

# Chapter 2: Exchange with production

Ch 32 in H. Varian 8<sup>th</sup> Ed.

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Inspired by [Michael D. Robinson](#), Mount Holyoke College

# Exchange Economies (revisited)

So far...

- No production, only endowments, so no description of how resources are converted to consumables.
- General equilibrium: all markets clear simultaneously.
- 1st and 2nd Fundamental Theorems of Welfare Economics.

Now

- Add input markets, output markets, describe firms' technologies, the distributions of firms' outputs and profits ... **That's not easy!**

# Outline

1. Robinson Crusoe's Economy
2. Robinson Crusoe as a Firm
3. Fundamental Theorems of Welfare Economics
4. Production Possibilities
5. Comparative Advantage

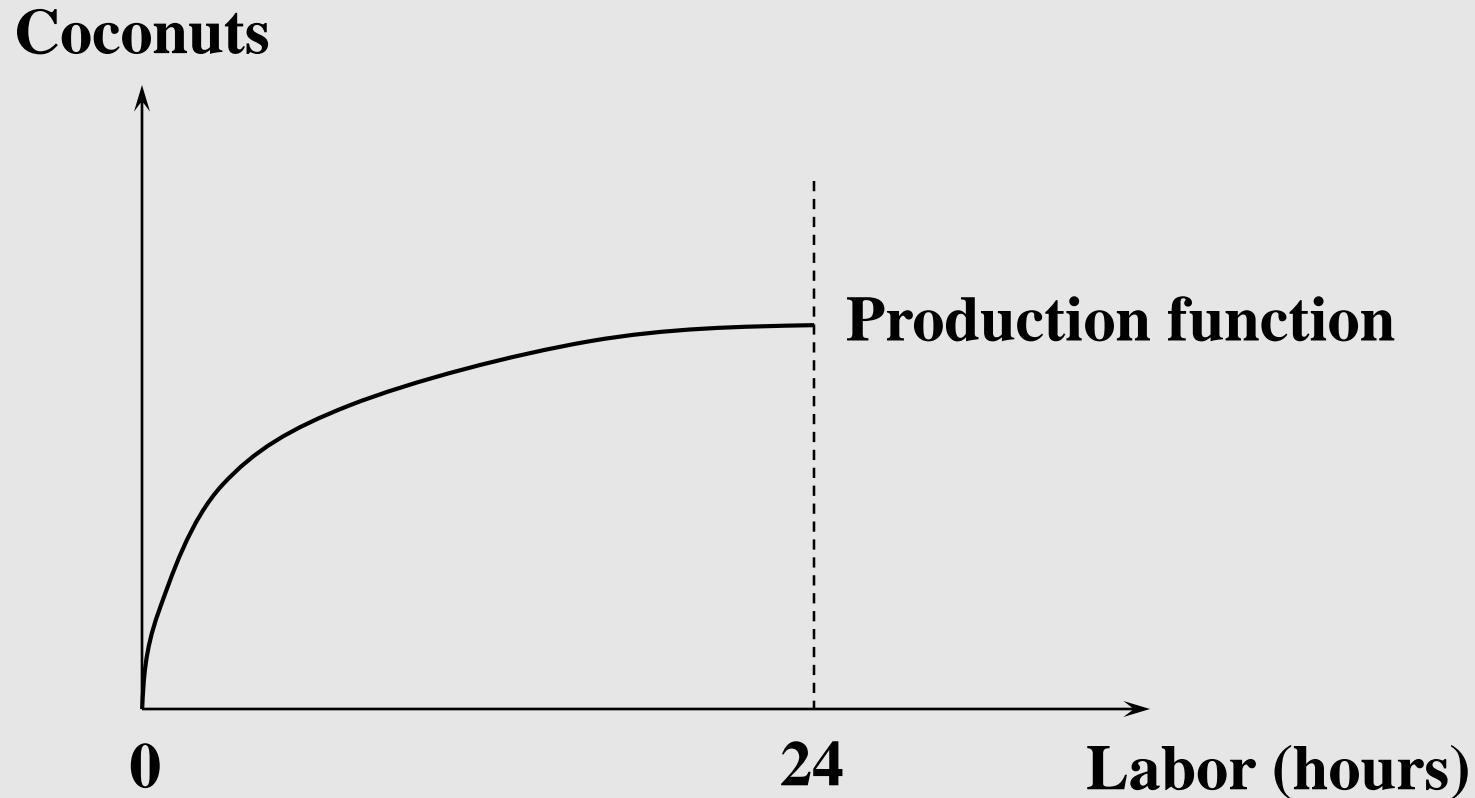
# 1. Robinson Crusoe's Economy

- One agent, RC.
- Endowed with a fixed quantity of one resource → 24 hours.
- Use time for labor (production) or leisure (consumption).
- Labor time =  $L$ . Leisure time =  $24 - L$ .
- What will RC choose?

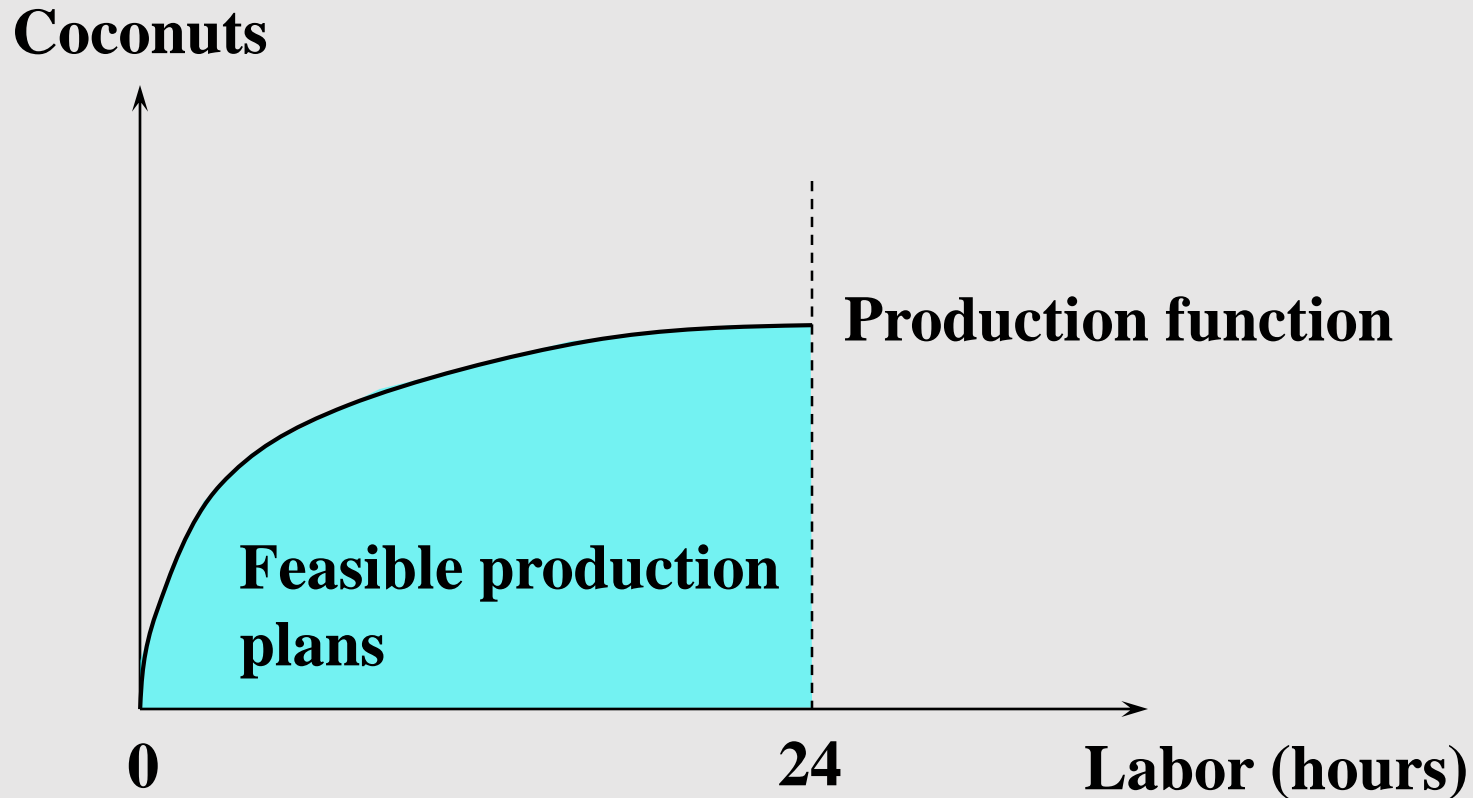
# Robinson Crusoe's Technology

- Technology: Labor produces output (coconuts) according to a concave production function.

# Robinson Crusoe's Technology



# Robinson Crusoe's Technology



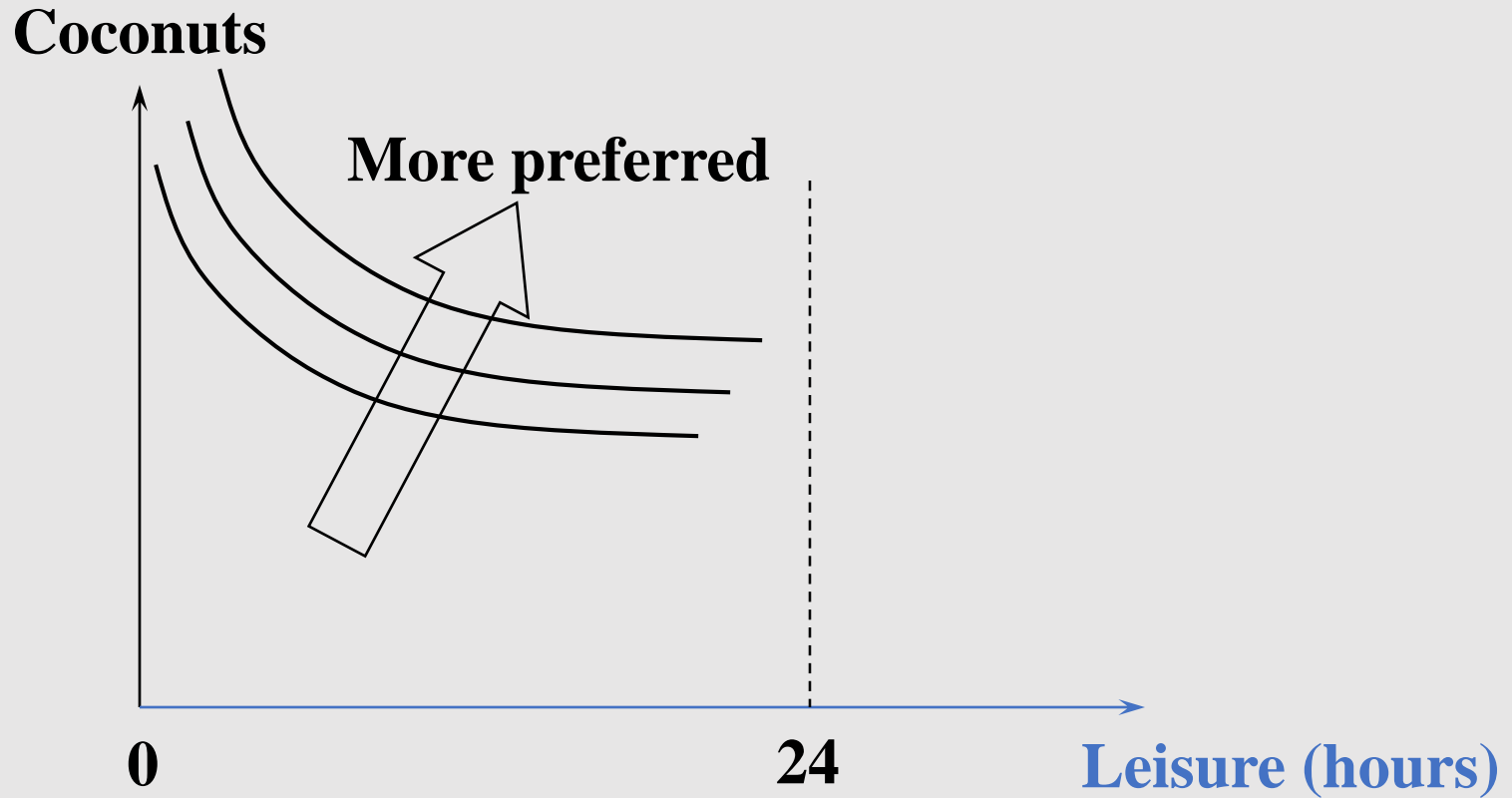
# Robinson Crusoe's Preferences

→ RC's preferences:

- coconut is a good
- leisure is a good

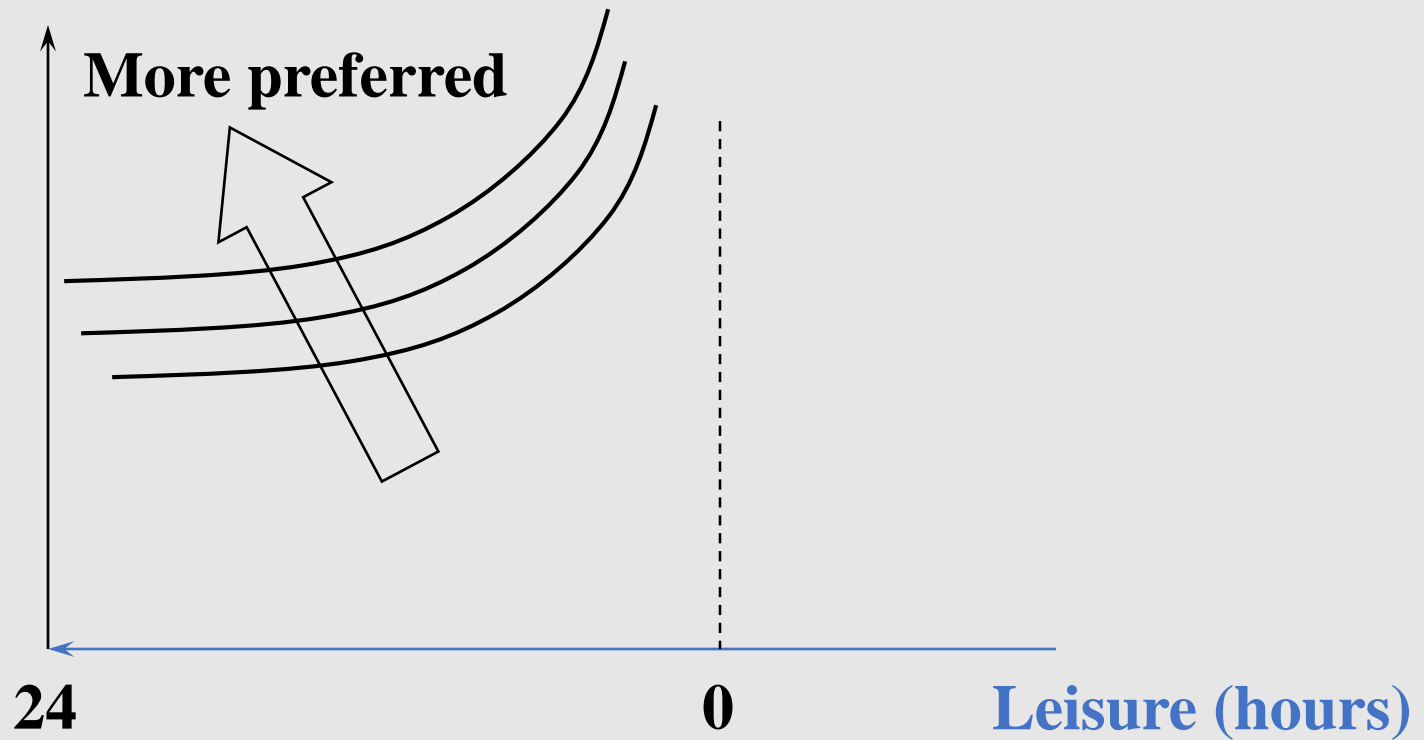


# Robinson Crusoe's Preferences

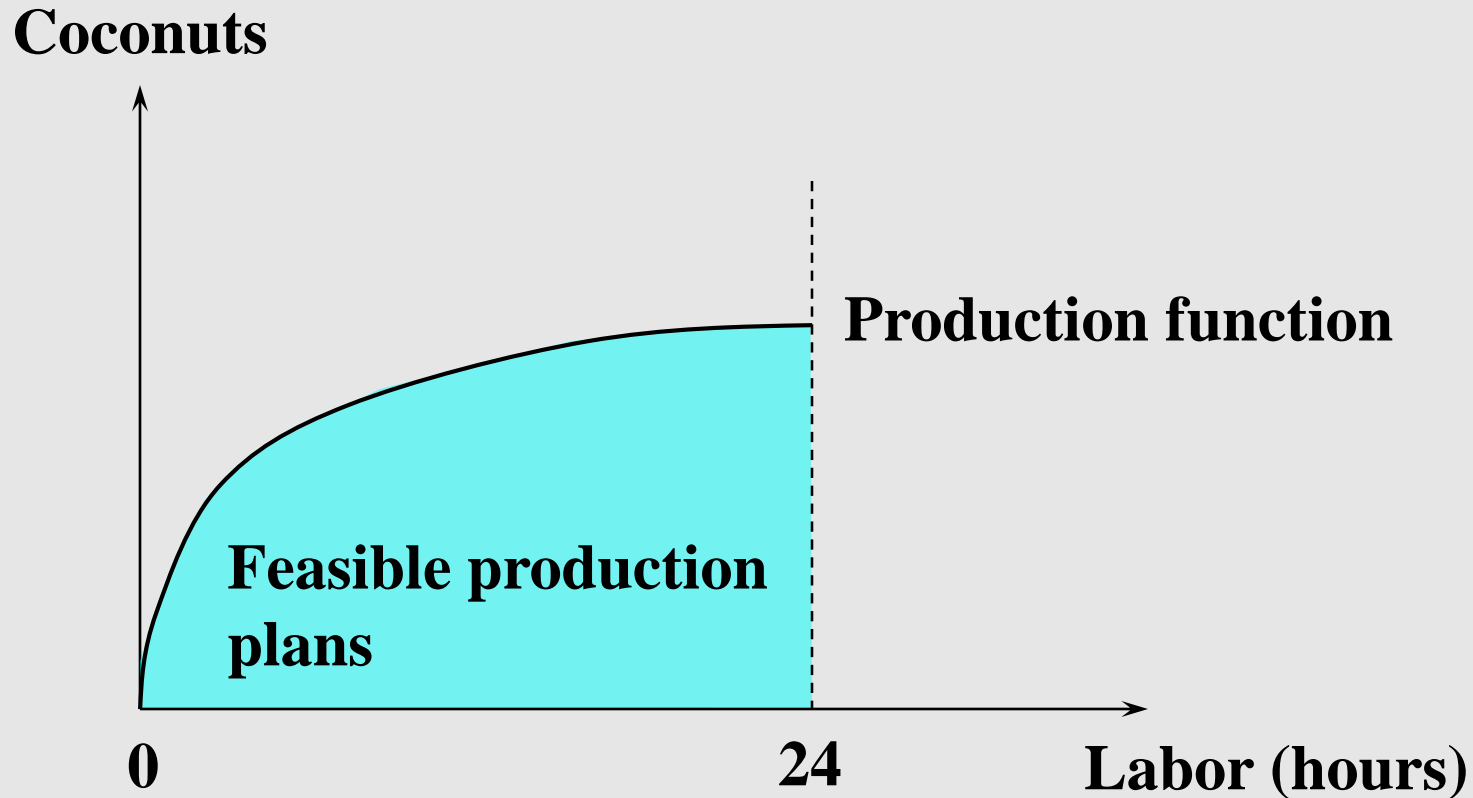


# Robinson Crusoe's Preferences

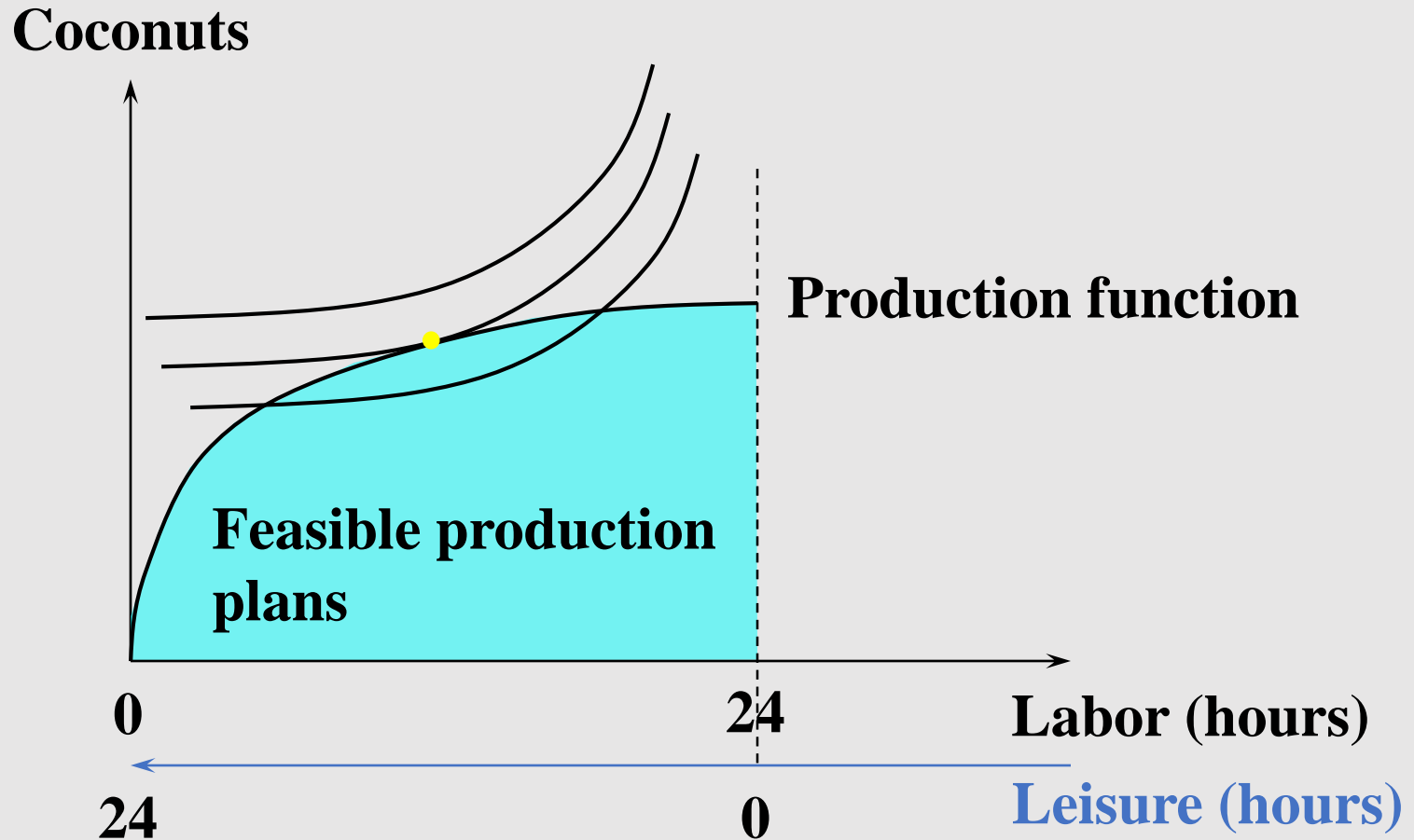
**Coconuts**



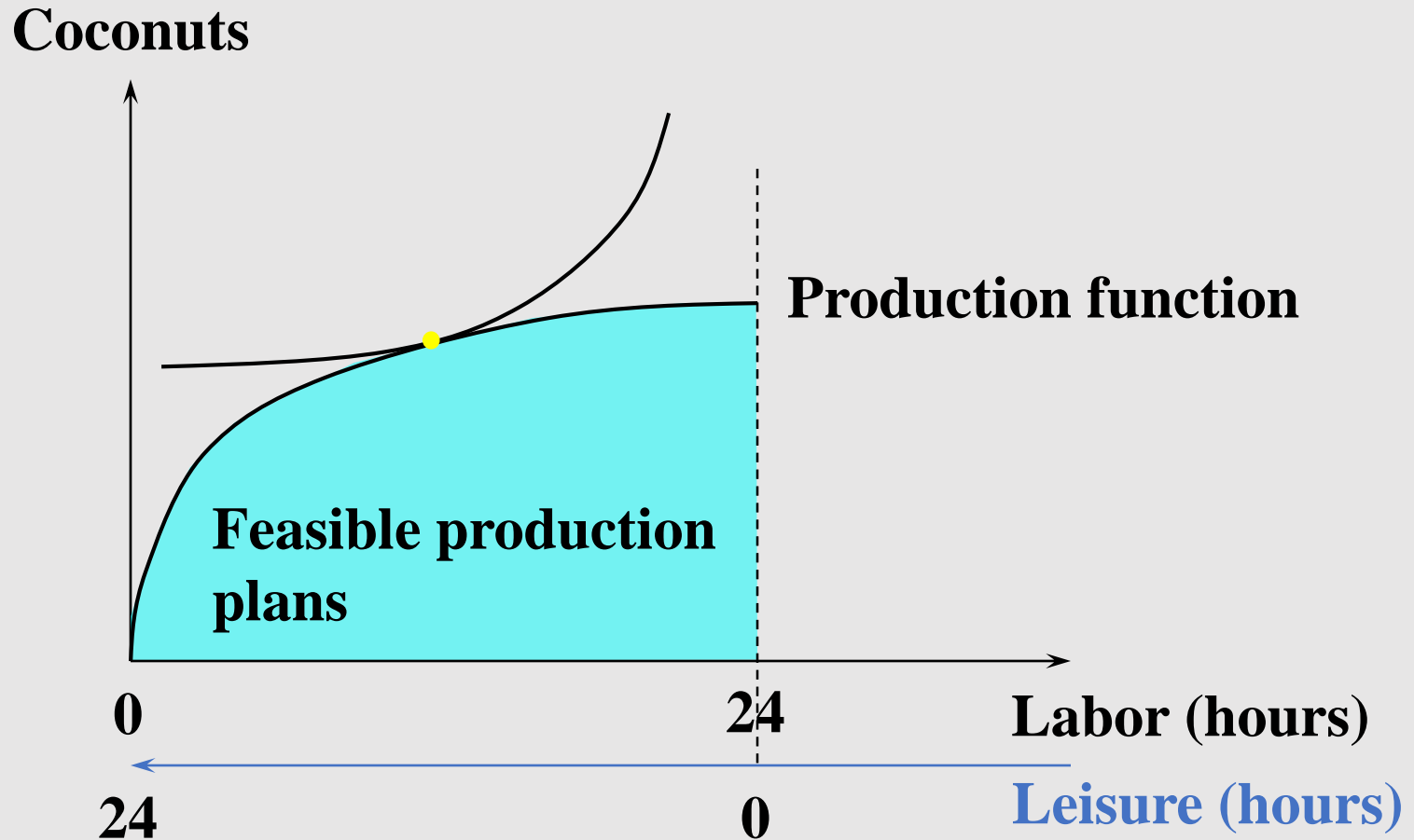
# Robinson Crusoe's Choice



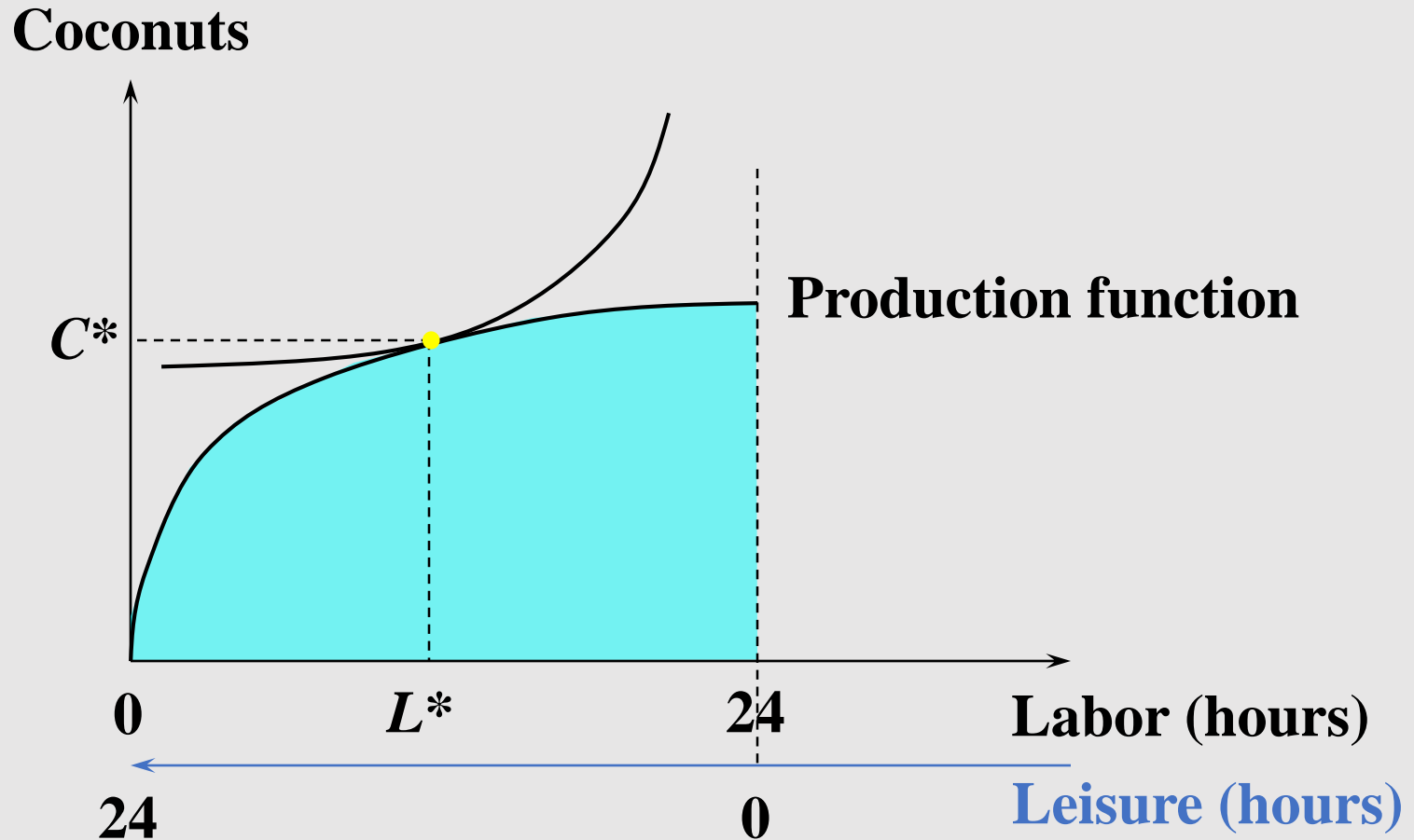
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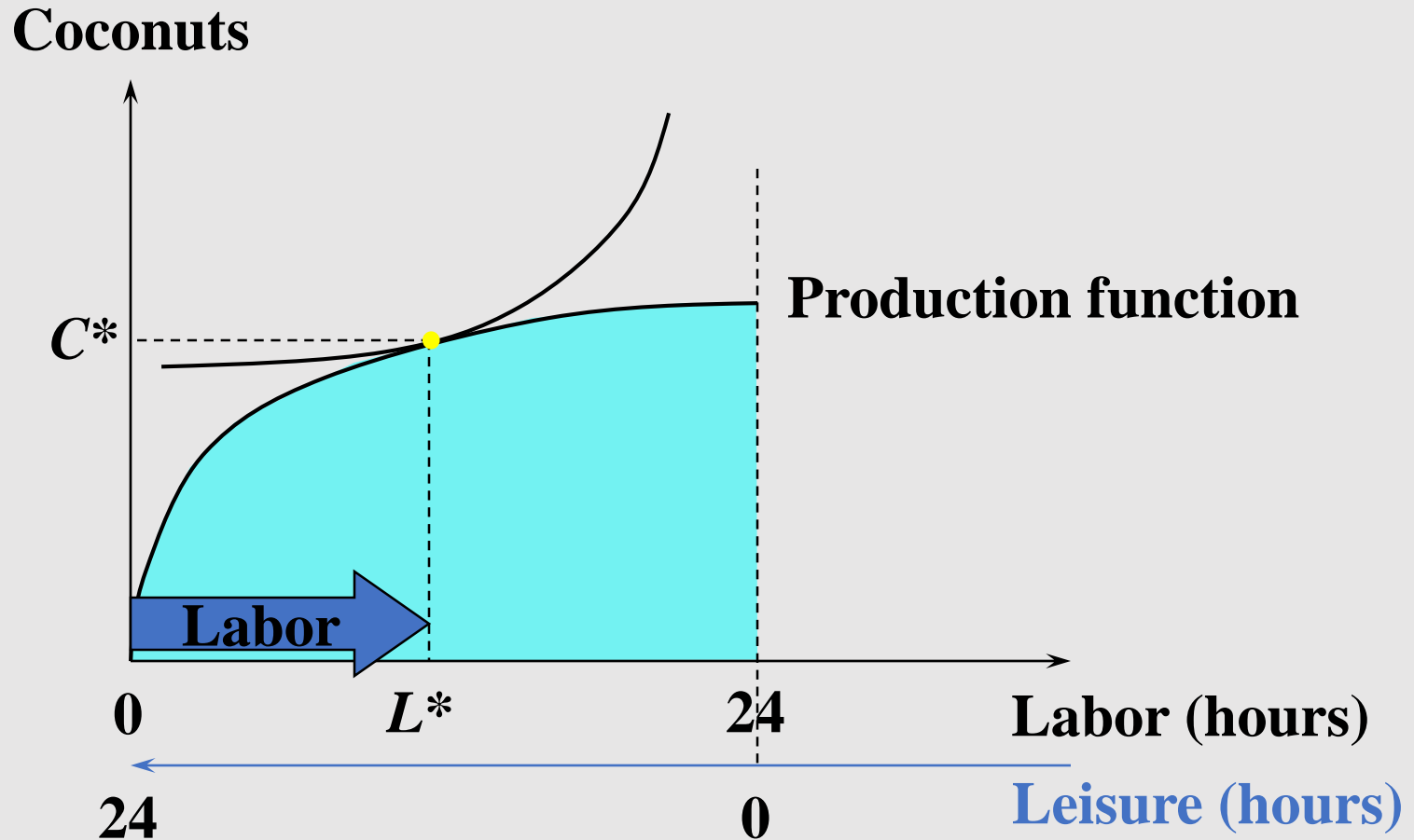
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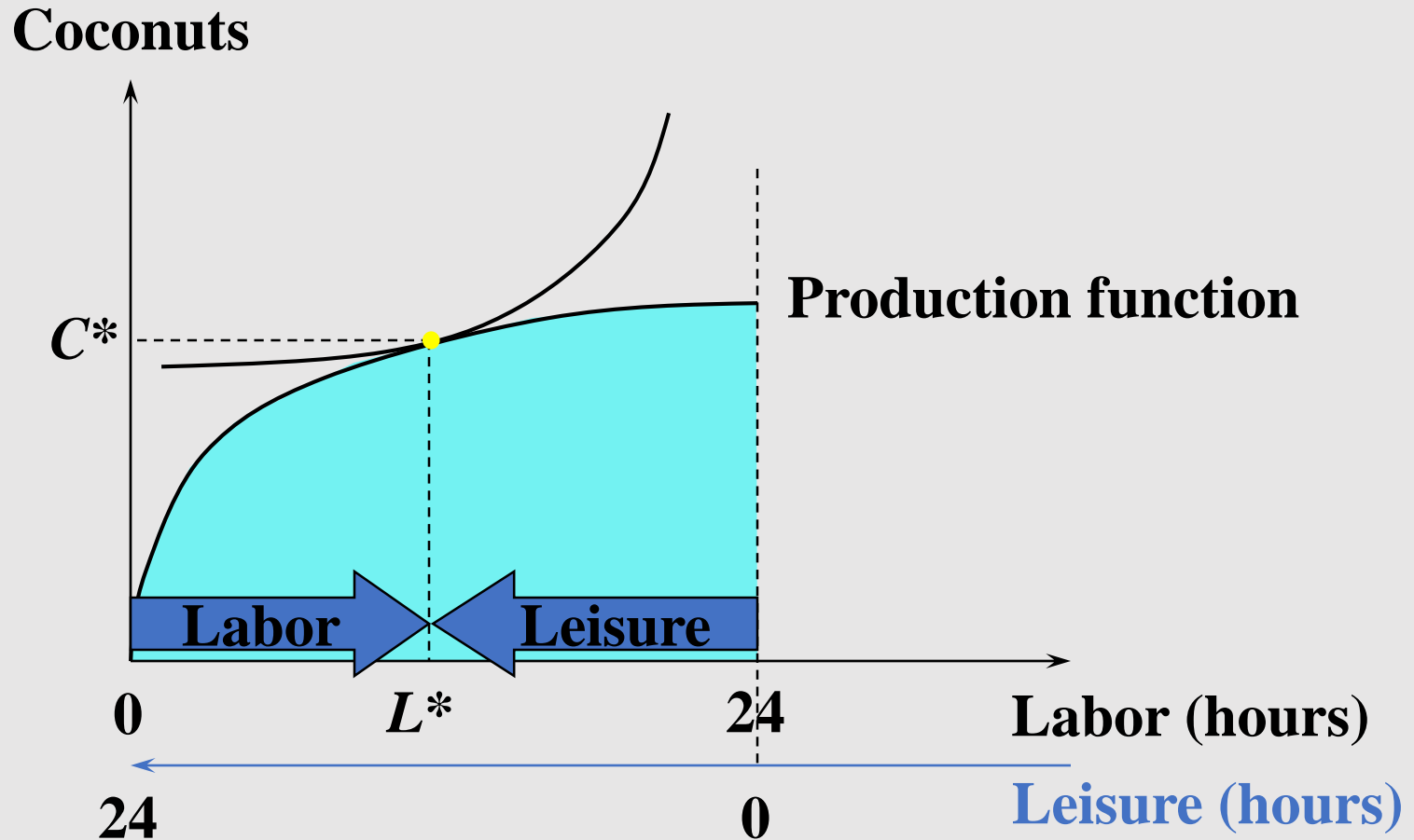
# Robinson Crusoe's Choice



# Robinson Crusoe's Choice

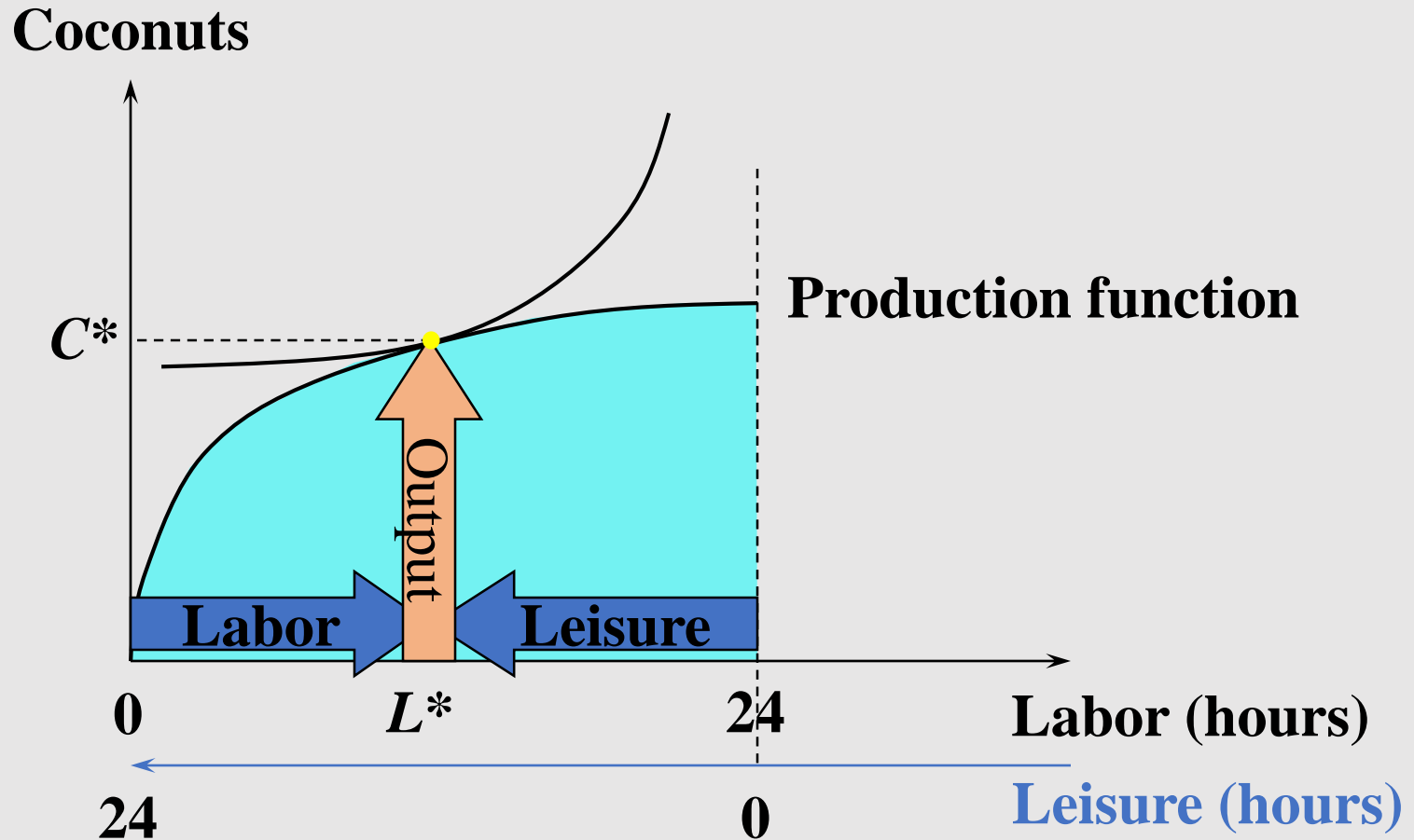


# Robinson Crusoe's Choice

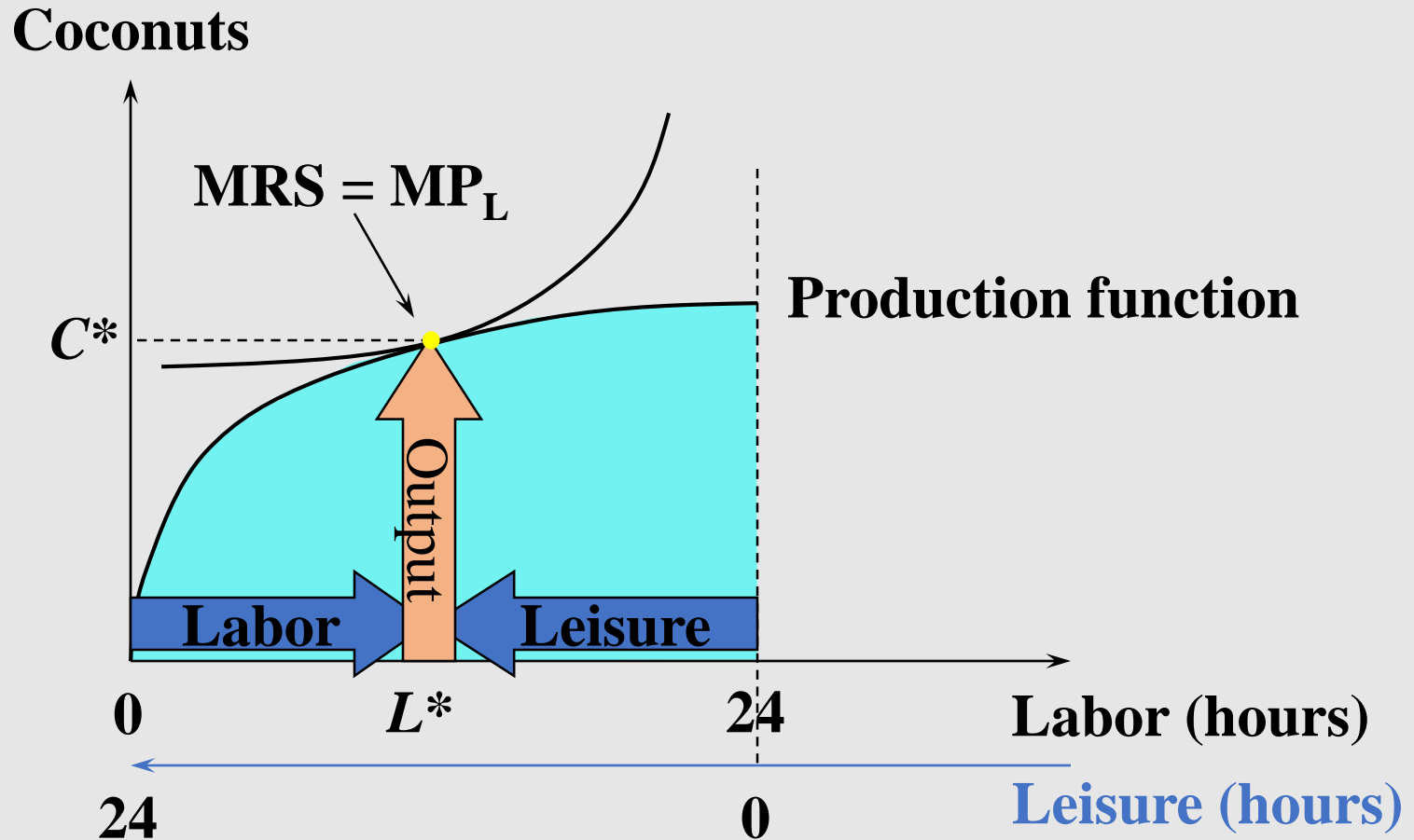




# Robinson Crusoe's Choice



# Robinson Crusoe's Choice



## 2. Robinson Crusoe as a Firm

- Now suppose RC is both a utility-maximizing consumer and a profit-maximizing firm.
- Use coconuts as the numeraire good; i.e. price of a coconut = \$1.
- RC's wage rate is  $w$ .
- Coconut output level is  $C$ .

# Robinson Crusoe as a Firm

→ RC's firm's profit is  $\pi = C - wL$ .

→  $\pi = C - wL \Leftrightarrow C = \pi + wL$ , the equation of an isoprofit line.

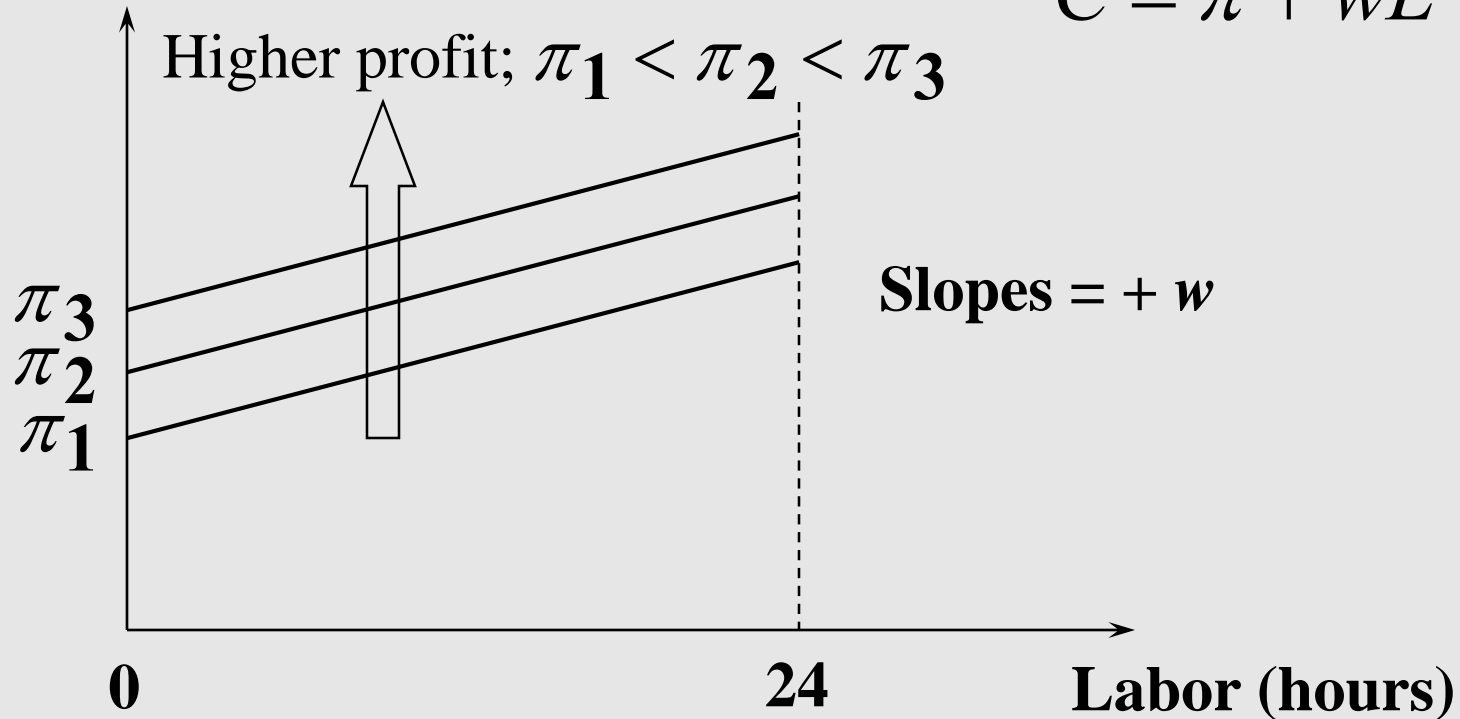
→ Slope =  $+ w$ .

→ Intercept =  $\pi$ .

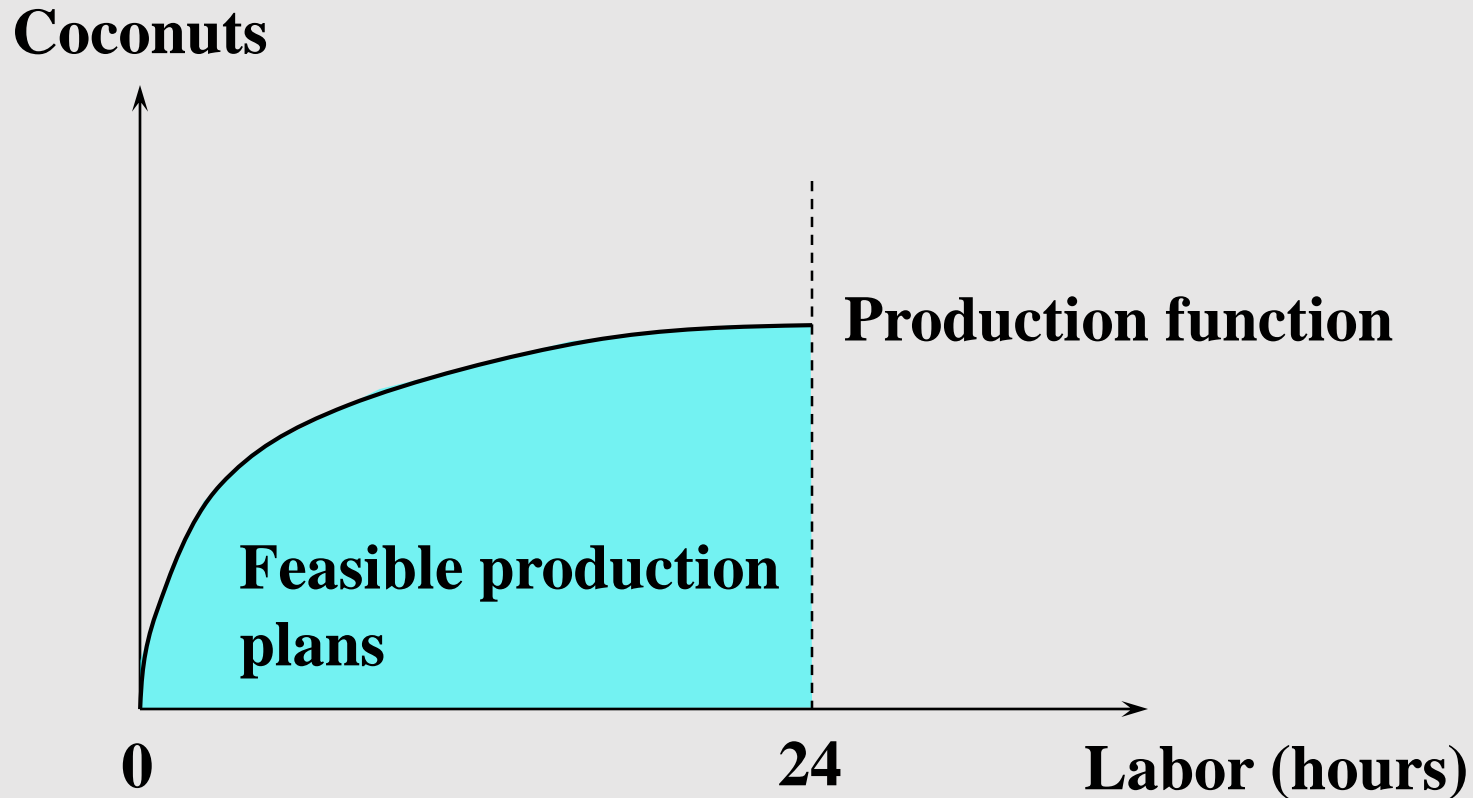
# Isoprofit Lines

Coconuts

$$C = \pi + wL$$

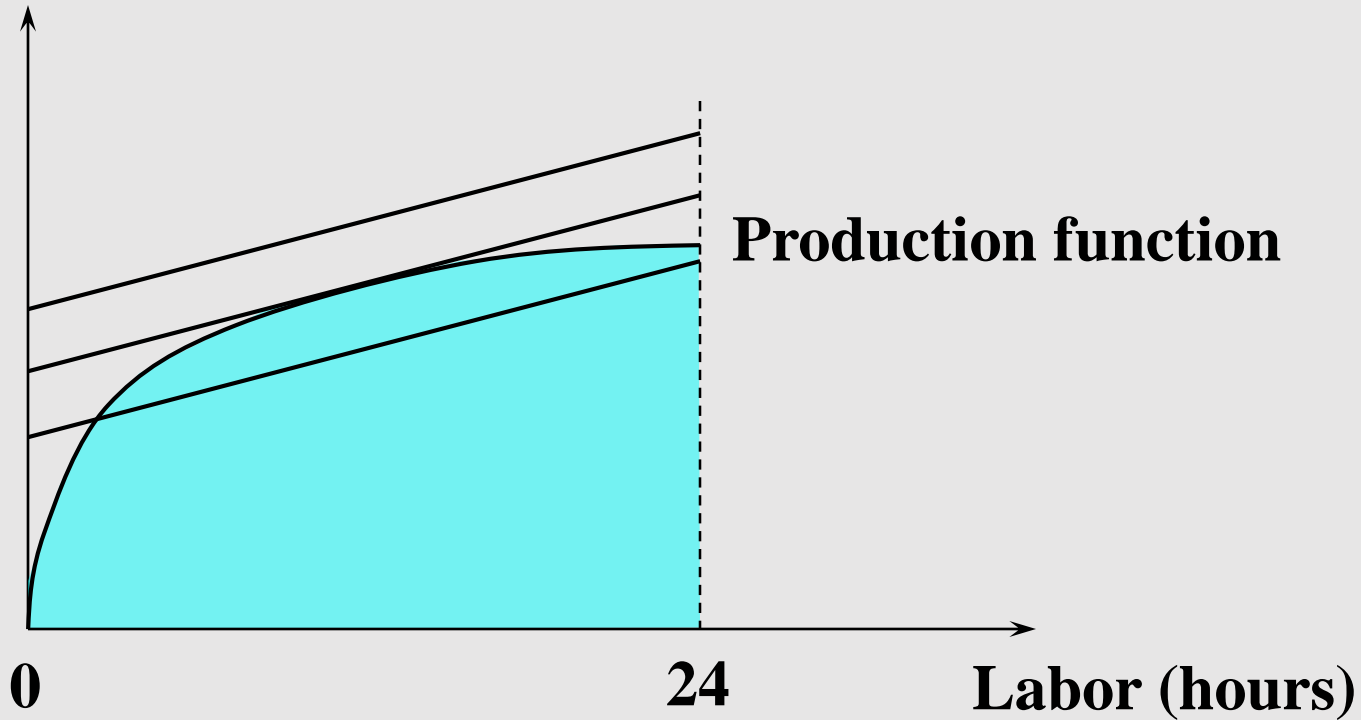


# Profit-Maximization



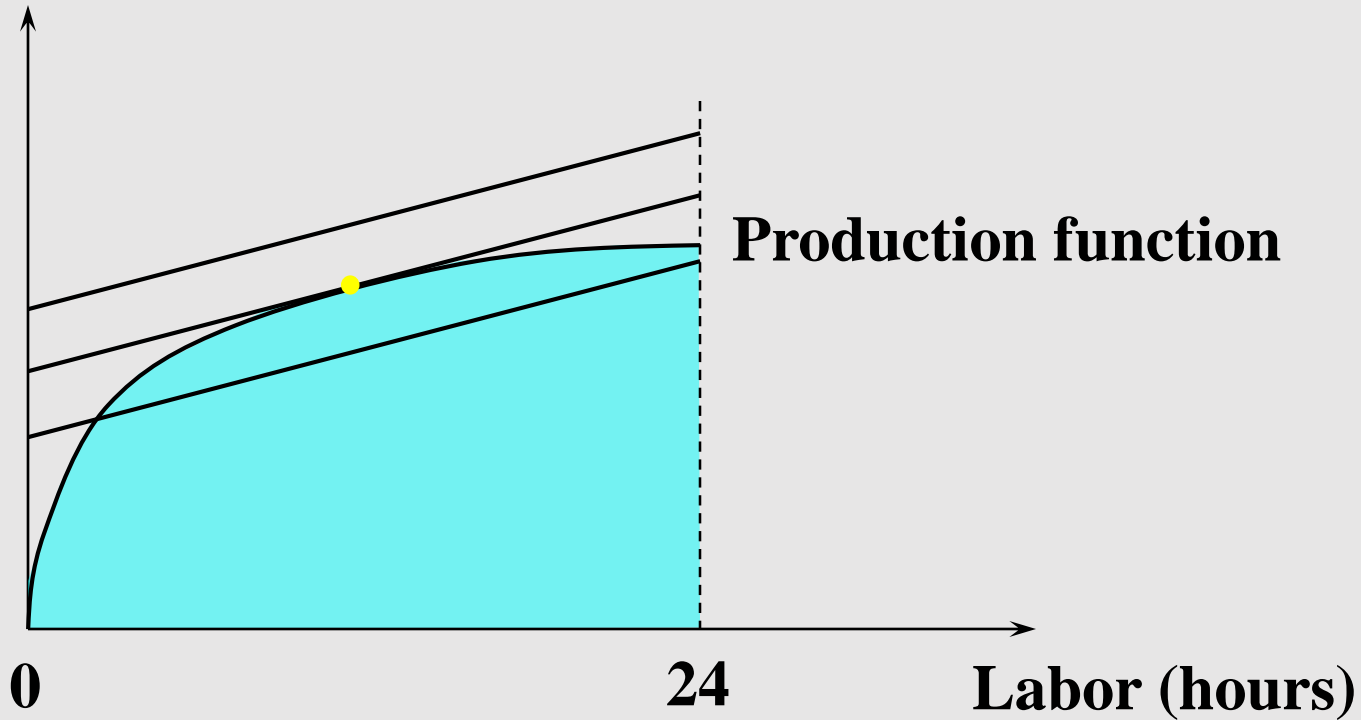
# Profit-Maximization

**Coconuts**



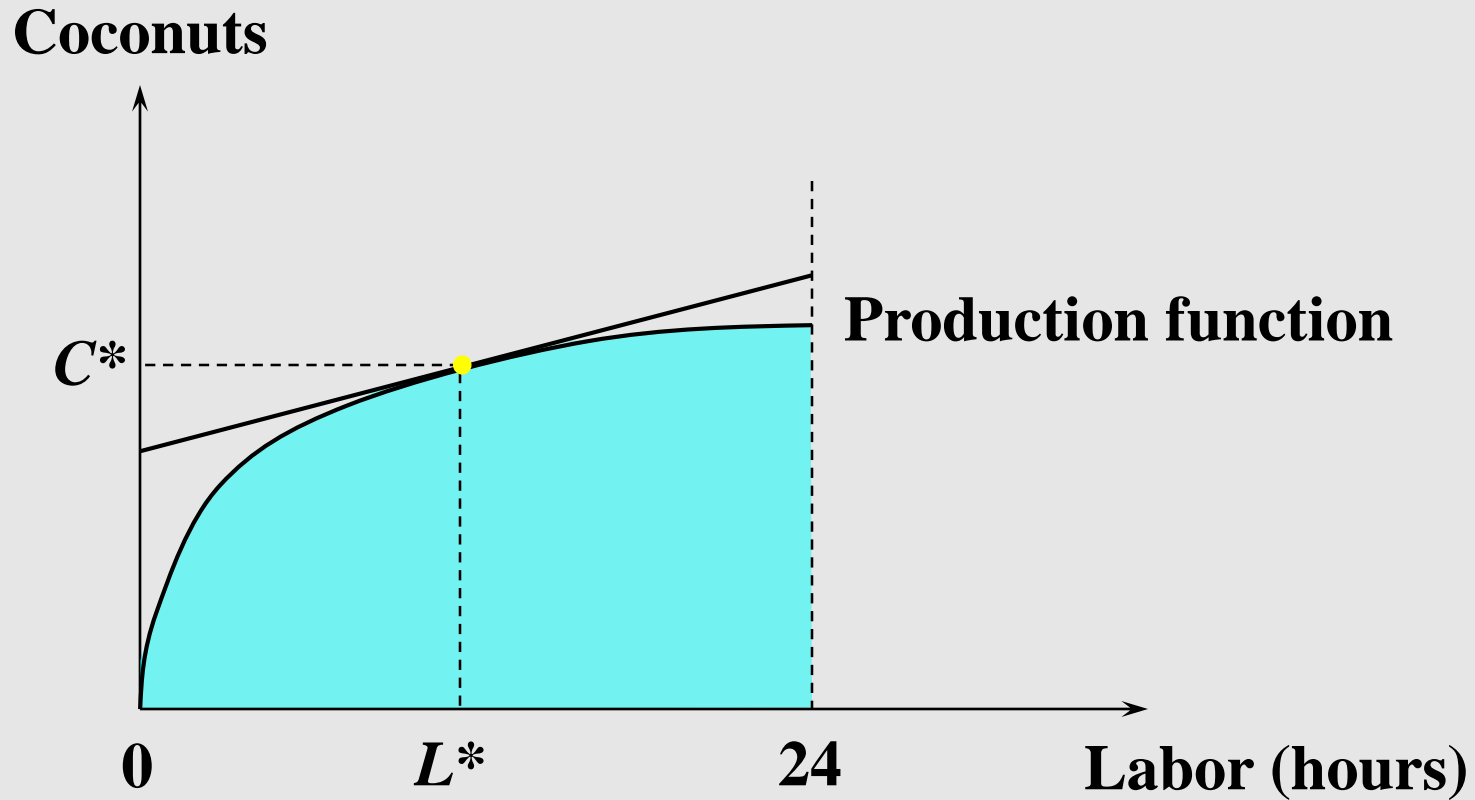
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**Coconuts**

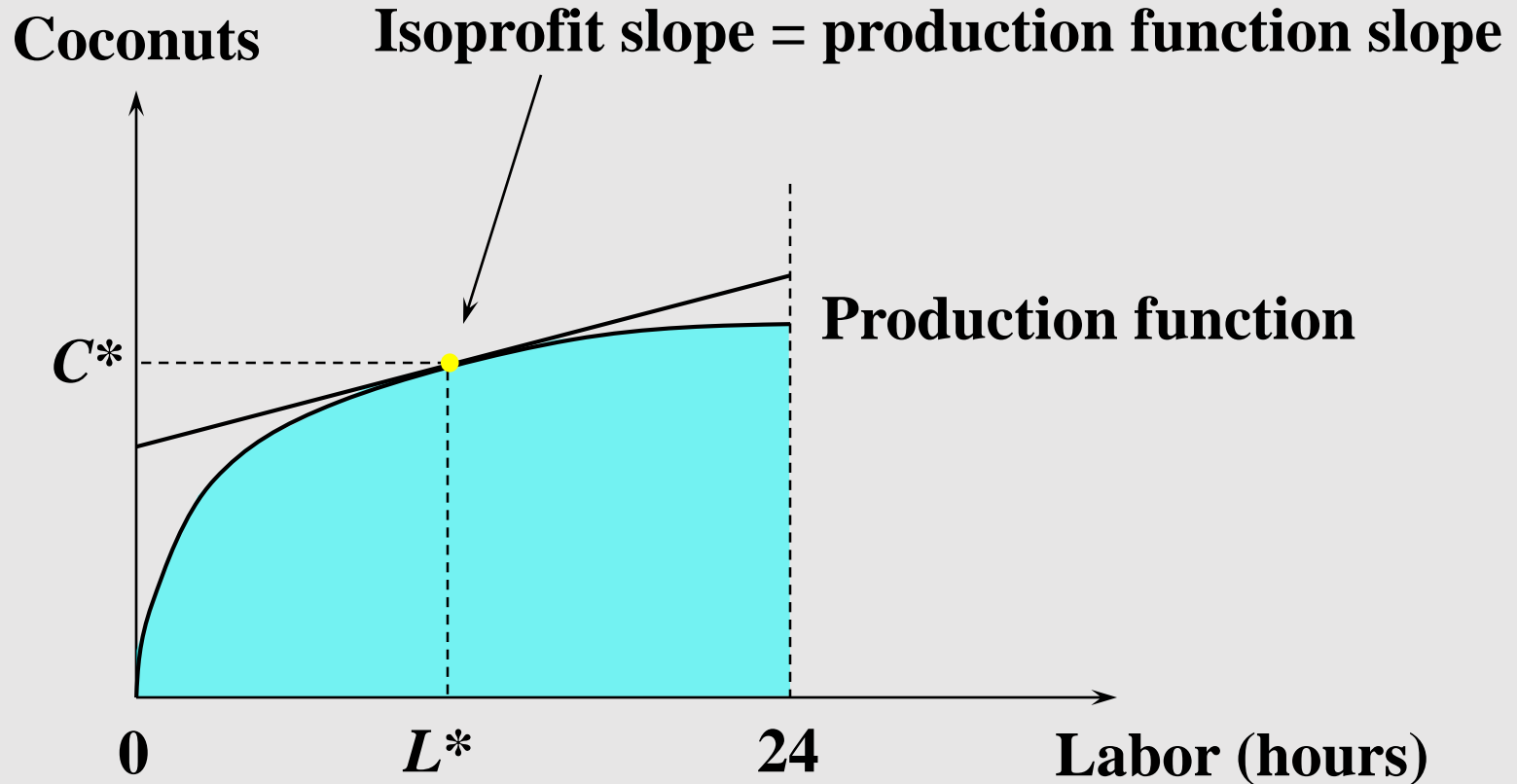




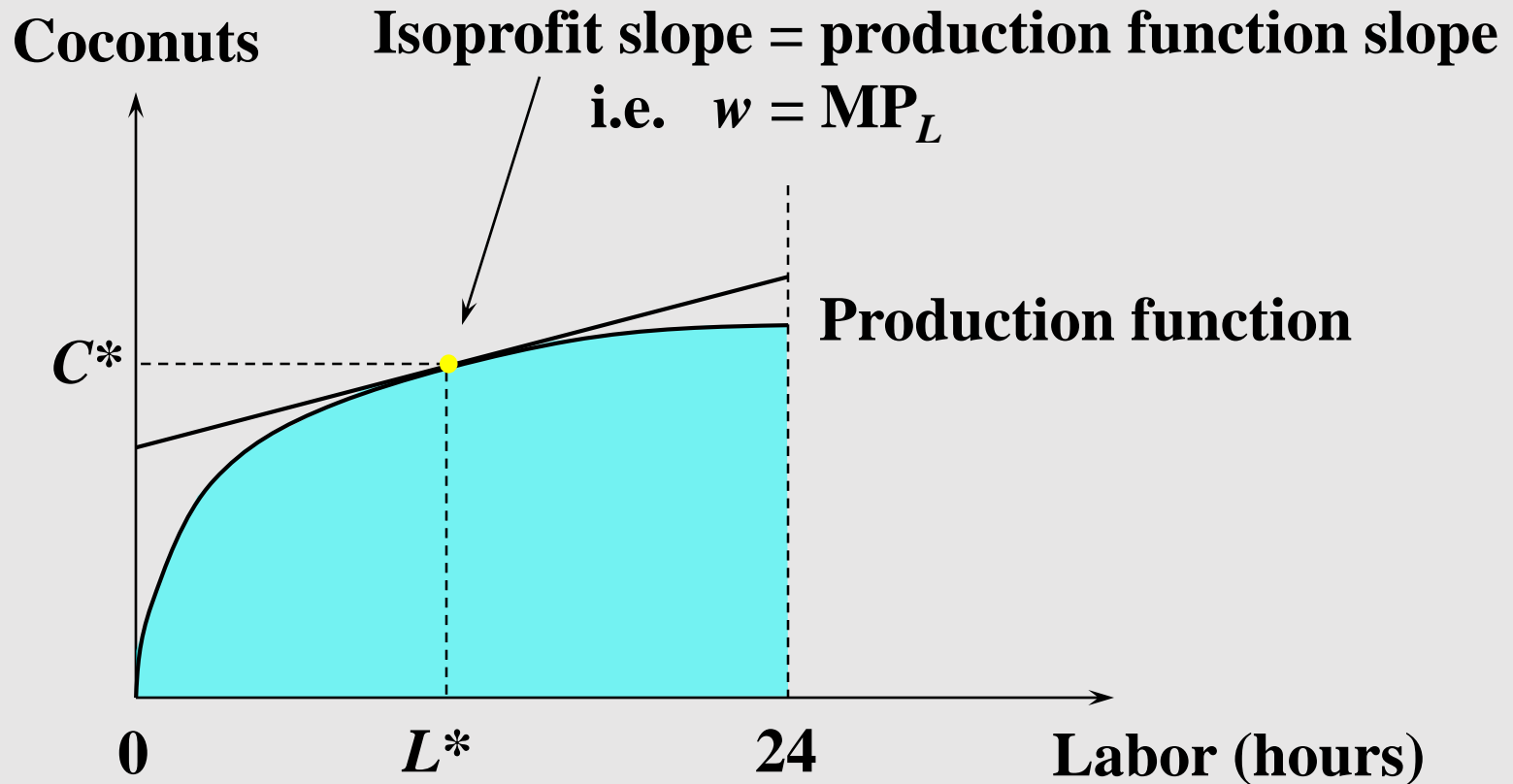
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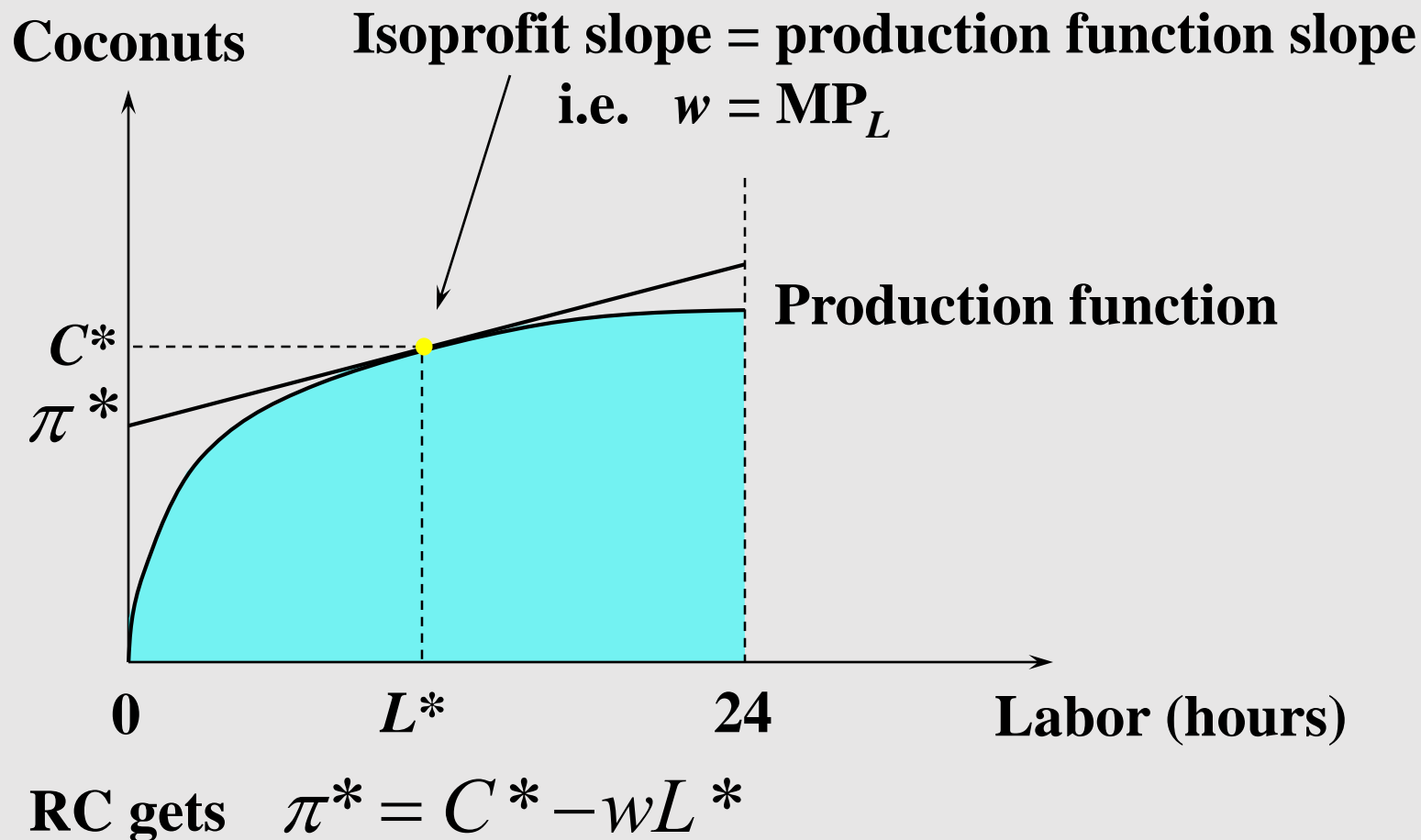
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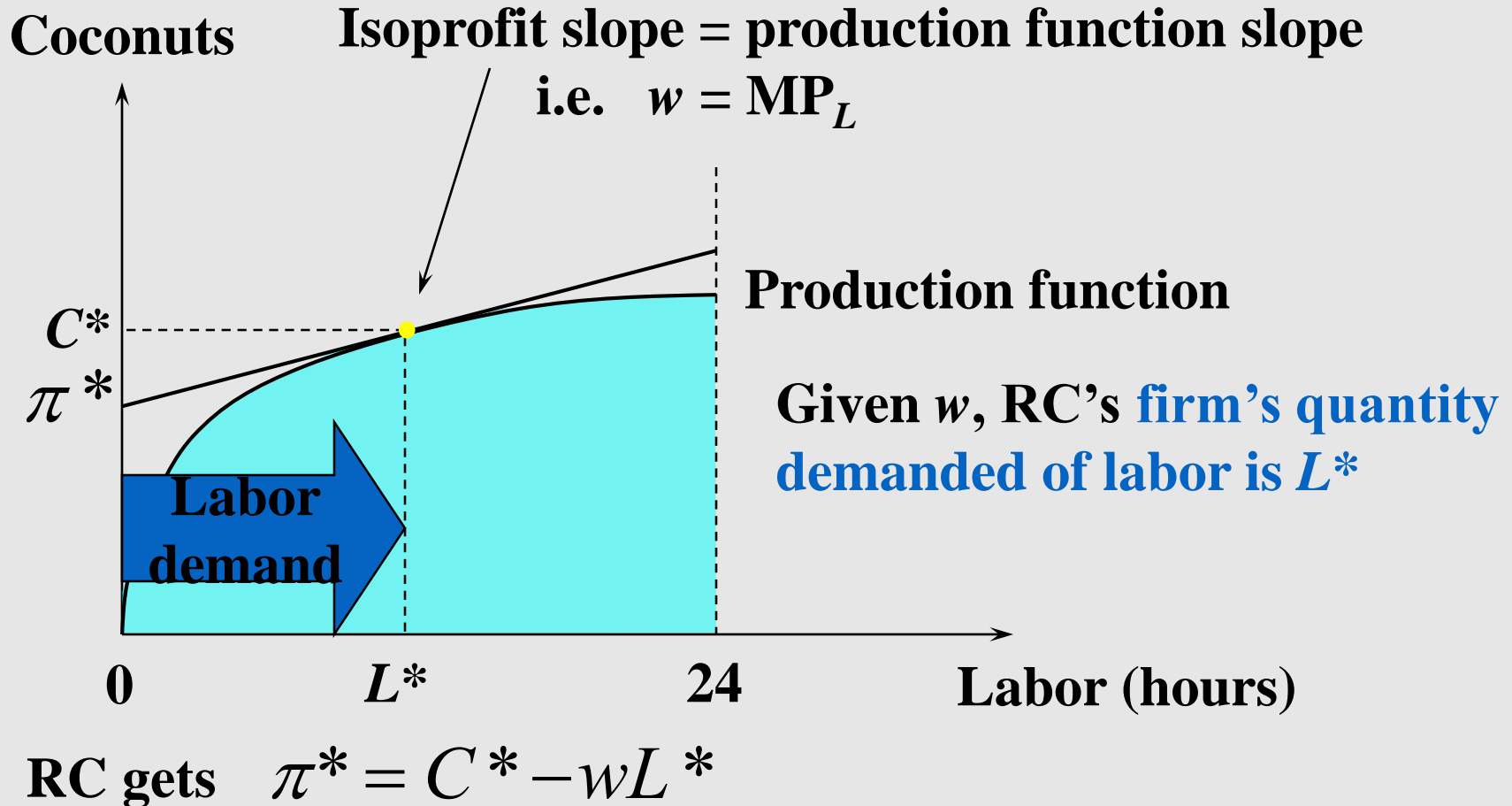
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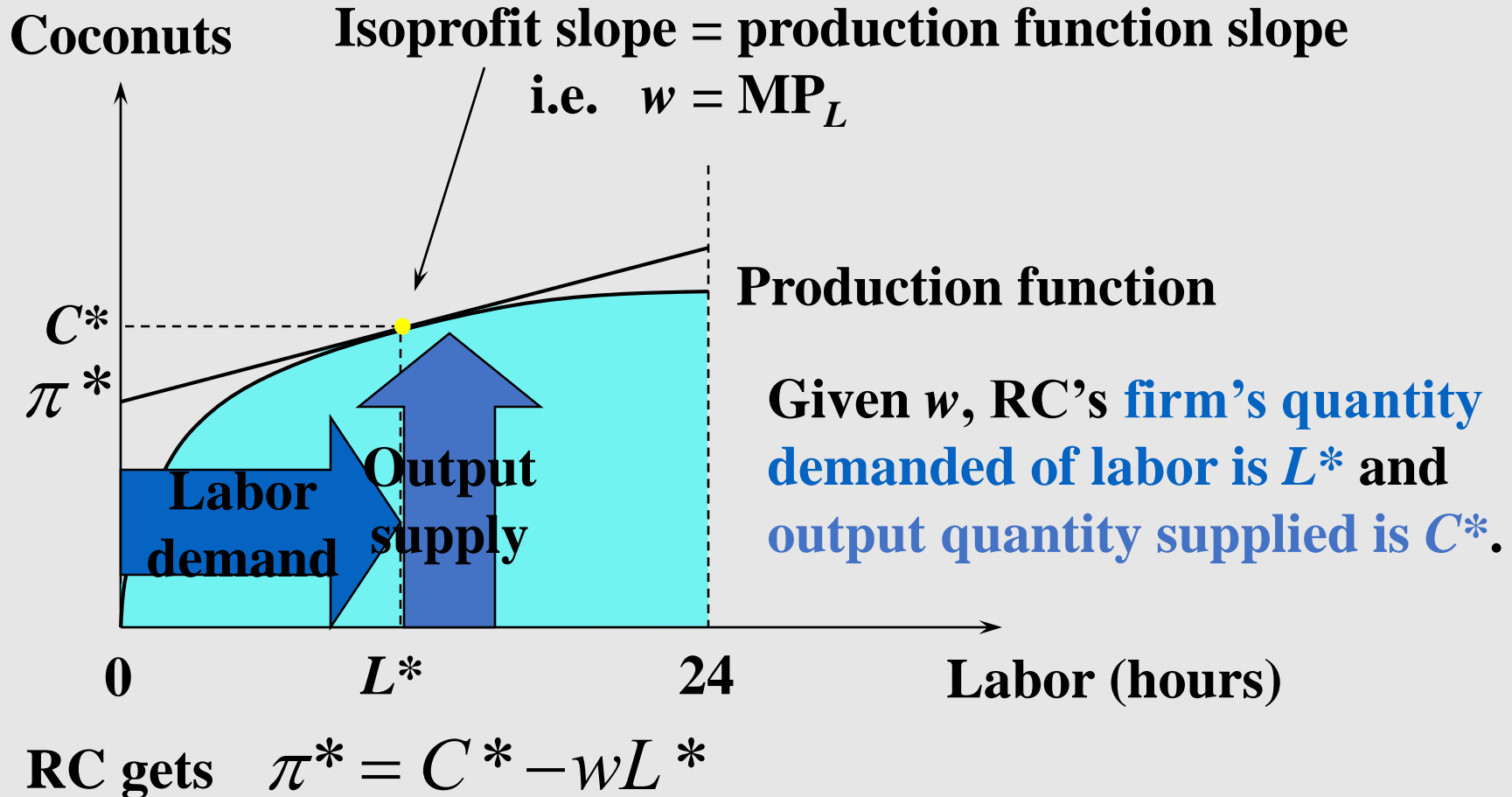
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# Profit-Maximization



# Profit-Maximization



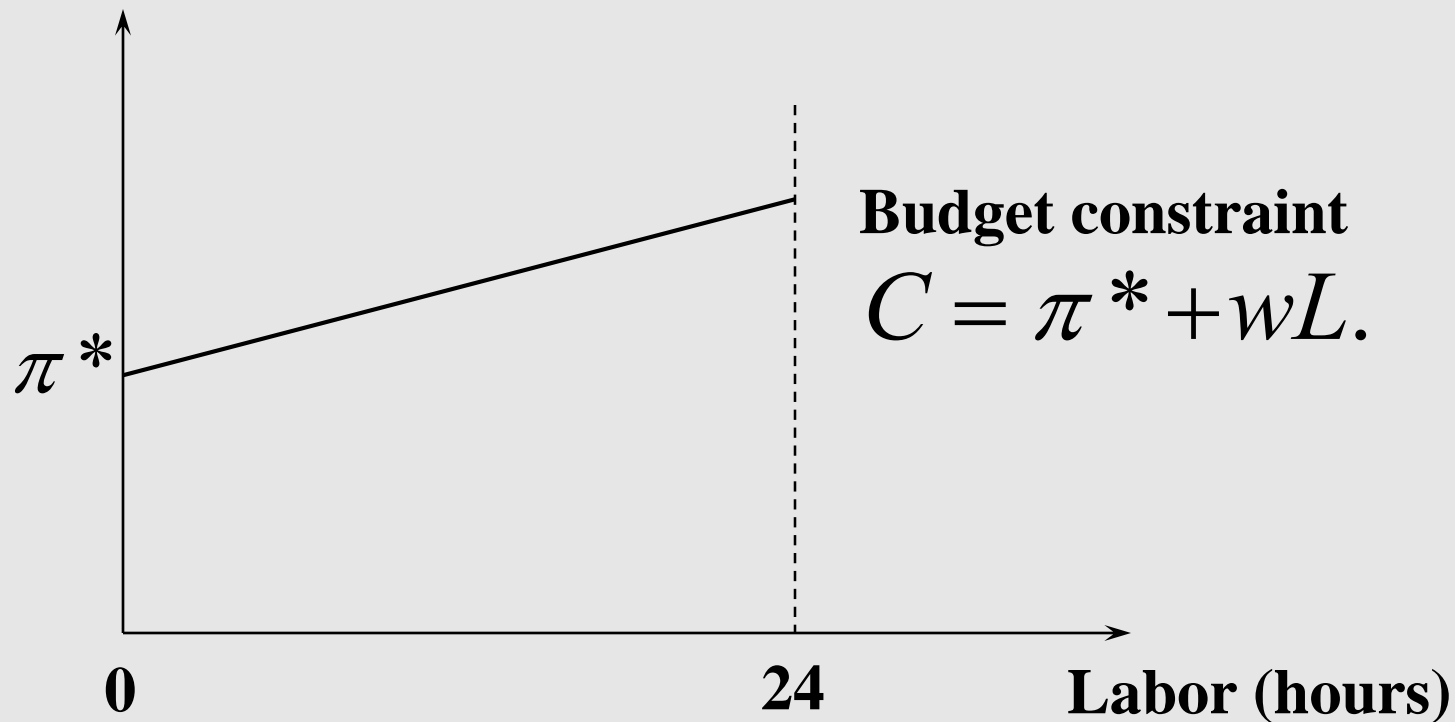
# Utility-Maximization

- Now consider RC as a consumer endowed with  $\pi^*$  who can work for  $w$  per hour.
- What is RC's most preferred consumption bundle?
- Budget constraint is

$$C = \pi^* + wL.$$

# Utility-Maximization

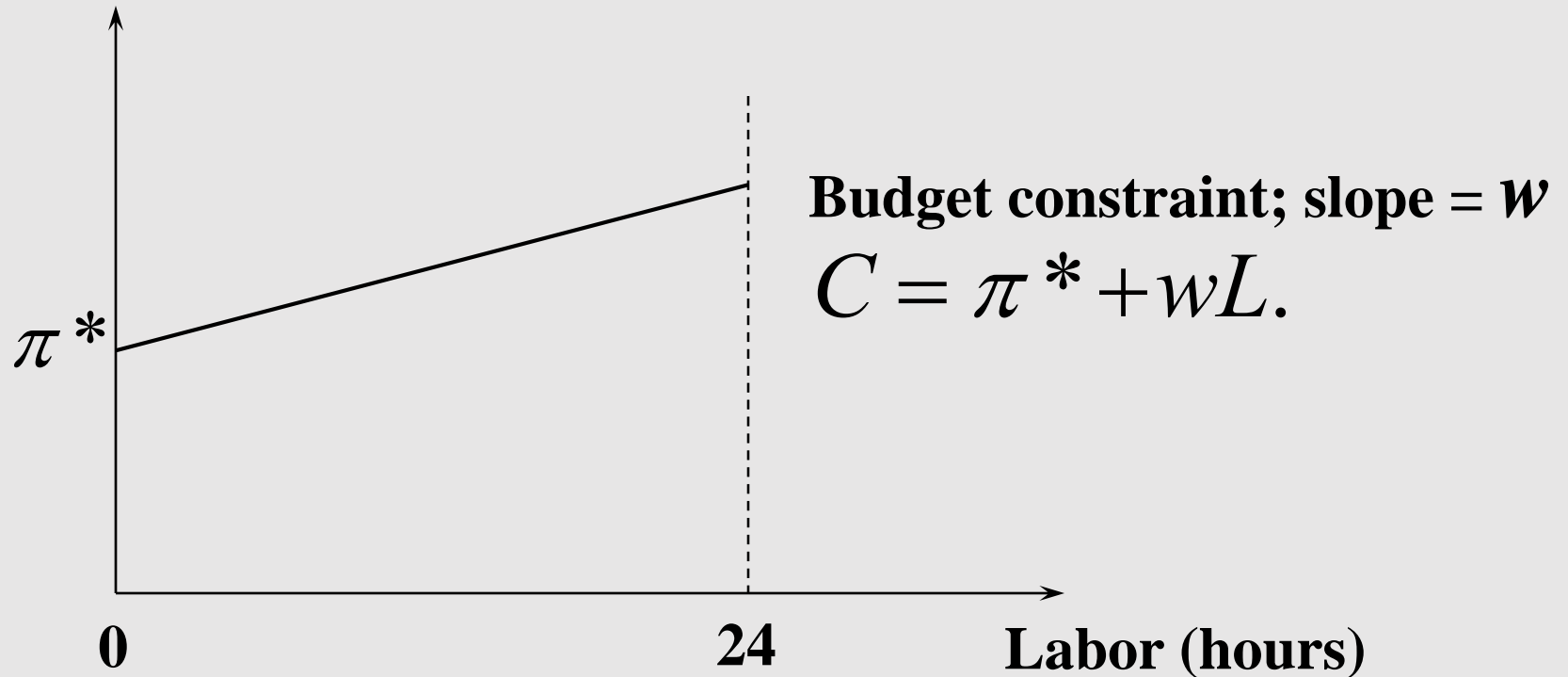
Coconuts





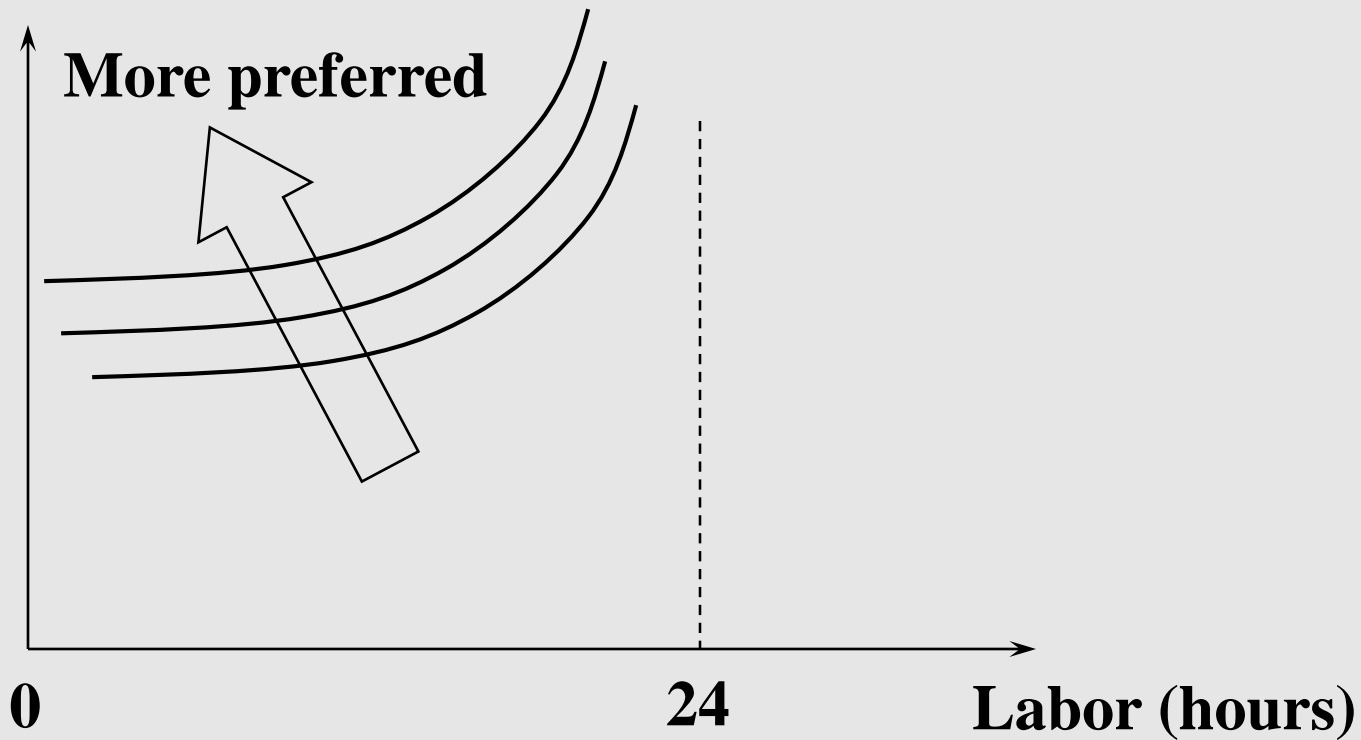
# Utility-Maximization

Coconuts



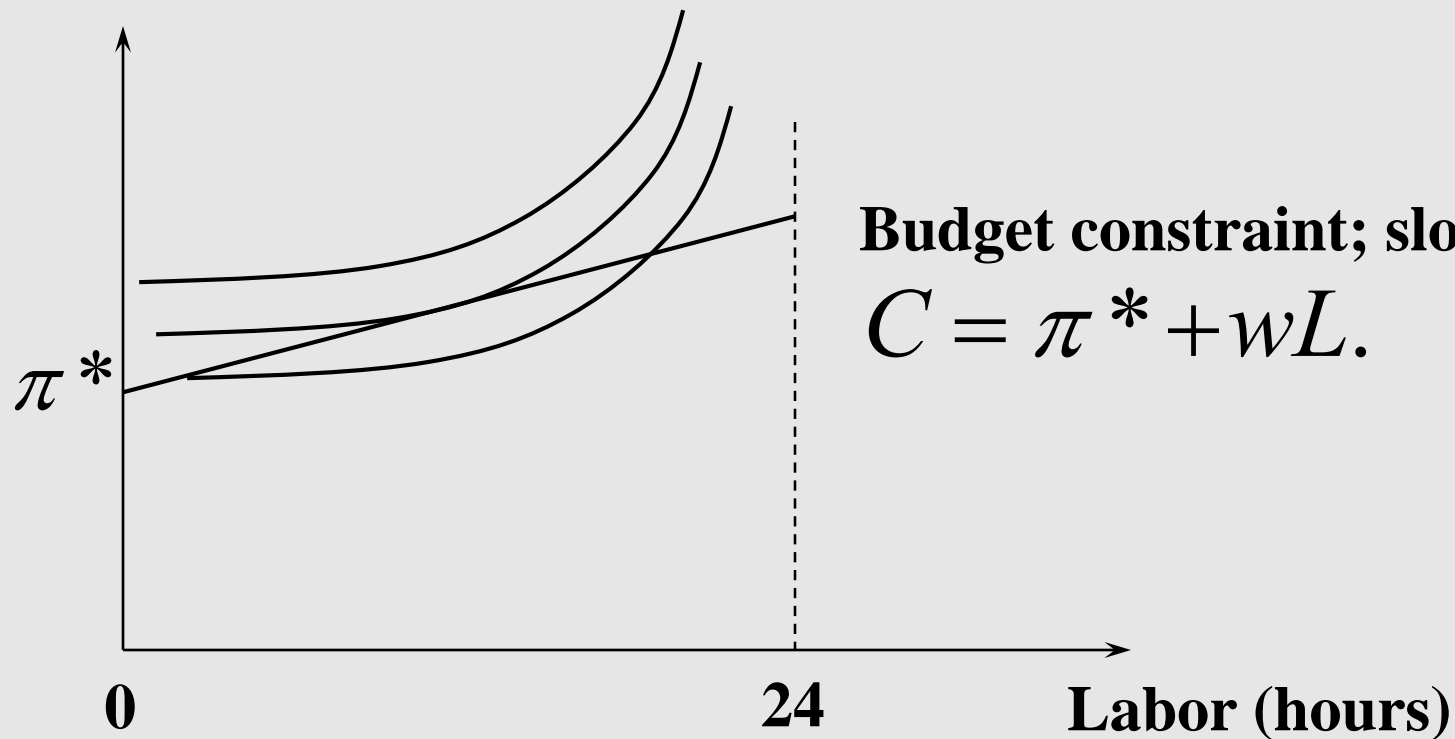
# Utility-Maximization

**Coconuts**



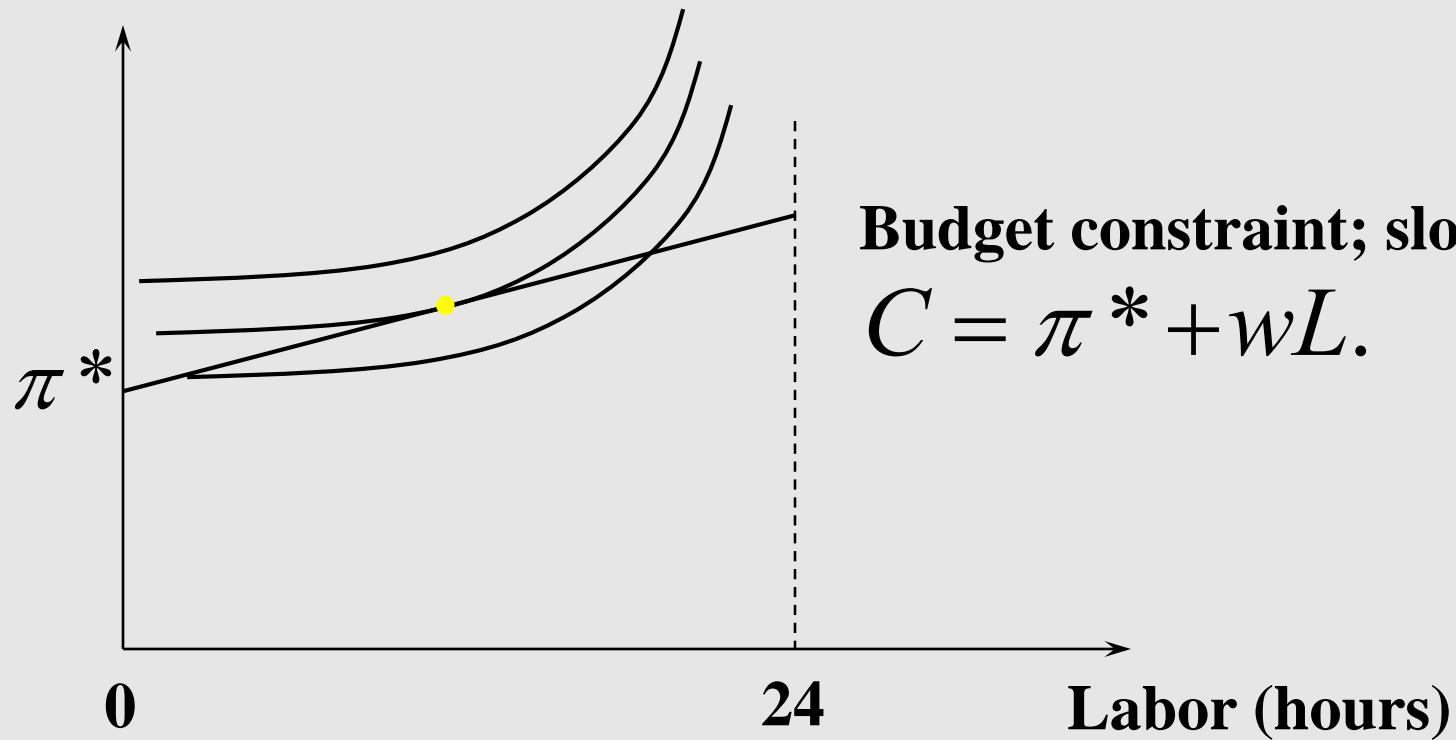
# Utility-Maximization

Coconuts



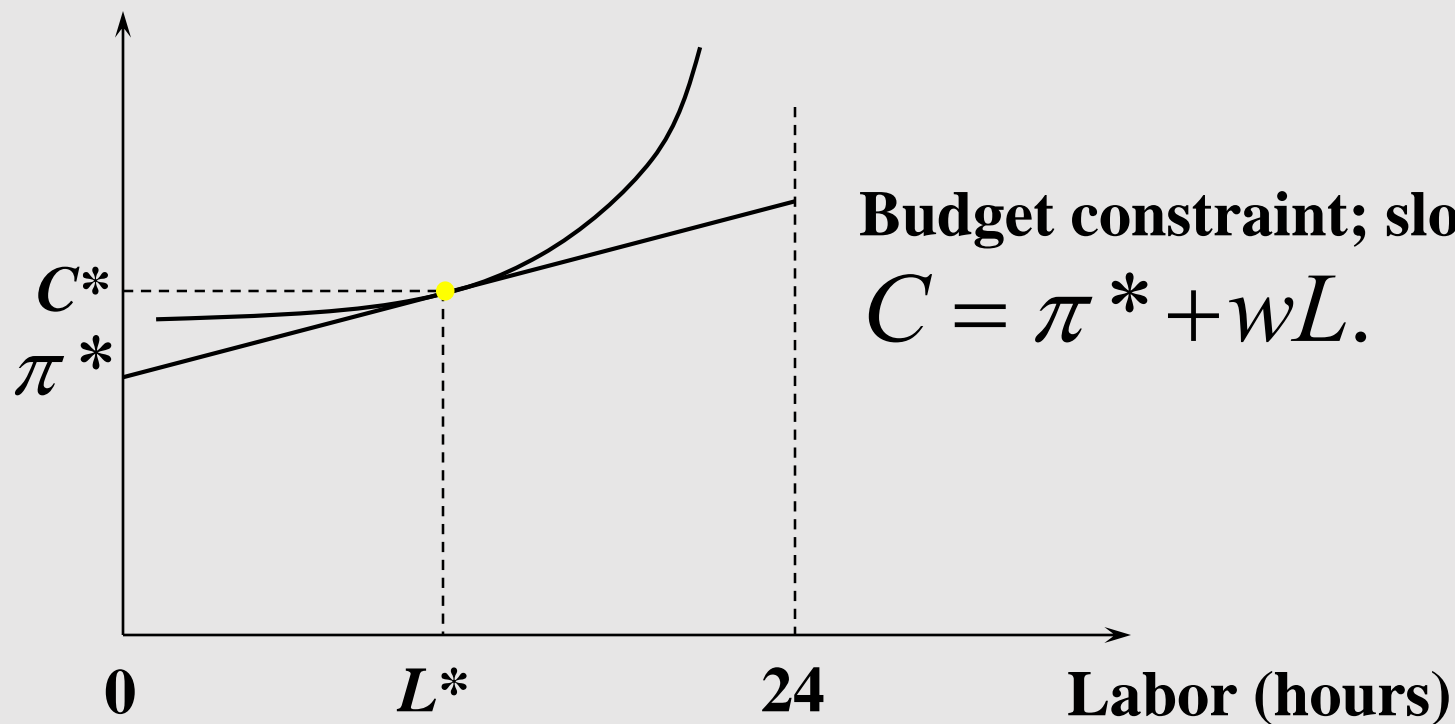
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Coconuts



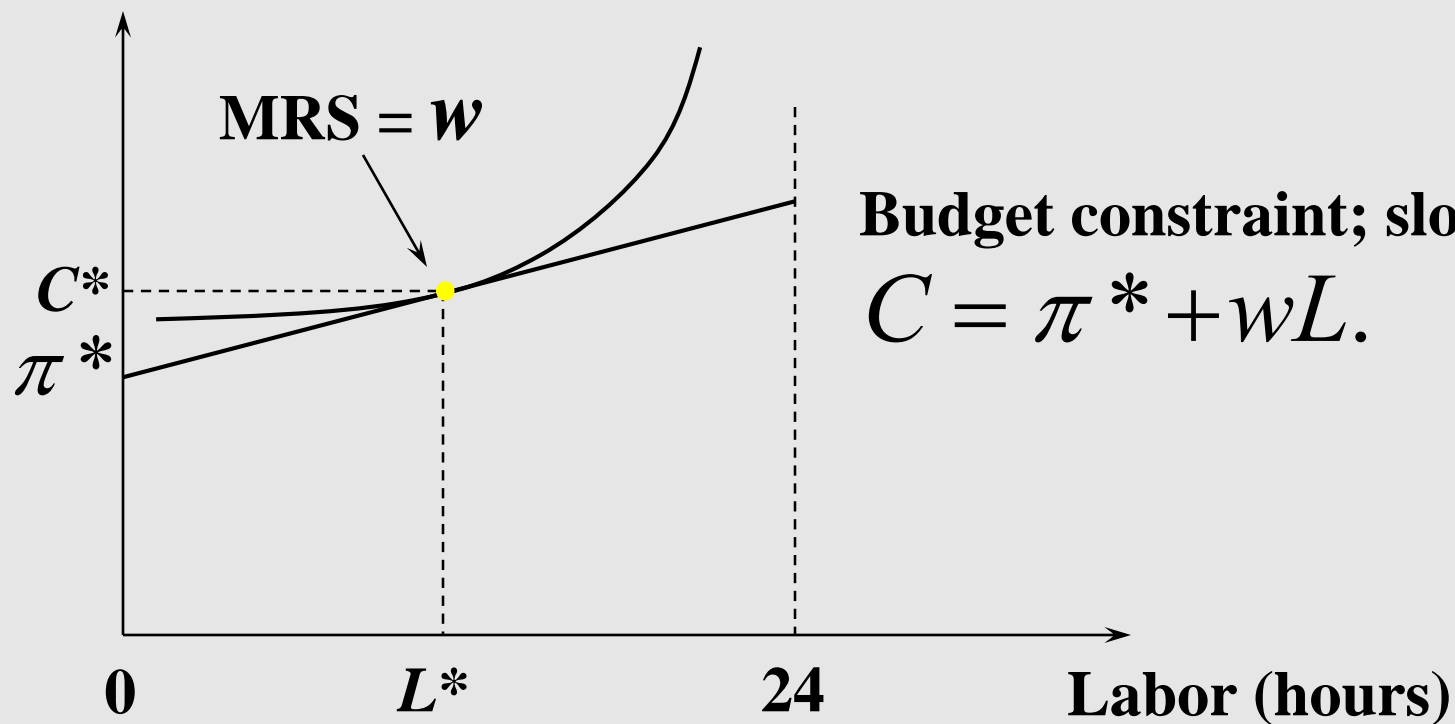
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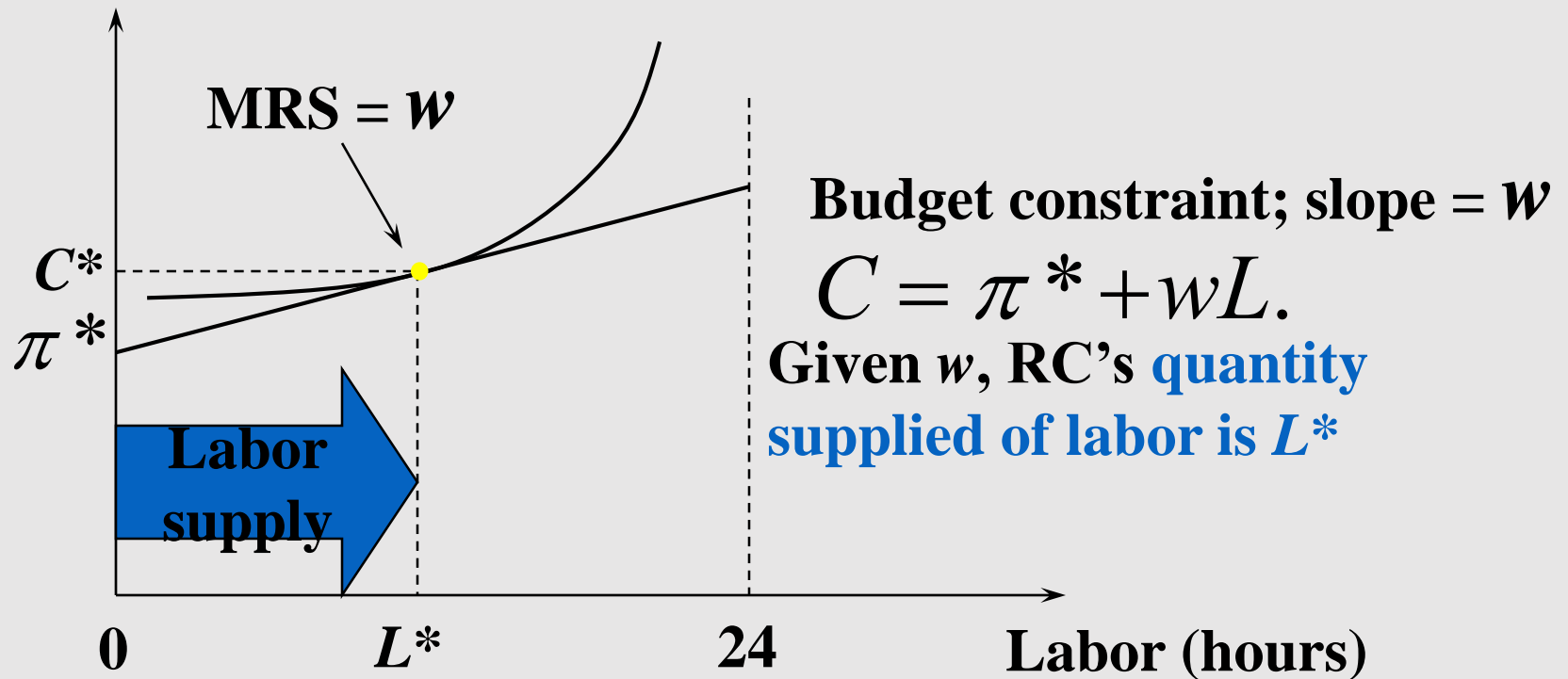
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Coconuts



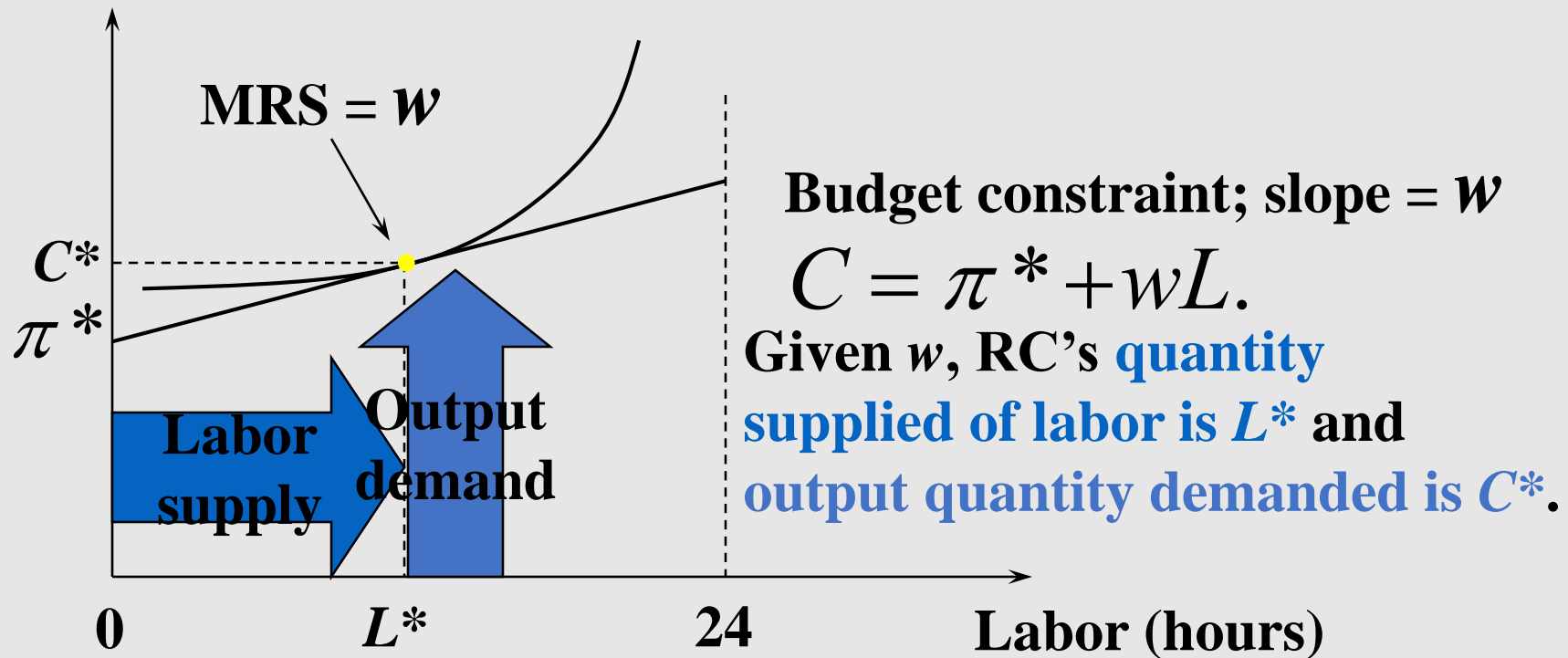
# Utility-Maximization

Coconuts



# Utility-Maximization

Coconuts





# Utility-Maximization & Profit-Maximization

→ Profit-maximization:

- $w = MP_L$
- quantity of output supplied =  $C^*$
- quantity of labor demanded =  $L^*$

→ Utility-maximization:

- $w = MRS$
- quantity of output demanded =  $C^*$
- quantity of labor supplied =  $L^*$

# Utility-Maximization & Profit-Maximization

→ Profit-maximization:

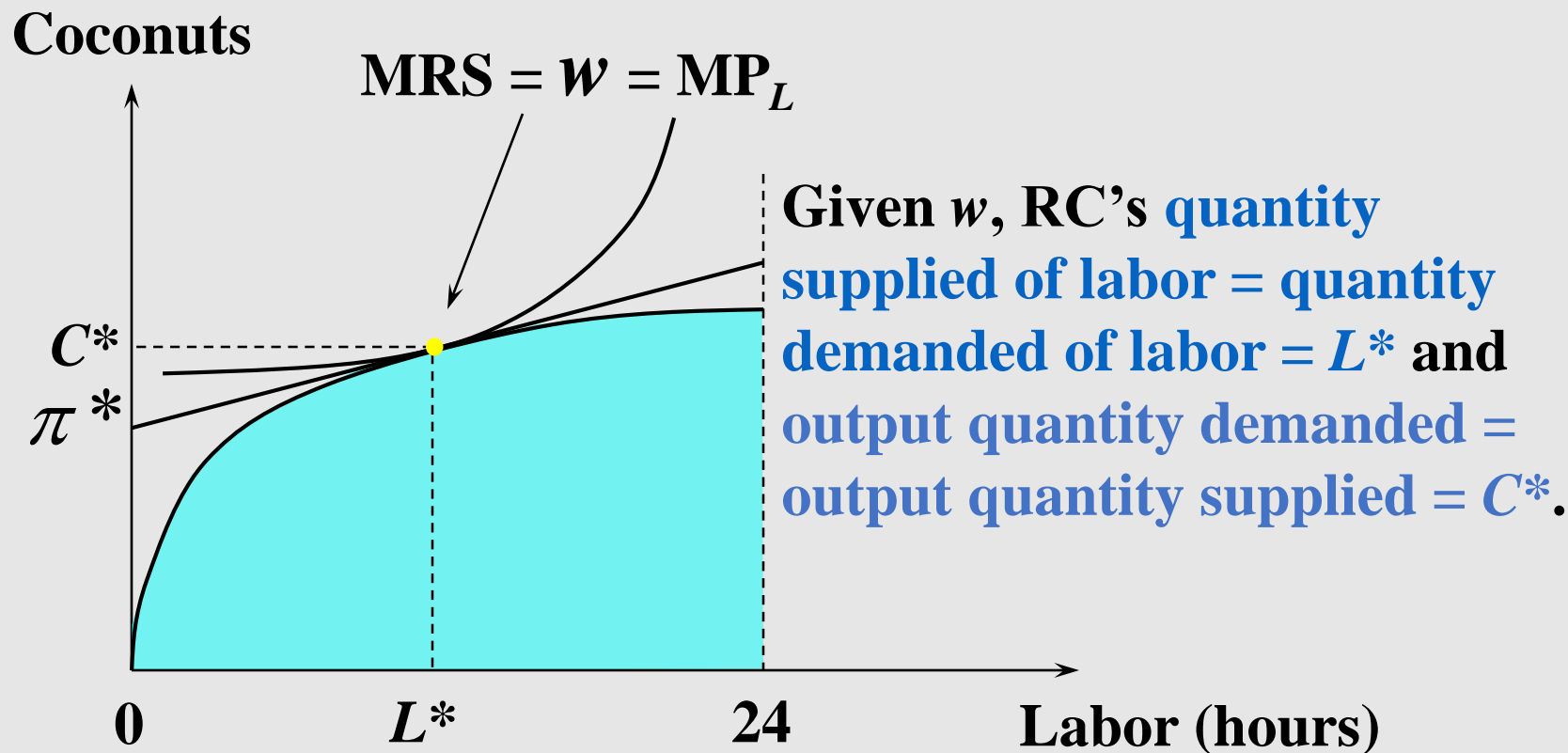
- $w = MP_L$
- quantity of output supplied =  $C^*$
- quantity of labor demanded =  $L^*$

→ Utility-maximization:

- $w = MRS$
- quantity of output demanded =  $C^*$
- quantity of labor supplied =  $L^*$

**Coconut and labor  
markets both clear.**

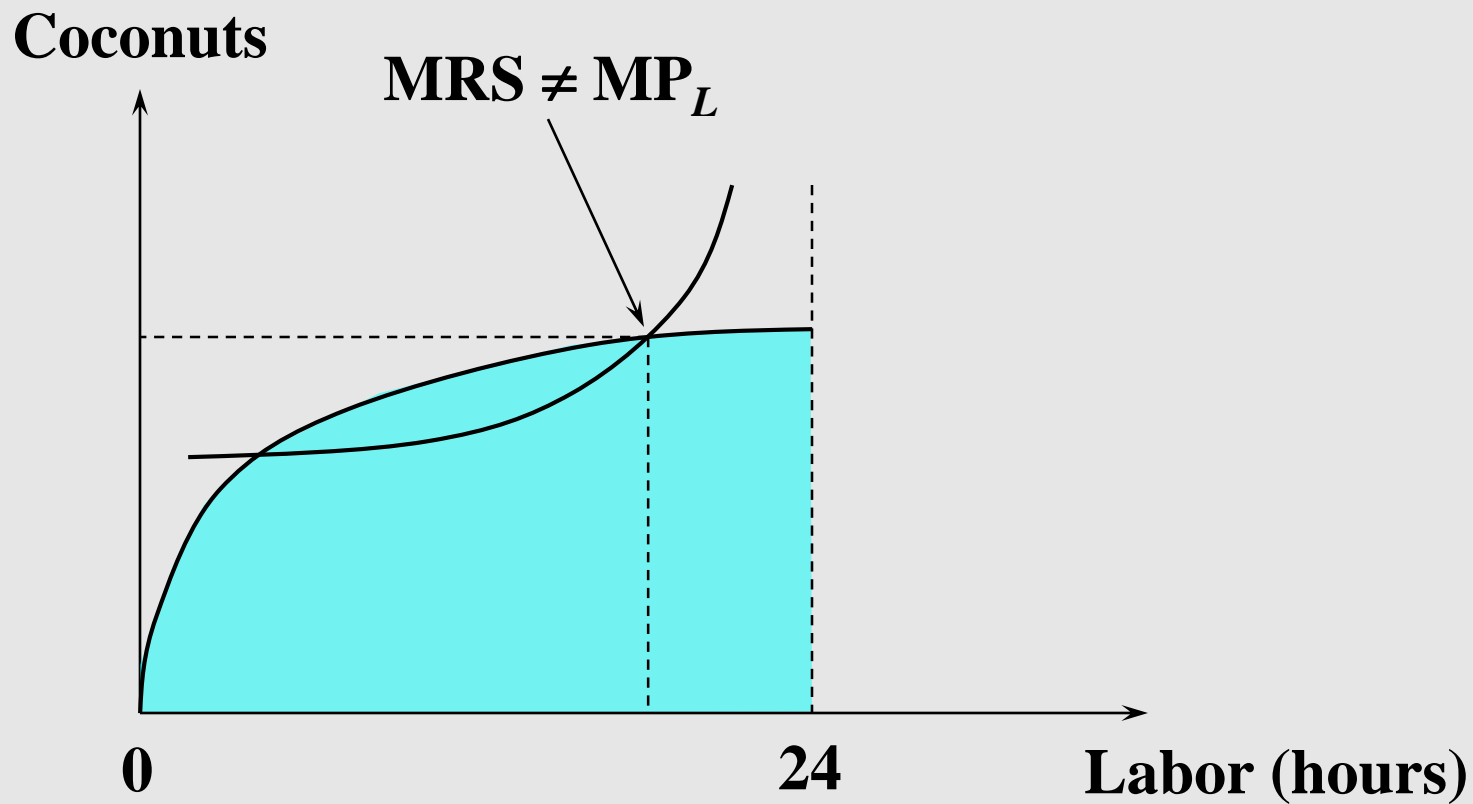
# Utility-Maximization & Profit-Maximization



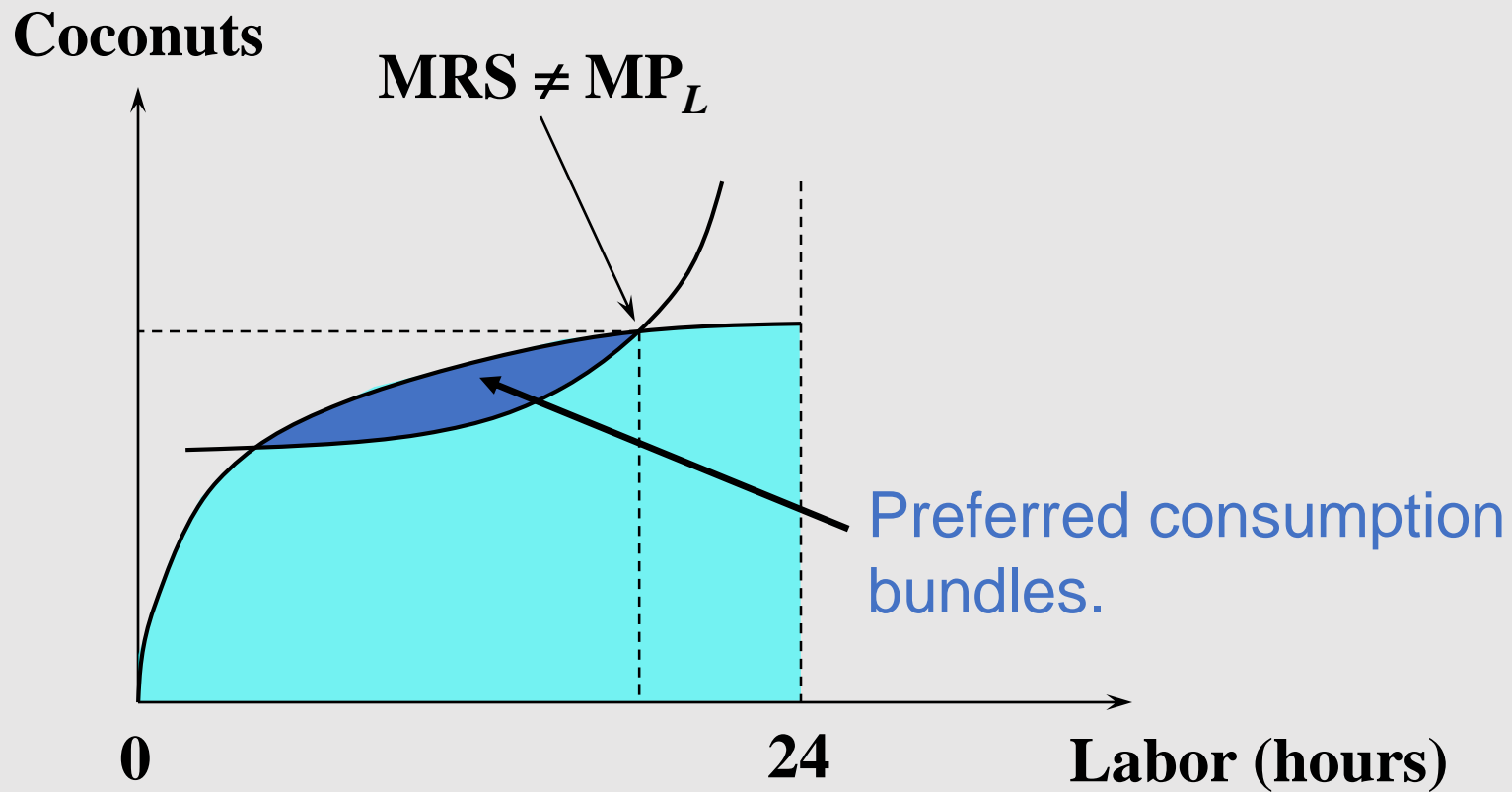
# Pareto Efficiency

→ Must have  $MRS = MP_L$ .

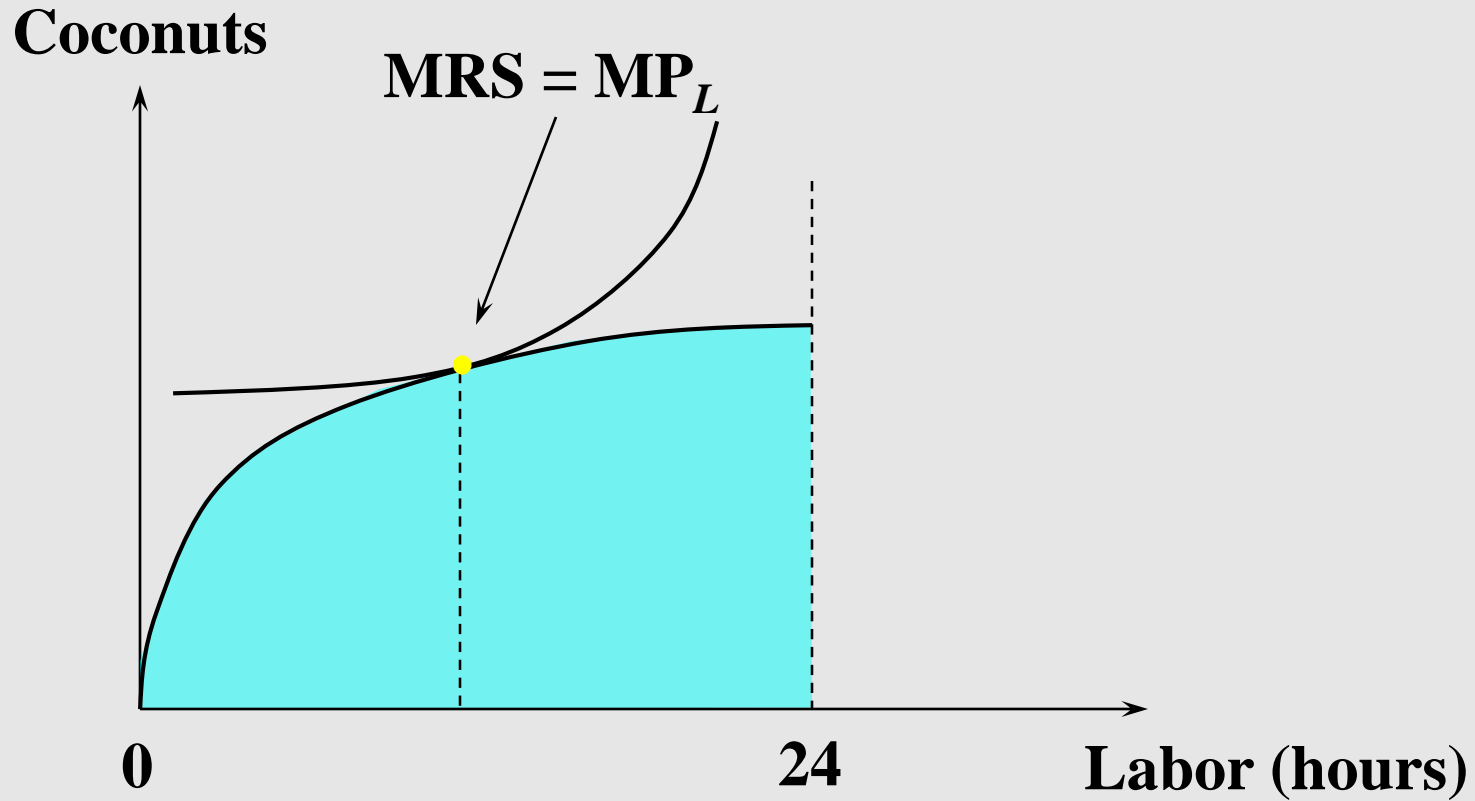
# Pareto Efficiency



# Pareto Efficiency



# Pareto Efficiency



# 3. Fundamental Theorems of Welfare Economics

## First Fundamental Theorem of Welfare Economics

- A competitive market equilibrium is Pareto efficient if
  - consumers' preferences are convex
  - there are no externalities in consumption or production.



# Fundamental Theorems of Welfare Economics

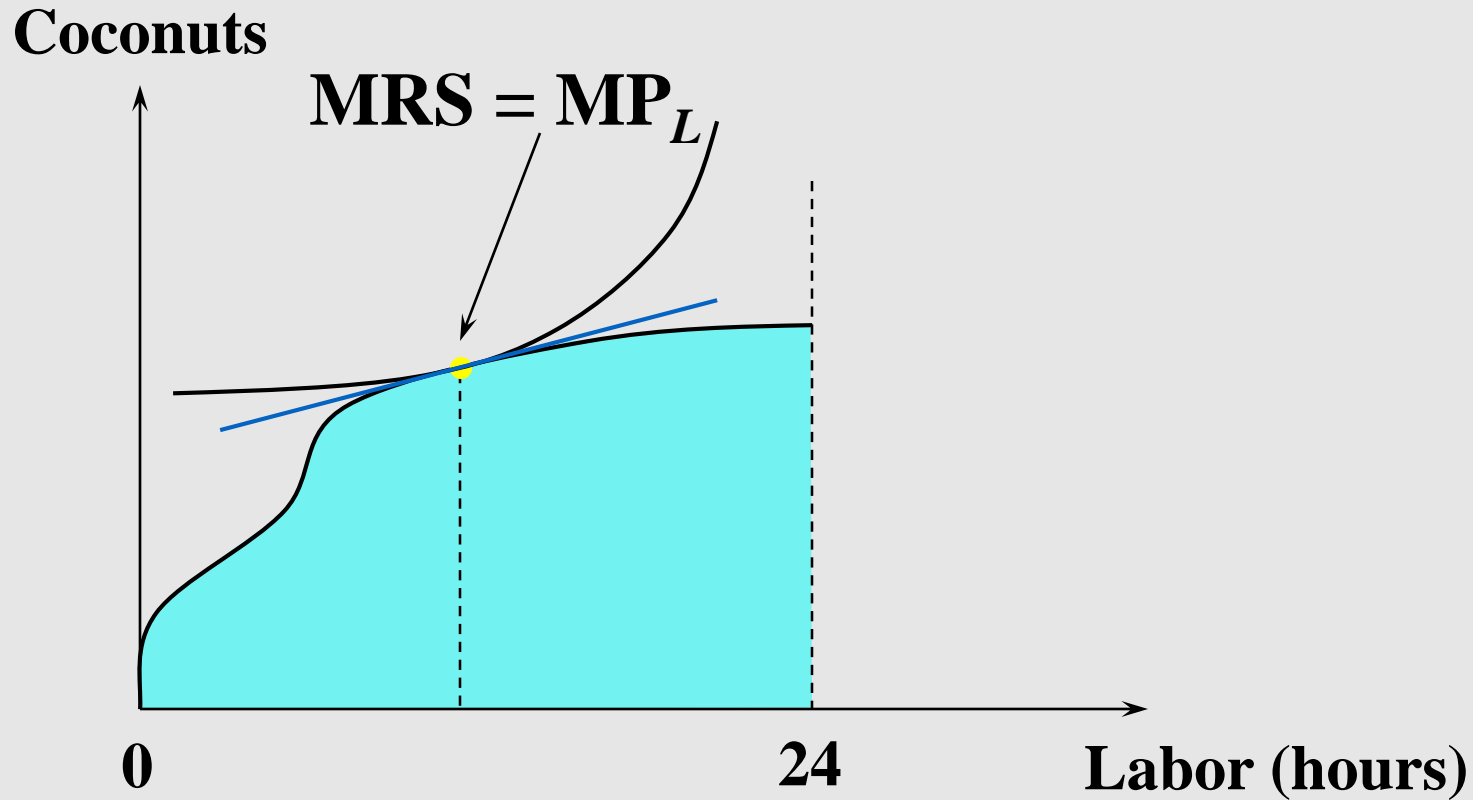
## Second Fundamental Theorem of Welfare Economics

- Any Pareto efficient economic state can be achieved as a competitive market equilibrium if
  - consumers' preferences are convex
  - firms' technologies are convex: feasible production plans are a convex set
    - For every two points in the set, the line that joins them belong to the set
  - there are no externalities in consumption or production.

# Non-Convex Technologies

- Do the Welfare Theorems hold if firms have non-convex technologies?
- The 1st Theorem does not rely upon firms' technologies being convex.

# Non-Convex Technologies

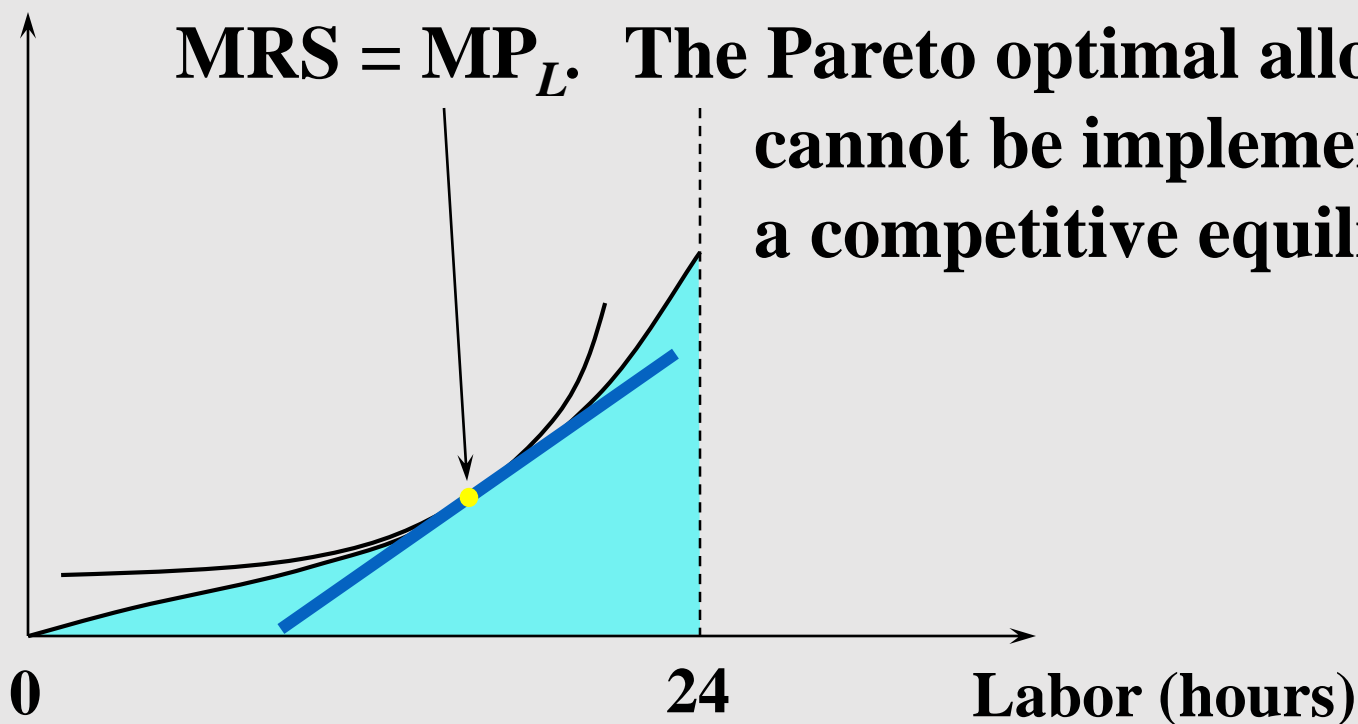


# Non-Convex Technologies

- Do the Welfare Theorems hold if firms have non-convex technologies?
- The 2nd Theorem **does** require that firms' technologies be convex.
  - This is, the feasible production plans are a convex set
    - For every two points in the set, the line that joins them belong to the set
  - This means that all situations with increasing returns to scale are ruled out.

# Non-Convex Technologies

Coconuts



## 4. Production Possibilities

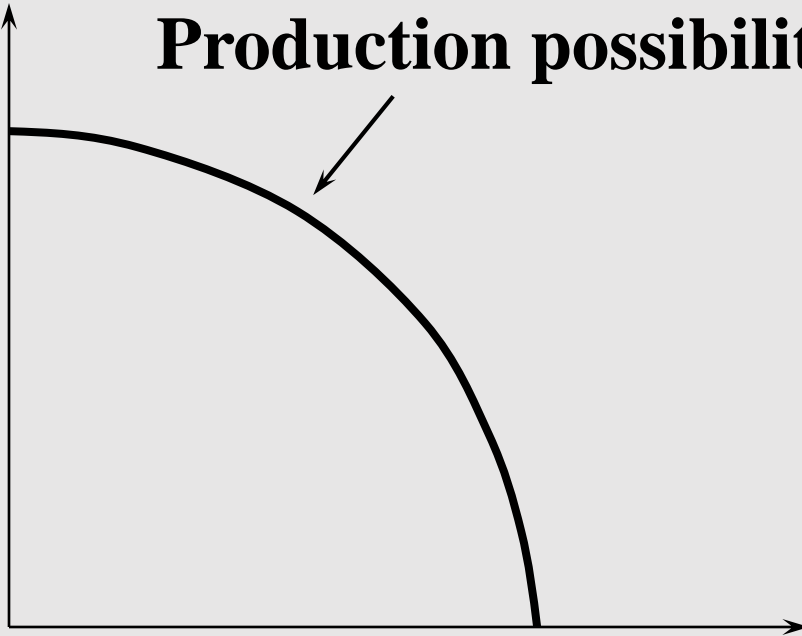
Let's generalize to two outputs

- Resource and technological limitations restrict what an economy can produce.
- The set of all feasible output bundles is the economy's **production possibility set**.
- The set's outer boundary is the **production possibility frontier**.
- Difference: a function of the two outputs, inputs are not in the graph anymore

# Production Possibilities

**Coconuts**

**Production possibility frontier (ppf)**



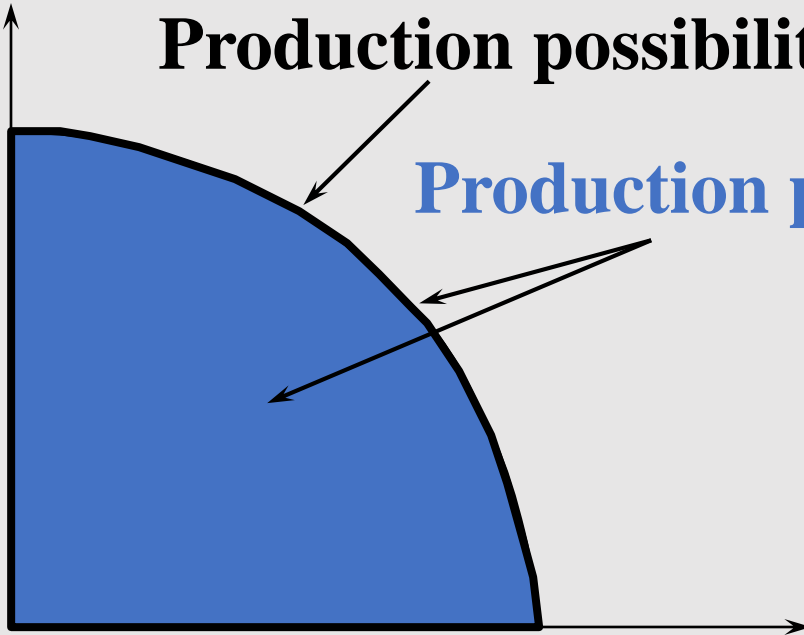
**Fish**

# Production Possibilities

**Coconuts**

**Production possibility frontier (ppf)**

**Production possibility set**

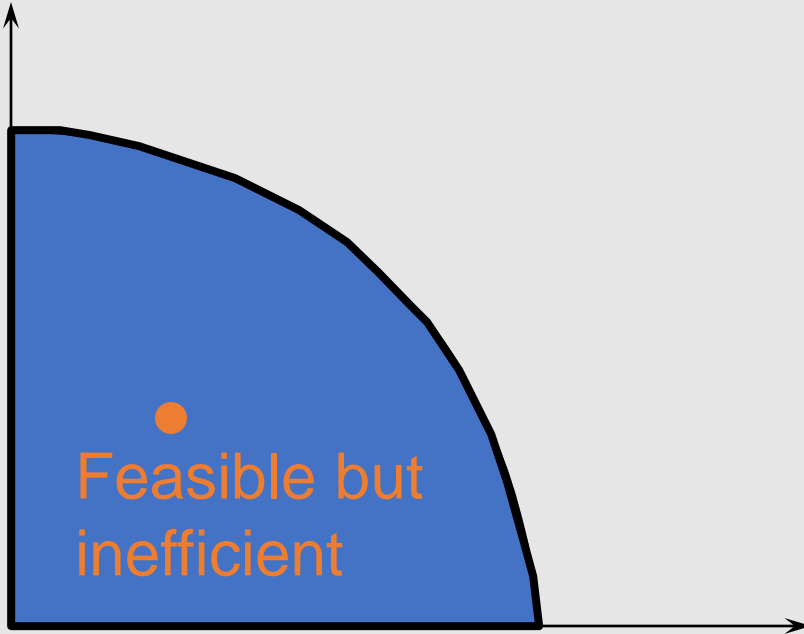


**Fish**



# Production Possibilities

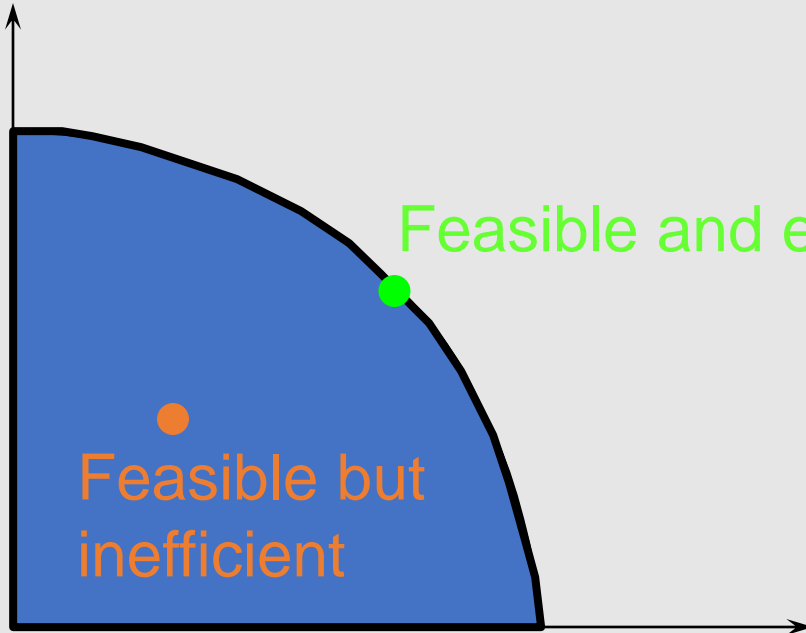
**Coconuts**



**Fish**

# Production Possibilities

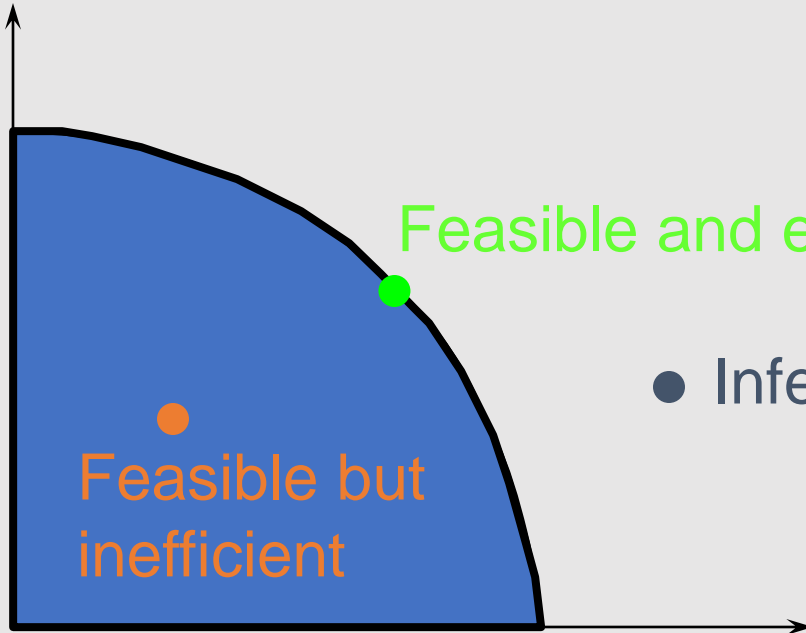
**Coconuts**



**Fish**

# Production Possibilities

**Coconuts**



Feasible and efficient

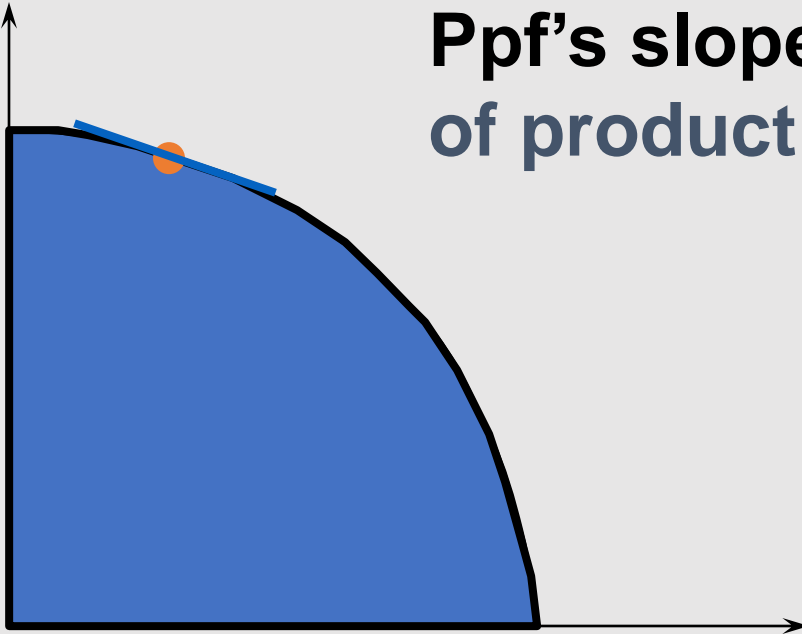
● Infeasible

Feasible but  
inefficient

**Fish**

# Production Possibilities

Coconuts

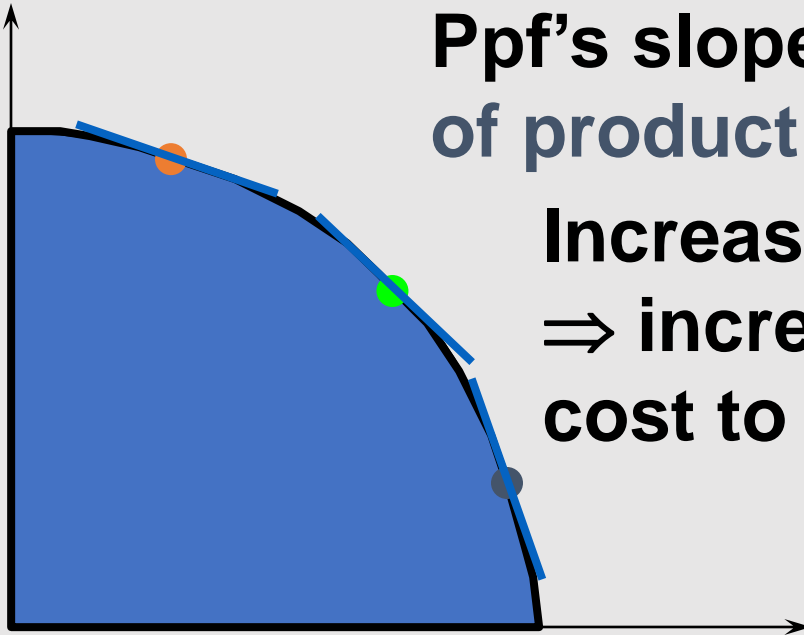


**Ppf's slope is the marginal rate of product transformation.**

Fish

# Production Possibilities

Coconuts



**Ppf's slope is the marginal rate of product transformation.**

**Increasingly negative MRPT  
⇒ increasing opportunity  
cost to specialization.**

Fish

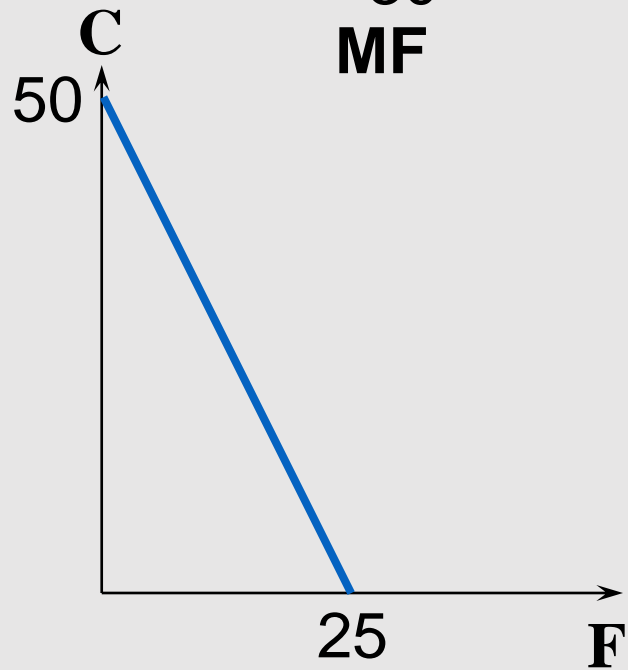
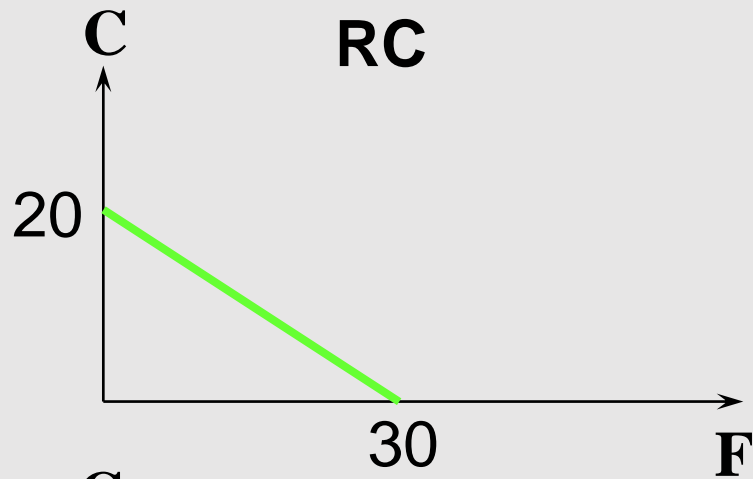
# Production Possibilities

- The construction of the ppf was quite simple since there was only one way to produce fish and coconuts.
- What if there is more than one way to produce each good?
- Let's add another worker with different skills in producing fish and coconuts.

## 5. Comparative Advantage

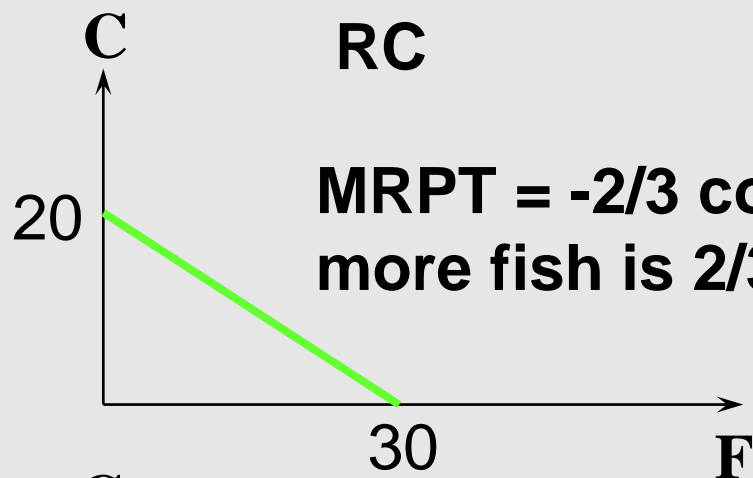
- Two agents, RC and Man Friday (MF).
- RC can produce at most 20 coconuts or 30 fish.
- MF can produce at most 50 coconuts or 25 fish.

# Comparative Advantage

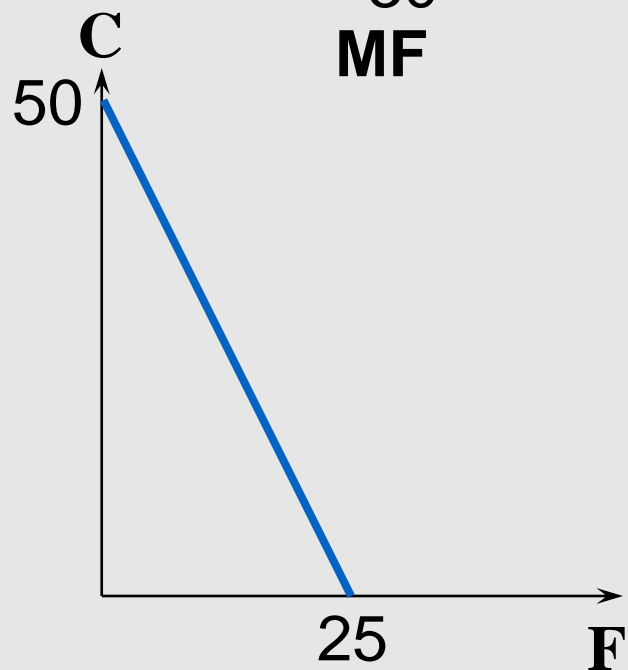




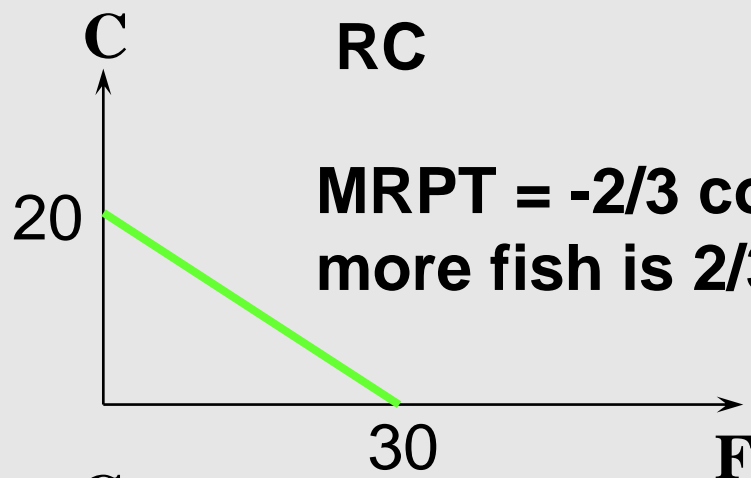
# Comparative Advantage



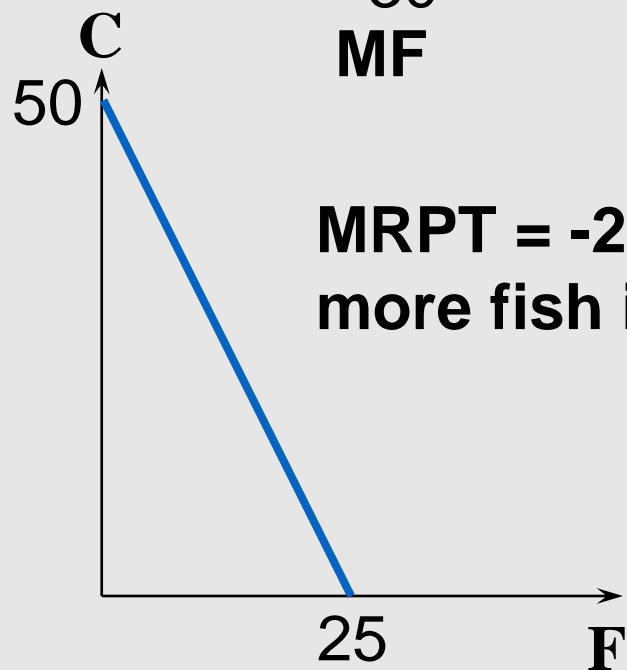
**MRPT =  $-\frac{2}{3}$  coconuts/fish so opp. cost of one more fish is  $\frac{2}{3}$  foregone coconuts.**



# Comparative Advantage

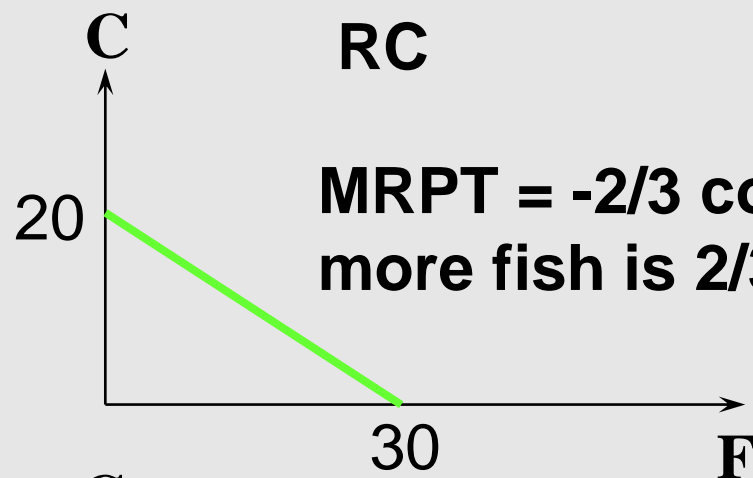


**MRPT =  $-\frac{2}{3}$  coconuts/fish so opp. cost of one more fish is  $\frac{2}{3}$  foregone coconuts.**



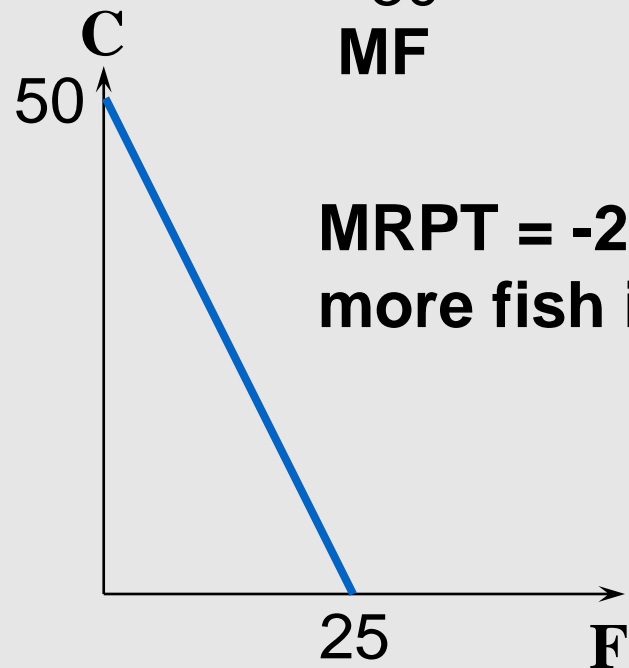
**MRPT =  $-2$  coconuts/fish so opp. cost of one more fish is 2 foregone coconuts.**

# Comparative Advantage



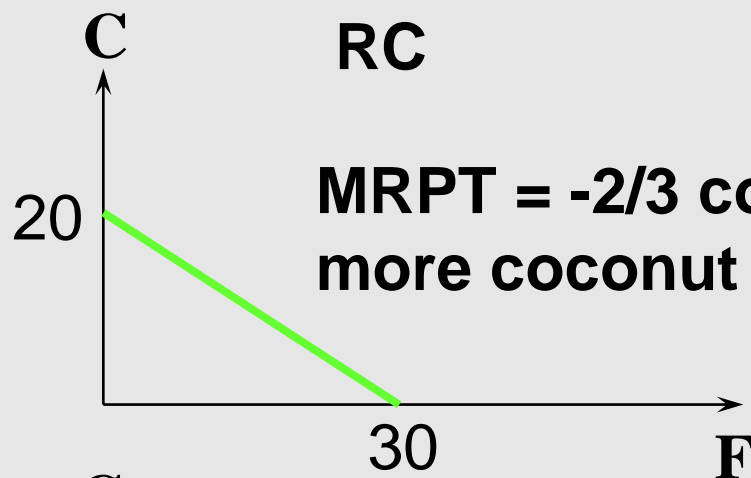
**MRPT =  $-2/3$  coconuts/fish so opp. cost of one more fish is  $2/3$  foregone coconuts.**

**RC has the comparative opp. cost advantage in producing fish.**

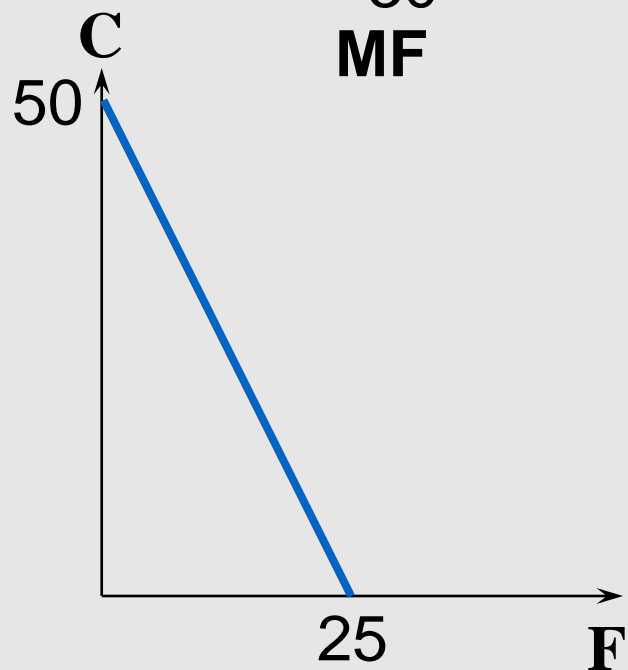


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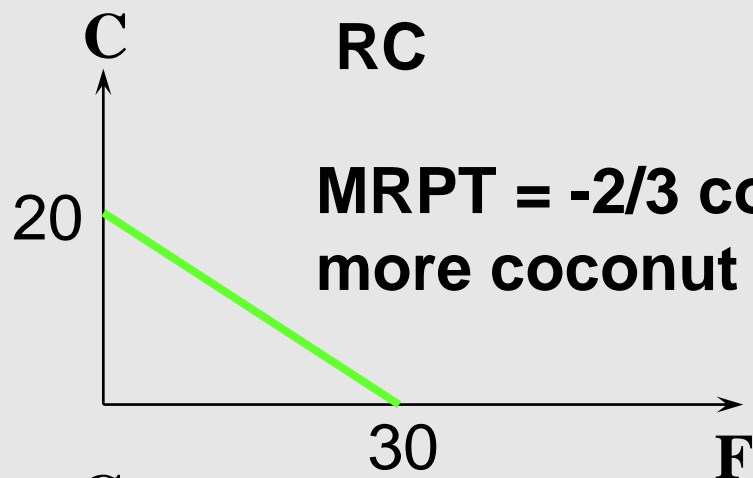
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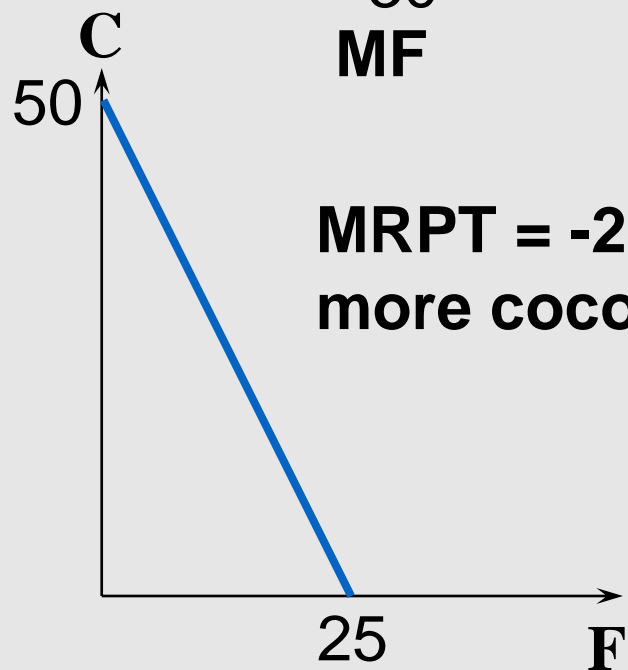
**MRPT =  $-\frac{2}{3}$  coconuts/fish so opp. cost of one more coconut is  $\frac{3}{2}$  foregone fish.**



# Comparative Advantage

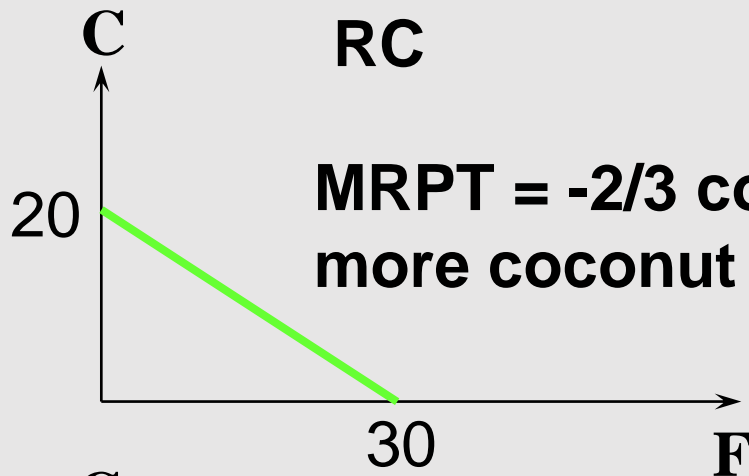


**MRPT =  $-\frac{2}{3}$  coconuts/fish so opp. cost of one more coconut is  $\frac{3}{2}$  foregone fish.**

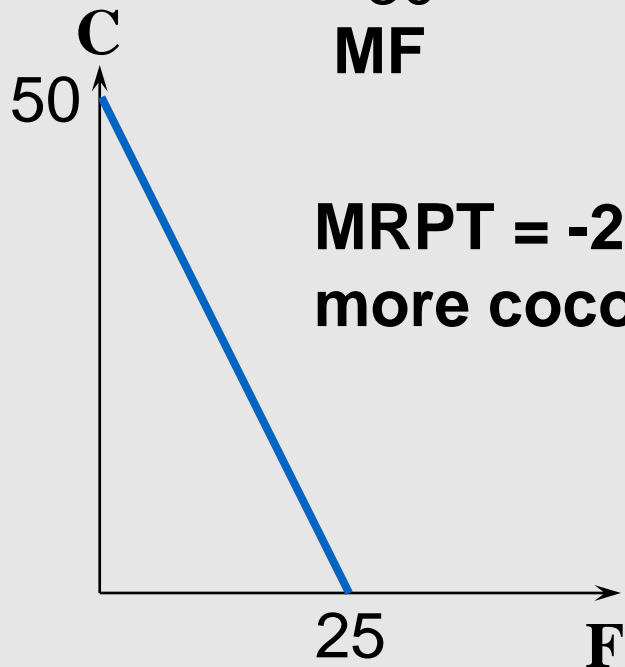


**MRPT =  $-2$  coconuts/fish so opp. cost of one more coconut is  $\frac{1}{2}$  foregone fish.**

# Comparative Advantage



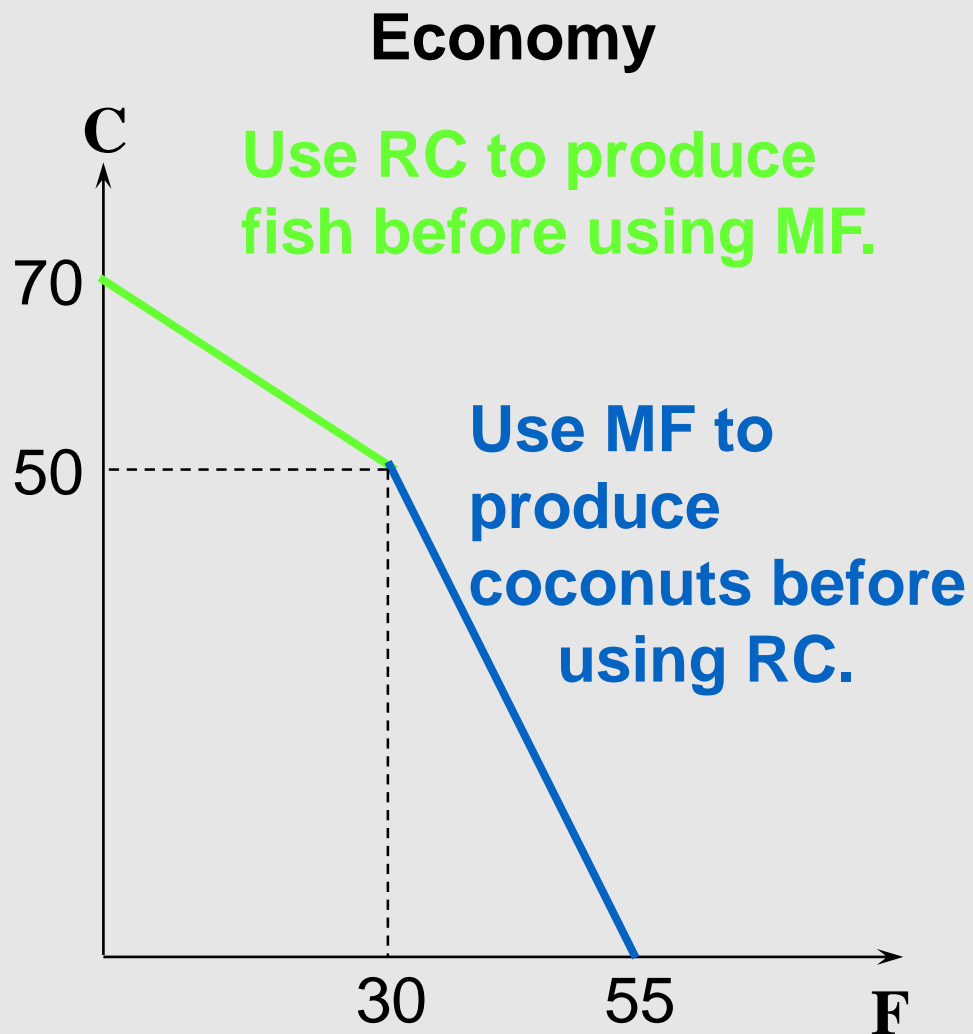
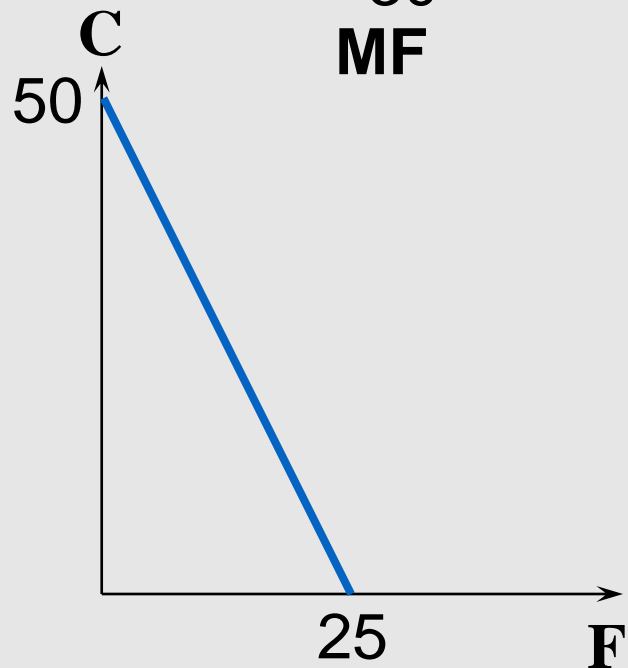
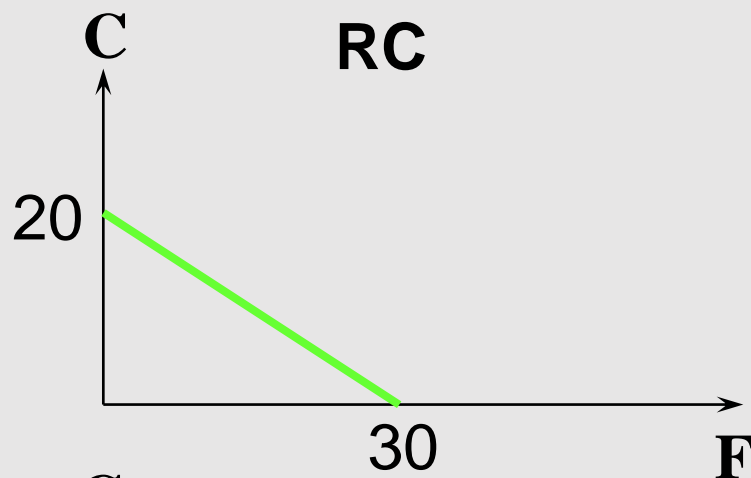
**MRPT =  $-\frac{2}{3}$  coconuts/fish so opp. cost of one more coconut is  $\frac{3}{2}$  foregone fish.**



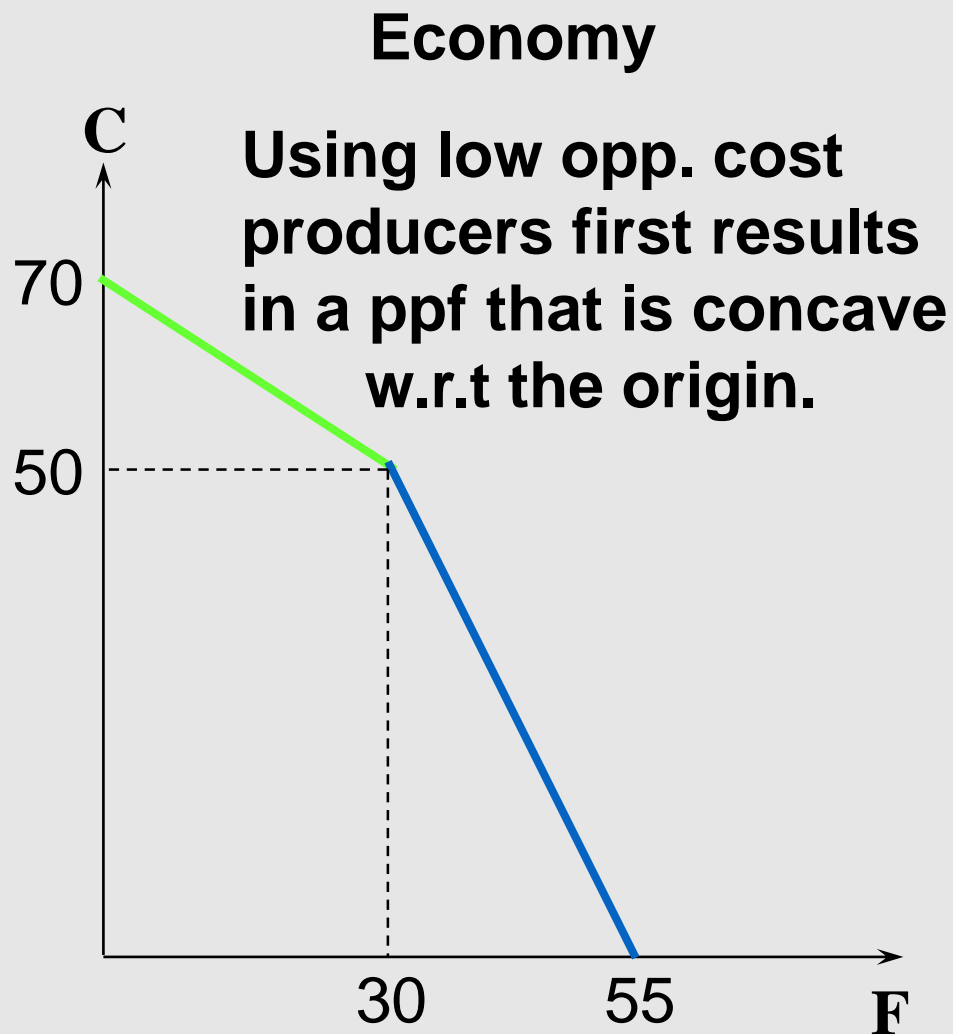
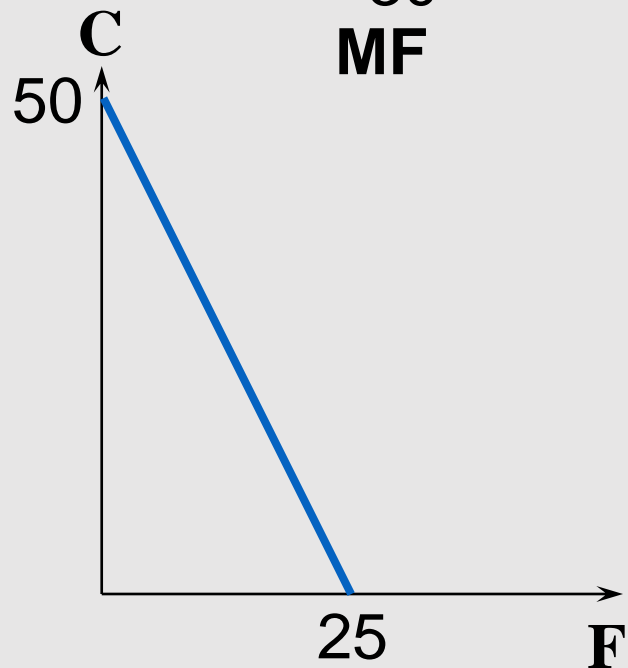
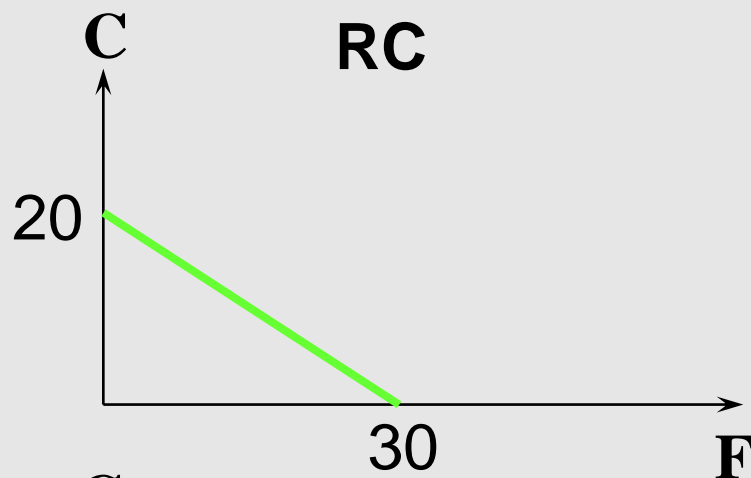
**MRPT =  $-2$  coconuts/fish so opp. cost of one more coconut is  $\frac{1}{2}$  foregone fish.**

**MF has the comparative opp. cost advantage in producing coconuts.**

# Comparative Advantage



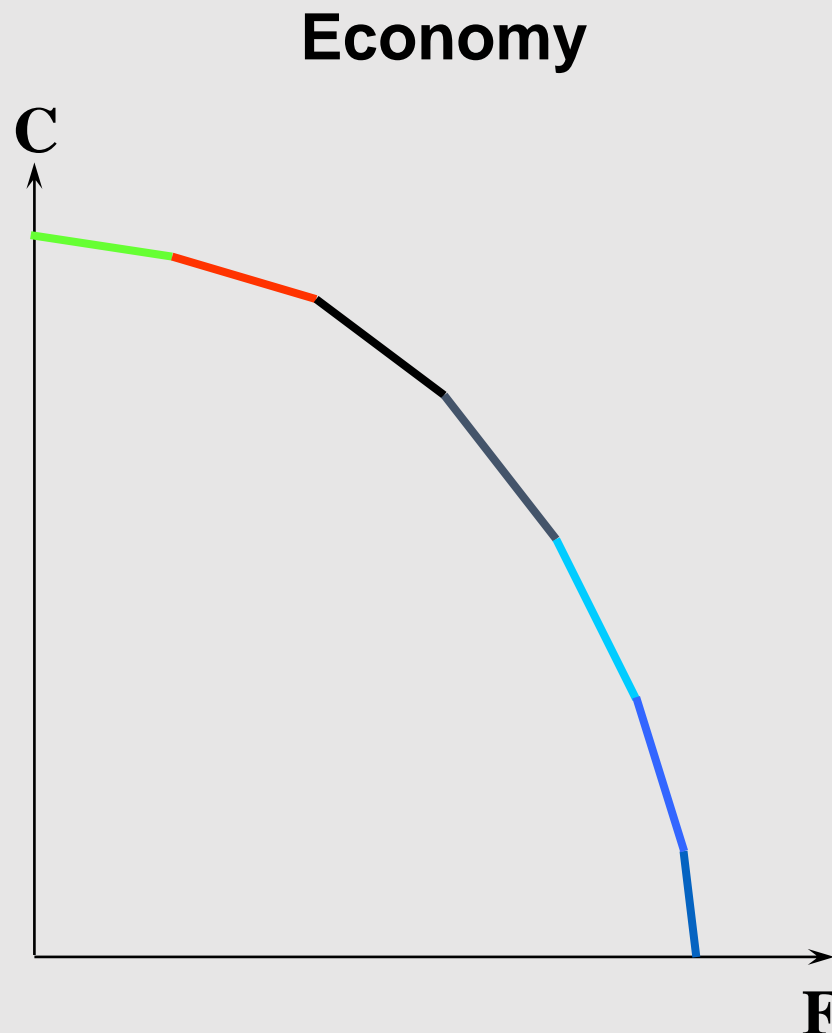
# Comparative Advantage





# Comparative Advantage

**More producers with  
different opp. costs  
“smooth out” the ppf.**

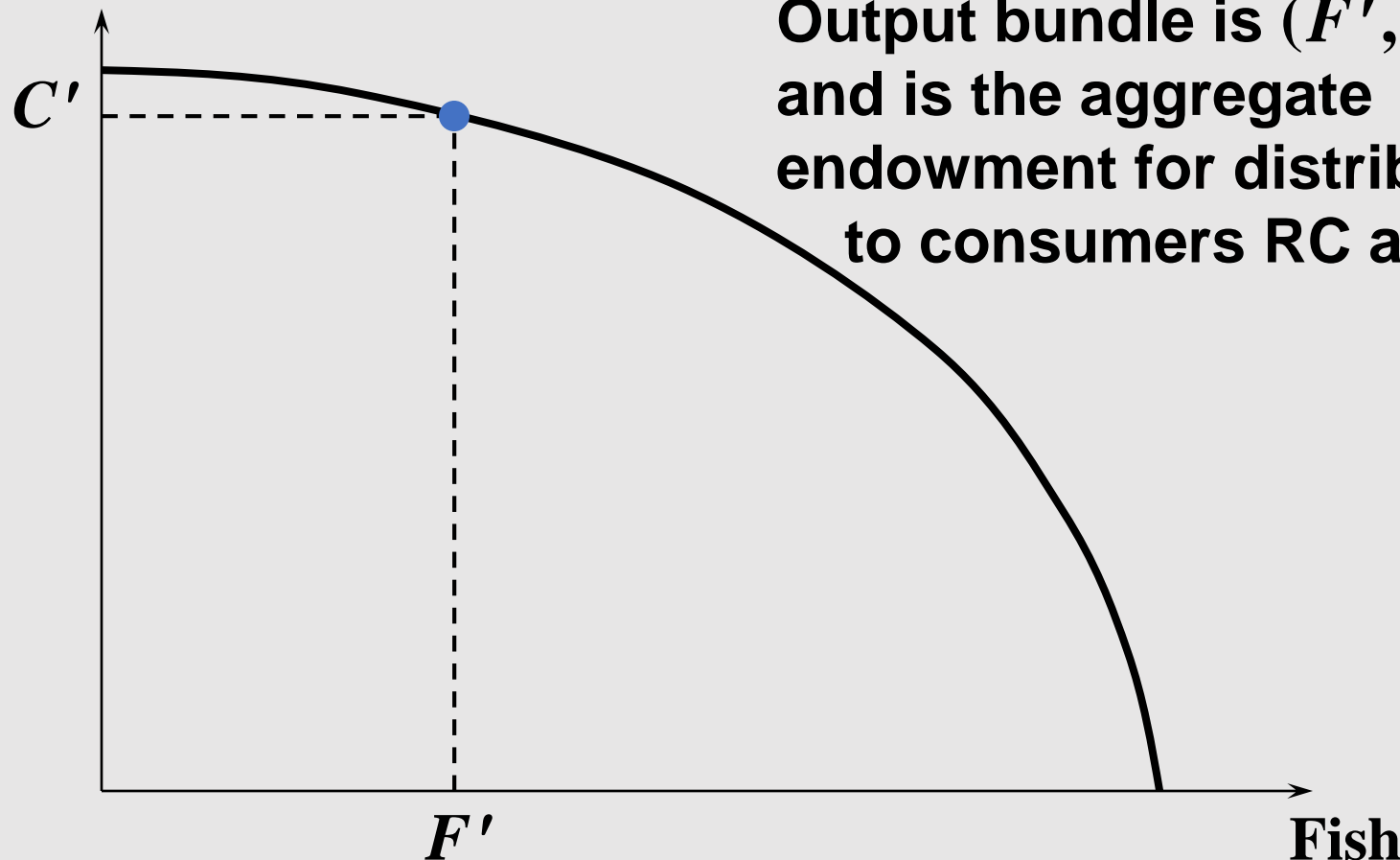


# Coordinating Production & Consumption

- The ppf contains many technically efficient output bundles.
- Which are Pareto efficient for consumers?

# Coordinating Production & Consumption

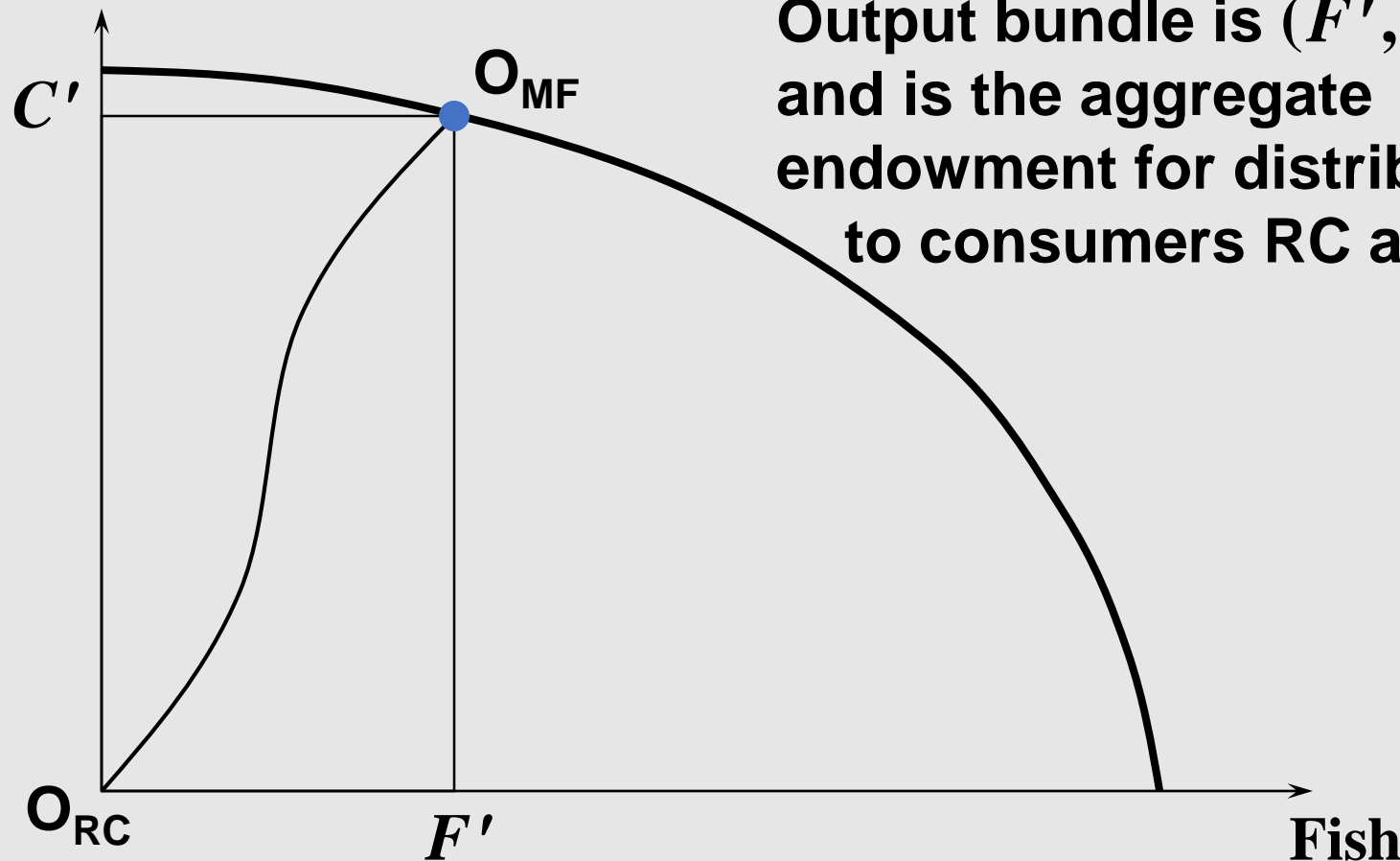
Coconuts



**Output bundle is  $(F', C')$   
and is the aggregate  
endowment for distribution  
to consumers RC and MF.**

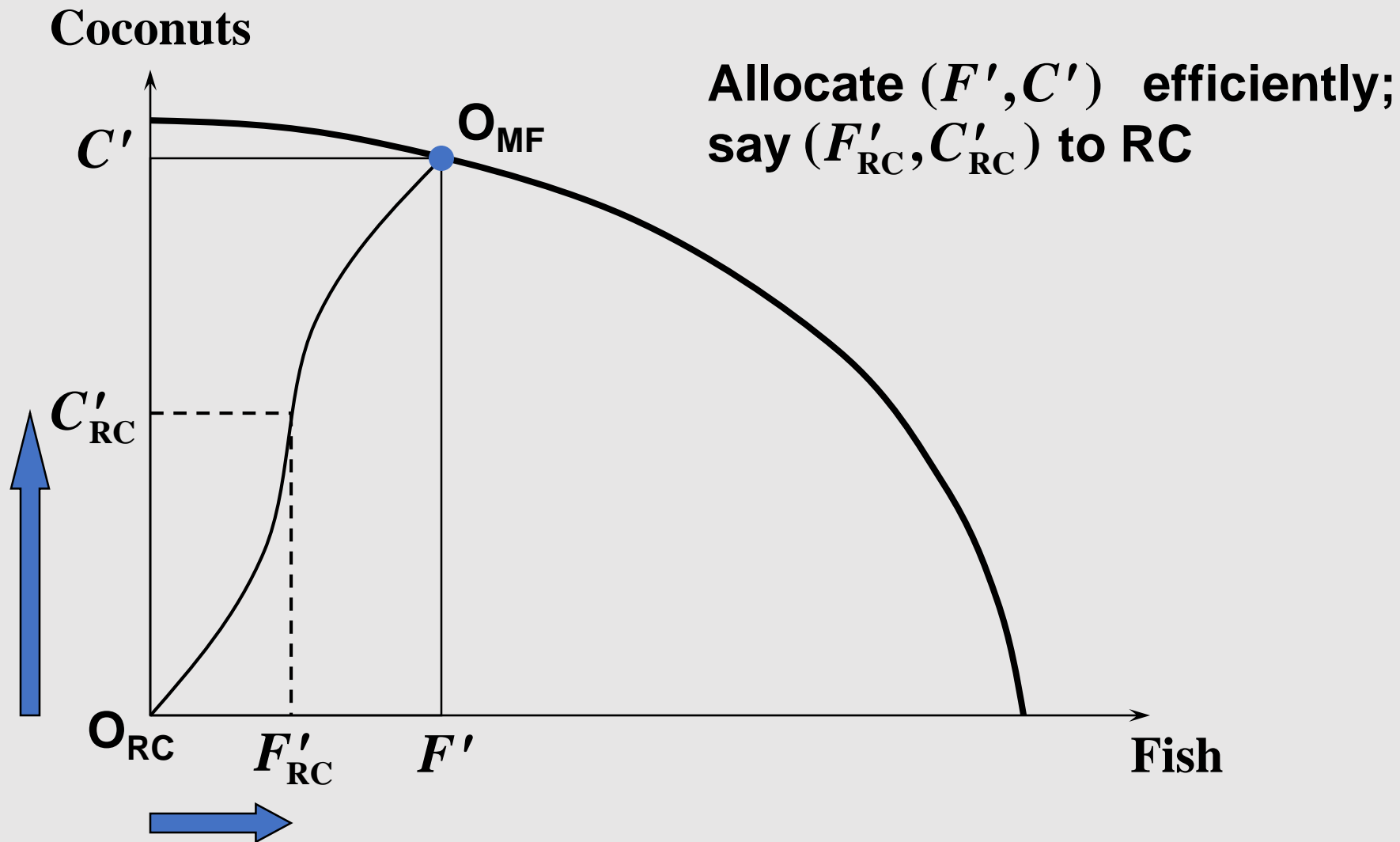
# Coordinating Production & Consumption

Coconuts

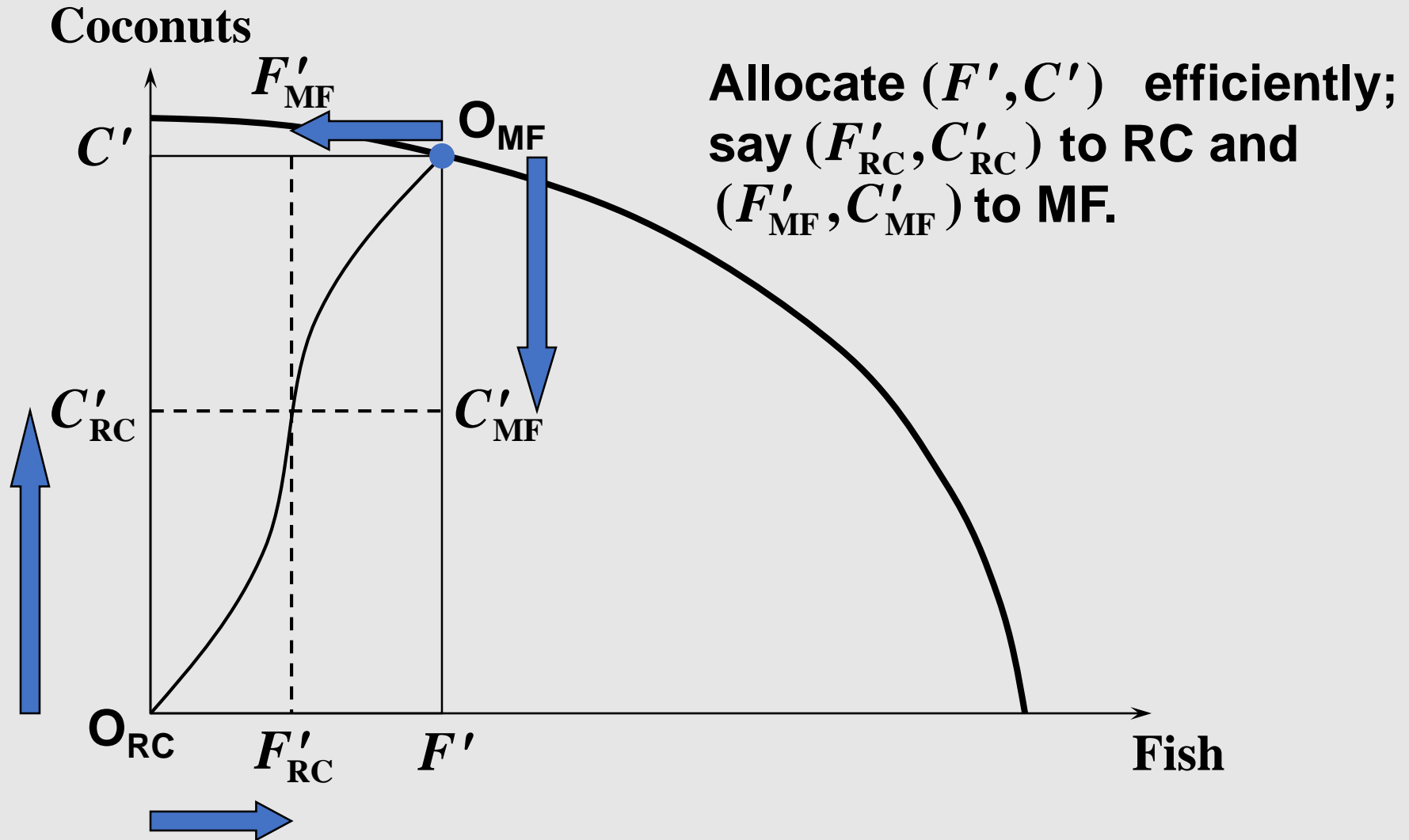


Output bundle is  $(F', C')$   
and is the aggregate  
endowment for distribution  
to consumers RC and MF.

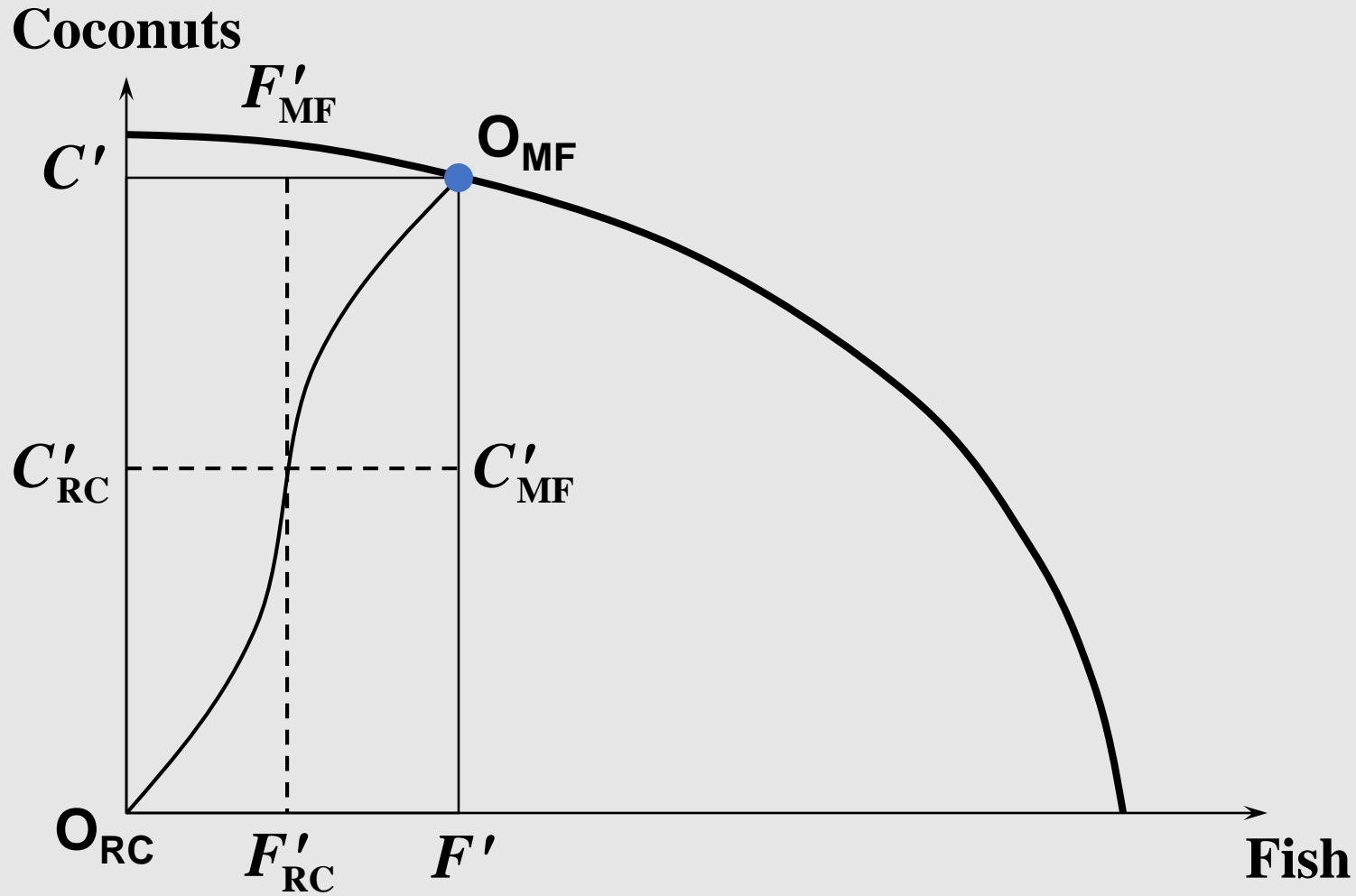
# Coordinating Production & Consumption



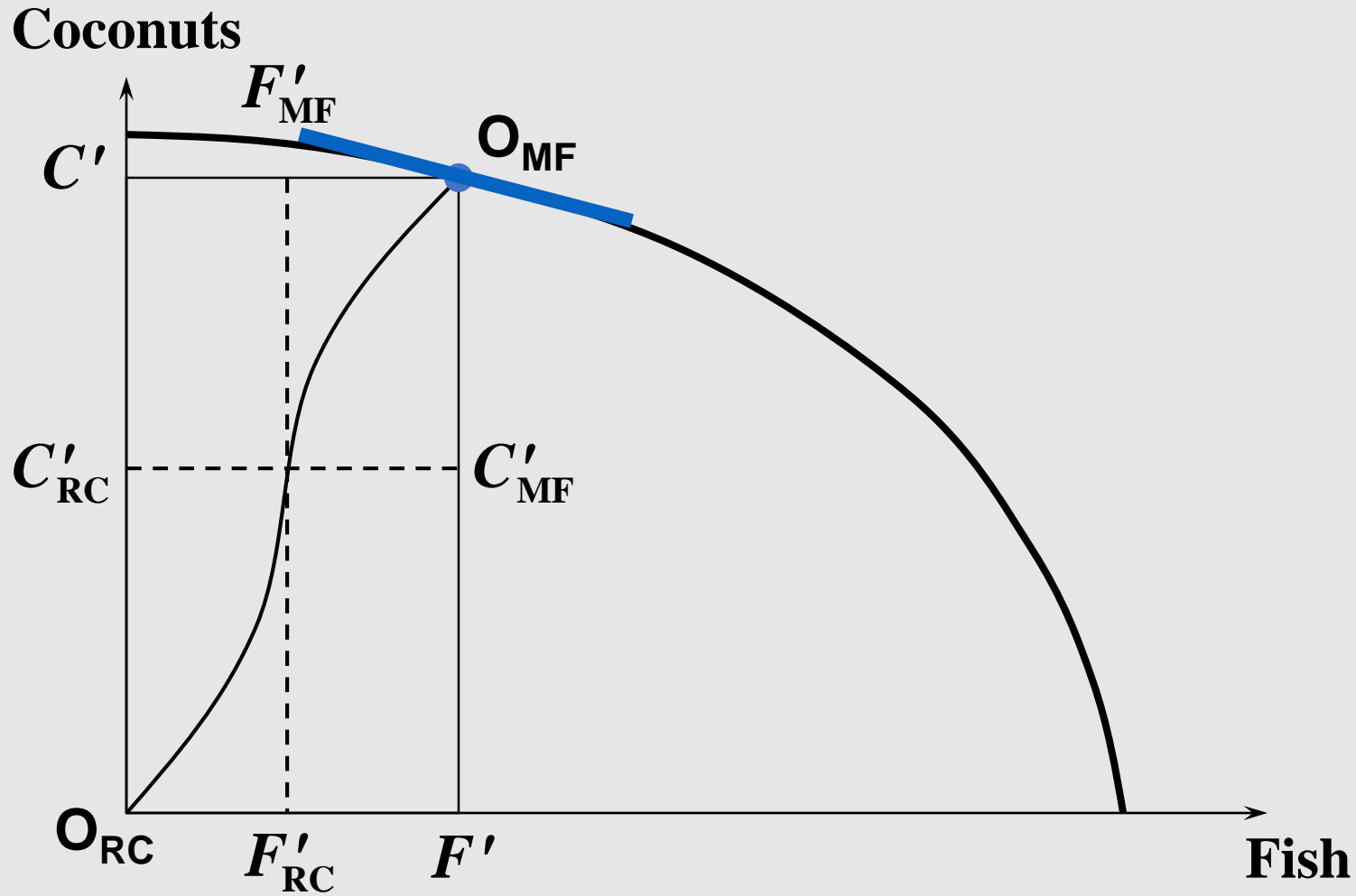
# Coordinating Production & Consumption



# Coordinating Production & Consumption

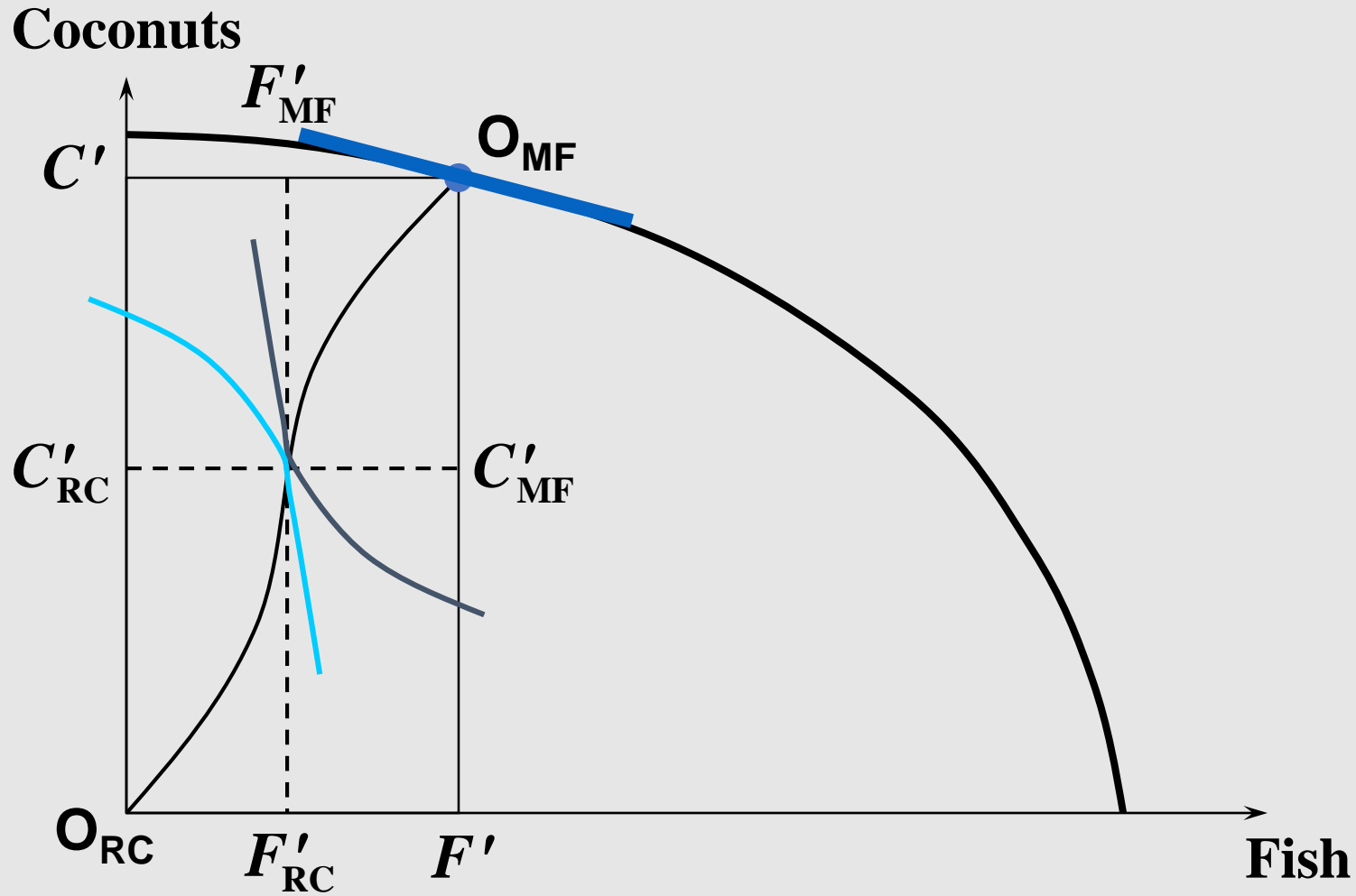


# Coordinating Production & Consumption

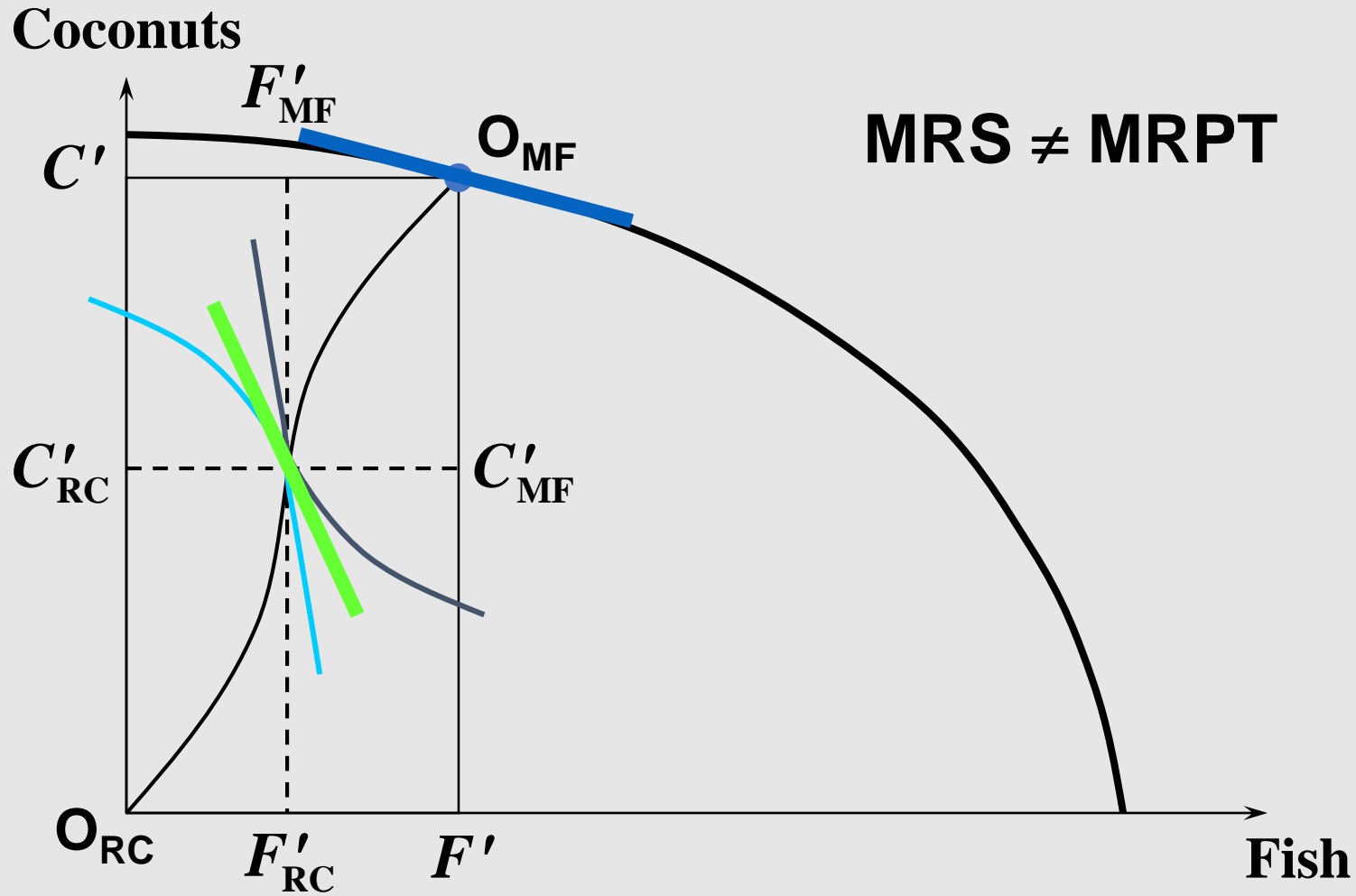




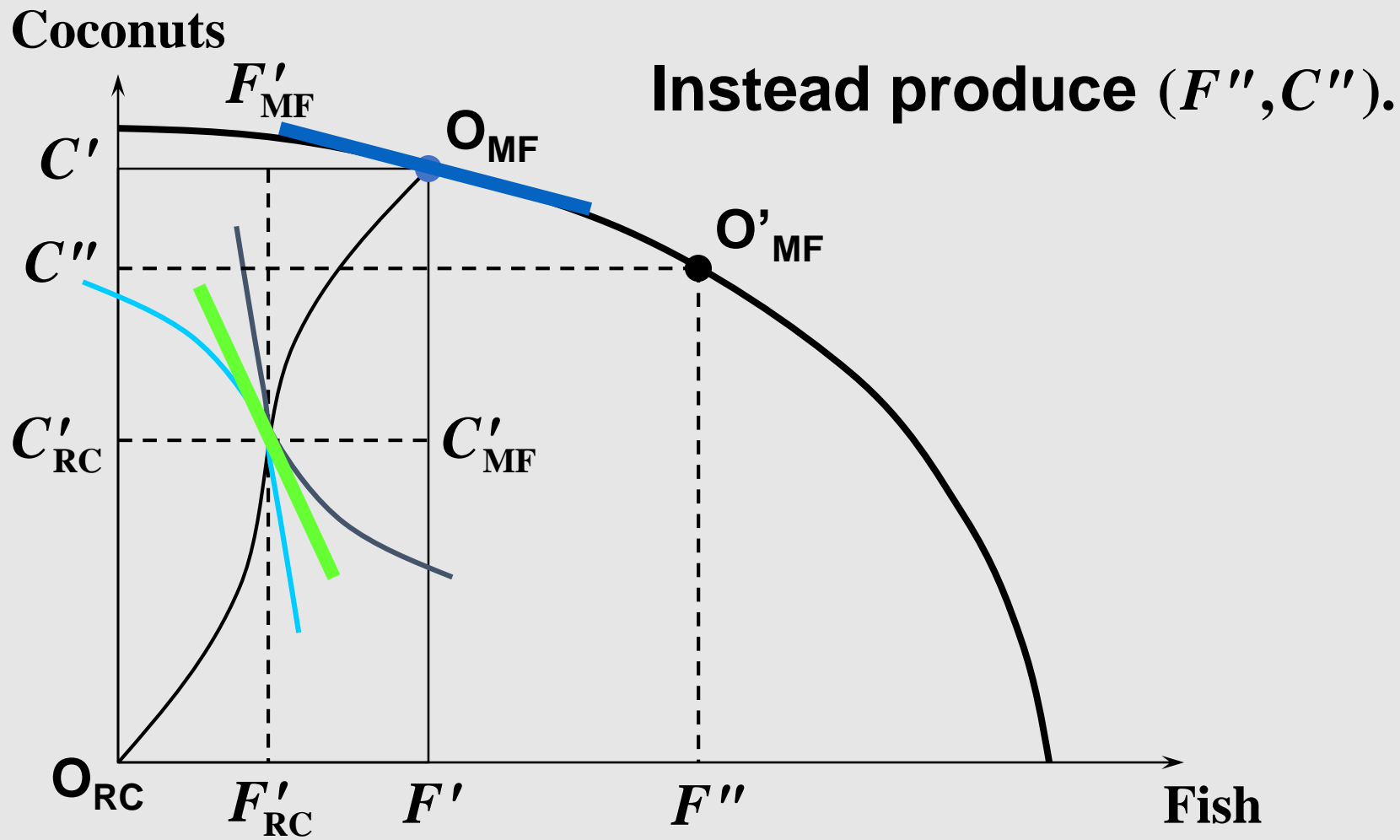
# Coordinating Production & Consumption



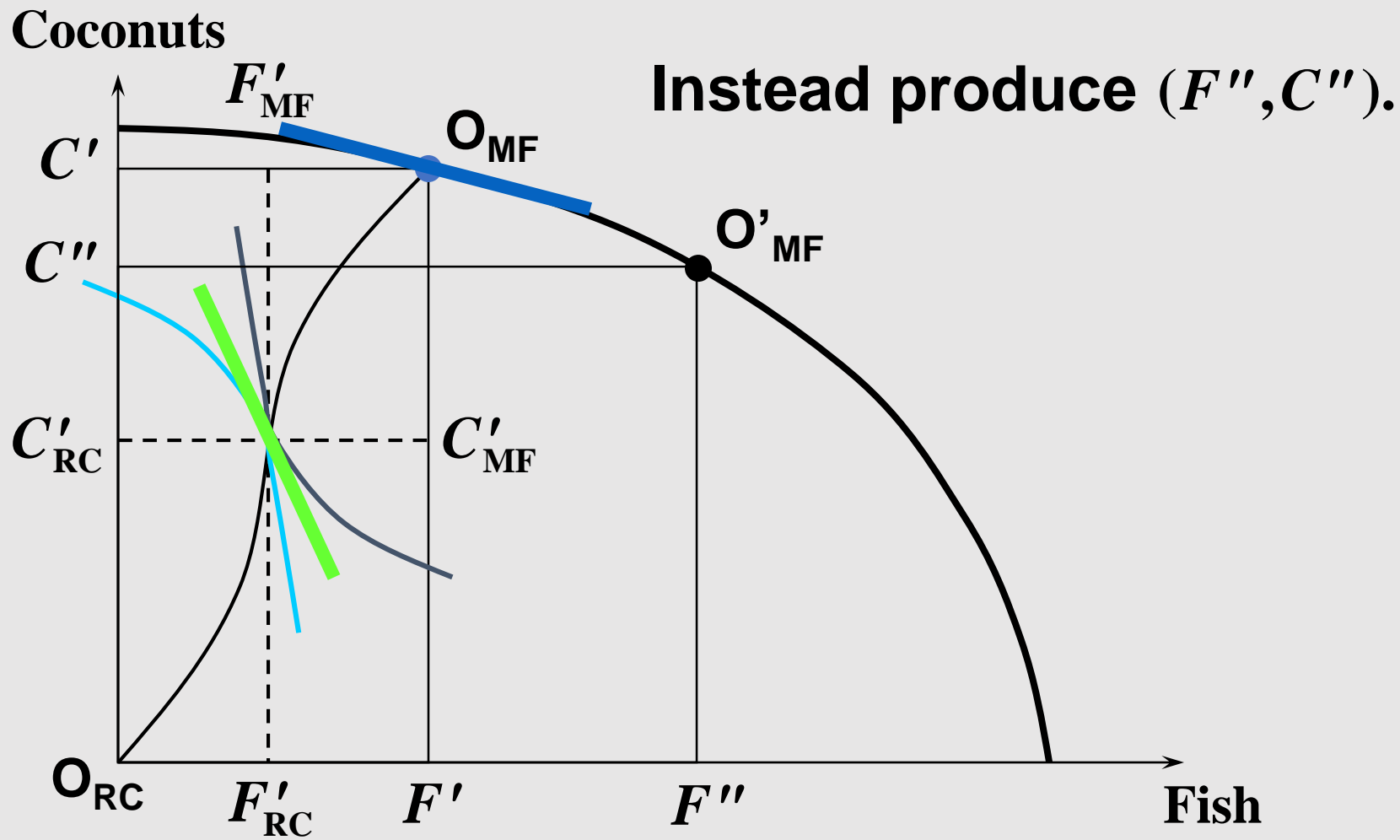
# Coordinating Production & Consumption



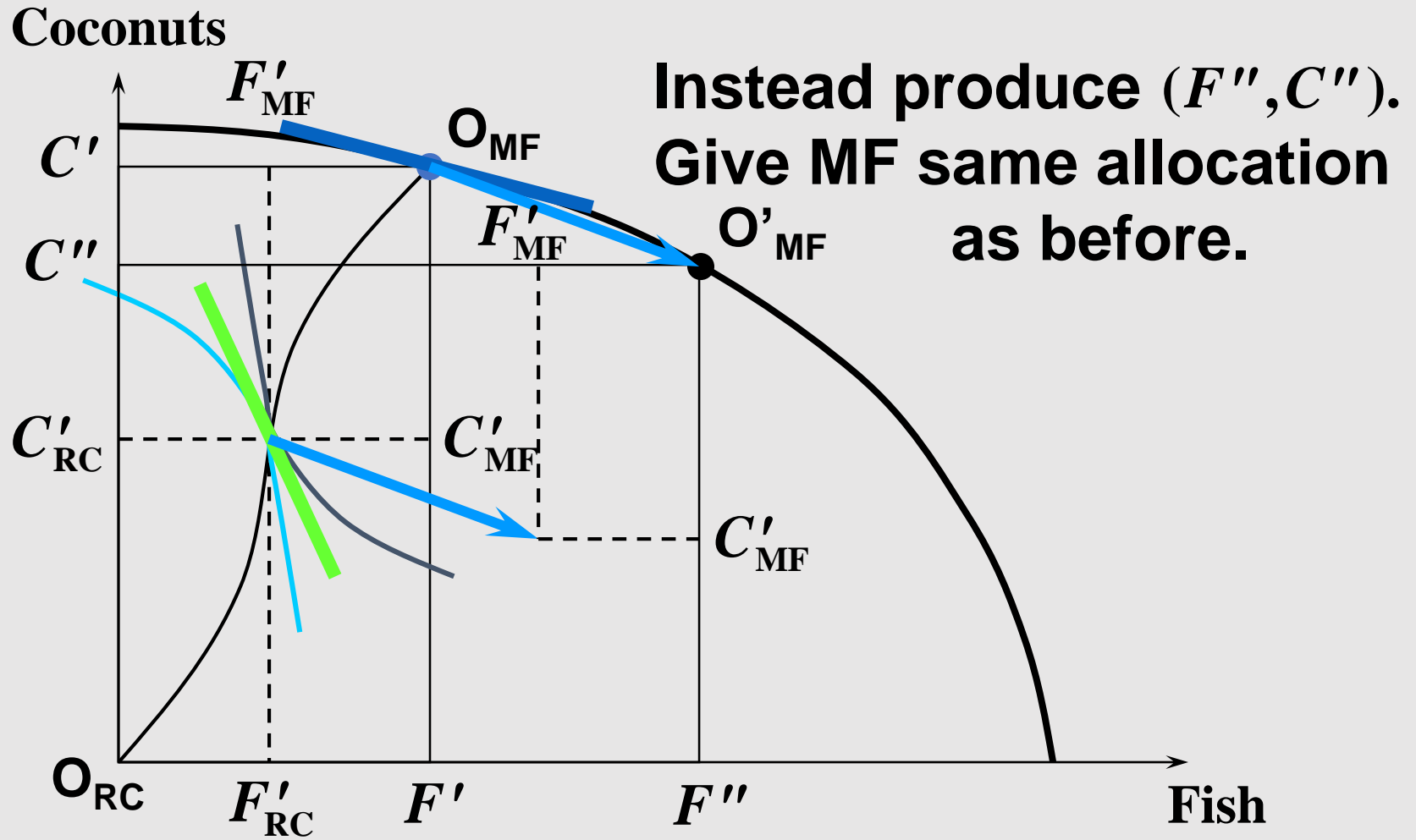
# Coordinating Production & Consumption



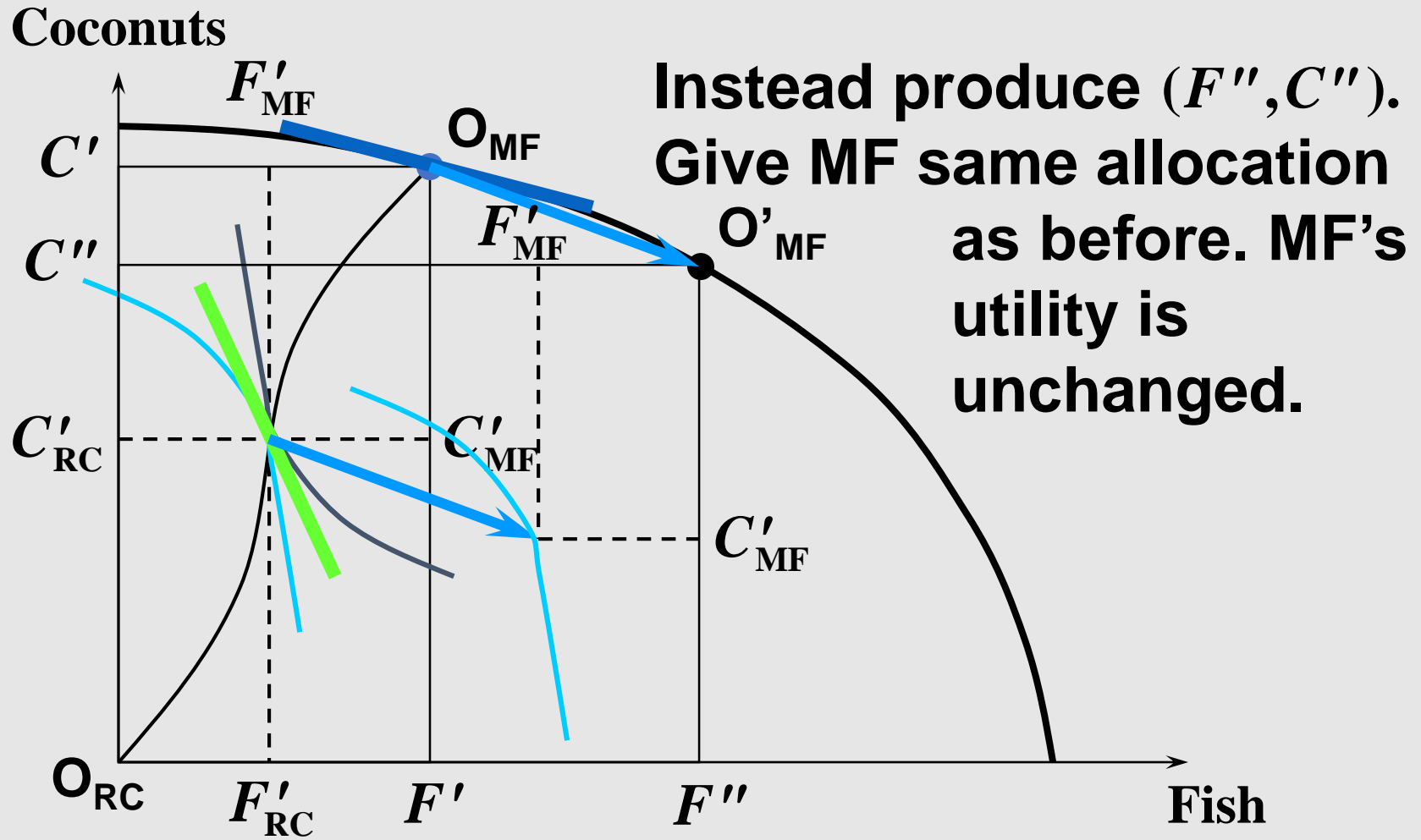
# Coordinating Production & Consumption



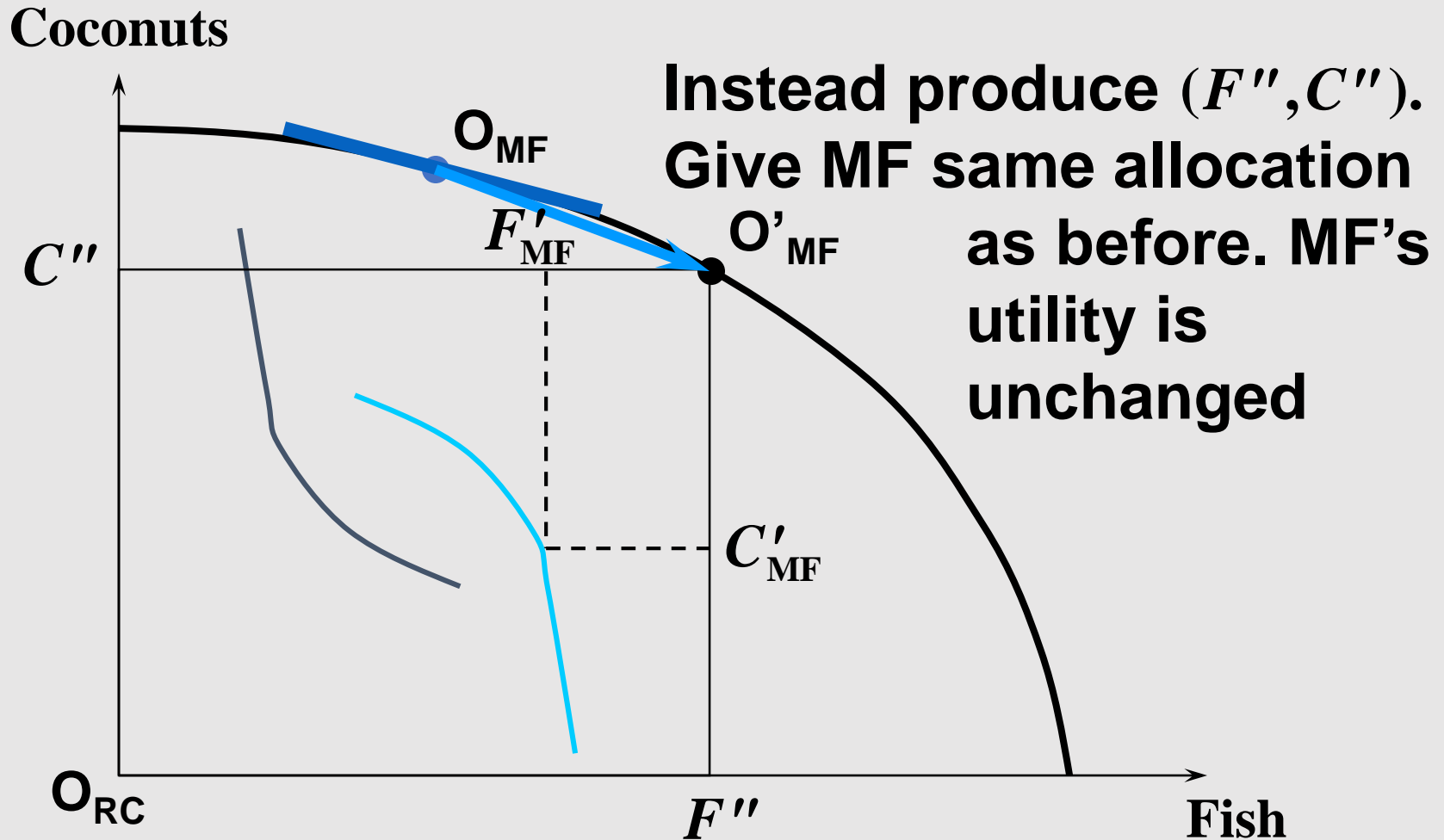
# Coordinating Production & Consumption



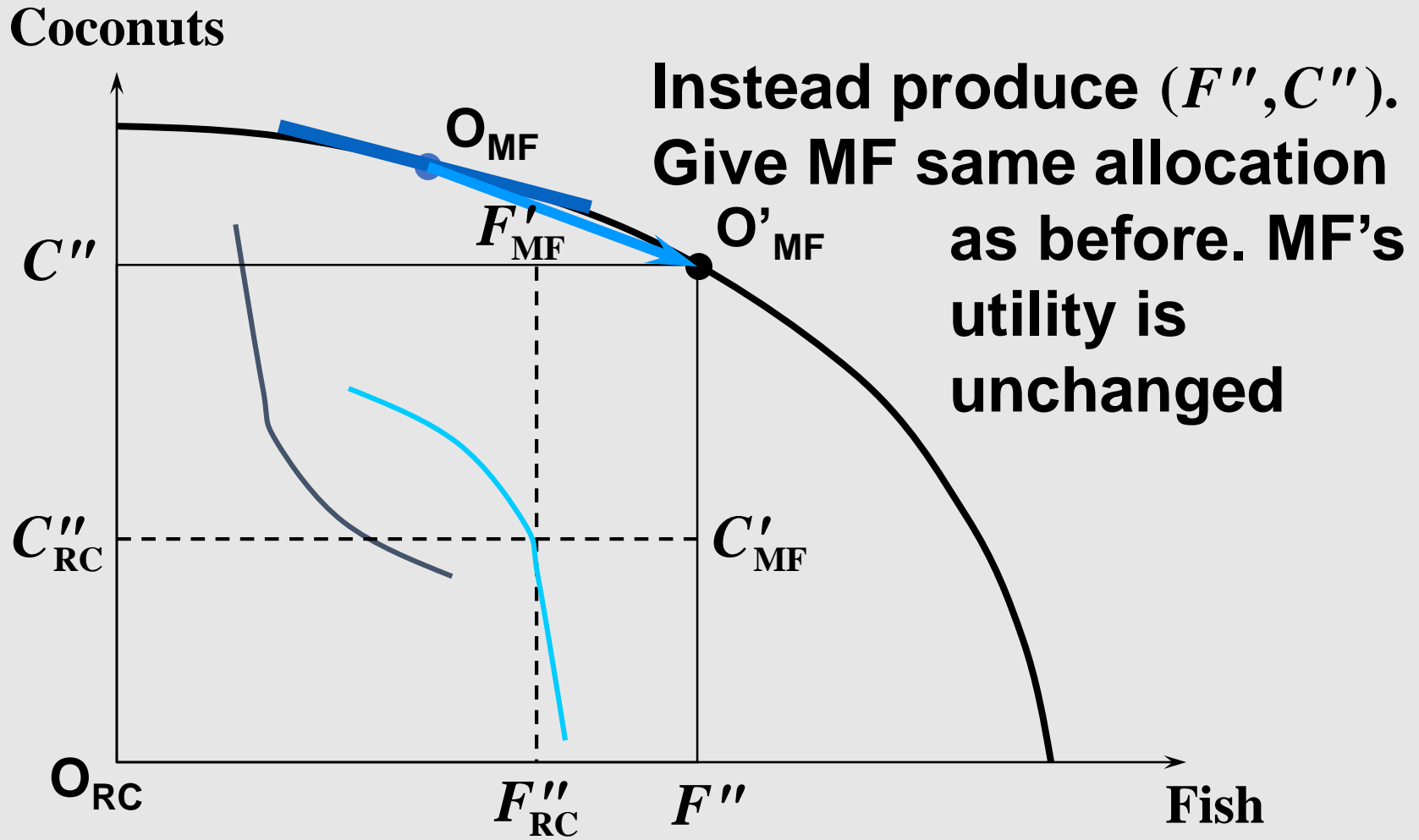
# Coordinating Production & Consumption



# Coordinating Production & Consumption

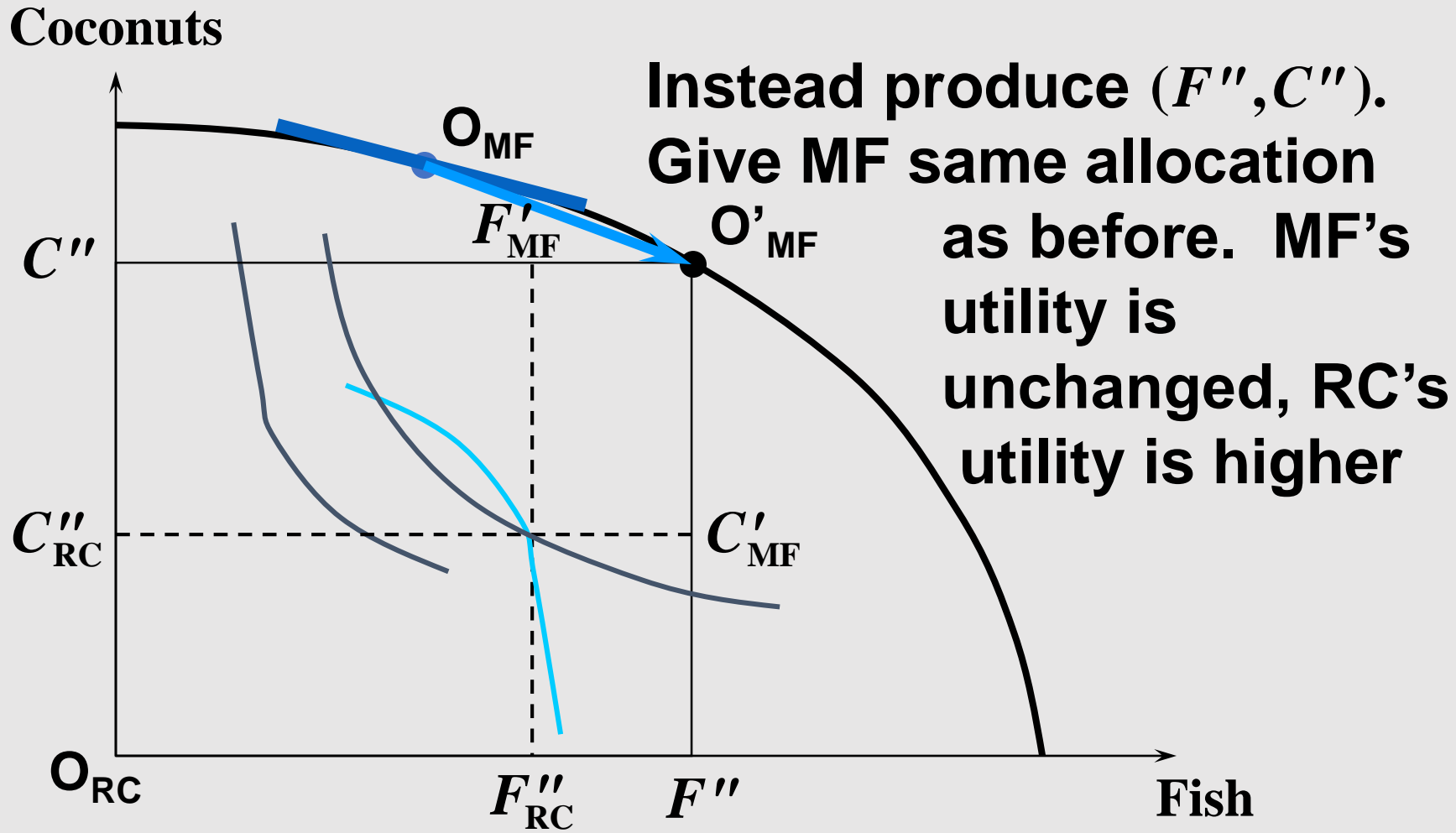


# Coordinating Production & Consumption

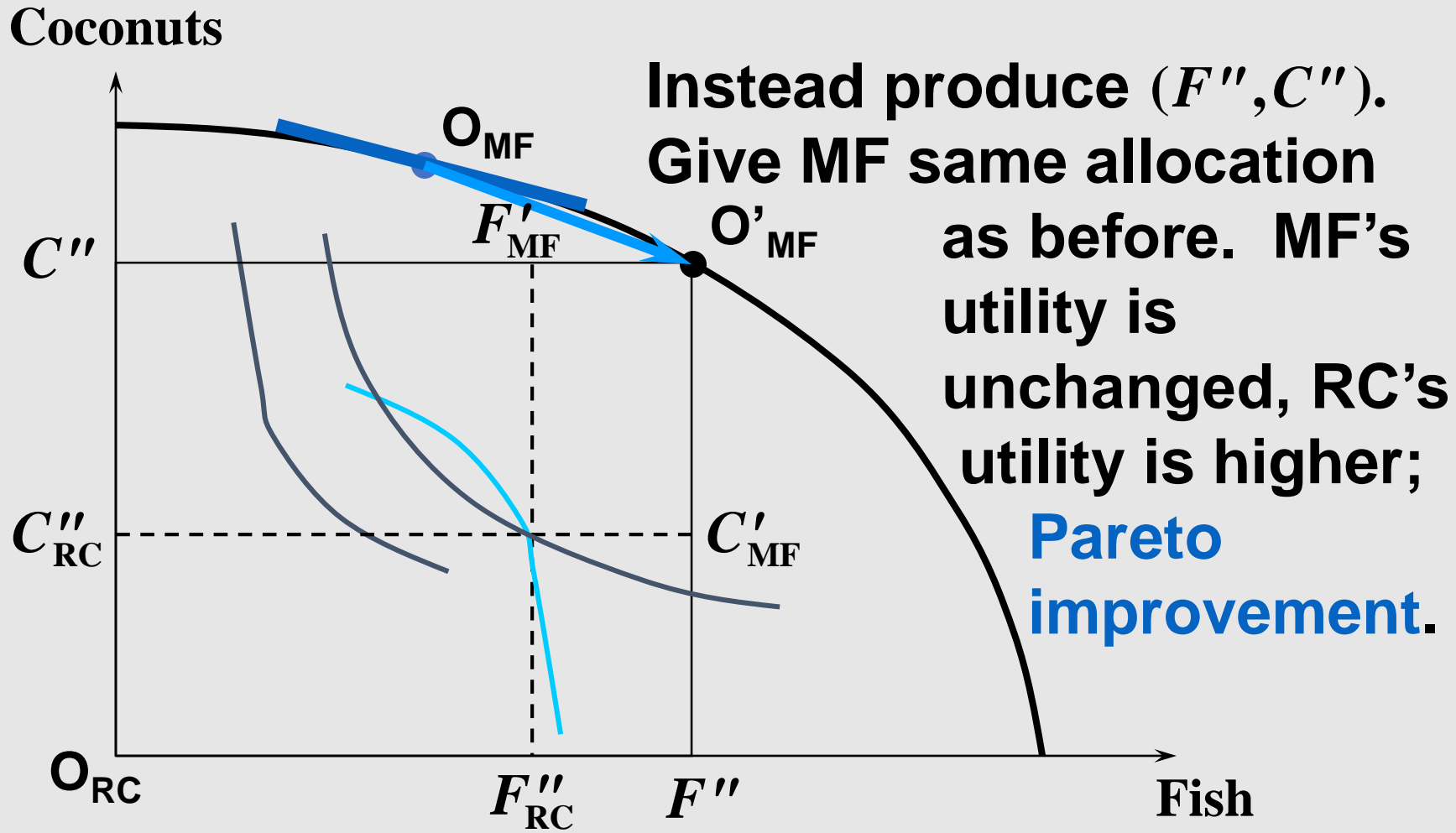




# Coordinating Production & Consumption



# Coordinating Production & Consumption

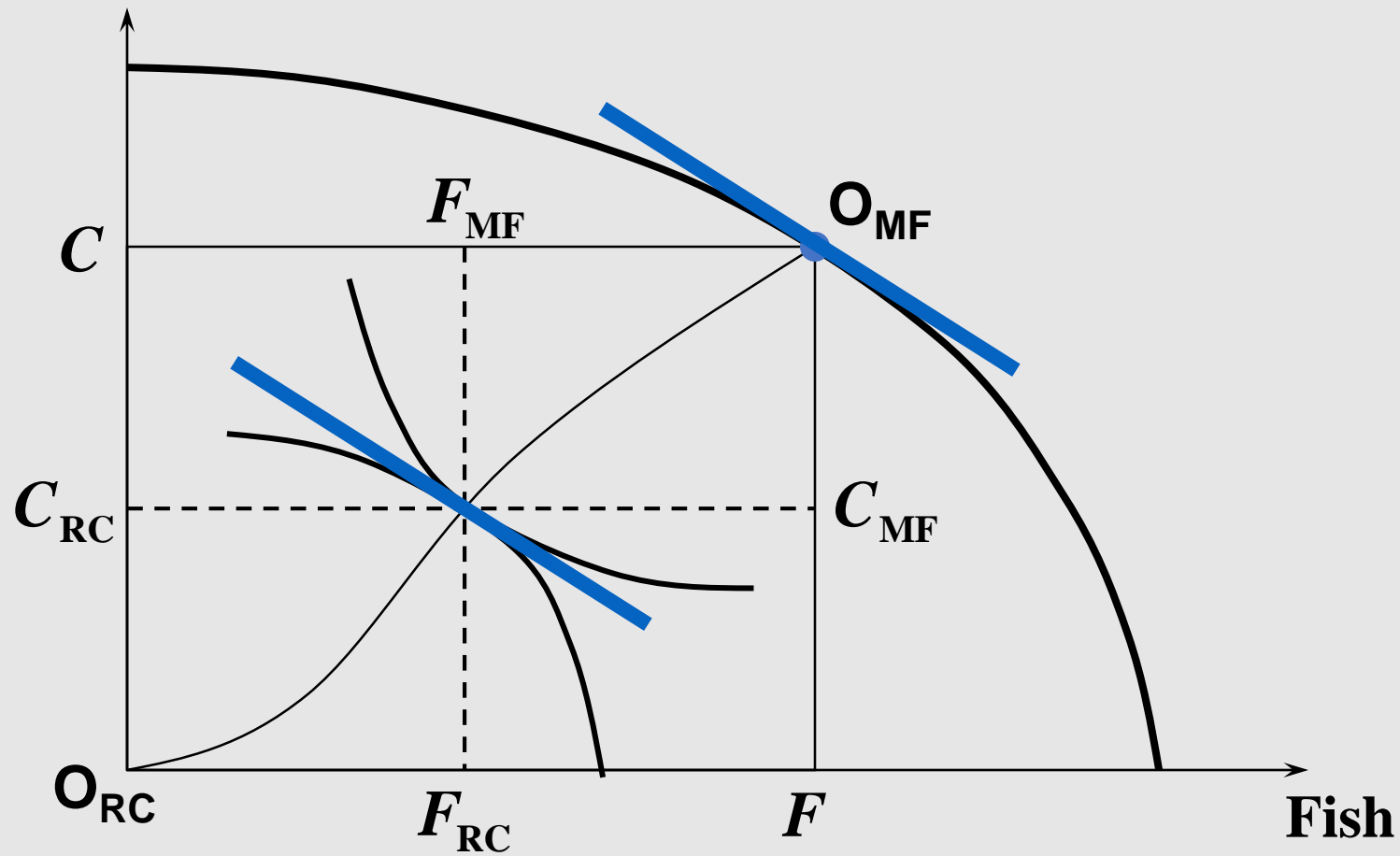


# Coordinating Production & Consumption

- $MRS \neq MRPT \Rightarrow$  inefficient coordination of production and consumption.
- Hence,  $MRS = MRPT$  is necessary for a Pareto optimal economic state.

# Coordinating Production & Consumption

Coconuts



# Decentralized Coordination of Production & Consumption

- RC and MF jointly run a firm producing coconuts and fish.
- RC and MF are also consumers who can sell labor.
- Price of coconut =  $p_C$ .
- Price of fish =  $p_F$ .
- RC's wage rate =  $w_{RC}$ .
- MF's wage rate =  $w_{MF}$ .

# Decentralized Coordination of Production & Consumption

→  $L_{RC}$ ,  $L_{MF}$  are amounts of labor purchased from RC and MF.

→ F irm's profit-maximization problem is choose  $C$ ,  $F$ ,  $L_{RC}$  and  $L_{MF}$  to

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

# Decentralized Coordination of Production & Consumption

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

**Isoprofit line equation is**

$$\text{constant } \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}$$

# Decentralized Coordination of Production & Consumption

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

**Isoprofit line equation is**

$$\text{constant } \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}$$

**which rearranges to**

$$C = \frac{\pi + w_{RC} L_{RC} + w_{MF} L_{MF}}{p_C} - \frac{p_F}{p_C} F.$$



# Decentralized Coordination of Production & Consumption

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

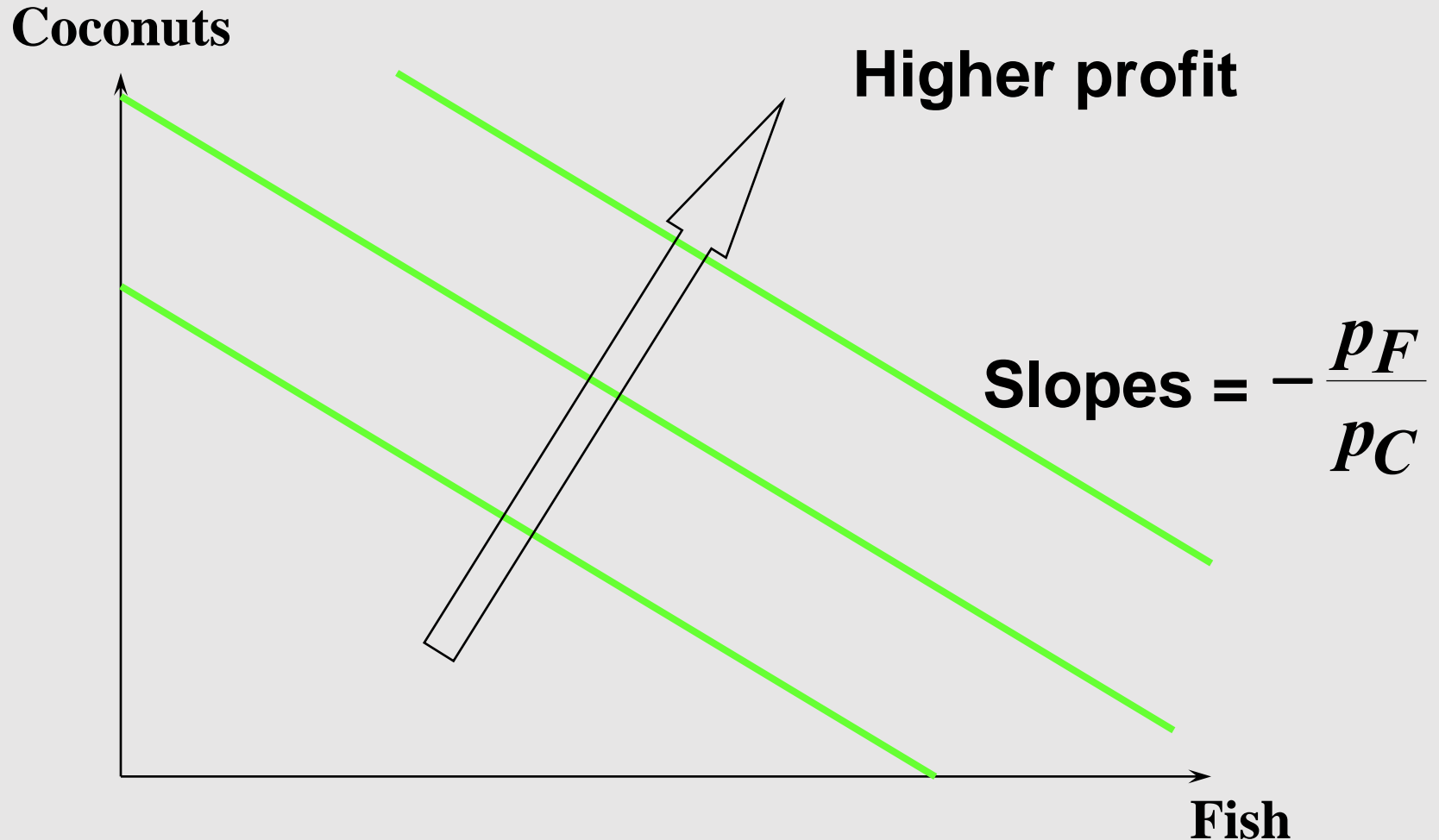
**Isoprofit line equation is**

$$\text{constant } \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}$$

**which rearranges to**

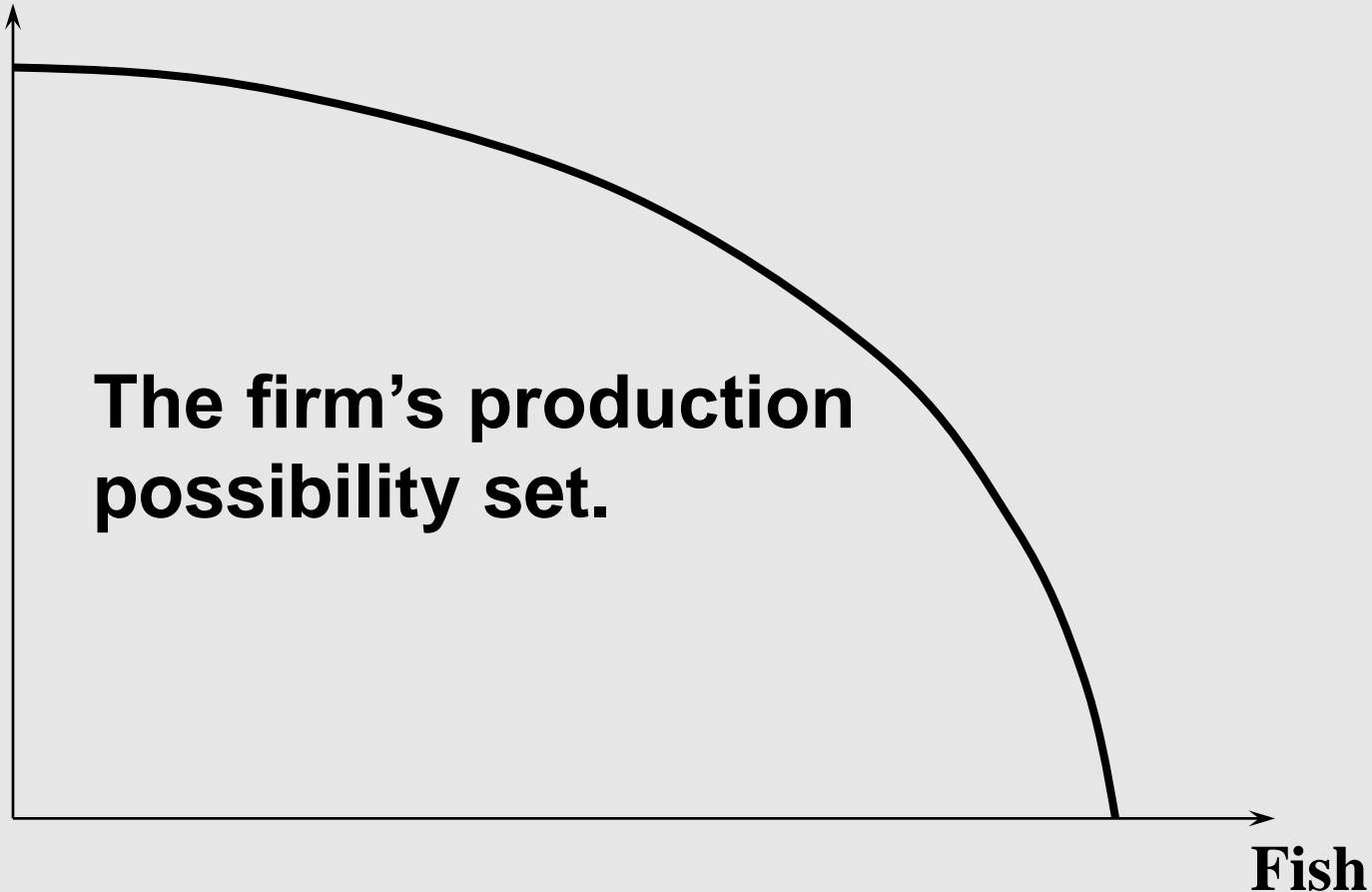
$$C = \underbrace{\frac{\pi + w_{RC} L_{RC} + w_{MF} L_{MF}}{p_C}}_{\text{Intercept}} - \underbrace{\frac{p_F}{p_C} F}_{\text{Slope}}.$$

# Decentralized Coordination of Production & Consumption



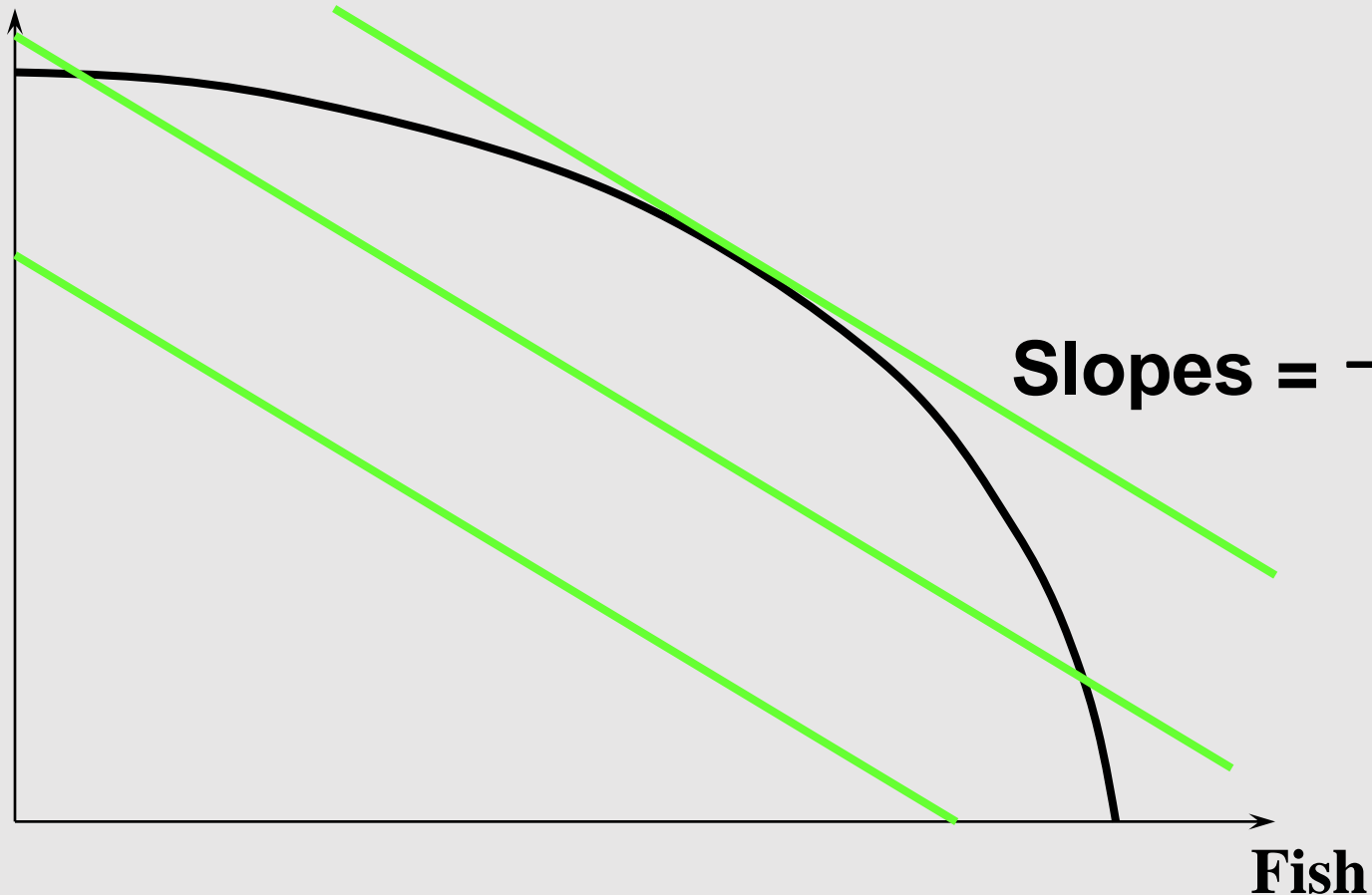
# Decentralized Coordination of Production & Consumption

**Coconuts**

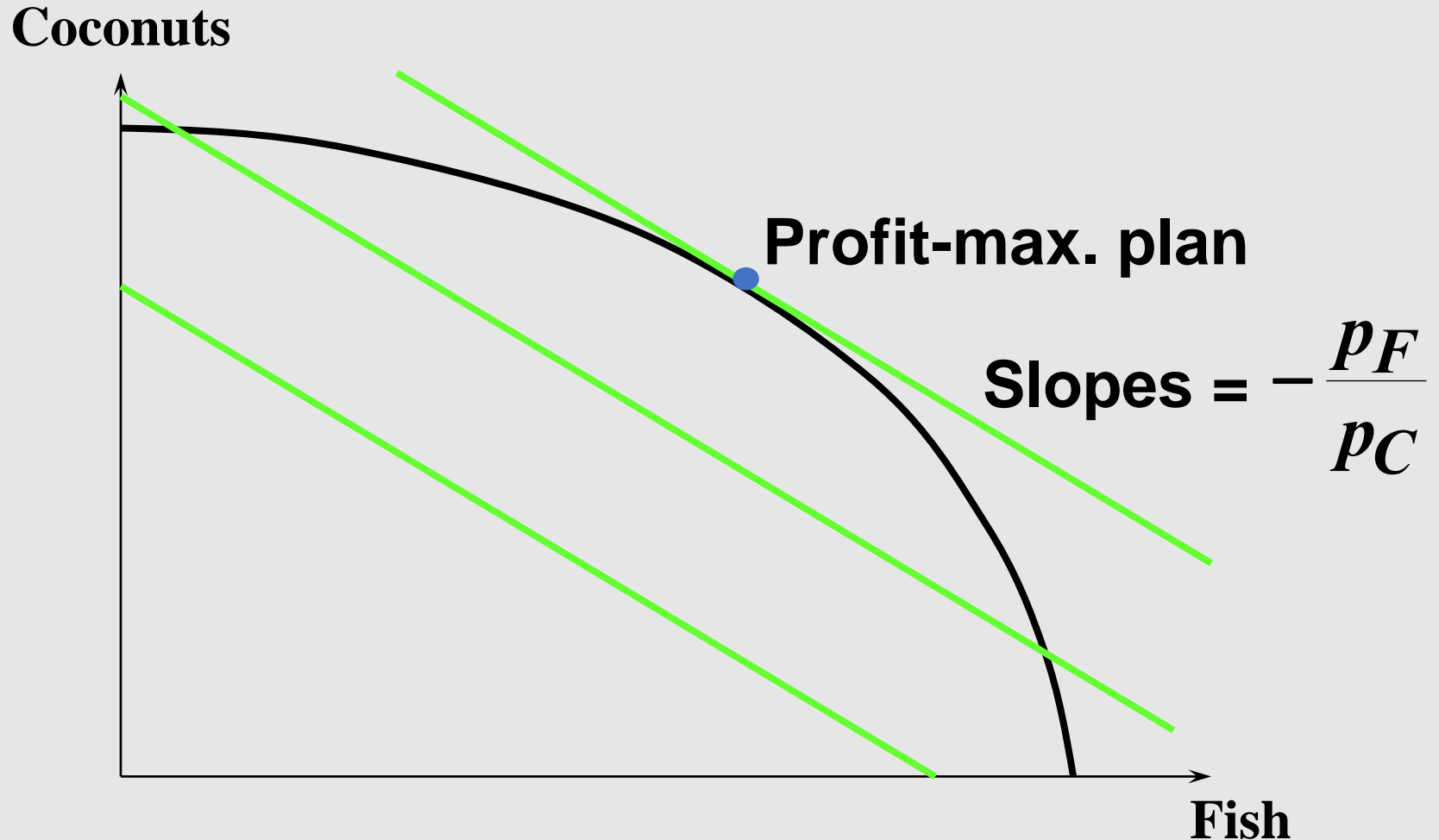


# Decentralized Coordination of Production & Consumption

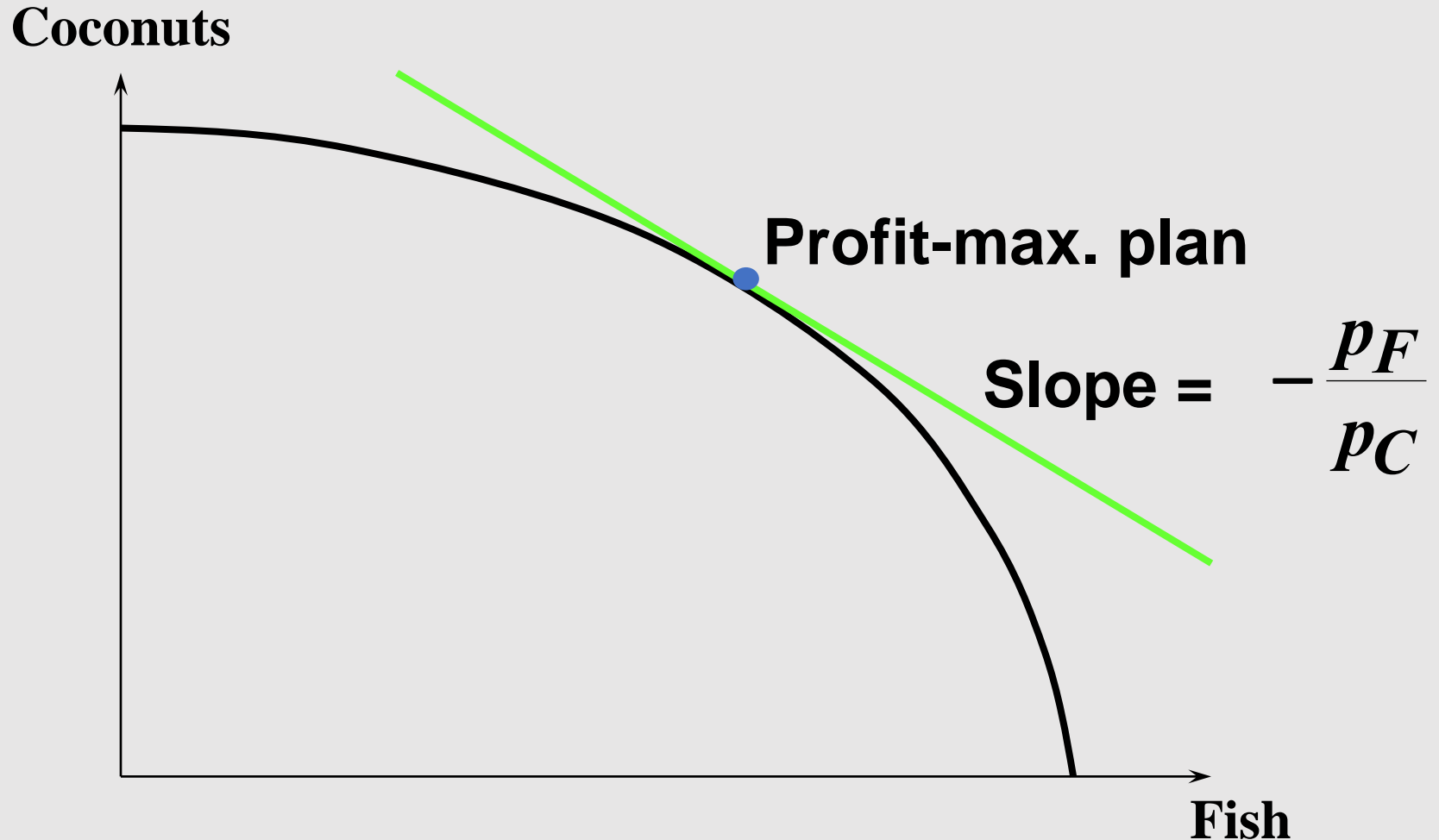
**Coconuts**



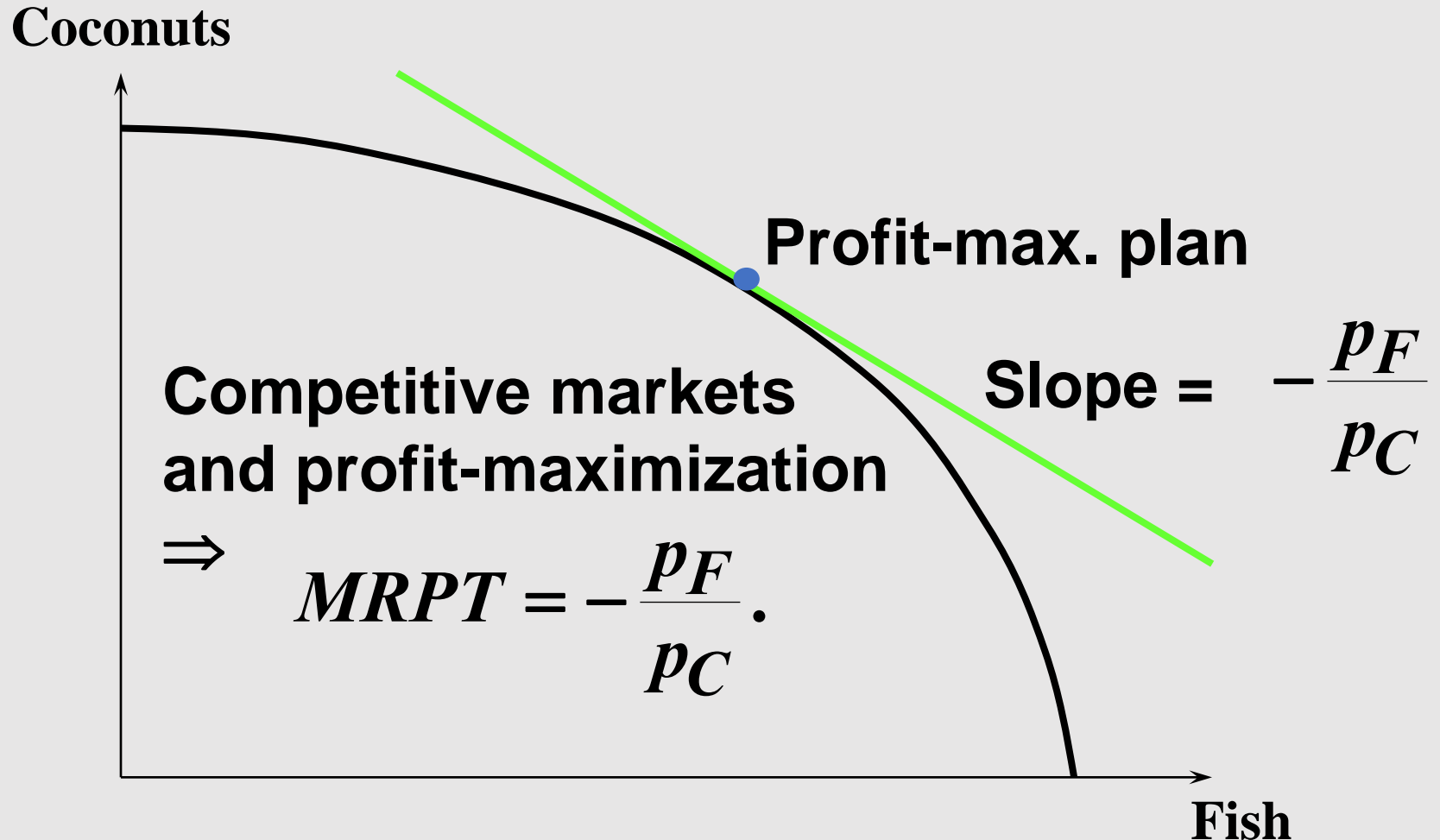
# Decentralized Coordination of Production & Consumption



# Decentralized Coordination of Production & Consumption



# Decentralized Coordination of Production & Consumption



# Decentralized Coordination of Production & Consumption

→ So competitive markets, profit-maximization, and utility maximization all together cause

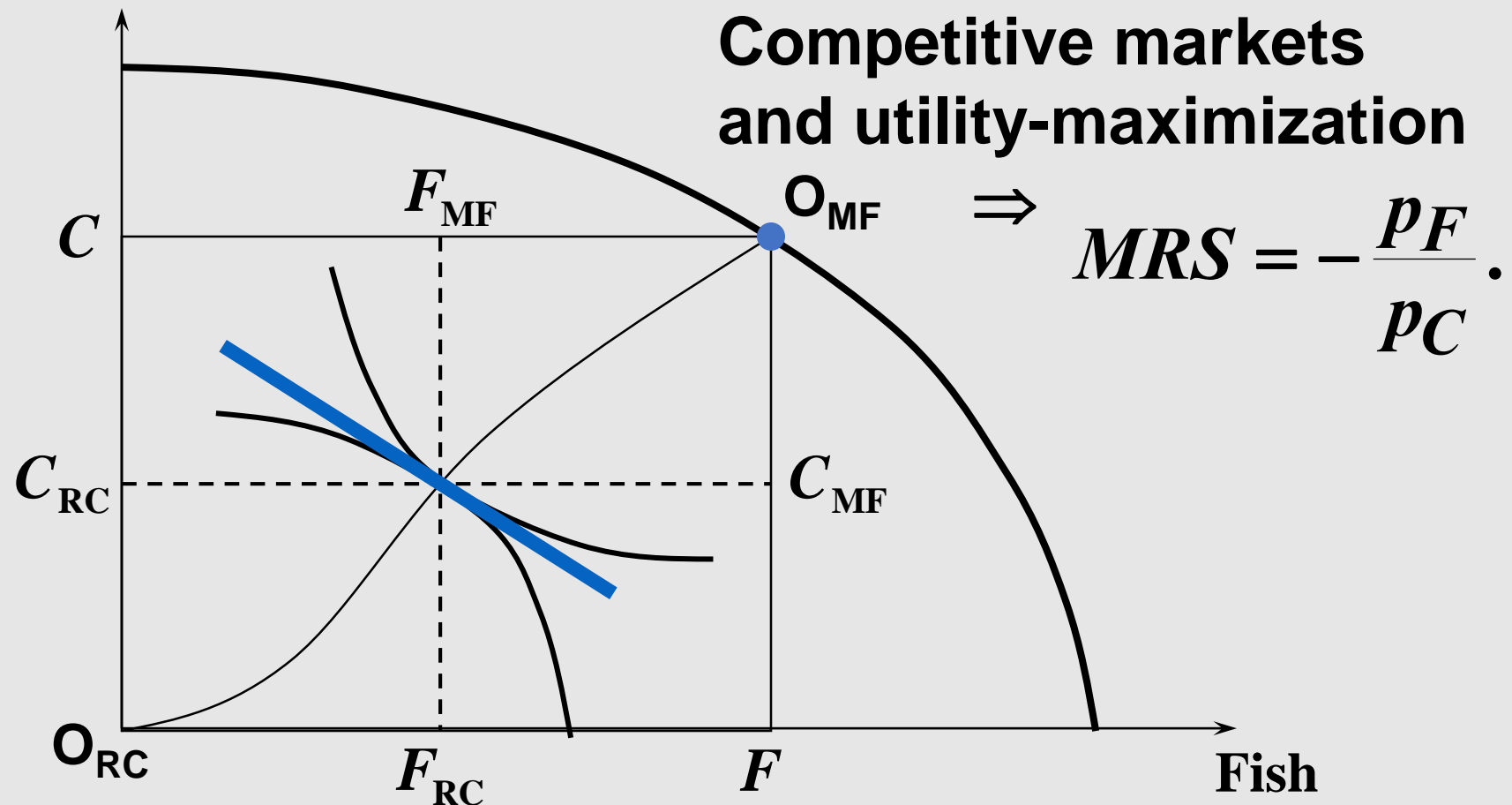
$$MRPT = -\frac{p_F}{p_C} = MRS,$$

→ The condition necessary for a Pareto optimal economic state.



# Decentralized Coordination of Production & Consumption

Coconuts



# Decentralized Coordination of Production & Consumption

