Problem Set 5: Large Samples

Exercise 1: Coins
Suppose a fair coin is flipped 100 times. Let X be the number of times the coin lands on heads.

1. Find the expected value and the variance of X
2. Use the Tchebysheff's inequality to find a bound on the probability that the number of times the coin lands on heads is at least 60 or at most 40.

Exercise 2: Coins 2
A coin is weighted so that its probability of landing on heads is 20%. Suppose the coin is flipped 20 times.

1. Use the Markov inequality to find a high bound for the probability that the coin lands on heads at least 16 times.
2. Now, find the same bound by using Tchebysheff's inequality
3. Use the Binomial Tables to find the true probability that the coin lands on heads at least 16 times.

Exercise 3: Cat's height
The average height of a cat is 25cm.

1. Given an upper bound on the probability that a certain cat is at least 35cm tall.
2. The standard deviation of this height distribution is 4cm. Find a lower bound on the probability that a certain cat is between 20 and 30cm tall.
3. Now assume this distribution is normal. Use a normal CDF table to repeat the calculation from part (2). How close was your bound to the true probability?

Exercise 4: A checkout counter
The service times for customers coming through a checkout counter in a retail store are independent random variables with mean 1.5 minutes and variance 1.0. Approximate the probability that 100 customers can be served in less than 2 hours of total service time.

Exercise 5: Neanderthal's height
An anthropologist wishes to estimate the average height of Neanderthal people. If the population standard deviation is assumed to be 6 centimeters, and if she randomly samples 100 people, find the probability that the difference between the sample mean and the true population mean will not exceed 1.5 centimeters.

Exercise 6: Wage discrimination?
Workers employed in a large service industry have an average wage of $7.00 per hour with a standard deviation of $0.50. The industry has 64 workers of a certain ethnic group. These workers have an average wage of $6.90 per hour. Is it reasonable to assume that the wage rate of the ethnic group is equivalent to that of a random sample of workers from those employed in the service industry? (Hint: Calculate the probability of obtaining a sample mean less than or equal to $6.90 per hour.)