Chapter 1: Exchange

Ch 31 in H. Varian 8th Ed.

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Inspired by Michael D. Robinson, Mount Holyoke College

Introduction

So far…

→ Demand and supply for goods has been seen as **independent** of other goods → **Partial equilibrium analysis**

Now…

→ Prices of good 1 can affect D and S of good 2

- If they are substitutes or complementary
- If people sell good A and increase their available income to buy good B

→ This is called: **General Equilibrium Analysis**

- Demand and supply interact in **several** markets to determine prices of **many** goods

Introduction: simplification assumptions

General Equilibrium analysis is a complex task. We will adopt several simplification assumptions:

1. Restrict to **competitive** markets: consumers and producers take prices as given

2. We restrict the analysis to **2 goods** and **2 consumers**

3. We restrict to the case of pure exchange

    1. Consumers have **fix endowments** and decide whether to trade with the other consumer
    2. For the moment, there is no production

Outline of the chapter

1. The Edgeworth Box
2. Adding preferences to the Box
3. Pareto Efficient Allocations
4. Trade in Competitive Markets
5. Two Theorems of Welfare Economics
6. Walras' Law

1. The Edgeworth Box

→ Edgeworth and Bowley devised a diagram, called an Edgeworth box, to show all possible allocations of the available quantities of goods 1 and 2 between two consumers A and B.

- **Consumption bundle**: $X^A = (x_A^1, x_A^2)$ how much consumer A consumes of each good. Alternatively, $X^B = (x_B^1, x_B^2)$
- **Allocation:** a pair of consumption bundles is an allocation. Ex: X^A and X^B
- **Initial endowment**: how much each consumer have of each good at the beginning (ω_A^1, ω_A^2) and (ω_B^1, ω_B^2)
- **Feasible allocation**: when the total amount of each good consumed is smaller or equal than the total amount available

$$x_A^1 + x_B^1 \leq \omega_A^1 + \omega_B^1$$
$$x_A^2 + x_B^2 \leq \omega_A^2 + \omega_B^2$$

The Edgeworth Box: example

- Two consumers, A and B.
- Their endowments of goods 1 and 2 are

$$\omega^{A} = (6,4) \quad \textbf{and} \quad \omega^{B} = (2,2).$$

- The total quantities available

are $\omega_1^A + \omega_1^B = 6 + 2 = 8$ **units of good 1**

and $\omega_2^A + \omega_2^B = 4 + 2 = 6$ **units of good 2.**

Starting an Edgeworth Box

Starting an Edgeworth Box

Height = $\omega_2^A + \omega_2^B = 4 + 2 = 6$

The dimensions of the box are the quantities available of the goods.

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

The Endowment Allocation

Height = $\omega_2^{\text{A}} + \omega_2^{\text{B}}$

$= 4 + 2$

$= 6$

The endowment allocation is

$\omega^{\text{A}} = (6,4)$

and

$\omega^{\text{B}} = (2,2).$

Width = $\omega_1^{\text{A}} + \omega_1^{\text{B}} = 6 + 2 = 8$

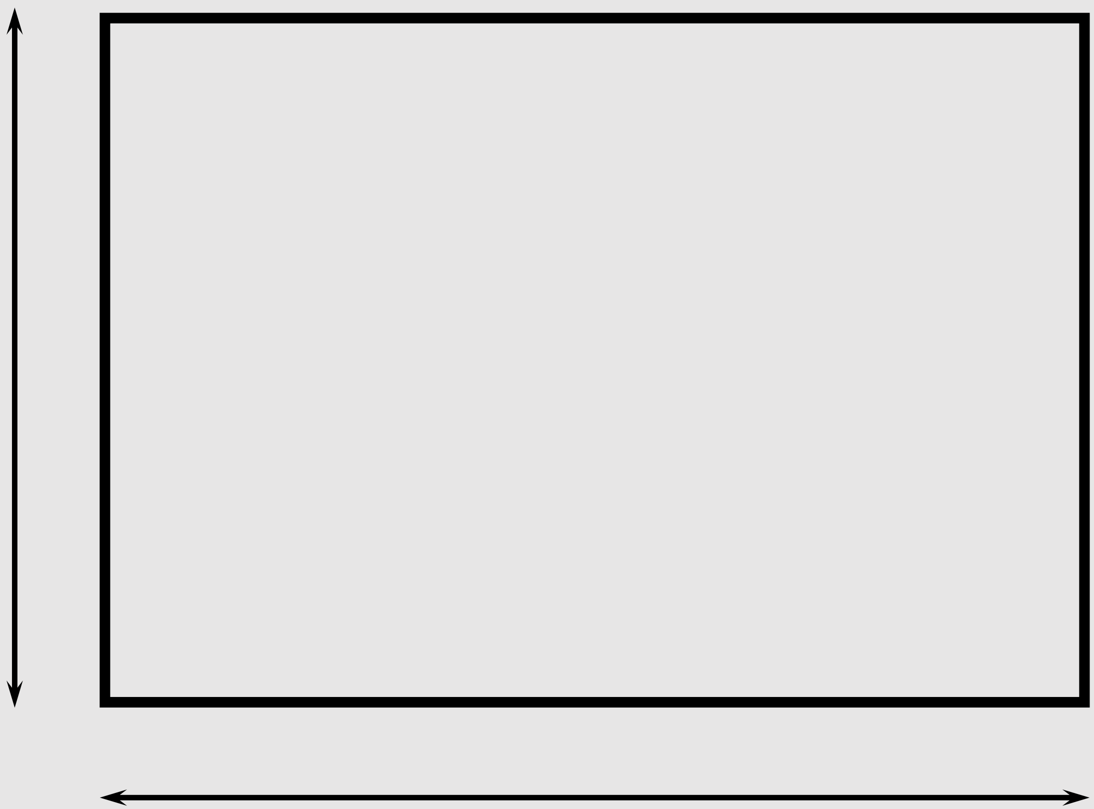
The Endowment Allocation

Height =

$\omega_2^{A} + \omega_2^{B}$

$= 4 + 2$

$= 6$

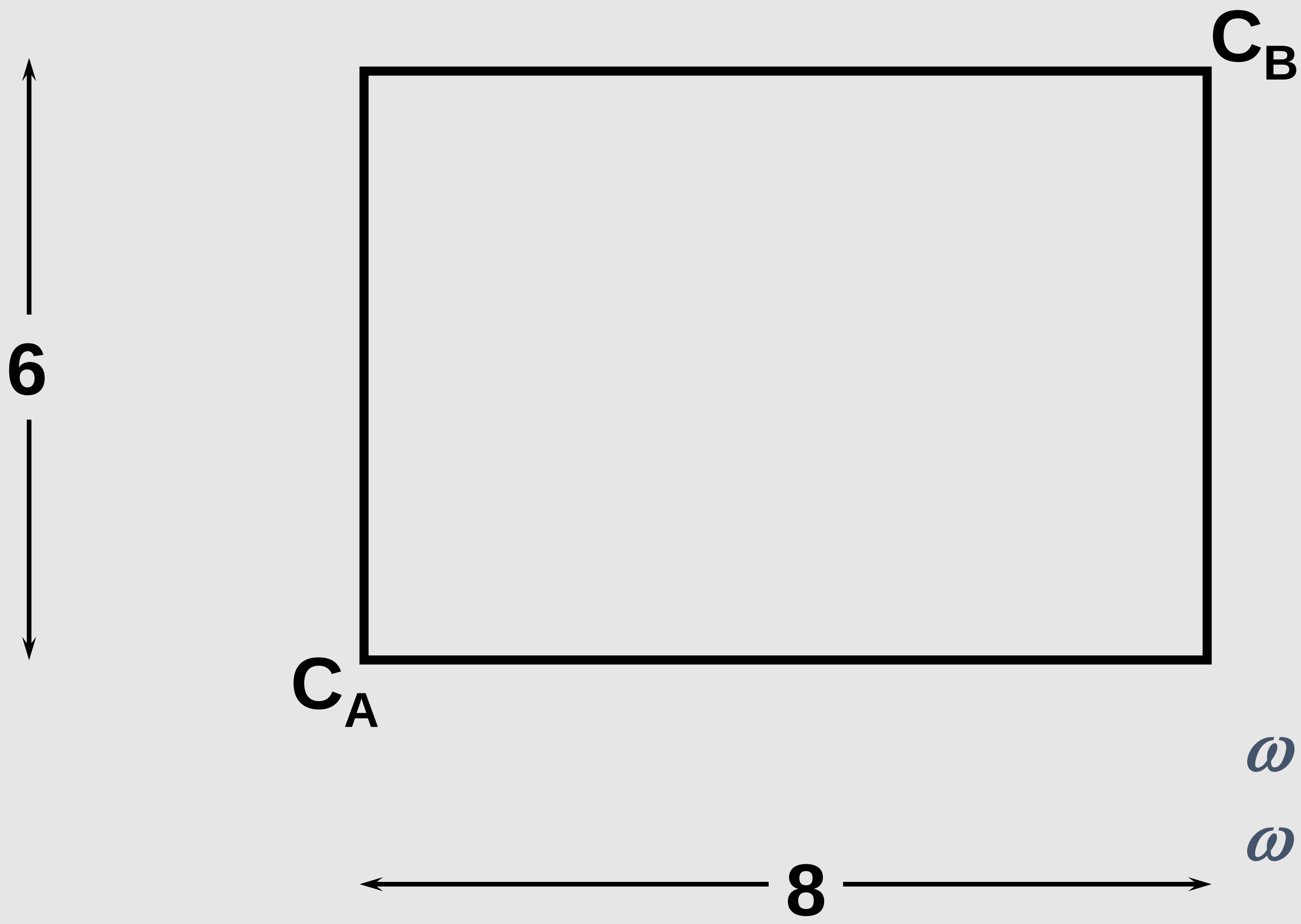


$\omega^{A} = (6,4)$

$\omega^{B} = (2,2)$

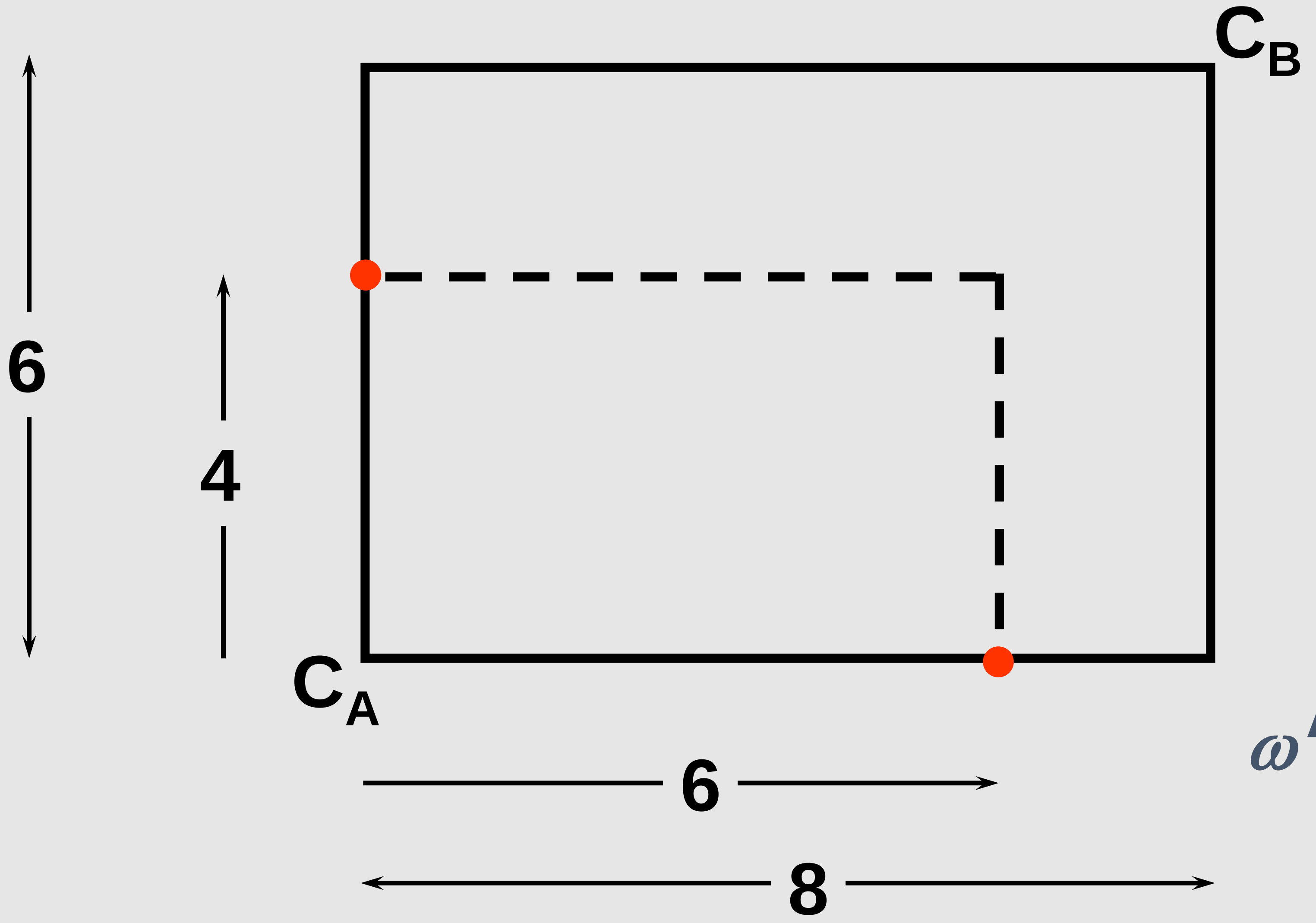
Width = $\omega_1^{A} + \omega_1^{B} = 6 + 2 = 8$

The Endowment Allocation



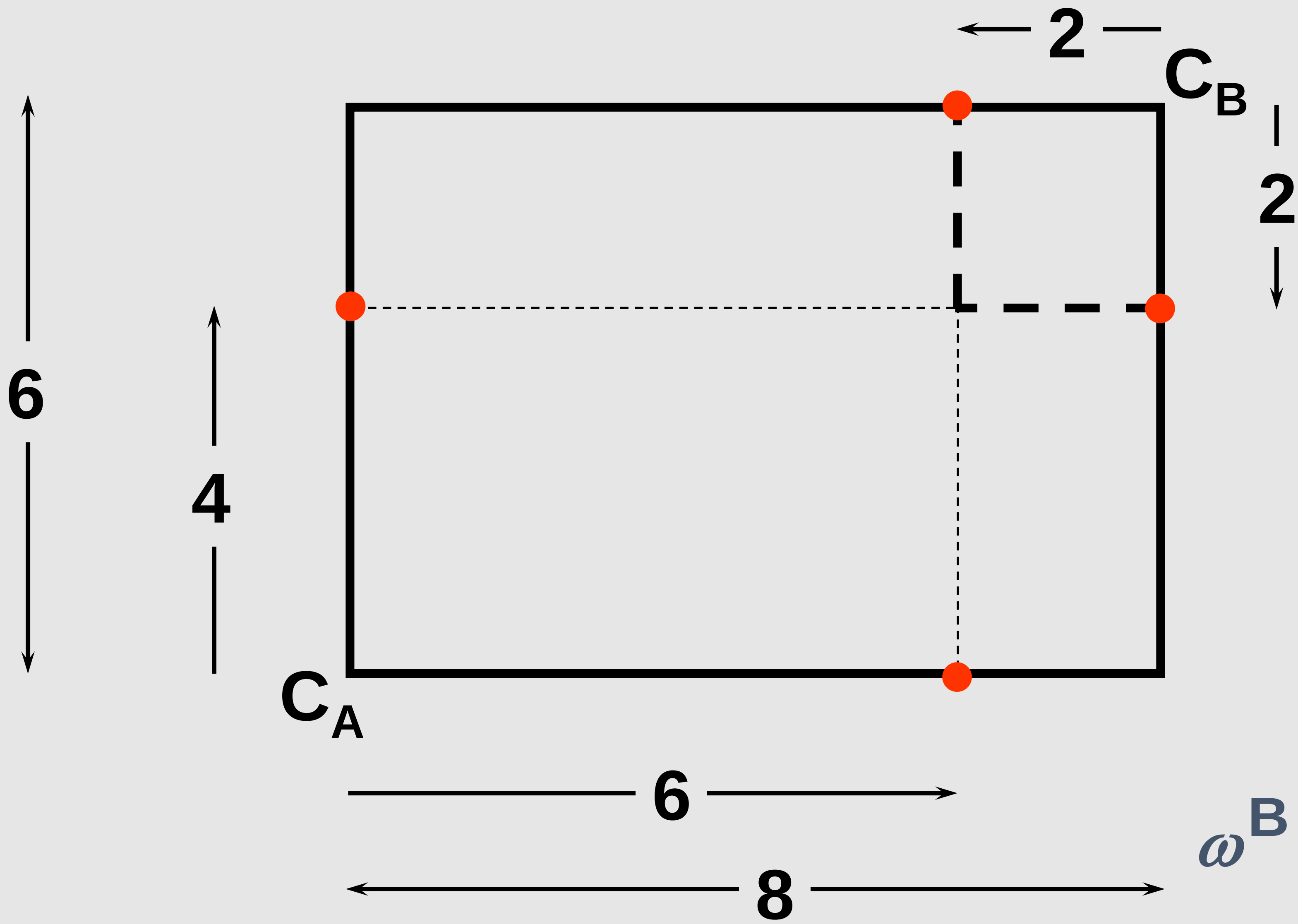
$\omega^A = (6,4)$
$\omega^B = (2,2)$

The Endowment Allocation



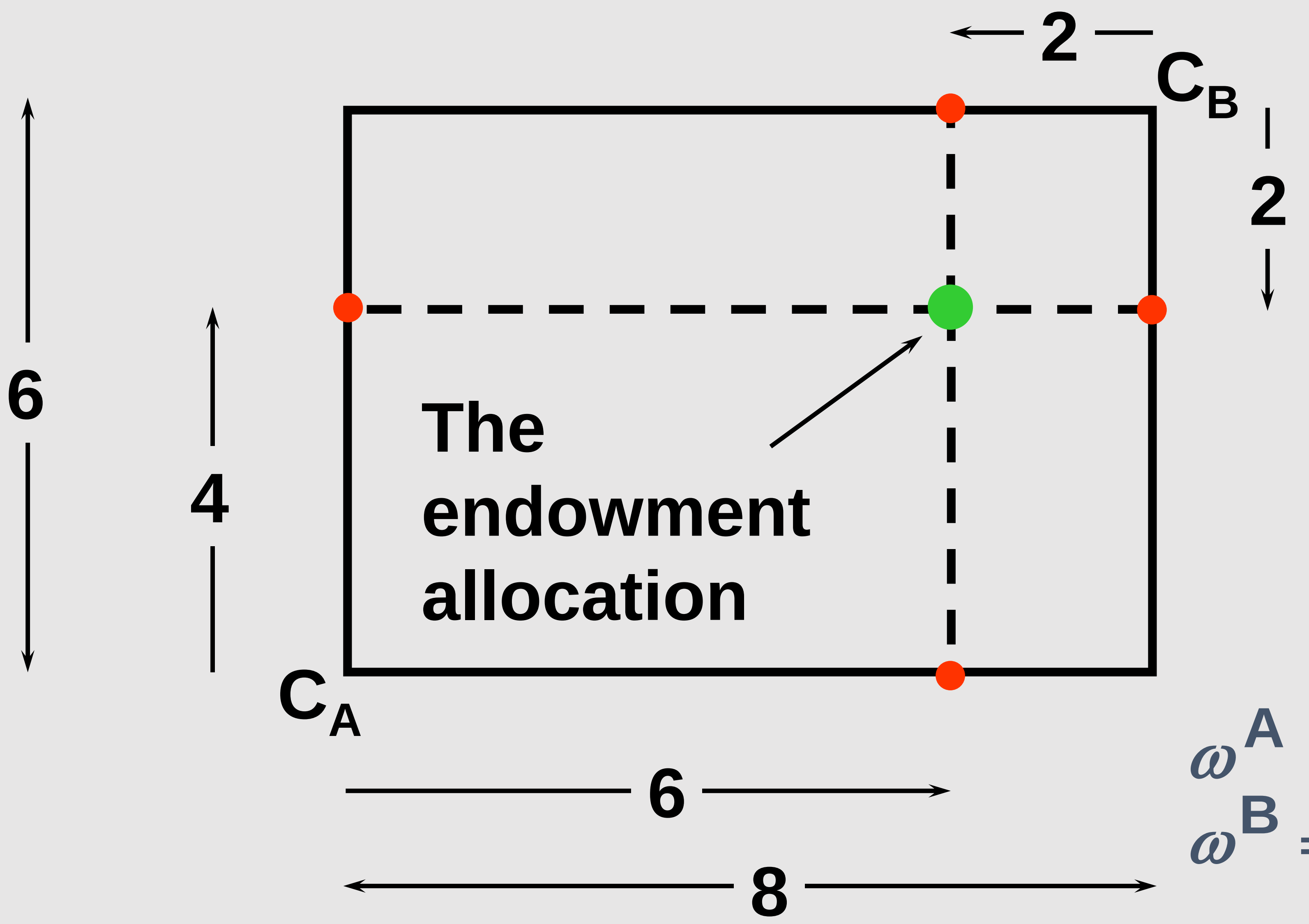
$$\omega^A = (6,4)$$

The Endowment Allocation



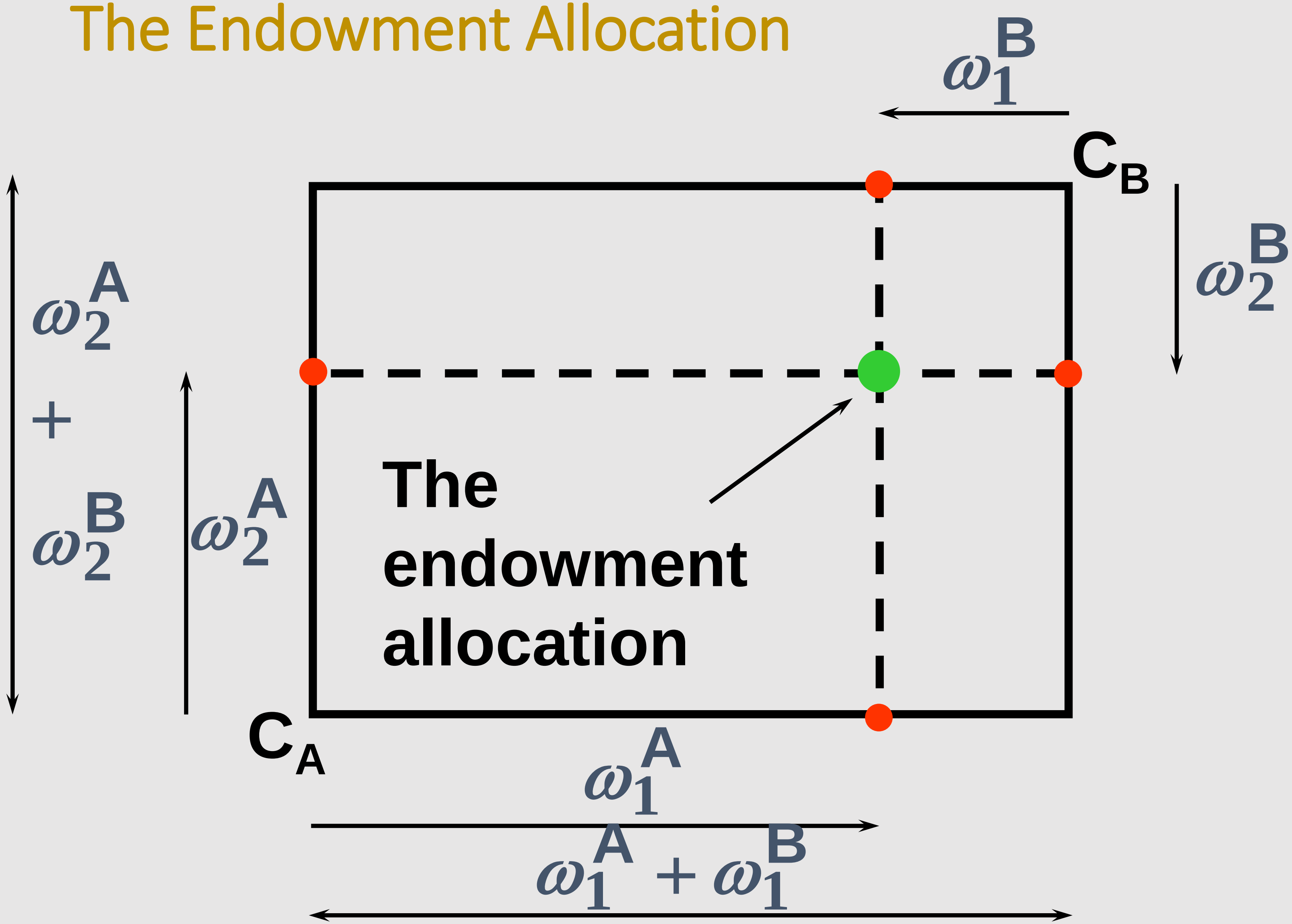
$\omega^{\mathrm{B}} = (2,2)$

The Endowment Allocation



$$\omega^A = (6,4)$$
$$\omega^B = (2,2)$$

The Endowment Allocation



Other Feasible Allocations

(x_1^A, x_2^A) denotes an allocation to consumer A.

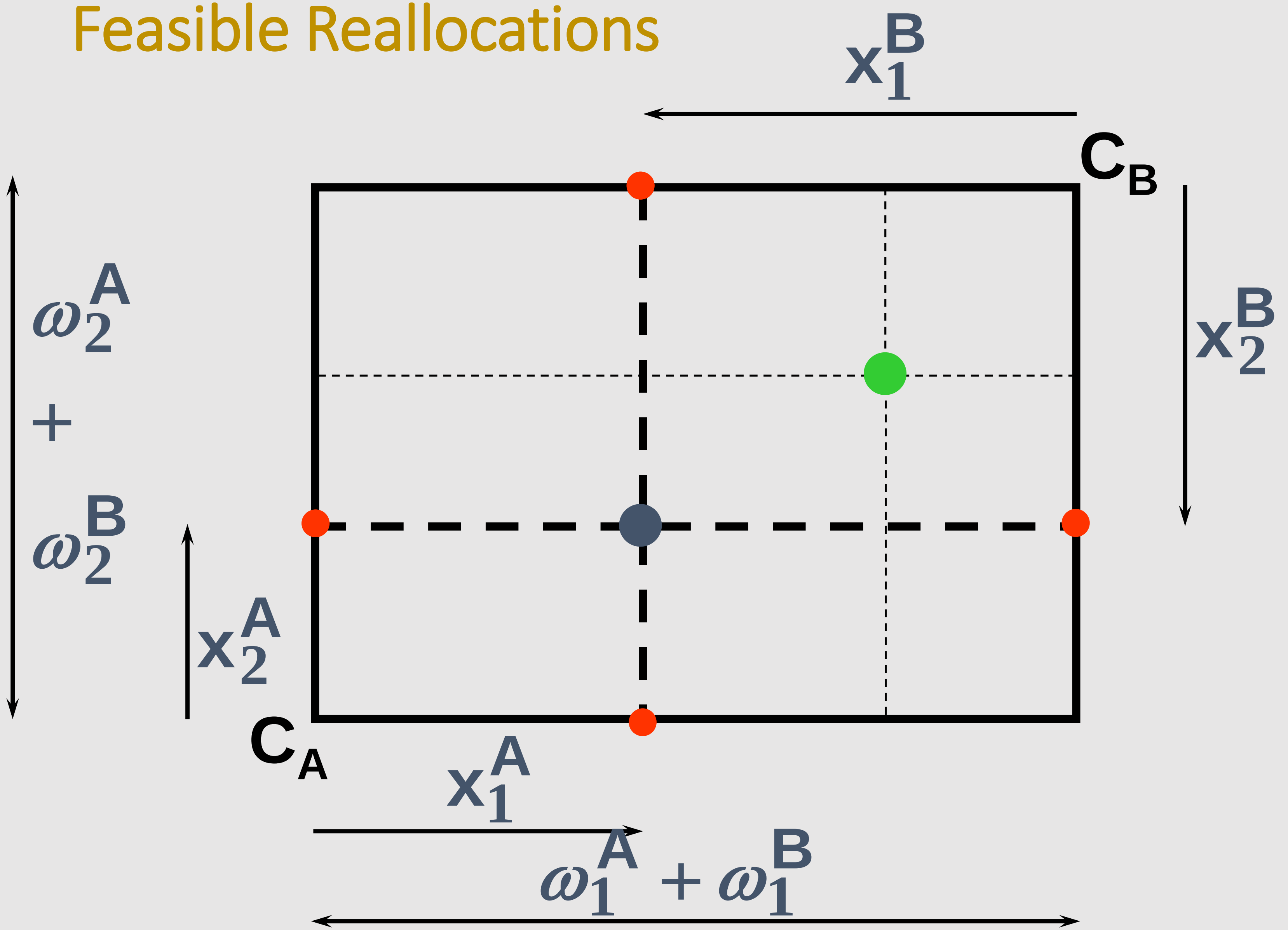
(x_1^B, x_2^B) denotes an allocation to consumer B.

→ Remember: An allocation is feasible if and only if

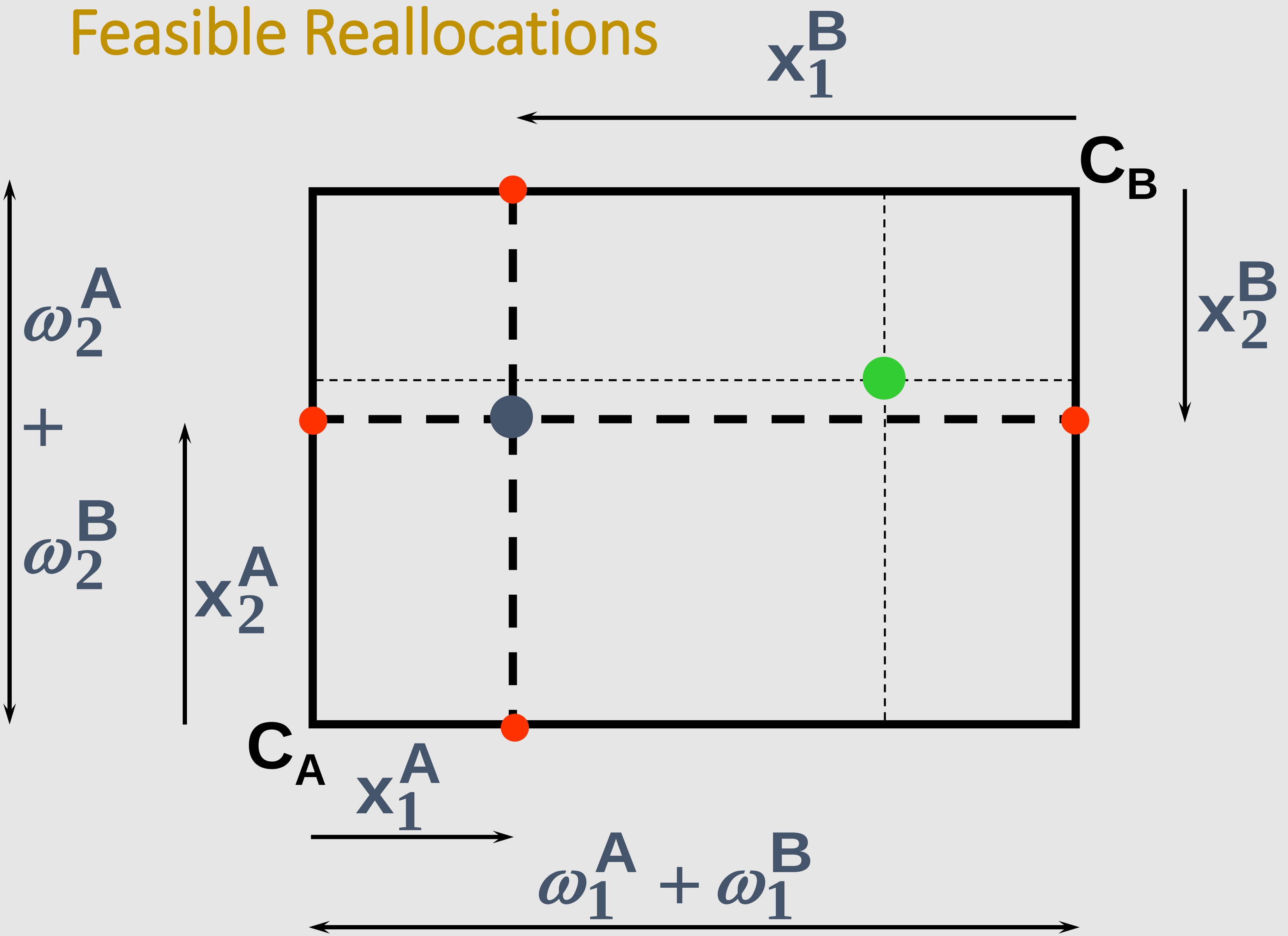
$$x_1^A + x_1^B \le \omega_1^A + \omega_1^B$$

and $$x_2^A + x_2^B \le \omega_2^A + \omega_2^B.$$

Feasible Reallocations



Feasible Reallocations



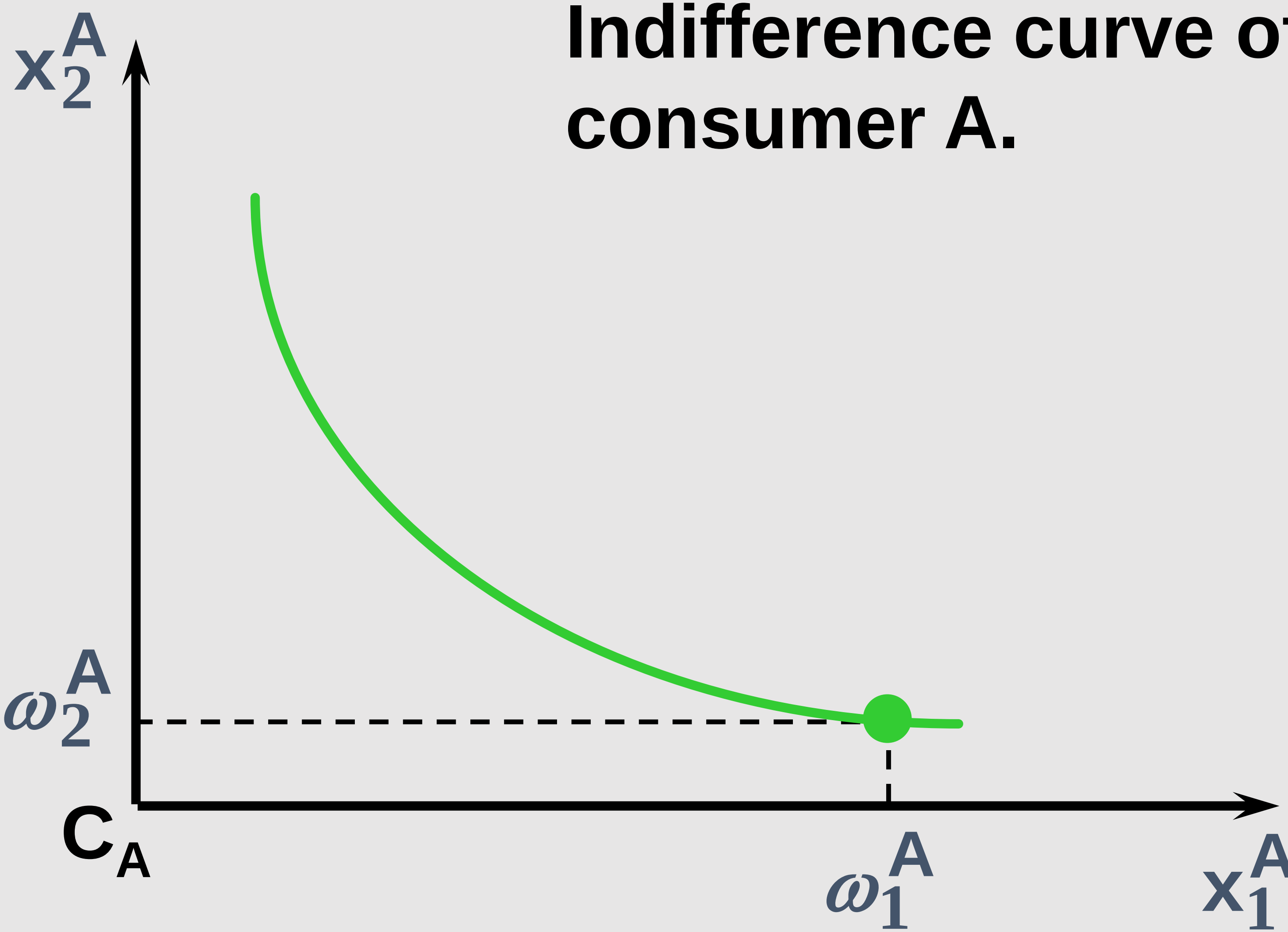
Feasible Reallocations

- All points in the box, including the boundary, represent feasible allocations of the combined endowments.
- Which allocations will be blocked by one or both consumers?
- Which allocations make both consumers better off?

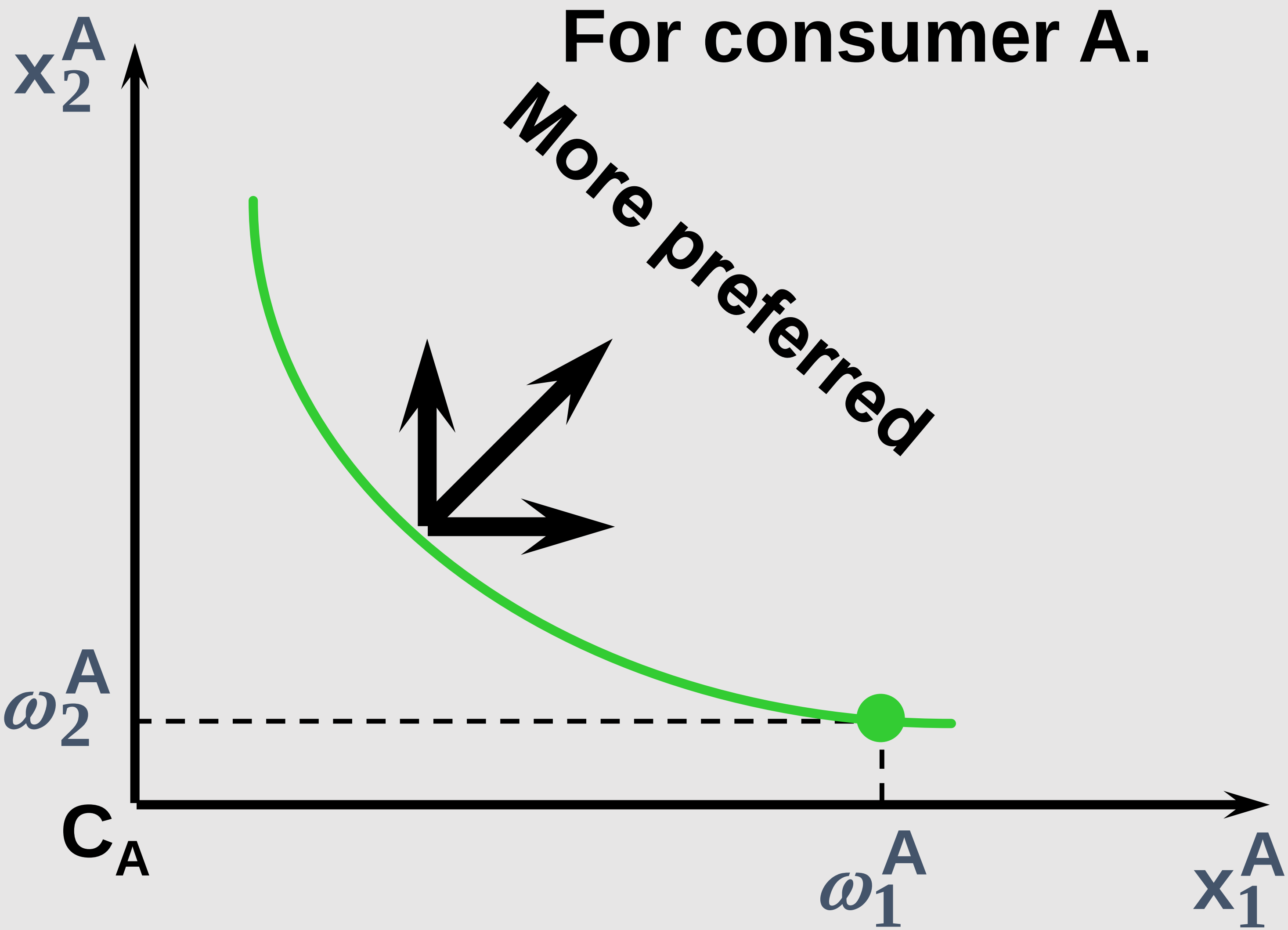
- To answer these questions, we need to account for **preferences** of consumer A and B.

2. Adding Preferences to the Box

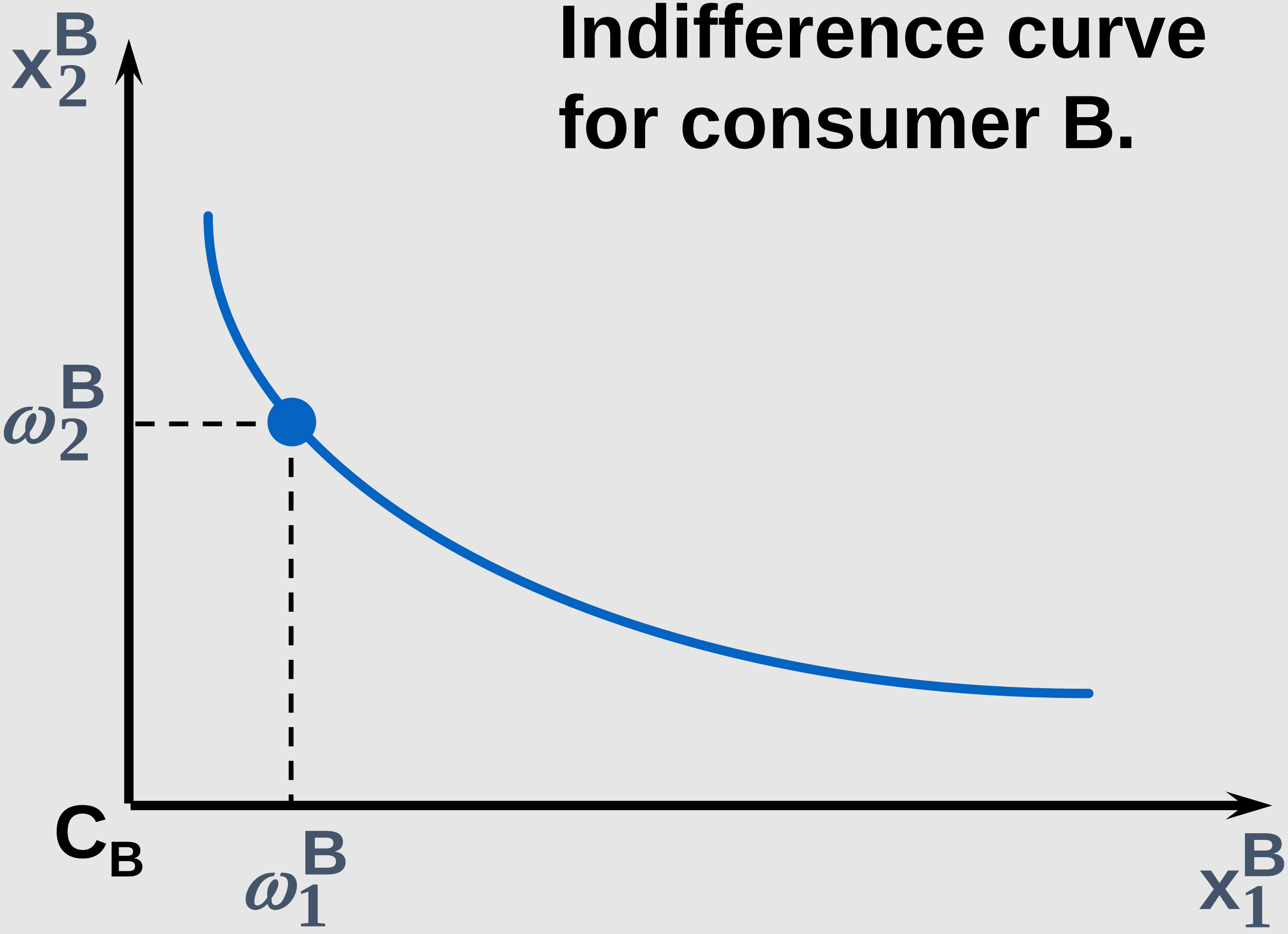
Indifference curve of consumer A.



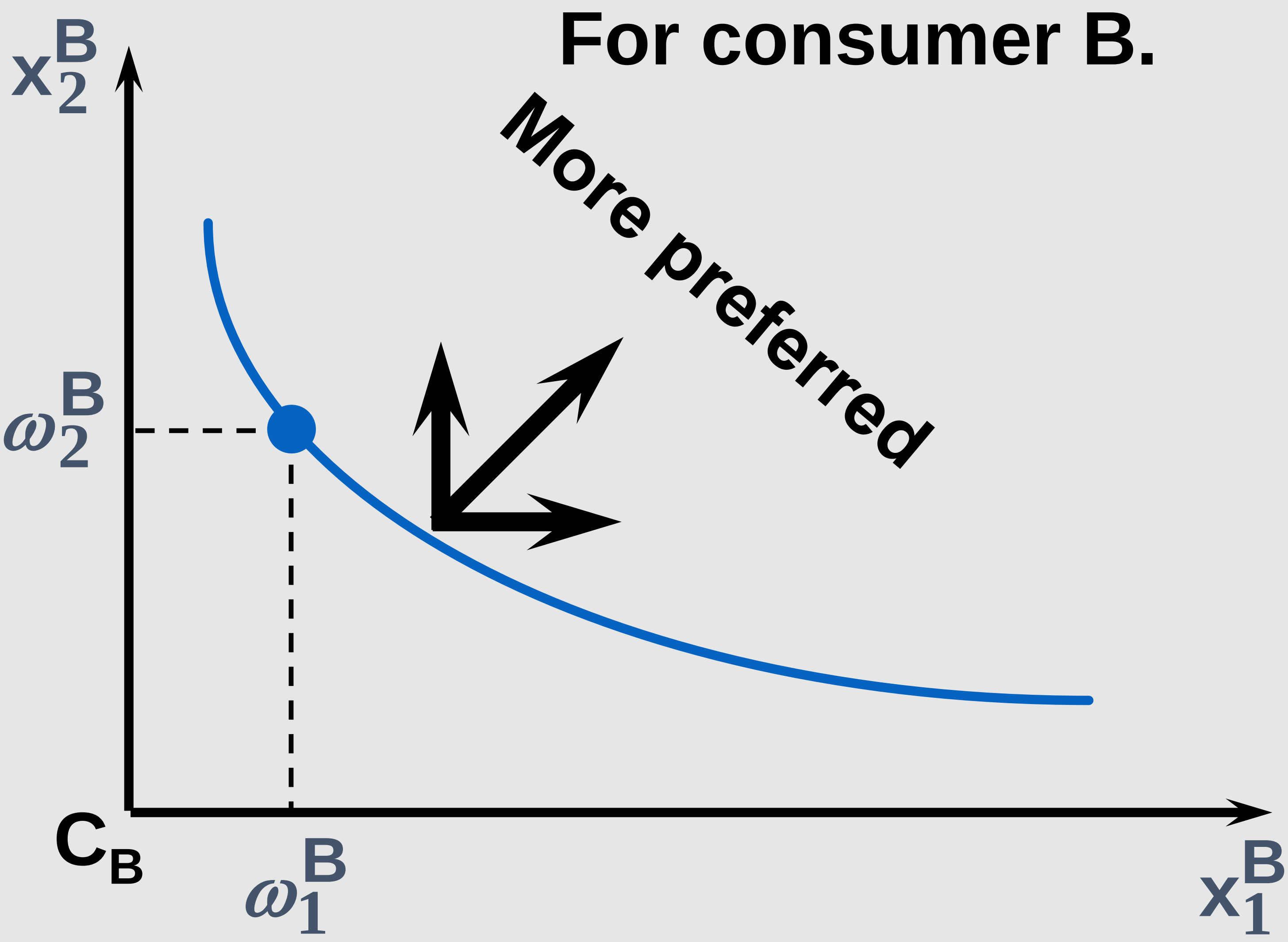
Adding Preferences to the Box



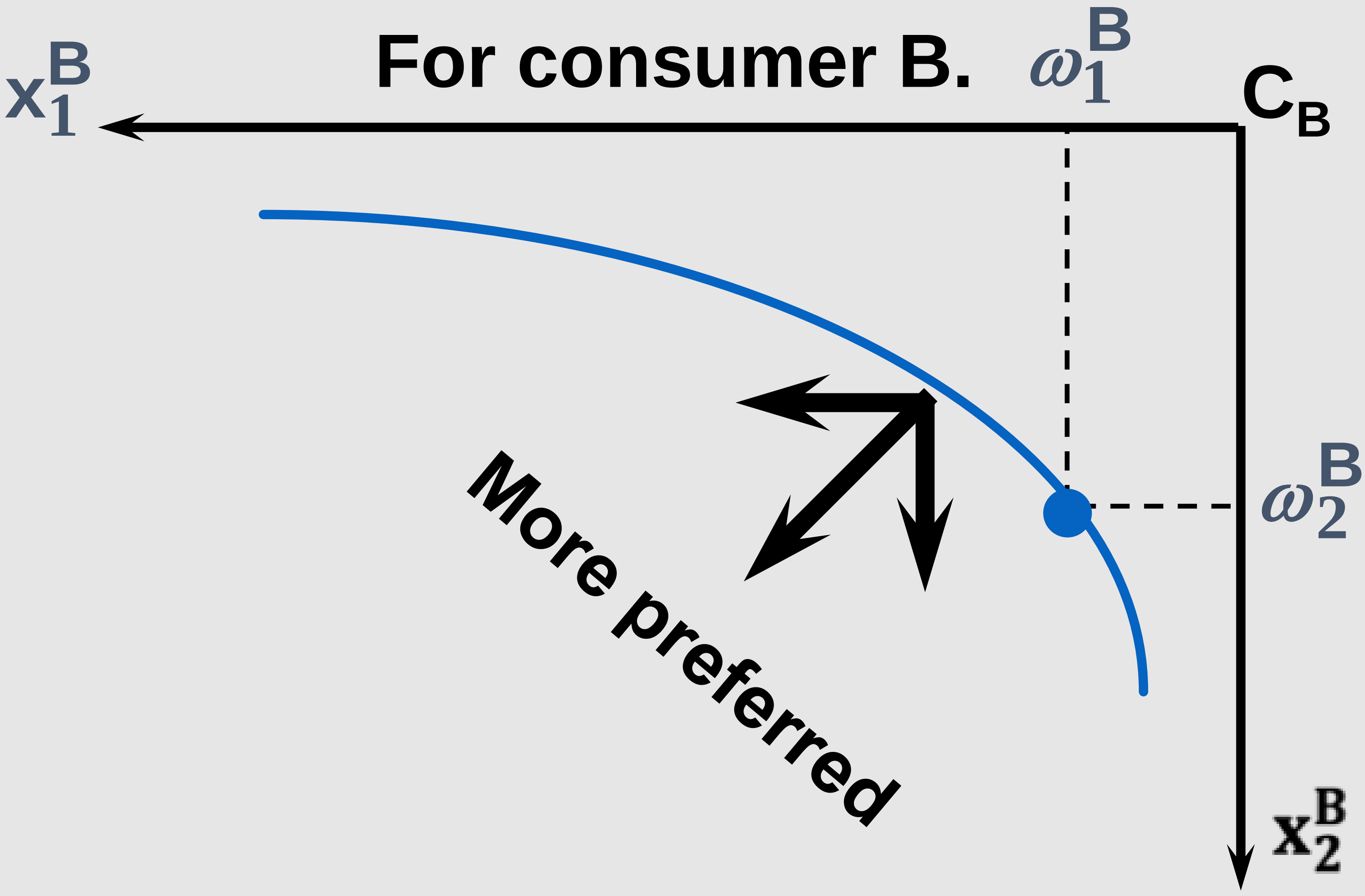
Adding Preferences to the Box



Adding Preferences to the Box



Adding Preferences to the Box



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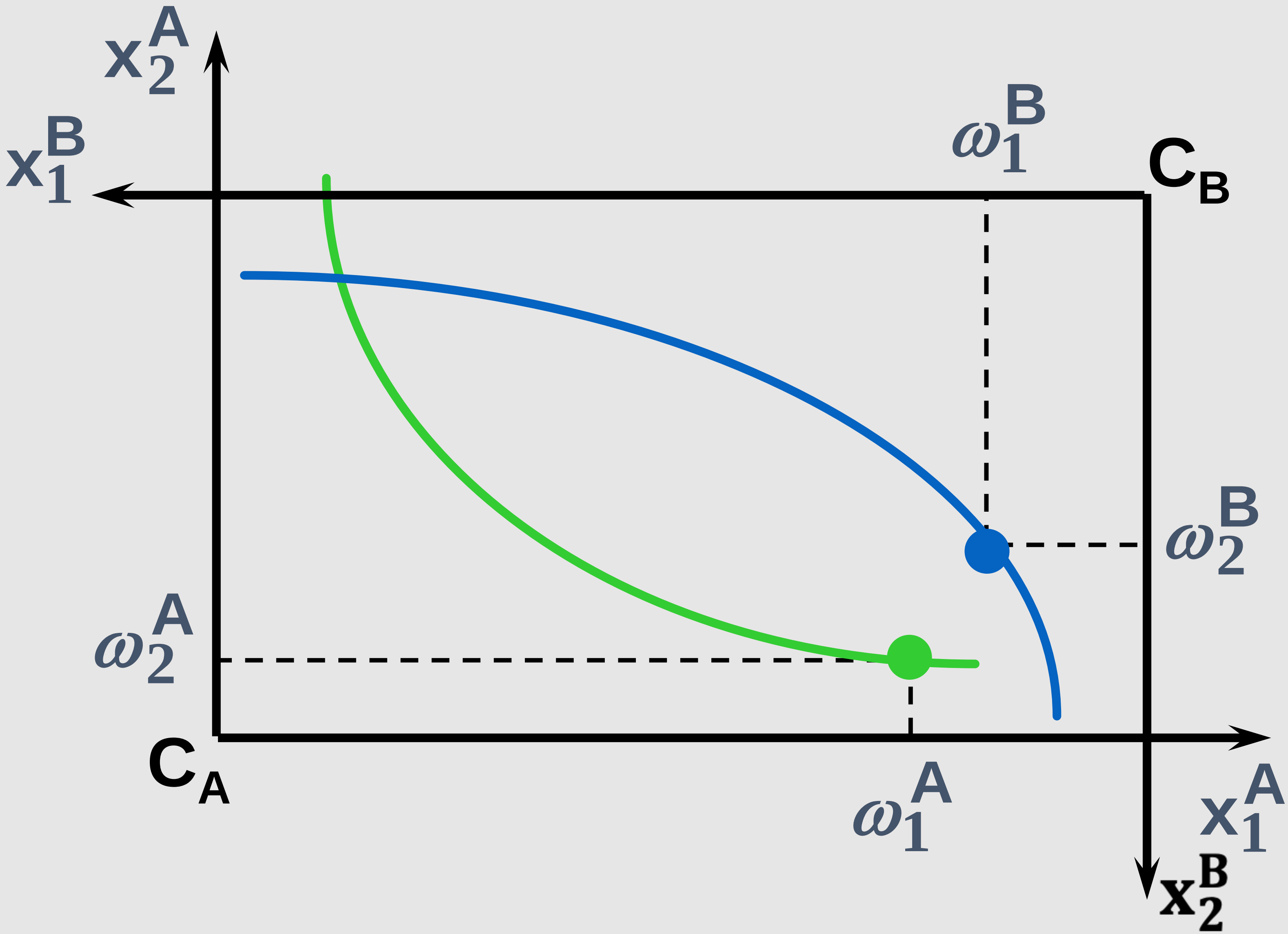

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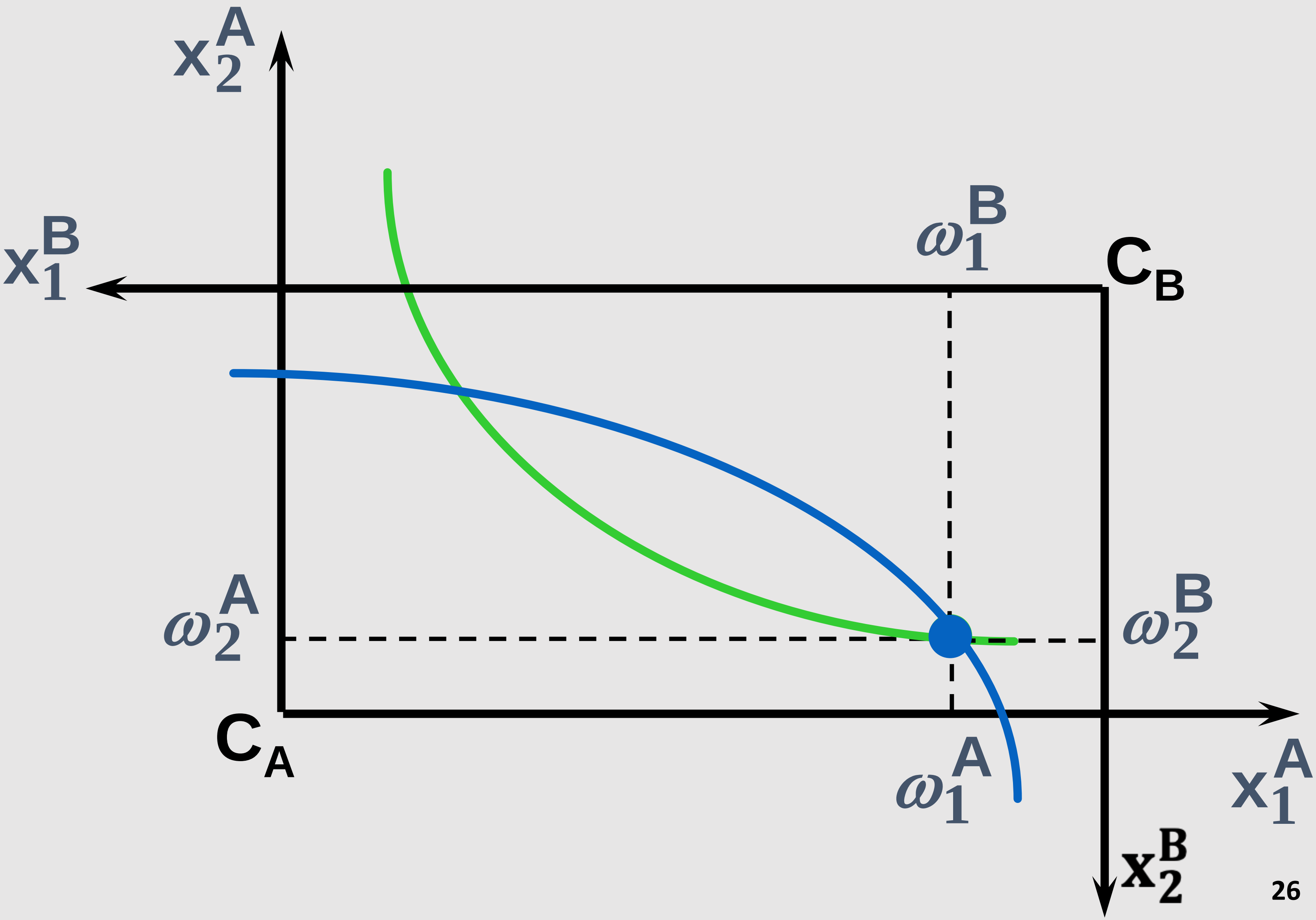
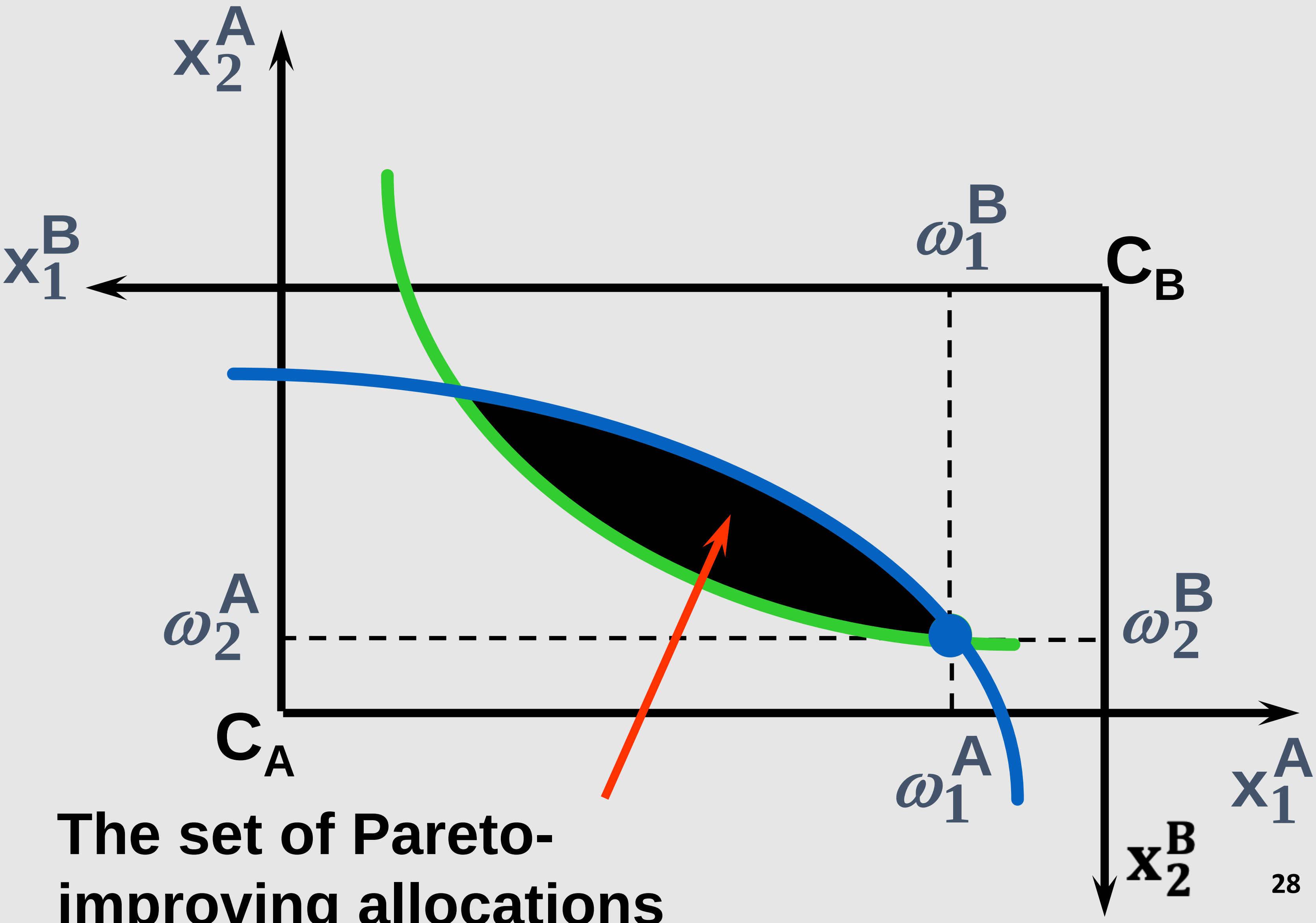


3. Pareto Efficient Allocation

- An allocation of the endowment that improves the welfare of a consumer without reducing the welfare of another is a Pareto-improving allocation.

- Where are the Pareto-improving allocations?

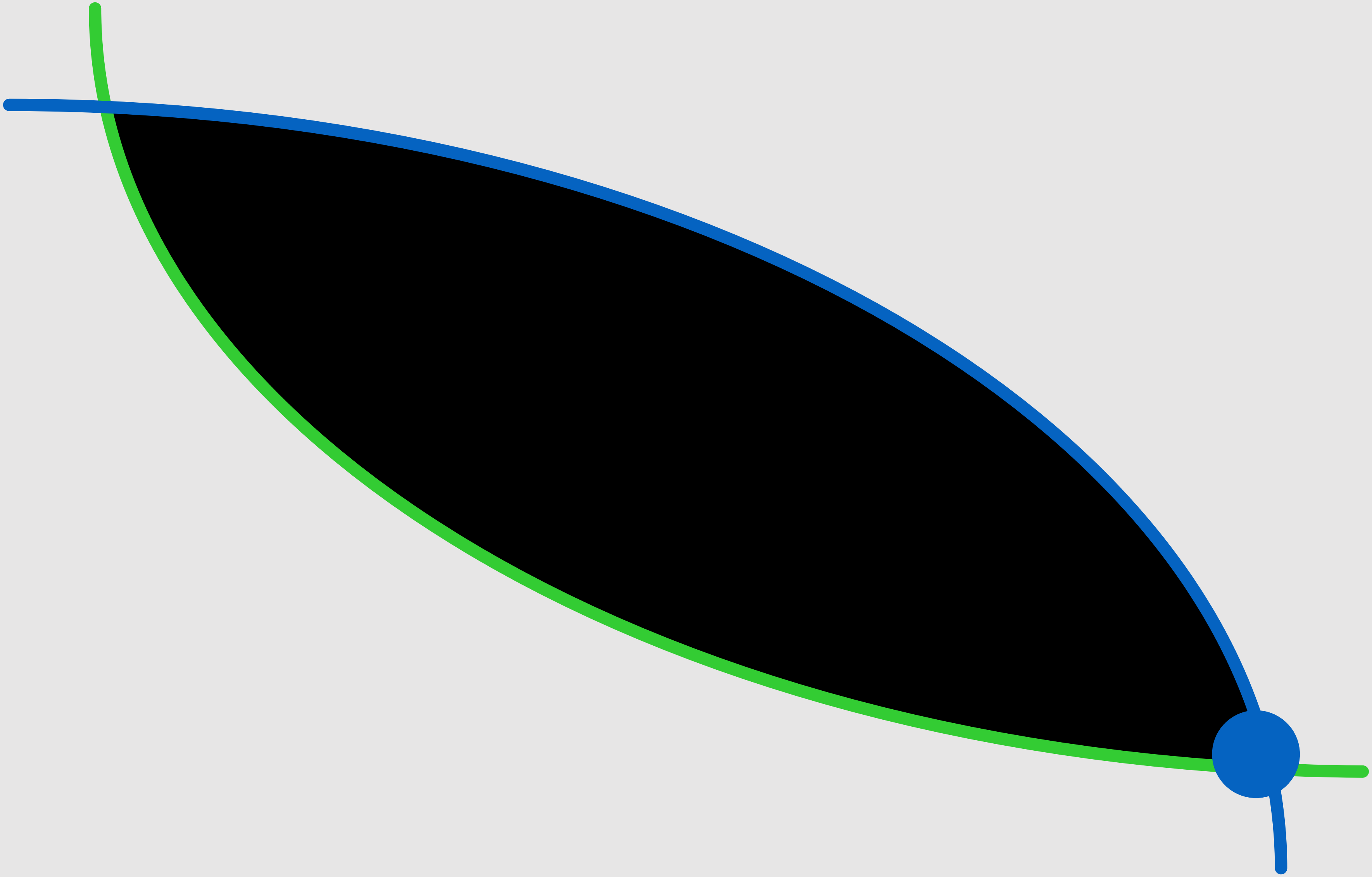
Pareto Efficient Allocation



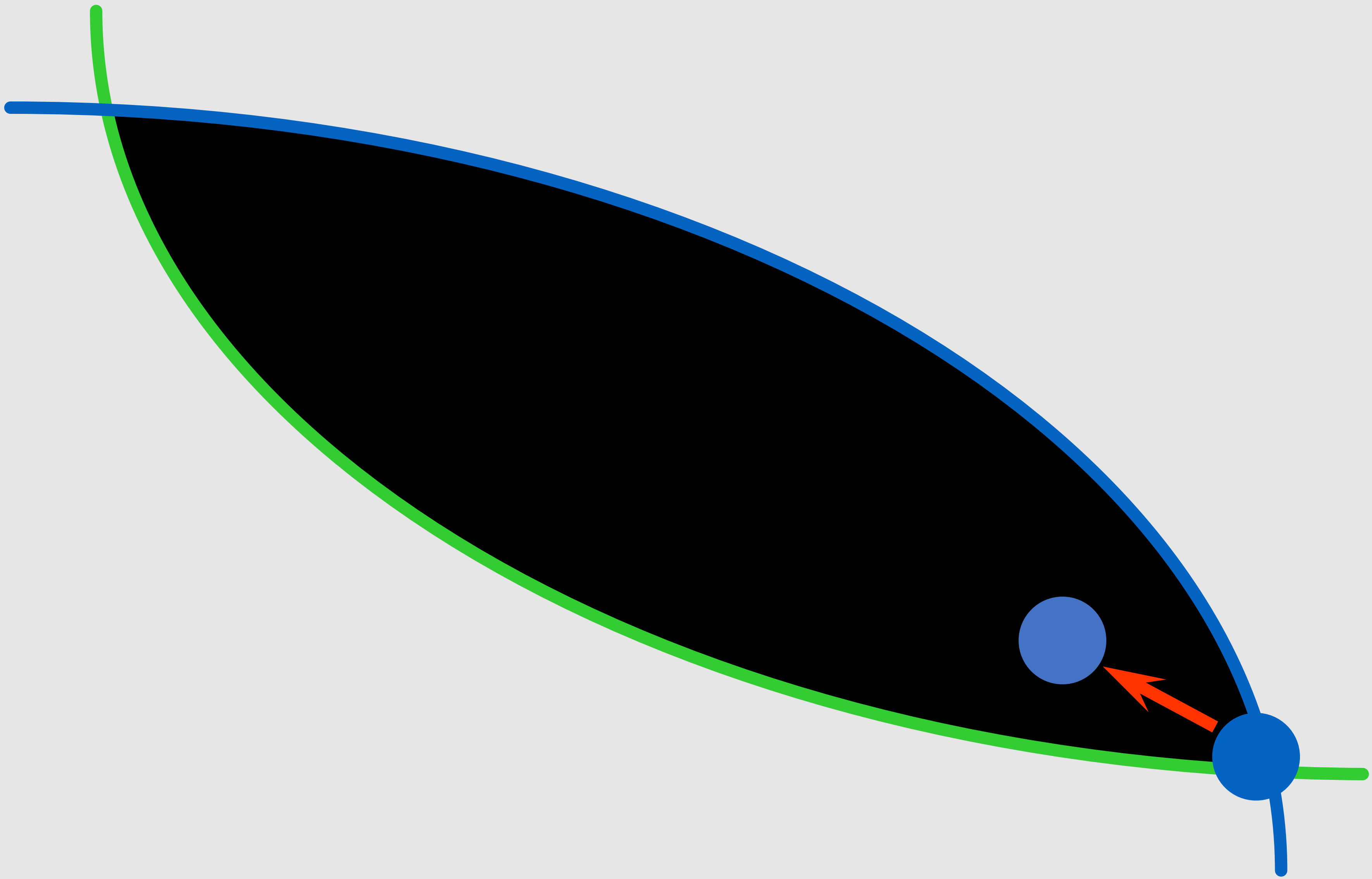
Pareto Efficient Allocation

- Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.
- But which particular Pareto-improving allocation will be the outcome of trade?

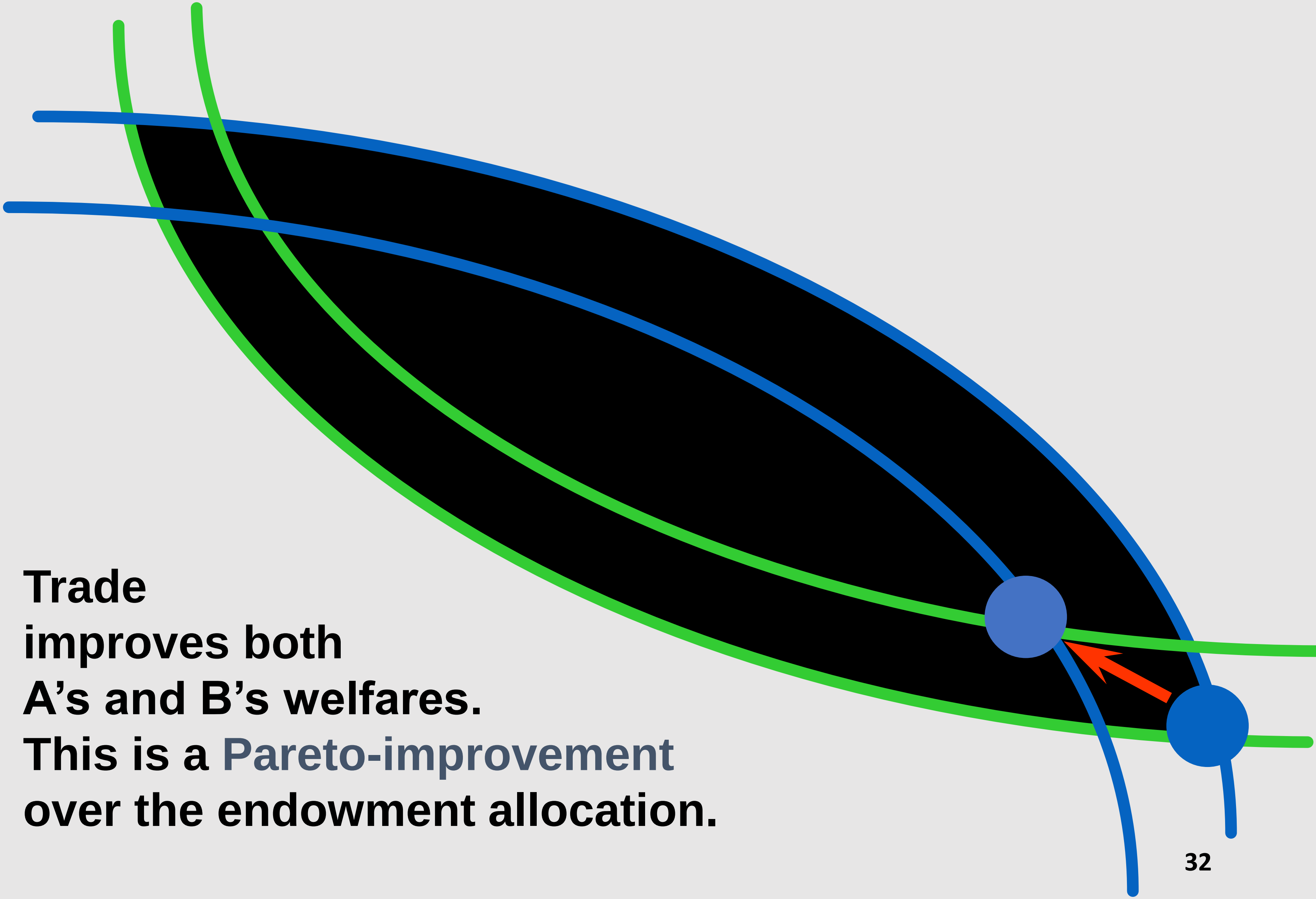
Pareto Efficient Allocation



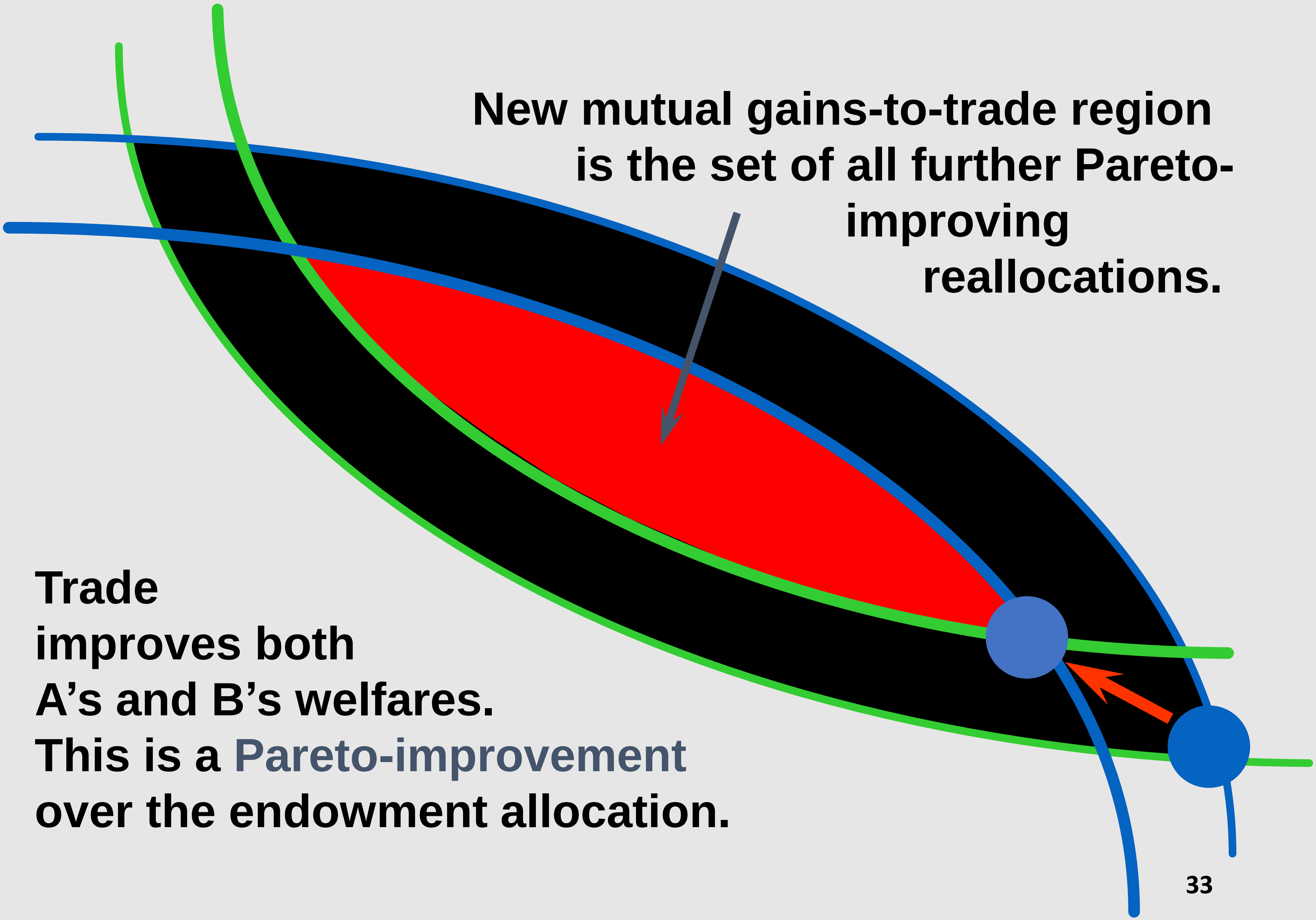
Pareto Efficient Allocation



Pareto Efficient Allocation



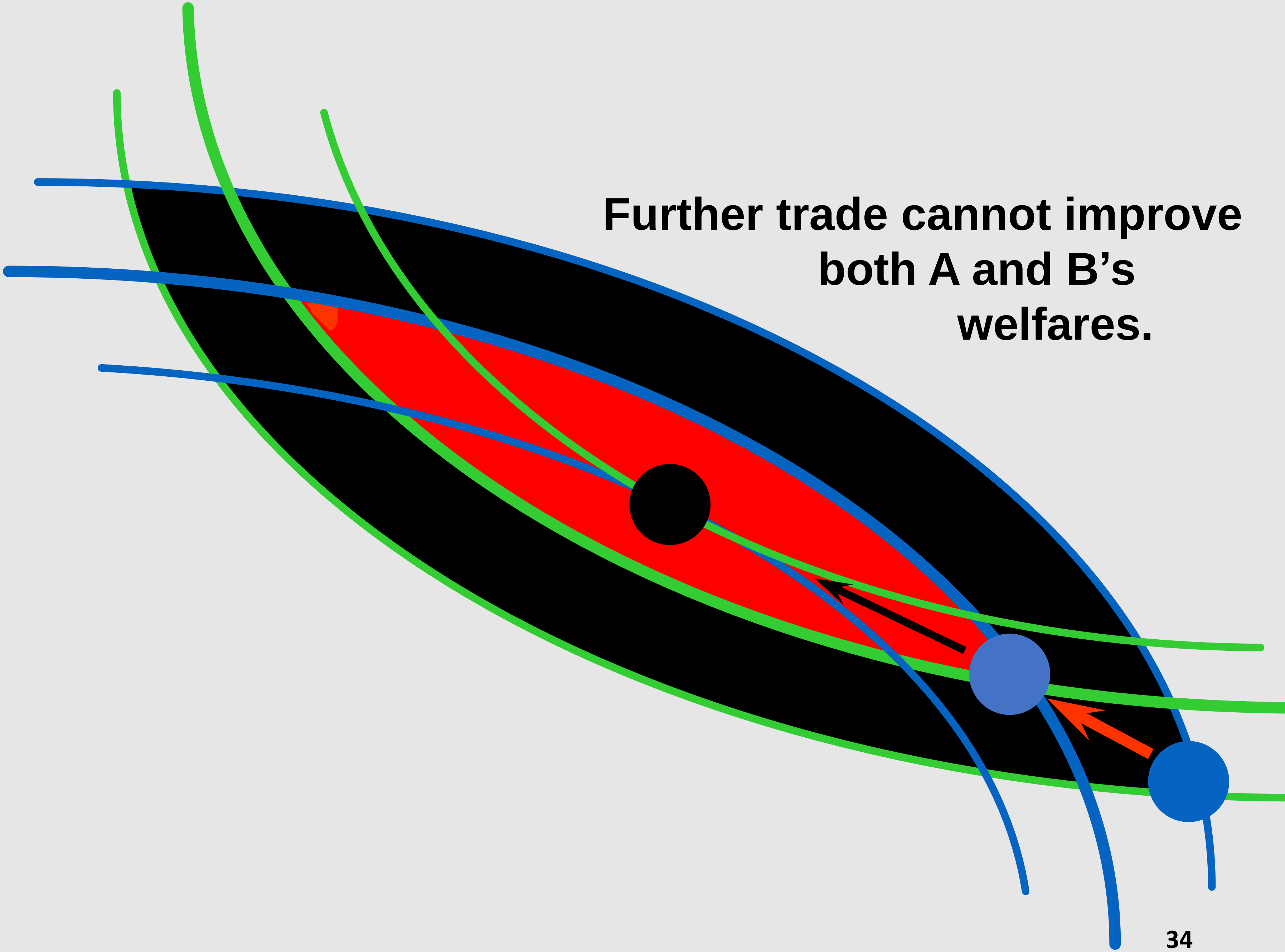
Trade
improves both
A's and B's welfares.
This is a Pareto-improvement
over the endowment allocation.



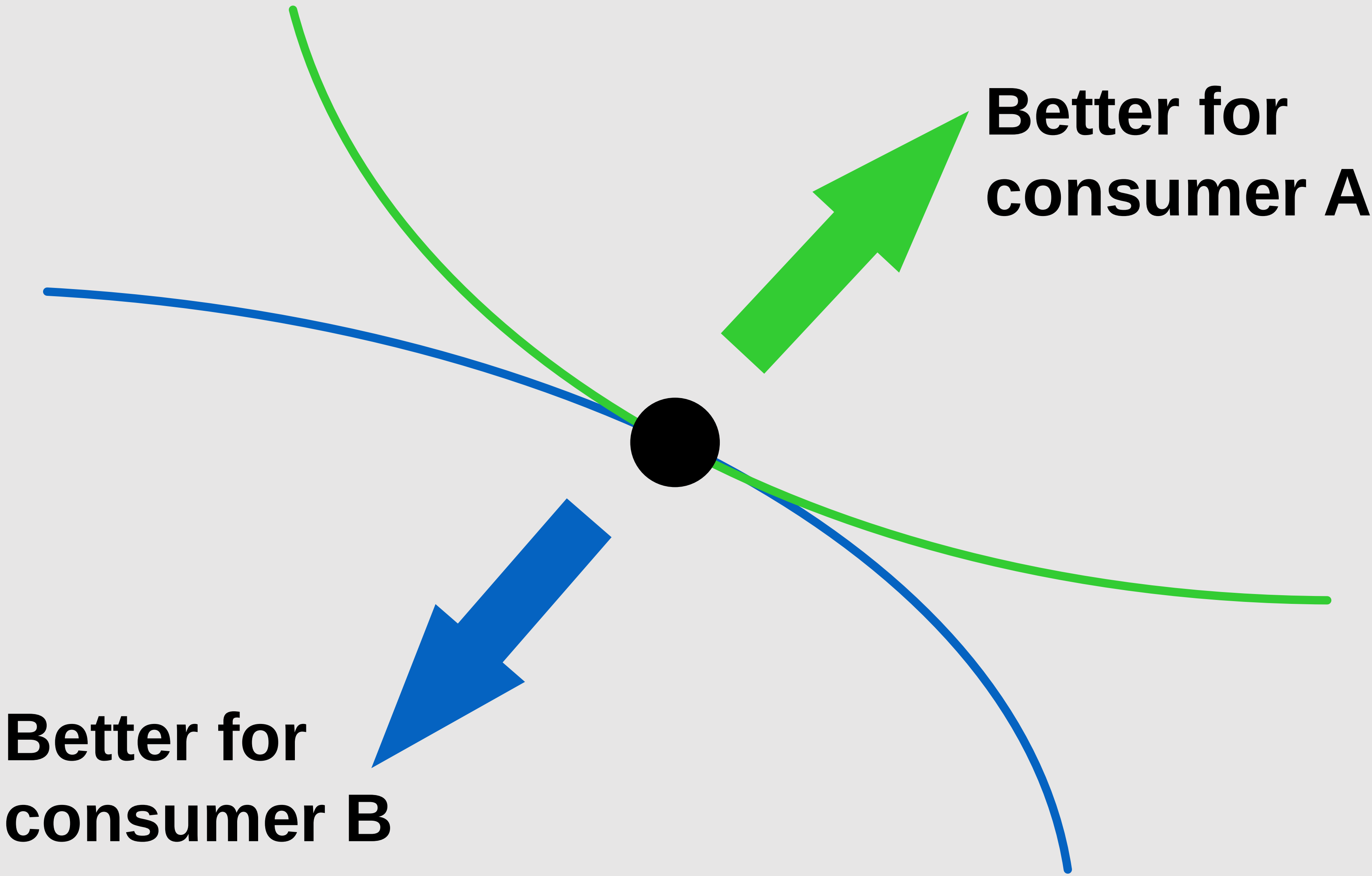
New mutual gains-to-trade region is the set of all further Pareto-improving reallocations.

Trade improves both A’s and B’s welfares.
This is a Pareto-improvement over the endowment allocation.

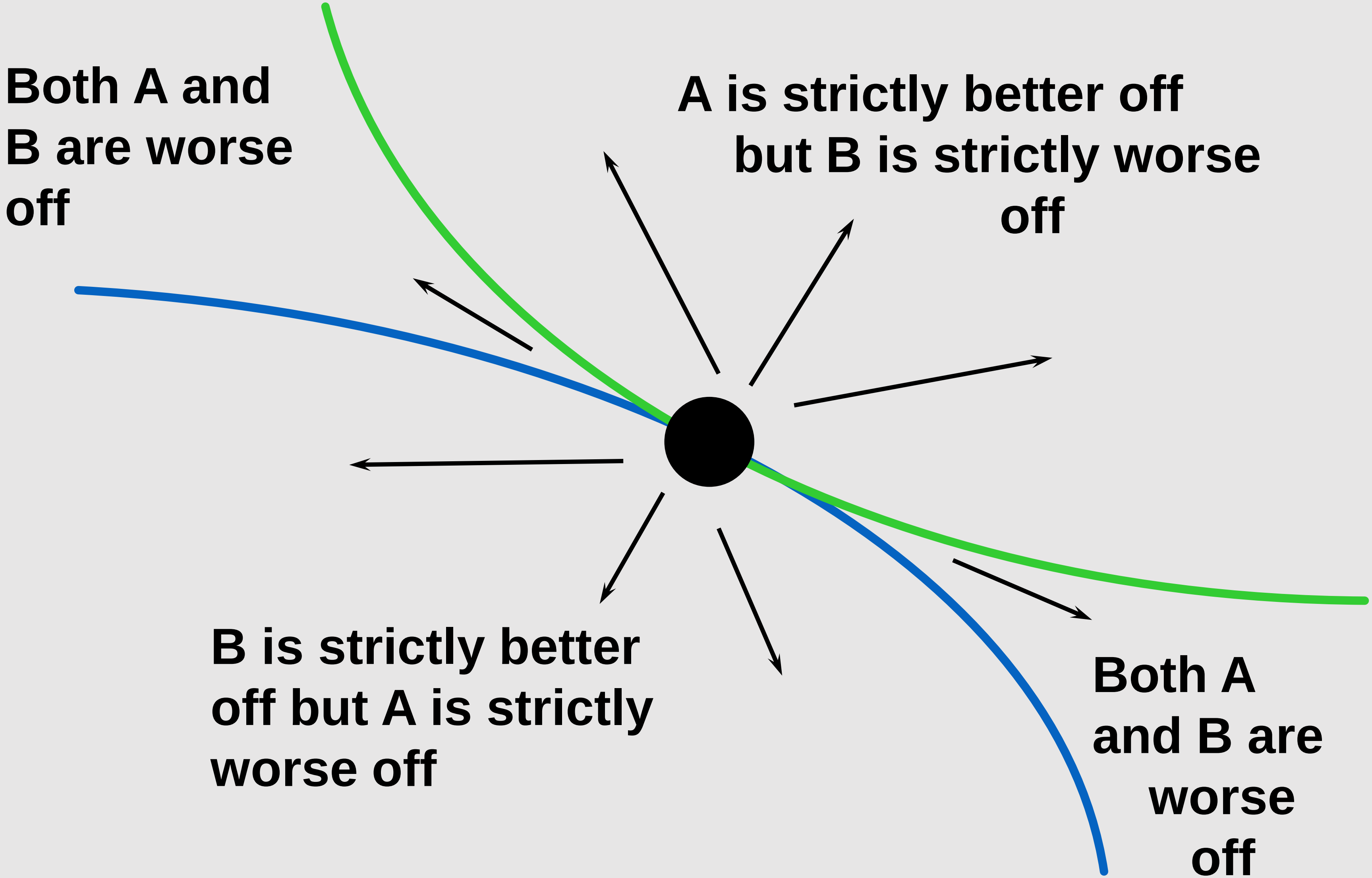
Further trade cannot improve both A and B's welfares.



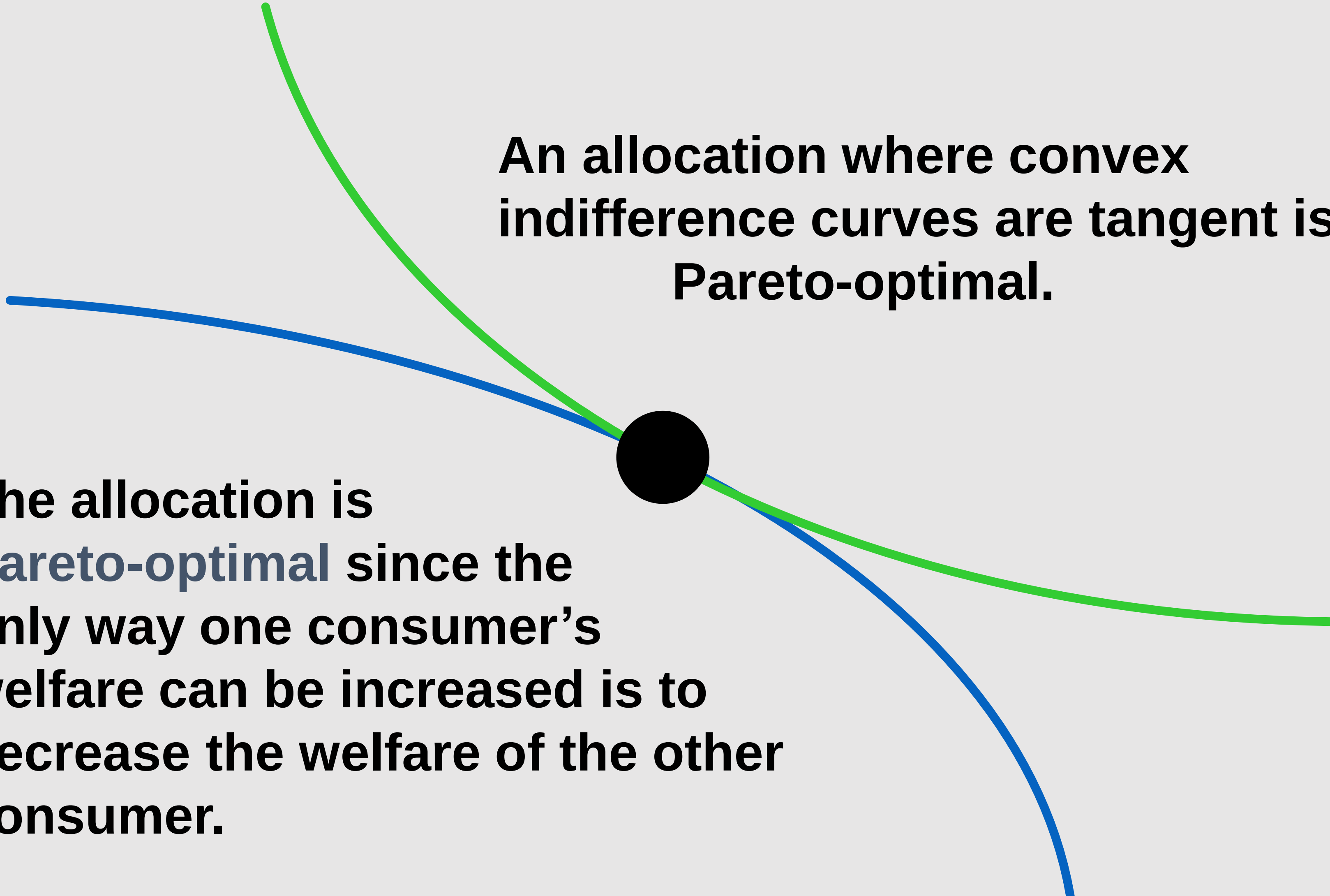
Pareto Efficient Allocation



Pareto Efficient Allocation



Pareto Efficient Allocation



An allocation where convex indifference curves are tangent is Pareto-optimal.

The allocation is Pareto-optimal since the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality or Pareto efficiency

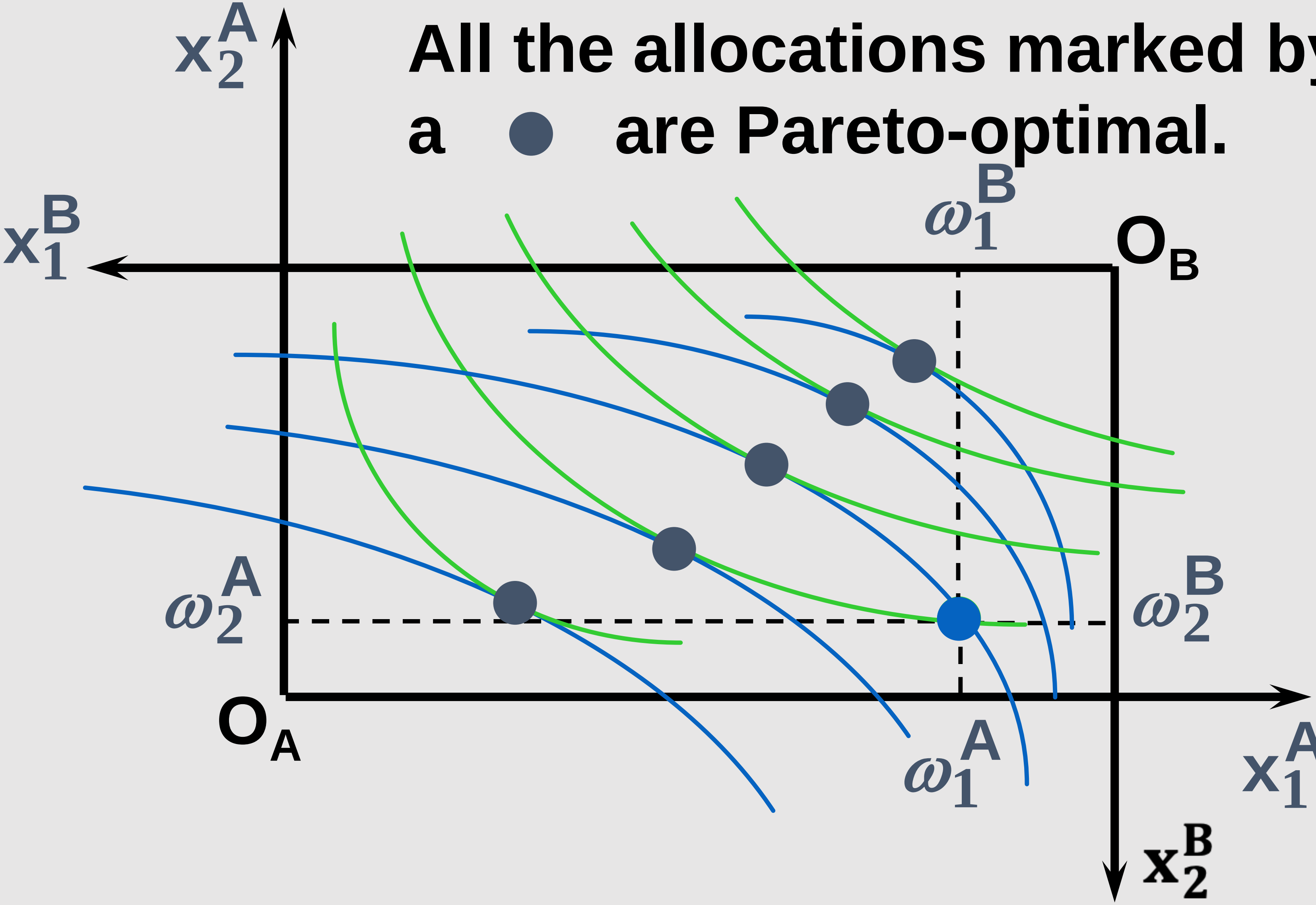
An allocation is Pareto efficient if:

1. There is no way to make all the people involved better off
2. There is no way to make some individual better off without making someone else worse off
3. All the gains from trade have been exhausted
4. There are no mutually advantageous trades to be made
5. $MRS^A = MRS^B$: the tangent of U^A and U^B are the same.

→ Where are all of the Pareto-optimal allocations of the endowment?

Pareto Efficient Allocation

All the allocations marked by a ● are Pareto-optimal.



The contract curve

Given the tangency condition:

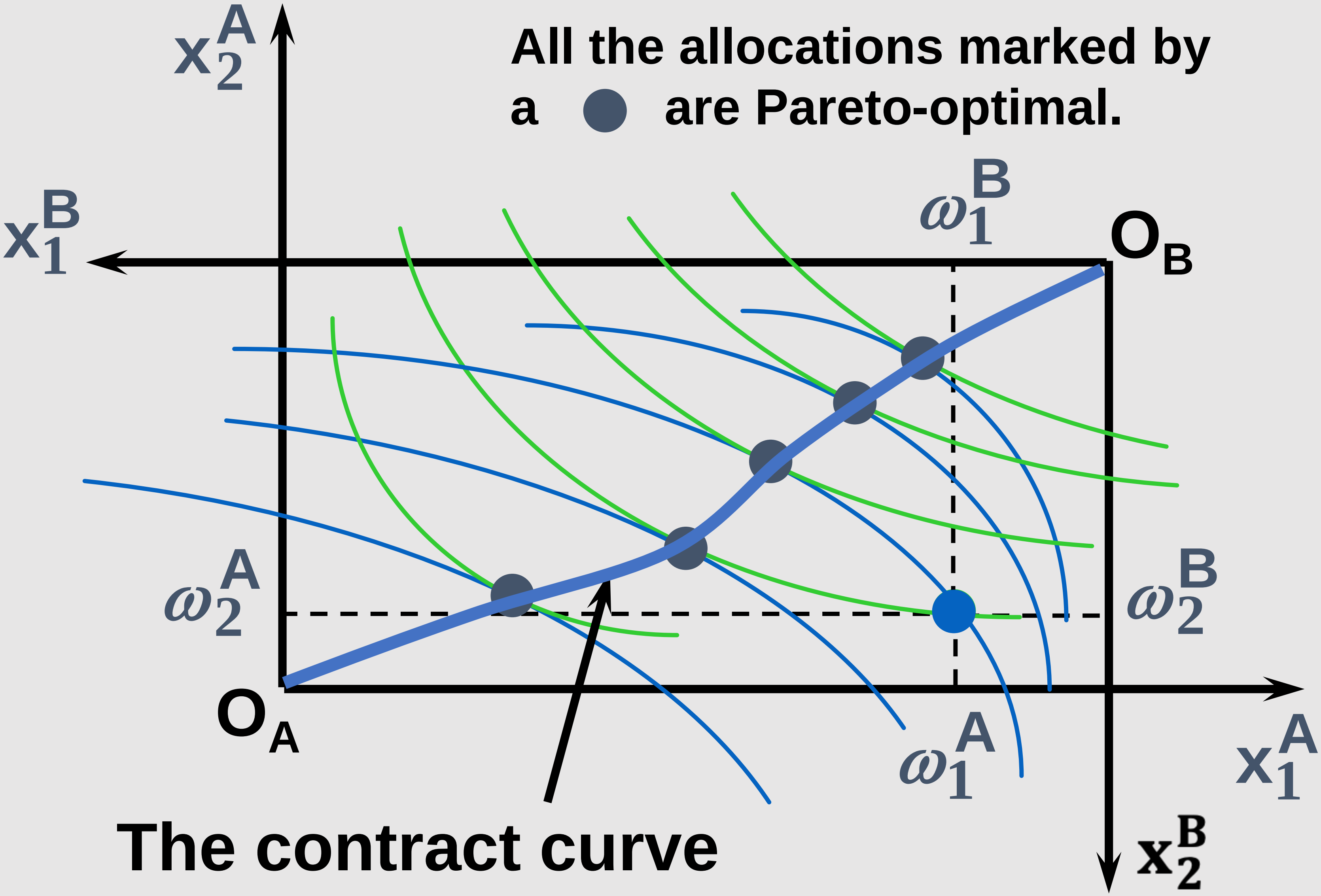
→ It is easy to see that there are many Pareto efficient allocations

→ For each indifference curve of A, we can draw an indifference curve for B so that to obtain a Pareto optimal allocation.

→ The set of all Pareto-optimal allocations is called the **contract curve** or the **Pareto set**

- A contract curve stretches from A's origin to B's origin
- Pareto set: all possible outcomes of mutually advantageous trade
- The contract curve or the Pareto set DO NOT DEPEND on the initial endowment
- To obtain the formula of the contract curve we need $MRS^A = MRS^B$ (tangency of the indifference curves)

The contract curve



4. Trade in Competitive Markets

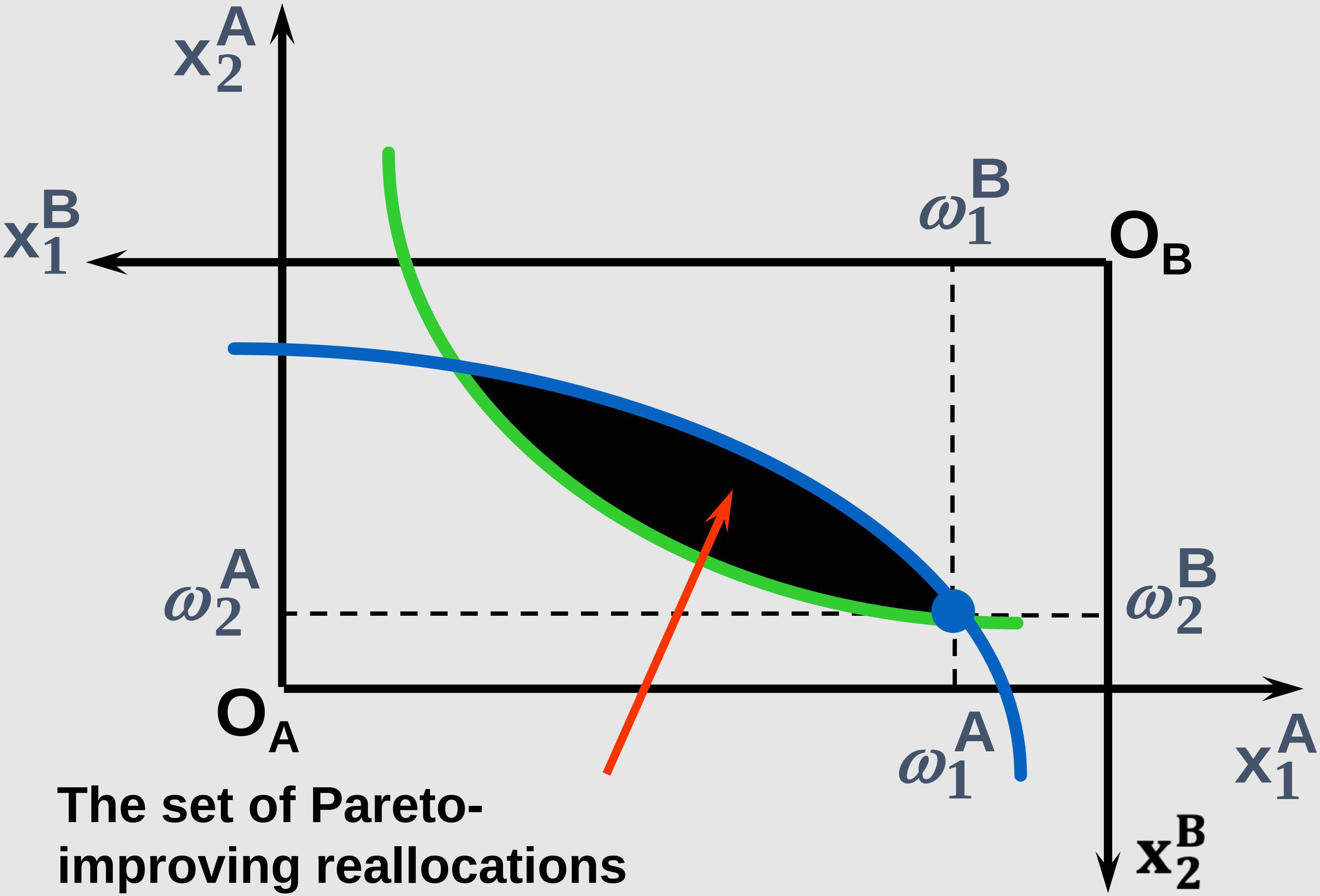
So far, we describe all possible efficient allocations.

→ But to which of the many allocations on the contract curve will consumers trade?

→ That depends upon how trade is conducted.

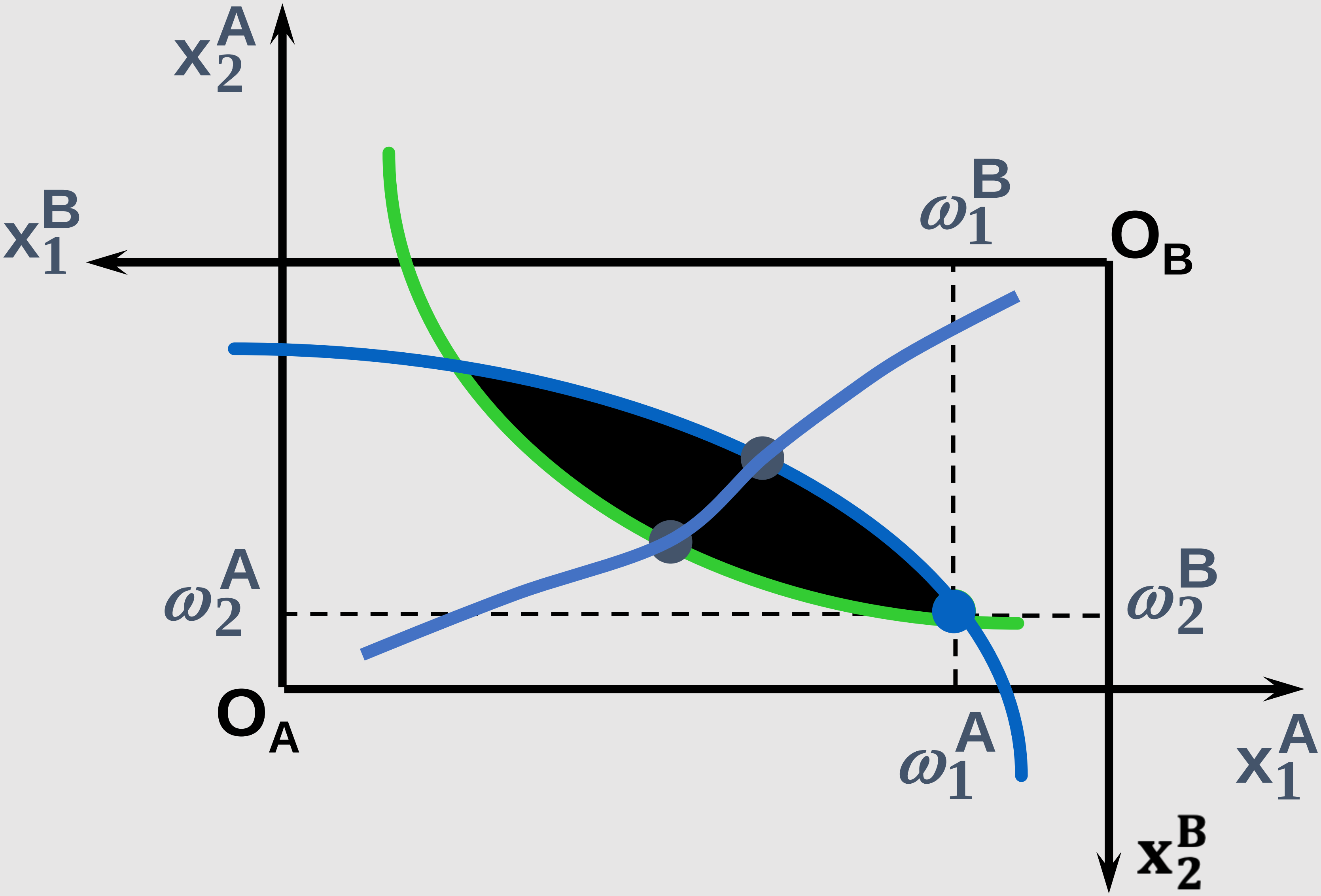
→ In perfectly competitive markets? By one-on-one bargaining?

The Core

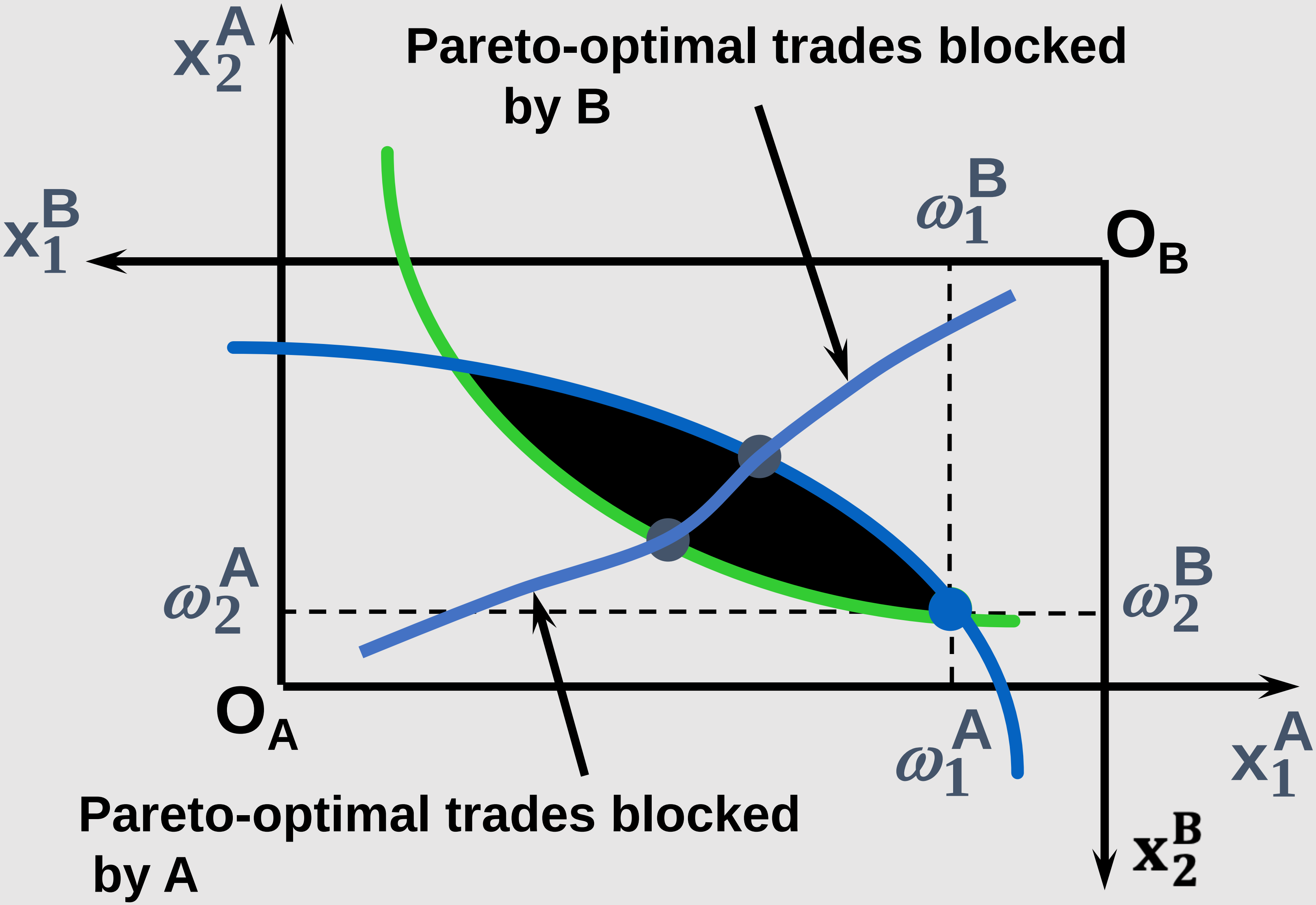


The set of Pareto-improving reallocations

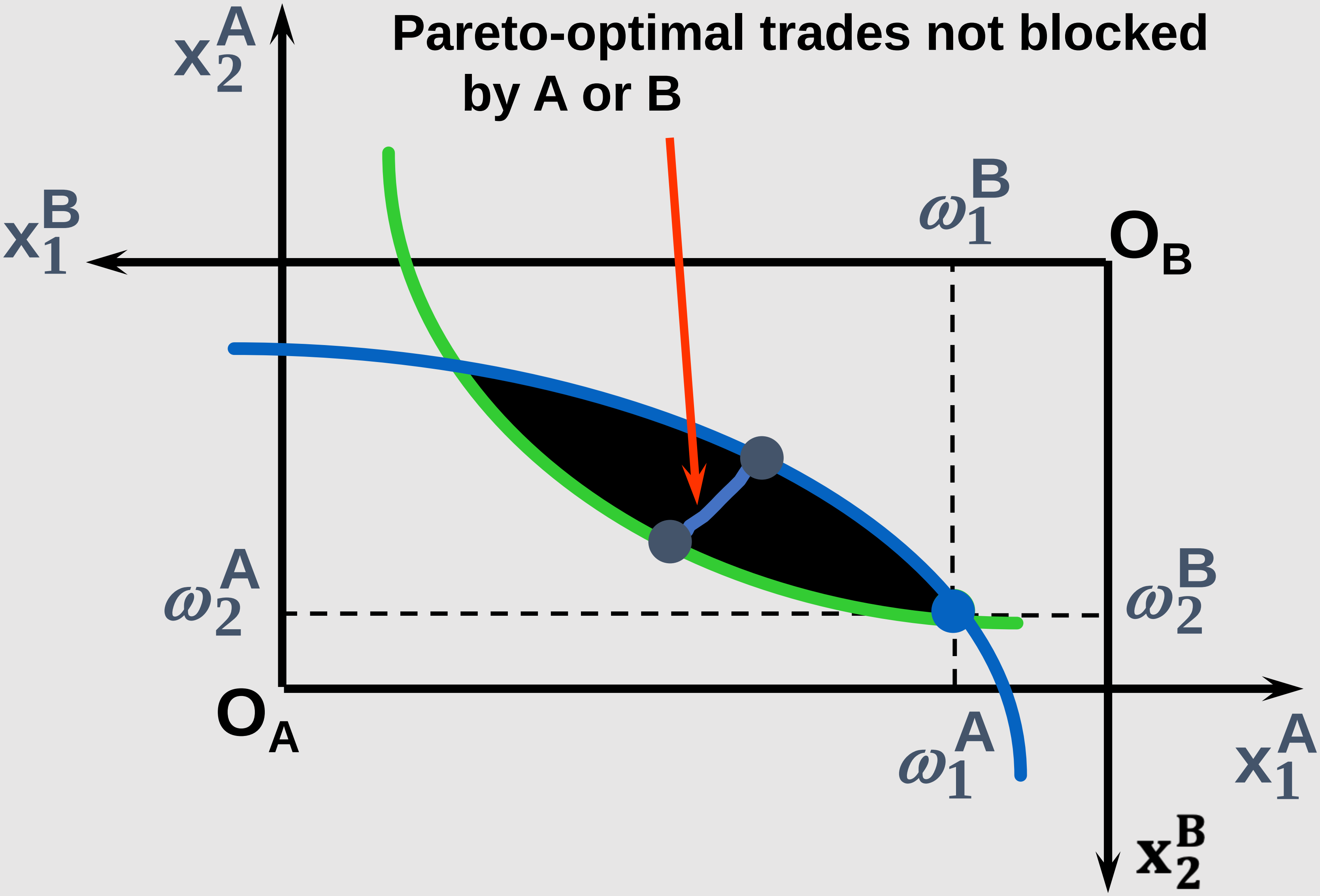
The Core



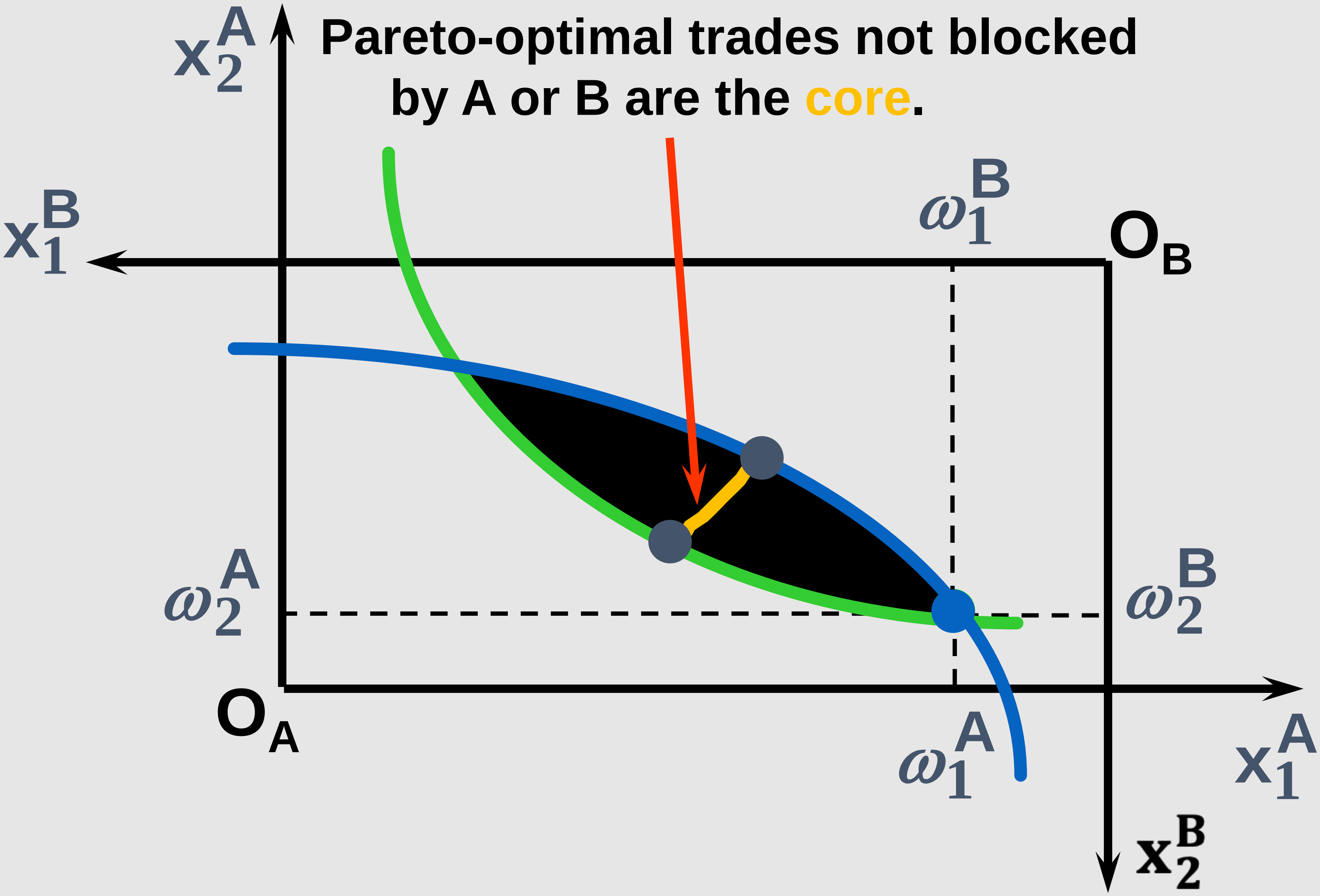
The Core


x_2^A
Pareto-optimal trades blocked
by B
ω_1^B
x_1^B
O_B
ω_2^A
ω_2^B
O_A
ω_1^A
x_1^A
Pareto-optimal trades blocked
by A
x_2^B

The Core


Pareto-optimal trades not blocked by A or B
x_2^A
x_1^B
ω_1^B
O_B
ω_2^A
ω_2^B
O_A
ω_1^A
x_1^A
x_2^B

The Core



x_2^A
Pareto-optimal trades not blocked by A or B are the core.
x_1^B
ω_1^B
O_B
ω_2^A
ω_2^B
O_A
ω_1^A
x_1^A
x_2^B

The Core

- The core is the set of all Pareto-optimal allocations that are welfare-improving for both consumers relative to their own endowments.

- Rational trade should achieve a core allocation.

- But which core allocation?

- Again, that depends upon the way trade is conducted.

Trade in Competitive Markets

→ Consider trade in perfectly **competitive** markets.

→ Each consumer is a **price-taker** trying to maximize her own utility given p_1, p_2 and her own endowment. That is:

→ We use the Consumer Maximization Problem

- Maximize utility
- Subject to a budget constraint
- Here the revenue R in the budget constraint is equivalent to how much the consumer can obtain when selling his initial endowment
- Consumers chooses X^A such that $MRS^A = \frac{p_1}{p_2}$

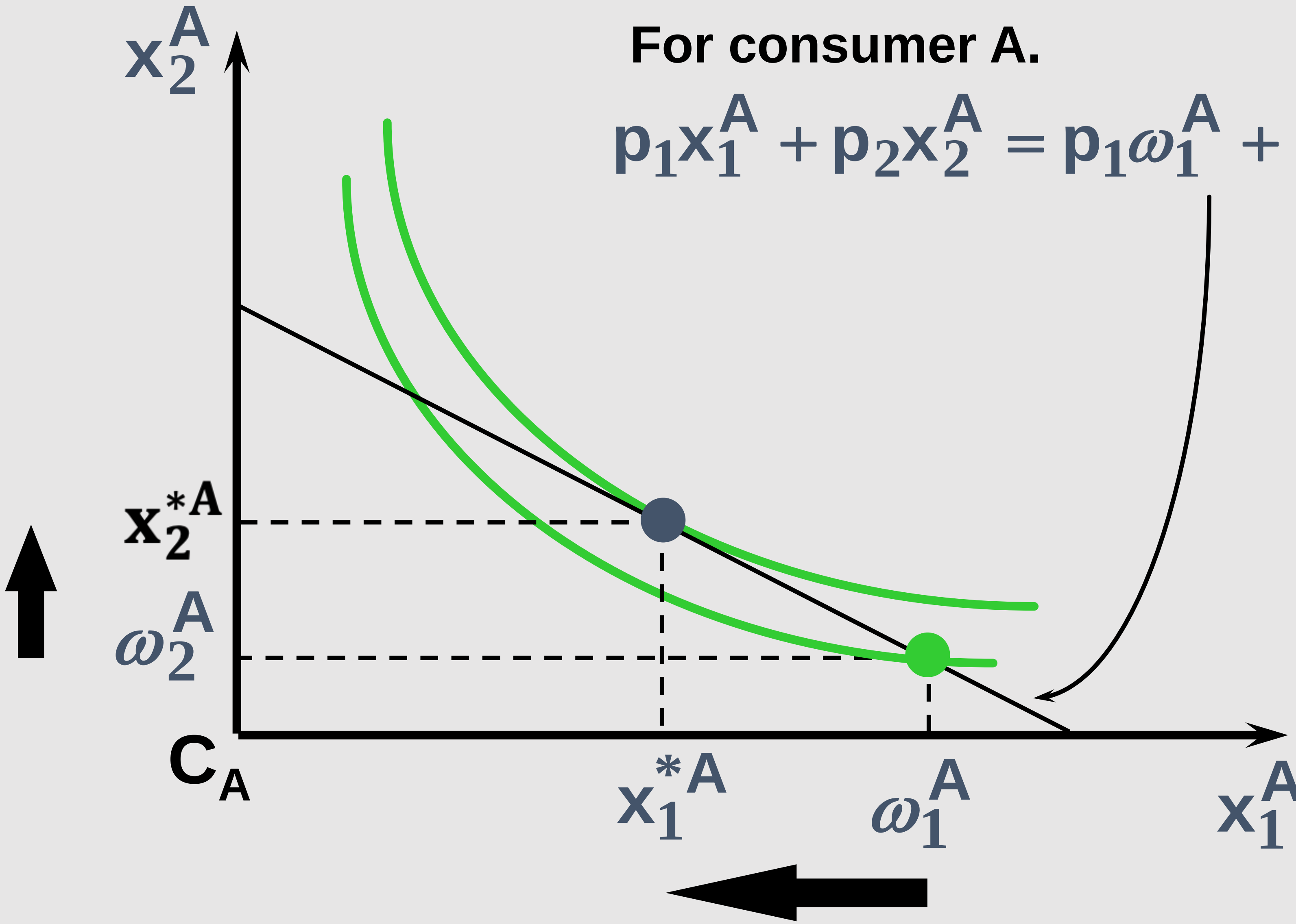
→ **(Gross) Demand:** total amount of good 1 that consumer A wants at the going prices. Noted by $x_A^1(p_1, p_2)$.

→ **Net Demand**: difference between this total demand and the initial endowment of good 1 that agent A holds. Noted by $e_A^1(p_1, p_2)$.

Trade in Competitive Markets

For consumer A.

$$p_1x_1^A + p_2x_2^A = p_1\omega_1^A + p_2\omega_2^A$$



Trade in Competitive Markets

- So given p_1 and p_2, consumer A's net demands for commodities 1 and 2 are

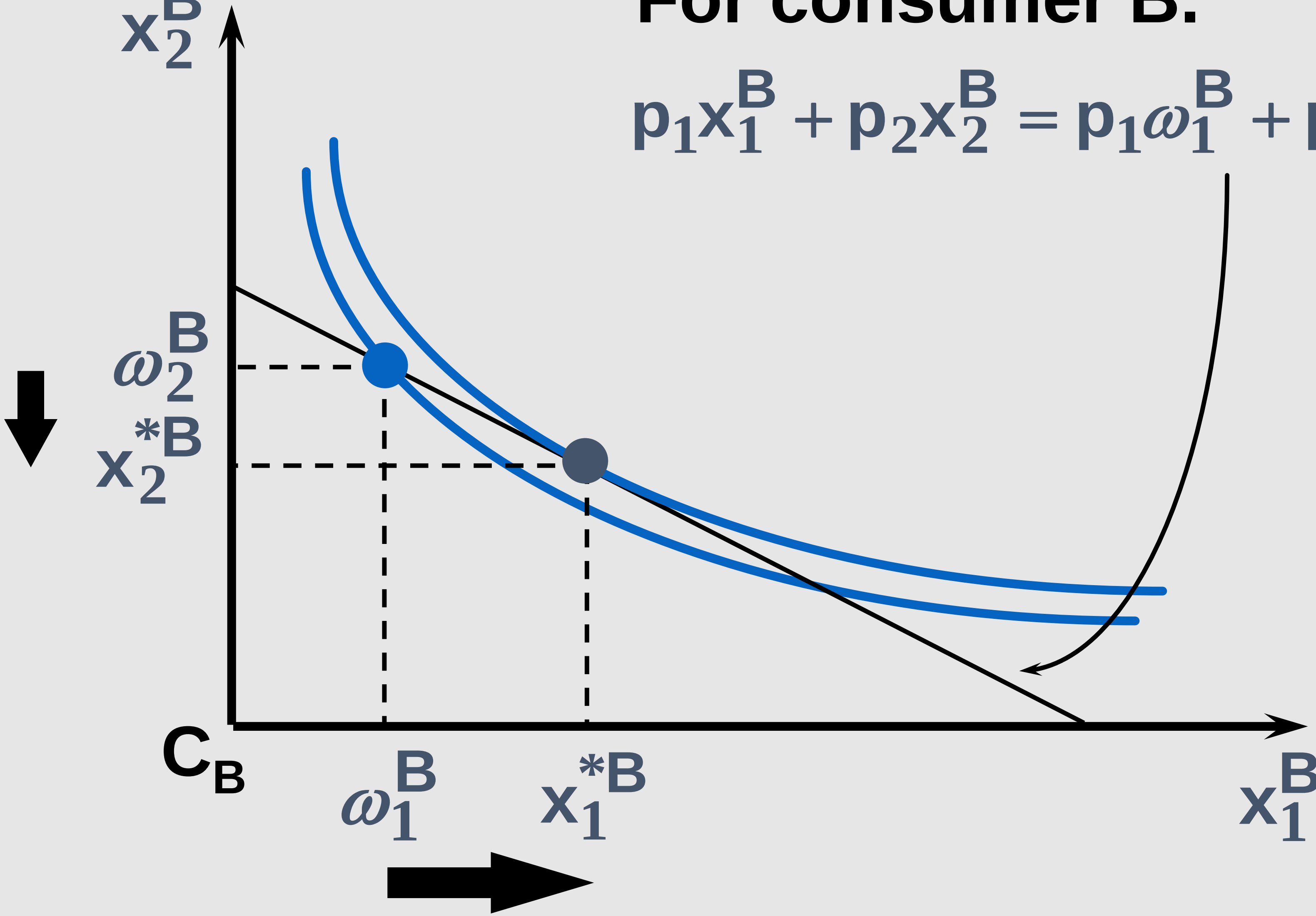
$$x_1^{*A} - \omega_1^A \quad \textbf{and} \quad x_2^{*A} - \omega_2^A .$$

- And, similarly, for consumer B …

Trade in Competitive Markets

For consumer B.

$$p_1x_1^B + p_2x_2^B = p_1\omega_1^B + p_2\omega_2^B$$



Trade in Competitive Markets

→ So given p_1 and p_2, consumer B's net demands for commodities 1 and 2 are

$$x_1^{*B} - \omega_1^B \quad \text{and} \quad x_2^{*B} - \omega_2^B.$$

Trade in Competitive Markets

→ A general equilibrium occurs when prices p_1 and p_2 cause both the markets for commodities 1 and 2 to clear.

→ Prices will adjust until the sum of net demands is null:

$$(x_1^{*A} - \omega_1^A) + (x_1^{*B} - \omega_1^B) = 0$$

$$\text{and } (x_2^{*A} - \omega_2^A) + (x_2^{*B} - \omega_2^B) = 0$$

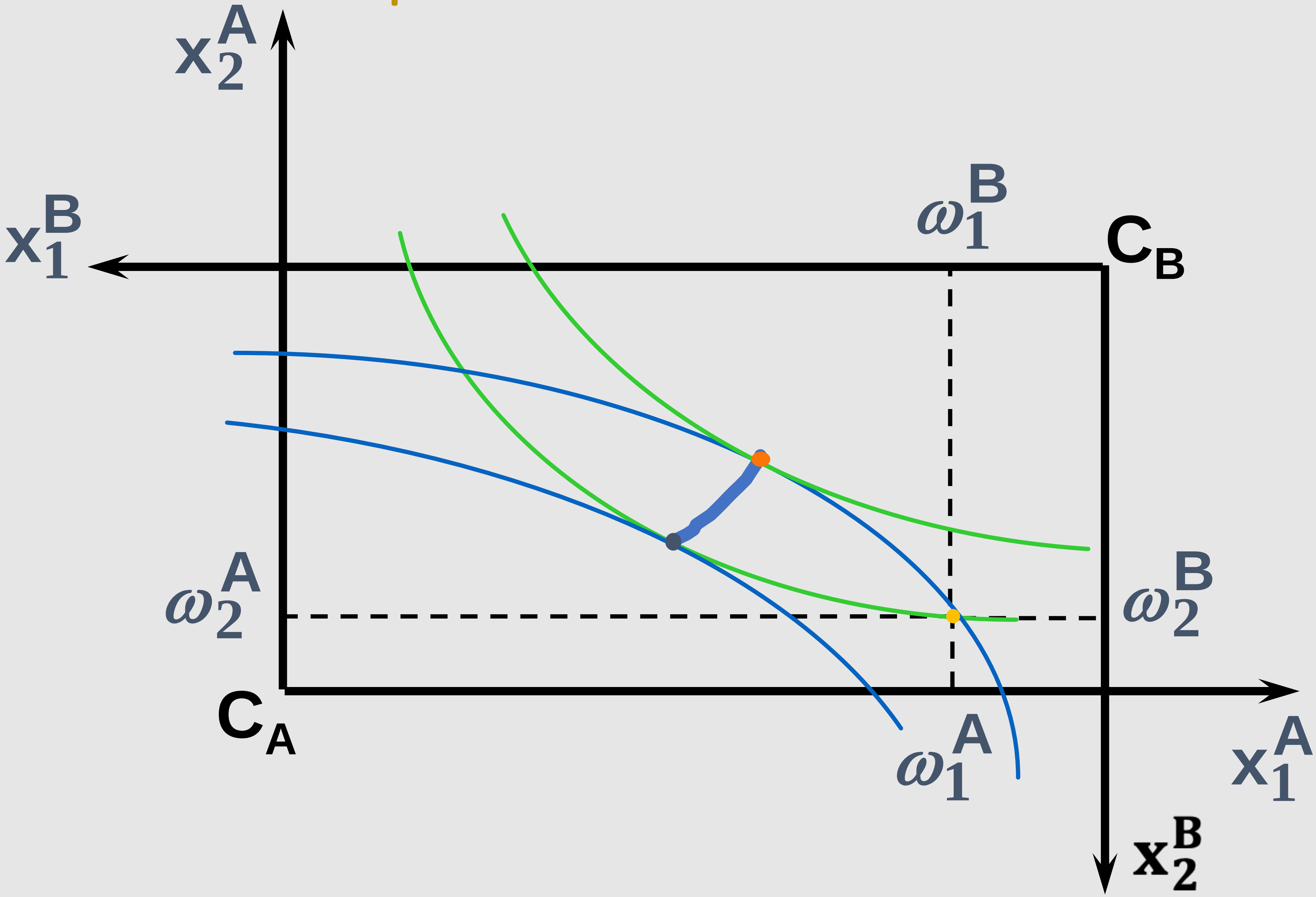
This is

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

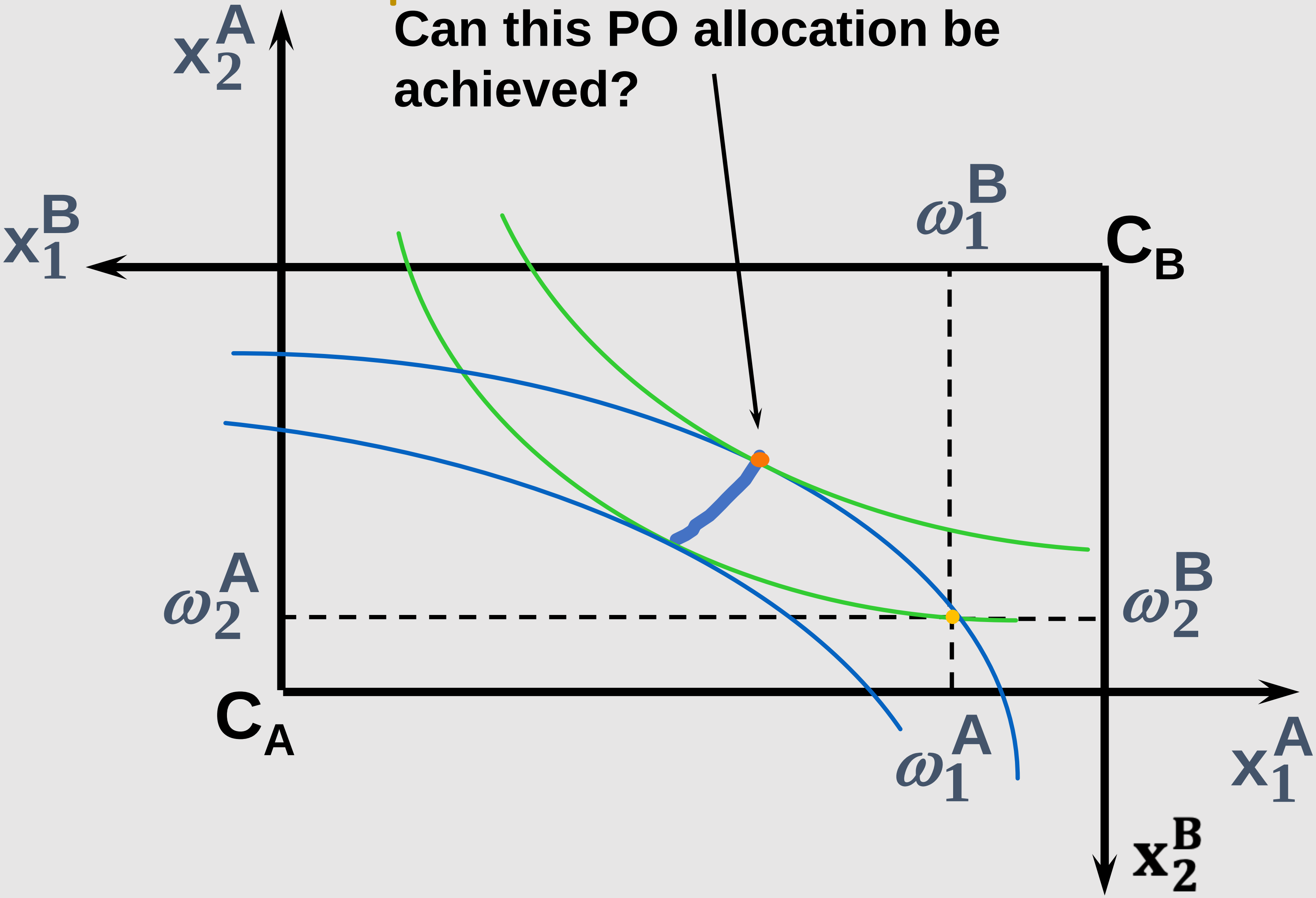
$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

→ A **market equilibrium**: set of prices such that each consumer is choosing his or her most-preferred affordable bundle, and all consumers' choices are compatible in the sense that demand equals supply in every market

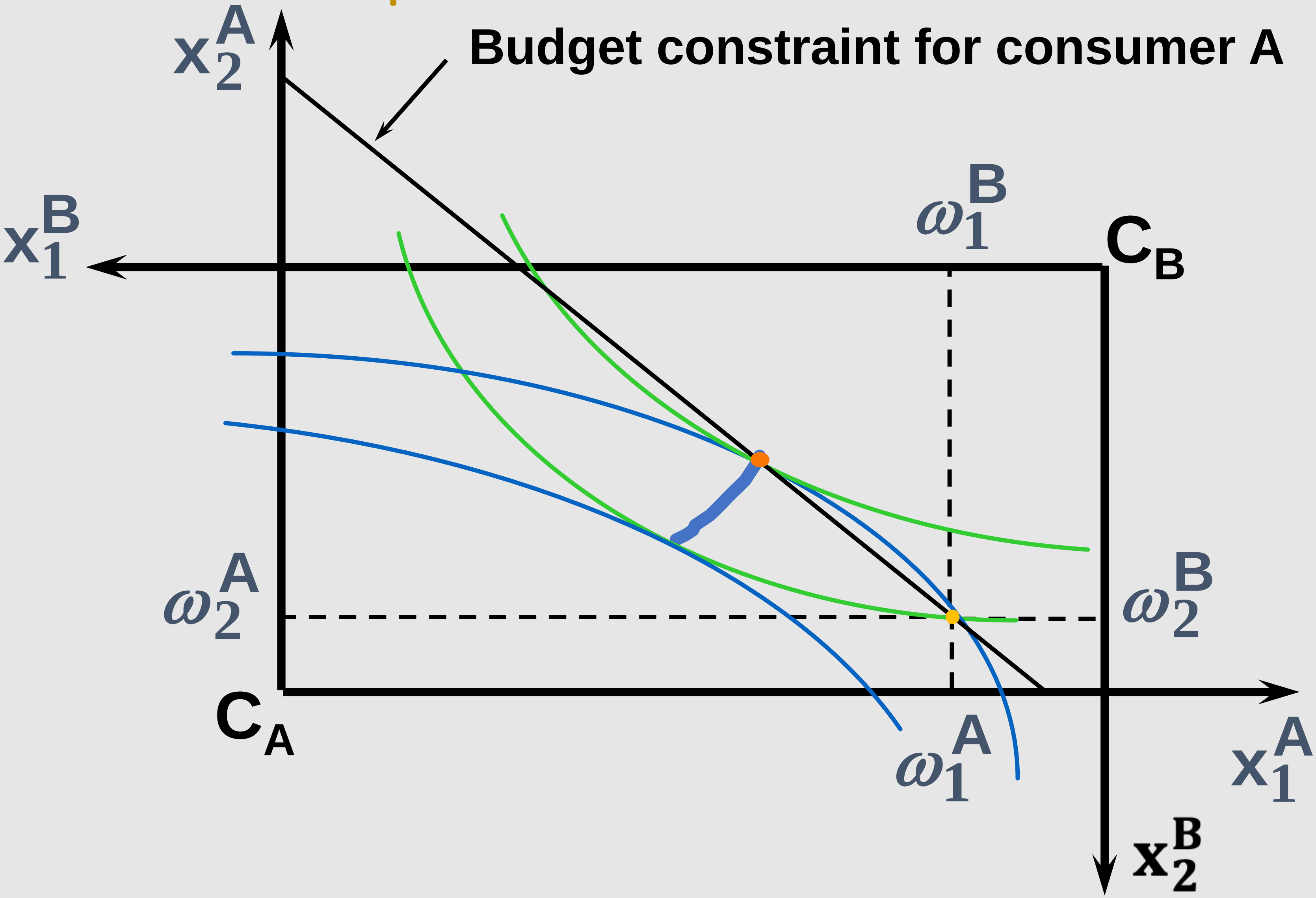
Trade in Competitive Markets



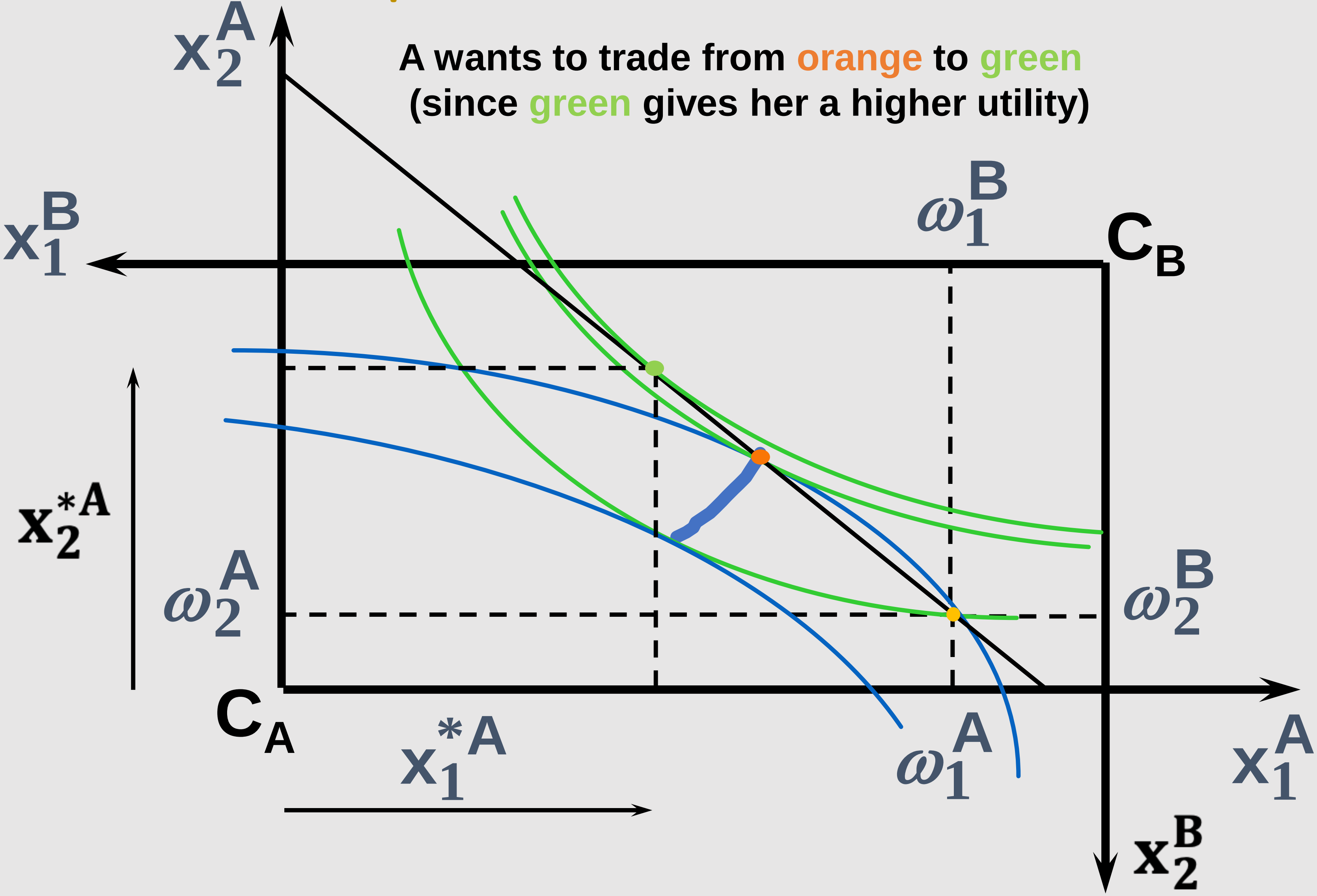
Trade in Competitive Markets



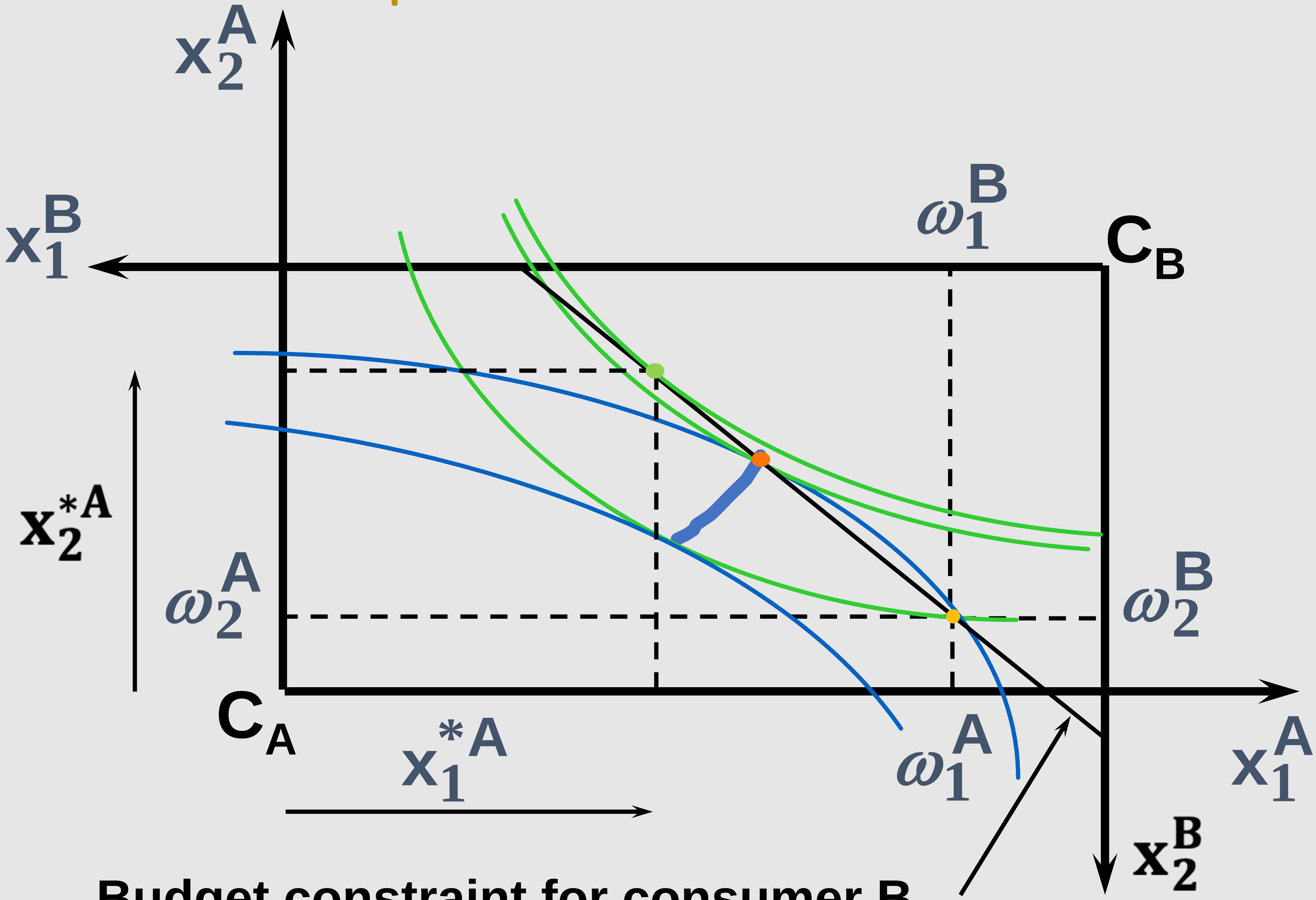
Trade in Competitive Markets



Trade in Competitive Markets

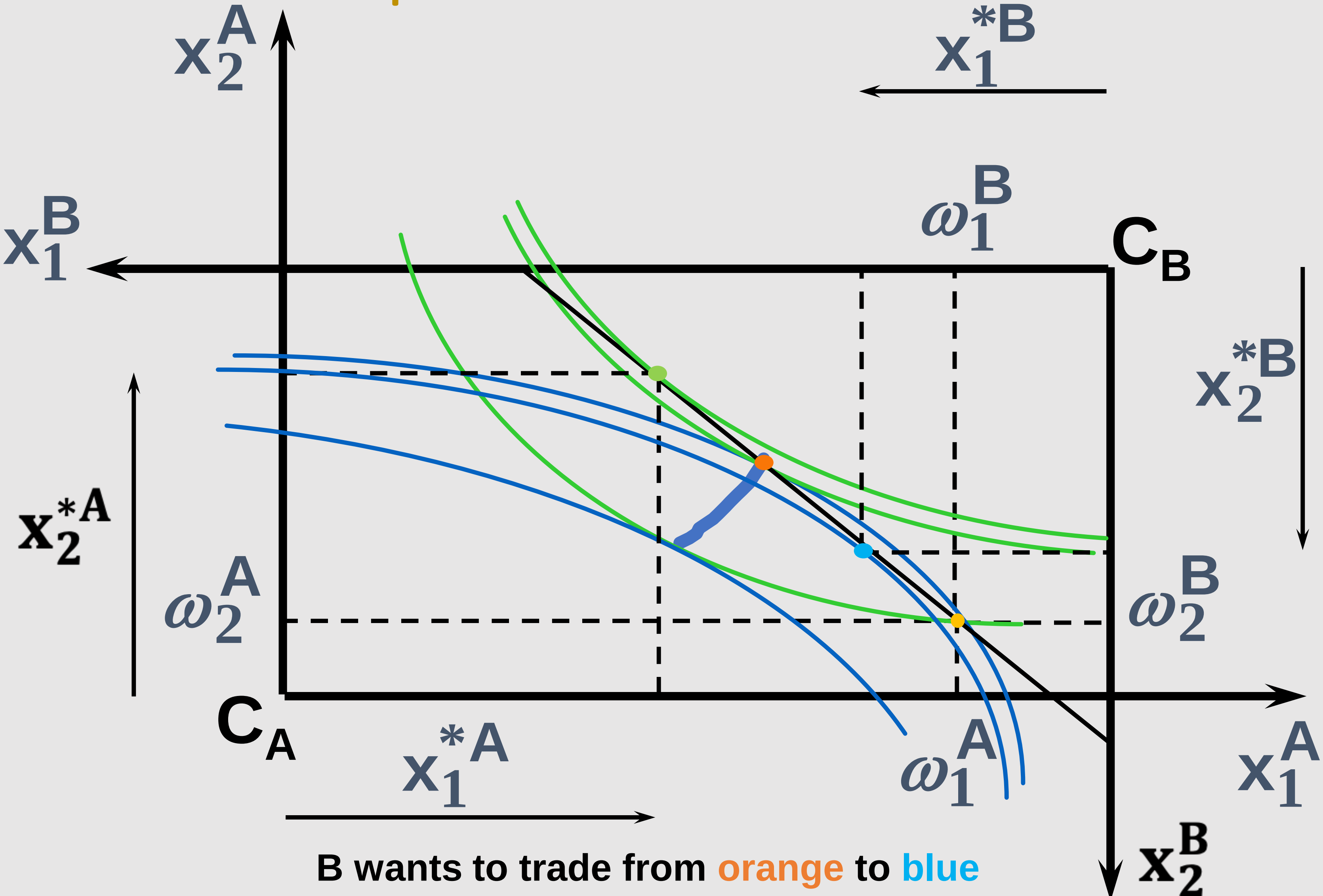


Trade in Competitive Markets



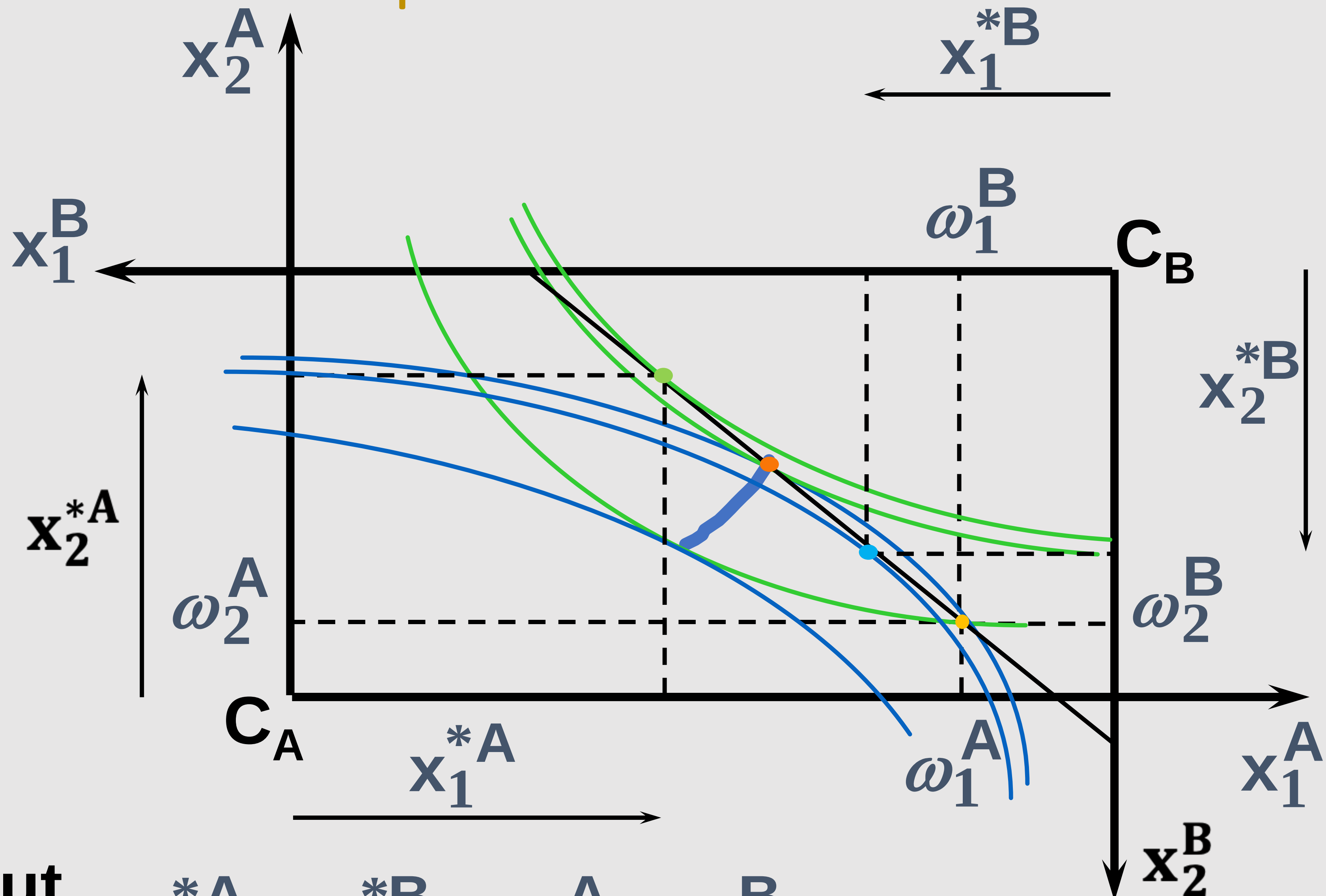
Budget constraint for consumer B

Trade in Competitive Markets



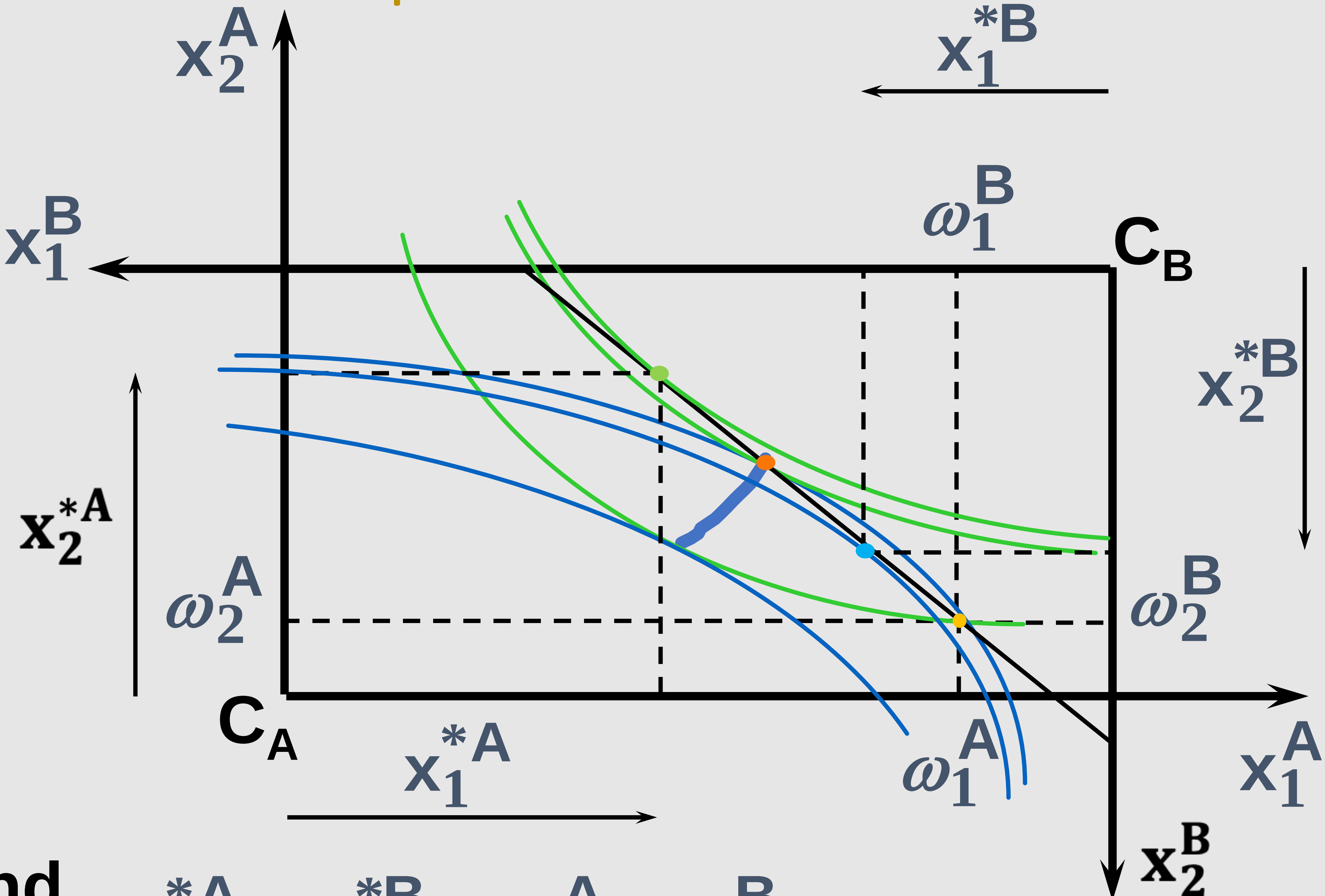
B wants to trade from orange to blue
(since blue gives her a higher utility)

Trade in Competitive Markets



But $x_1^{*A} + x_1^{*B} < \omega_1^A + \omega_1^B$

Trade in Competitive Markets



and $x_2^{*A} + x_2^{*B} > \omega_2^A + \omega_2^B$

Trade in Competitive Markets

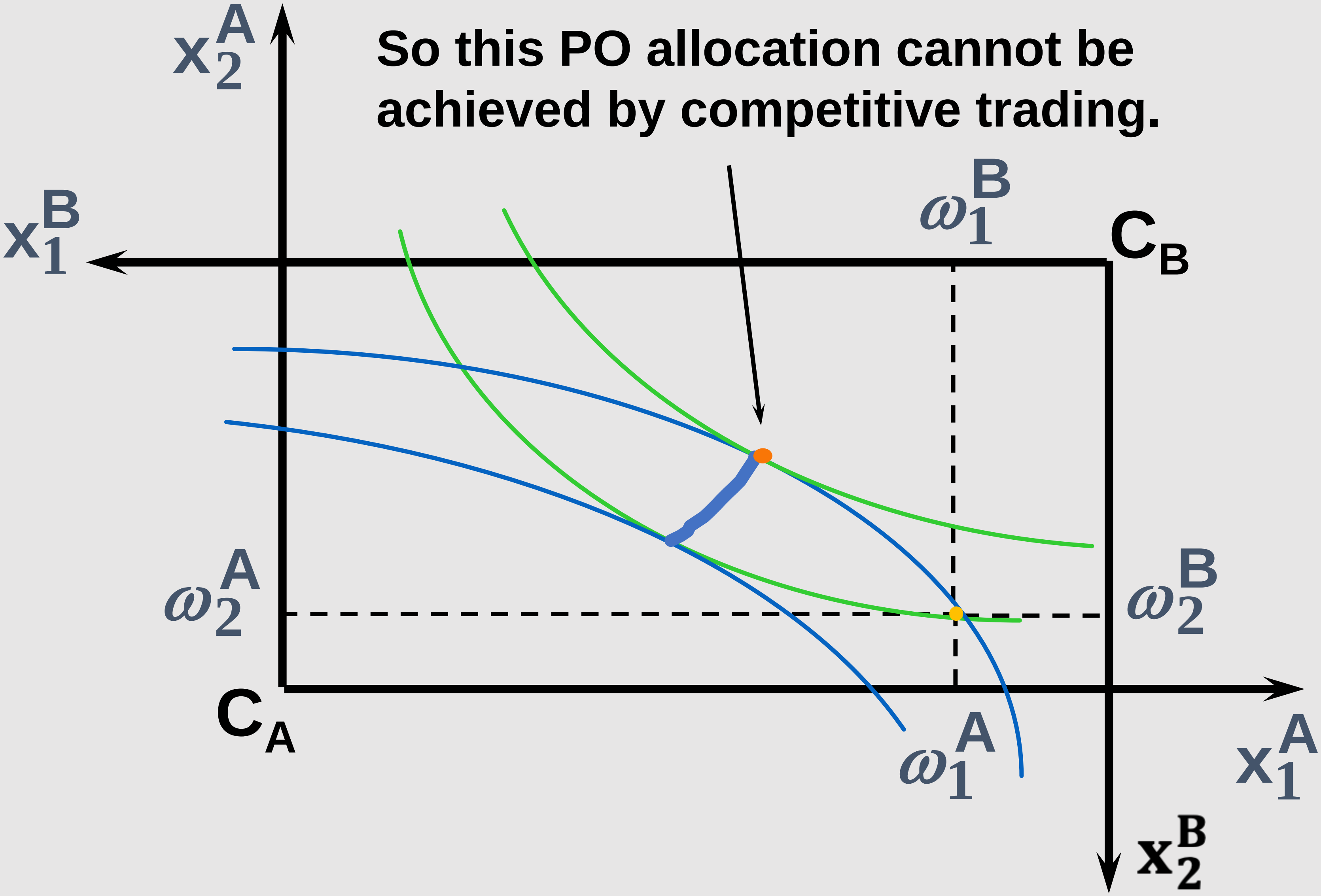
→ So at the given prices p_1 and p_2 there is an

- excess supply of commodity 1
- excess demand for commodity 2.

→ Neither market clears so the prices p_1 and p_2 do not cause a general equilibrium.

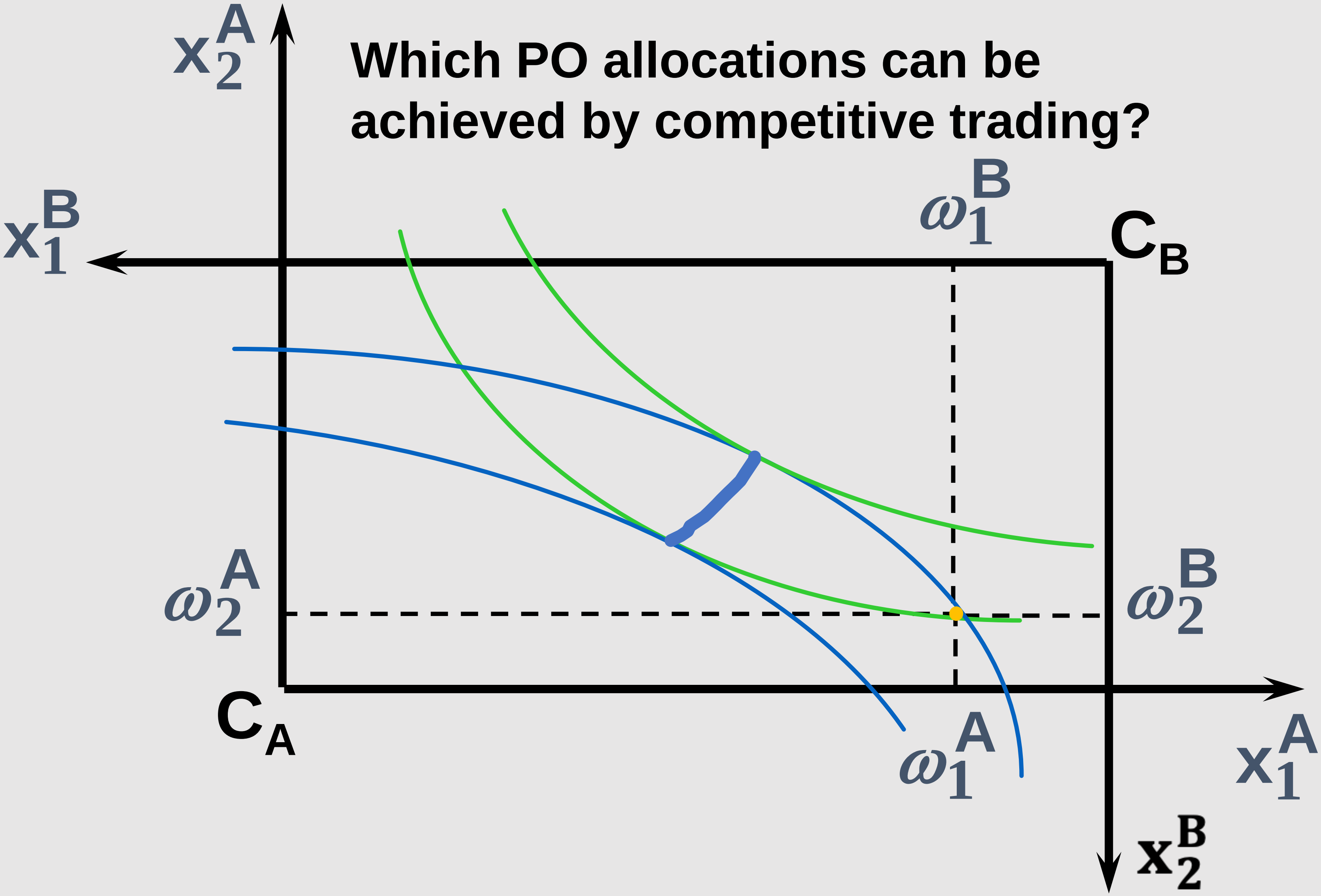
→ The market is in **disequilibrium**

Trade in Competitive Markets



Trade in Competitive Markets

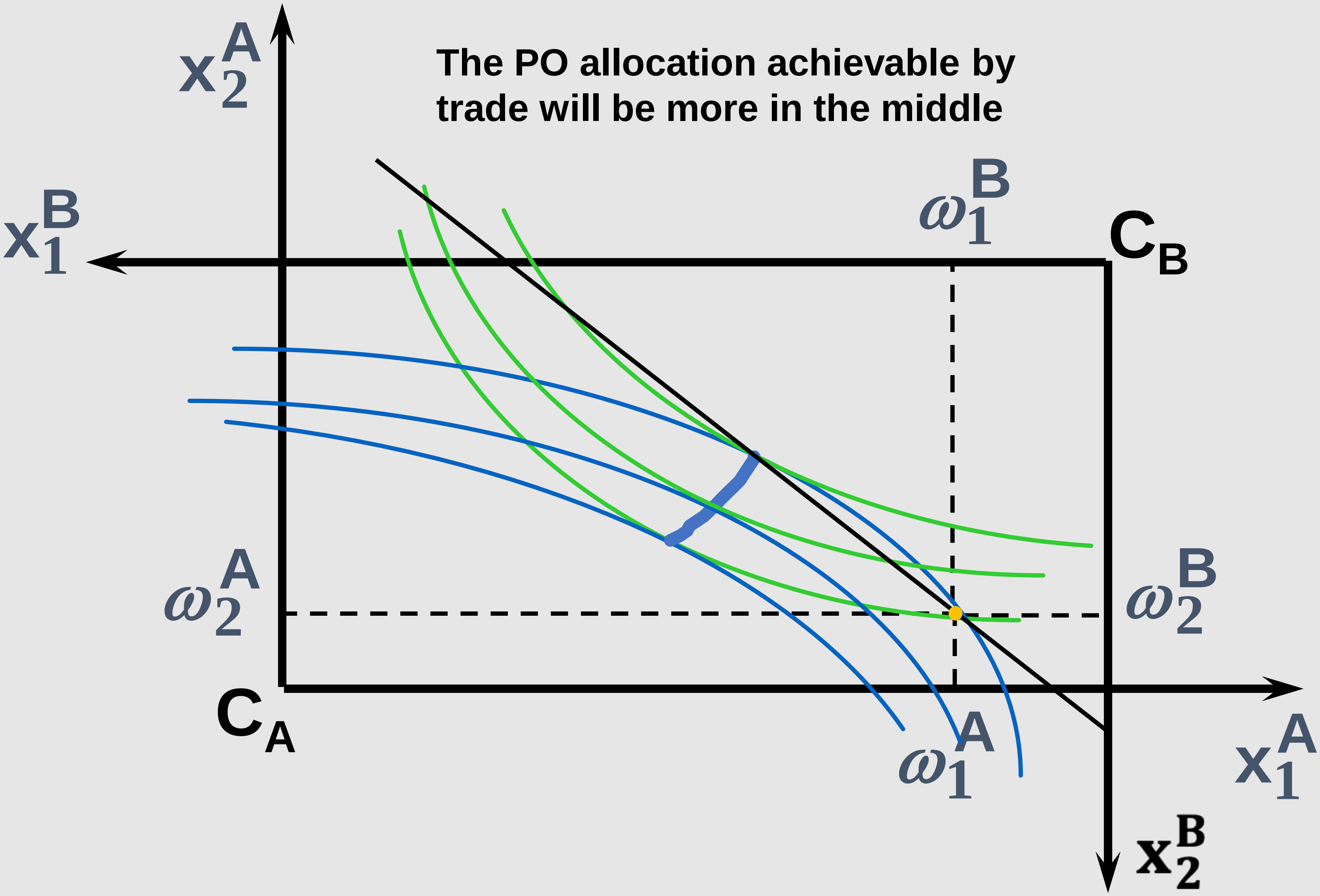
Which PO allocations can be achieved by competitive trading?



Trade in Competitive Markets

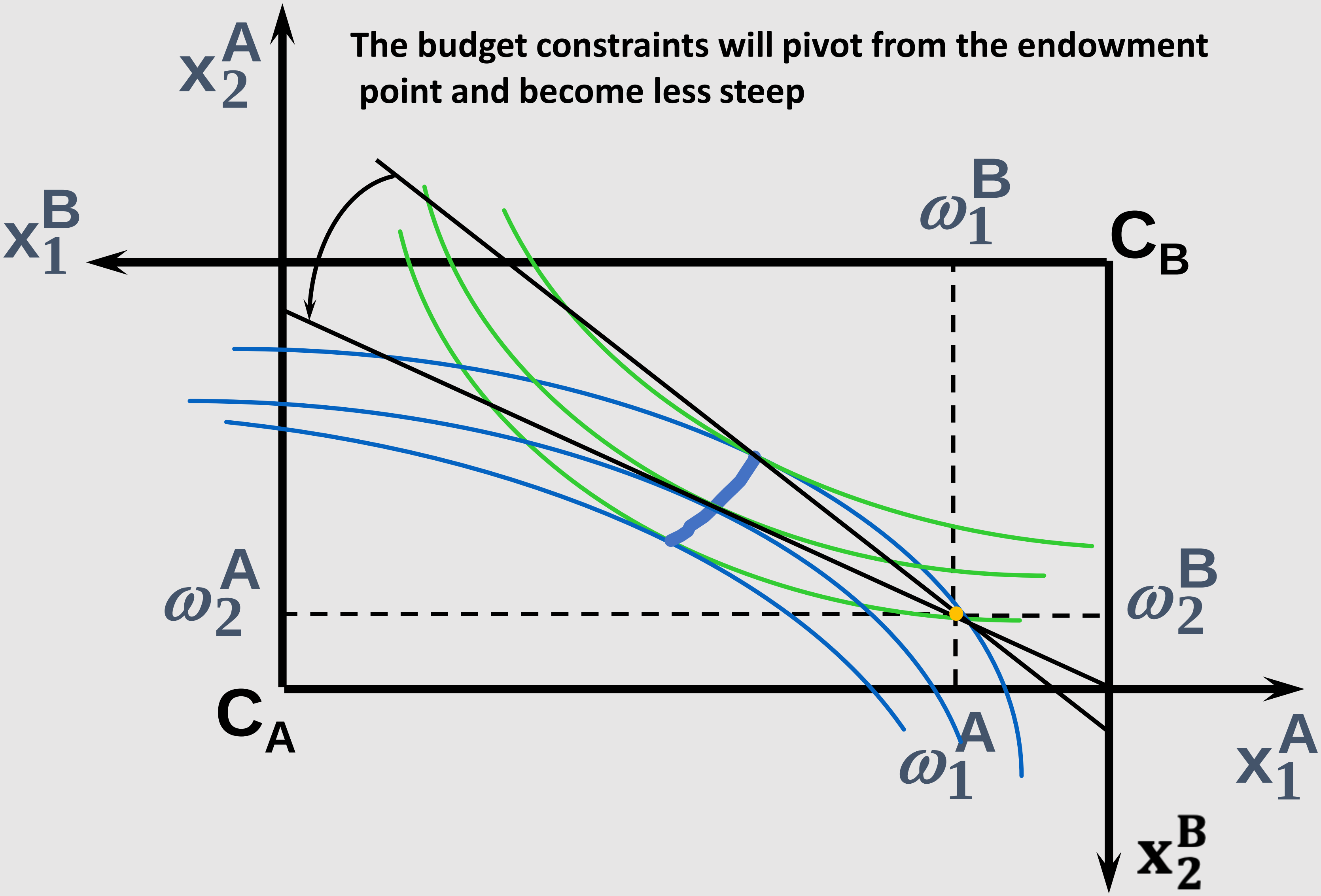
- → Since there is an excess demand for commodity 2, p_2 will rise.
- → Since there is an excess supply of commodity 1, p_1 will fall.
- → The slope of the budget constraints is - p_1/p_2 so the budget constraints will pivot from the endowment point and become less steep.

Trade in Competitive Markets

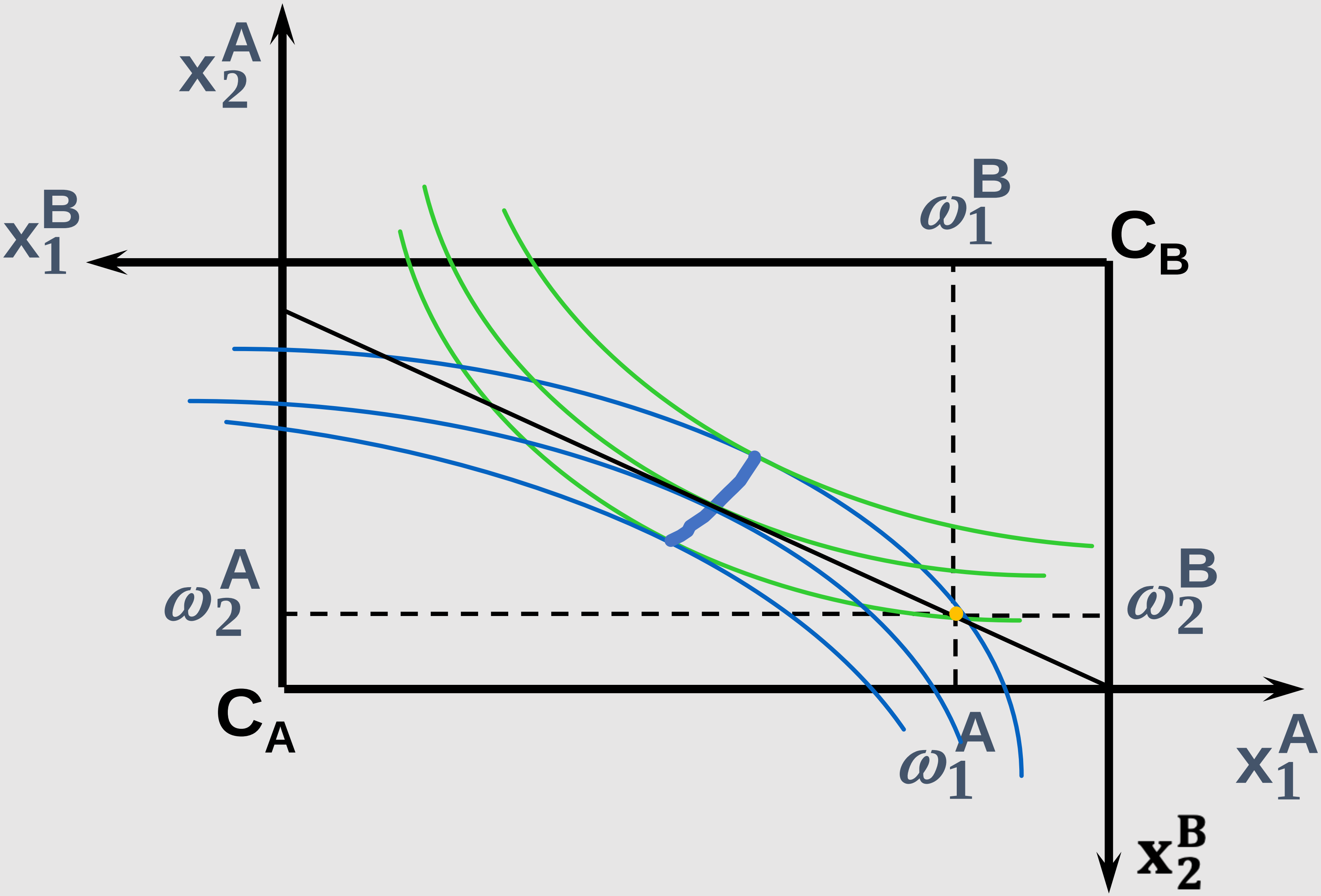


Trade in Competitive Markets

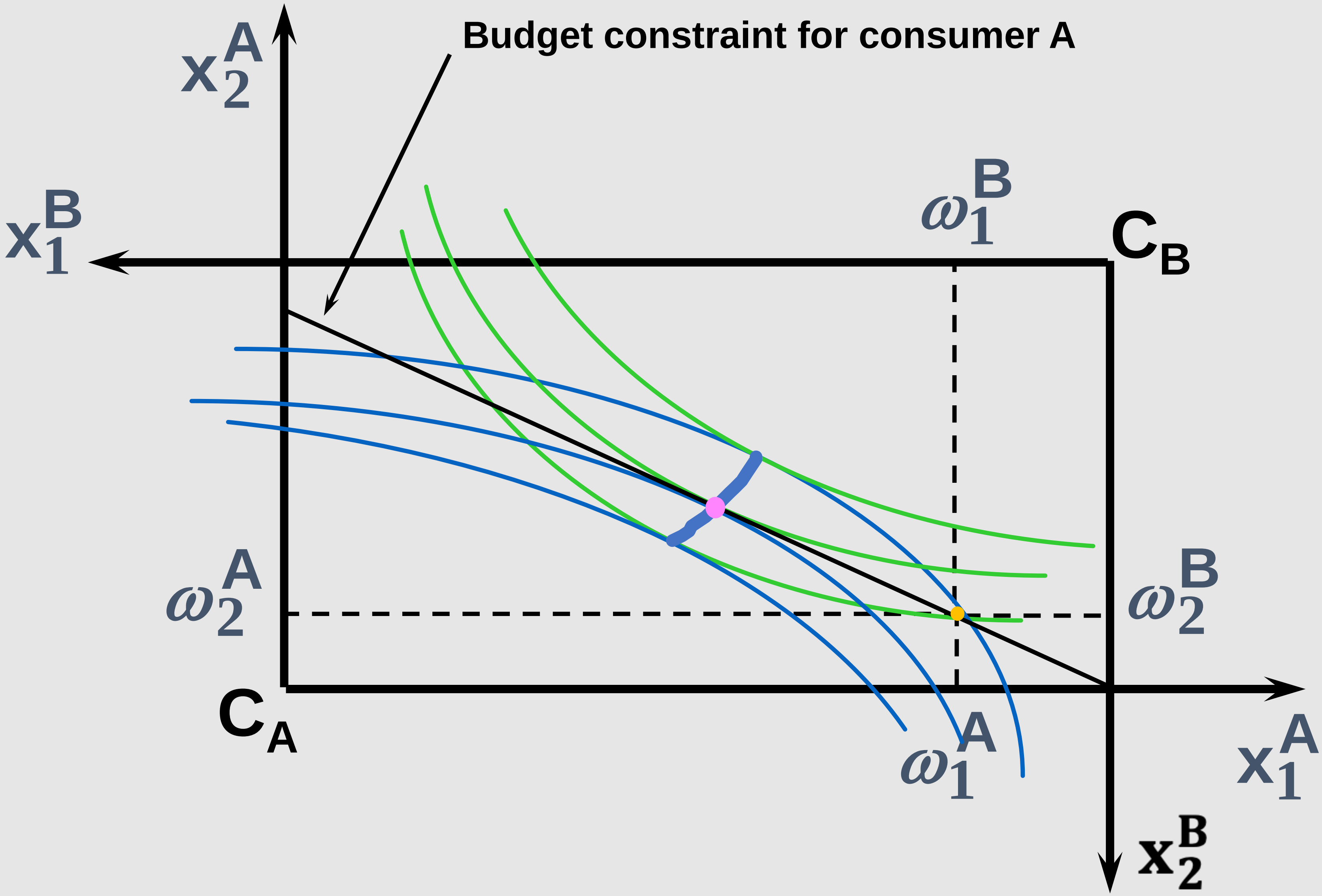
The budget constraints will pivot from the endowment point and become less steep



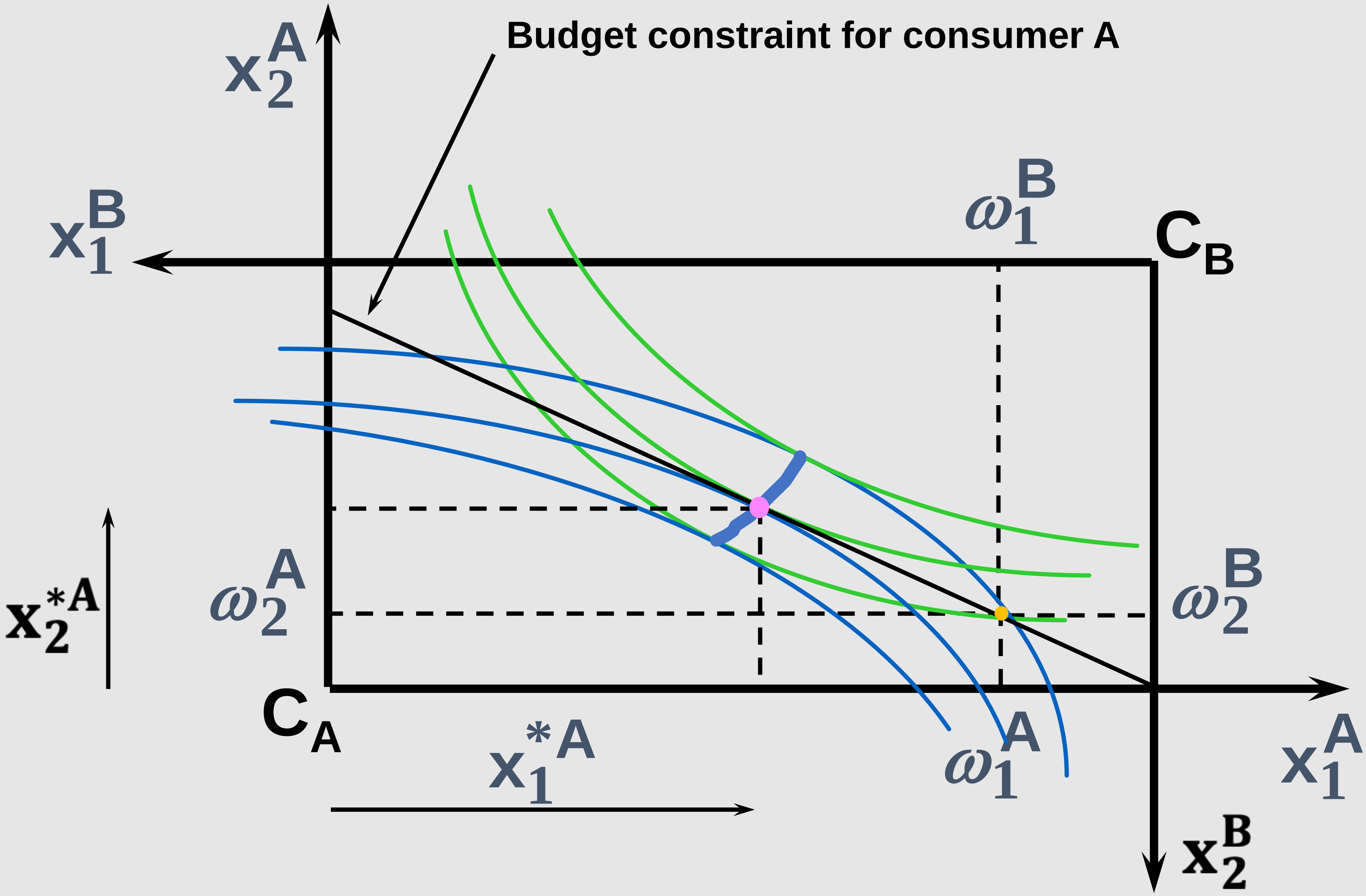
Trade in Competitive Markets



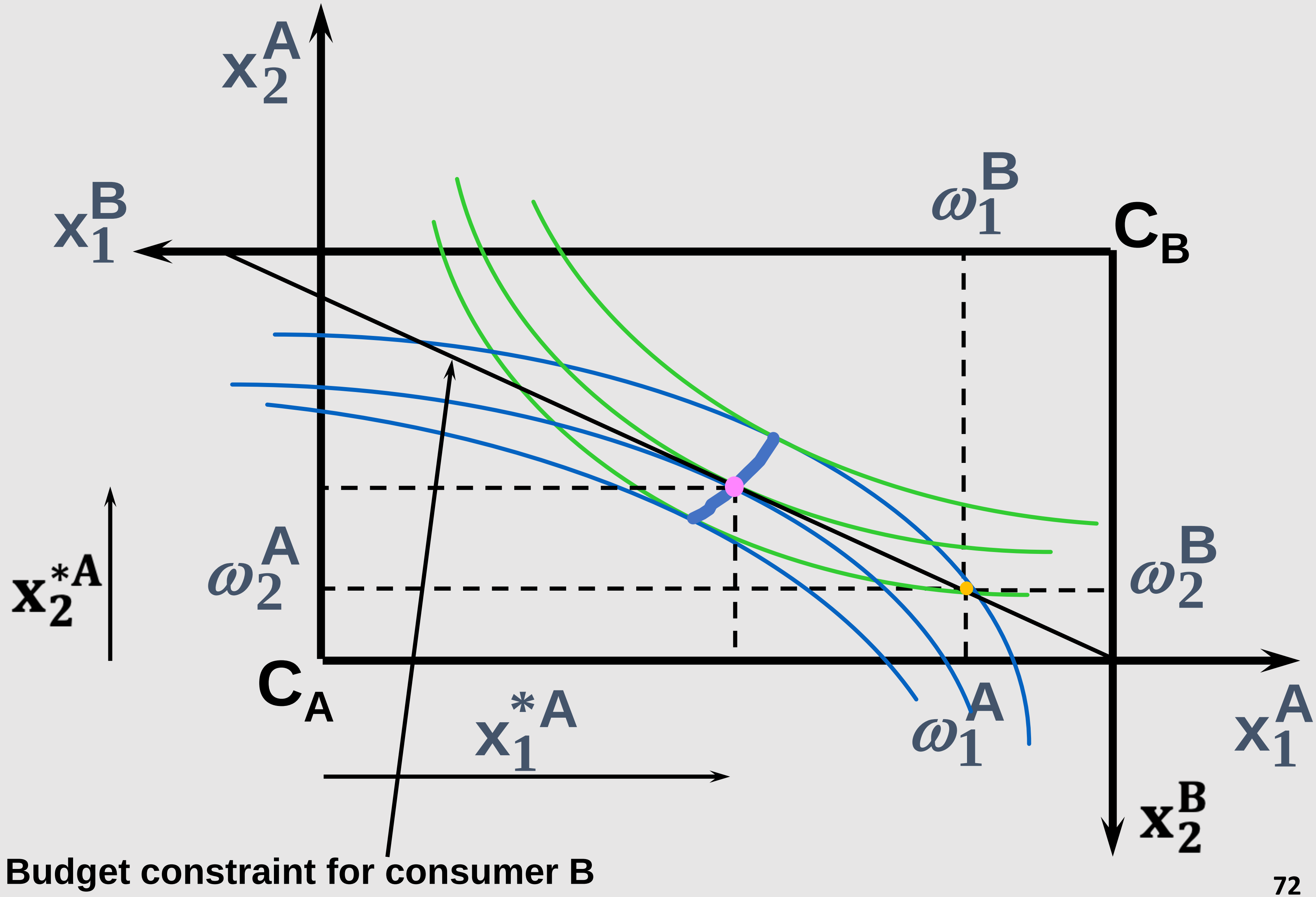
Trade in Competitive Markets



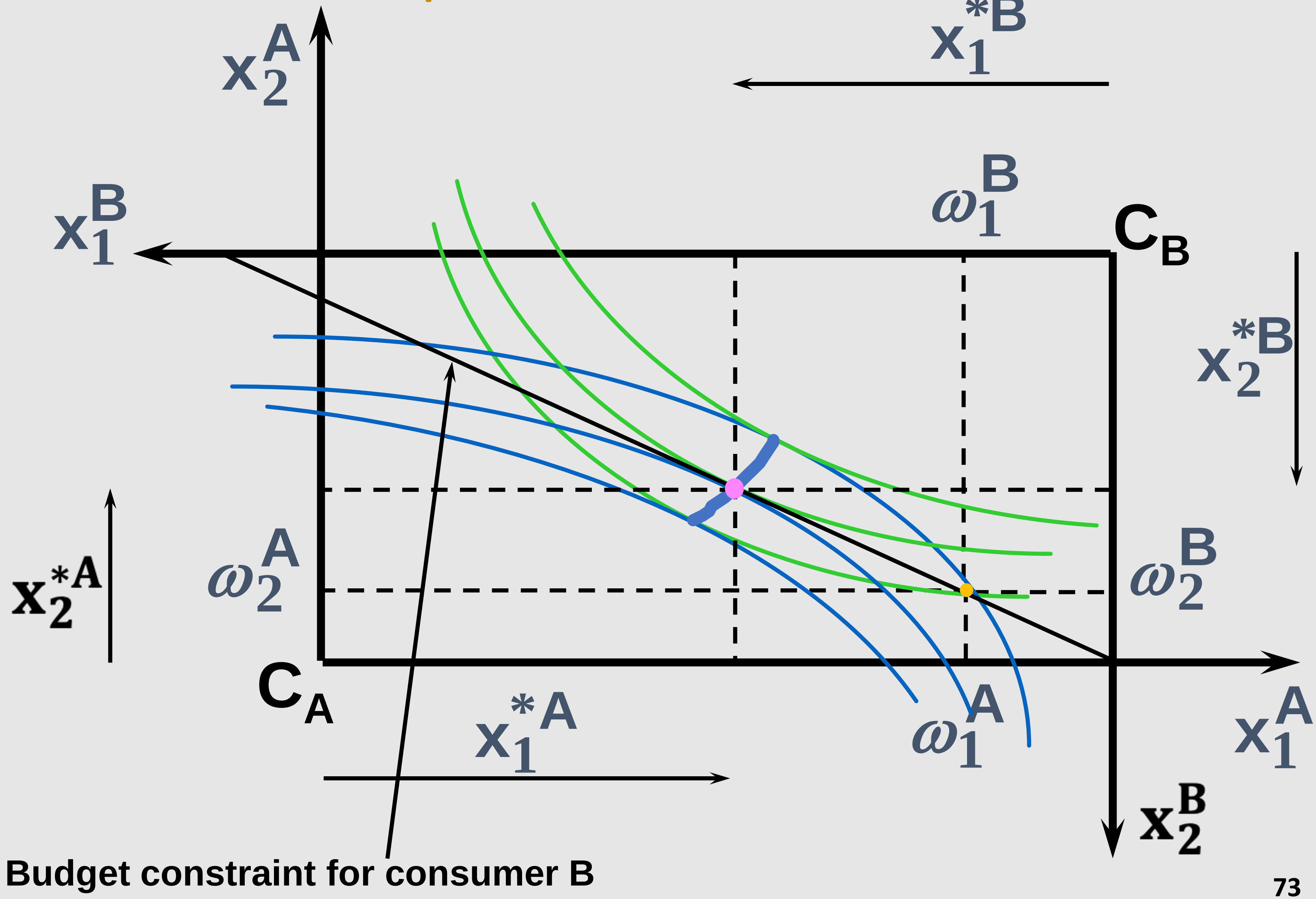
Trade in Competitive Markets



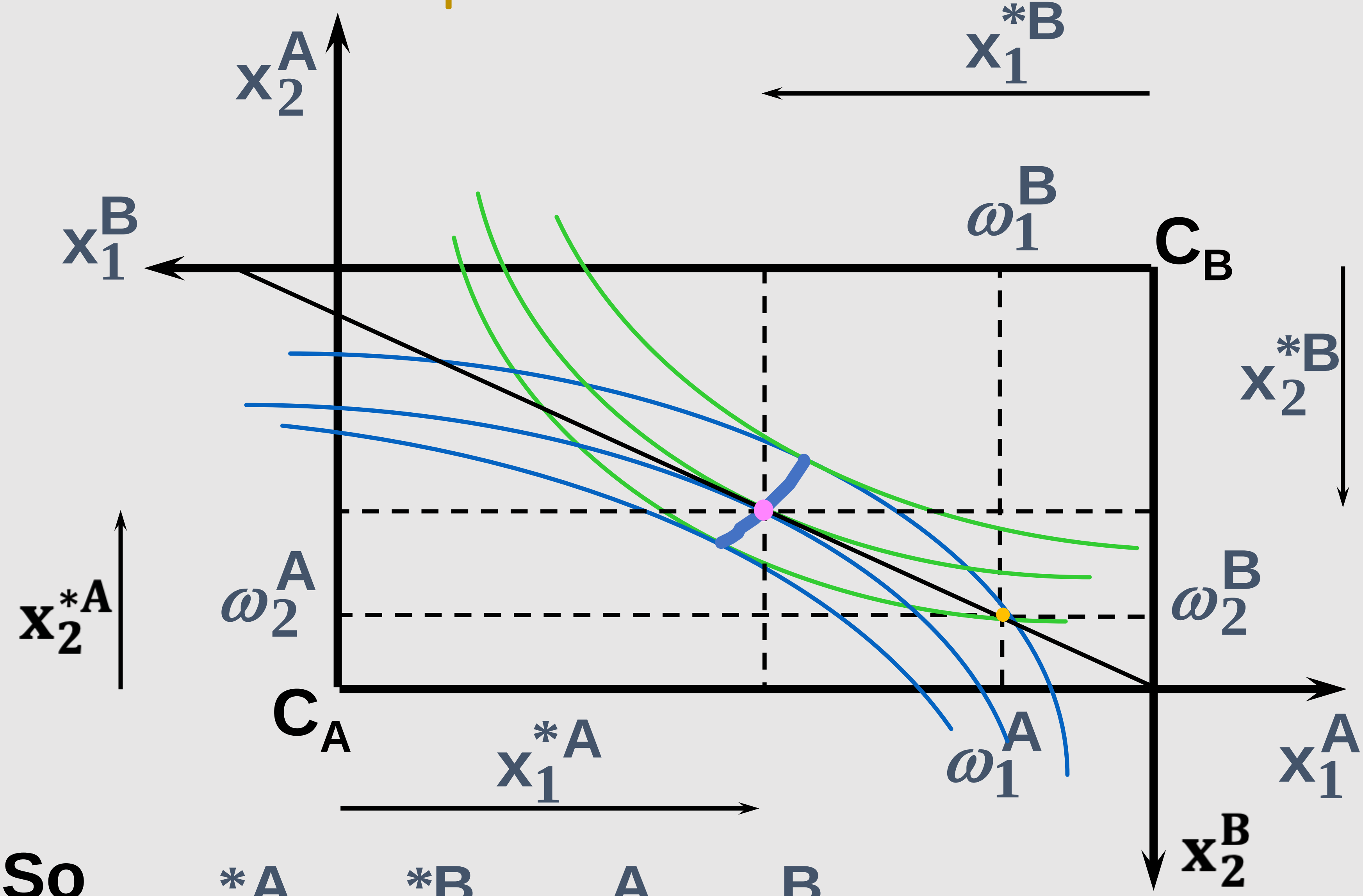
Trade in Competitive Markets



Trade in Competitive Markets

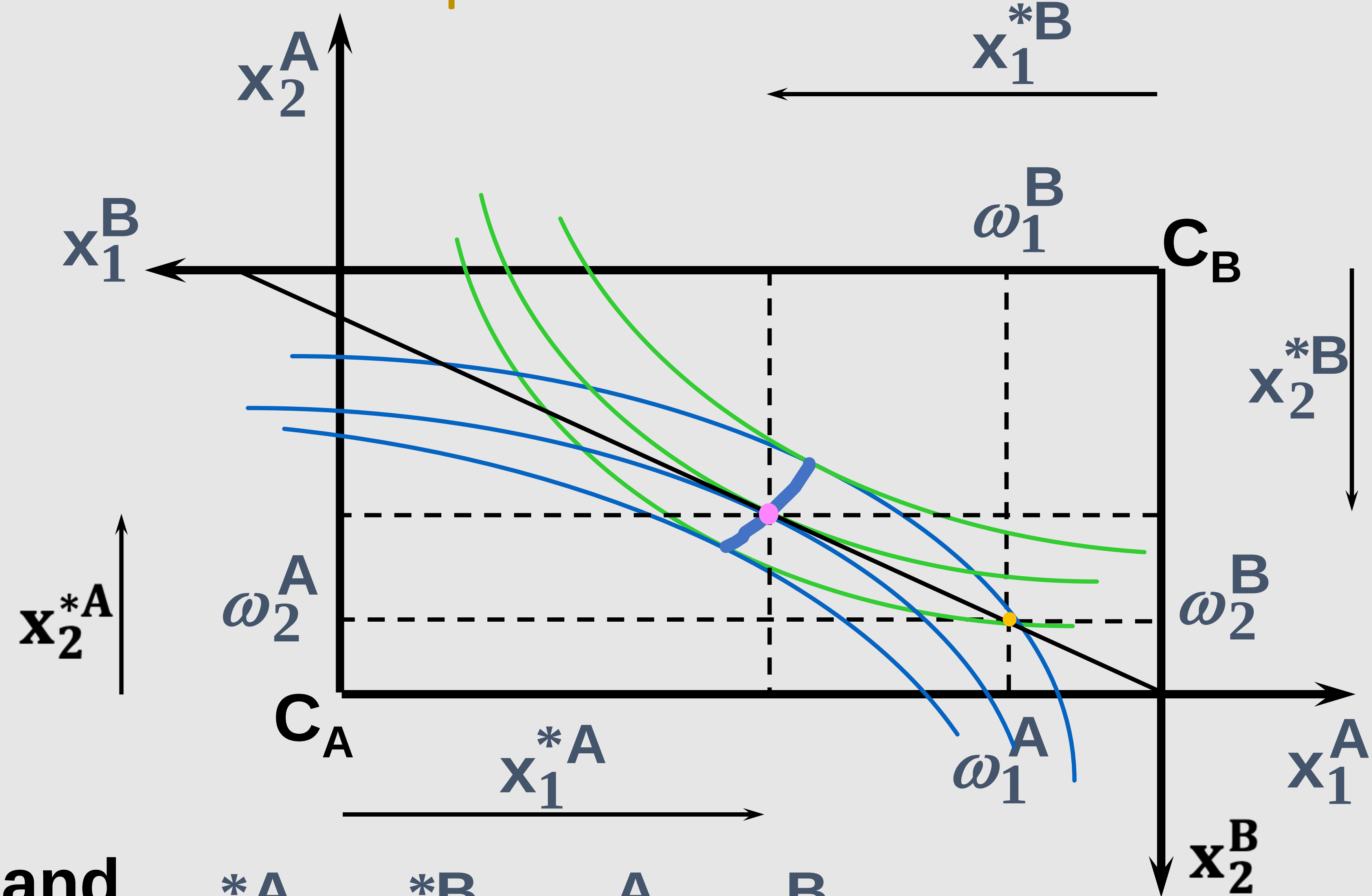


Trade in Competitive Markets



So $x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$

Trade in Competitive Markets



and $$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

5. Two Theorems of Welfare Economics

→ At the new prices p_1 and p_2 both markets clear; there is a **general equilibrium.**

→ Trading in competitive markets achieves a particular Pareto-optimal allocation of the endowments.

First Fundamental Theorem of Welfare Economics:

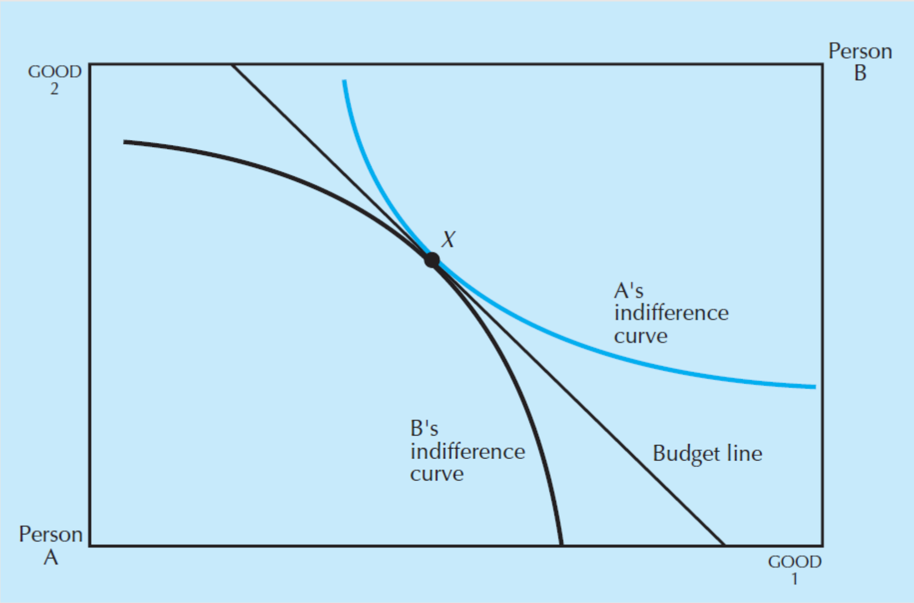
→ Trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy’s endowment.

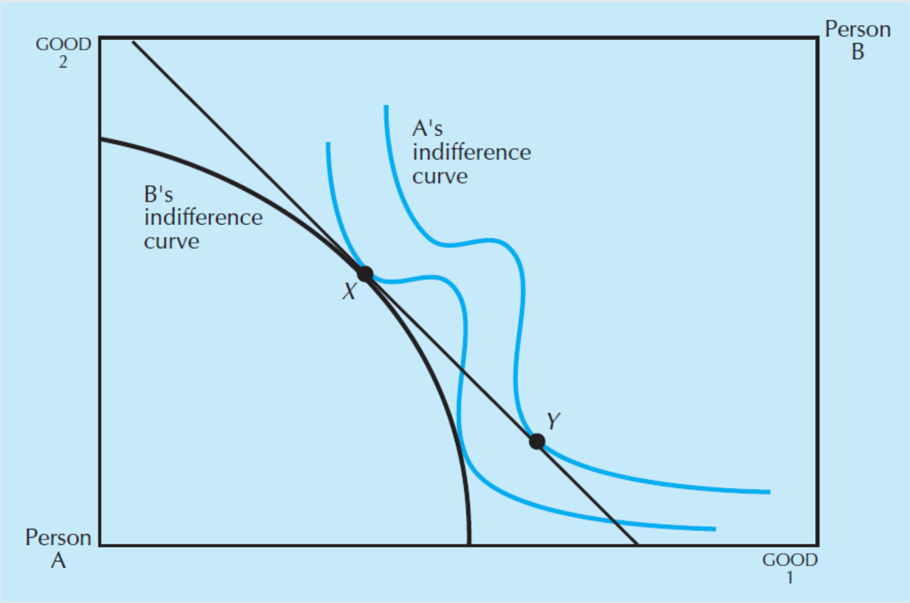
→ **All market equilibria are Pareto efficient**

– A competitive market will exhaust all gains from trade

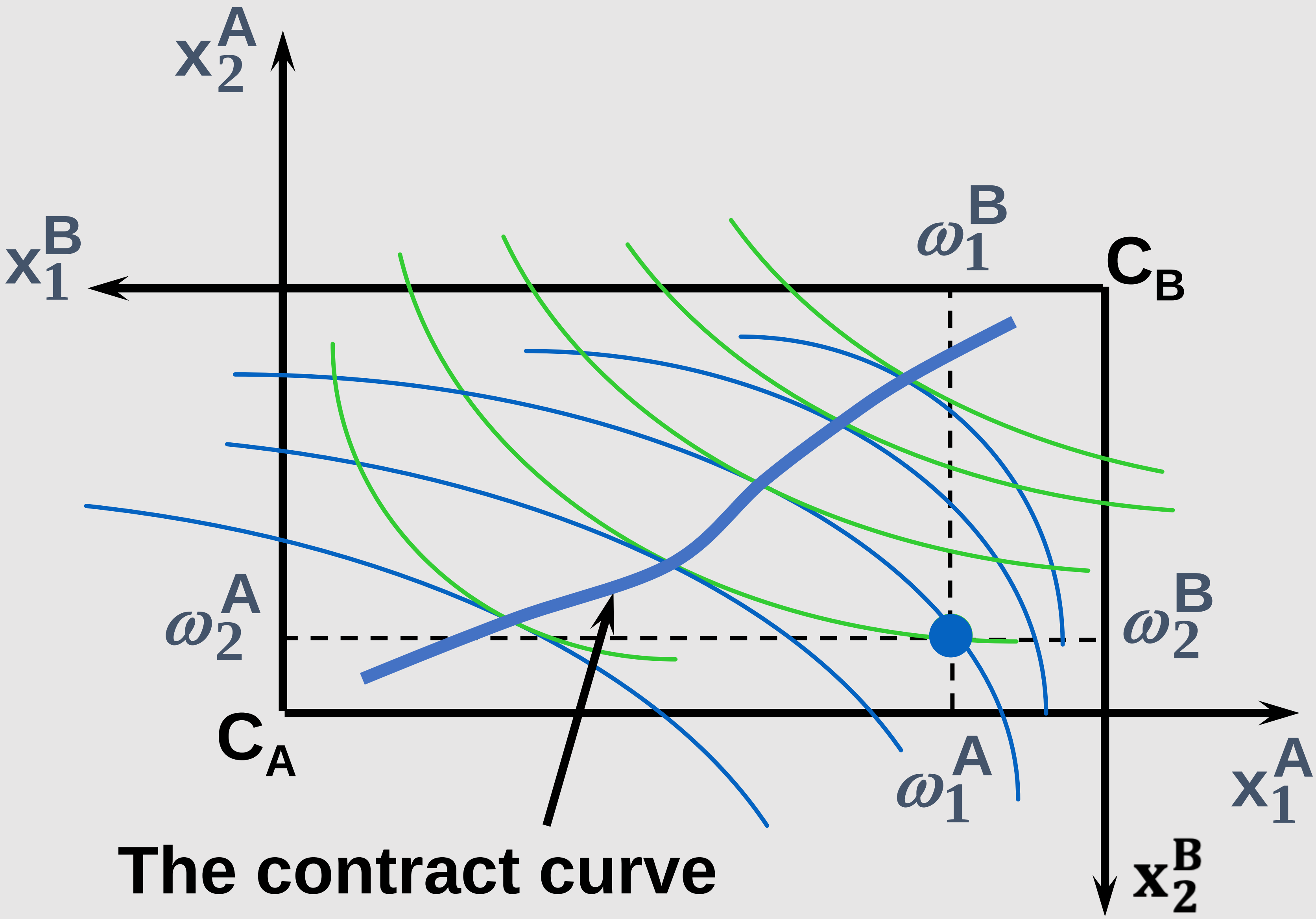
Second Fundamental Theorem of Welfare Economics

→ If consumers’ preferences are convex, for any Pareto-optimal allocation there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.

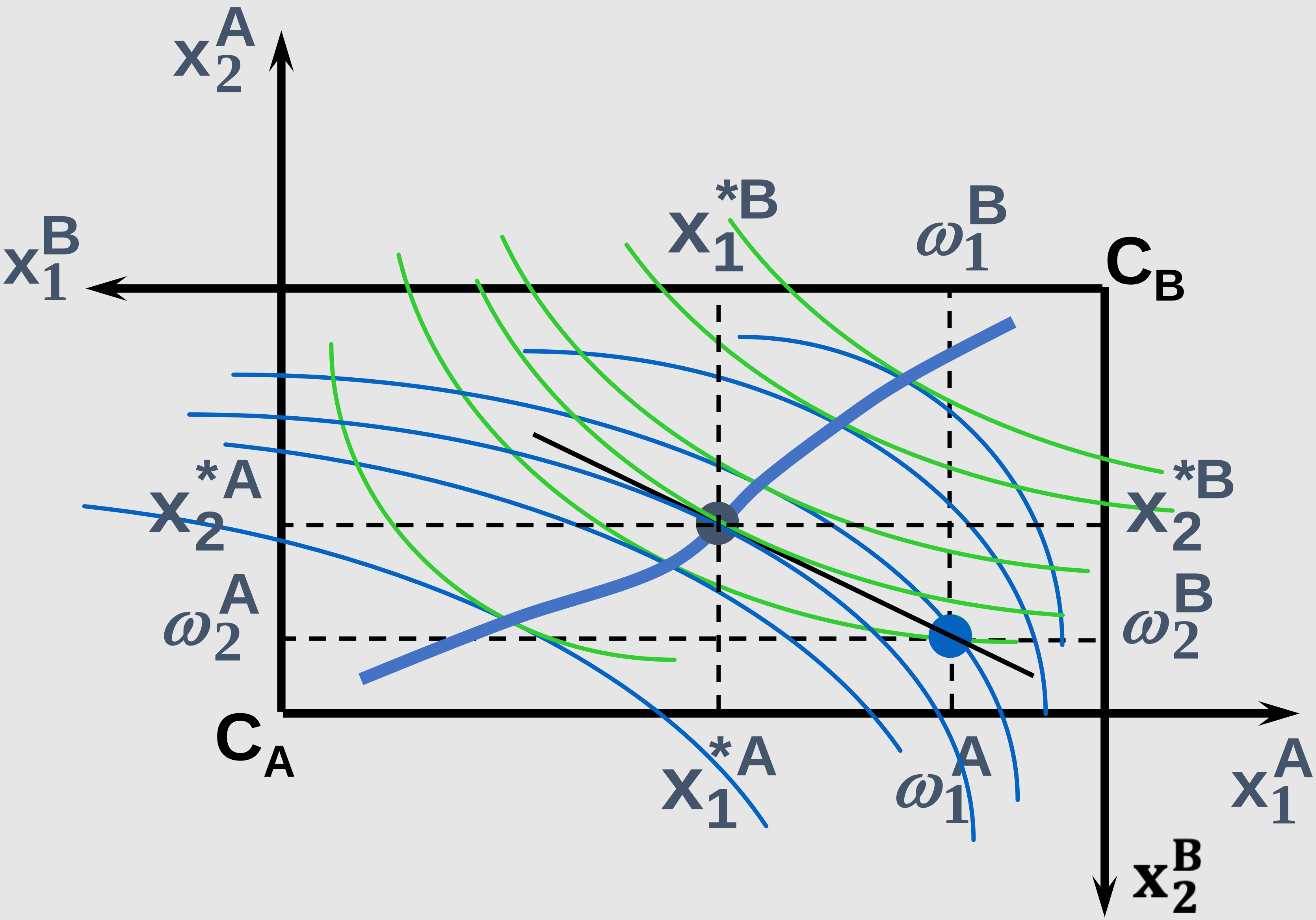




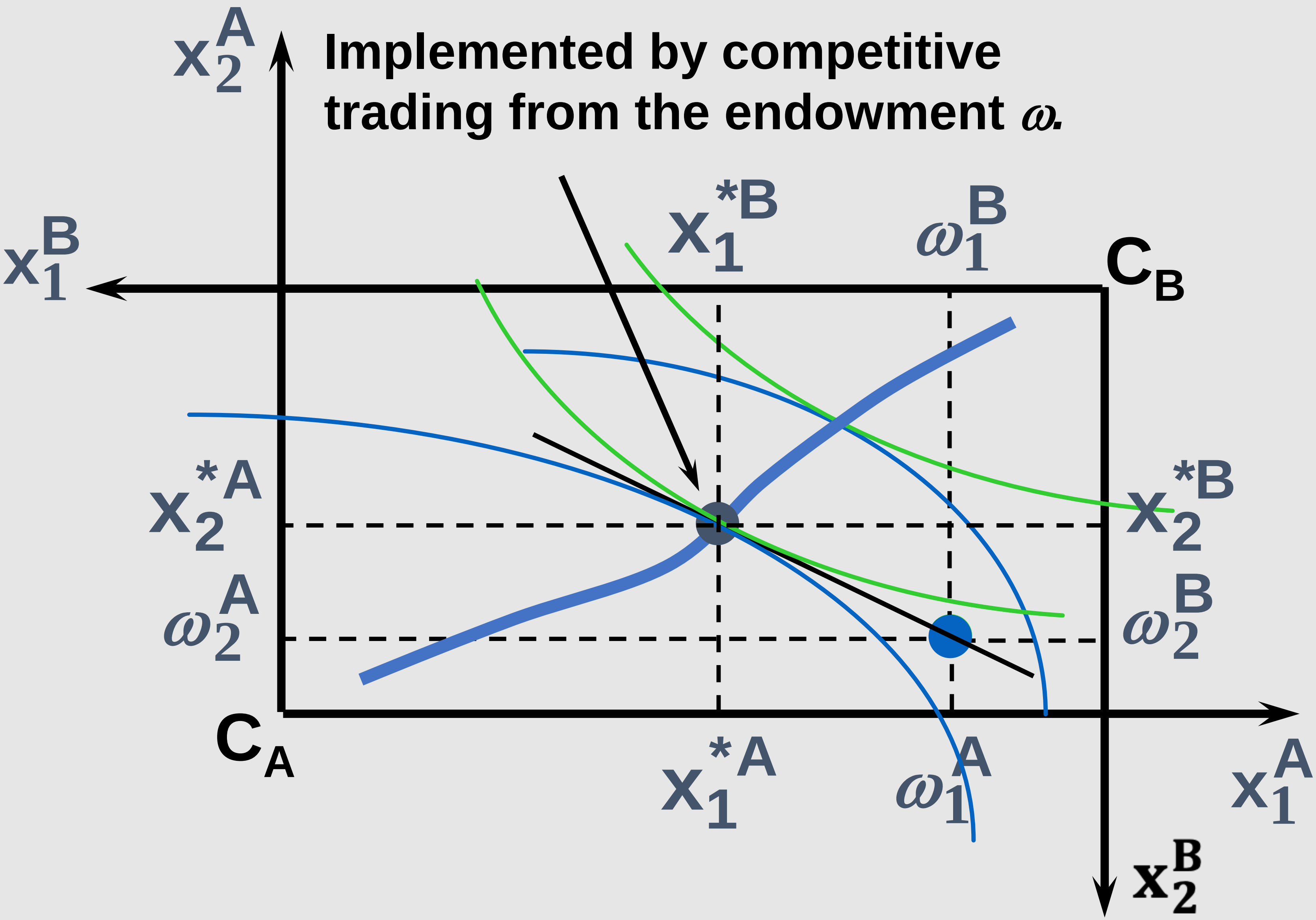
Second Fundamental Theorem



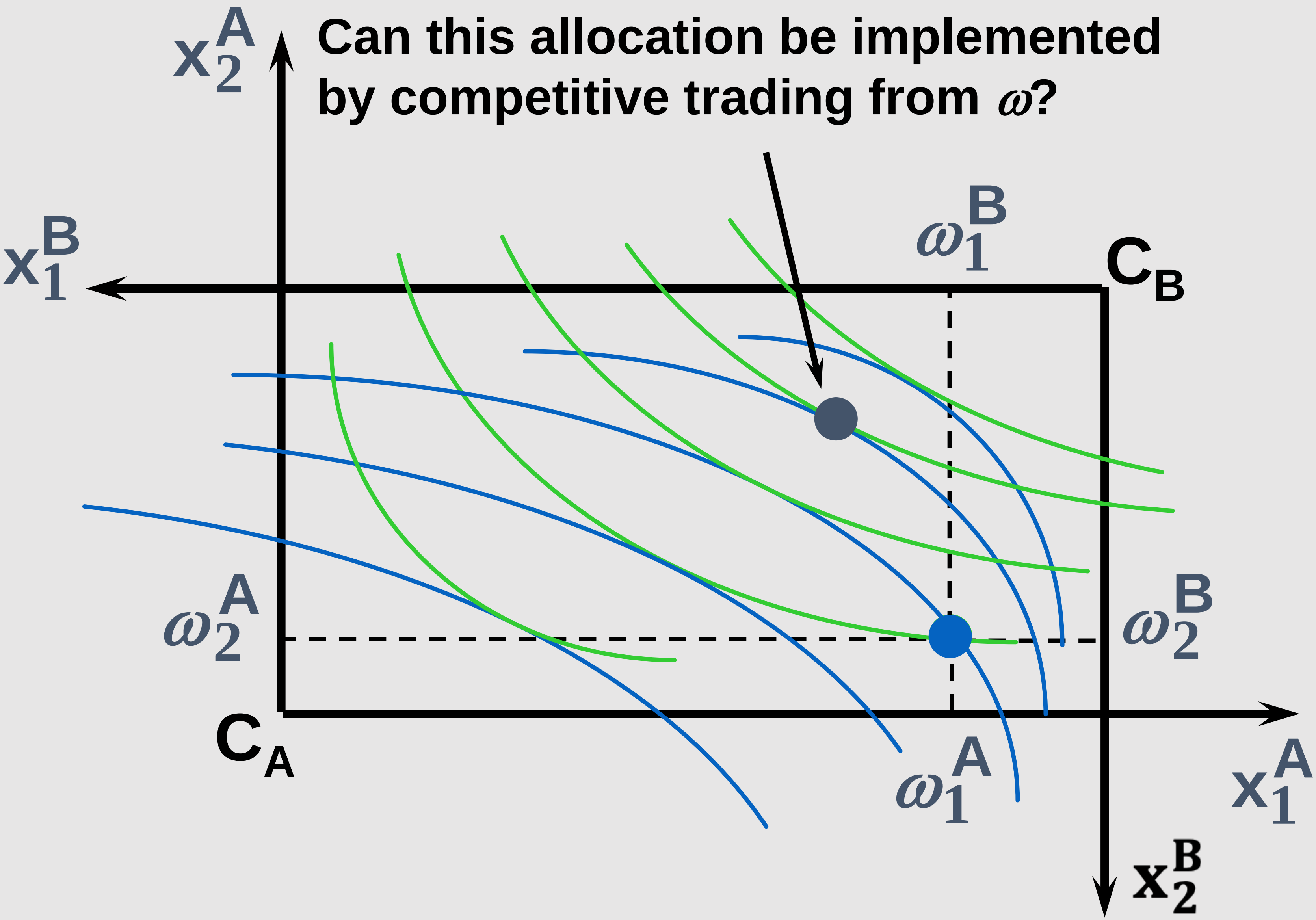
Second Fundamental Theorem



Second Fundamental Theorem

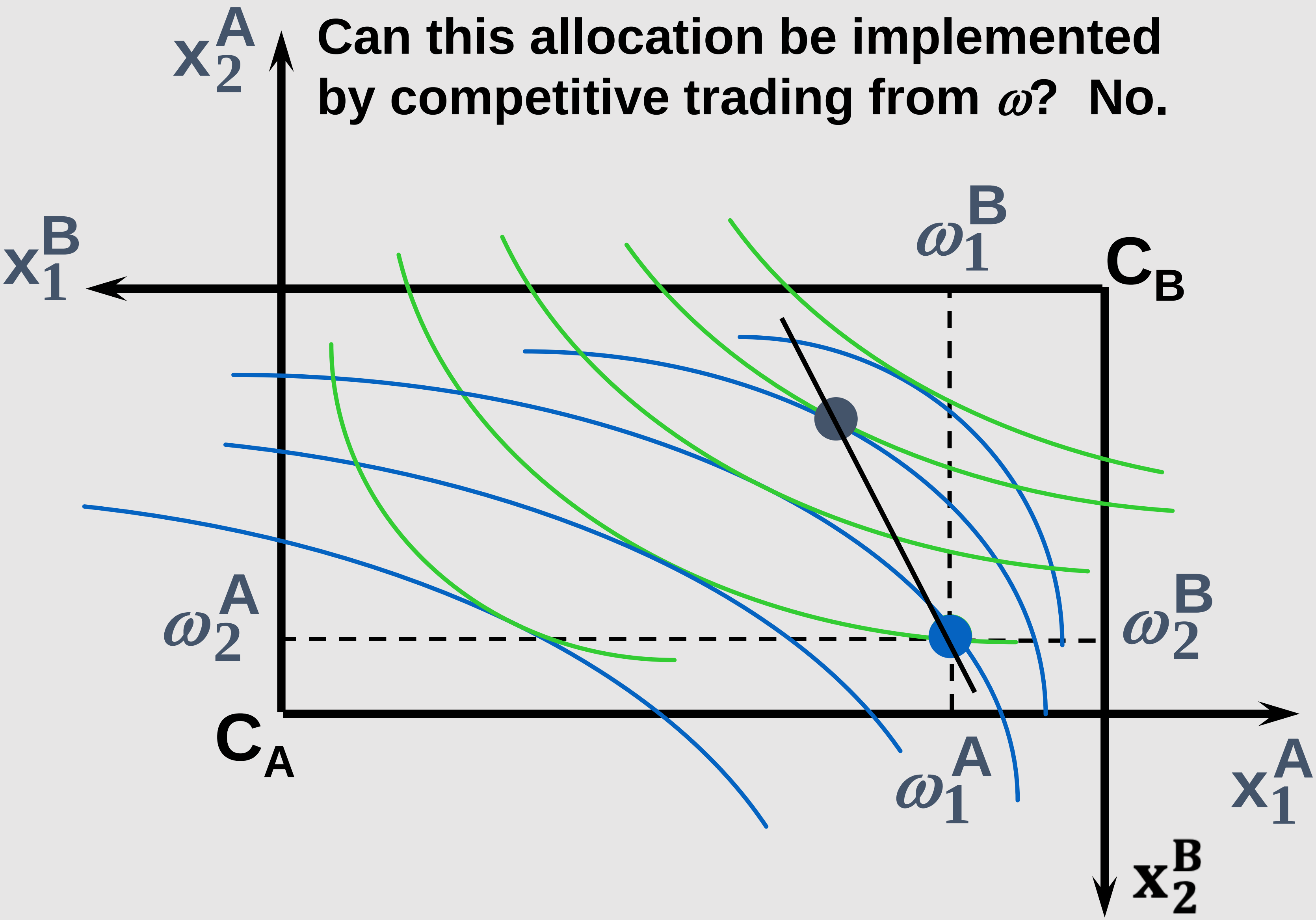


Second Fundamental Theorem

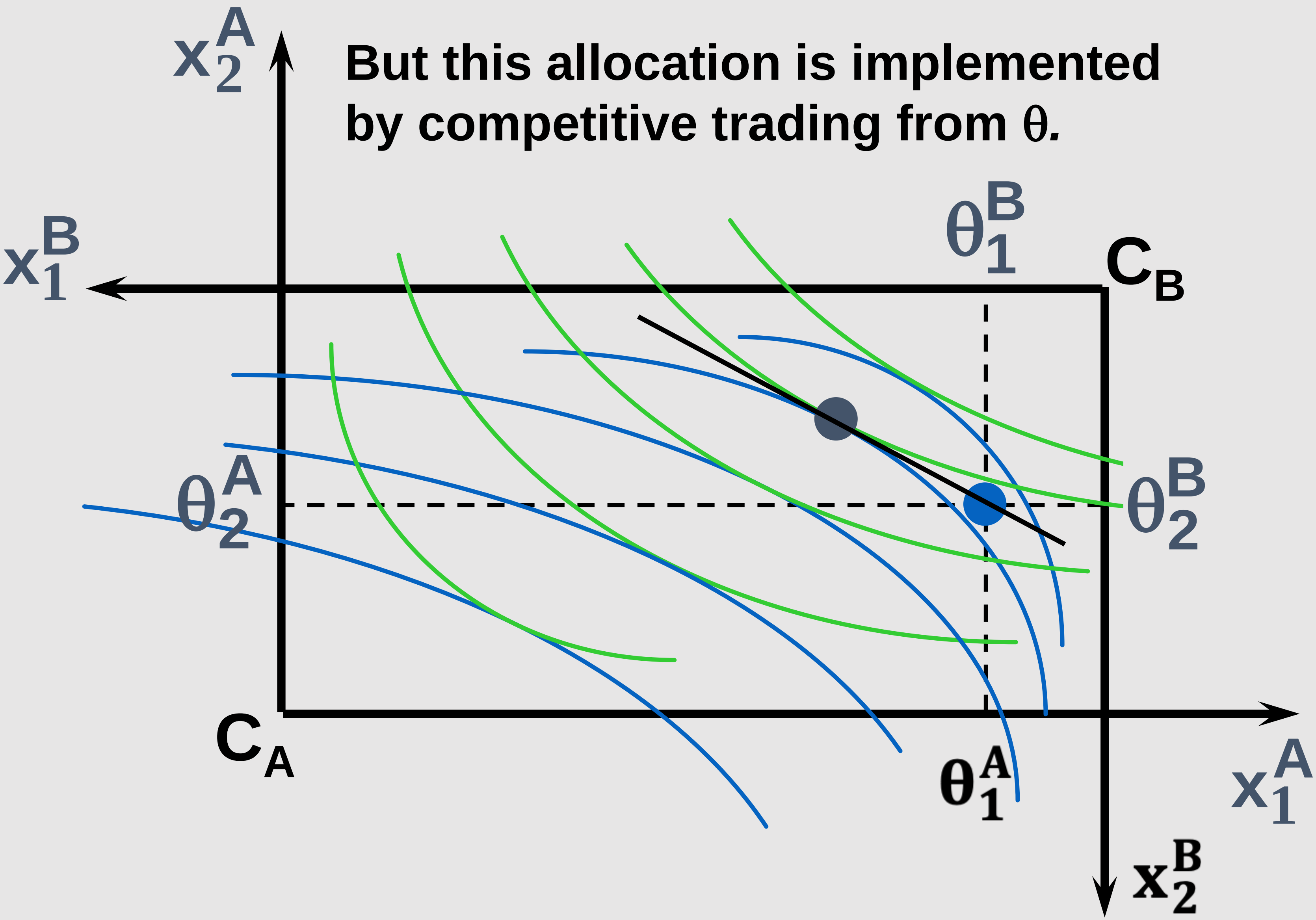


Second Fundamental Theorem

Can this allocation be implemented by competitive trading from ω? No.



Second Fundamental Theorem



6. Walras' Law

→ Walras' Law is an **identity**; i.e. a statement that is true for any positive prices (p_1,p_2), whether these are equilibrium prices or not.

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) = 0$$

→ Meaning: the value of aggregate net demand (z_1 and z_2 is the net demand of both consumers A and B) is identically zero for all possible choices of prices, not just equilibrium prices

Proof

→ Every consumer's preferences are well-behaved (convex) so, for any positive prices (p_1,p_2), each consumer spends all of his budget.

→ For consumer A: $p_1 x_A^1(p_1, p_2) + p_2 x_A^2(p_1, p_2) = p_1\, \omega_A^1 + p_2 \omega_A^2$

→ Let's rewrite as: $p_1[x_A^1(p_1, p_2) - \omega_A^1] + p_2[x_A^2(p_1, p_2) - \omega_A^2] = 0$

6. Walras' Law

→ This equation says that the value of agent A's net demand is zero. Since

$e_A^1 = x_A^1(p_1, p_2) - \omega_A^1$ we can rewrite

$$p_1 e_A^1(p_1, p_2) + p_2 e_A^2(p_1, p_2) = 0$$

→ The same is true for consumer B, we have that

$$p_1 e_B^1(p_1, p_2) + p_2 e_B^2(p_1, p_2) = 0$$

→ Adding the equations for agent A and agent B together and using the definition of aggregate net demand, $z_1 = e_A^1 + e_B^1$ and $z_2 = e_A^2 + e_B^2$, we have

$$p_1[e_A^1(p_1, p_2) + e_B^1(p_1, p_2)] + p_2[e_A^2(p_1, p_2) + e_B^2(p_1, p_2)] = 0$$

→ Then

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) = 0$$

Implications of Walras' Law

1. The value of how much A wants to buy of good 1 plus the value of how much she wants to buy of good 2 must equal zero.
   - (Of course the amount that she wants to buy of *one* of the goods must be negative—that is, she intends to sell some of one of the goods to buy more of the other.)
2. If demand equals supply in one market, demand must also equal supply in the other market.