Problem Set 2: Discrete Random Variables

Exercise 1: Measurements of a random variable
Let Y be a random variable with $p(y)$ given in the accompanying table. Find $E(Y)$, $E(1/Y)$, $E(Y^2 - 1)$, and $V(Y)$.

y	1	2	3	4
p(y)	.4	.3	.2	.1

Exercise 2: Two dice
Suppose that two balanced dice are rolled. Determine and sketch the probability distribution of each of the following random variables.

1. Let X denote the absolute value of the difference between the two numbers that appear.
2. Let Y denote the product of the two numbers that appear.
3. Let Z denote the number of even numbers that appear. Find and plot the cdf of Z.

Exercise 3: A binomial distribution

Recall that a random variable X has the binomial distribution if

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

where n is the number of trials and p is the chance of success. For the following questions about the binomial distribution, do the following: Define p, n, what the specific "trials" are, what "success" is. Then write down the relevant distribution and answer the specific question.

1. If 25 percent of the balls in a certain box are red, and if 15 balls are selected from the box at random, with replacement, what is the probability that more than four red balls will be obtained?
2. Suppose an economist is organizing a survey of American minimum wage workers, and is interested in understanding how many workers that earn the minimum wage are teenagers. Suppose further that one out of every four minimum wage workers is a teenager. If the economist finds 80 minimum wage workers for his survey, what's the probability that he interviews exactly 14 teenagers? 35 teenagers?
3. A city has 5000 children, including 800 who have not been vaccinated for measles. Sixty-five of the city's children are enrolled in a day care center. Suppose the municipal health department sends a doctor and nurse to the day care center to immunize any child who has not already been vaccinated. Find a formula for the probability that exactly k of the children at the day care center have not been vaccinated.

Exercise 4: Electrical fuses

Suppose that a lot of 5000 electrical fuses contains 5% defectives. If a sample of 5 fuses is tested, find the probability of observing at least one defective.

Exercise 5: The new formula
The manufacturer of a low-calorie dairy drink wishes to compare the taste appeal of a new formula (formula B) with that of the standard formula (formula A). Each of four judges is given three glasses in random order, two containing formula A and the other containing formula B. Each judge is asked to state which glass he or she most enjoyed. Suppose that the two formulas are equally attractive. Let Y be the number of judges stating a preference for the new formula.

1. Find the probability function for Y .
2. What is the probability that at least three of the four judges state a preference for the new formula?
3. Find the expected value of Y
4. Find the variance of Y

Exercise 6: A weighted coin
Suppose you flip a weighted coin (probability of heads is 0.6) 8 times.

1. What is the probability that you get a particular ordering of 5 heads and 3 tails?
2. What is the probability that you get 5 heads?
3. Suppose now that you have a hat with two coins, one weighted as above and one fair. You choose one at random and flip that one 8 times. Let Y be the number of heads in 8 flips. What is the PDF of Y?
4. What is the probability that you chose the fair coin given that $Y = 6$, this is $P(\text{Fair}|Y = 6)$?

Exercise 7: Tree seeding
A certain type of tree has seedlings randomly dispersed in a large area, with the mean density of seedlings being approximately five per square yard. If a forester randomly locates ten 1-square-yard sampling regions in the area, find the probability that none of the regions will contain seedlings. What is the expected value of the number of seeding per region? and the variance?

Exercise 8: Industrial accidents
Industrial accidents occur according to a Poisson process with an average of three accidents per month. During the last two months, ten accidents occurred. Does this number seem highly improbable? What is the probability of observing a number of accidents equal or higher than 10? Hint: Use Table 3 in Appendix to avoid tedious calculations.