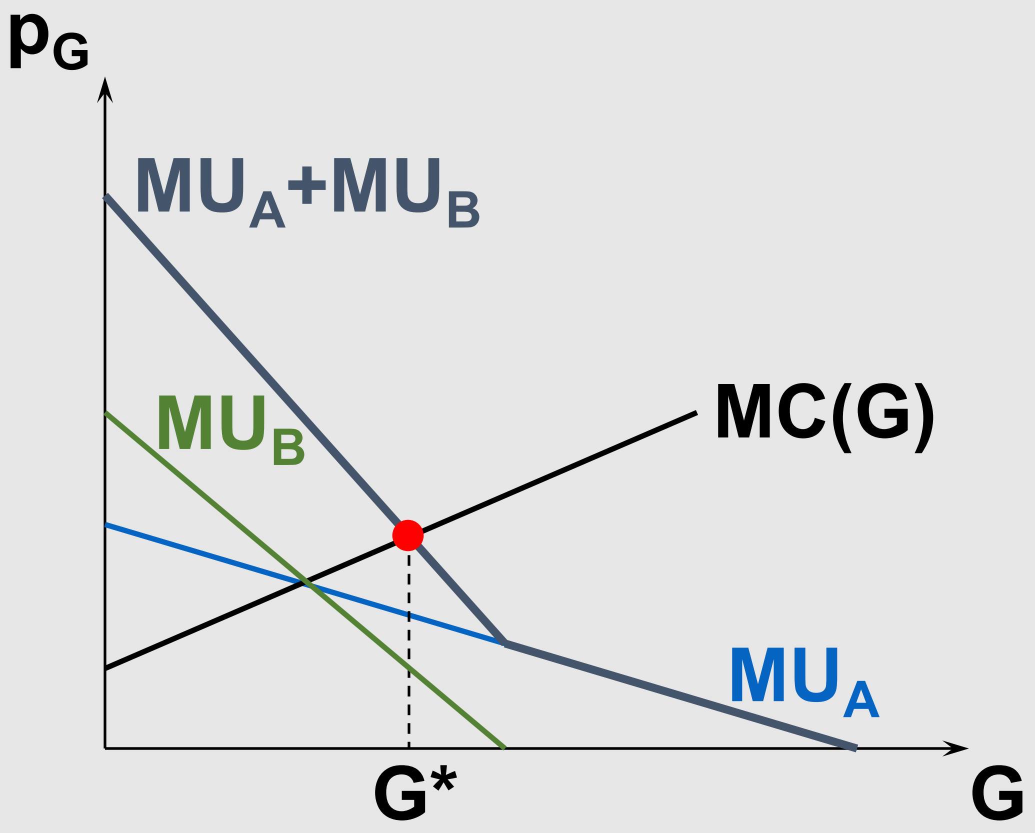
# Quasilinear Preferences and Public Goods



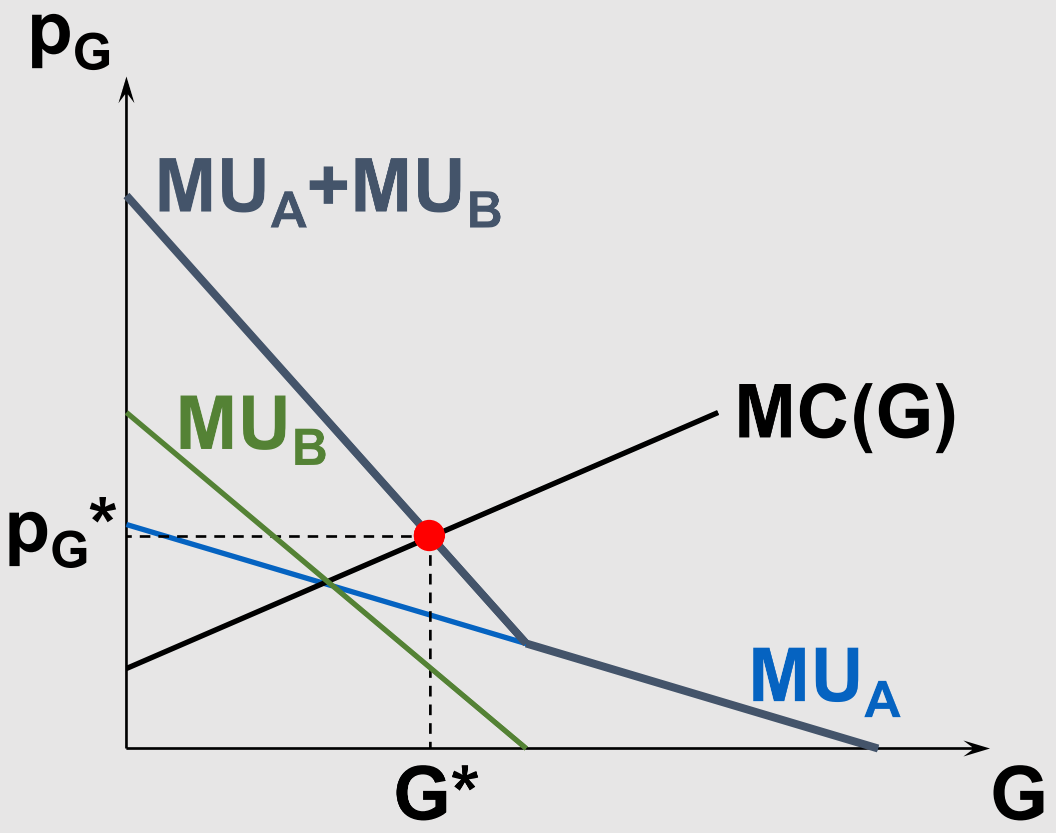
# Quasilinear Preferences and Public Goods



# Quasilinear Preferences and Public Goods

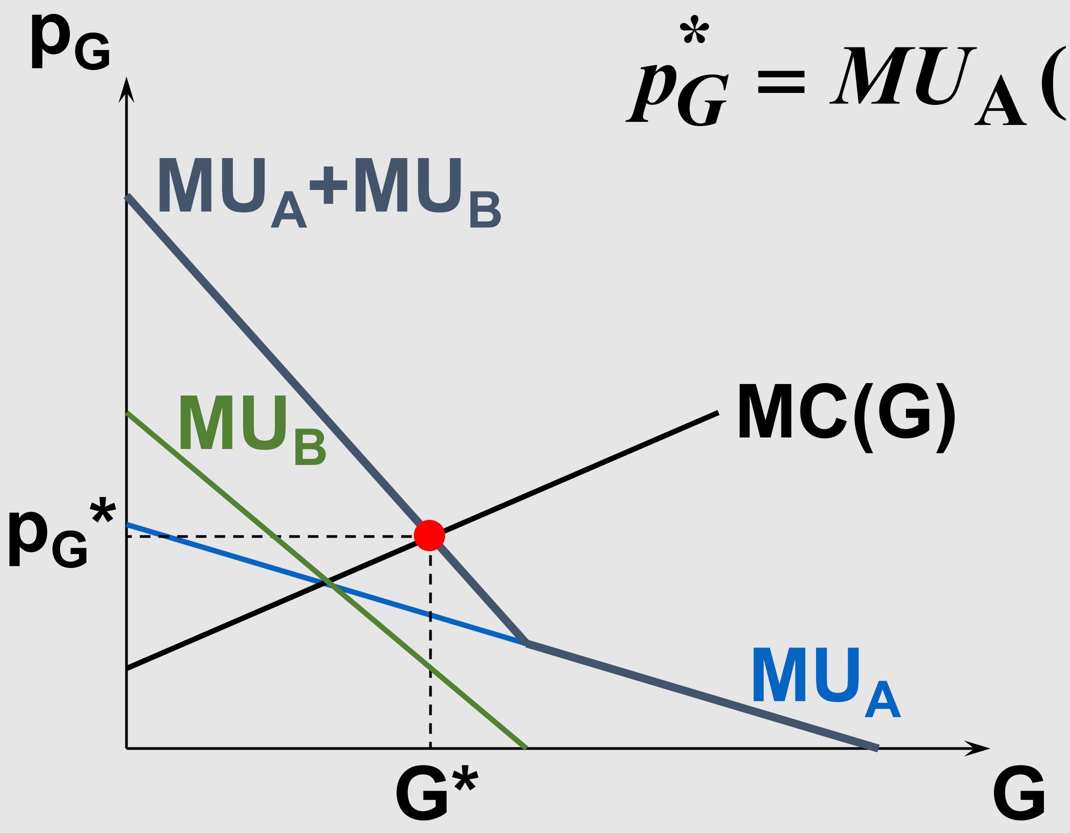


# Quasilinear Preferences and Public Goods



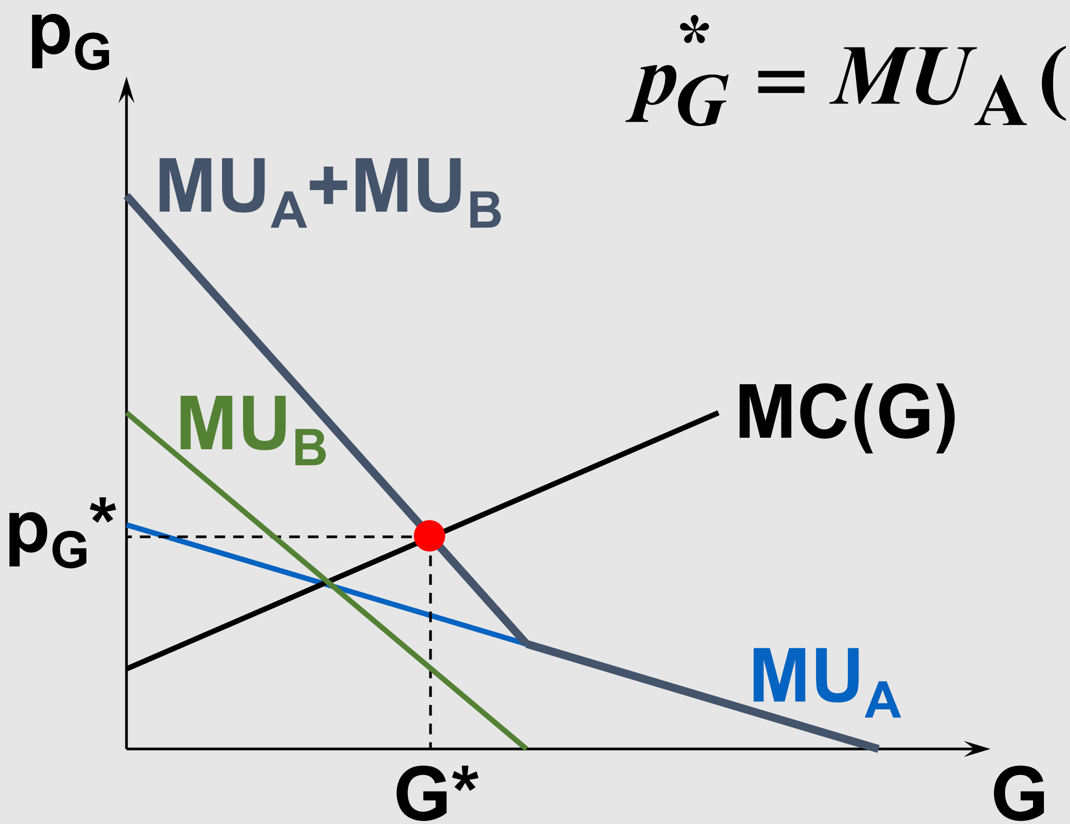
# Quasilinear Preferences and Public Goods

$$p_G^* = MU_A(G^*) + MU_B(G^*)$$



# Quasilinear Preferences and Public Goods

$$p_G^* = MU_A(G^*) + MU_B(G^*)$$




**Efficient public good supply requires A & B to state truthfully their marginal valuations.**

# 5. Free-Riding Revisited

→ When is free-riding ***individually*** rational?

Suppose that

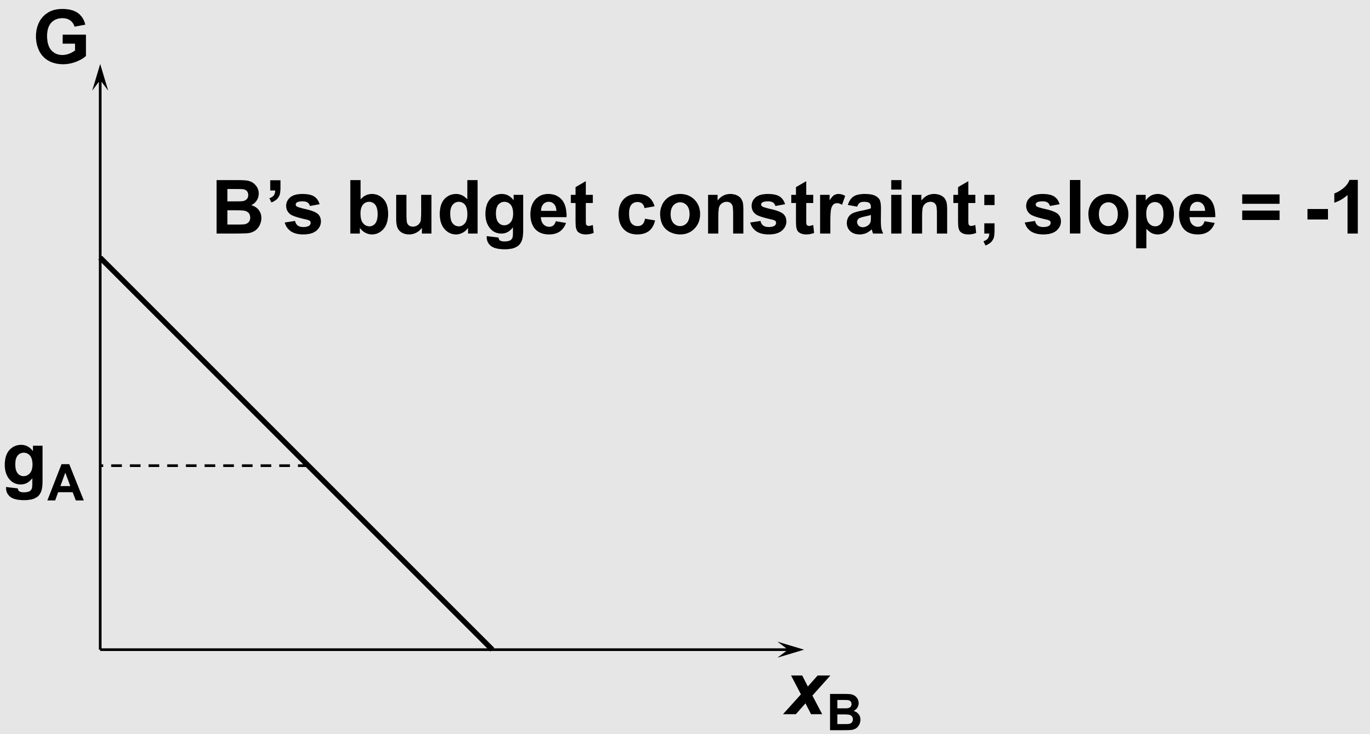
→ Individuals can contribute only positively to public good supply; nobody can lower the supply level. $g_1 \geq 0$

Given that A contributes $g_A$ units of public good, B's problem is

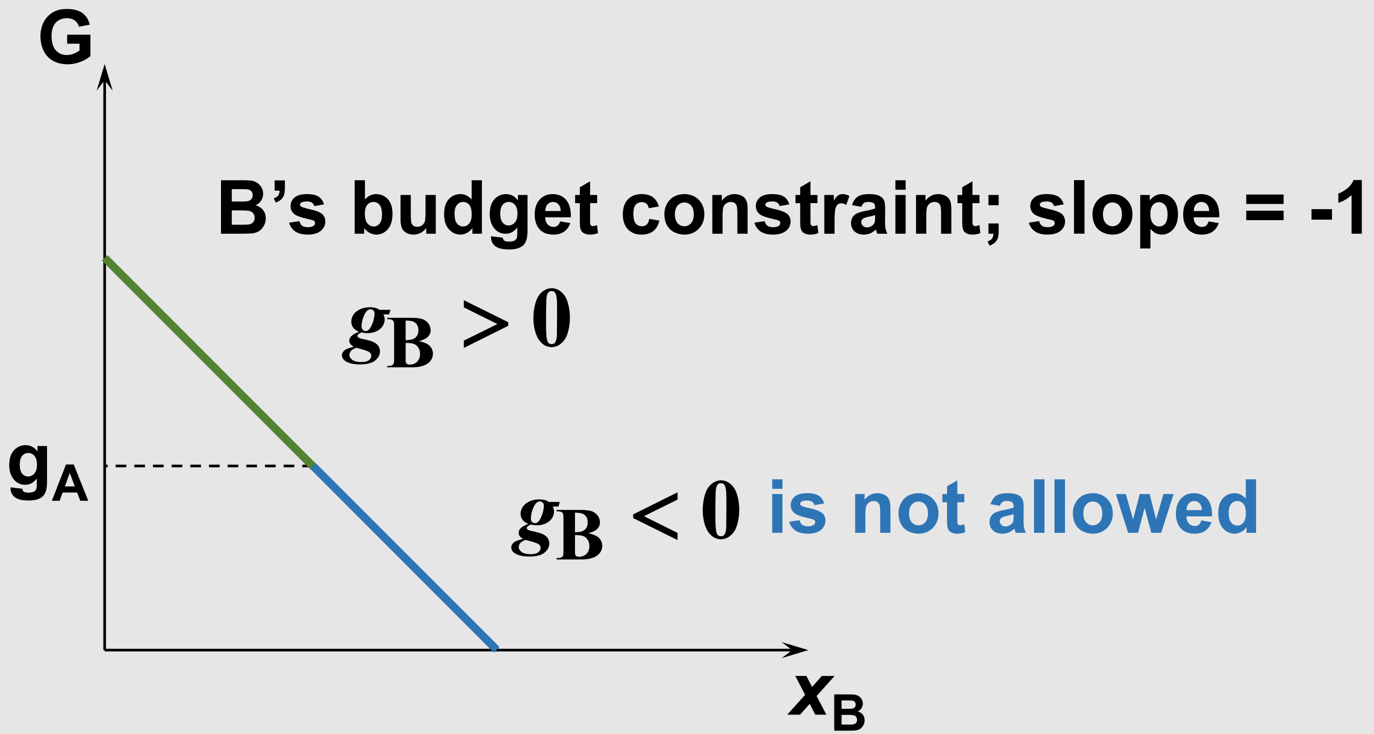
$$\max_{x_B, g_B} U_B(x_B, g_A + g_B)$$

Subject to $x_B + g_B = w_B$ and $g_B \geq 0$

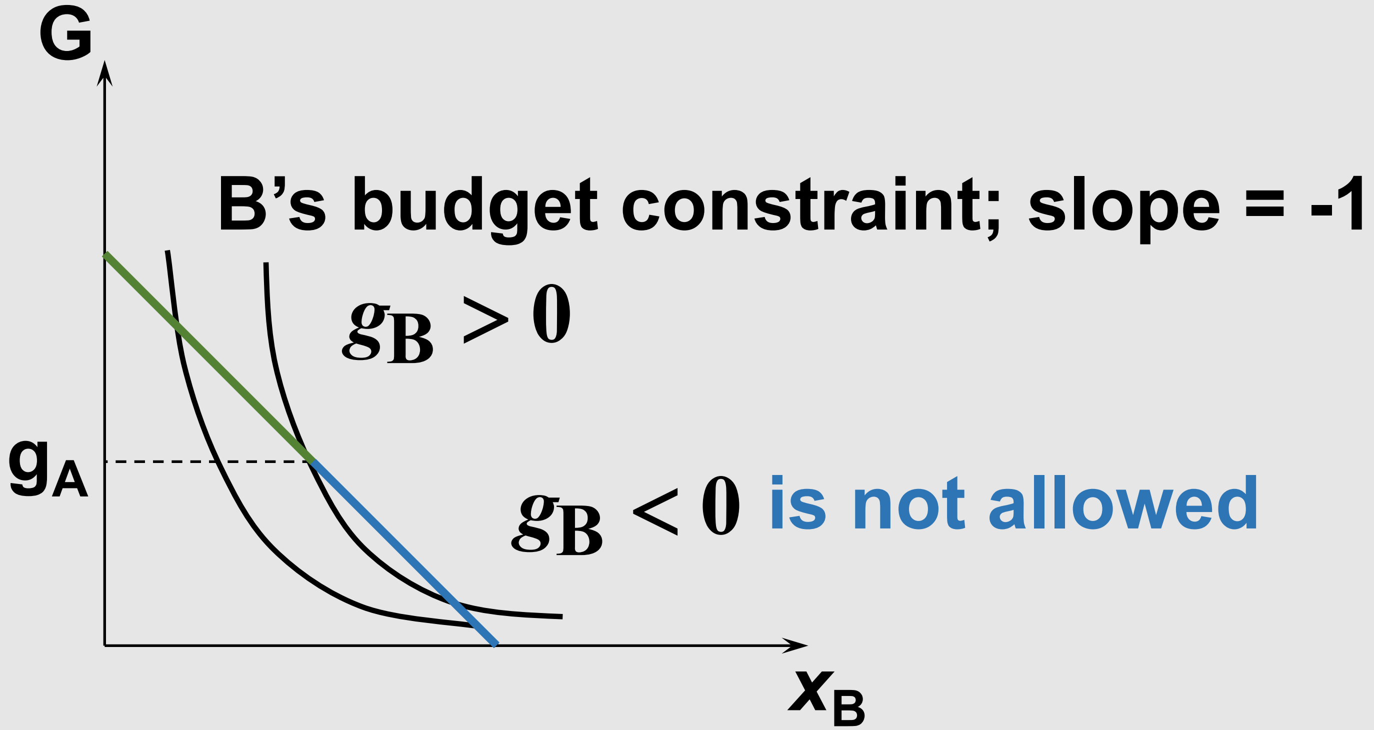
# Free-Riding Revisited



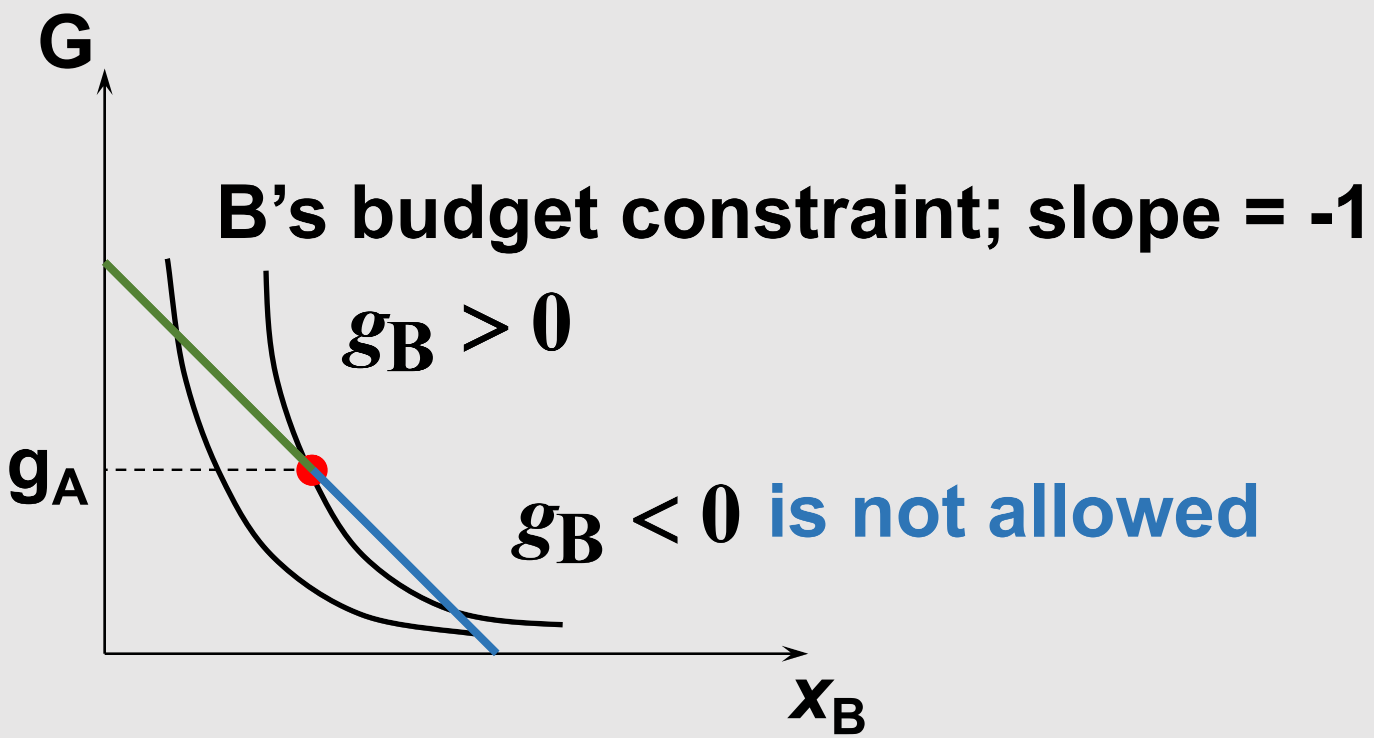
# Free-Riding Revisited



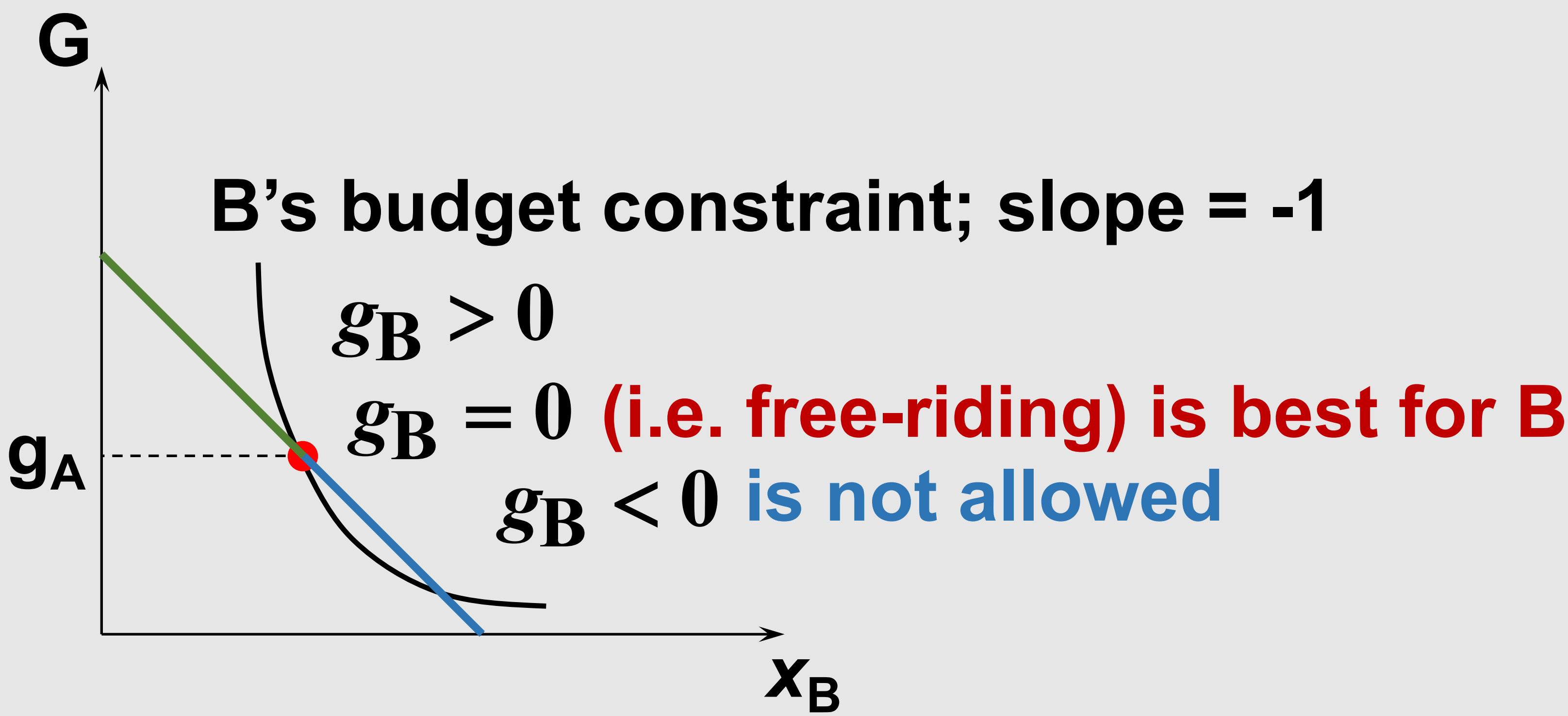
# Free-Riding Revisited



# Free-Riding Revisited



# Free-Riding Revisited




**Free-riding is rational in such cases.**

# Free-Riding Intuition

- → Free rider problem: When an investment has a personal cost but a common benefit, individuals will underinvest.

- → Because of the free rider problem, the private market undersupplies public goods

Another way to see it:

- → Private provision of a public good creates a positive externality (as everybody else benefits)
  - Goods with positive externalities are under-supplied by the market

# Free-Riding Intuition

- 2 individuals with identical utility functions defined on x private good (cookies) and G public good (fireworks)

$$G = g_A + g_A$$

- Utility of individual i is $\mathrm{U_i} = 2\log(\mathrm{x_i}) + \log(g_A + g_B)$
- Budget $x_i + g_i = 100$
- Individual A chooses $\mathrm{g_A}$ to maximize $2\log(100 - \mathrm{g_A}) + \log(g_A + g_B)$ taking $g_B$ as given
- First order condition:
- $-\frac{2}{(100-\mathrm{g_A})} + \frac{1}{g_A+g_B} = 0 \qquad \mathrm{g_A} = \frac{100-2g_B}{3}$
- Note that $g_A$ goes down with $g_B$ due to the free rider problem (called the reaction curve)
- Symmetrically, we have $\mathrm{g}_B = \frac{100-2g_A}{3}$

# Can Private Provision Overcome Free Rider Problem?

→ The free rider problem does not lead to a complete absence of private provision of public goods. Private provision works better when:

1. Some Individuals Care More than Others:
   - Private provision is particularly likely to surmount the free rider problem when individuals are not identical, and when some individuals have an especially high demand for the public good.
   - Ex: driveway between a mansion and a shelter
2. Altruism:
   - When individuals value the benefits and costs to others in making their consumption choices.
   - Ex: parteon.com for independent creative content

# 6. Collective decision mechanisms

For private goods

→ A competitive market mechanisms will achieve a Pareto-optimal allocation

- Important assumption: an individual’s consumption did not affect other people’s utility

For public goods

→ Utilities of the individuals are linked since everyone consumes the same amount of the public good

- Market provision will not be Pareto efficient
- Alternatives:
  - Command mechanism
  - Voting system

# Voting system

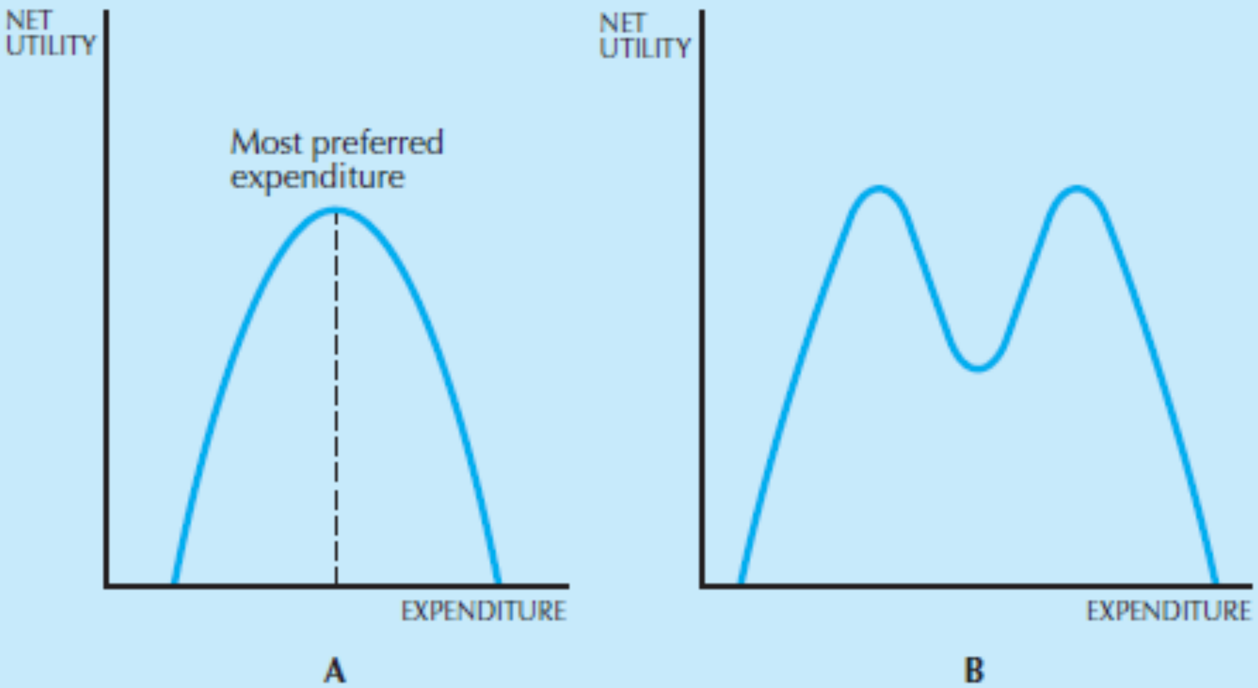
→ Let's imagine that the consumers are voting about the size of some public good

- Same problems with voting than in Chapter 3…
- Non transitivity
- Sensitive to manipulation

→ What restrictions on preferences will allow us to rule them in (or out)?

- **Single-peaked preferences:** net utility of expenditure on the public good rises at first due to the benefits of the public good but then eventually falls, due to the costs of providing it

# Single peaked preferences




**Shapes of preferences.** Single-peaked preferences are shown in panel A and multiple peaked preferences in panel B.

# Voting system

→ With single-peaked preferences, there will never be intransitivity

→ The chosen result is the median expenditure: one-half of the population wants to spend more, and one-half wants to spend less

Is it Pareto efficient?

→ In general, No. Since it doesn't say anything about *how much more.* Efficiency takes this kind of information into account,

→In most cases, voting will not lead to an efficient outcome.

# Voting system

Problem

→ Consumers may not have good incentives to report true utility values

→ Challenge: determine TRUE individual utility functions

→ A scheme that makes it rational for individuals to reveal truthfully their private valuations of a public good is a **revelation mechanism**.

→ E.g. the Groves-Clarke taxation scheme

# 7. Lindahl prices

→ With Public goods, everyone must consume the same quantity of G

→ Idea: can we come up with "individualized prices" that vary across individuals reflecting their willingness to pay? → **Lindahl Prices**

→ With ordinary private goods: people all face the same price; they choose the quantity they wish to consume.

→ With a pure public good (opposite situation): everyone consumes the same quantity of the good, but people's prices are different.

→ Lindahl price is a **hypothetical** price that a person would be willing to pay for a little more of the good

# Lindahl prices

→ Rule (with 2 consumers):

$$p_G^A(G, R_A) + p_G^B(G, R_B) = c$$

Conditions for Lindahl equilibrium (general)

- Public good must be fully financed $\sum p_G^i = c$
- All individuals must demand the same quantity of G

→ Definition of equilibrium

- Equilibrium is a set of prices such that all persons demand the same level of the public good.

# Lindahl prices - example

- 2 consumers

$$U^A(x_A, G) = 2\ln(x_A) + \ln(G)$$
$$U^B(x_B, G) = \ln(x_B) + 2\ln(G)$$

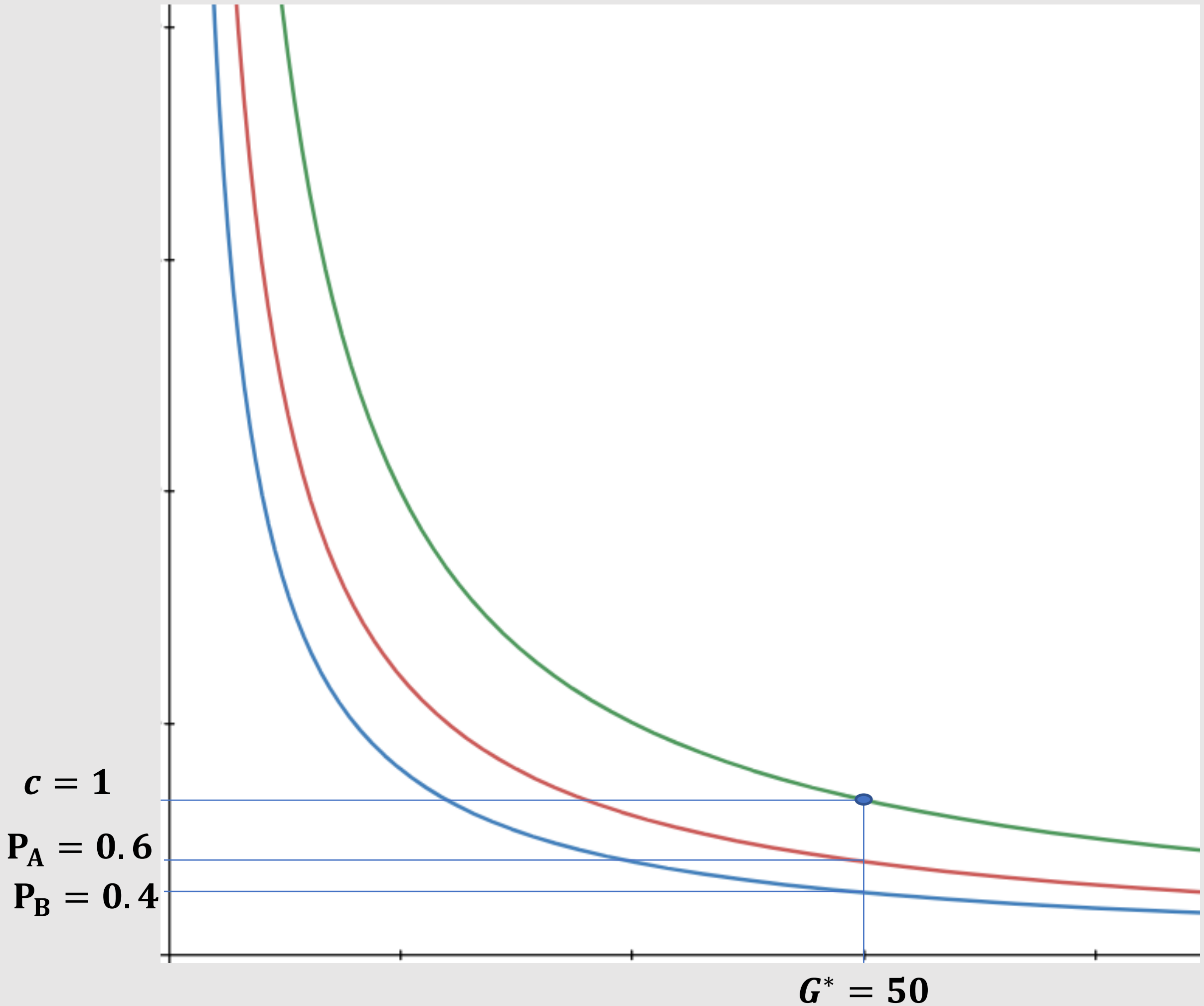
- Budget constraint is $\mathrm{x_i} + \mathrm{p_i G} = R_i$
- Find inverse demand functions using $\mathrm{MRS}^i_{Gx} = p_i$
- We obtain

$p_A = \frac{1}{3}\frac{1}{G}R_A$ and $p_B = \frac{2}{3}\frac{1}{G}R_B$

# Lindahl prices - example

- Let’s asume c=1. Then, we need to find G such that $\mathrm{p_A} + \mathrm{p_B} = 1$
- We obtain $G = \frac{R_A}{3} + \frac{2}{3} R_B$
- If $\mathrm{R_A} = 90$ and $\mathrm{R_B} = 30$
- Then $\mathrm{G} = 50$ and $\mathrm{p_A} = 0.6$ and $\mathrm{p_B} = 0.4$

# Lindahl prices - example




$$p_A = \frac{30}{G}$$

$$p_B = \frac{20}{G}$$

$$c = \frac{30}{G} + \frac{20}{G}$$

# Lindahl prices - intuition

→ Putting together the optimal condition and the Lindahl prices condition:

$$\mathrm{MRS}^{i}_{Gx} = p_i \qquad \text{and} \qquad p^{A}_{G}(G, R_A) + p(G, R_B) = c$$

→ We obtain that

$$\sum \mathrm{MRS}^{i}_{Gx} = \sum p_i = c$$

→ Hence, the Lindahl equilibrium satisfies the Samuelson Rule and outcome is Pareto efficient.

→ Intuition: each individual bears only a fraction of the cost of the Public Good, so people end up picking the right amount of the PG

→ General Conclusion: efficiency can be attained with public goods by the use of personalized prices.