Problem Set 3: Continuous Random Variables

Exercise 1: A public fountain

A public fountain has two pumps, each of which can pump up to 10,000 liters of water in a month. The total amount of water pumped at the fountain in a month is a random variable Y (measured in 10,000 liters) with a probability density function given by

$$f(y) = \begin{cases} y & \text{if } 0 < y < 1 \\ 2 - y & \text{if } 1 \leq y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

1. Graph $f(y)$.

2. Find $F(y)$.

3. Find the probability that the station will pump between 8000 and 10,000 liters in a particular month.

4. Given that the fountain pumped more than 10,000 liters in a particular month, find the probability that the fountain pumped more than 15,000 liters during the month.

Exercise 2: Expected Value

If Y has density function

$$f(y) = \begin{cases} 0.2 & -1 < y \leq 0, \\ 0.2 + 1.2y, & 0 < y \leq 1, \\ 0, & \text{elsewhere}, \end{cases}$$

Find the mean of Y. Find the variance.

Exercise 3: A parachutist

If a parachutist lands at a random point on a line between markers A and B...

1. Find the probability that she is closer to A than to B if A=0 and B=100

2. Find the probability that her distance to A is more than three times her distance to B.

Exercise 4: Normal distributions

Use Table 4 to find the following probabilities for a standard normal random variable Z:

1. $P(0 \leq Z \leq 1.2)$

2. $P(-.9 \leq Z \leq 0)$

3. $P(Z > z_0) = .5$.

4. $P(Z < z_0) = .8643$.

5. $P(-z_0 < Z < z_0) = .90$.

Exercise 5: Apple juice bottles

A company that manufactures and bottles apple juice uses a machine that automatically fills 33cl bottles. There is some variation, however, in the amounts of liquid dispensed into the bottles that are filled. The amount dispensed has been observed to be approximately normally distributed with mean 33cl and standard deviation 2cl.

1. Bottles have a maximum capacity of 36cl, find the proportion of bottles that will spill.

2. The company knows that clients complain if bottles have less than 29cl, how likely is that?

Exercise 6: Ore samples

For certain ore samples, the proportion Y of impurities per sample is a random variable with density function equal to:

$$f(y) = \begin{cases} \frac{3}{2}y^2 + y & \text{if } 0 \leq y \leq 1, \\ 0 & \text{elsewhere.} \end{cases}$$

The dollar value of each sample is $W = 5 - 0.5Y$. Find the mean of W.

Exercise 7: Biking a mountain

The cycle time for bikes reaching a mountain summit is uniformly distributed over the interval 50 to 70 minutes.

1. What is the probability that the cycle time exceeds 65 minutes if it is known that the cycle time exceeds 55 minutes?

2. Find the mean of the cycle time Y.

Exercise 8: College scores

Scores on an examination are assumed to be normally distributed with mean $\mu = 78$ and variance $\sigma^2 = 36$.

1. What is the probability that a person taking the examination scores higher than 72?

2. Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?

3. What must be the cutoff point for passing the examination if the examiner wants only the top 28.1% of all scores to be passing?

4. Approximately what proportion of students have scores 5 or more points above the score that cuts off the lowest 25%?

5. If it is known that a student's score exceeds 72, what is the probability that his or her score exceeds 84?

Exercise 9: Find the CFD/PDF

1. Suppose that a random variable has a PDF that is proportional to x on the interval $[0, 1]$. Write down a formula for this PDF. What is the corresponding CDF?

2. Now suppose that the random variable has a CDF that is proportional to x on the interval $[0, 1]$. Write down a formula for the CDF. What is the corresponding PDF?