Chapter 2: Exchange with production

Ch 32 in H. Varian 8th Ed.

Slides by Mariona Segú, CYU Cergy Paris Université

Inspired by Michael D. Robinson, Mount Holyoke College

Exchange Economies (revisited)

So far…

→ No production, only endowments, so no description of how resources are converted to consumables.

→ General equilibrium: all markets clear simultaneously.

→ 1st and 2nd Fundamental Theorems of Welfare Economics.

Now

→ Add input markets, output markets, describe firms' technologies, the distributions of firms' outputs and profits … That's not easy!

Outline

1. Robinson Crusoe's Economy
2. Robinson Crusoe as a Firm
3. Fundamental Theorems of Welfare Economics
4. Production Possibilities
5. Comparative Advantage

1. Robinson Crusoe's Economy

→ One agent, RC.

→ Endowed with a fixed quantity of one resource → 24 hours.

→ Use time for labor (production) or leisure (consumption).

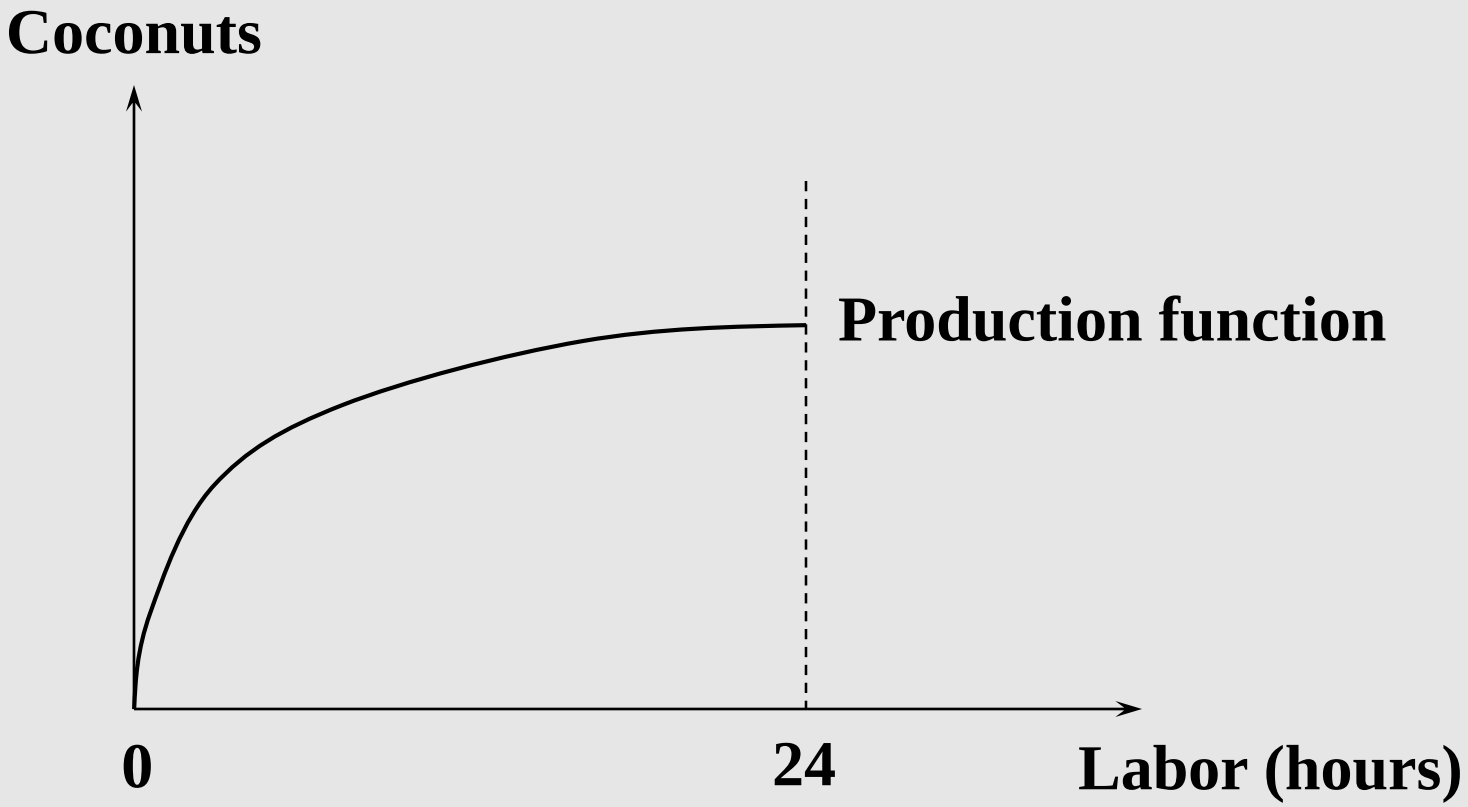
→ Labor time = L. Leisure time = 24 - L.

→ What will RC choose?

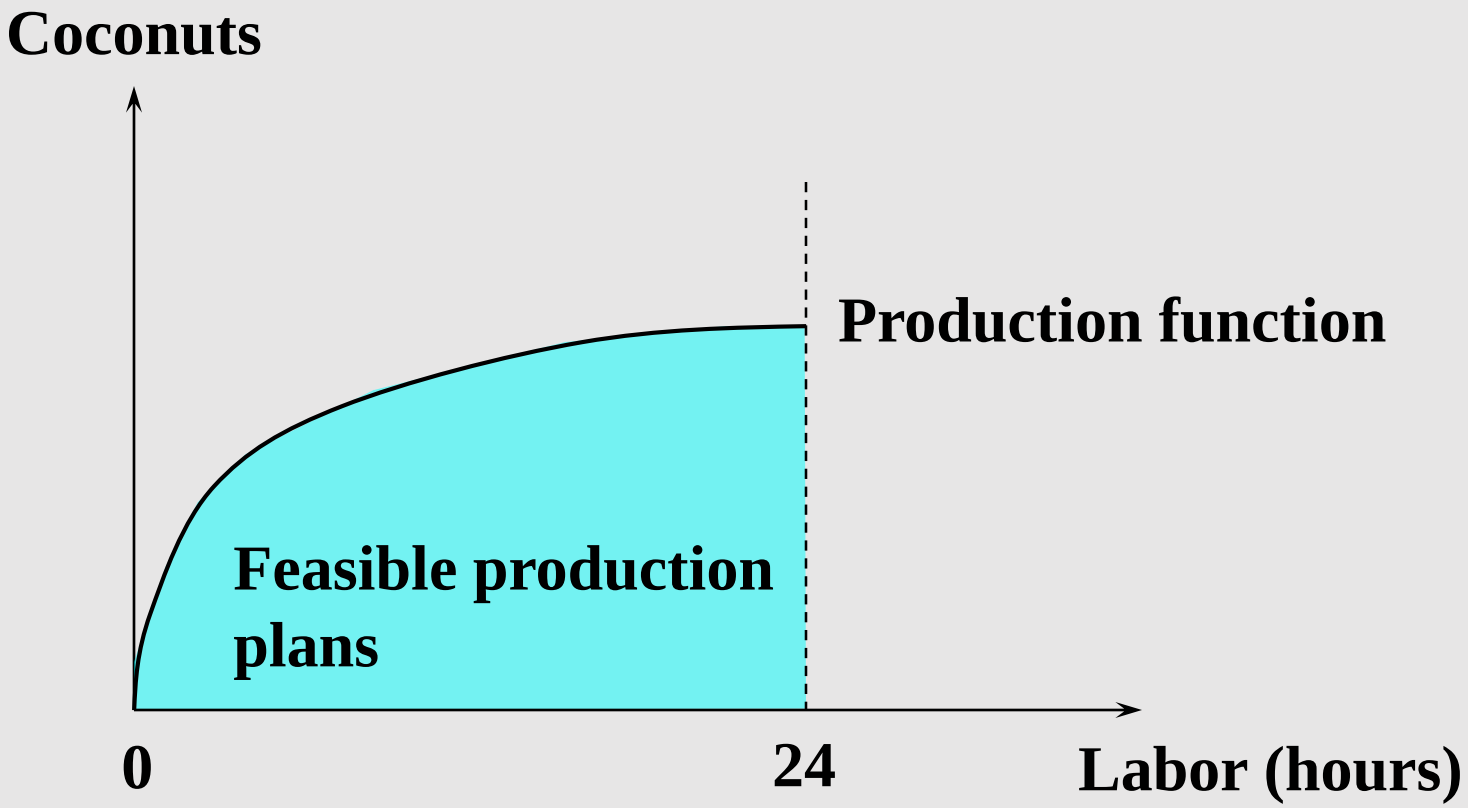
Robinson Crusoe's Technology

→ Technology: Labor produces output (coconuts) according to a concave production function.

Robinson Crusoe's Technology



Robinson Crusoe's Technology

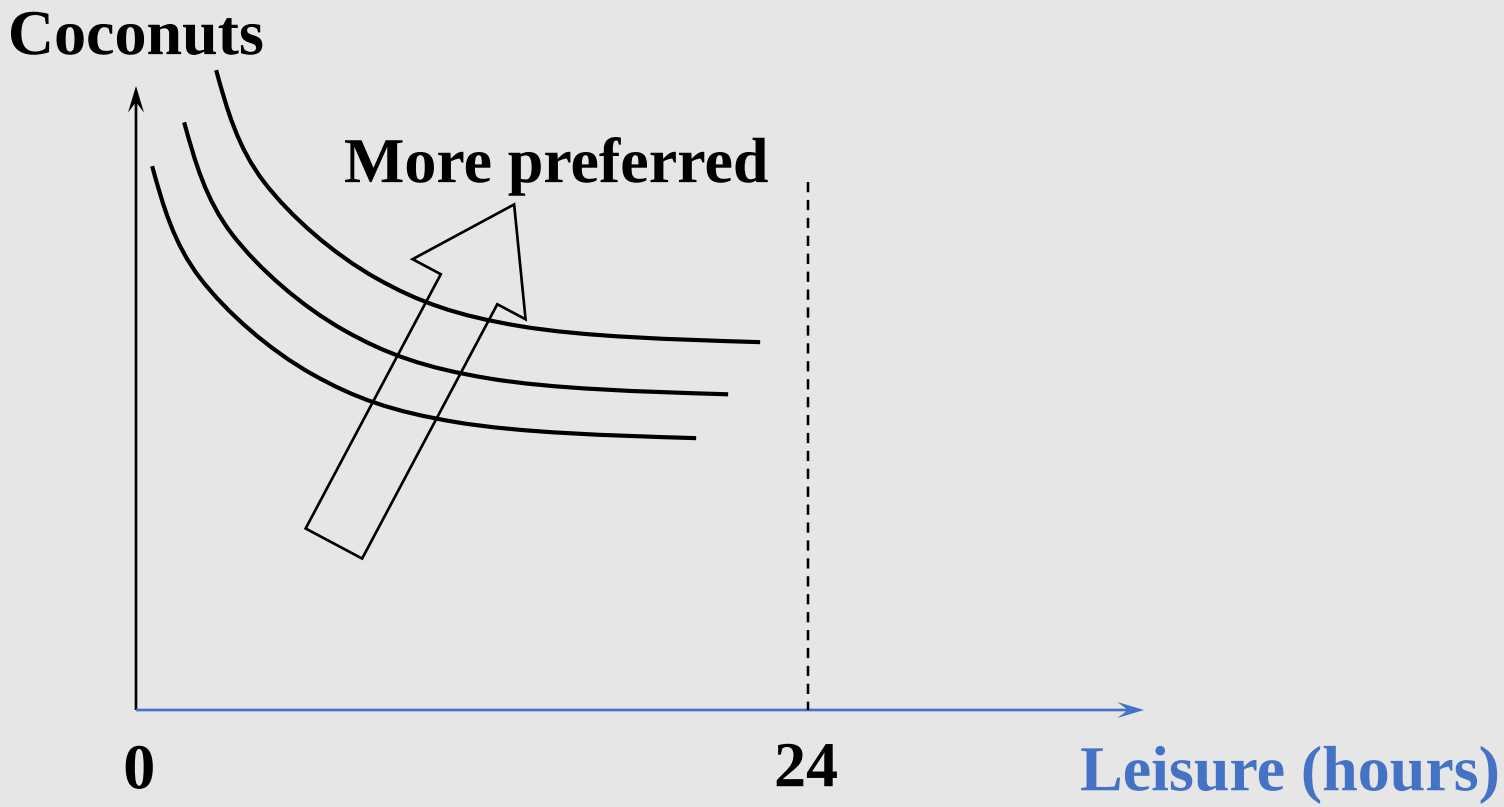


Robinson Crusoe's Preferences

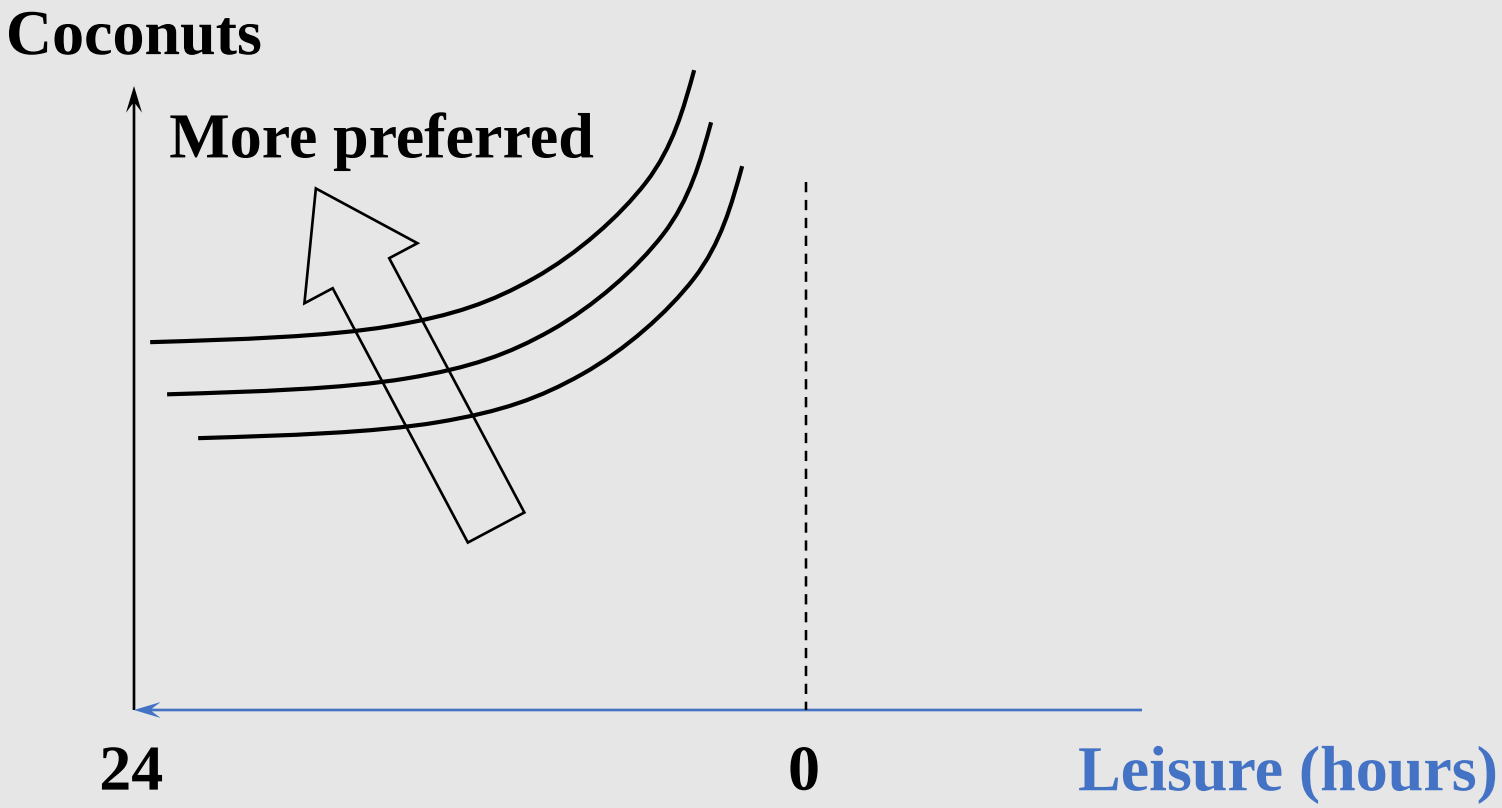
→ RC's preferences:

- coconut is a good
- leisure is a good

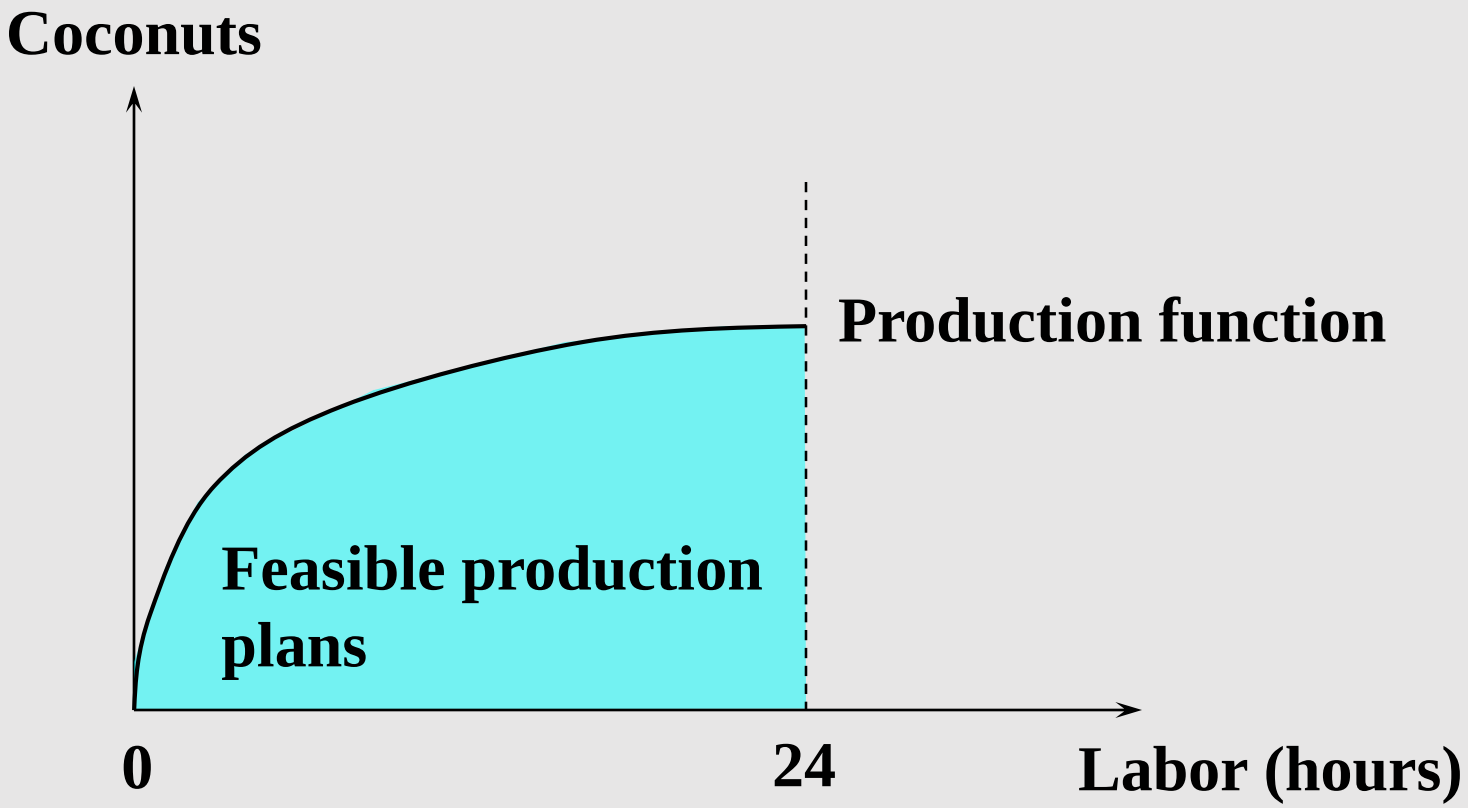
Robinson Crusoe's Preferences



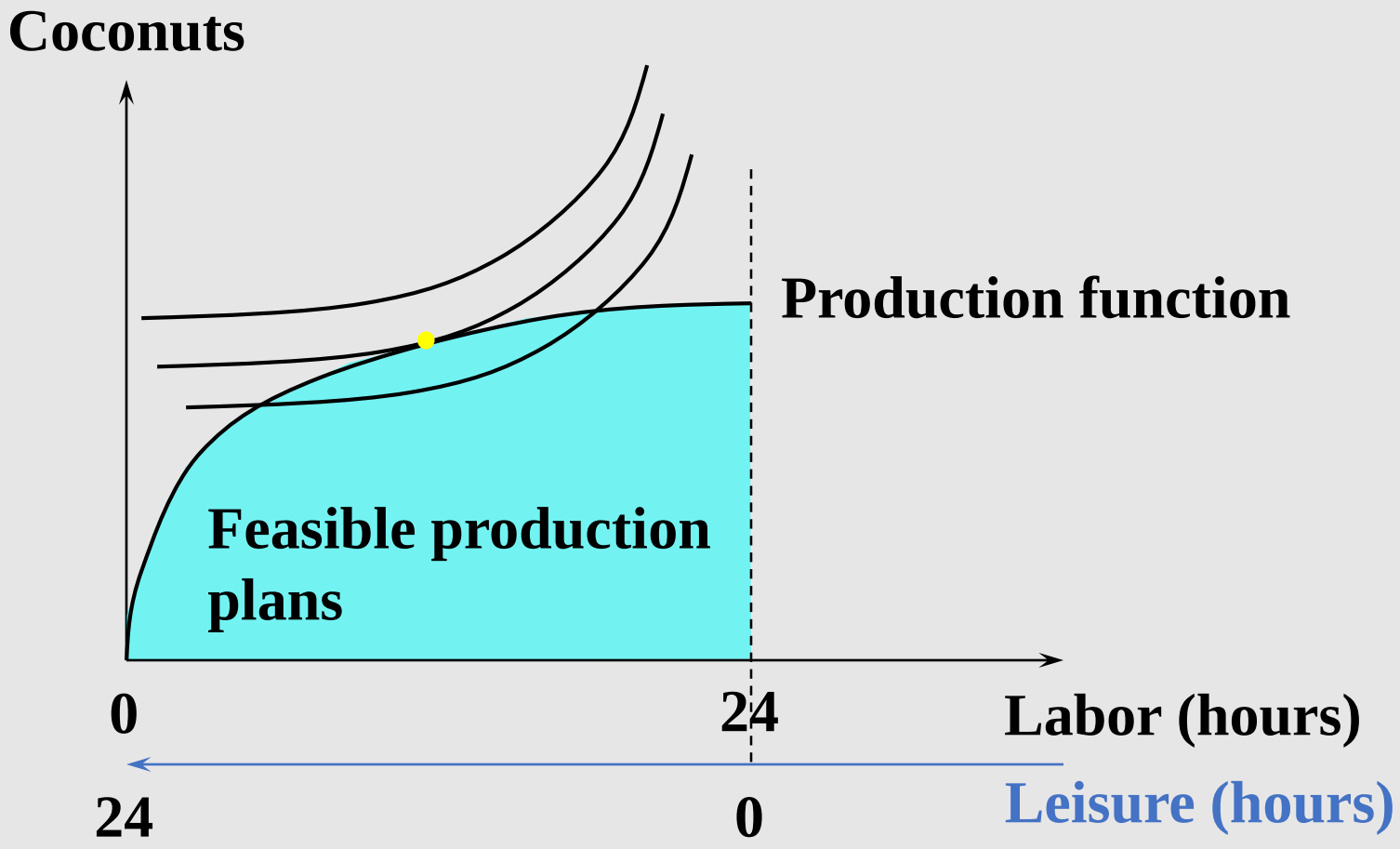
Robinson Crusoe's Preferences



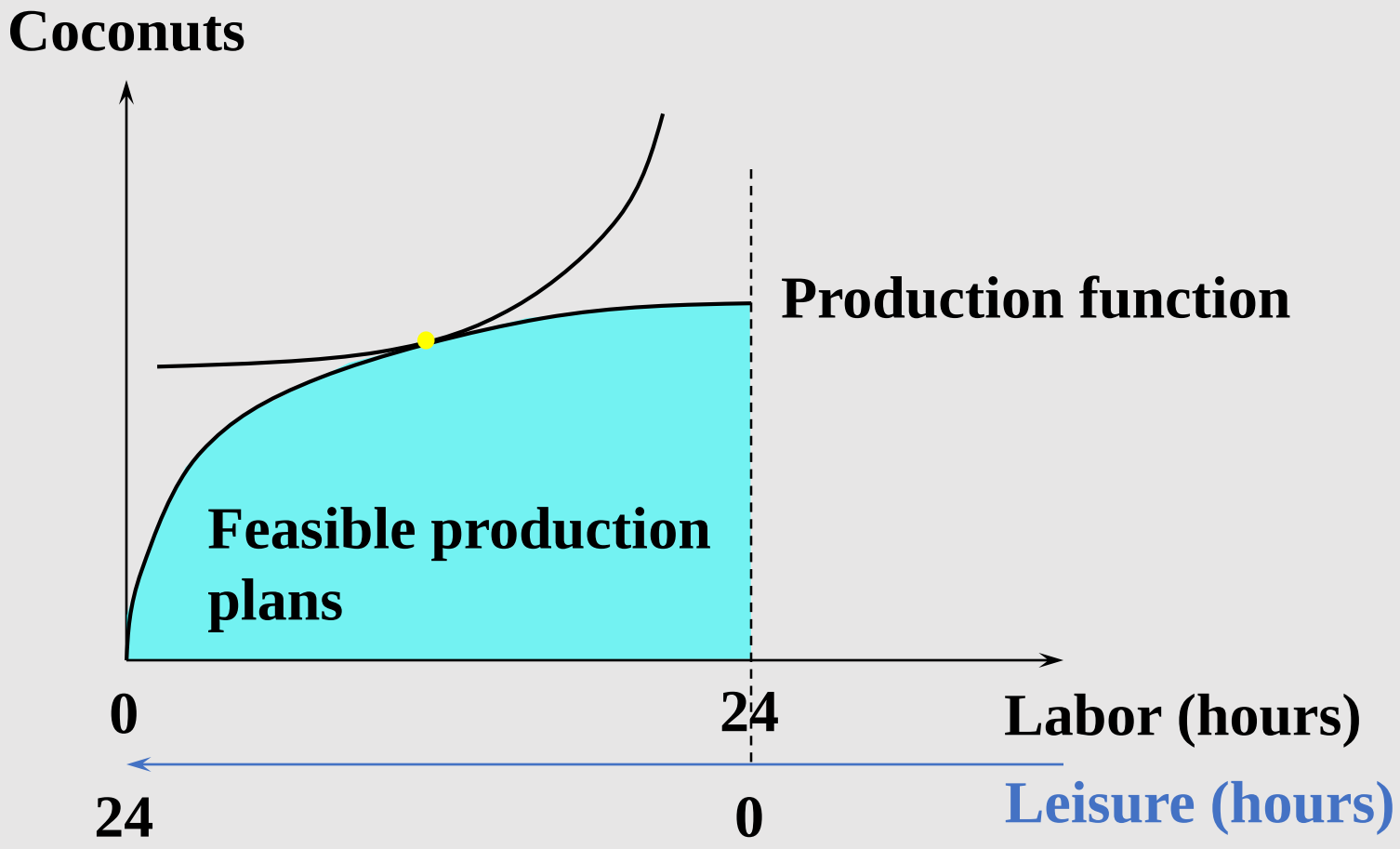
Robinson Crusoe's Choice



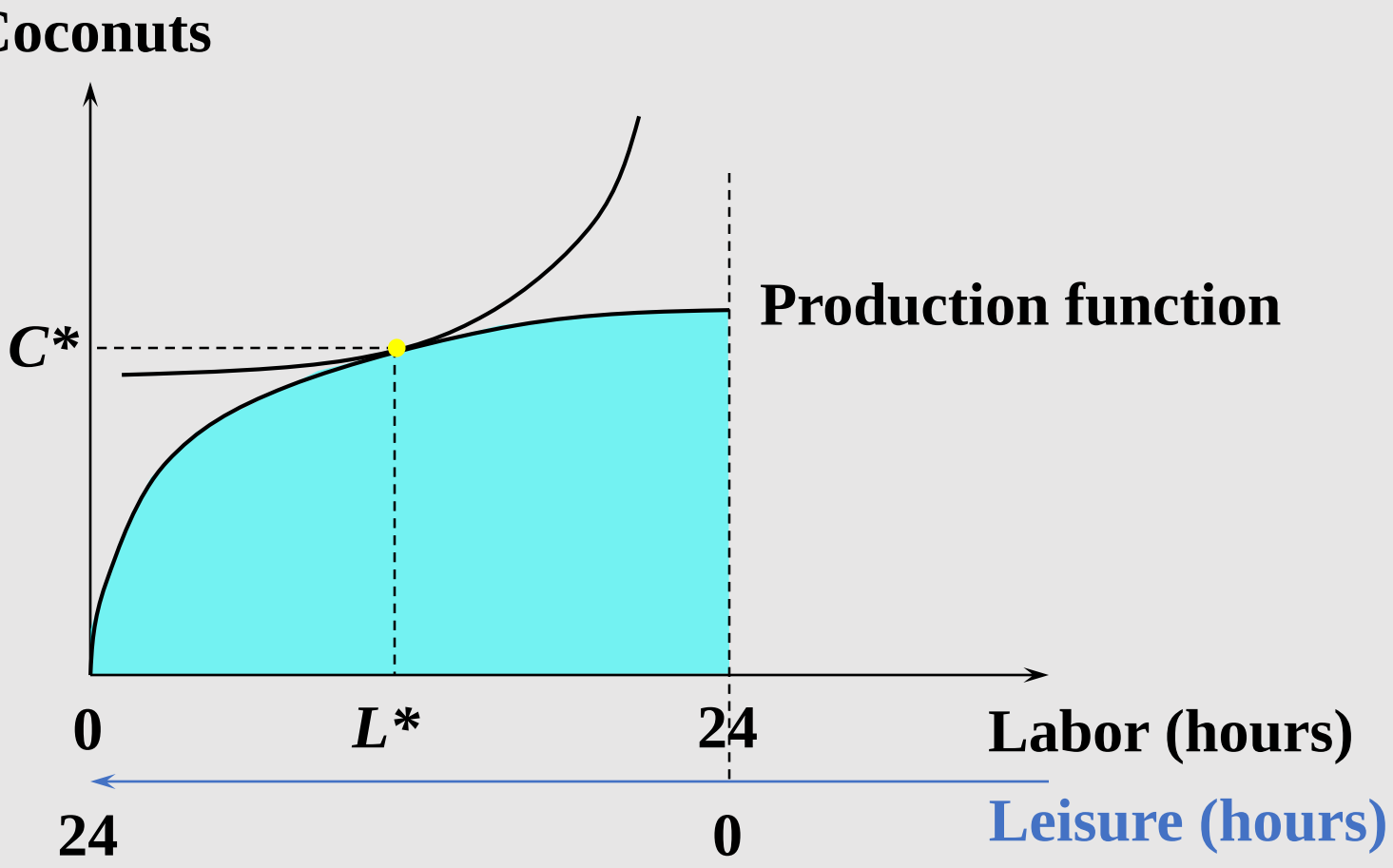
Robinson Crusoe's Choice



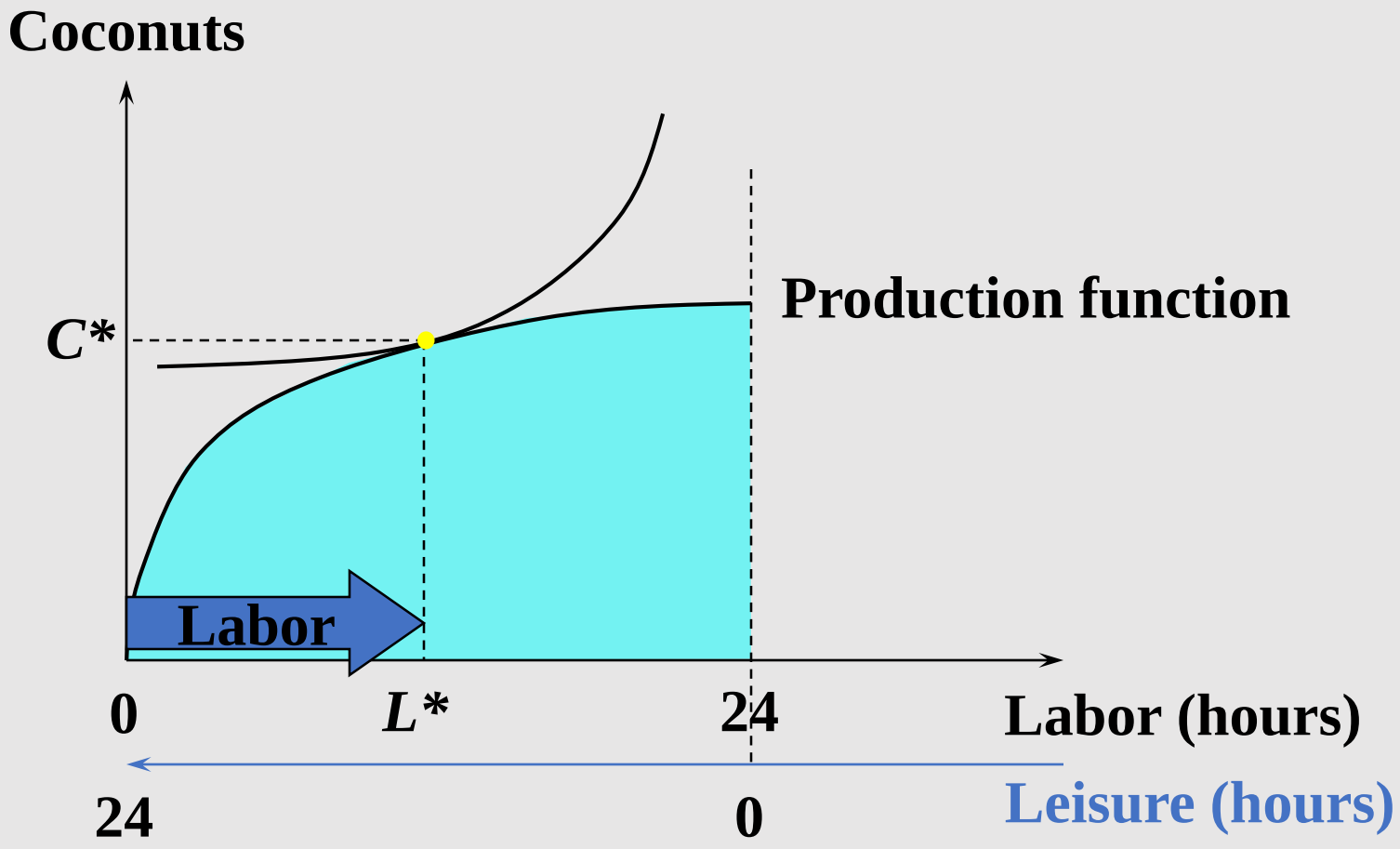
Robinson Crusoe's Choice



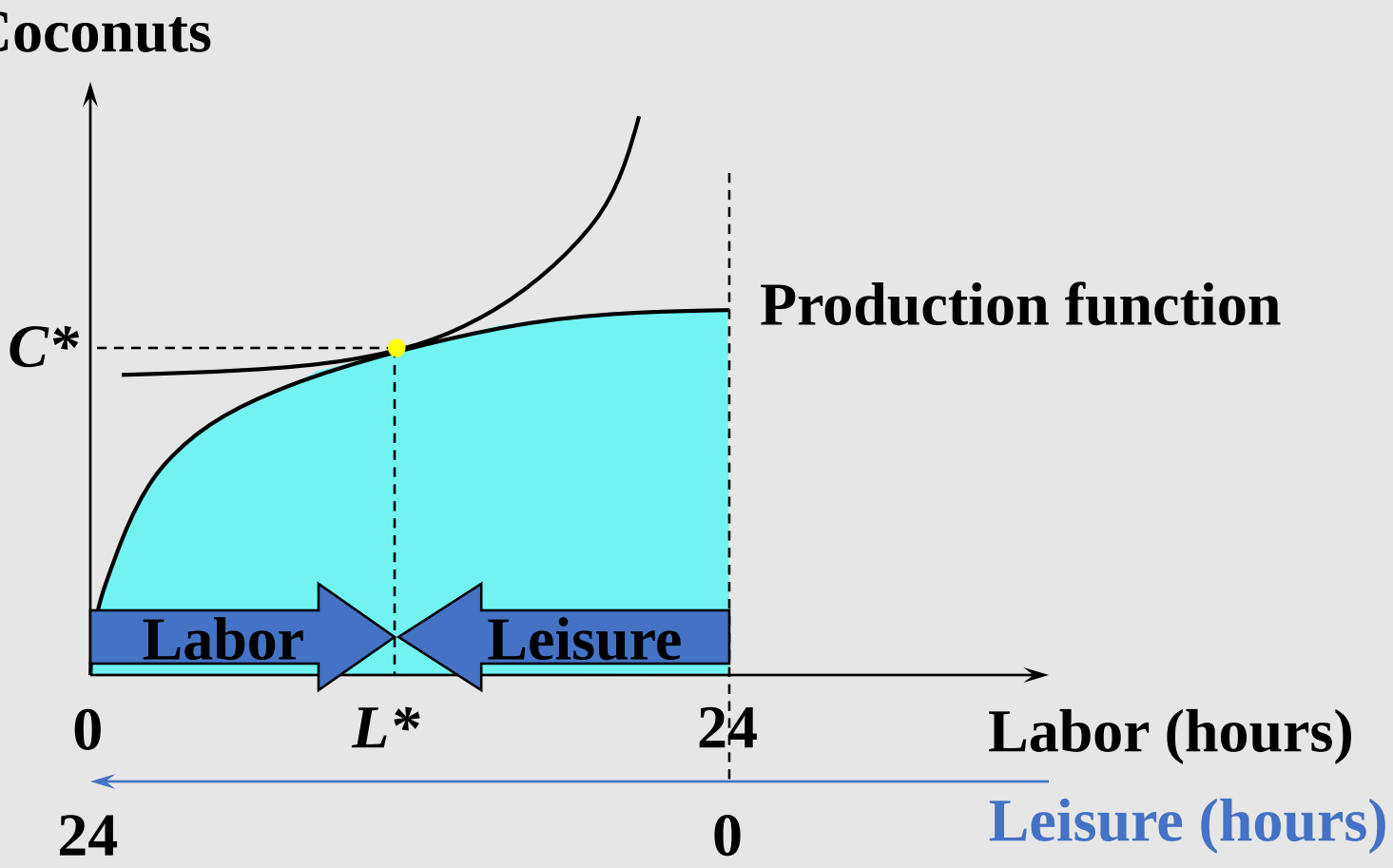
Robinson Crusoe's Choice



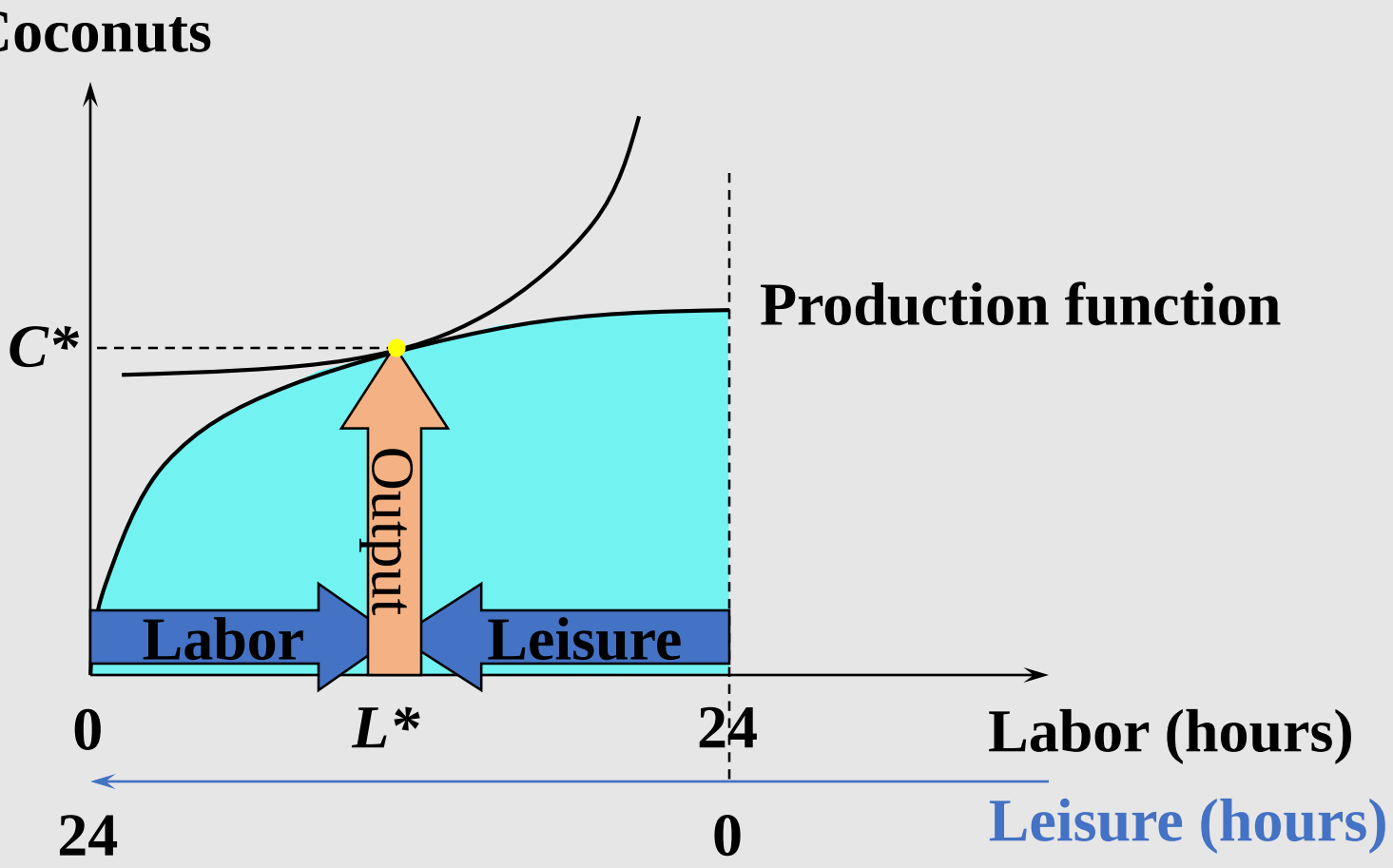
Robinson Crusoe's Choice



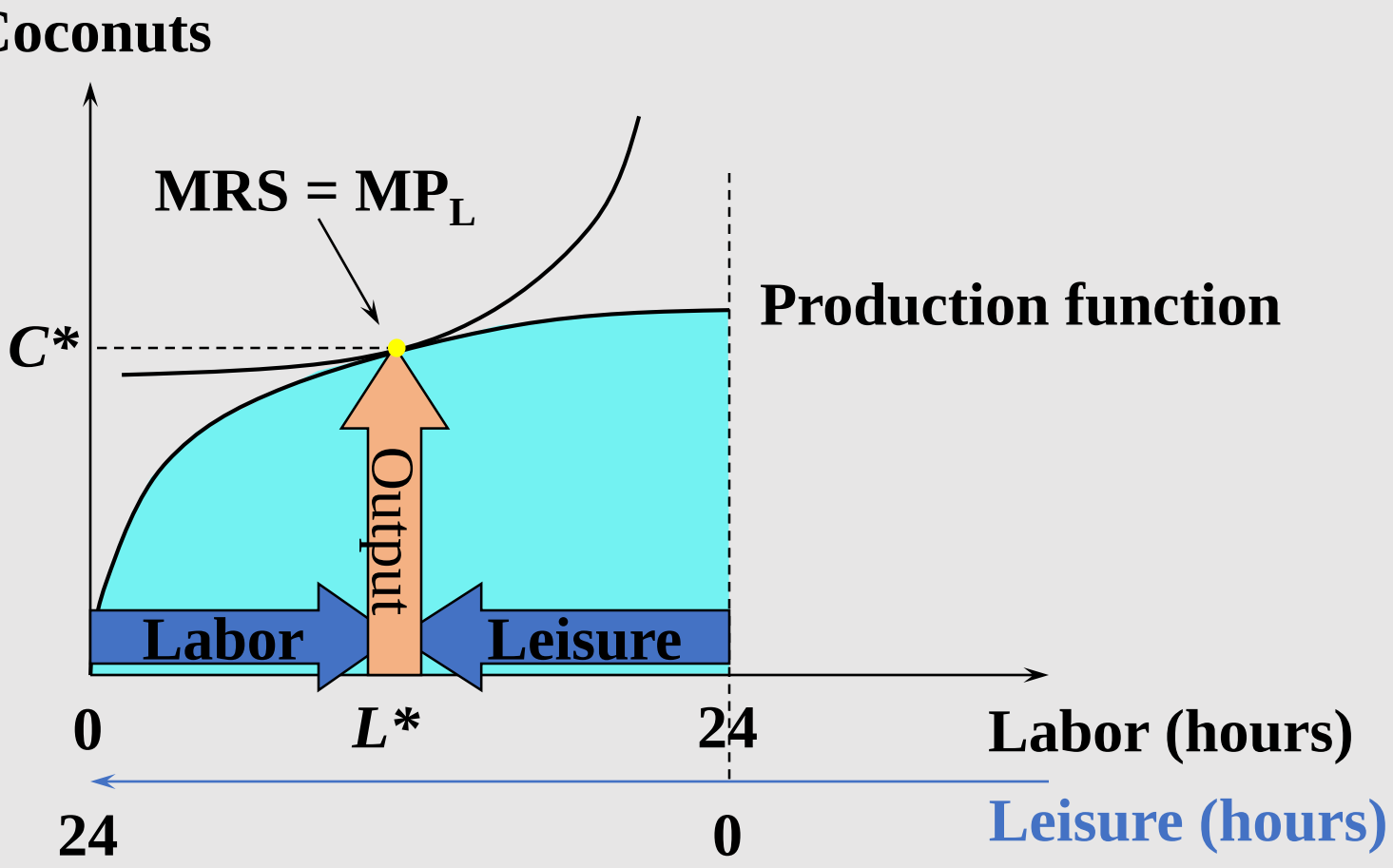
Robinson Crusoe's Choice



Robinson Crusoe's Choice



Robinson Crusoe's Choice



2. Robinson Crusoe as a Firm

→ Now suppose RC is both a utility-maximizing consumer and a profit-maximizing firm.

→ Use coconuts as the numeraire good; i.e. price of a coconut = $1.

→ RC's wage rate is w.

→ Coconut output level is C.

Robinson Crusoe as a Firm

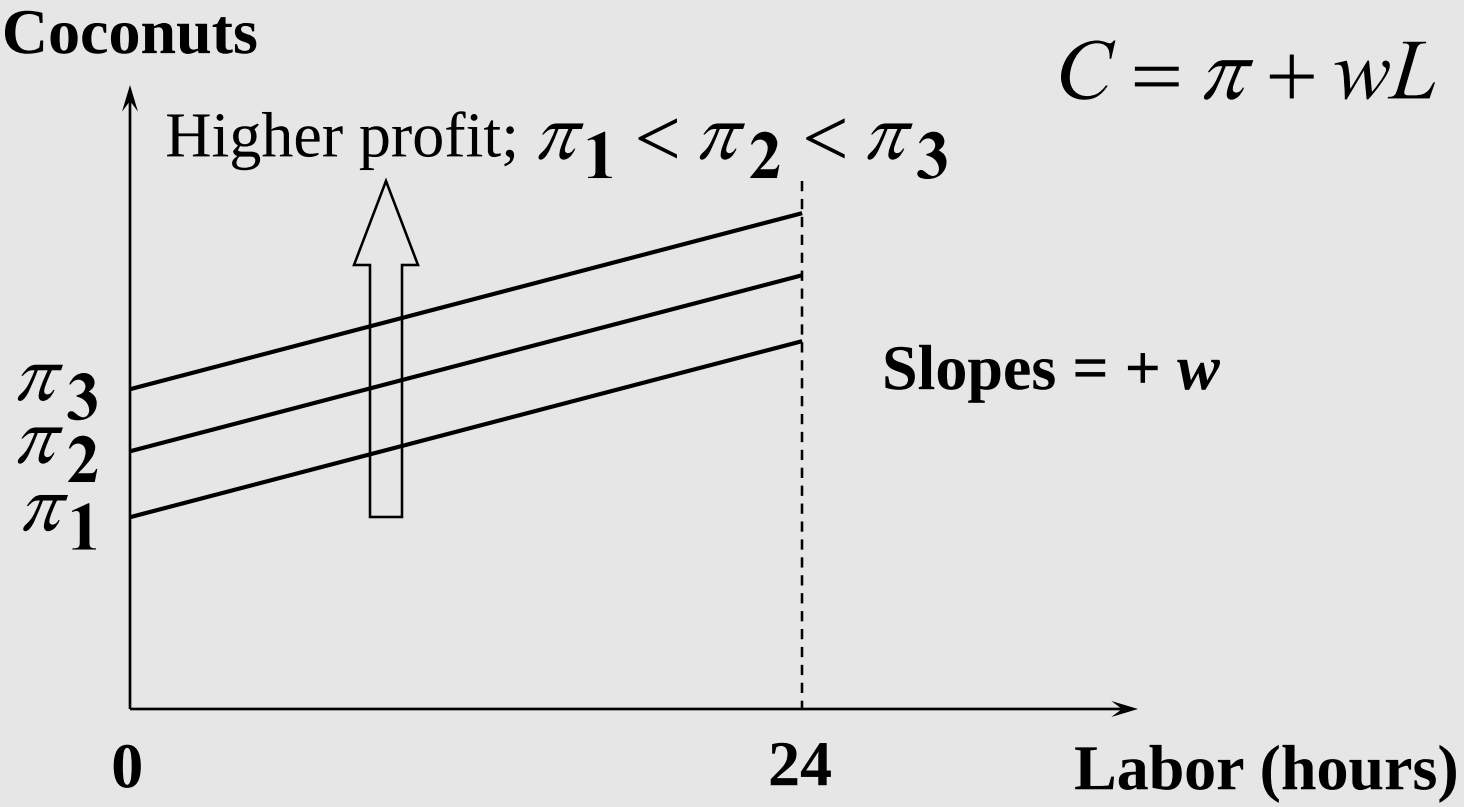
→ RC's firm's profit is $\pi = C - wL$.

→ $\pi = C - wL \Leftrightarrow C = \pi + wL$, the equation of an isoprofit line.

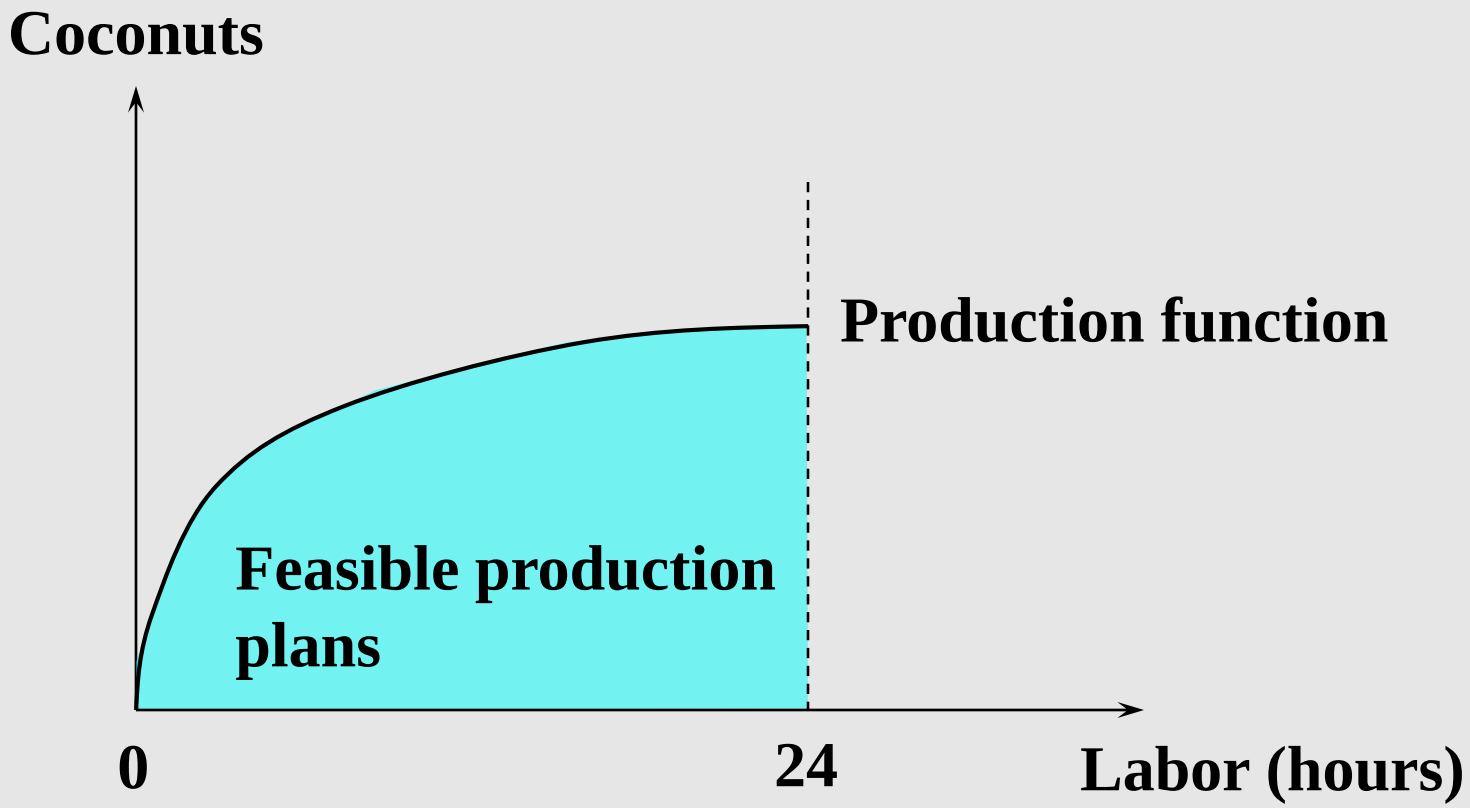
→ Slope = + w .

→ Intercept = π .

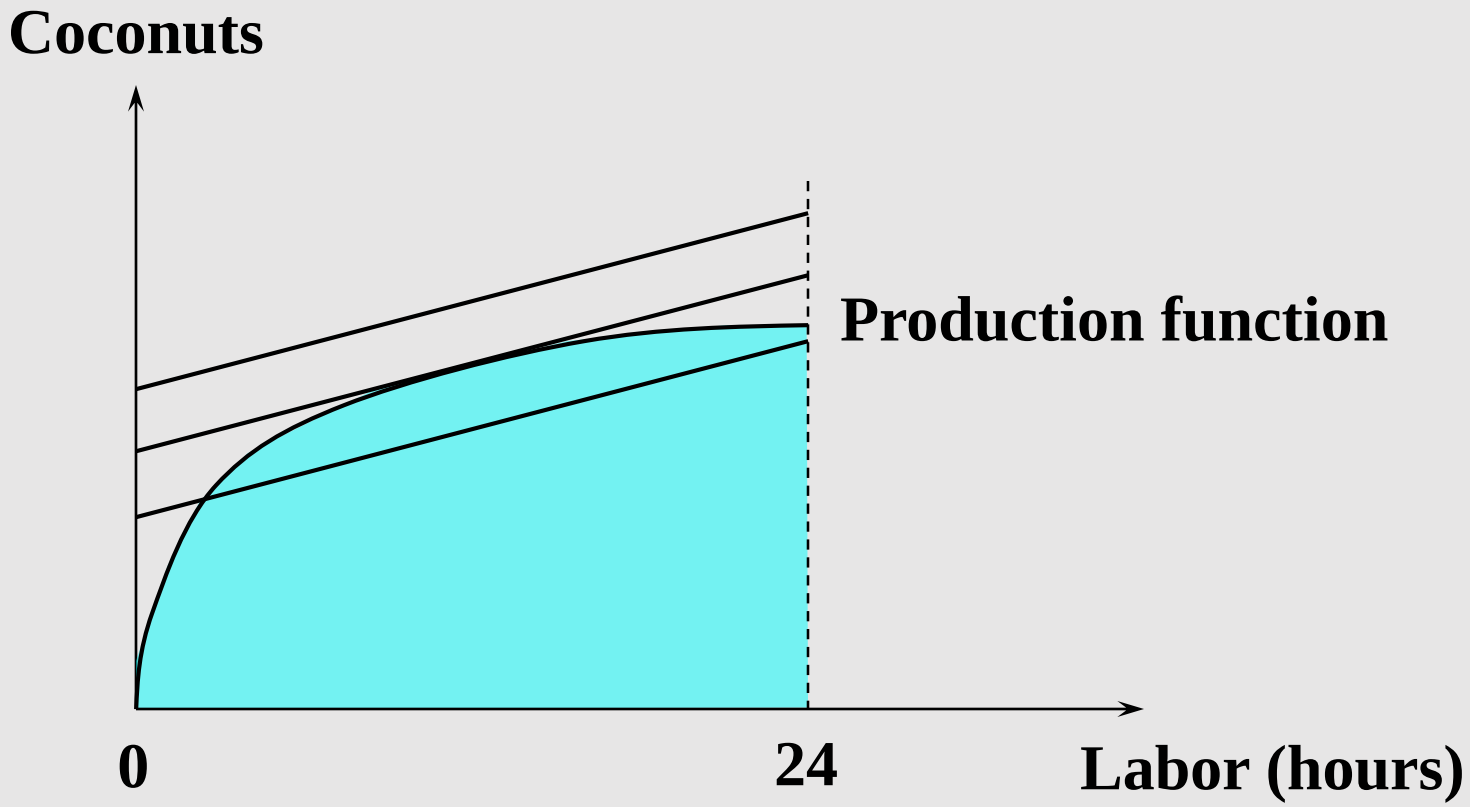
Isoprofit Lines



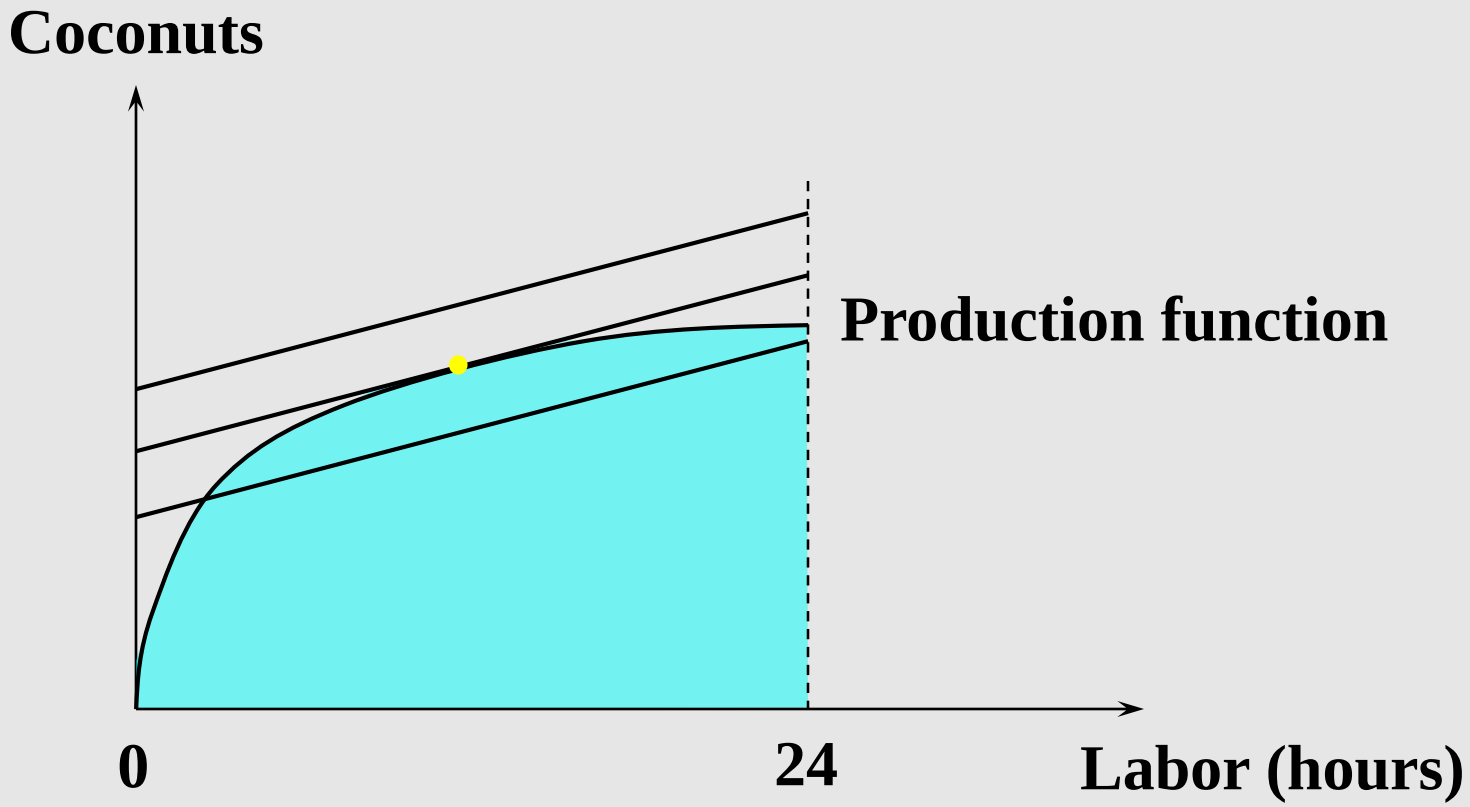
Profit-Maximization



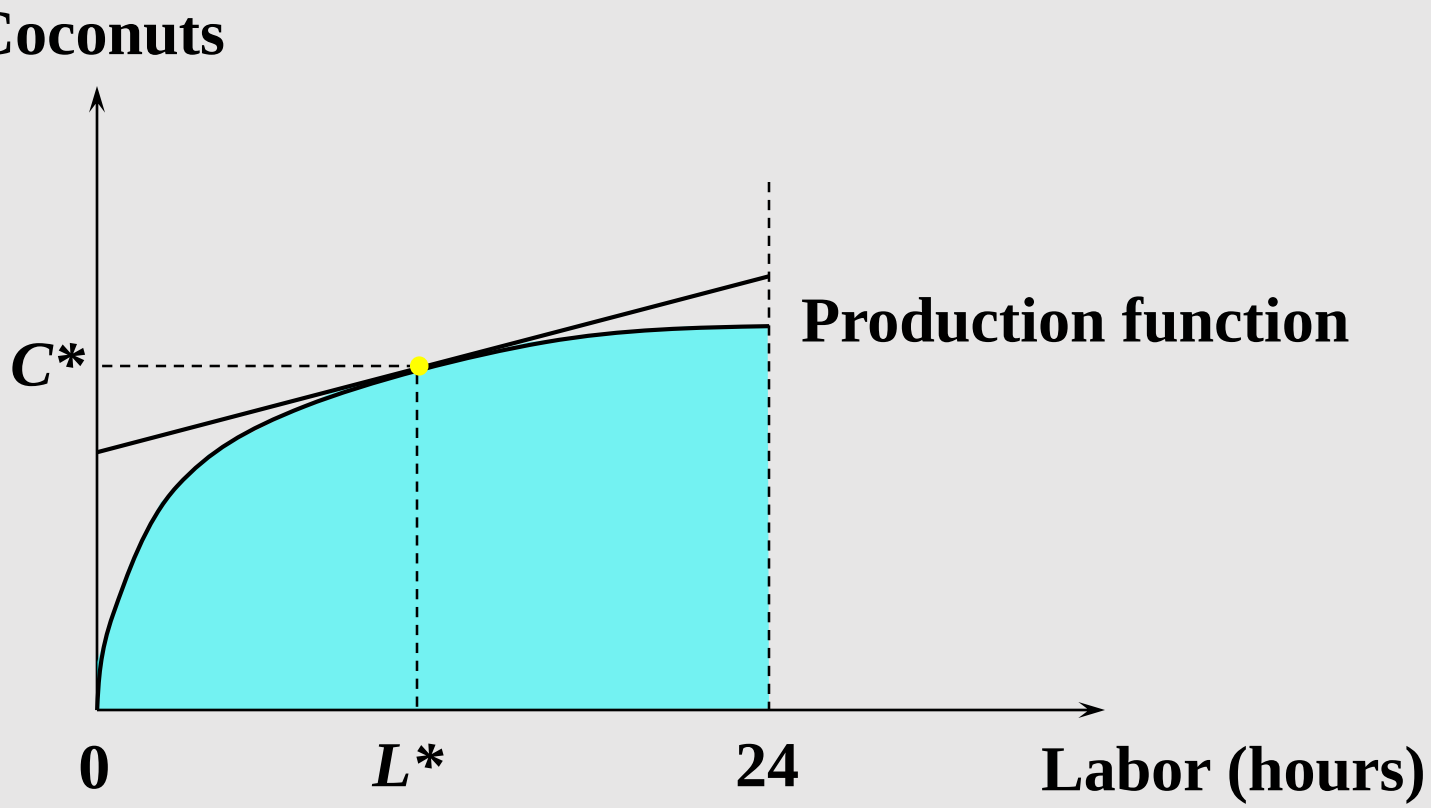
Profit-Maximization



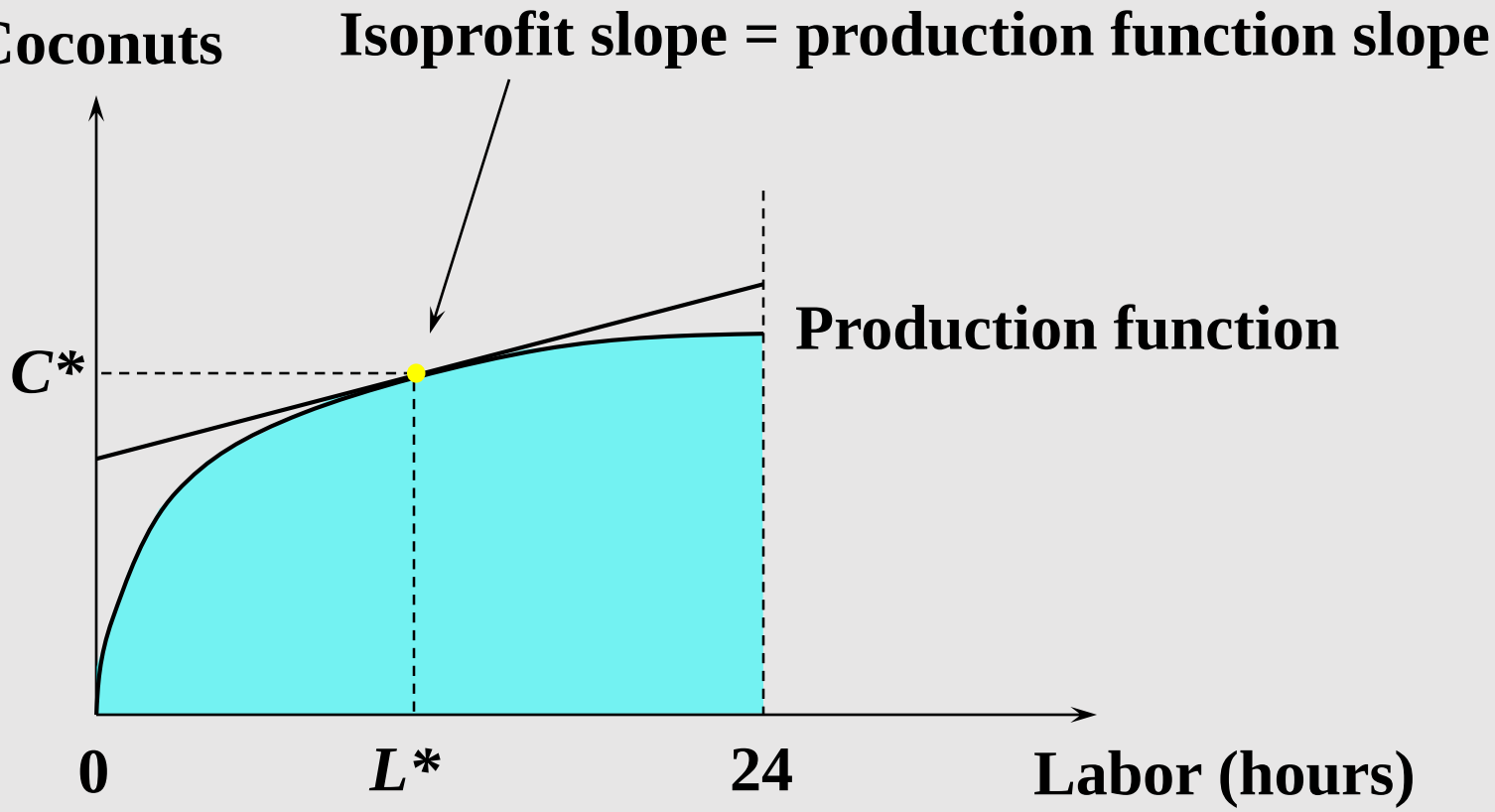
Profit-Maximization



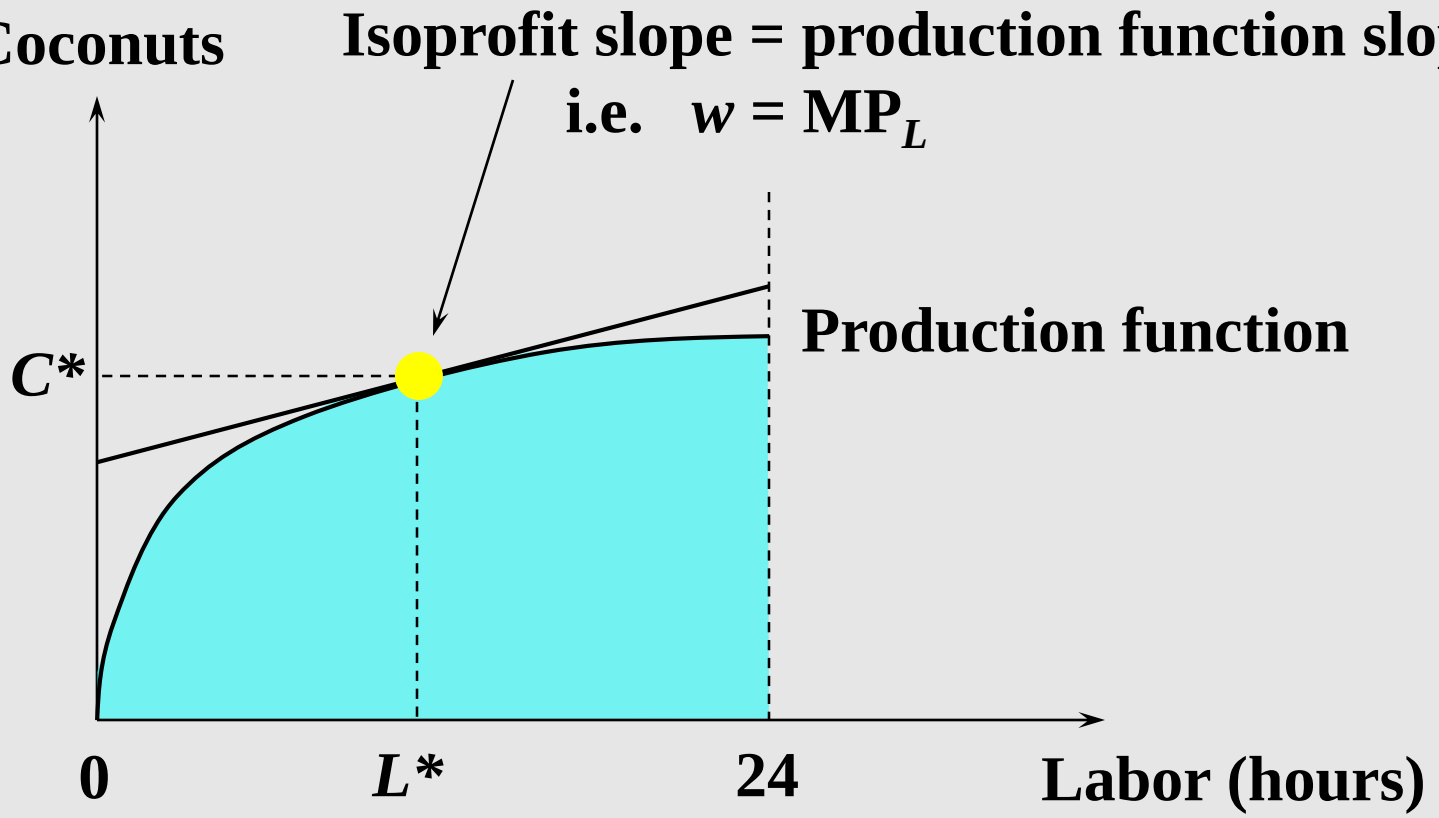
Profit-Maximization



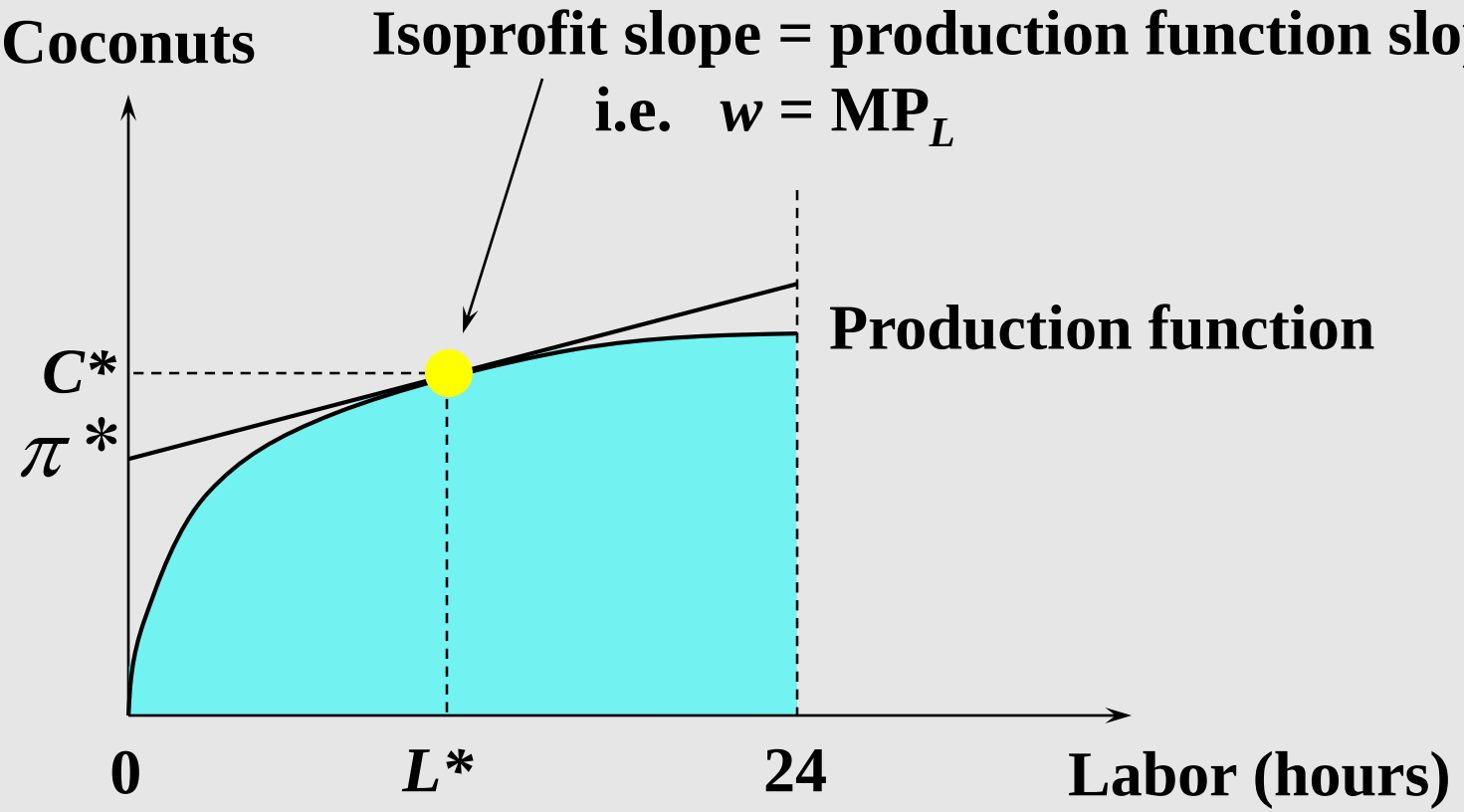
Profit-Maximization



Profit-Maximization

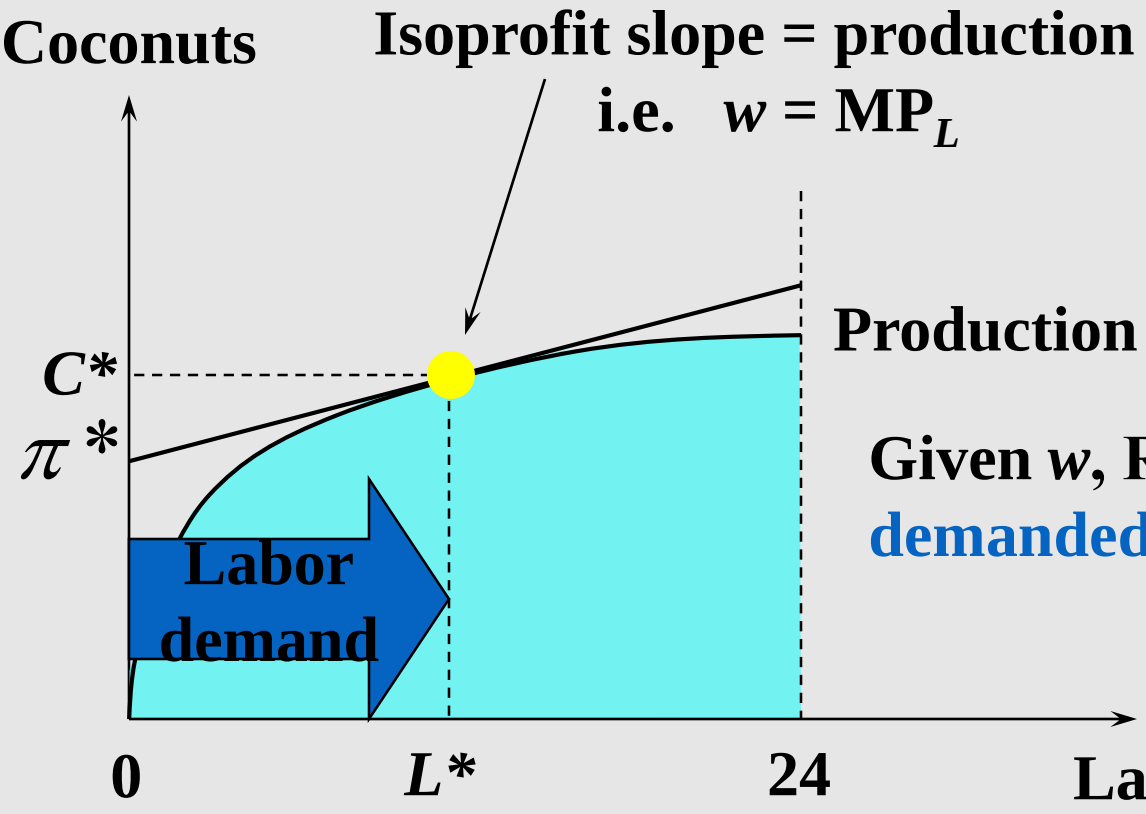


Profit-Maximization



RC gets $\pi^* = C^* - wL^*$

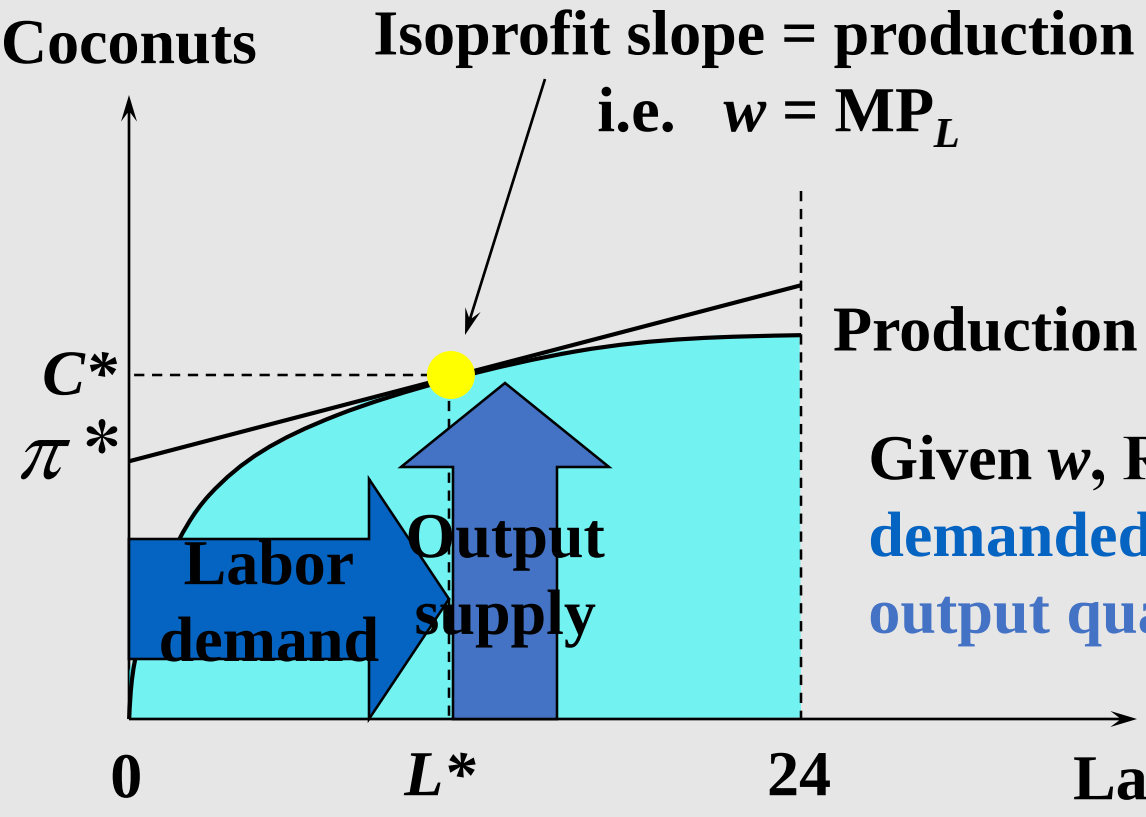
Profit-Maximization



Given *w*, RC's **firm's quantity demanded of labor is *L*****

RC gets $\pi^* = C^* - wL^*$

Profit-Maximization



Given *w*, RC's **firm's quantity demanded of labor is *L**** and **output quantity supplied is *C****.

RC gets $\pi^* = C^* - wL^*$

Utility-Maximization

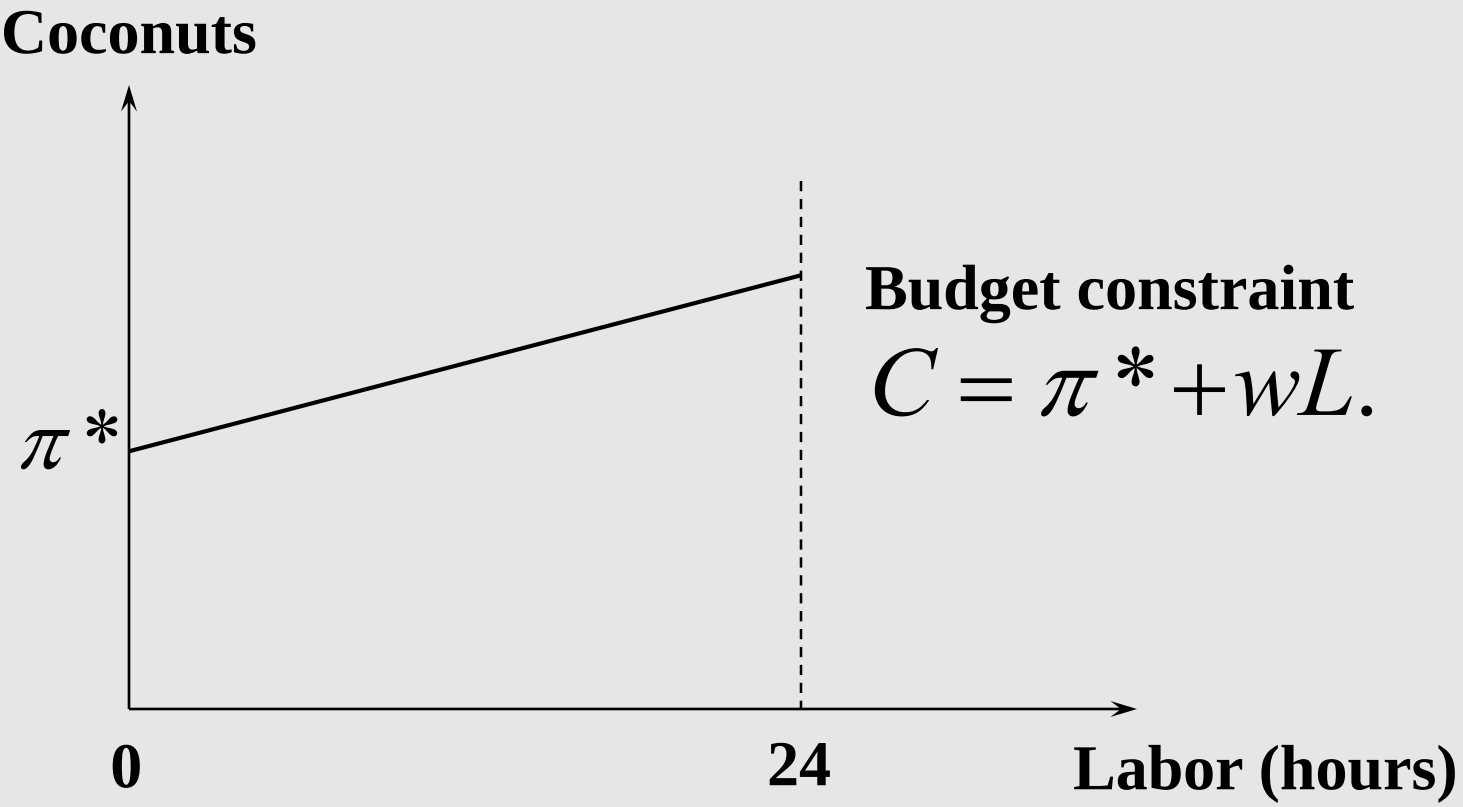
→ Now consider RC as a consumer endowed with $\$\pi^*$ who can work for $\$w$ per hour.

→ What is RC's most preferred consumption bundle?

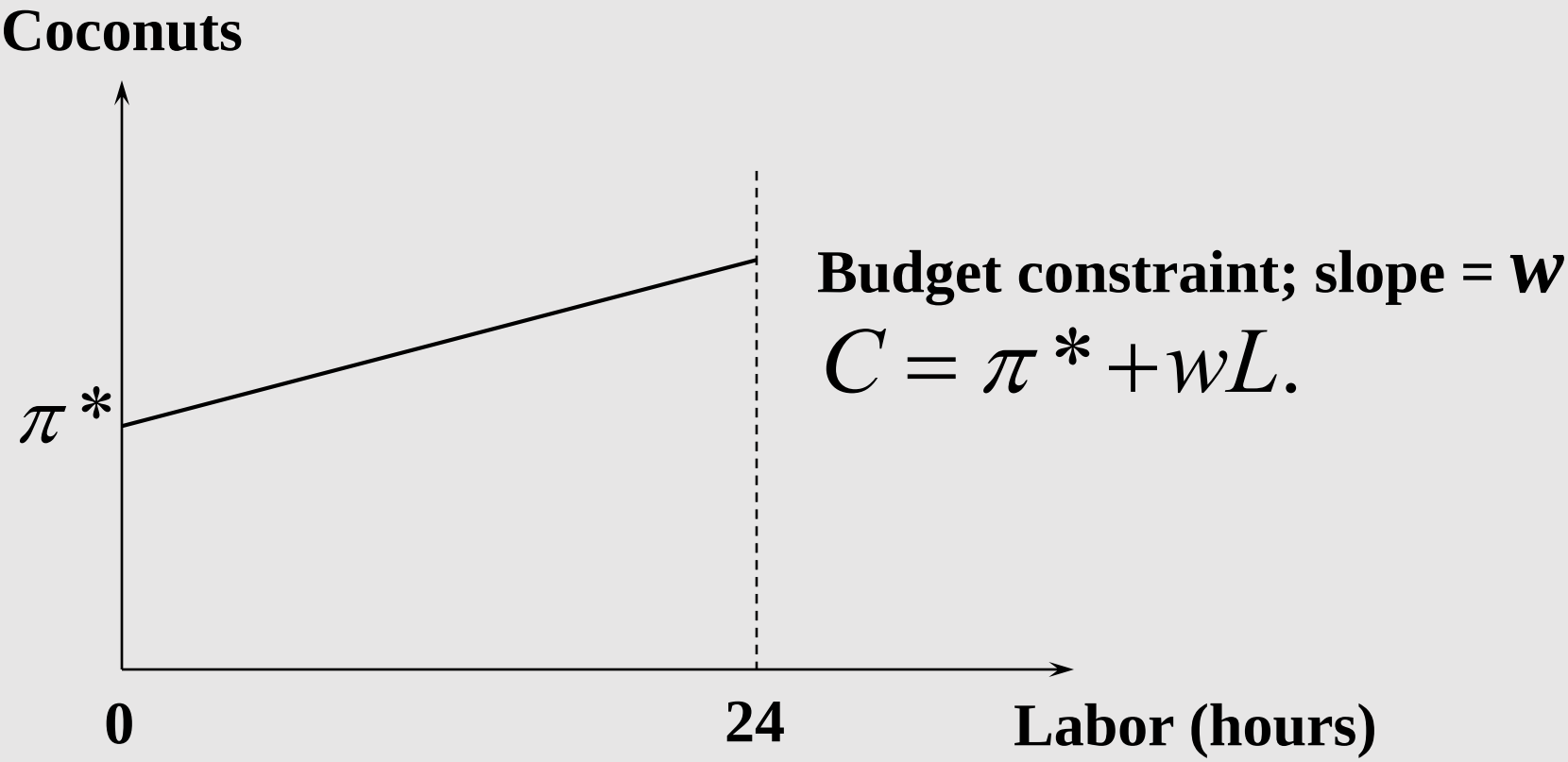
→ Budget constraint is

$$C = \pi^* + wL.$$

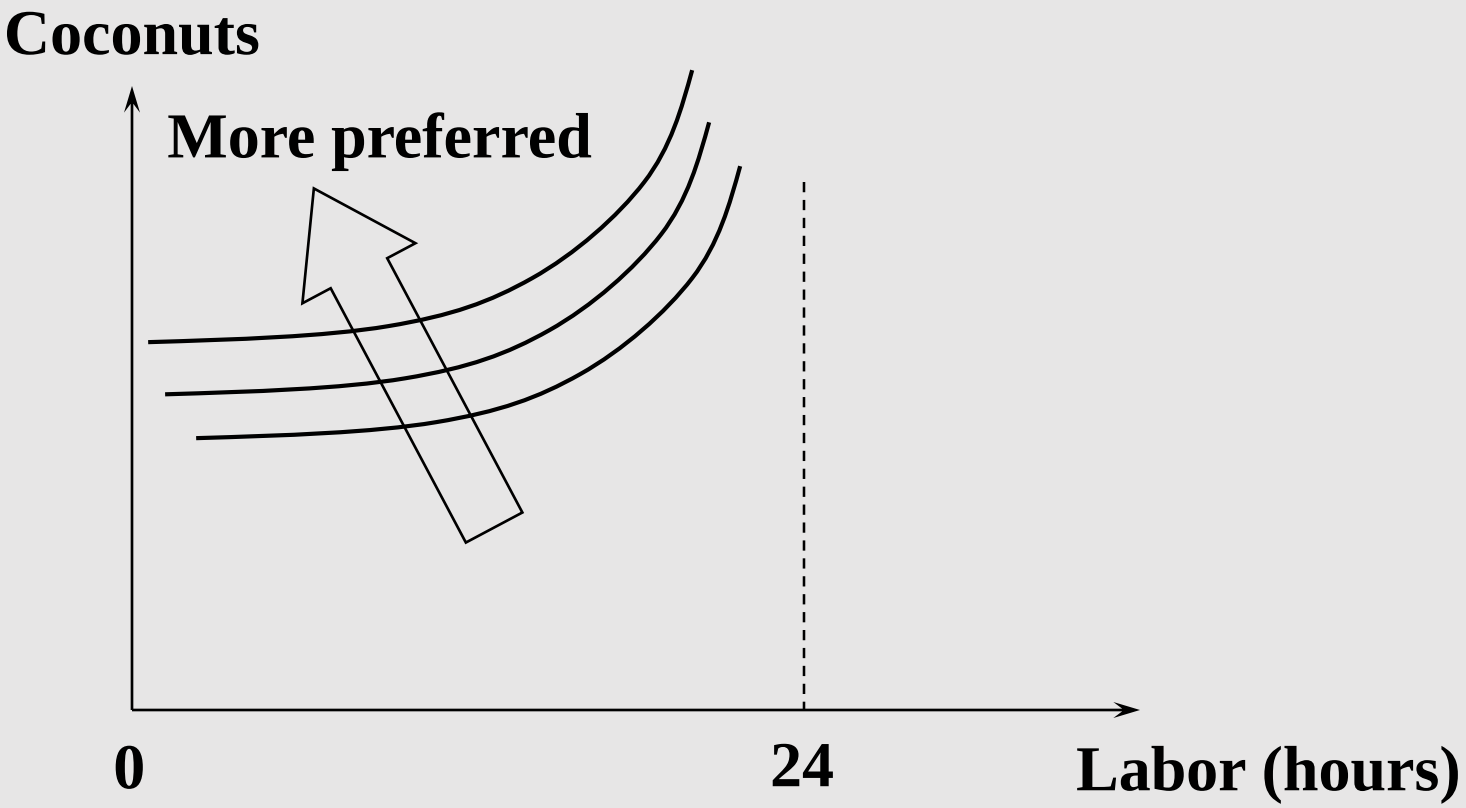
Utility-Maximization



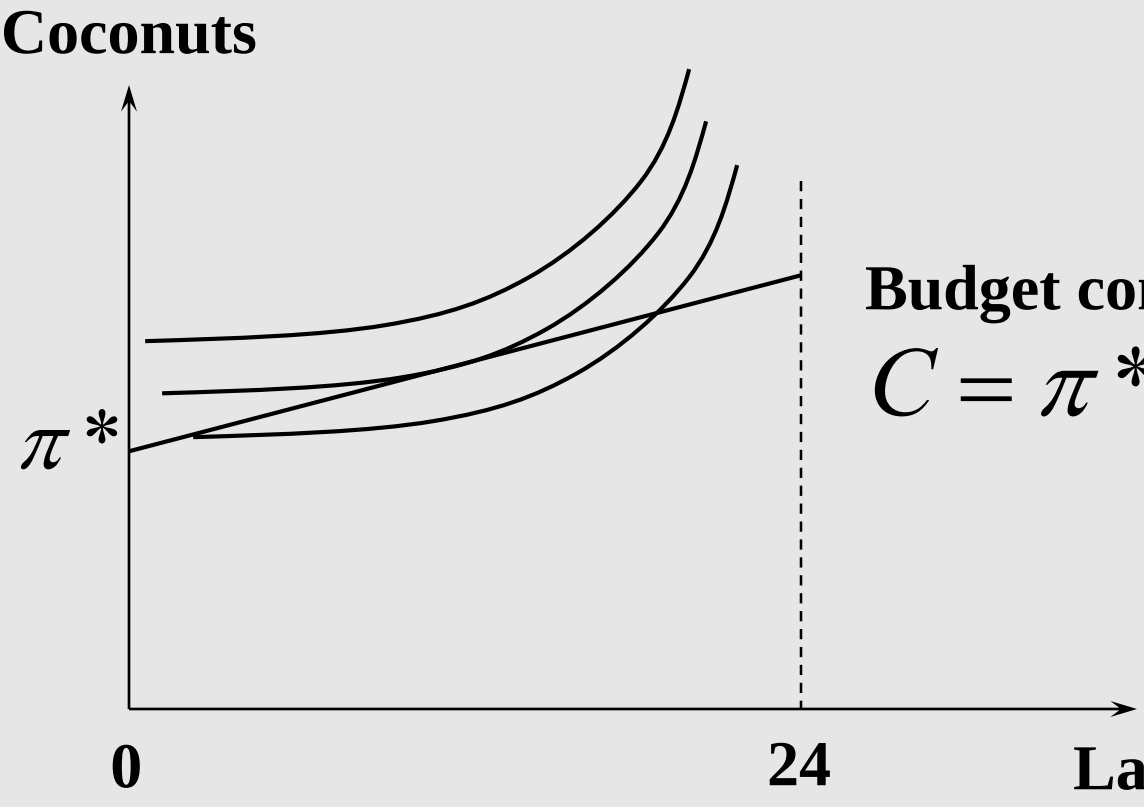
Utility-Maximization



Utility-Maximization



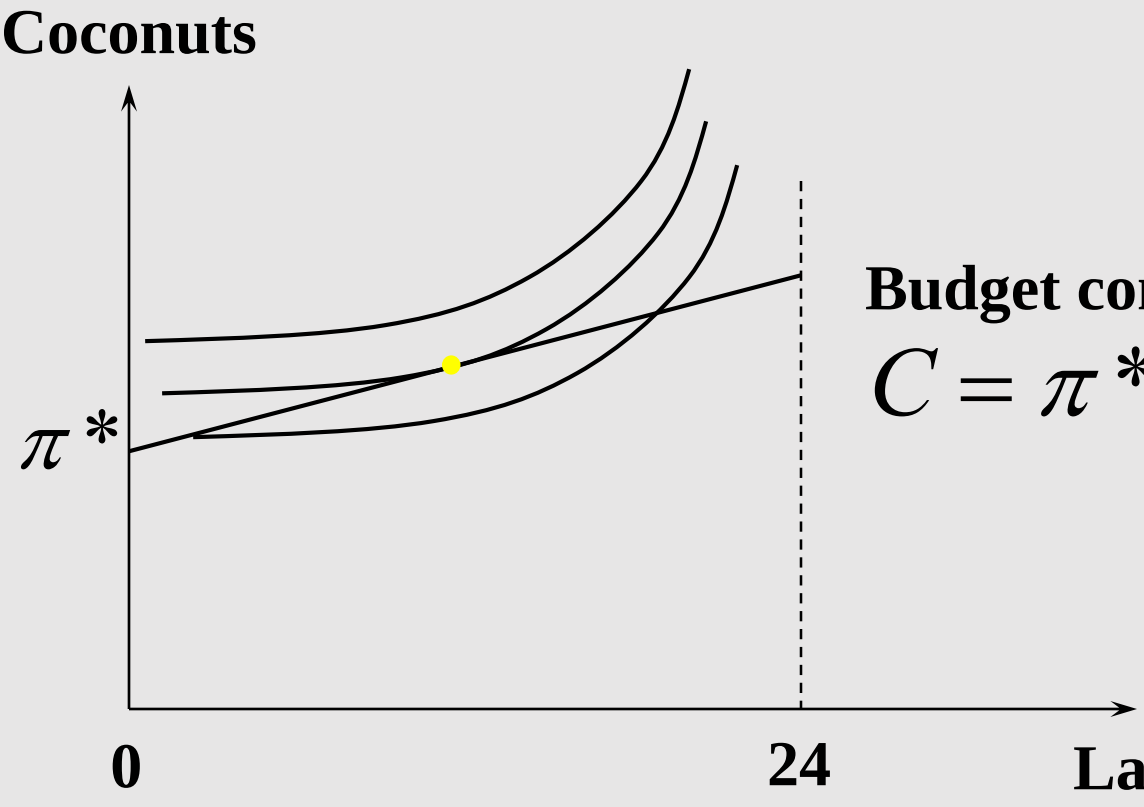
Utility-Maximization



Budget constraint; slope = *w*

$$C = \pi^* + wL.$$

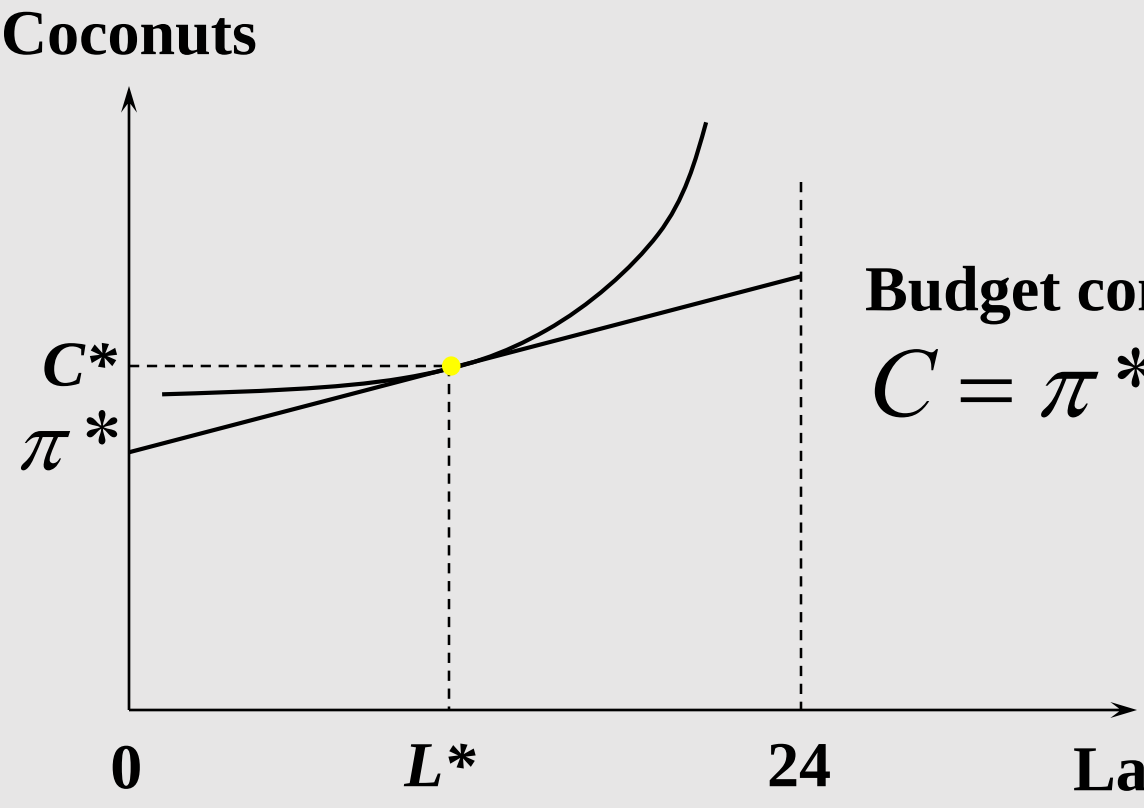
Utility-Maximization



Budget constraint; slope = *w*

$$C = \pi^* + wL.$$

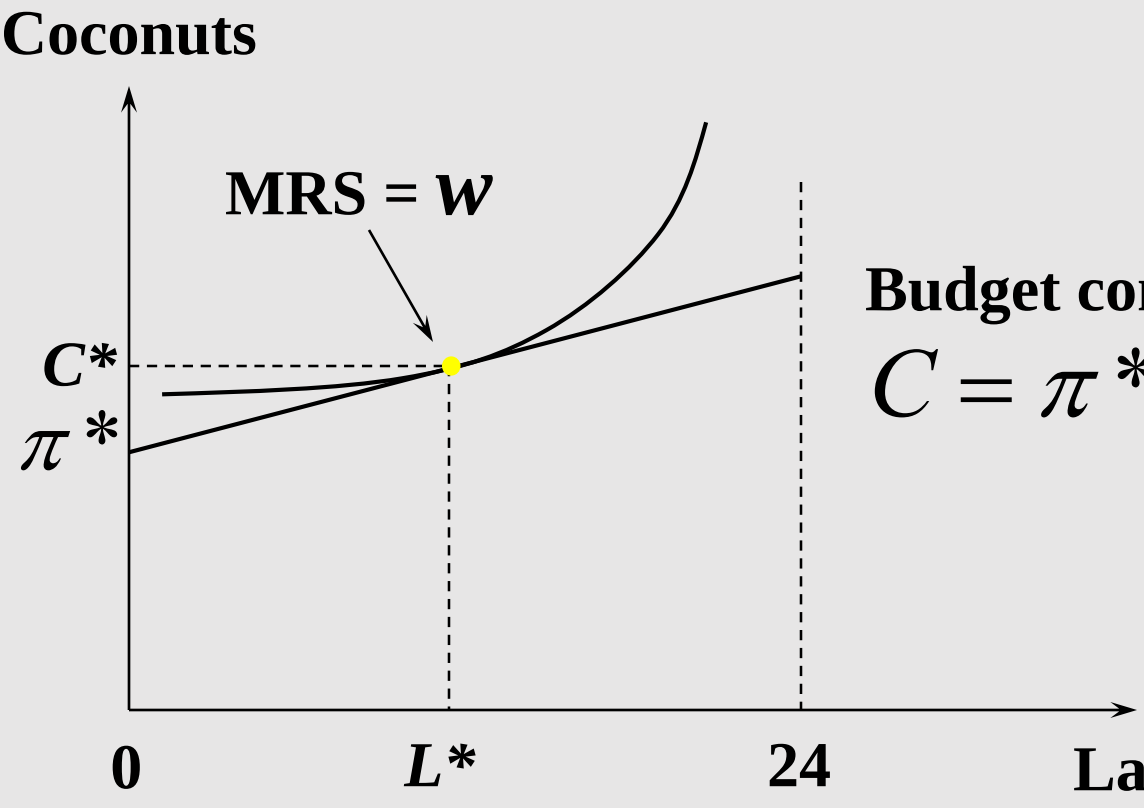
Utility-Maximization



Budget constraint; slope = *w*

$$C = \pi^* + wL.$$

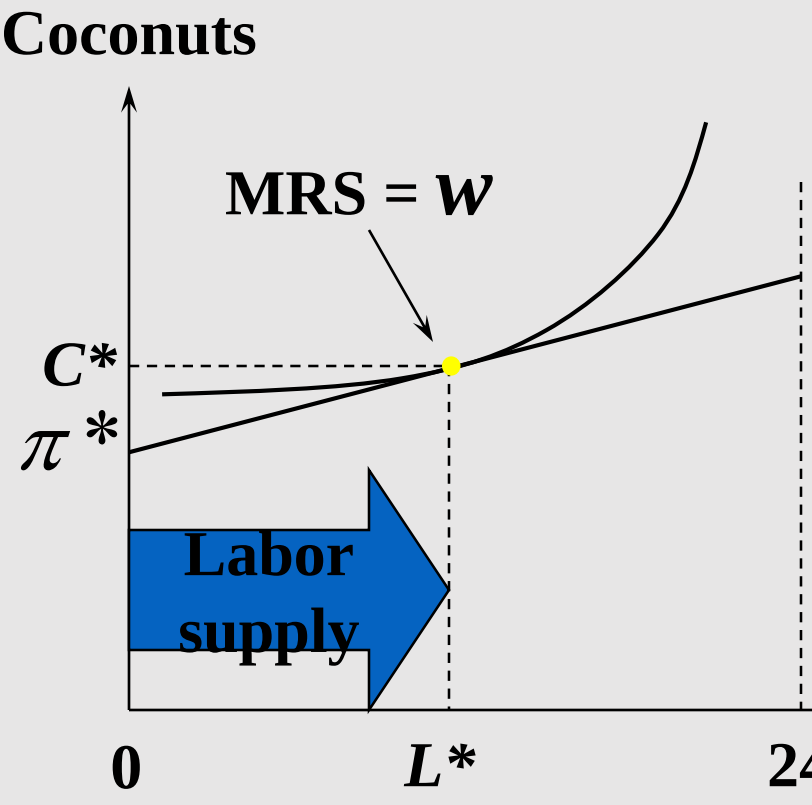
Utility-Maximization



Budget constraint; slope = *w*

$$C = \pi^* + wL.$$

Utility-Maximization

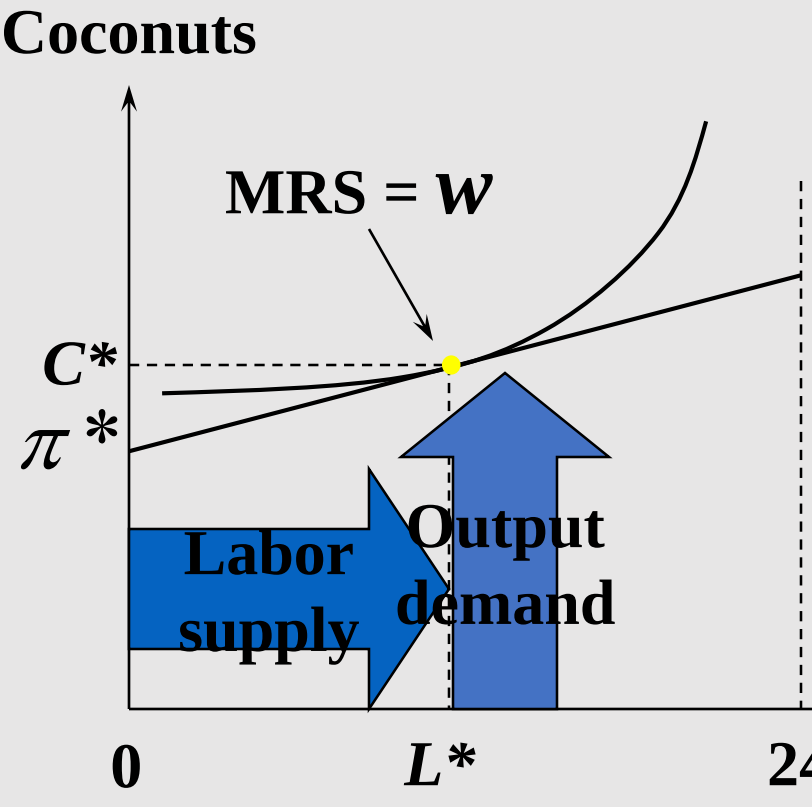


Budget constraint; slope = *w*

$$C = \pi^* + wL.$$

Given *w*, RC's quantity supplied of labor is *L

Utility-Maximization



Budget constraint; slope = *w*

$$C = \pi^* + wL.$$

Given *w*, RC's quantity supplied of labor is *L** **and output quantity demanded is *C****.

Utility-Maximization & Profit-Maximization

→ Profit-maximization:

- w = MP_L
- quantity of output supplied = *C**
- quantity of labor demanded = *L**

→ Utility-maximization:

- w = MRS
- quantity of output demanded = *C**
- quantity of labor supplied = *L**

Utility-Maximization & Profit-Maximization

→ Profit-maximization:

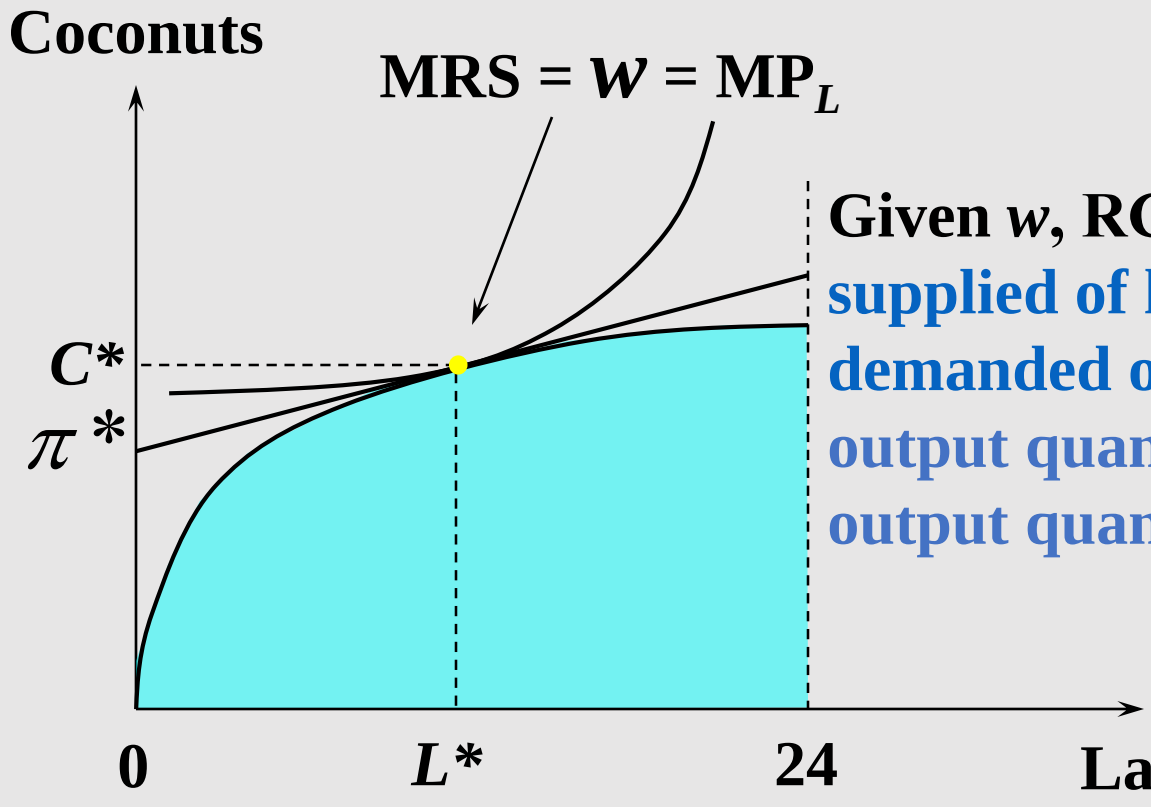
- $w = MP_L$
- quantity of output supplied = C^*
- quantity of labor demanded = L^*

→ Utility-maximization:

- w = MRS
- quantity of output demanded = C^*
- quantity of labor supplied = L^*

Coconut and labor markets both clear.

Utility-Maximization & Profit-Maximization

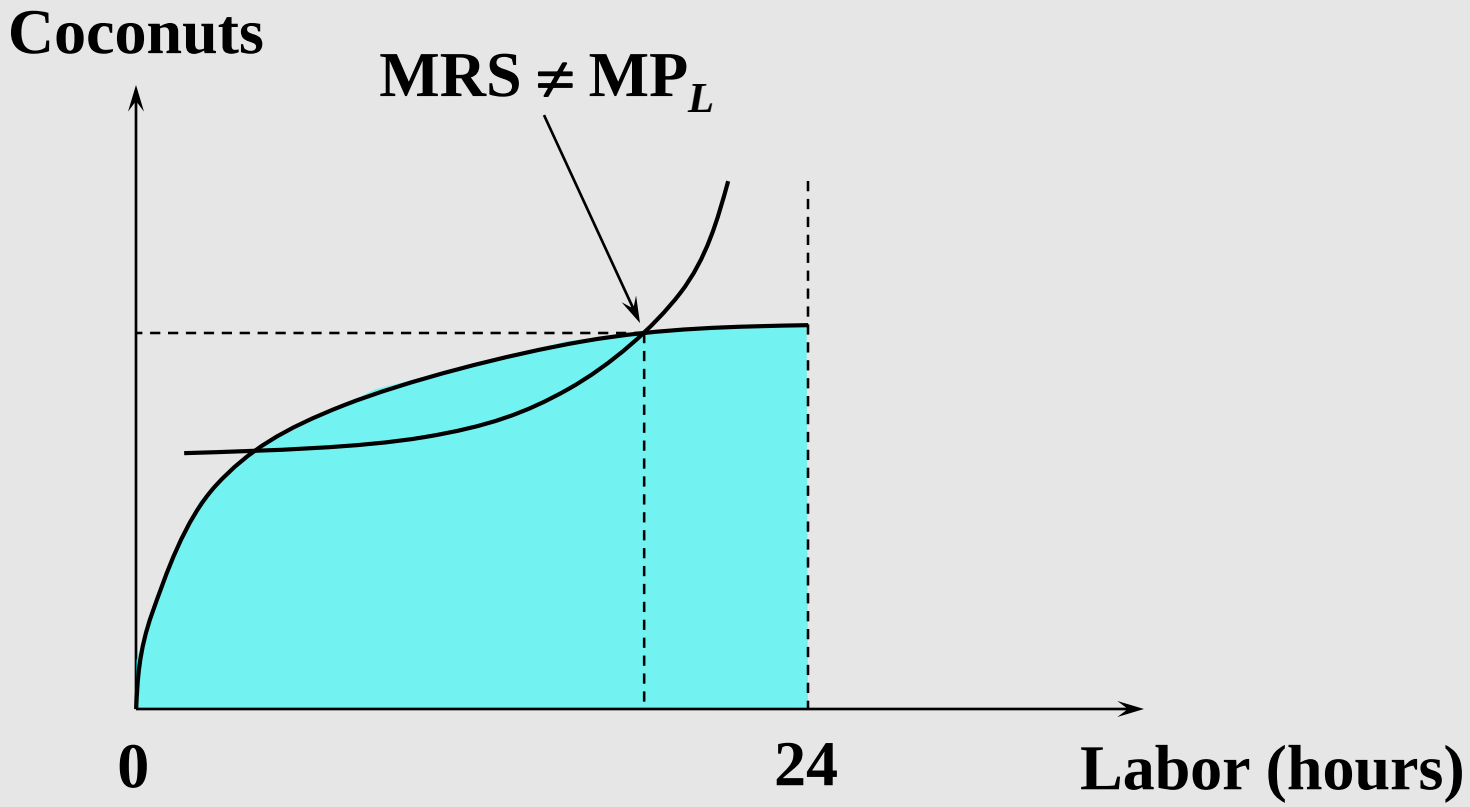


Given *w*, RC's quantity supplied of labor = quantity demanded of labor = *L** **and output quantity demanded = output quantity supplied = *C****.**

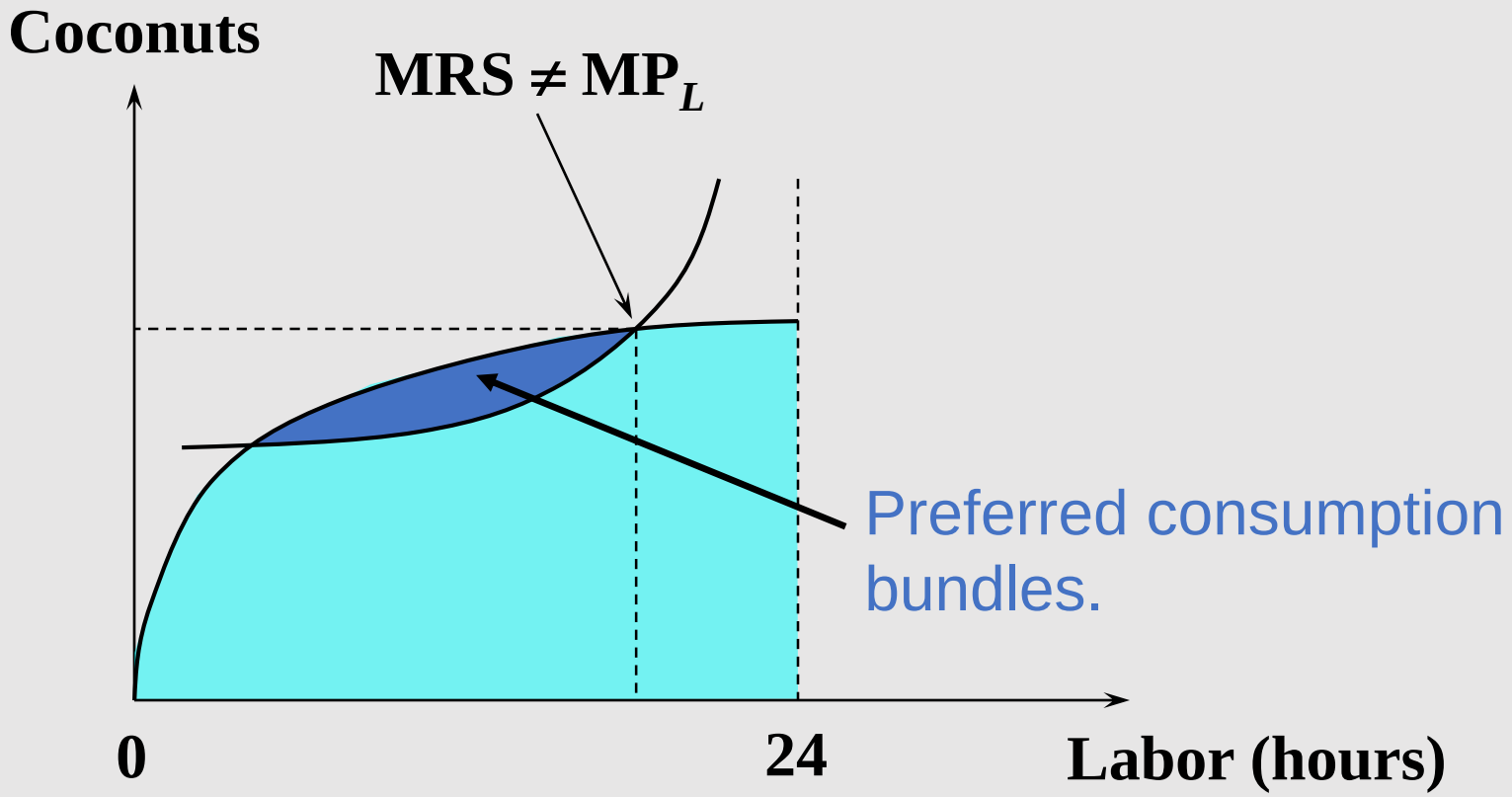
Pareto Efficiency

→ Must have MRS = MP_L.

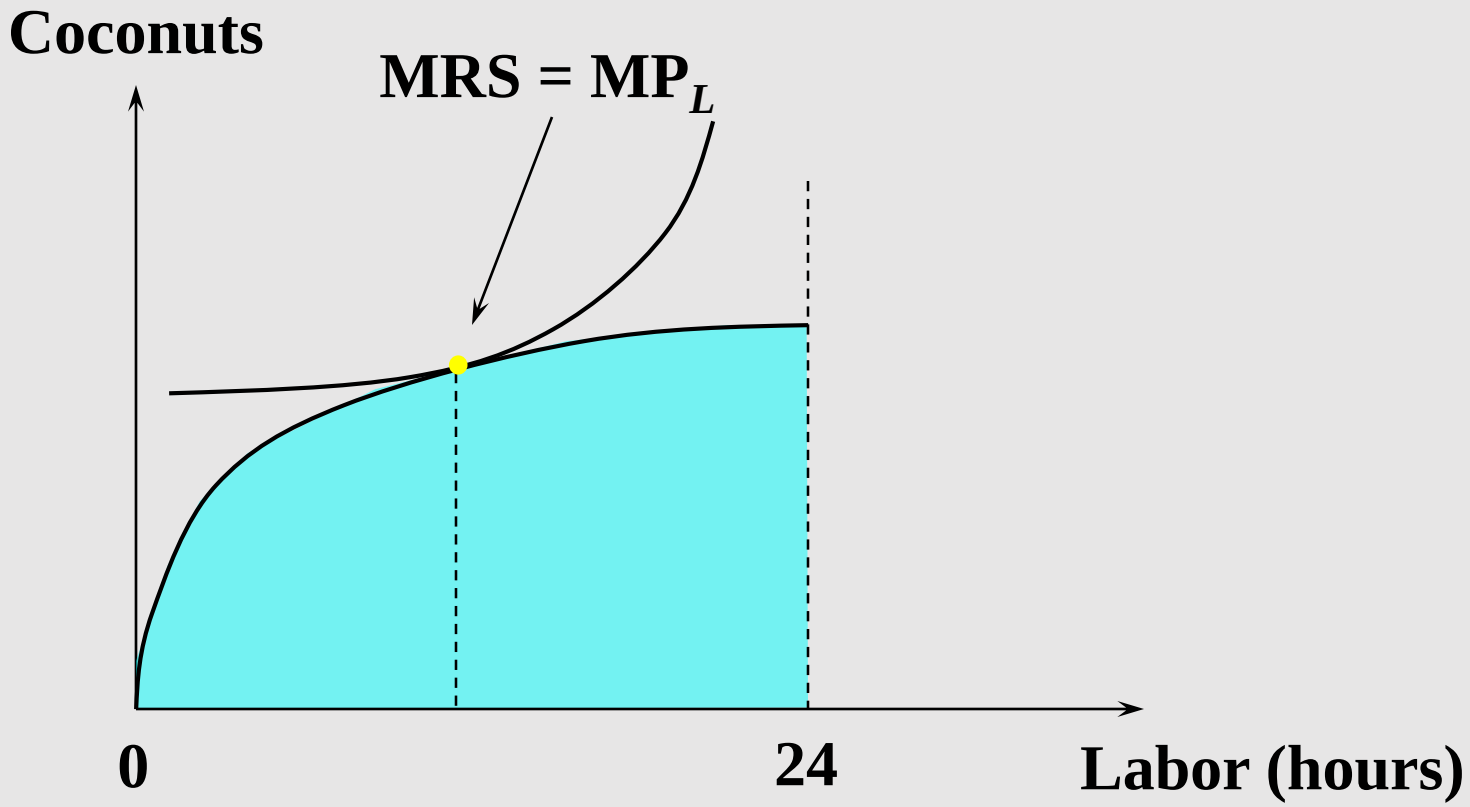
Pareto Efficiency



Pareto Efficiency



Pareto Efficiency



3. Fundamental Theorems of Welfare Economics

First Fundamental Theorem of Welfare Economics

→A competitive market equilibrium is Pareto efficient if

- consumers' preferences are convex
- there are no externalities in consumption or production.

Fundamental Theorems of Welfare Economics

Second Fundamental Theorem of Welfare Economics

→ Any Pareto efficient economic state can be achieved as a competitive market equilibrium if

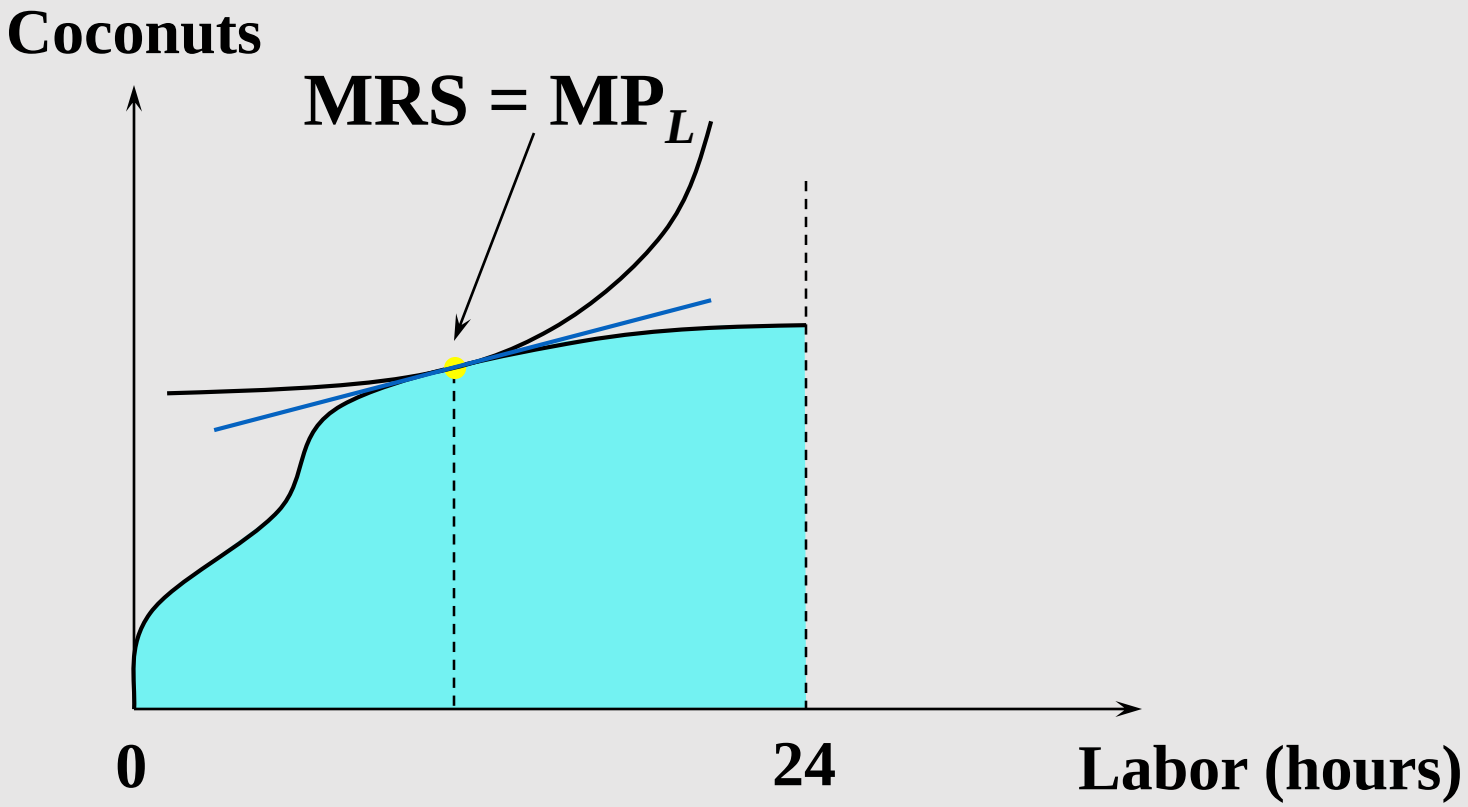
- consumers' preferences are convex
- firms' technologies are convex: feasible production plans are a convex set
  - For every two points in the set, the line that joins them belong to the set
- there are no externalities in consumption or production.

Non-Convex Technologies

→ Do the Welfare Theorems hold if firms have non-convex technologies?

→ The 1st Theorem does not rely upon firms' technologies being convex.

Non-Convex Technologies



Non-Convex Technologies

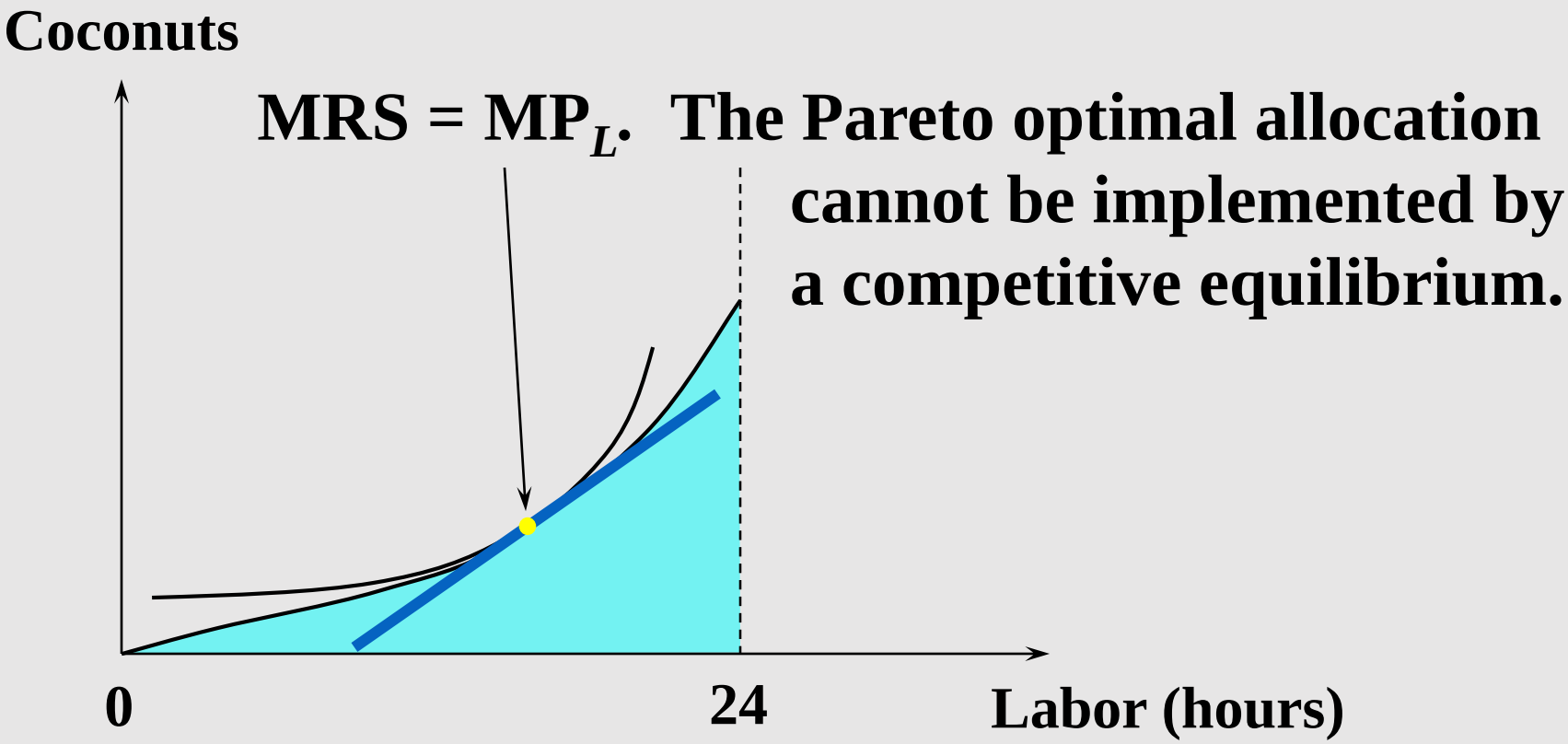
→ Do the Welfare Theorems hold if firms have non-convex technologies?

→ The 2nd Theorem **does** require that firms' technologies be convex.

- This is, the feasible production plans are a convex set
  - For every two points in the set, the line that joins them belong to the set

- This means that all situations with increasing returns to scale are ruled out.

Non-Convex Technologies



4. Production Possibilities

Let’s generalize to two outputs

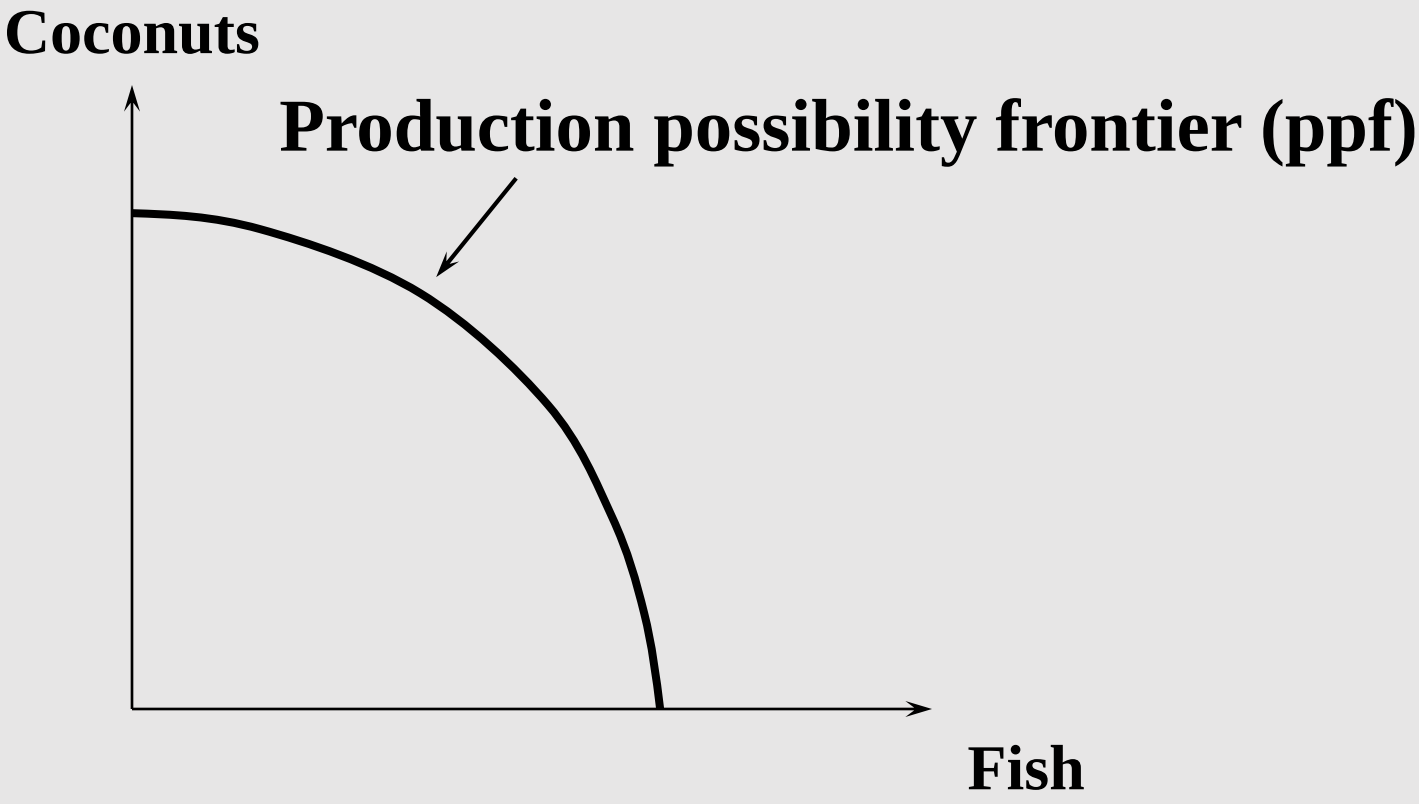
→ Resource and technological limitations restrict what an economy can produce.

→ The set of all feasible output bundles is the economy’s **production possibility set.**

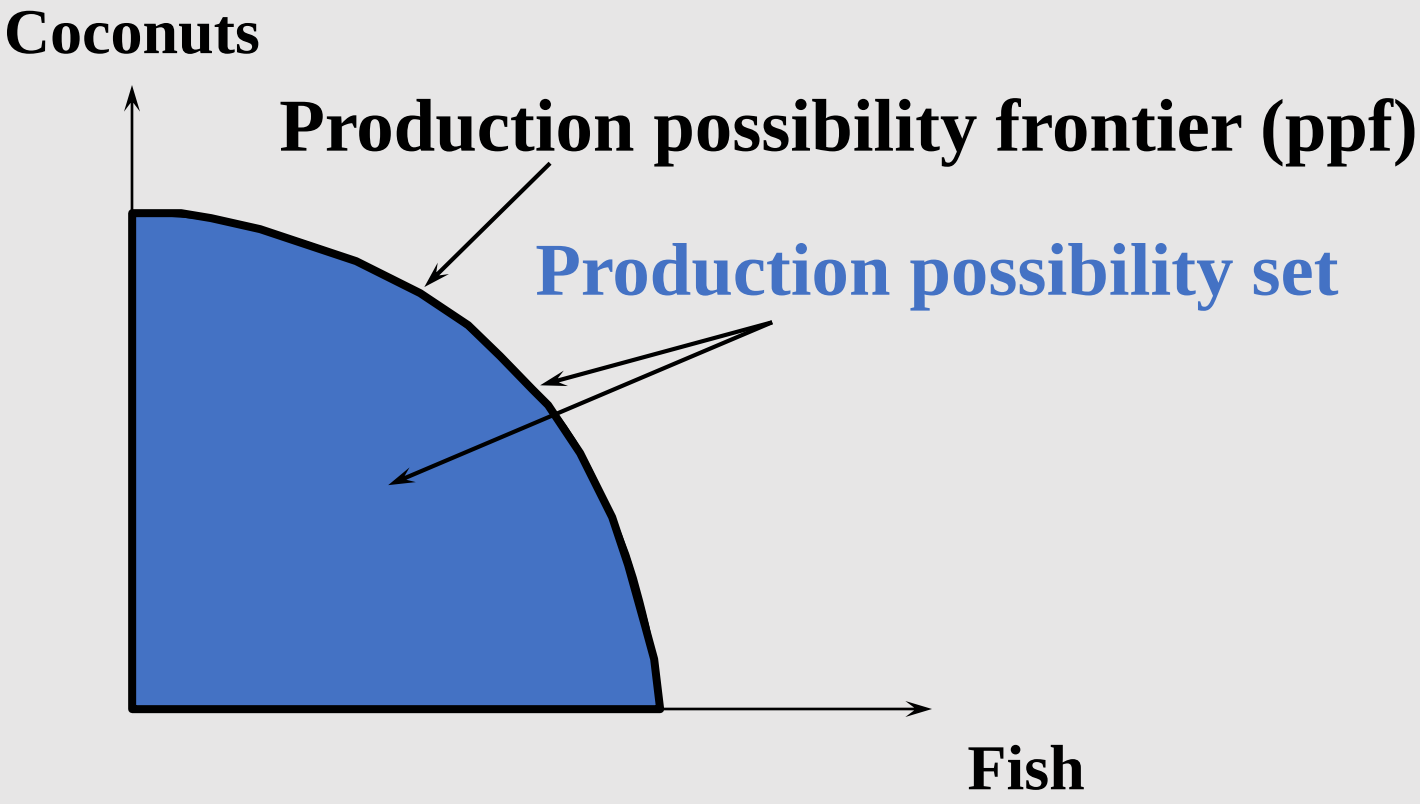
→ The set’s outer boundary is the **production possibility frontier**.

→ Difference: a function of the two outputs, inputs are not in the graph anymore

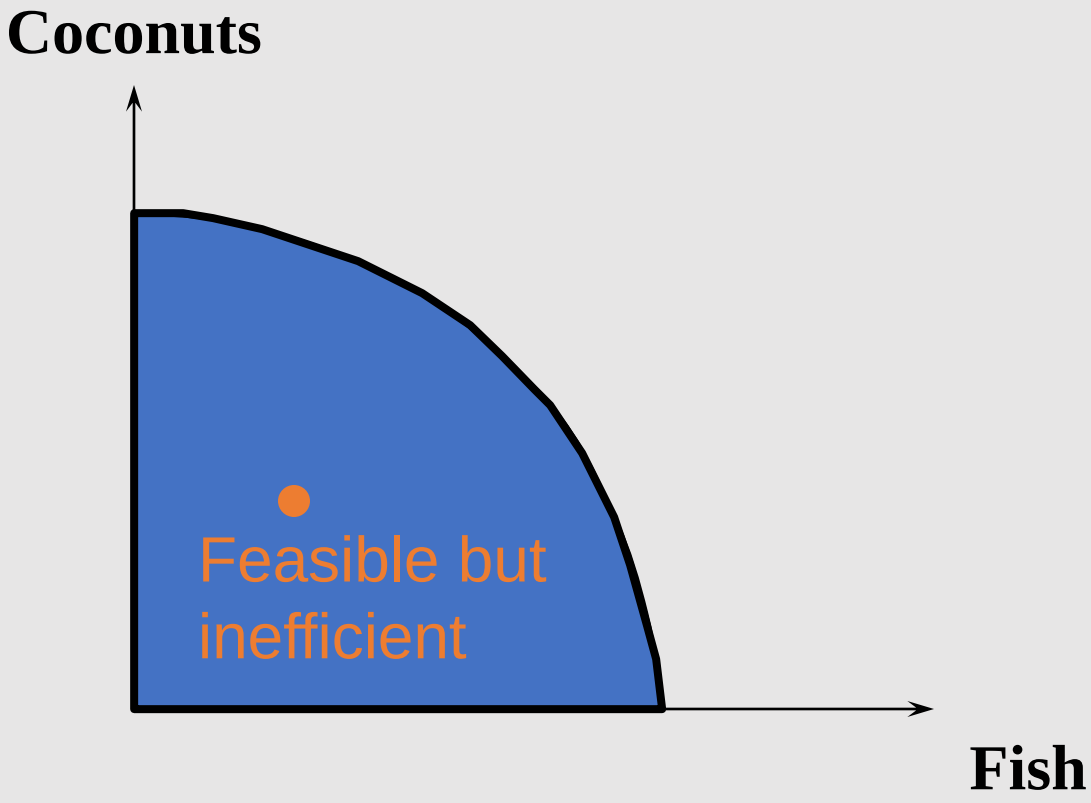
Production Possibilities



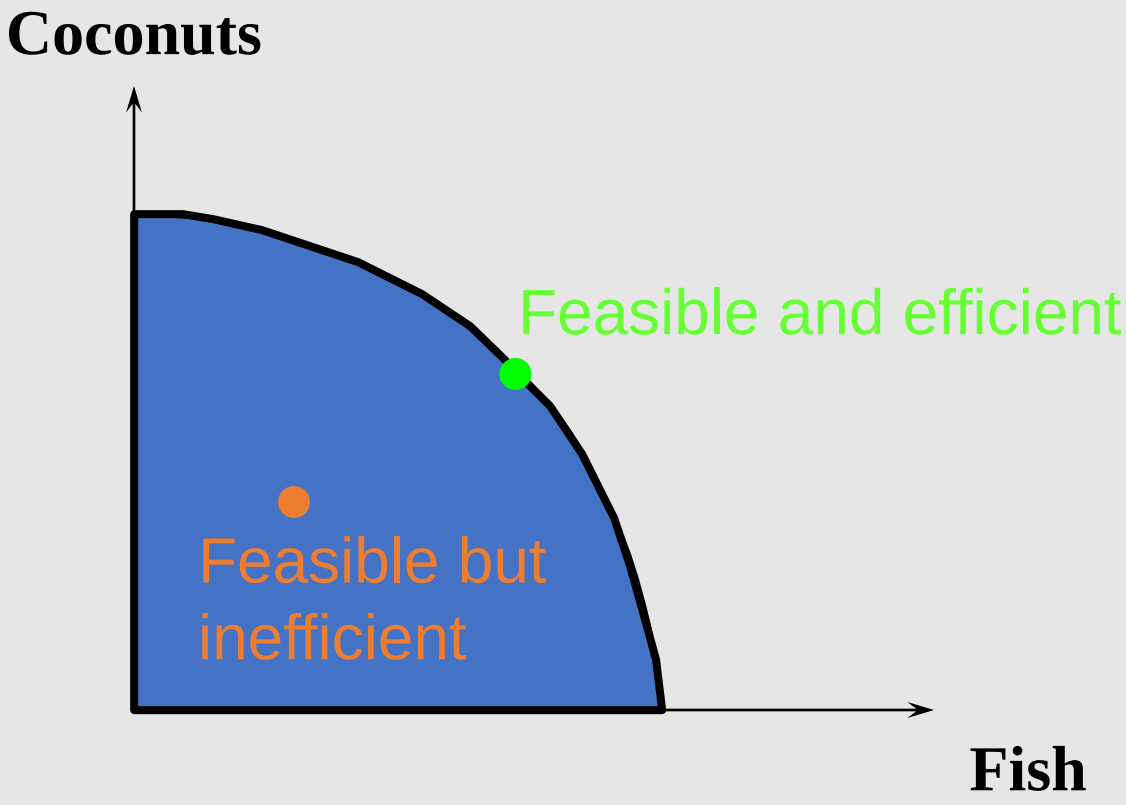
Production Possibilities



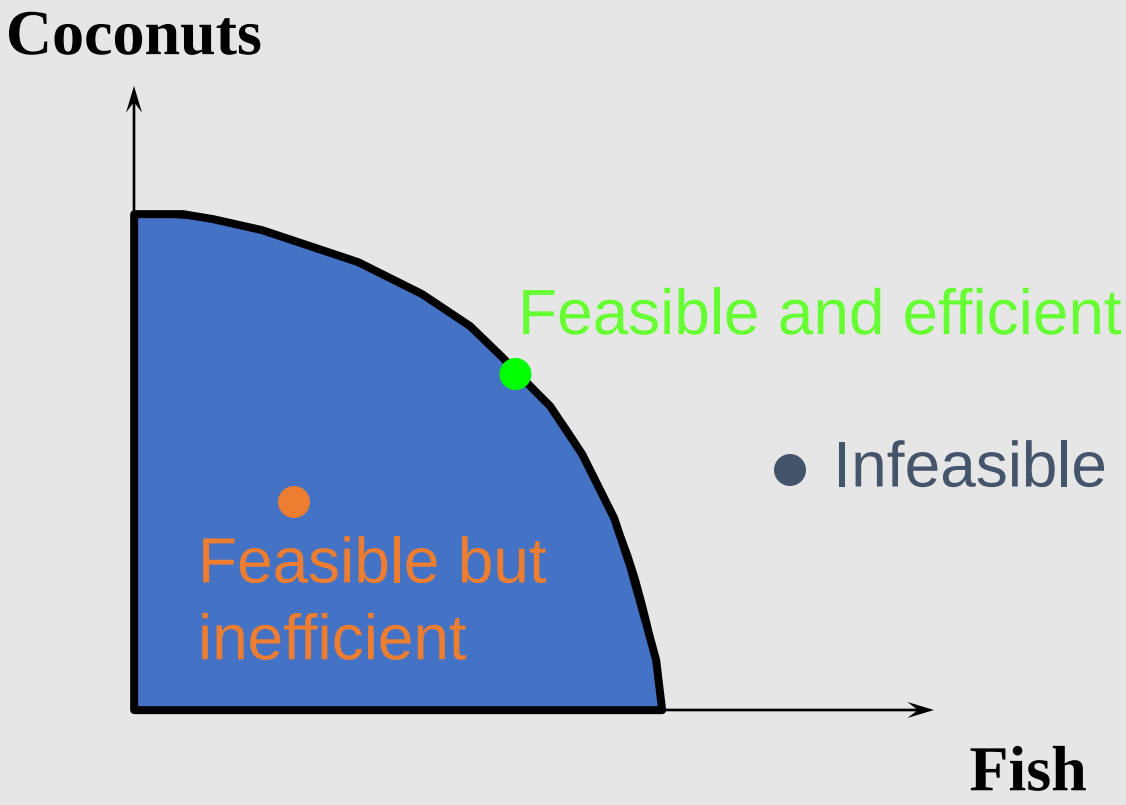
Production Possibilities



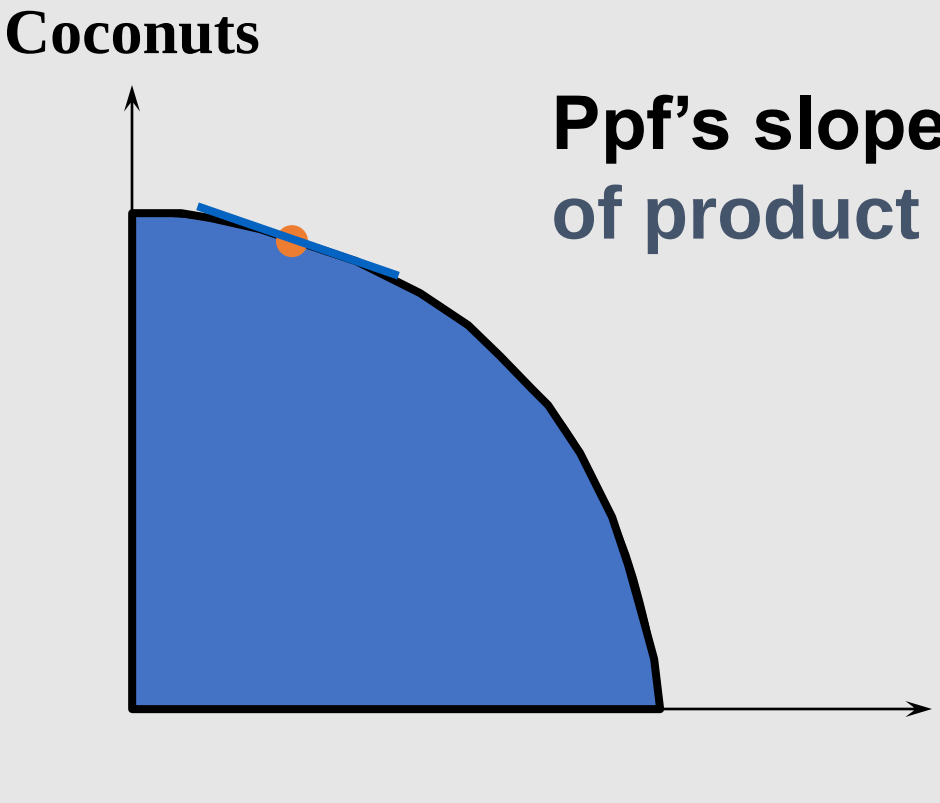
Production Possibilities



Production Possibilities

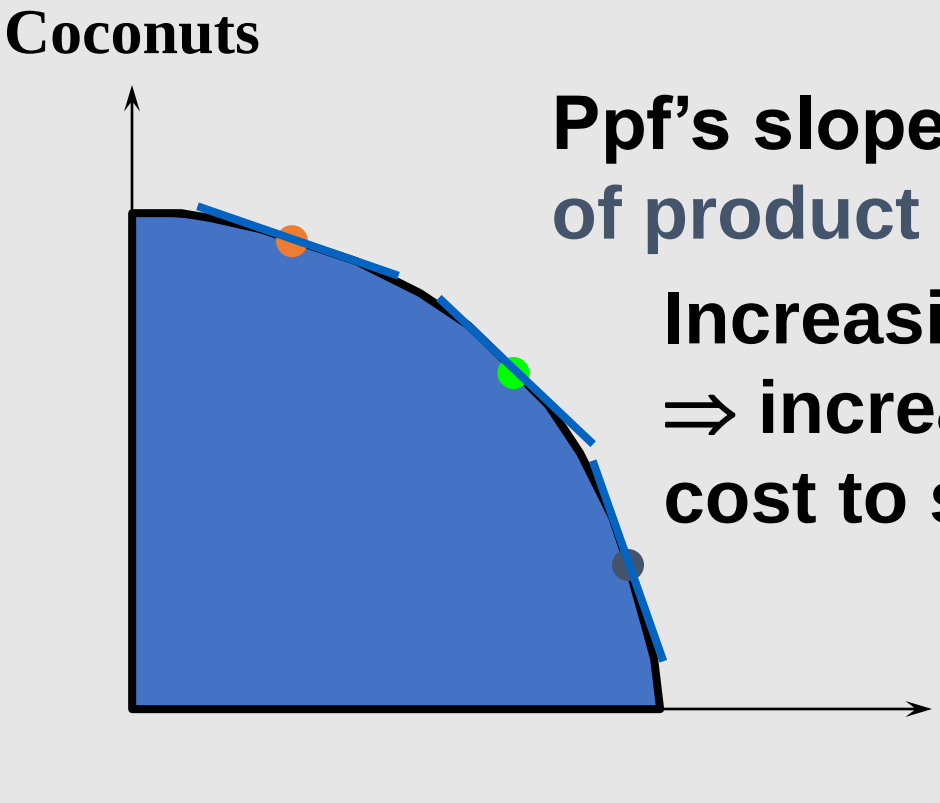


Production Possibilities



Ppf's slope is the marginal rate of product transformation.

Production Possibilities



Ppf's slope is the marginal rate of product transformation.

Increasingly negative MRPT ⇒ increasing opportunity cost to specialization.

Production Possibilities

→ The construction of the ppf was quite simple since there was only one way to produce fish and coconuts.

→ What if there is more than one way to produce each good?

→ Let's add another worker with different skills in producing fish and coconuts.

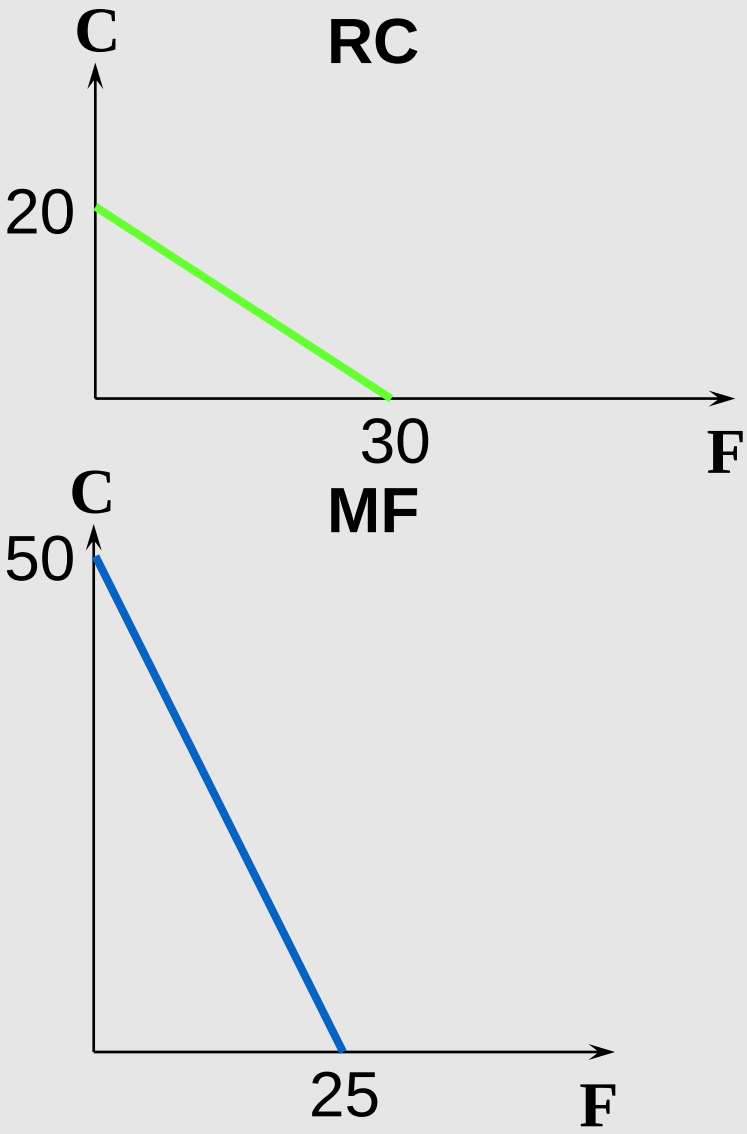
5. Comparative Advantage

→ Two agents, RC and Man Friday (MF).

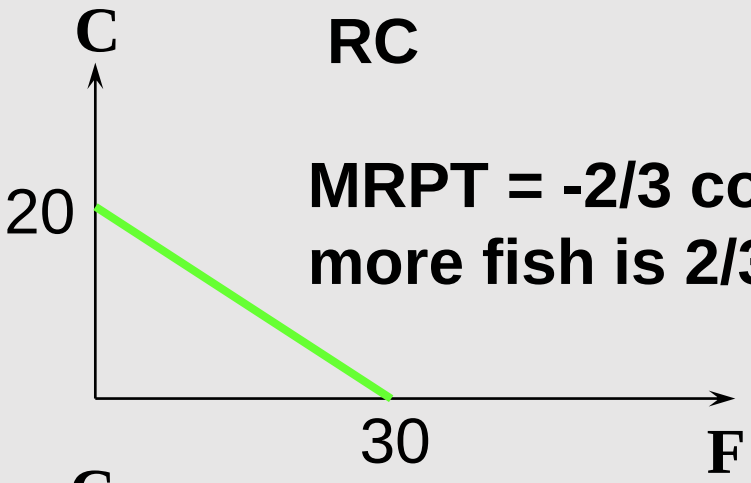
→ RC can produce at most 20 coconuts or 30 fish.

→ MF can produce at most 50 coconuts or 25 fish.

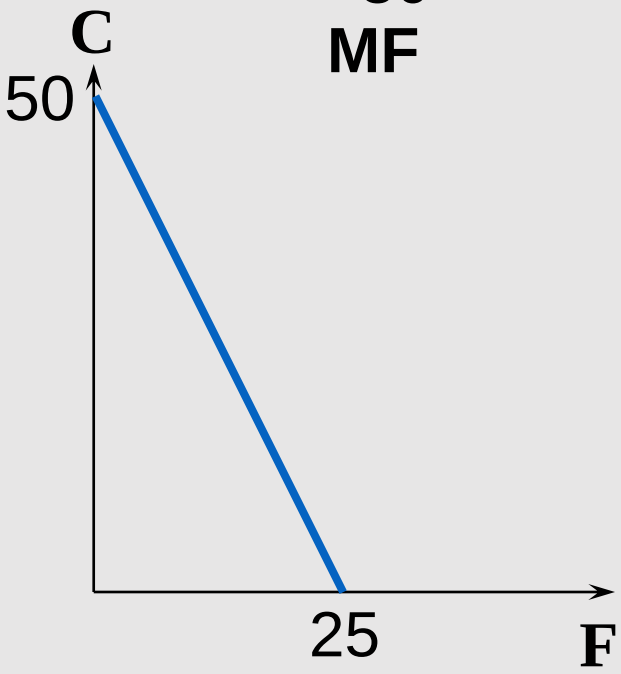
Comparative Advantage



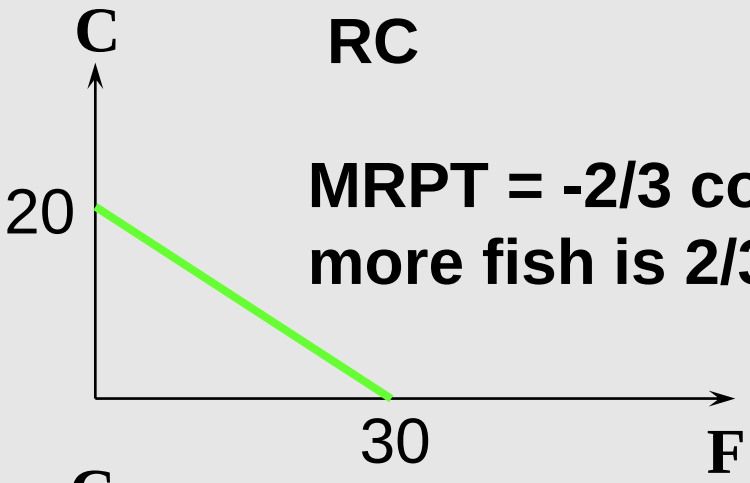
Comparative Advantage



MRPT = -2/3 coconuts/fish so opp. cost of one more fish is 2/3 foregone coconuts.

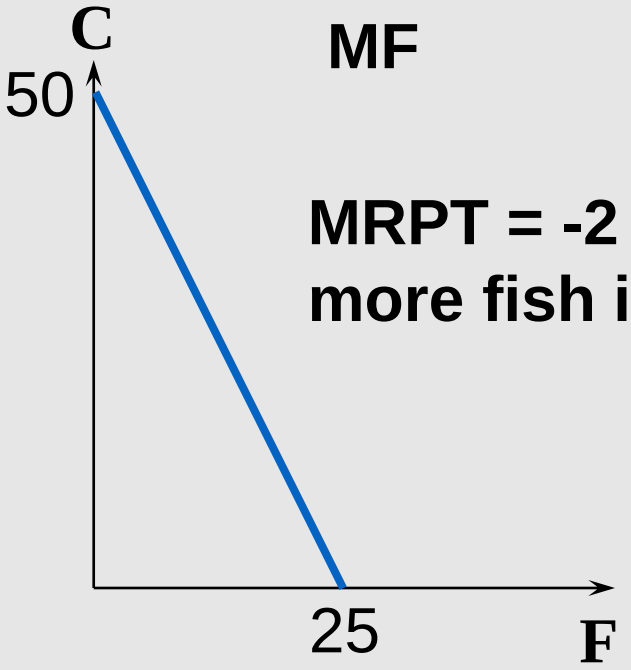


Comparative Advantage



RC

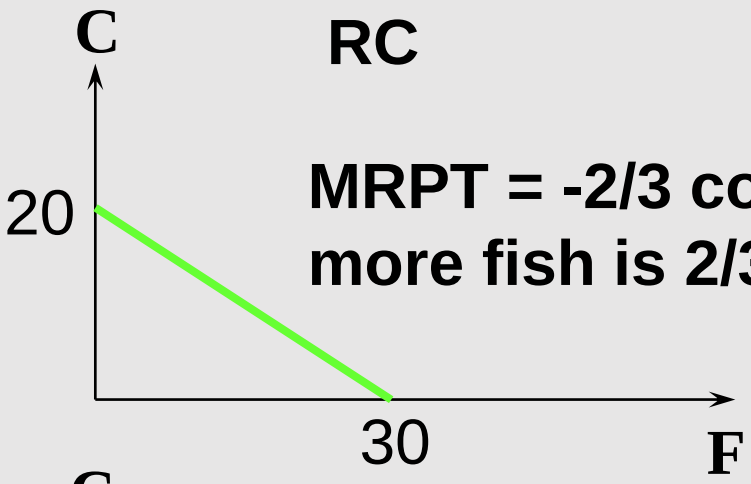
MRPT = -2/3 coconuts/fish so opp. cost of one more fish is 2/3 foregone coconuts.



MF

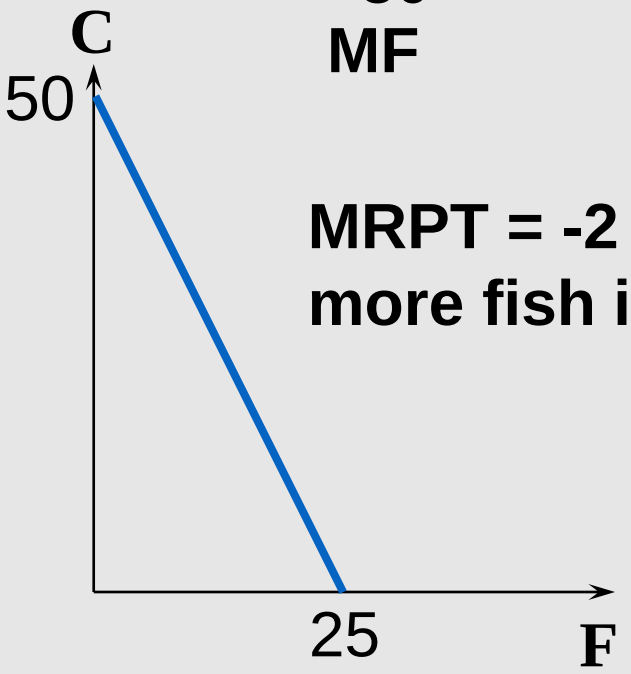
MRPT = -2 coconuts/fish so opp. cost of one more fish is 2 foregone coconuts.

Comparative Advantage



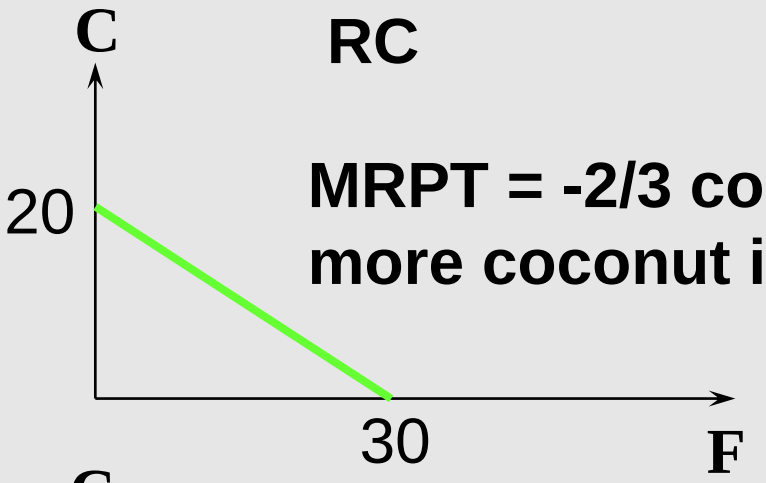
MRPT = -2/3 coconuts/fish so opp. cost of one more fish is 2/3 foregone coconuts.

RC has the comparative opp. cost advantage in producing fish.

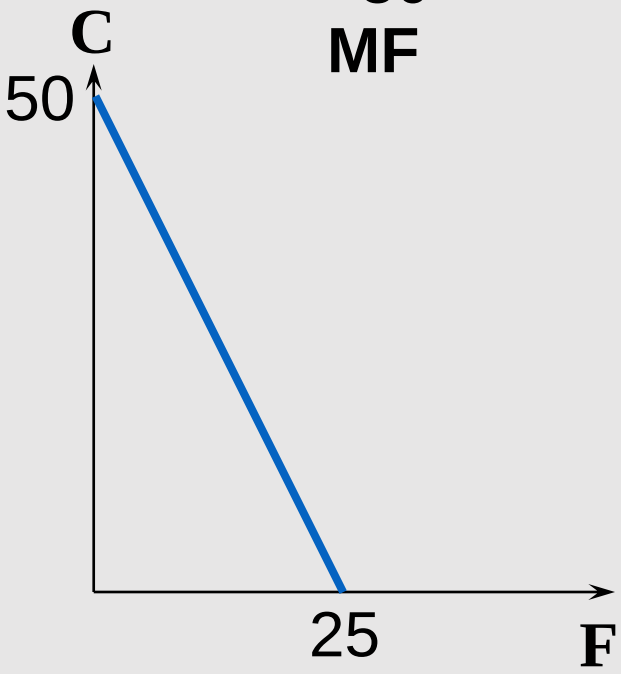


MRPT = -2 coconuts/fish so opp. cost of one more fish is 2 foregone coconuts.

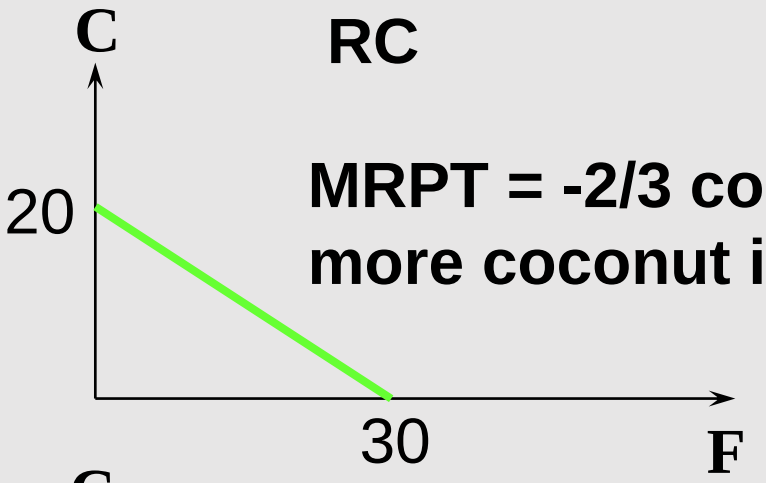
Comparative Advantage



MRPT = -2/3 coconuts/fish so opp. cost of one more coconut is 3/2 foregone fish.

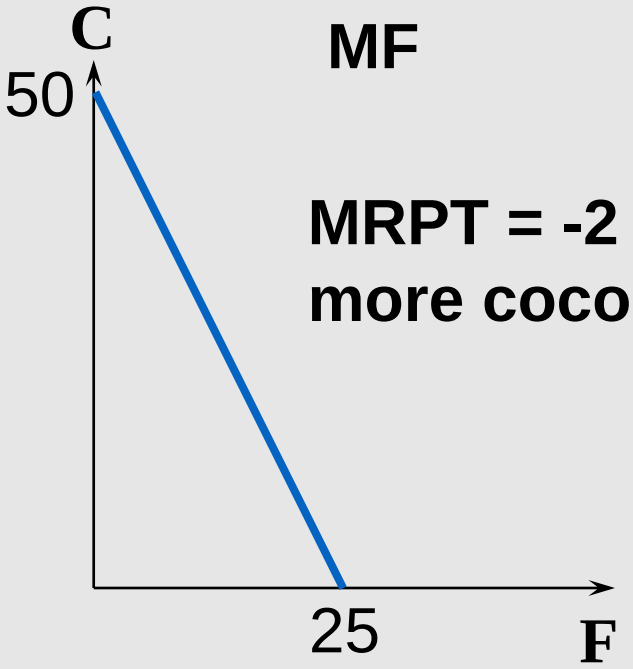


Comparative Advantage



RC

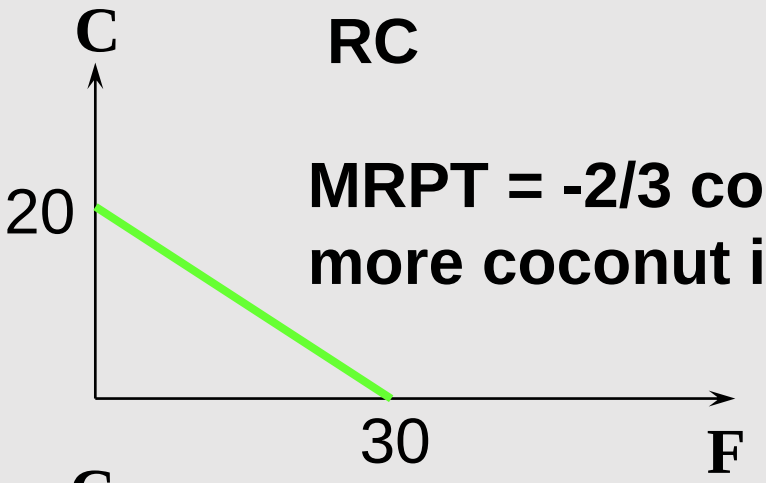
MRPT = -2/3 coconuts/fish so opp. cost of one more coconut is 3/2 foregone fish.



MF

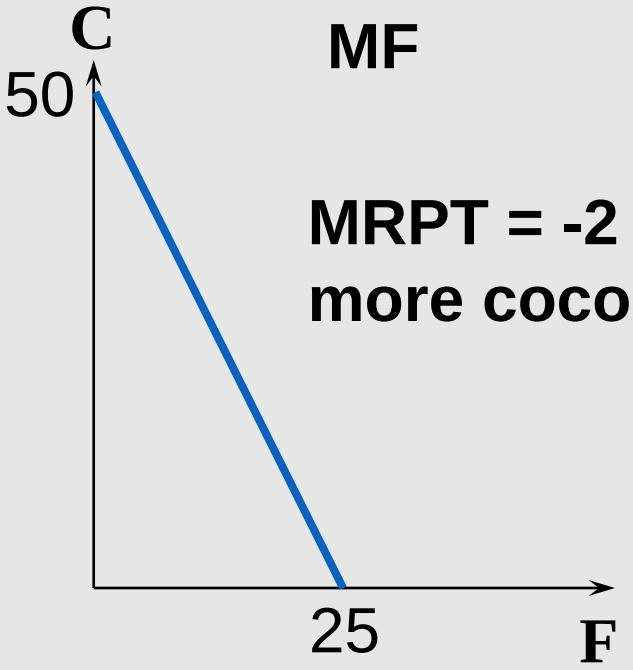
MRPT = -2 coconuts/fish so opp. cost of one more coconut is 1/2 foregone fish.

Comparative Advantage



RC

MRPT = -2/3 coconuts/fish so opp. cost of one more coconut is 3/2 foregone fish.

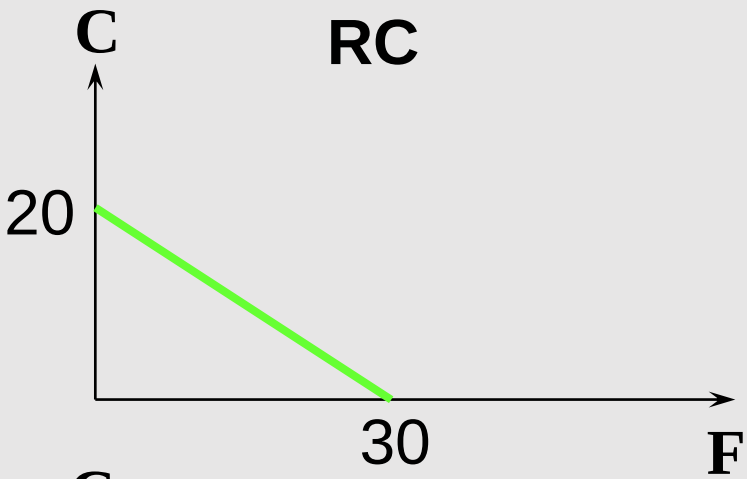


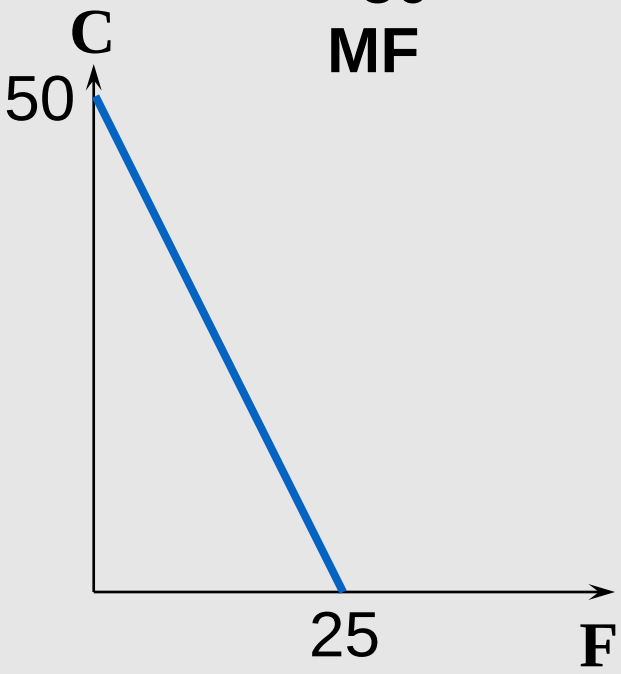
MF

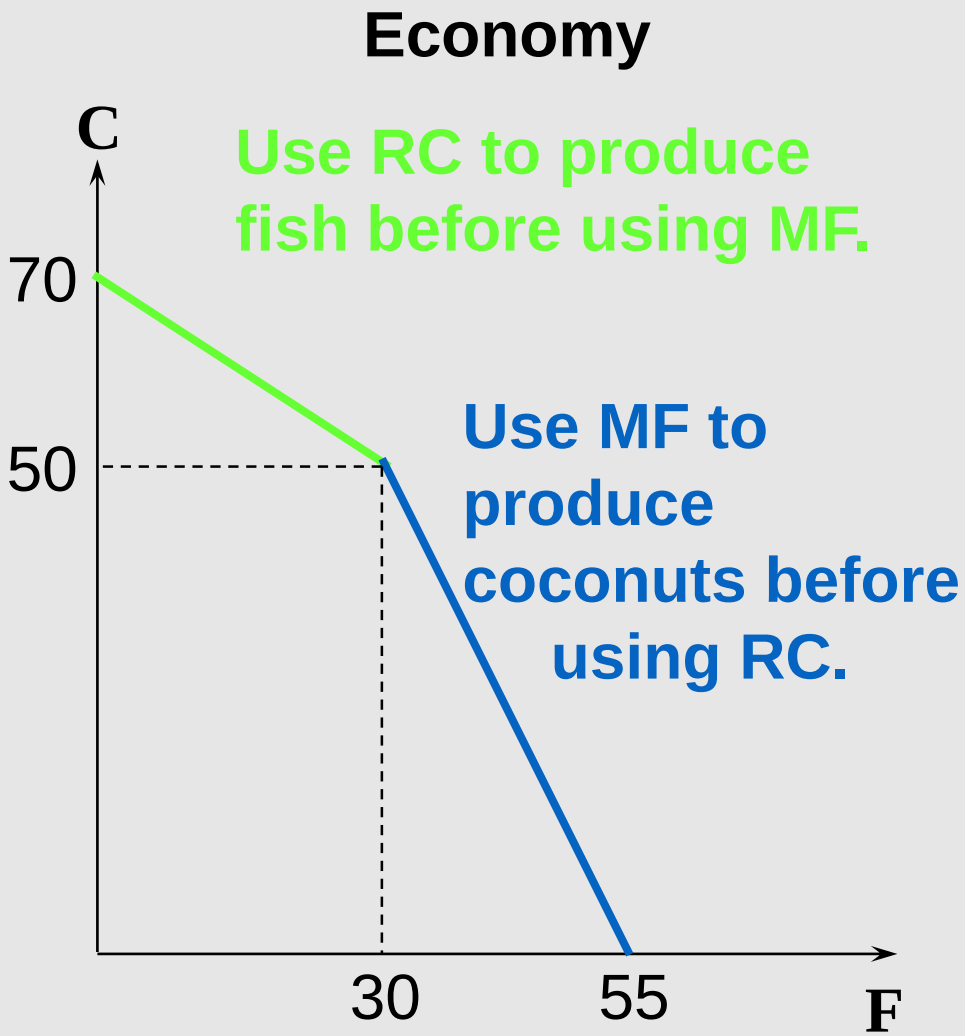
MRPT = -2 coconuts/fish so opp. cost of one more coconut is 1/2 foregone fish.

MF has the comparative opp. cost advantage in producing coconuts.

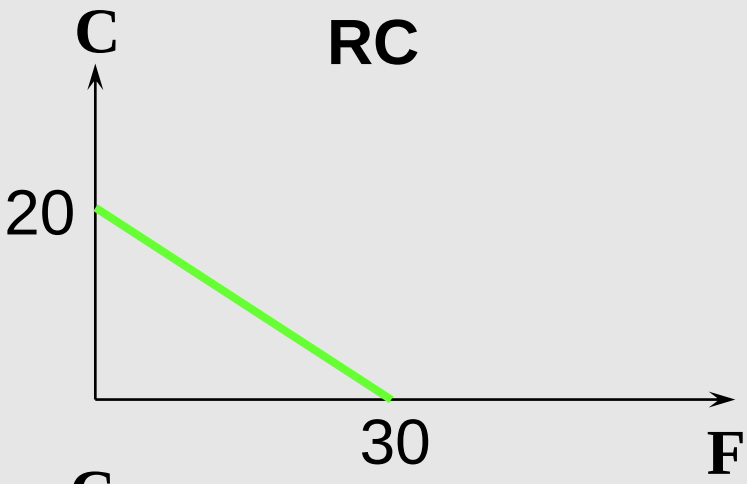
Comparative Advantage

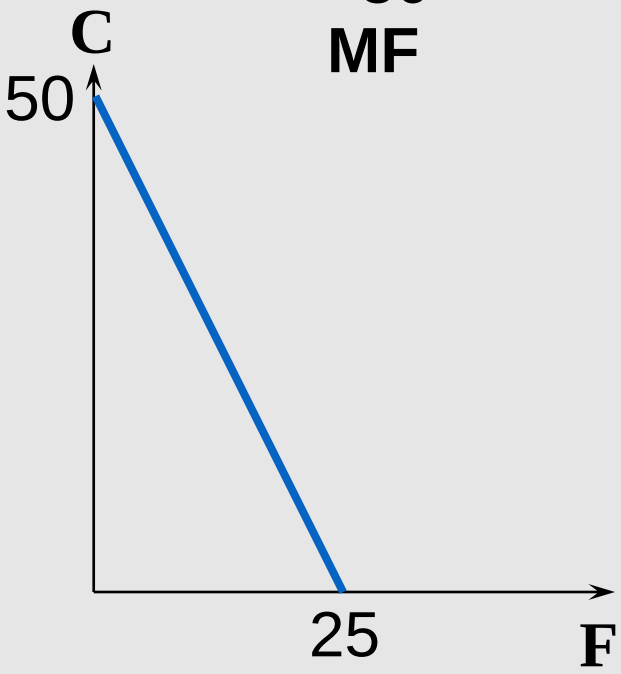






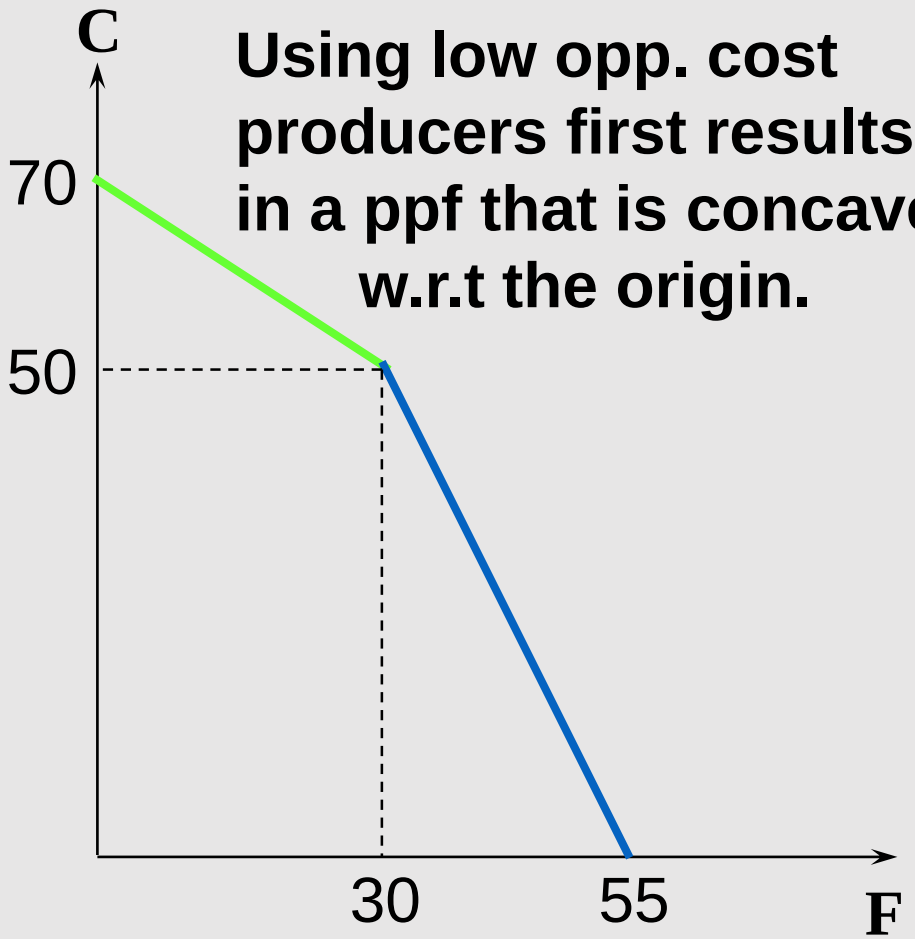
Comparative Advantage





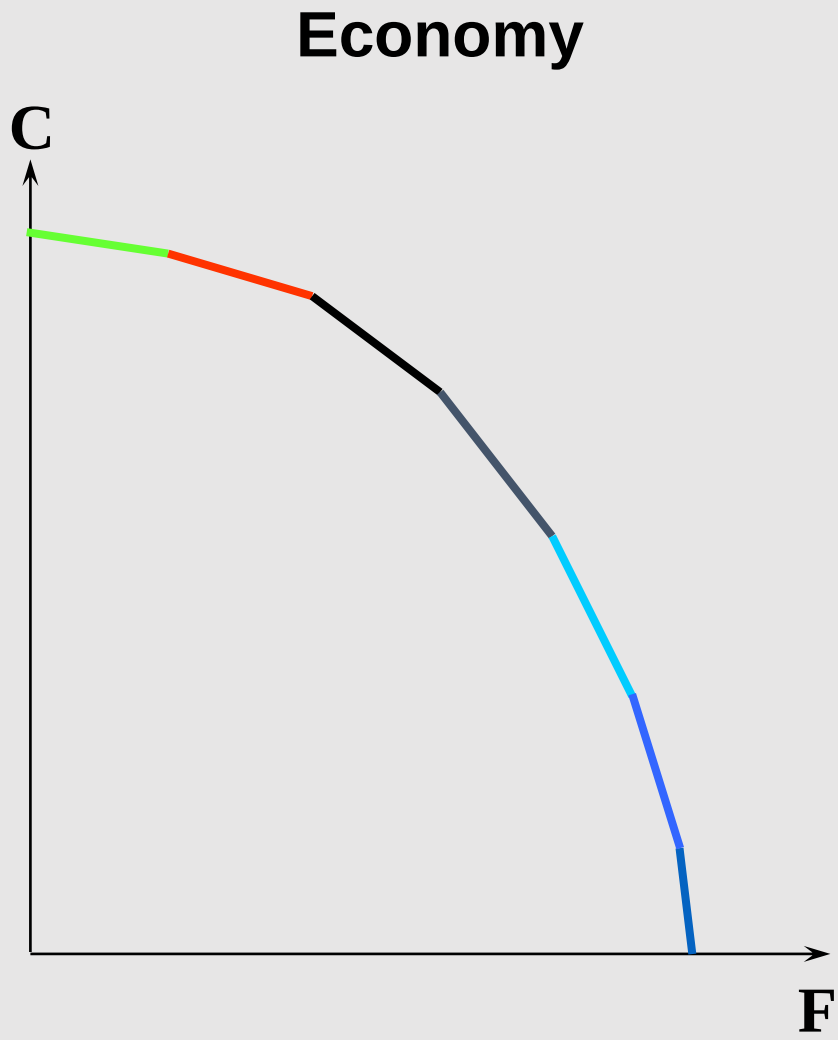
Economy

Using low opp. cost producers first results in a ppf that is concave w.r.t the origin.



Comparative Advantage

More producers with different opp. costs "smooth out" the ppf.

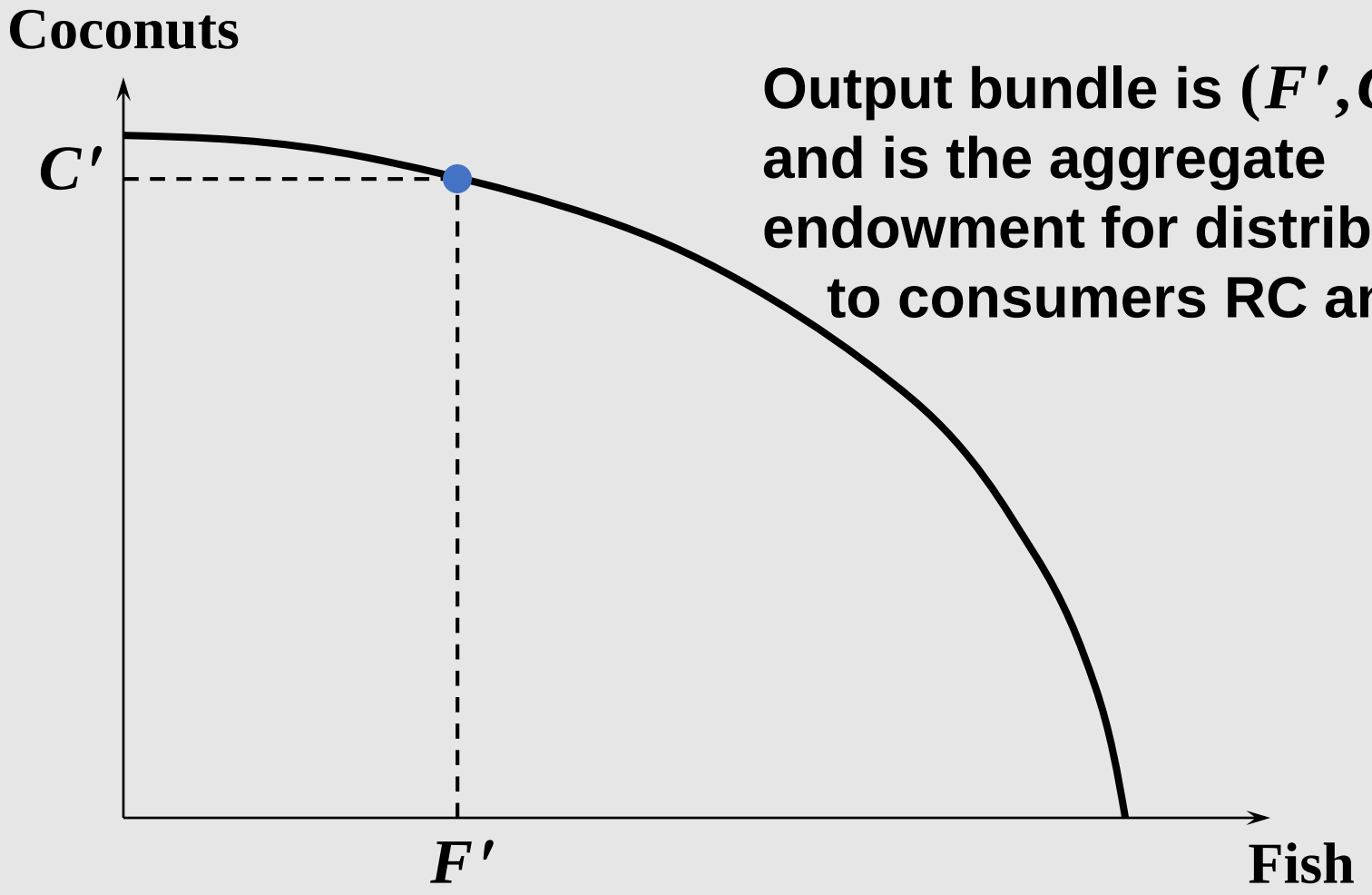


Coordinating Production & Consumption

→ The ppf contains many technically efficient output bundles.

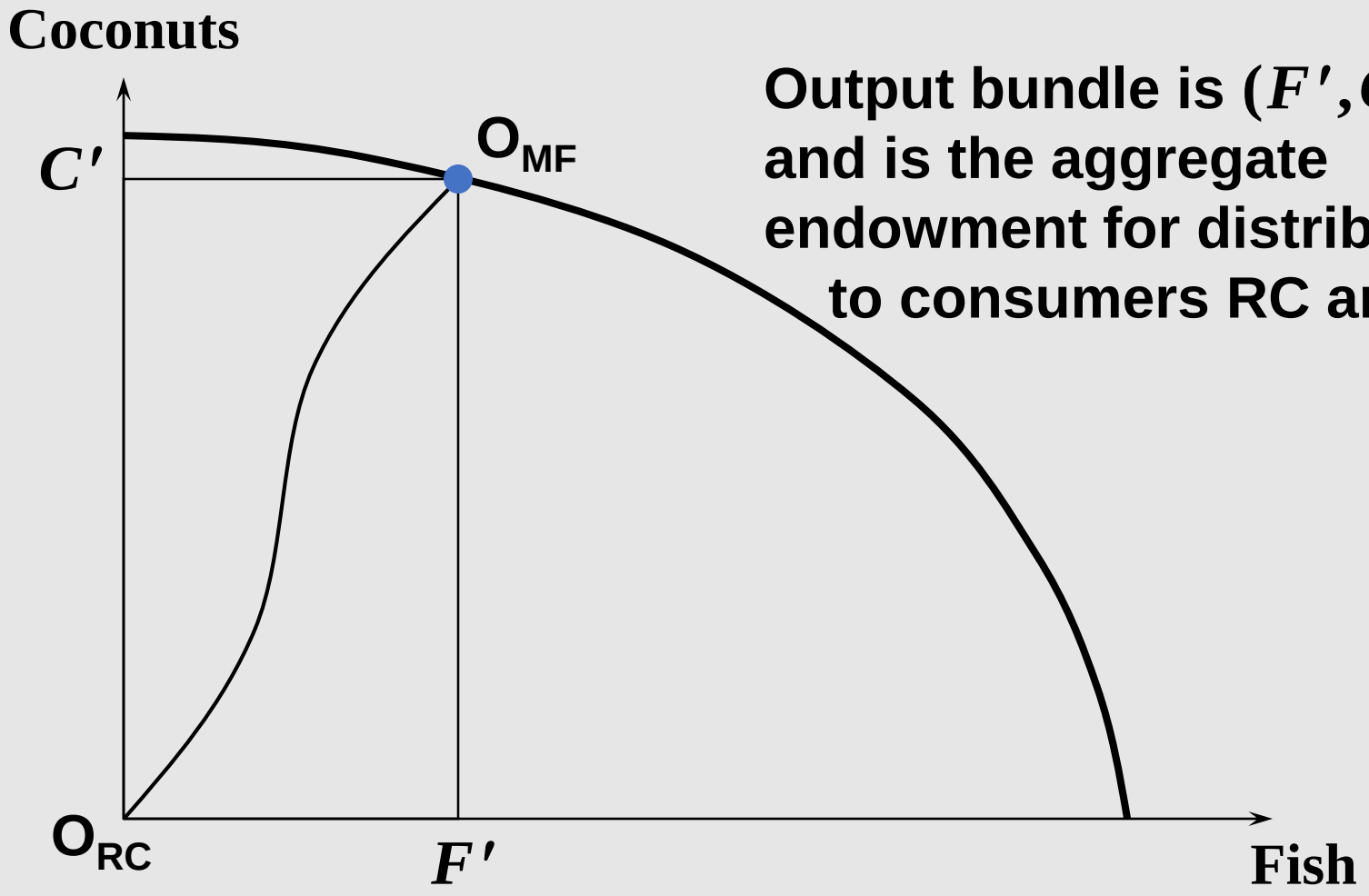
→ Which are Pareto efficient for consumers?

Coordinating Production & Consumption



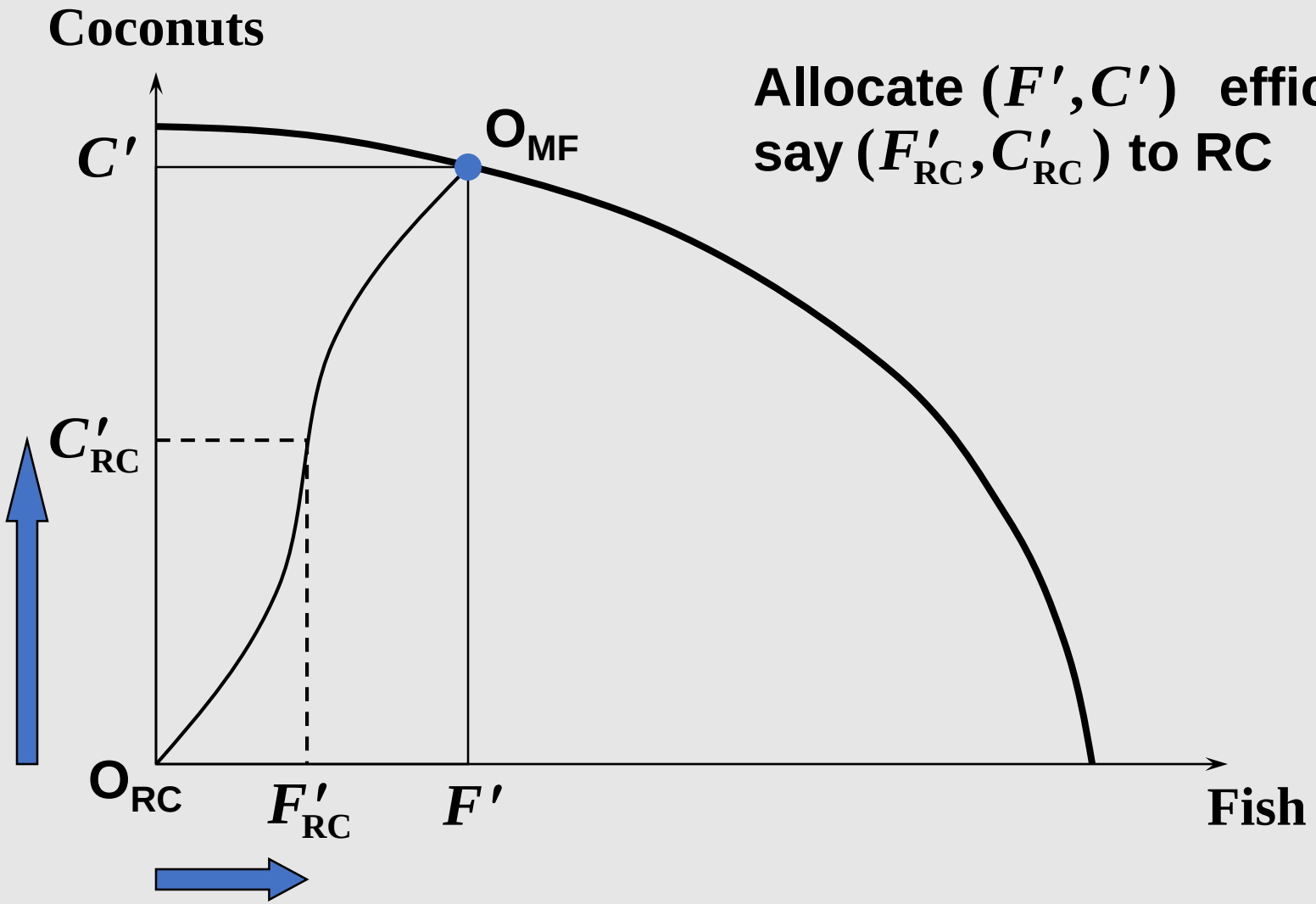
Output bundle is (F', C') and is the aggregate endowment for distribution to consumers RC and MF.

Coordinating Production & Consumption



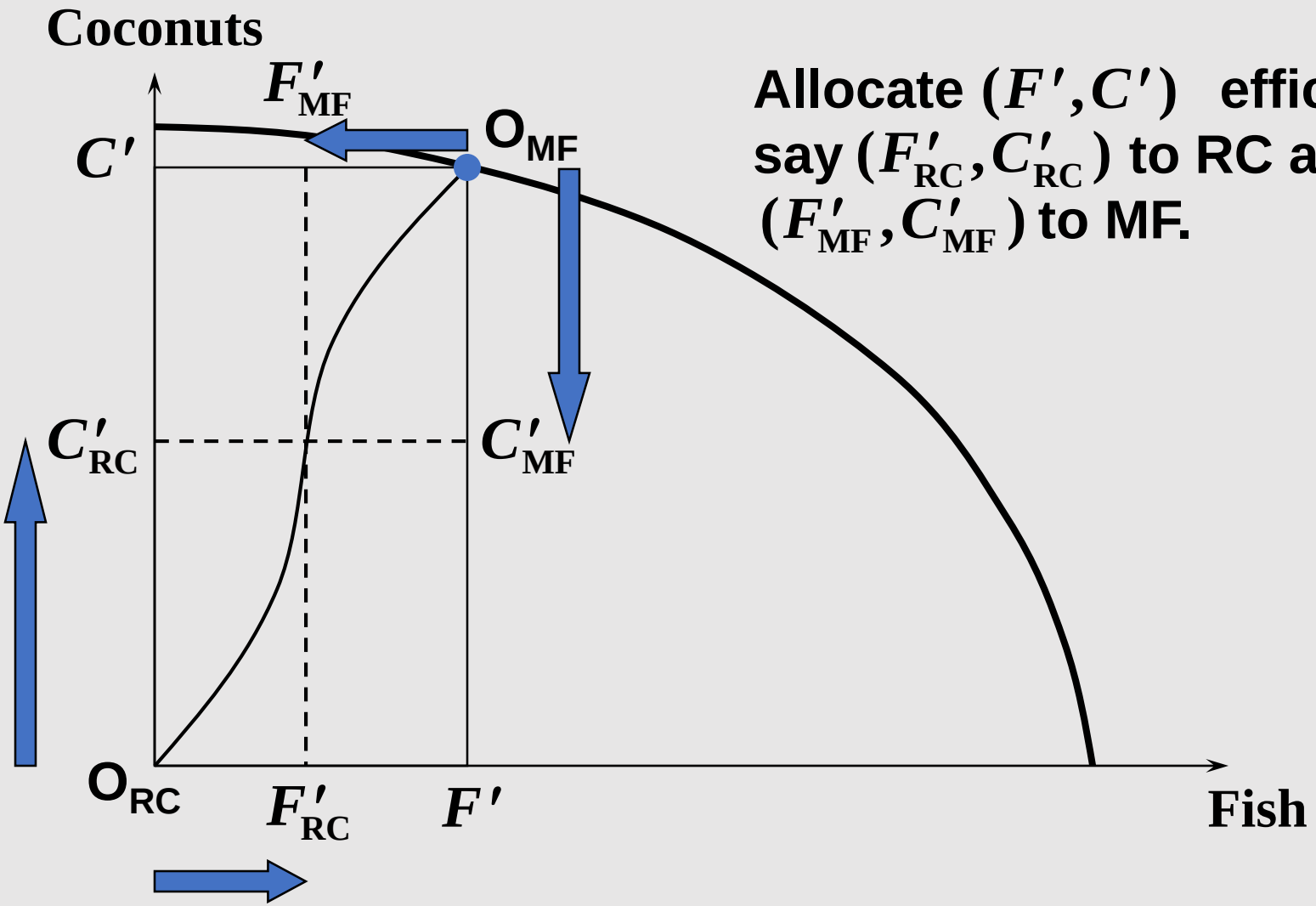
Output bundle is (F', C') and is the aggregate endowment for distribution to consumers RC and MF.

Coordinating Production & Consumption



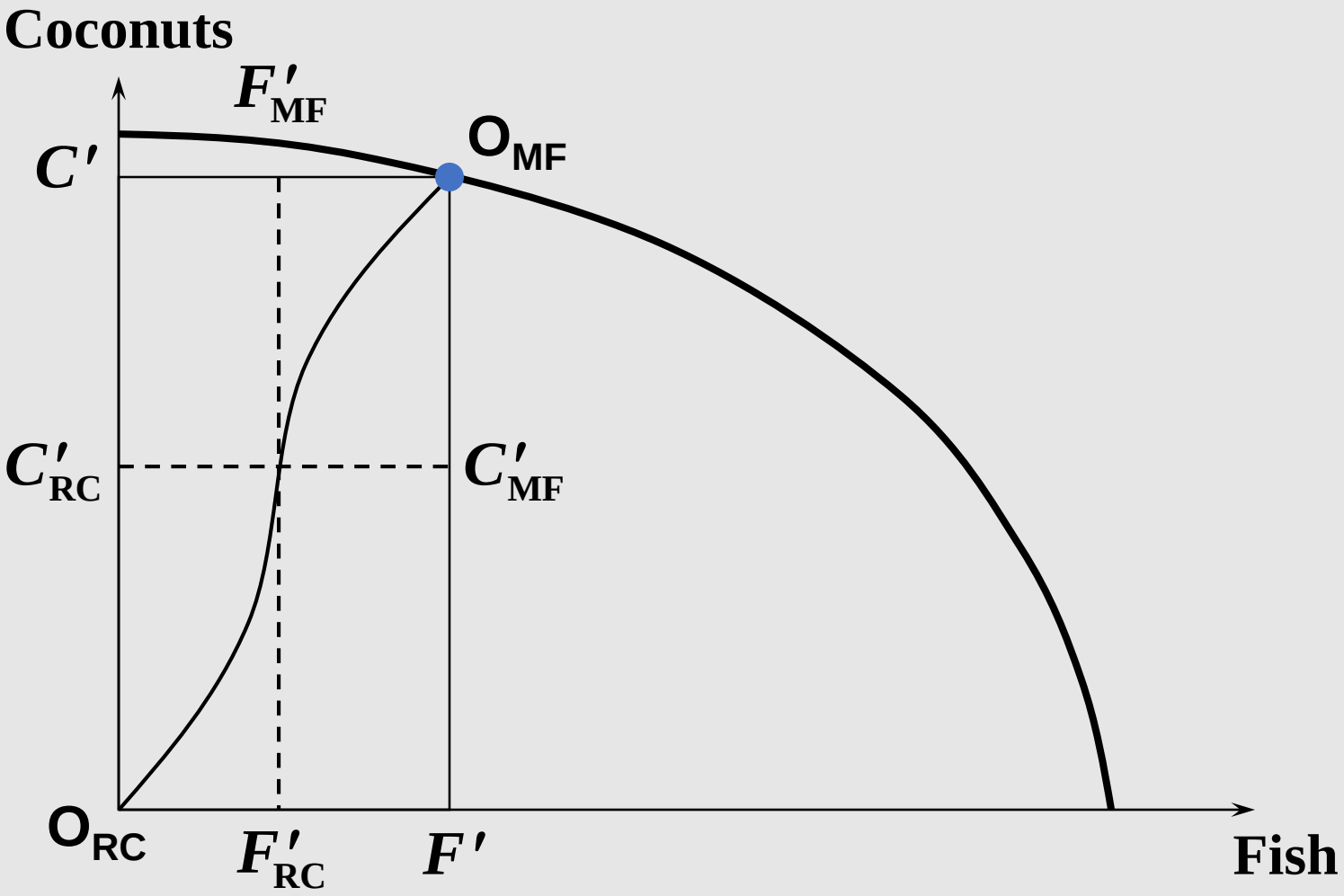
Allocate (F', C') efficiently; say (F'_{RC}, C'_{RC}) to RC

Coordinating Production & Consumption

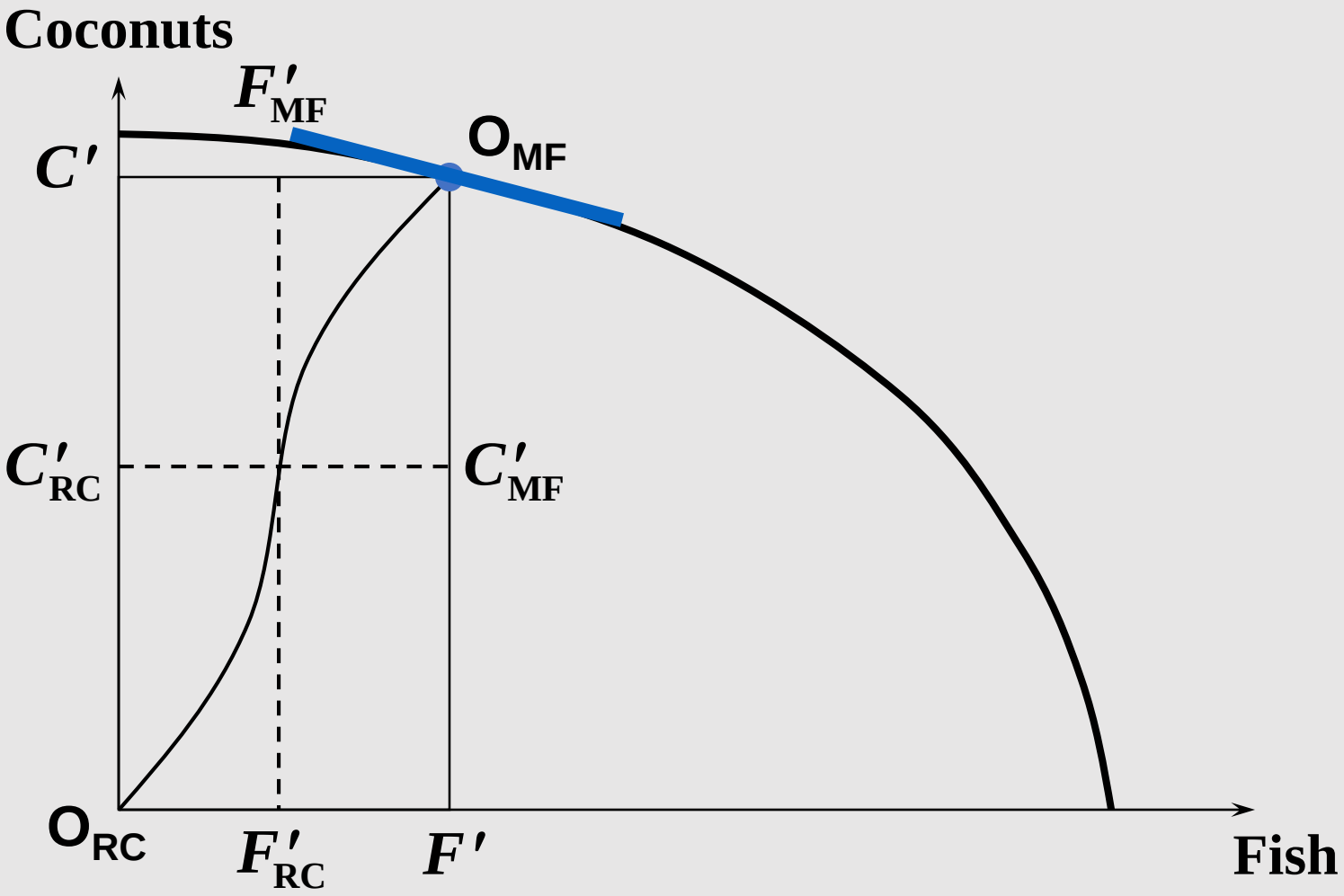


Allocate (F', C') efficiently; say (F'_{RC}, C'_{RC}) to RC and (F'_{MF}, C'_{MF}) to MF.

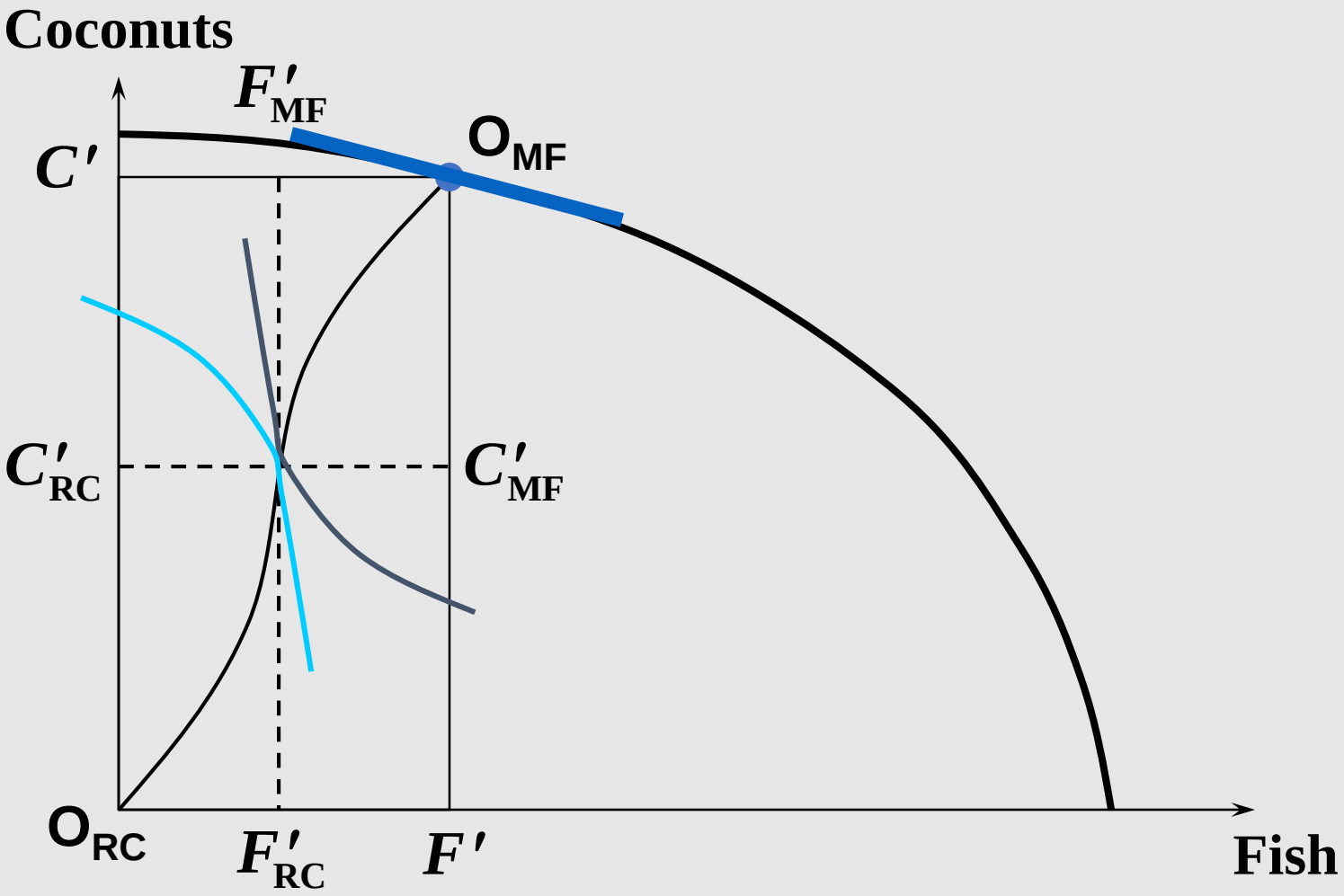
Coordinating Production & Consumption



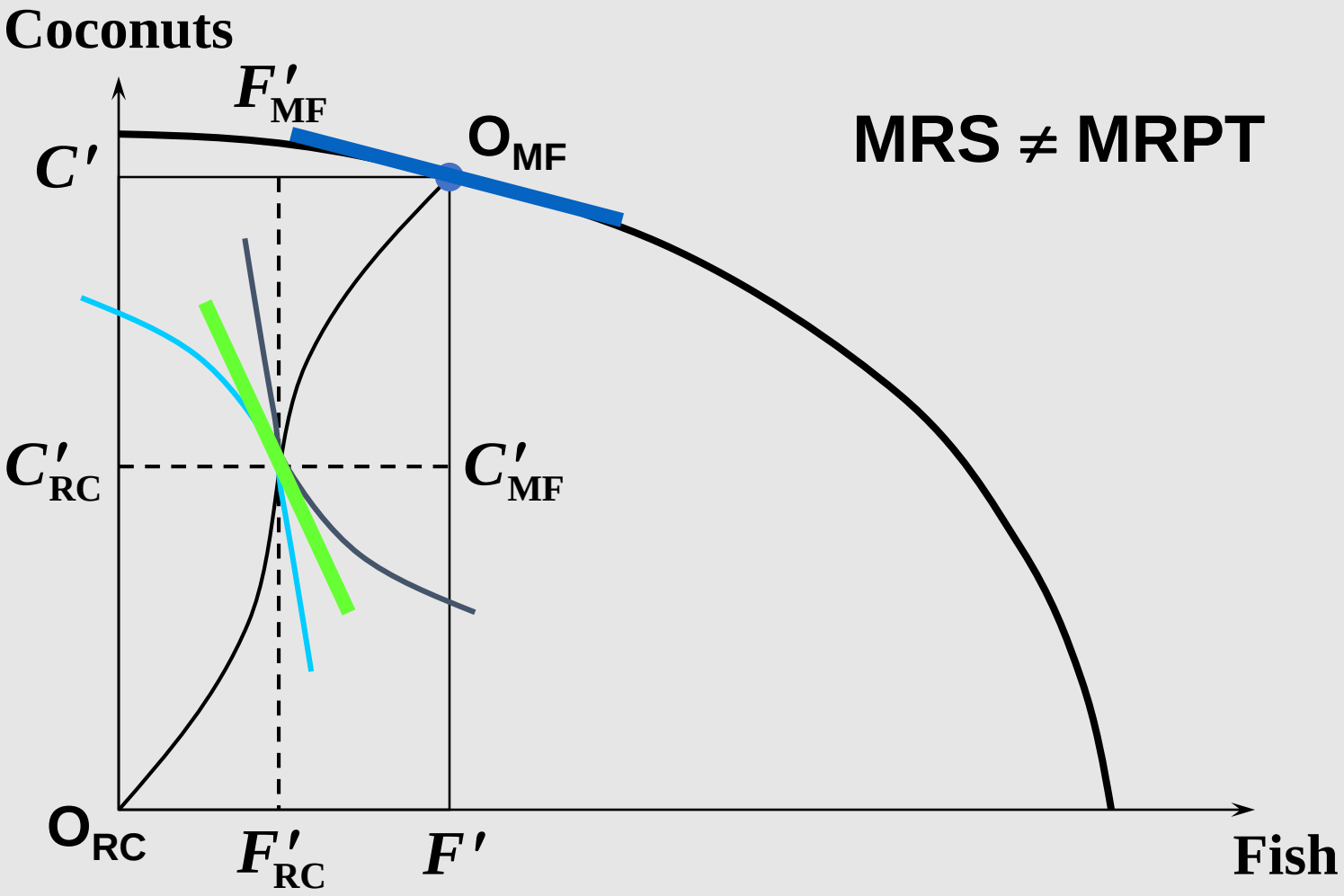
Coordinating Production & Consumption



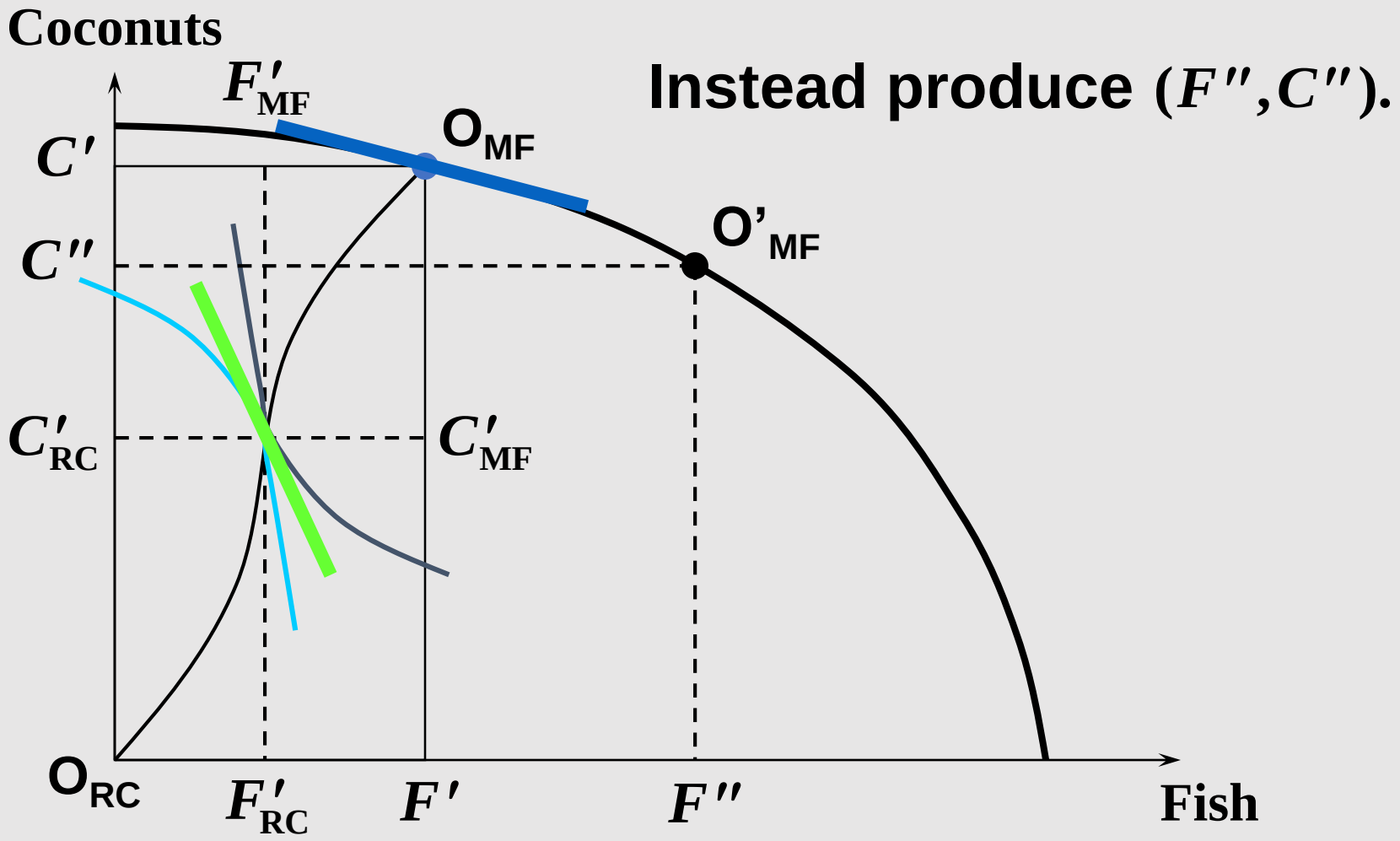
Coordinating Production & Consumption



Coordinating Production & Consumption

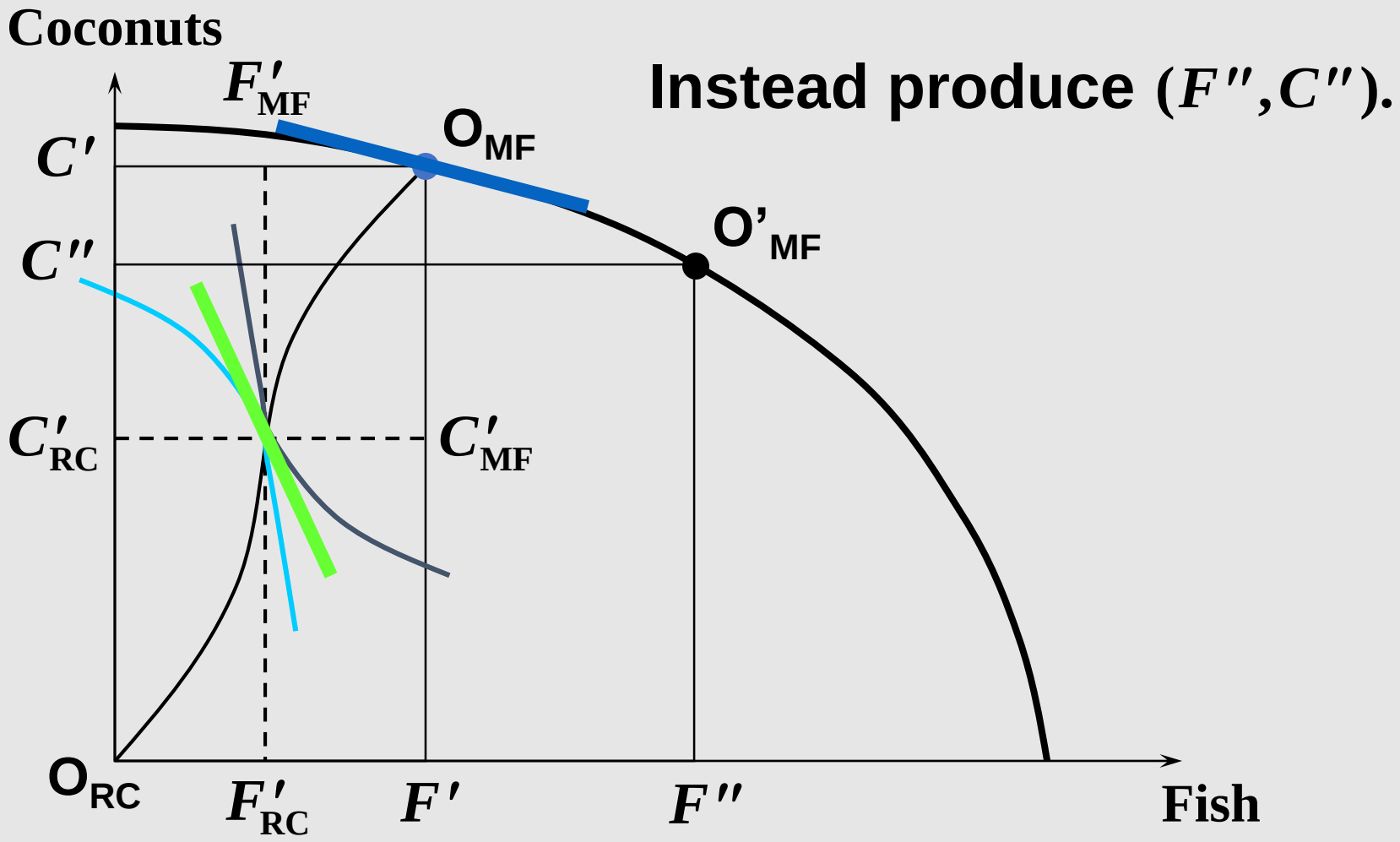


Coordinating Production & Consumption



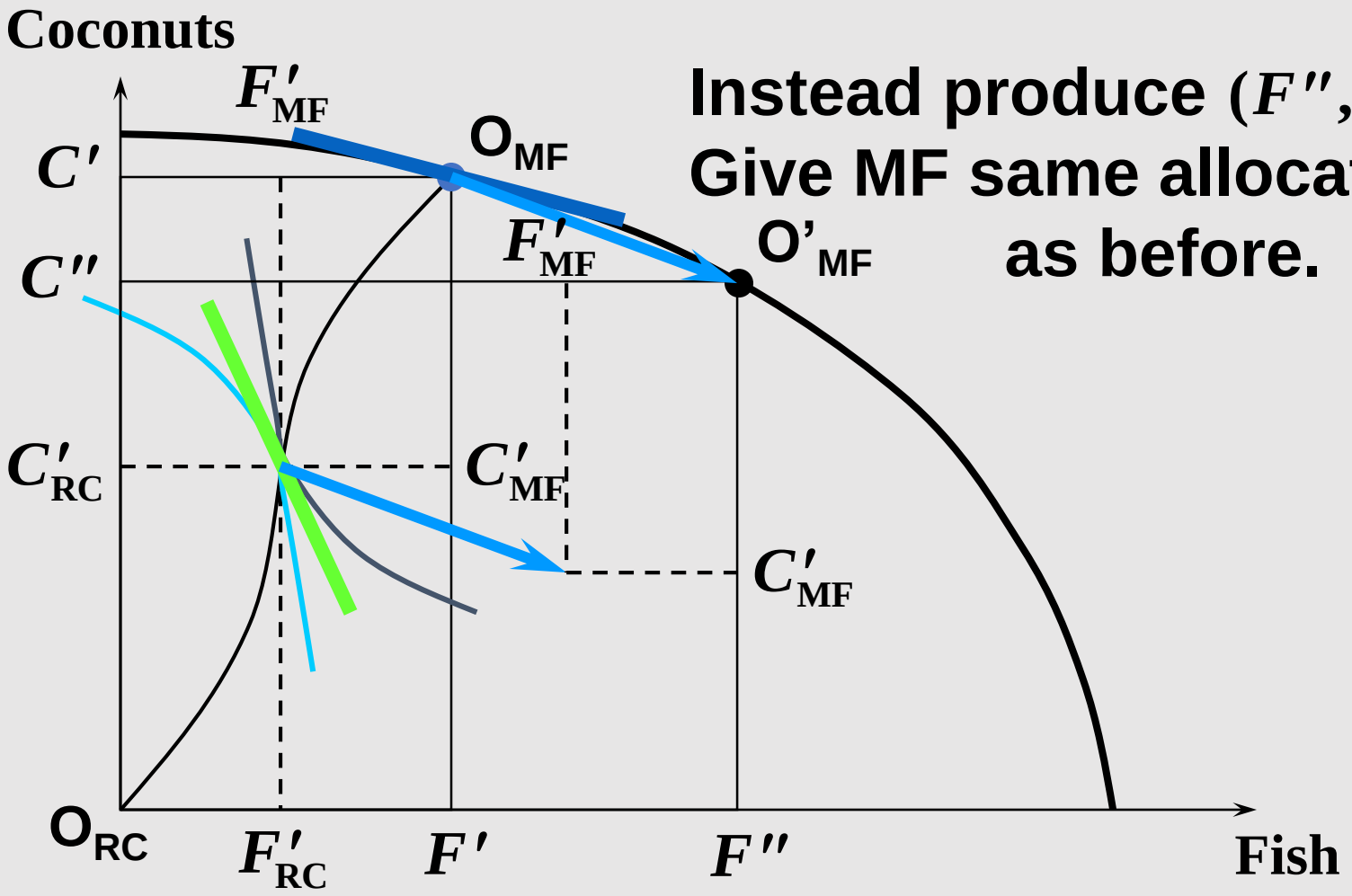
Instead produce (F'', C'').

Coordinating Production & Consumption



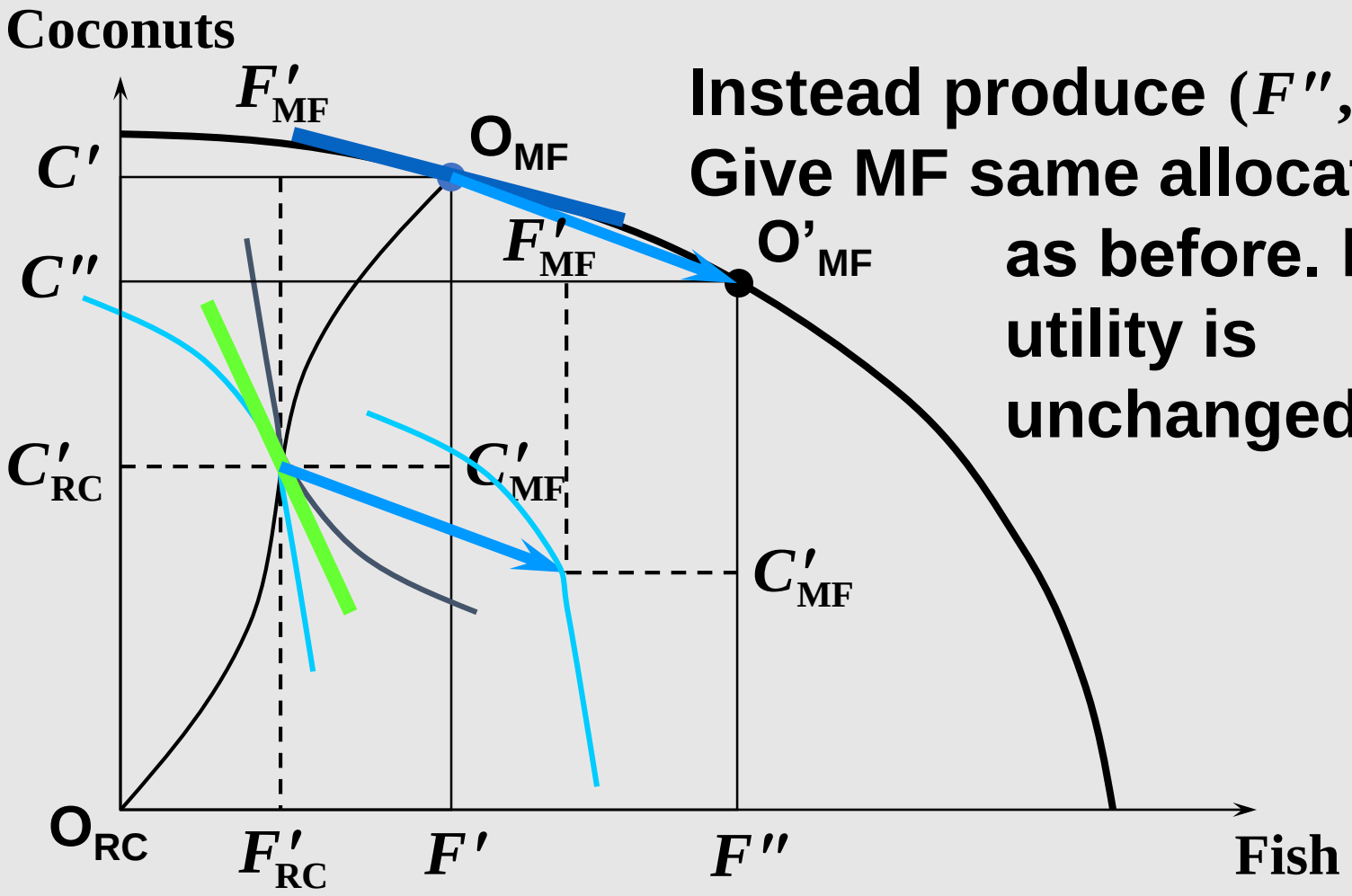
Instead produce (F'', C'').

Coordinating Production & Consumption



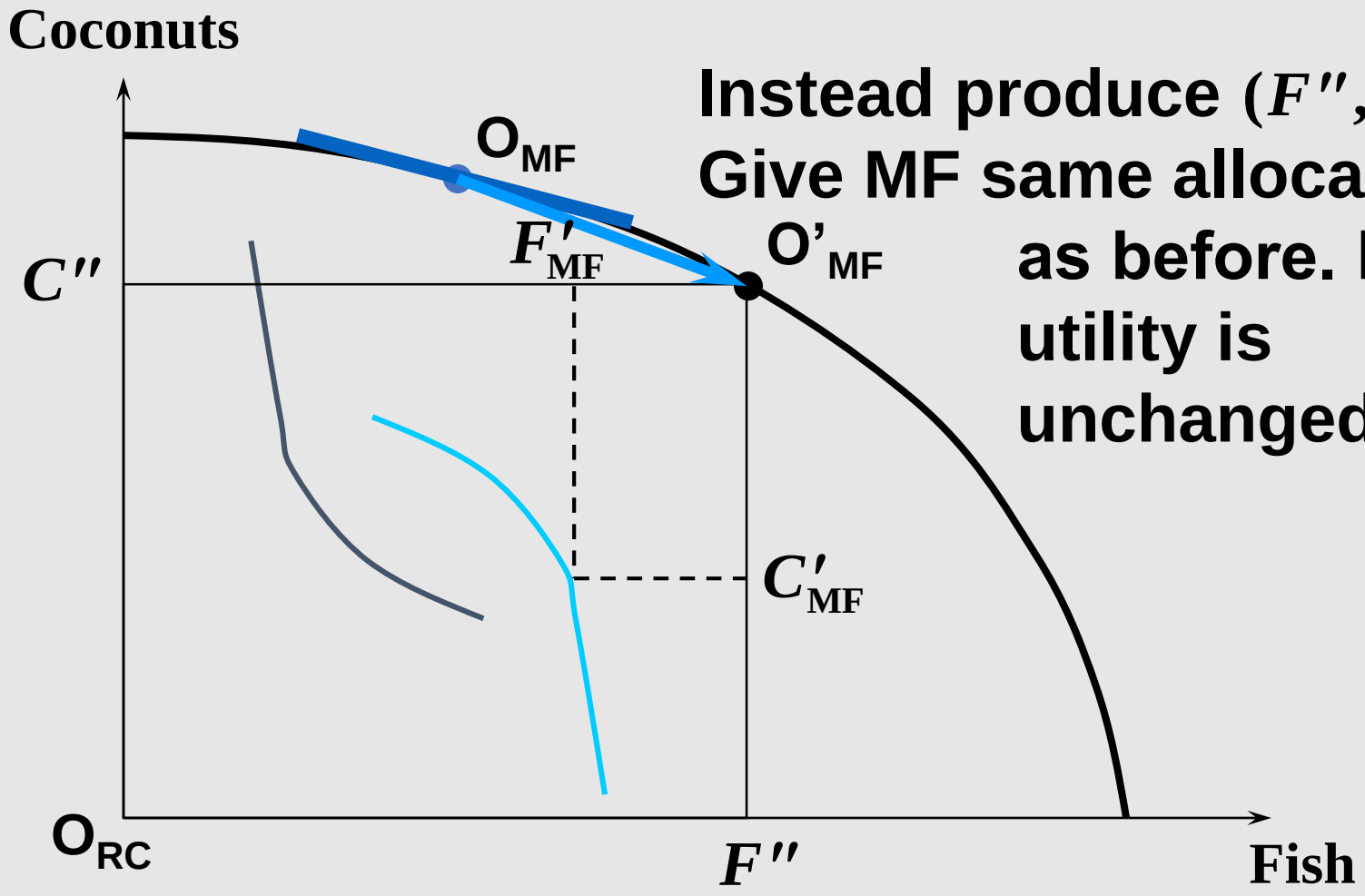
Instead produce (F'', C''). Give MF same allocation as before.

Coordinating Production & Consumption



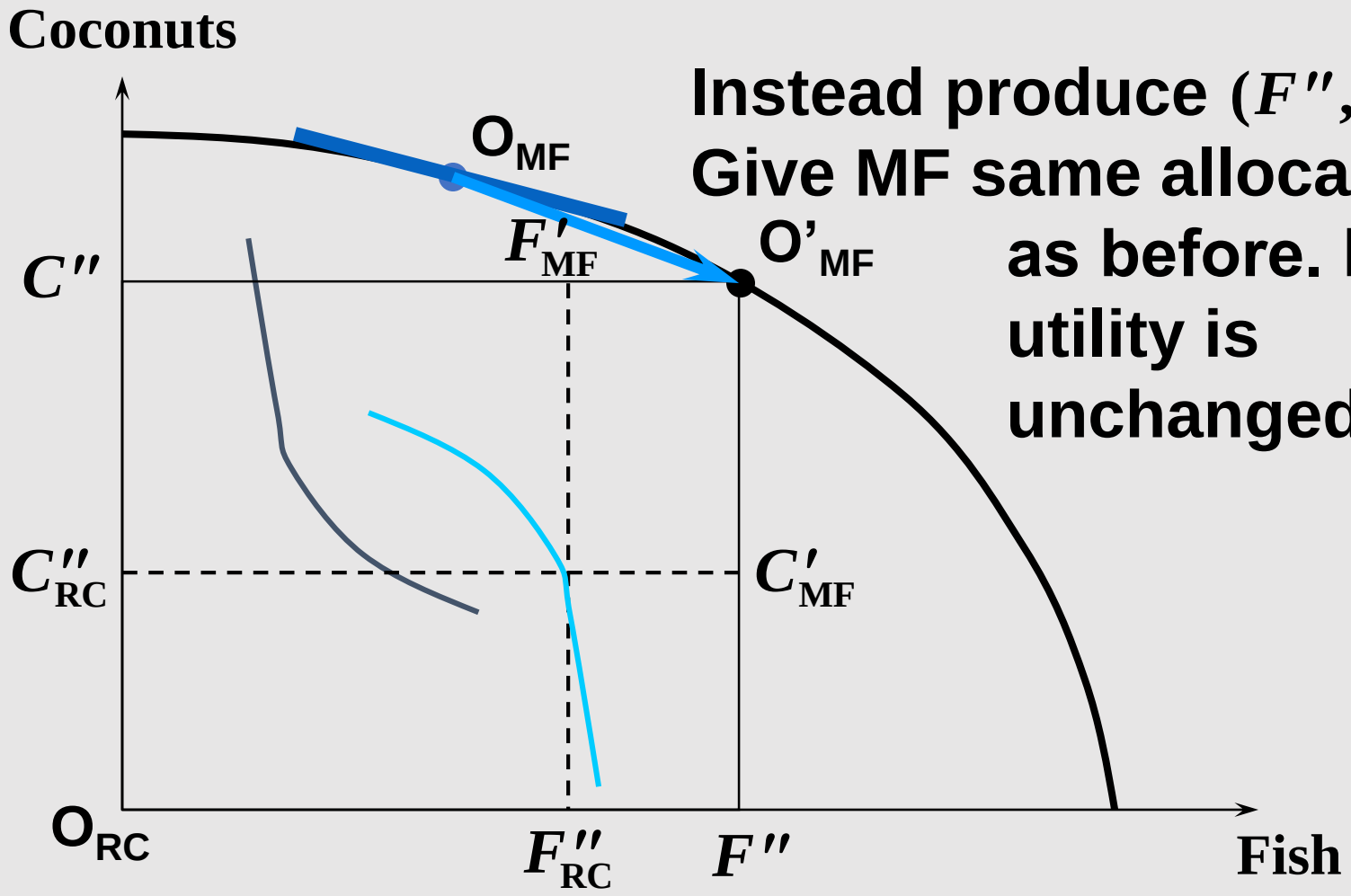
Instead produce (F'',C''). Give MF same allocation as before. MF's utility is unchanged.

Coordinating Production & Consumption



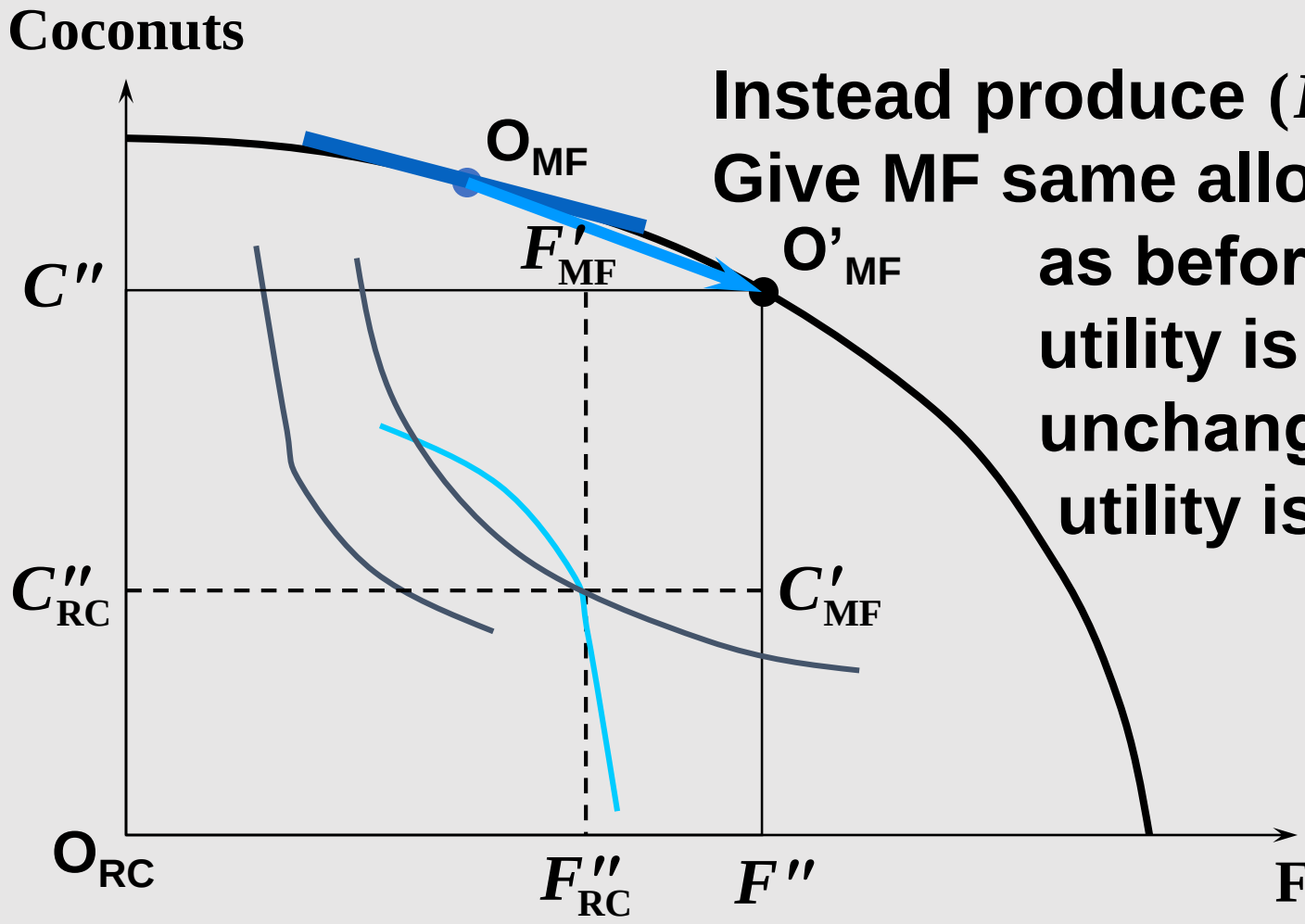
Instead produce (F'', C''). Give MF same allocation as before. MF's utility is unchanged

Coordinating Production & Consumption



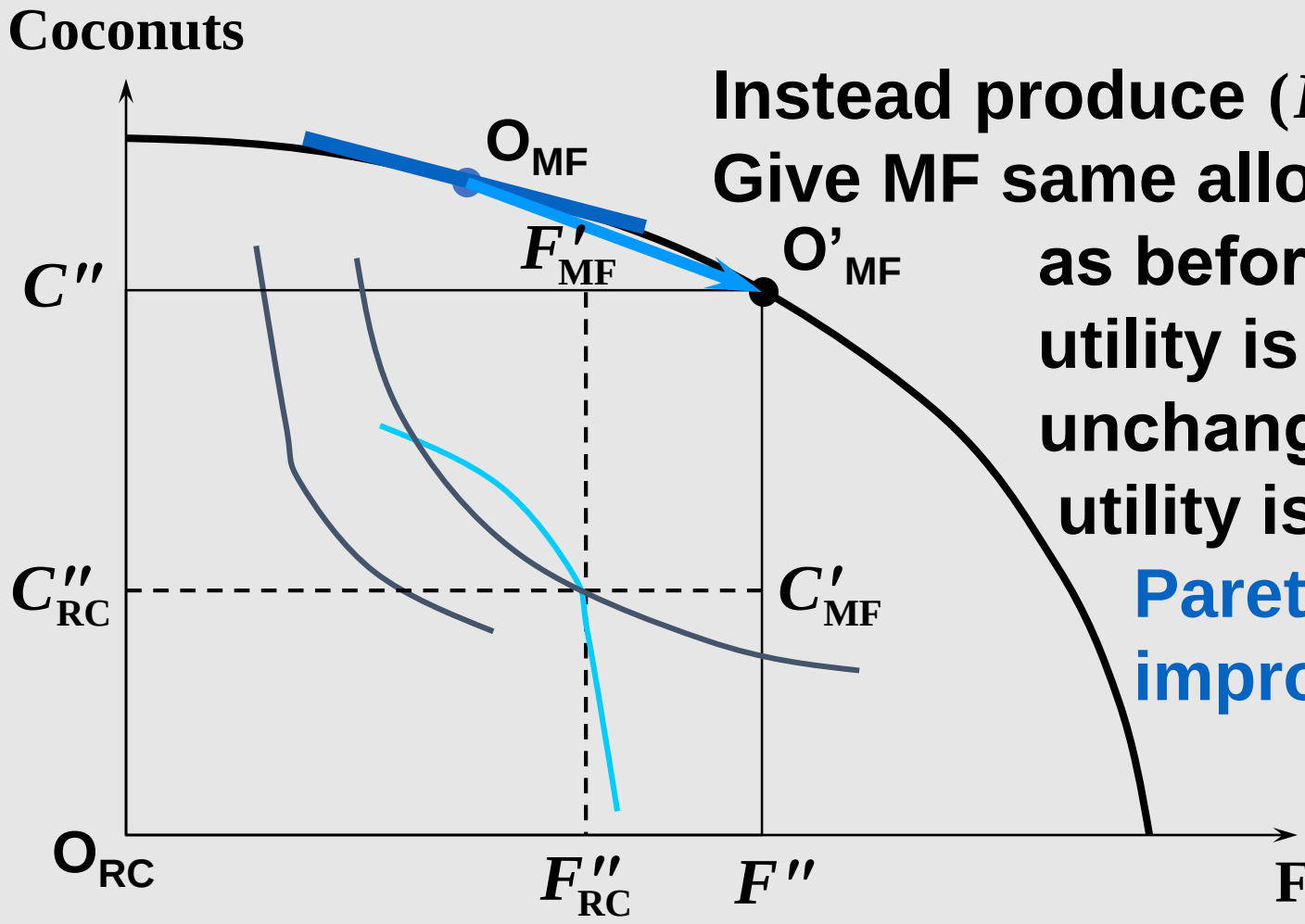
Instead produce (F'', C''). Give MF same allocation as before. MF's utility is unchanged

Coordinating Production & Consumption



Instead produce (F'', C''). Give MF same allocation as before. MF's utility is unchanged, RC's utility is higher

Coordinating Production & Consumption



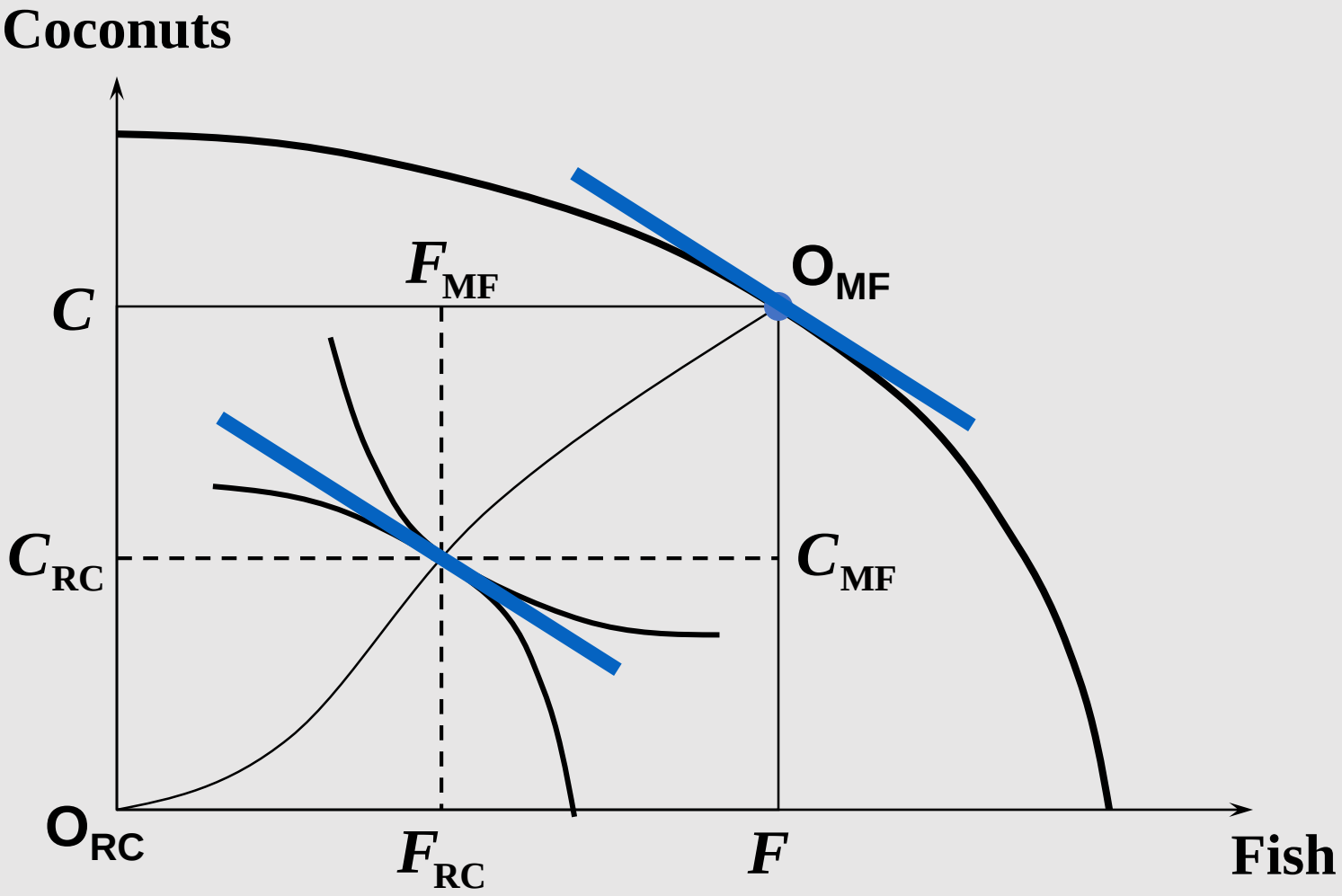
Instead produce (F'', C''). Give MF same allocation as before. MF's utility is unchanged, RC's utility is higher; Pareto improvement.

Coordinating Production & Consumption

→ MRS $\neq$ MRPT $\Rightarrow$ inefficient coordination of production and consumption.

→ Hence, MRS = MRPT is necessary for a Pareto optimal economic state.

Coordinating Production & Consumption



Decentralized Coordination of Production & Consumption

→ RC and MF jointly run a firm producing coconuts and fish.

→ RC and MF are also consumers who can sell labor.

→ Price of coconut = p_C.

→ Price of fish = p_F.

→ RC's wage rate = w_{RC}.

→ MF's wage rate = w_{MF}.

Decentralized Coordination of Production & Consumption

→ L_{RC}, L_{MF} are amounts of labor purchased from RC and MF.

→F irm's profit-maximization problem is choose *C*, *F*, L_{RC} and L_{MF} to

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

Decentralized Coordination of Production & Consumption

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

Isoprofit line equation is

$$\text{constant } \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}$$

Decentralized Coordination of Production & Consumption

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

Isoprofit line equation is

$$\text{constant } \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}$$

which rearranges to

$$C = \frac{\pi + w_{RC} L_{RC} + w_{MF} L_{MF}}{p_C} - \frac{p_F}{p_C} F.$$

Decentralized Coordination of Production & Consumption

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

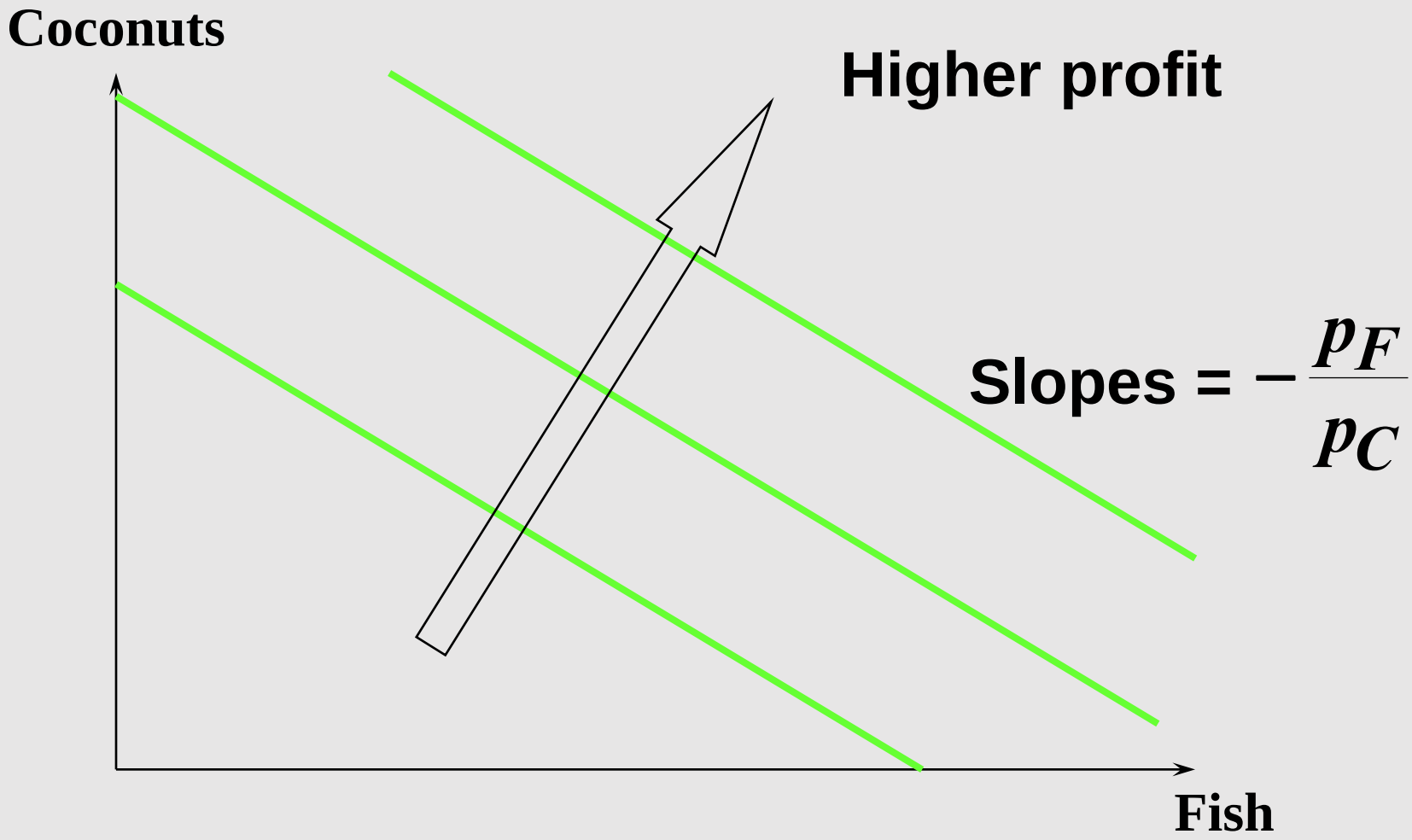
Isoprofit line equation is

$$\text{constant } \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}$$

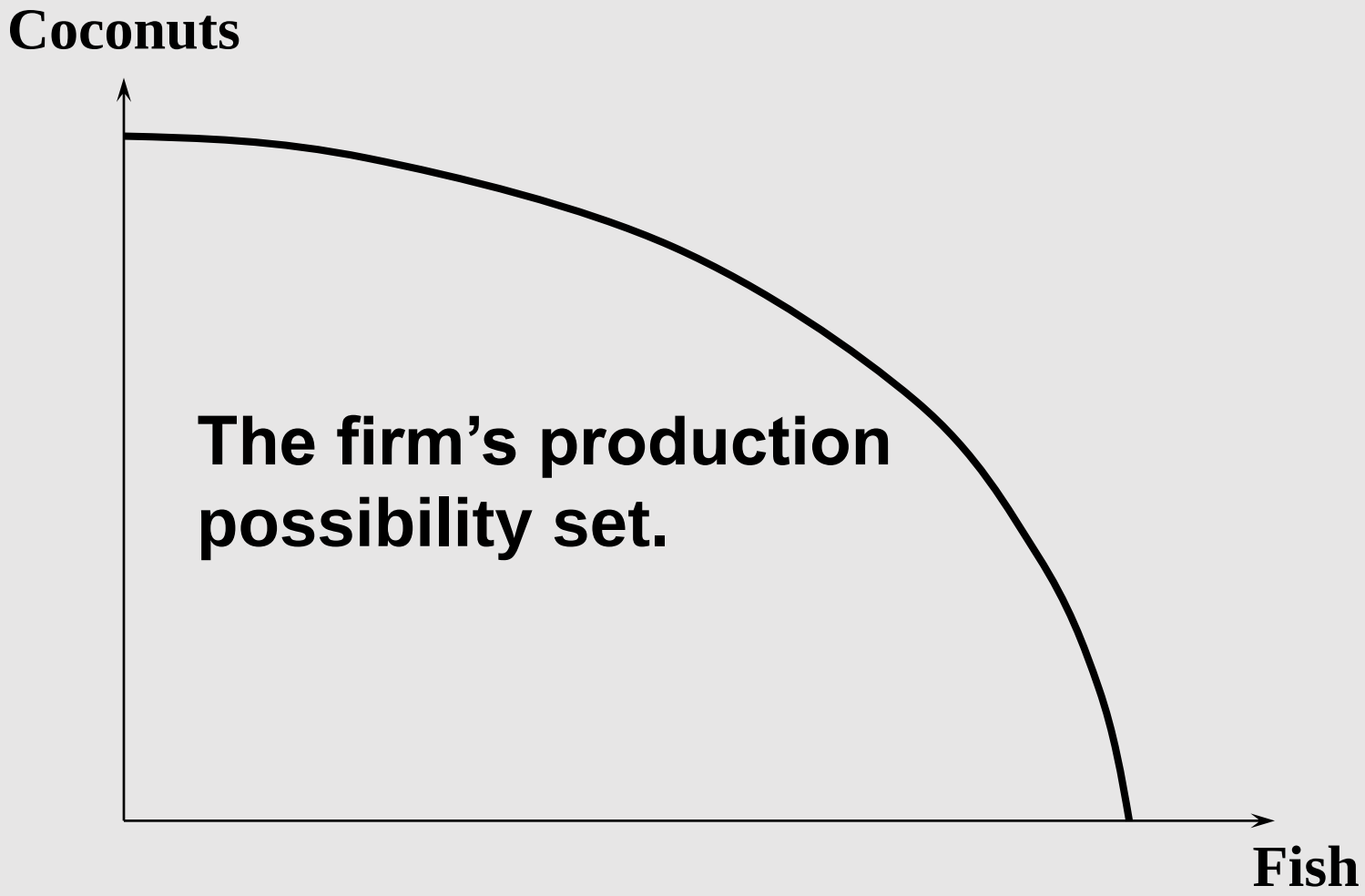
which rearranges to

$$C = \underbrace{\frac{\pi + w_{RC} L_{RC} + w_{MF} L_{MF}}{p_C}}_{\text{Intercept}} \underbrace{- \frac{p_F}{p_C}}_{\text{Slope}} F.$$

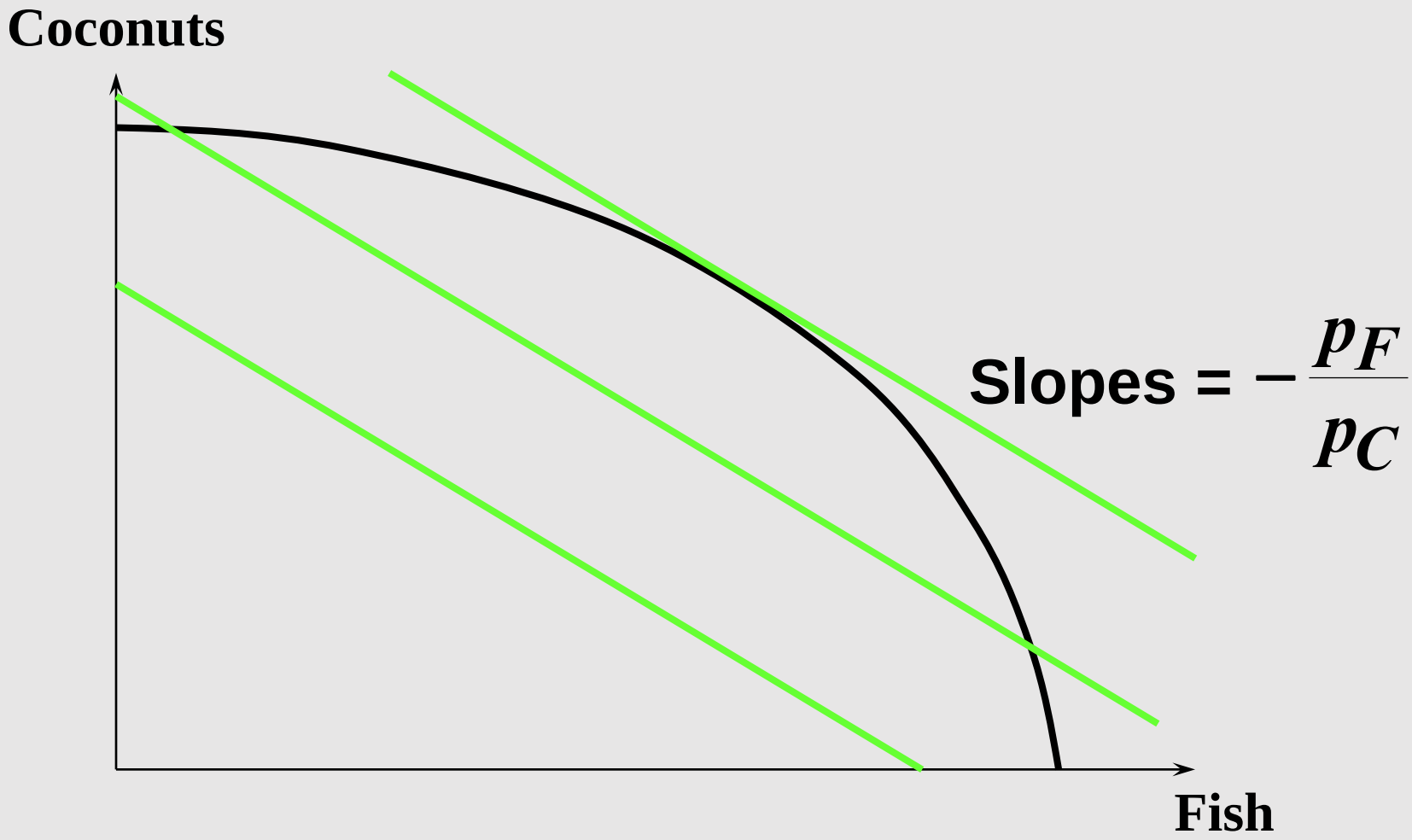
Decentralized Coordination of Production & Consumption



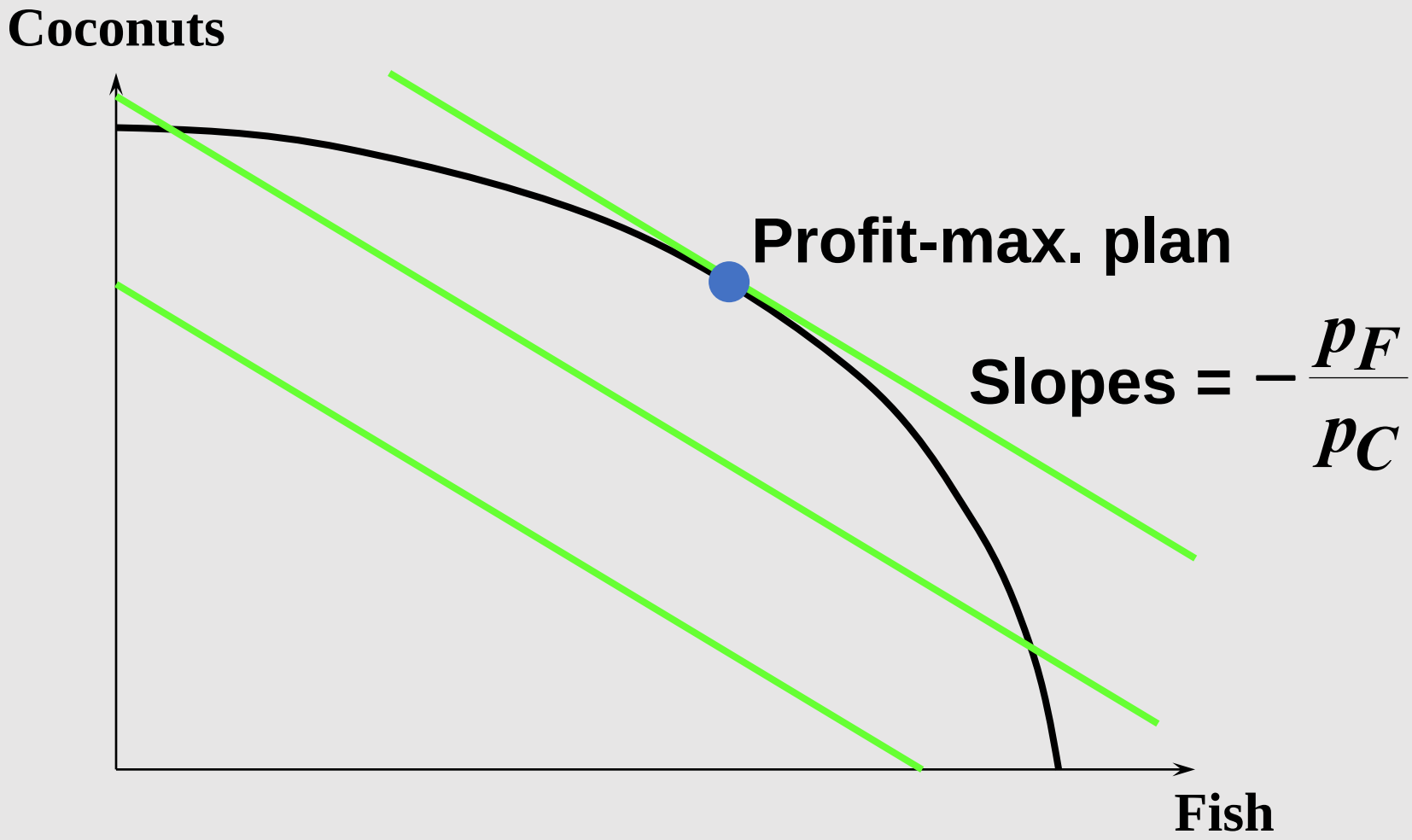
Decentralized Coordination of Production & Consumption



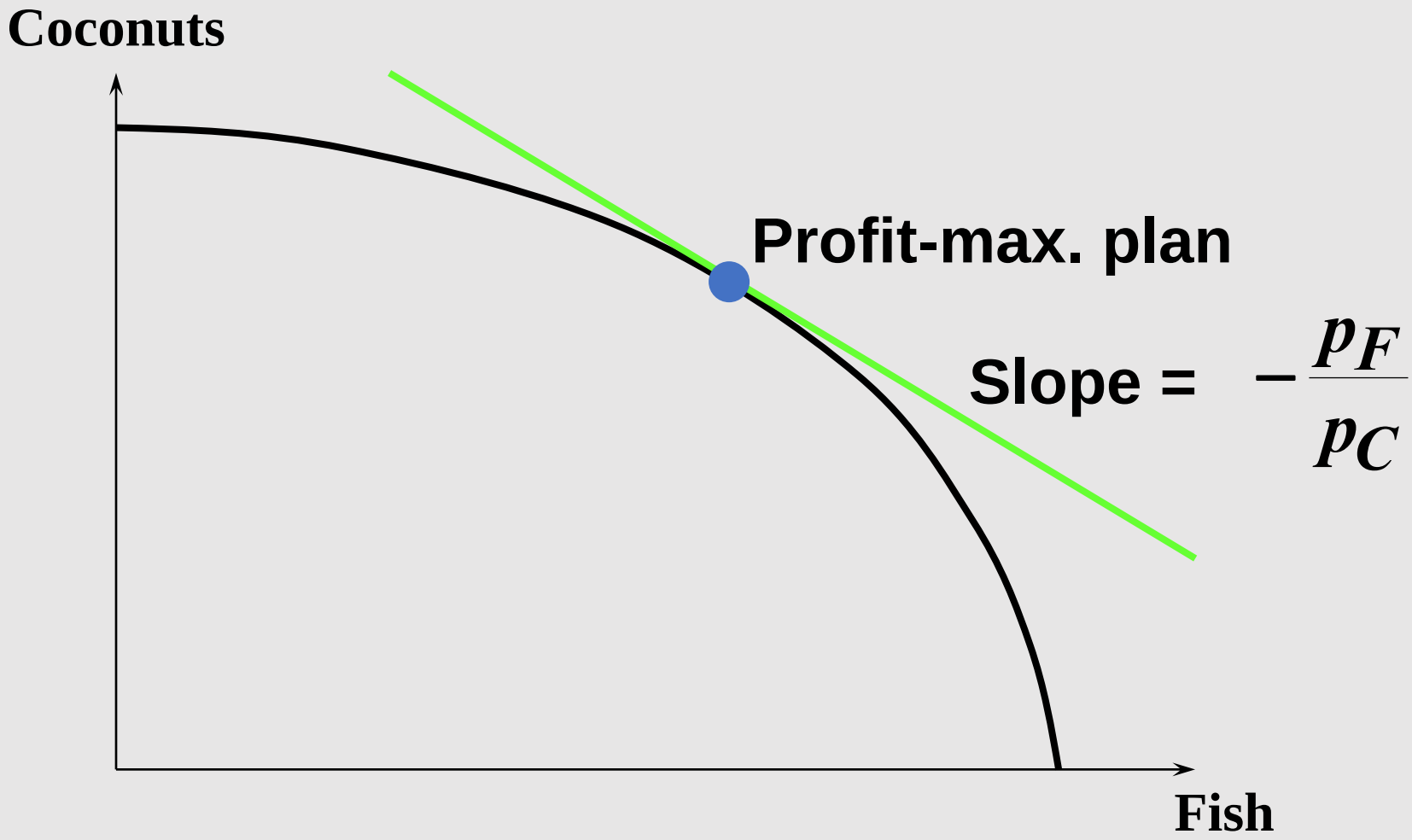
Decentralized Coordination of Production & Consumption



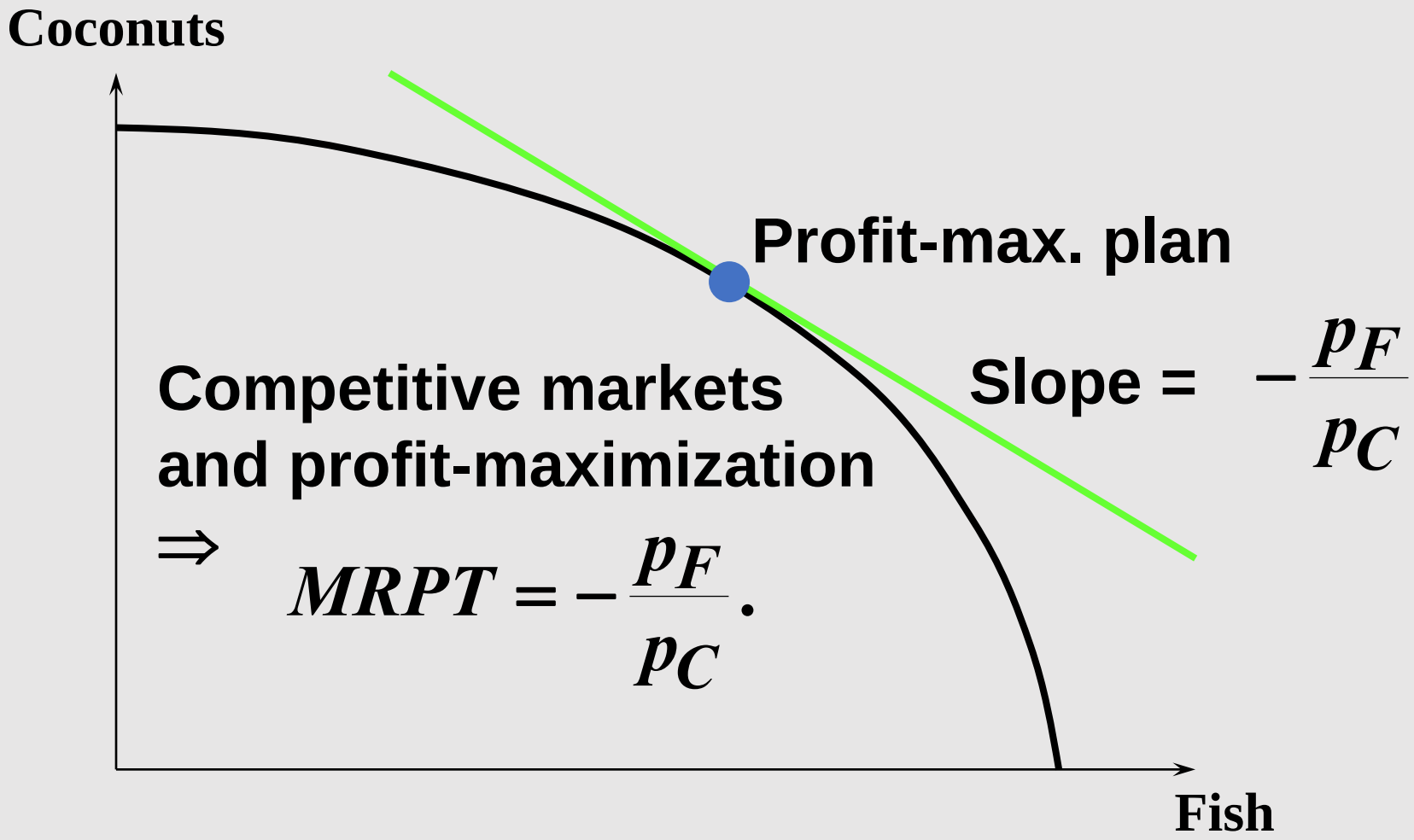
Decentralized Coordination of Production & Consumption



Decentralized Coordination of Production & Consumption



Decentralized Coordination of Production & Consumption



Decentralized Coordination of Production & Consumption

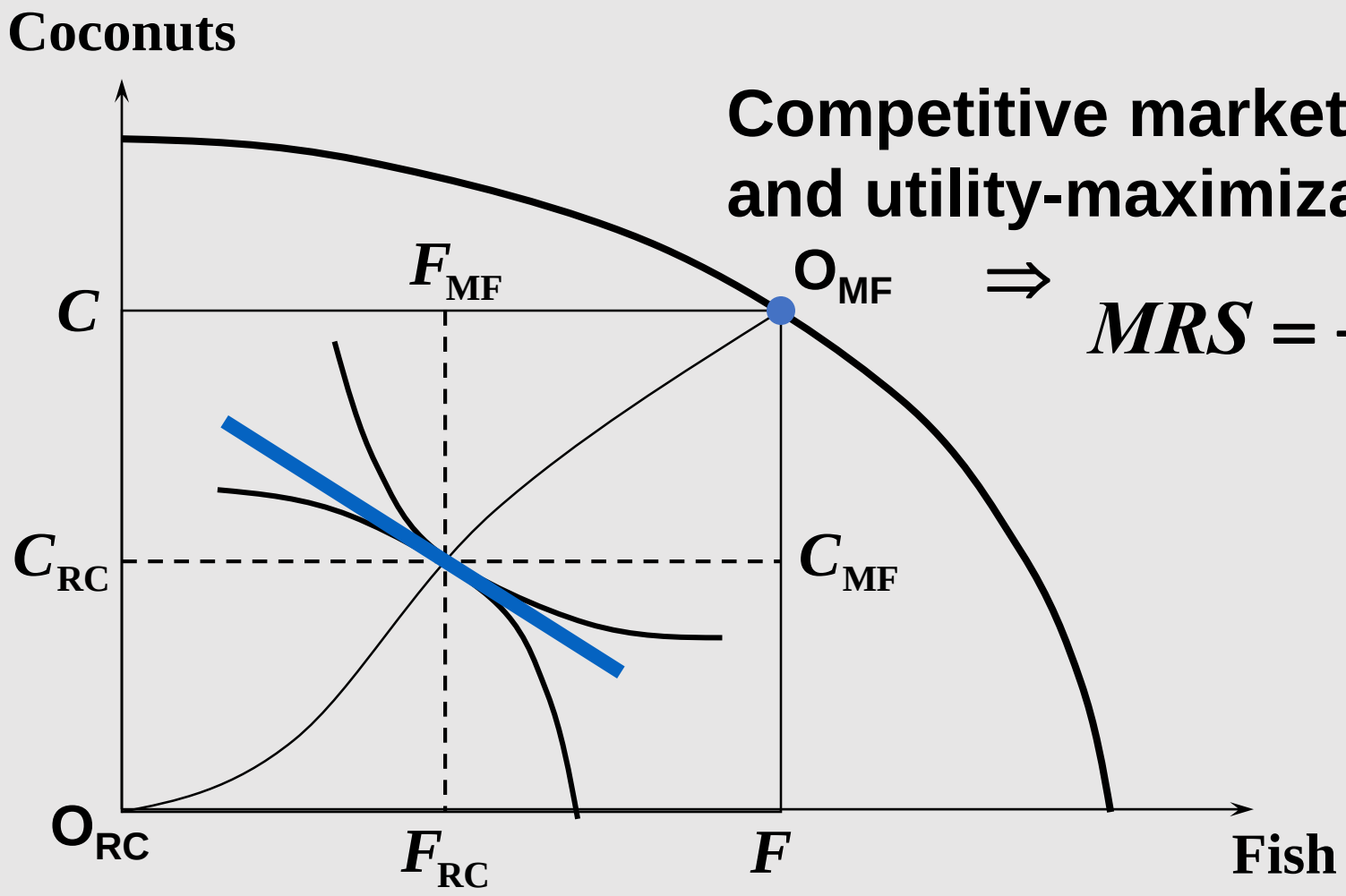
→ So competitive markets, profit-maximization, and utility maximization all together cause

$$MRPT = -\frac{p_F}{p_C} = MRS,$$

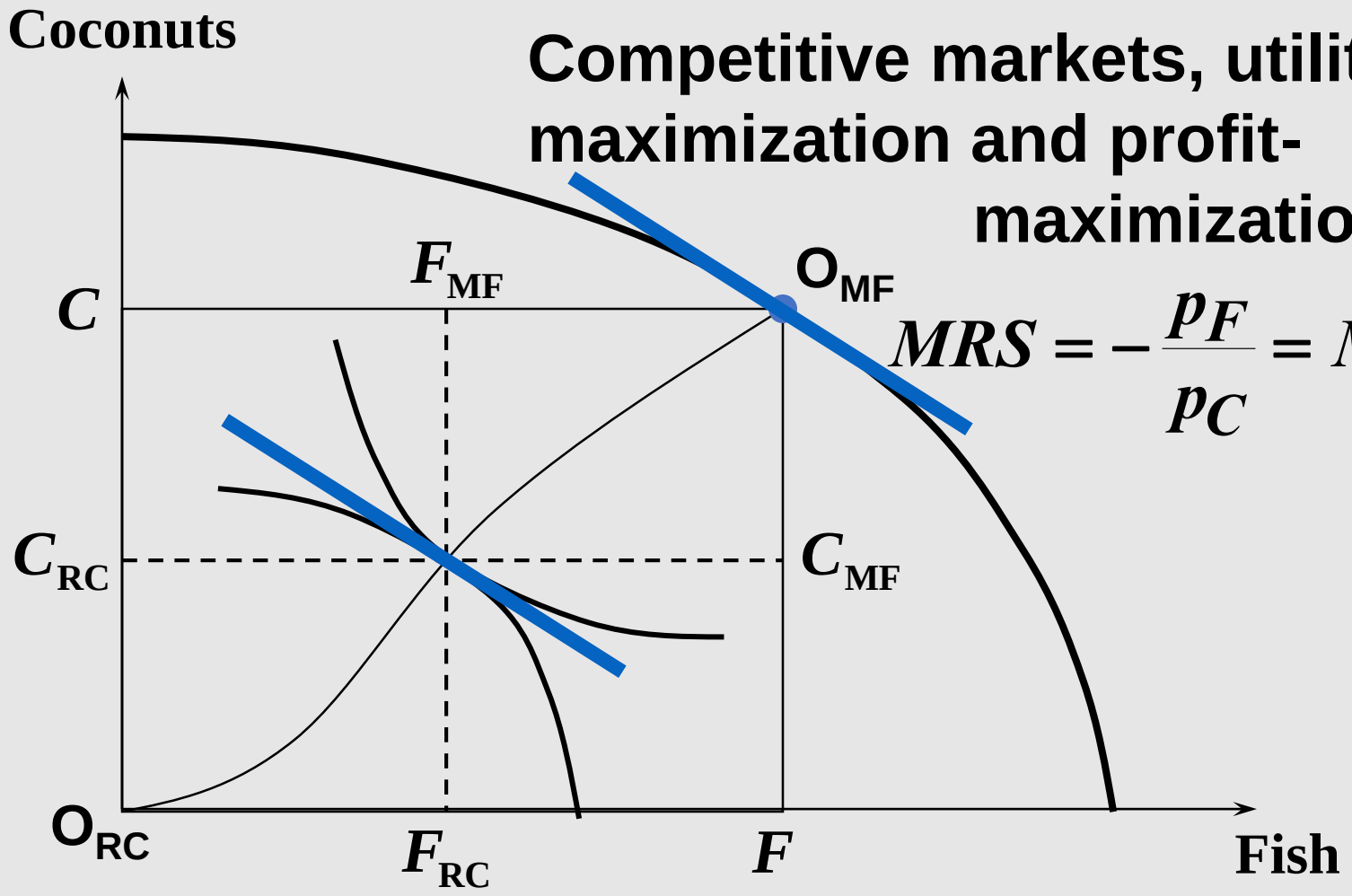
→

the condition necessary for a Pareto optimal economic state.

Decentralized Coordination of Production & Consumption



Decentralized Coordination of Production & Consumption



Competitive markets, utility-maximization and profit-maximization $\Rightarrow$

$$MRS = -\frac{p_F}{p_C} = MRPT.$$