

Problem Set 1: Exchange

Exercise 1: Exchanging bread and milk

Fatima (F) and Romuald (R) are the two individuals of a pure exchange economy whose only goods are bread and milk. The initial endowments are $\omega^F = (5, 5)$ and $\omega^R = (5, 5)$. The utility functions are assumed to be of the form $U^F = (m_F b_F)^2 + 2m_F b_F$ and $U^R = m_R^{3/4} + b_R^{1/4}$.

1. Compute the marginal rate of substitution for Fatima.
2. Compute the marginal rate of substitution for Romuald.
3. In the Edgeworth box, draw approximately an indifference curve for each consumer. Place the initial endowment. Is the initial endowment a Pareto optimum?

Exercise 2: One Thousand and One Nights

Let us consider a pure exchange economy with two individuals, Scheherazade and Shahryār, and two goods, stories and the days (nights) of life. Scheherazade's endowment consists of 500 stories and one night of life expectancy, while Shahryār's endowment consists of 0 stories and 2000 nights of life expectancy. Shahryār's happiness is linked to a precise distribution of the amount of stories and life expectancy, according to the following utility function: $U_{Sha} = \min\{\text{Number of stories}, \text{Number of nights}/2\}$, a non-increasing function of its arguments. Scheherazade wants to survive but cannot live without stories. Her happiness can be measured by the following utility function $U_{She} = (\text{Number of stories}) \times (\text{Number of nights})$.

1. Draw an Edgeworth box and place the initial endowment allocation. Sketch some indifference curves. In particular, draw the indifference curves through the initial endowment allocation.
2. Draw the area representing mutually beneficial allocations. Recall the definition of a Pareto-efficient allocation.

Exercise 3: Exchanging Fruits

In an exchange economy, two consumers Amélie et Baptiste want to trade with Oranges (good 1) and Kiwis (good 2). Their utility functions are the following:

$$\text{For Amélie: } U_A(x_1^A, x_2^A) = x_1^A x_2^A$$

$$\text{For Baptiste: } U_B(x_1^B, x_2^B) = 3x_1^B x_2^B$$

Their initial endowment is $W^A = (\omega_1^A, \omega_2^A) = (3, 10)$ and $W^B = (\omega_1^B, \omega_2^B) = (12, 5)$

1. Draw the Edgeworth Box and represent the indifference curves for the following values:

$$U_A = 9, U_A = 30 \text{ and } U_A = 64$$

$$U_B = 147, U_B = 180 \text{ and } U_B = 432$$

2. Place the initial endowment. Using simple calculation, can you find an allocation that will leave both Amelie and Baptiste better off?
3. Draw the area containing Pareto-improving allocations. What can you conclude with respect to the initial endowment?
4. Draw the contract curve and find its equation

Let's call prices of goods 1 and 2 p_1 and p_2 . R_i is the revenue of each consumer with $i=A,B$.

5. Find the (gross) demand function for each consumer
6. Find the net demand functions
7. Which condition(s) need to hold for this economy to be on a general equilibrium?
8. Compute the exchange needed to take place to obtain the general equilibrium
9. Compute the consumer's utility at the general equilibrium