Problem Set 3: Welfare

Exercise 1: Welfare maximizarion

In a given economy, there are 2 consumers (A and B) and two goods (x and y). One possible social welfare criterion is the sum of weighted utilities, as in:

$W(x, y) = \theta u_A(x, y) + (1 - \theta)u_B(x, y)$

where θ is the weight given to the utility of consumer A

We know that the utility of the consumers is:

$U_A(x^A, y^A) = logx^A + logy^A$ and $U_B(x^B, y^M) = 2logx^B + logy^B$

1. Determine the allocation that maximizes welfare as a function of $\theta, \overline{x}, \overline{y}$, with $\overline{x} = x^A + x^B$ and $\overline{y} = y^A + y^B$.

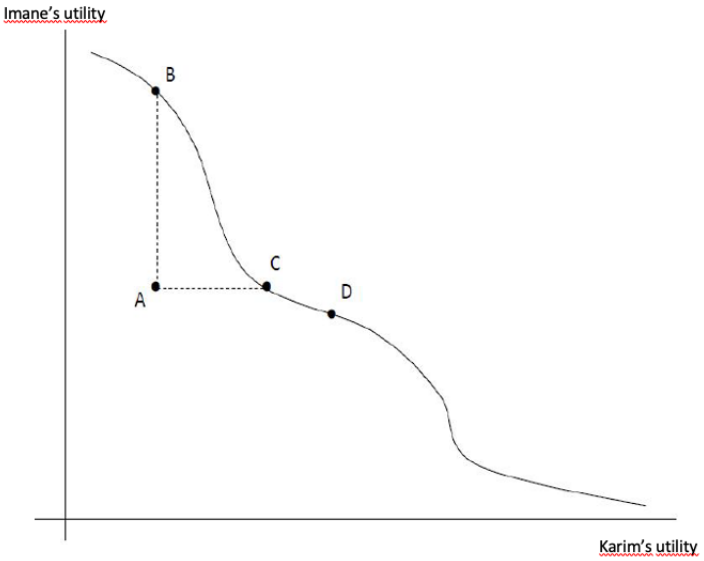
2. Find if the allocations that maximizes W also satisfies the Pareto-optimality condition $MRS_A = MRS_B$.

3. Assume that $(\overline{x}, \overline{y}) = (30, 60)$. What is the welfare maximizing allocation if $\theta = 0.5$? Depict this situation in an Edgeworth box diagram. What is the utility of each consumer? And the total welfare?

4. Who will win if θ increases to 0.75?

Exercise 2: Pareto-improving

The following graph is the utility frontier of the utilities of consumers Imane and Karim. It represents the maximum utility that a consumer can get, given the utility of the other.



1. Does going from allocation A to allocation B represent a Pareto-improving change?

2. Does going from allocation A to allocation C represent a Pareto-improving change?

3. Does going from allocation A to allocation D represent a Pareto-improving change?

Exercise 3: Rich and Poor: Rawls and Bentham

Consider a country where half of the population is identically rich, and the other half is identically poor. The GDP is 800 billion piasters and the population is 200 million. The inhabitants all have the same utility function, a function of income (Table 1).

Table 1: Utility function of a given inhabitant

Income	0	1000	2000	3000	4000	5000	6000	7000	8000
Utility	0	11	21	30	38	45	48	50	51
Marginal Utility									

1. Graphically represent the utility function. Complete the row in the table for marginal utility, and plot marginal utility as a function of income.

2. Adopt the utilitarian criterion, establishing that the social good is the sum of individual utilities. Determine the level of social welfare attained when individuals in the poor half of the population receive 0, 1000, 2000, 3000 or 4000 piasters (the rich half receives the remaining part of the GDP). Which allocation maximizes social welfare? Explain the role of marginal utility in determining the outcome.

3. Now adopt the Rawlsian approach, according to which the social welfare corresponds to the level of utility of the most disadvantaged individual. In the same way as in the previous question, determine the level of social welfare achieved for each level of income. Which allocation maximizes social welfare?

4. Plot in a graph all the possible combinations of utility. On this same graph, draw the allocations chosen using the two approaches from the previous questions, drawing the social indifference curve for each social criterion. Comment.

5. Suppose that the initial allocation is 600 billion piasters to the rich and 200 billion to the poor. The technology used to redistribute is inefficient, because to transfer 100 billion piasters to the rich, we need to extract 200 billion piasters from the rich (and so on: 400 for 200, 600 for 300,...). Describe all the possible combinations of utilities. Which allocation maximizes social welfare according to the utilitarian criterion? Which one is chosen by Rawls' criterion?