Chapter 5: Public Goods

Ch 36 in H. Varian 8th Ed.

Slides by Mariona Segú, CYU Cergy Paris Université

Inspired by Michael D. Robinson, Mount Holyoke College

and by Stefanie Stantcheva, Harvard

Introduction

So far…

→ Externalities can be solved if property right are properly assigned

→ But… not all externalities are solved like that, especially when there are more than 2 agents.

- Smoke example with 2 nonsmokers: Now, agents need to coordinate on how much clean air to sell.
- This can lead to coordination problems… (pollution in a whole country)

→ Public goods are one example of consumption externality

- Everyone must consume the same amount of the good
- Even if they have different preferences about it
- What is the optimal amount of public good?
- How can we socially decide about public goods?

Public Goods – Definition

→ A good is **purely public** if it is both **nonexcludable** and **nonrival** in consumption.

- Nonexcludable: all consumers can consume the good. Individuals cannot deny each other the opportunity to consume a good.
- Nonrival: each consumer can consume all of the good. One individual's consumption of a good does not affect another's opportunity to consume the good

→ **Impure public goods**: Goods that satisfy the two public good conditions (non-rival in consumption and non-excludable) to some extent, but **not fully**.

Public Goods – Examples

- Broadcast radio and TV programs.
- National defense.
- Public highways.
- Reductions in air pollution.
- National parks.

Reservation Prices

- A consumer's reservation price for a unit of a good is his maximum willingness-to-pay for it.
- Consumer's wealth is w
- Utility of not having the good is $\mathrm{U}(w, 0)$
- Utility of paying p for the good is $\mathrm{U}(w - p, 1)$
- Reservation price is r such that $\mathrm{U}(w, 0) = \mathrm{U}(w - r, 1)$
  - Meaning, the price r such that consumer is indifferent between paying r and having the good and not having the good

Reservation Prices: An Example

- Consumer utility is $U(x_1, x_2) = x_1(x_2 + 1)$
- Utility of not buying a unit of good 2 is

$$V(w, 0) = \frac{w}{p_1}(0 + 1) = \frac{w}{p_1}$$

- Utility of buying one unit of good 2 at price p is

$$V(w - p, 1) = \frac{w-p}{p_1}(1 + 1) = \frac{2(w-p)}{p_1}$$

Reservation Prices: An Example

- Reservation price r is such that $\mathrm{V}(w, 0\ ) = \mathrm{V}(w - r, 1\ )$

- This is

$$\frac{w}{p_1} = \frac{2(w-r)}{p_1} \qquad \text{hence} \qquad r=\frac{w}{2}$$

- Reservation price of each person will depend on that person's wealth:
  - the maximum amount that an individual will be willing to pay will depend to some degree on how much that individual is able to pay.

Outline

1. When Should a Public Good Be Provided?
2. Free-Riding
3. Variable Public Good Quantities
4. Quasilinear Preferences and Public Goods
5. Free-Riding Revisited
6. Collective decision mechanisms
7. Lindahl prices

1. When Should a Public Good Be Provided?

Example: two roommates A and B decide whether or not to purchase a TV. TV will be in the living room and enjoyed by both of them

- The TV is a public good that costs c.
- Individual wealth is w_A and w_B
- Individual payments for providing the public good are g_A and g_B.
- Each person's money left to spend on private consumption is x_A and x_B
- Budget constraint is then $x_A + g_A = w_A$ and $x_B + g_B = w_B$
- The TV is purchased if $g_A + g_B \geq c$

When Should a Public Good Be Provided?

- Utilities depend on private good and on public good

$$U_A(x_A, G) \text{ and } U_B(x_B, G)$$

- 2 possible allocations:
  - No TV: $(w_A, w_B, 0)$
  - TV is bought: $(x_A, x_B, 1)$ with $x_i = w_i - g_i$ for i=A,B

- TV should be bought when both people are better off having the TV and paying their share than not having the TV

- This is: payments must be individually rational, i.e.

$$U_A(w_A, 0) < U_A(w_A - g_A, 1)$$
$$U_B(w_B, 0) < U_B(w_B - g_B, 1)$$

When Should a Public Good Be Provided?

→ Using the reservation wage definition…

$$U_A(w_A, 0) = U_A\,(w_A - r_A, 1) < U_A(w_A - g_A, 1)$$

$$U_B(w_B, 0) = U_B(w_B - r_B, 1) < U_B(w_B - g_B, 1)$$

→ Hence, $w_A - r_A < w_A - g_A$ and $w_B - r_B < w_B - g_B$

→ Which implies $r_A > g_A$ and $r_B > g_B$

1. The contribution to the public good needs to be smaller than the reservation price → **necessary condition**

2. The sum of willingnesses to pay must be greater than the cost of the TV → **sufficient condition**

$$r_A + r_B > g_B + g_A = c$$

When Should a Public Good Be Provided?

If $r_A + r_B > g_B + g_A = c$, then it is Pareto-efficient to provide the public good.

→ Provision of public good only depends on individual's willingness to pay and on the total cost of the good

→ Provision of public good generally depends on each individuals' wealth (since r_i depends generally on wealth)

- One exception is with quasilinear preferences

Private Provision of a Public Good?

- Suppose $r_A > c$ and $r_B < c$
- Then A would supply the good even if B made no contribution.
- B then enjoys the good for free; **free-riding**.

Private Provision of a Public Good?

- Suppose $r_A < c$ and $r_B < c$
- Then neither A nor B will supply the good alone.
- Yet, if $r_A + r_B > c$ also, then it is Pareto-improving for the good to be supplied.
- Still, A and B may try to free-ride on each other, causing no good to be supplied.

2. Free-Riding

- Suppose A and B each have just two actions – individually supply a public good, or not.
- Cost of supply c = $100.
- Reservation price to A from the good = $80.
- Reservation price to B from the good = $65.
- $80 + $65 > $100, so supplying the good is Pareto-improving.
- Payoffs for each consumer are
  - $P_i = r_i - c$ if i buys TV
  - $P_i = r_i$ if the other buys TV
  - $P_i = 0$ if no one buys TV

Free-Riding

Suppose only one player can buy the good…

		Player B: Buy	Player B: Don't Buy
Player A	Buy	-\$20, -\$35	-\$20, \$65
	Don't Buy	\$80, -\$35	\$0, \$0

Free-Riding

		Player B	
		Buy	Don't Buy
Player A	Buy	-$20, -$35	-$20, $65
	Don't Buy	$80, -$35	$0, $0

(Don't' Buy, Don't Buy) is the unique NE.

Free-Riding

Player A	Player B: Buy	Player B: Don't Buy
Buy	-\$20, -\$35	-\$20, \$65
Don't Buy	\$80, -\$35	\$0, \$0

But (Don't' Buy, Don't Buy) is inefficient.

Free-Riding

- Now allow A and B to make contributions to supplying the good.
- E.g. A contributes \$60 and B contributes \$40.

- Payoffs for each consumer are
  - $P_i = r_i - c_i$ if both contribute to the TV
  - $P_i = -c_i$ if only i contributes to the TV
  - $P_i = 0$ if no one contributes to the TV

Free-Riding

		Player B	
		Contribute	Don't Contribute
Player A	Contribute	\$20, \$25	-\$60, \$0
	Don't Contribute	\$0, -\$40	\$0, \$0

Free-Riding

		Player B	
		Contribute	Don't Contribute
Player A	Contribute	\$20, \$25	-\$60, \$0
	Don't Contribute	\$0, -\$40	\$0, \$0

Two NE: (Contribute, Contribute) and (Don't Contribute, Don't Contribute).

Free-Riding

- So allowing contributions makes possible supply of a public good when no individual will supply the good alone.
- But what contribution scheme is best?
- And free-riding can persist even with contributions.
  - Ex: incentive to lie about willingness to pay

3. Variable Public Good Quantities

- E.g. how many broadcast TV programs, or how much land to include into a national park.
- $c(G)$ is the production cost of G units of public good.
- Two individuals, A and B.
- Private consumptions are x_A, x_B.

Variable Public Good Quantities

→ Budget allocations must satisfy

$$x_A + x_B + c(G) = w_A + \ w_B$$

→ For consumer A, the maximization problem is:

$$\max_{x_A, x_B, G} U_A(x_A, G)$$

Subject to $\quad x_A + x_B + c(G) = w_A + \ w_B$

And fixing U_B to $\overline{u_B}$, we add that $U_B(x_B, G) = \overline{u_B}$

The solution to this problem is such that

$$|MRS_A| + |MRS_B| = MC(G)$$

This is the **Samuelson rule**

Variable Public Good Quantities

$$|MRS_A| + |MRS_B| = \left|\frac{\Delta x_A}{\Delta G}\right| + \left|\frac{\Delta x_B}{\Delta G}\right| = \frac{MU_G}{MU_{x_A}} + \frac{MU_G}{MU_{x_B}} = MC(G)$$

- MRS_A & MRS_B are A & B's marg. rates of substitution between the private and public goods.
- MC(G) is the marginal cost of providing an extra unit of G
- This is the Pareto efficiency condition for public good supply
- Remember: the public good is nonrival in consumption, so 1 extra unit of public good is fully consumed by both A and B.
- Why?

Variable Public Good Quantities

→ To understand why, let's suppose $|MRS_A| + |MRS_B| < MC(G)$

→ Ex: $|MRS_A| = \frac{1}{4}$ $|MRS_B| = \frac{1}{2}$ and $MC = 1$

→ If the public good is reduced by 1 unit, A needs to be compensated with ¼ units of private good and B by ½. Hence, there is still ¼ of a dollar left over.

→ Making 1 less public good unit releases more private good than the compensation payment requires $\Rightarrow$ Pareto-improvement from reduced G.

→ Meaning that both consumers will be better off if G is reduced

Variable Public Good Quantities

- Now, let's suppose $|MRS_A| + |MRS_B| > MC(G)$
- Ex: $|MRS_A| = \frac{2}{3}$ $|MRS_B| = \frac{1}{2}$ and $MC = 1$
- A would give up 2/3 units of private good to get 1 unit of G and B would give up ½. If they give up more 2/3 and ½ we have more than enough to produce 1 more G and we could give back some of the extra money.
- Making 1 more public good unit is Pareto-improving
- Meaning that both consumers will be better off if G is increased

Variable Public Good Quantities

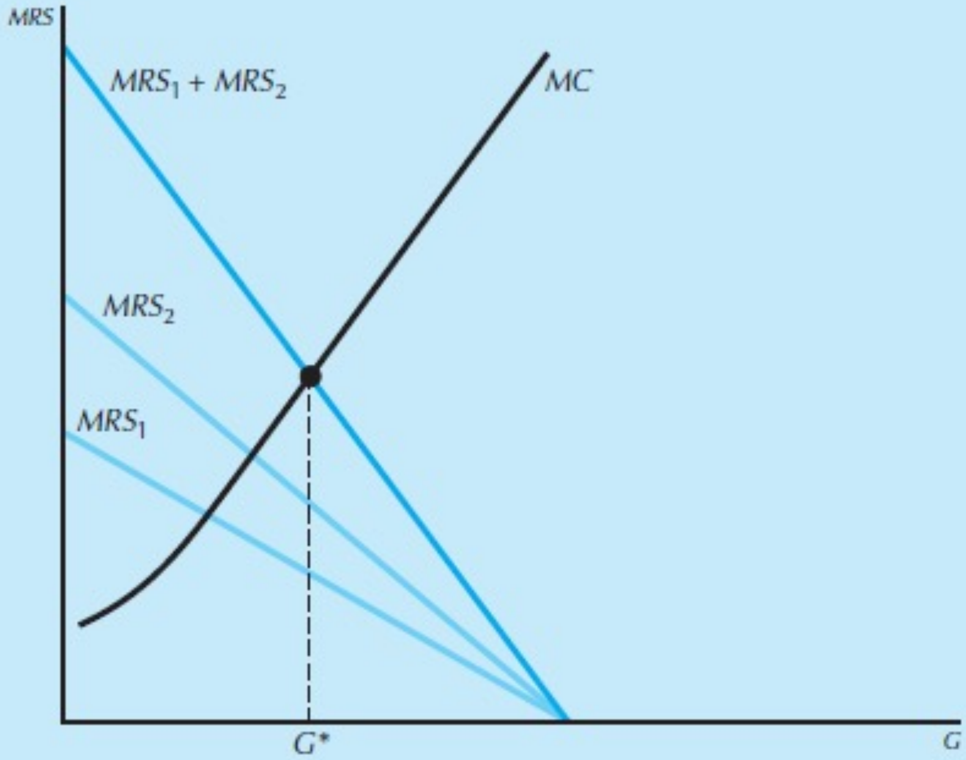
→ Hence, necessarily, efficient public good production requires

$$|MRS_A| + |MRS_B| = MC(G)$$

→ Suppose there are n consumers; i = 1,…,n. Then efficient public good production requires

$$\sum_{i=1}^{n} |MRS_i| = MC(G)$$

Variable Public Good Quantities



Determining the efficient amount of a public good. The sum of the marginal rates of substitution must equal the marginal cost.

4. Quasilinear Preferences and Public Goods

→ Two consumers, A and B.

$U_i(x_i, G) = x_i + f_i(G)$ for i=A,B

→ Marginal utility of the private good is always 1

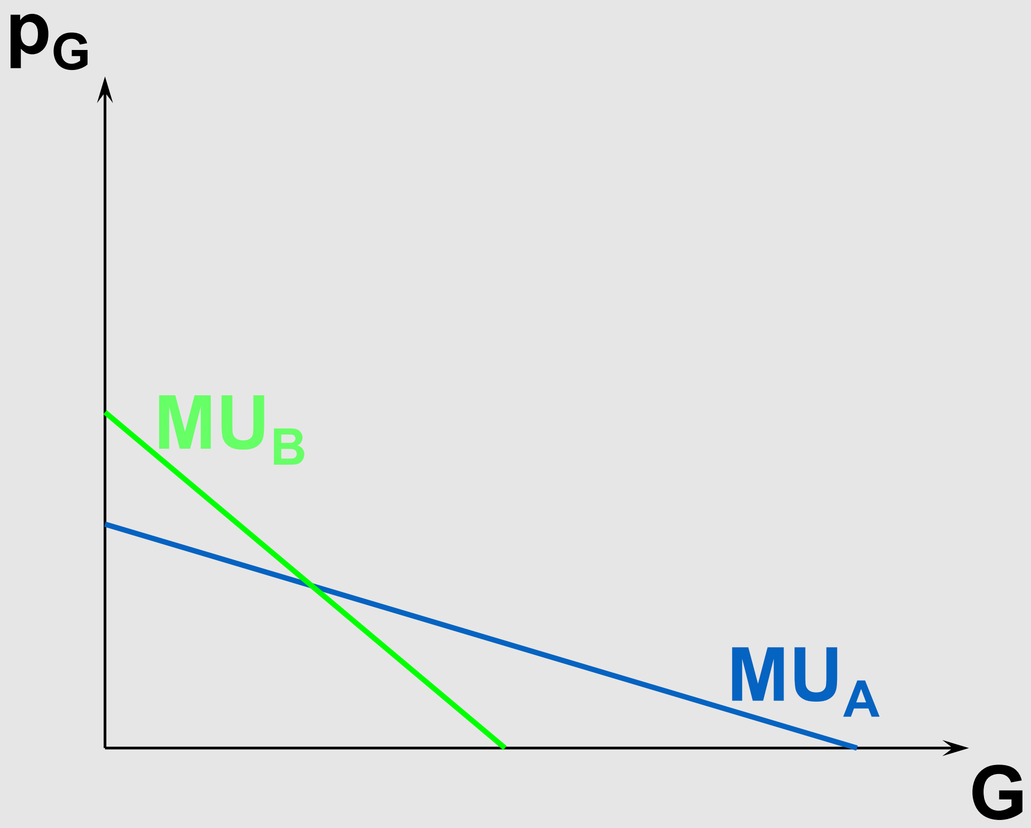
→ Utility-maximization requires

$$|MRS_i| = \frac{\frac{\Delta U_i(x_1, G)}{\Delta G}}{\frac{\Delta U_i}{\Delta x_1}} = \frac{\Delta U_i(x_1, G)}{\Delta G} = f'_i(G)$$

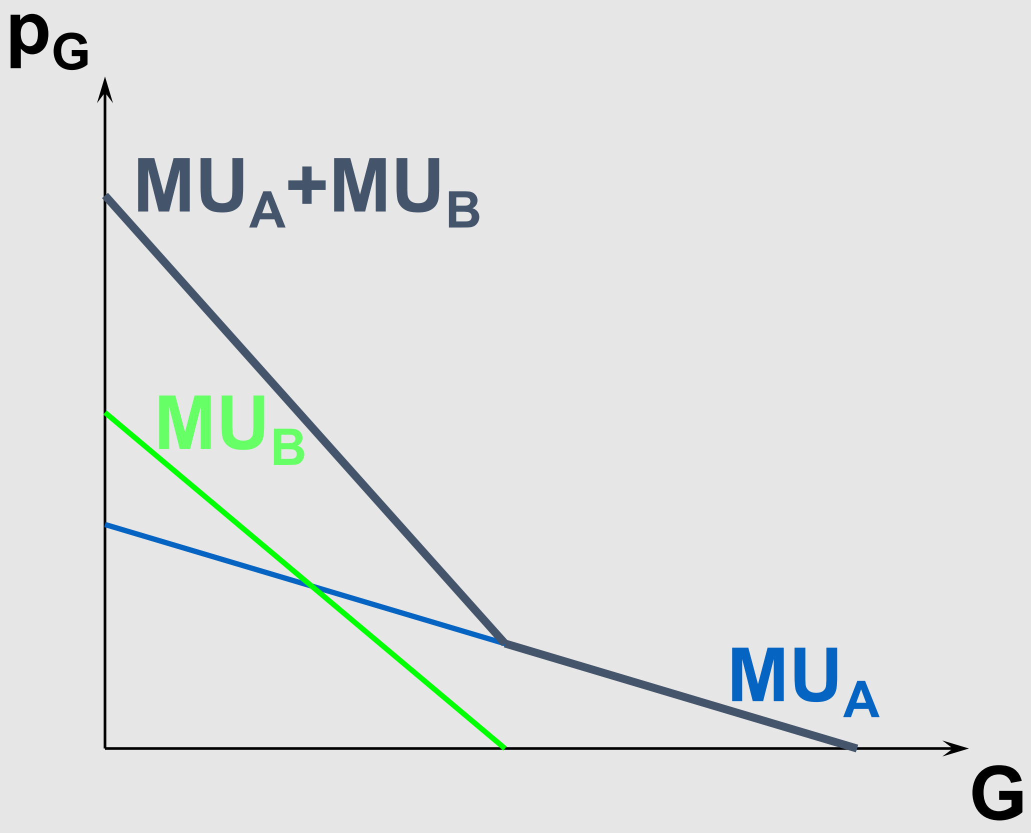
Since $|MRS_A| + |MRS_B| = MC(G)$

Then $f'_A(G) + f'_B(G) = MC(G)$

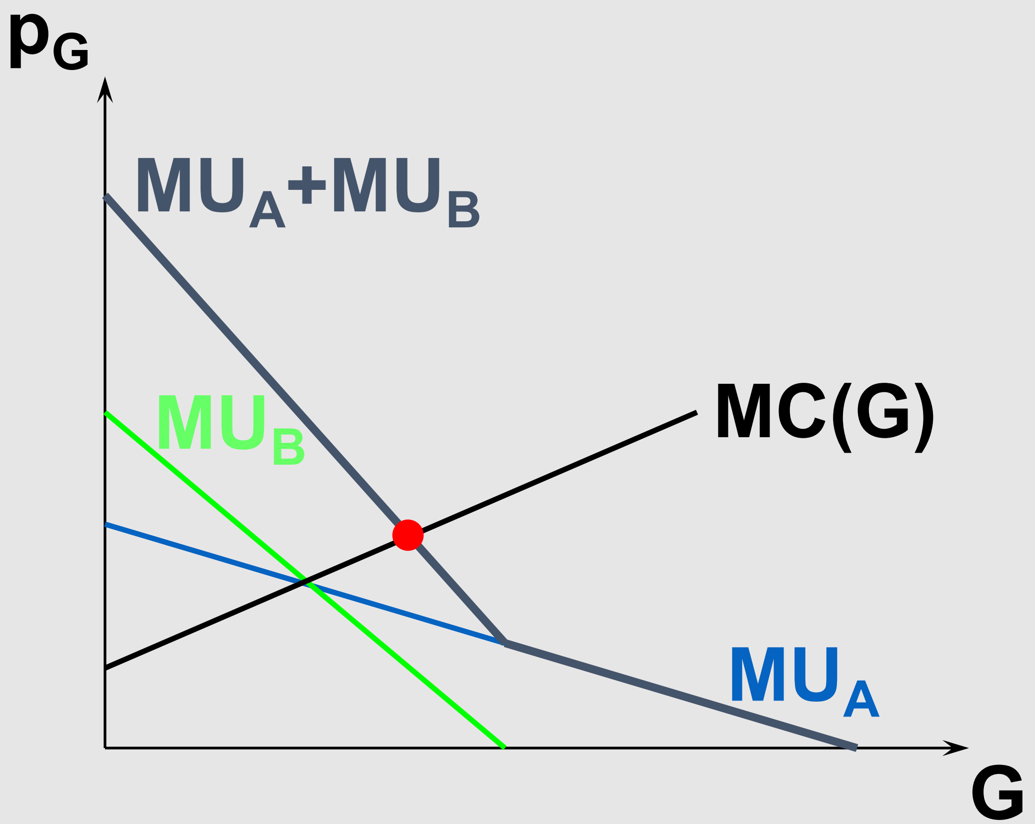
Quasilinear Preferences and Public Goods



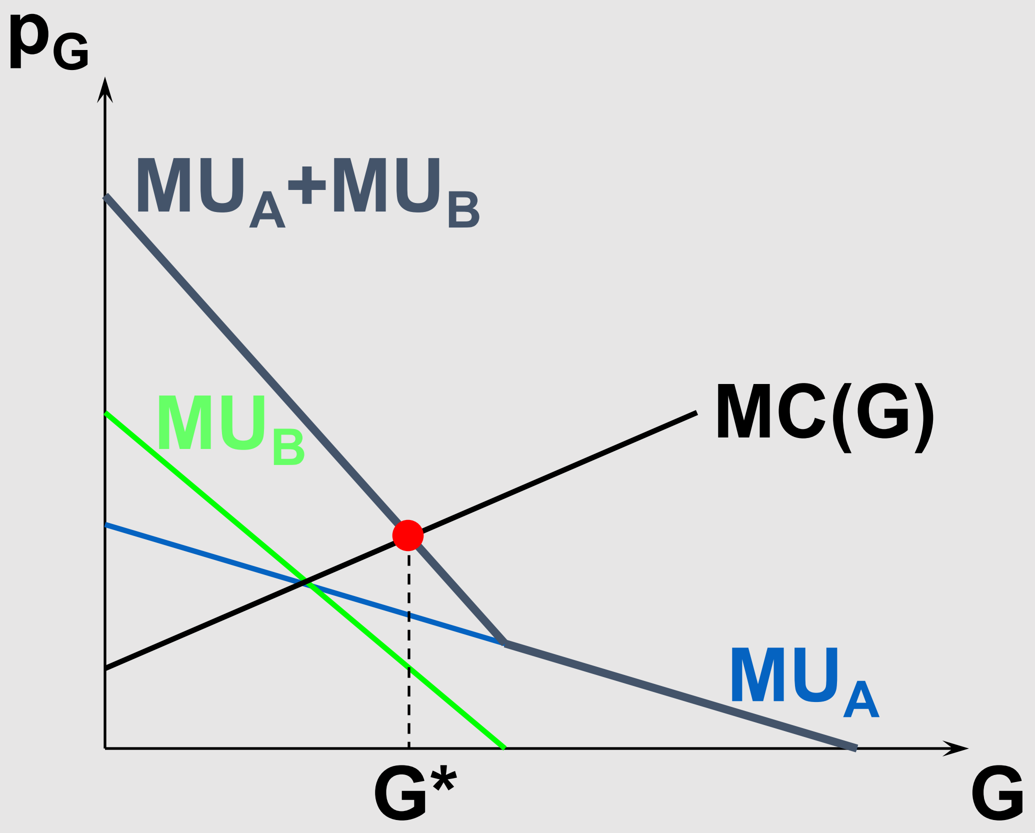
Quasilinear Preferences and Public Goods



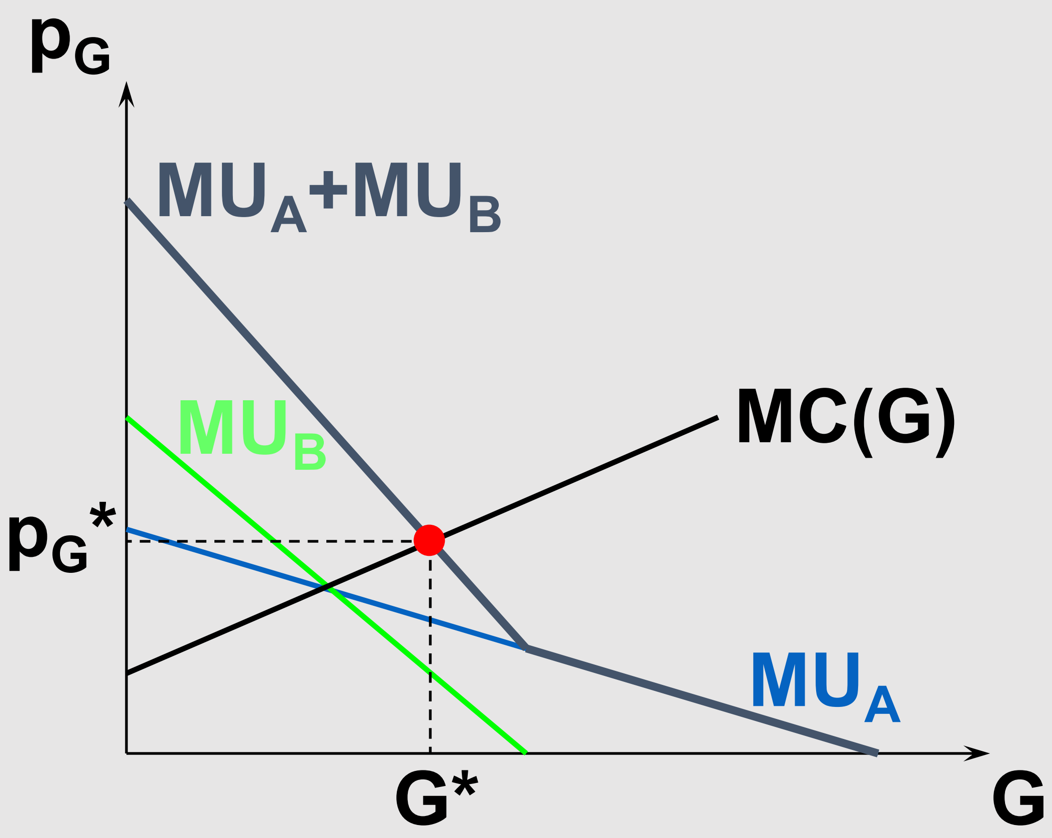
Quasilinear Preferences and Public Goods



Quasilinear Preferences and Public Goods

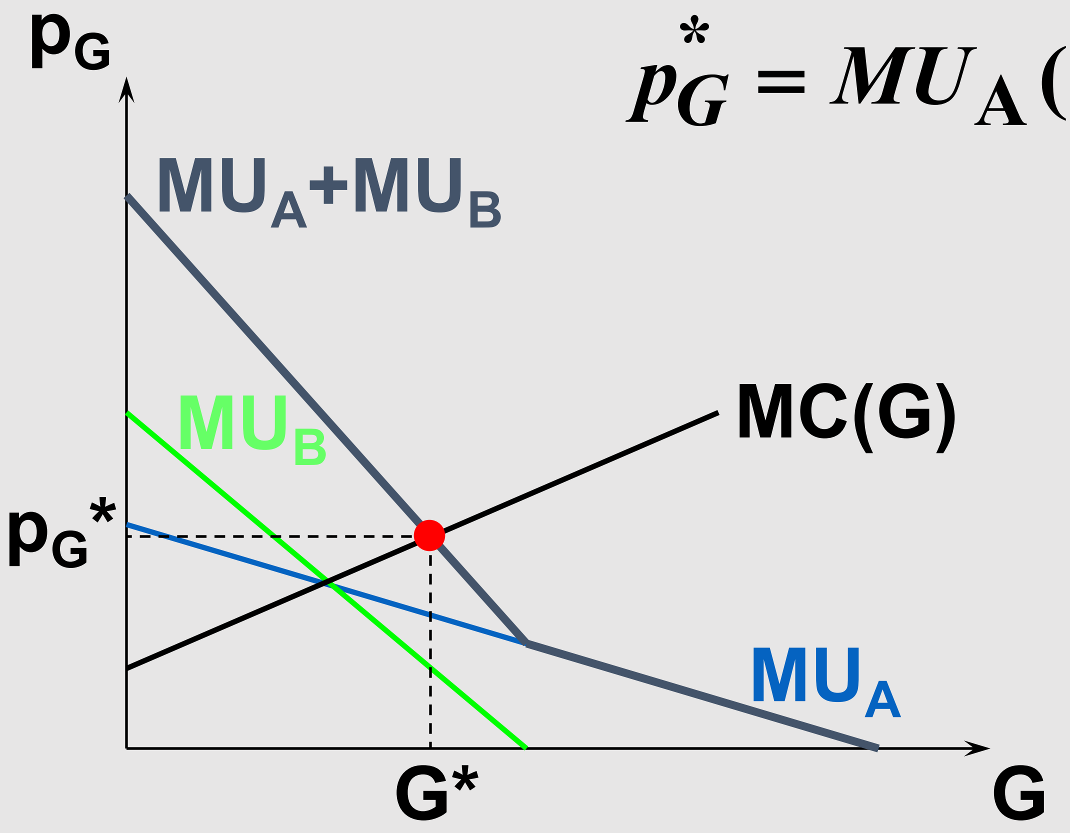


Quasilinear Preferences and Public Goods



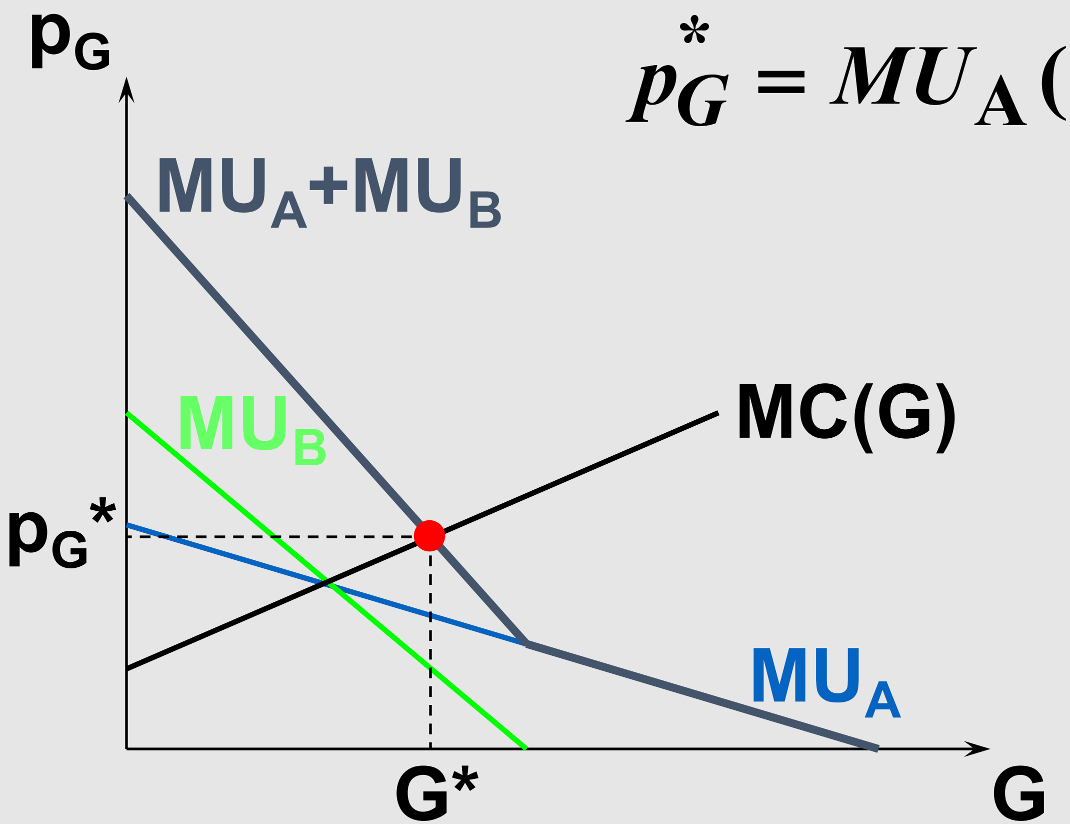
Quasilinear Preferences and Public Goods

$$p_G^* = MU_A(G^*) + MU_B(G^*)$$



Quasilinear Preferences and Public Goods

$$p_G^* = MU_A(G^*) + MU_B(G^*)$$



Efficient public good supply requires A & B to state truthfully their marginal valuations.

5. Free-Riding Revisited

→ When is free-riding individually rational?

Suppose that

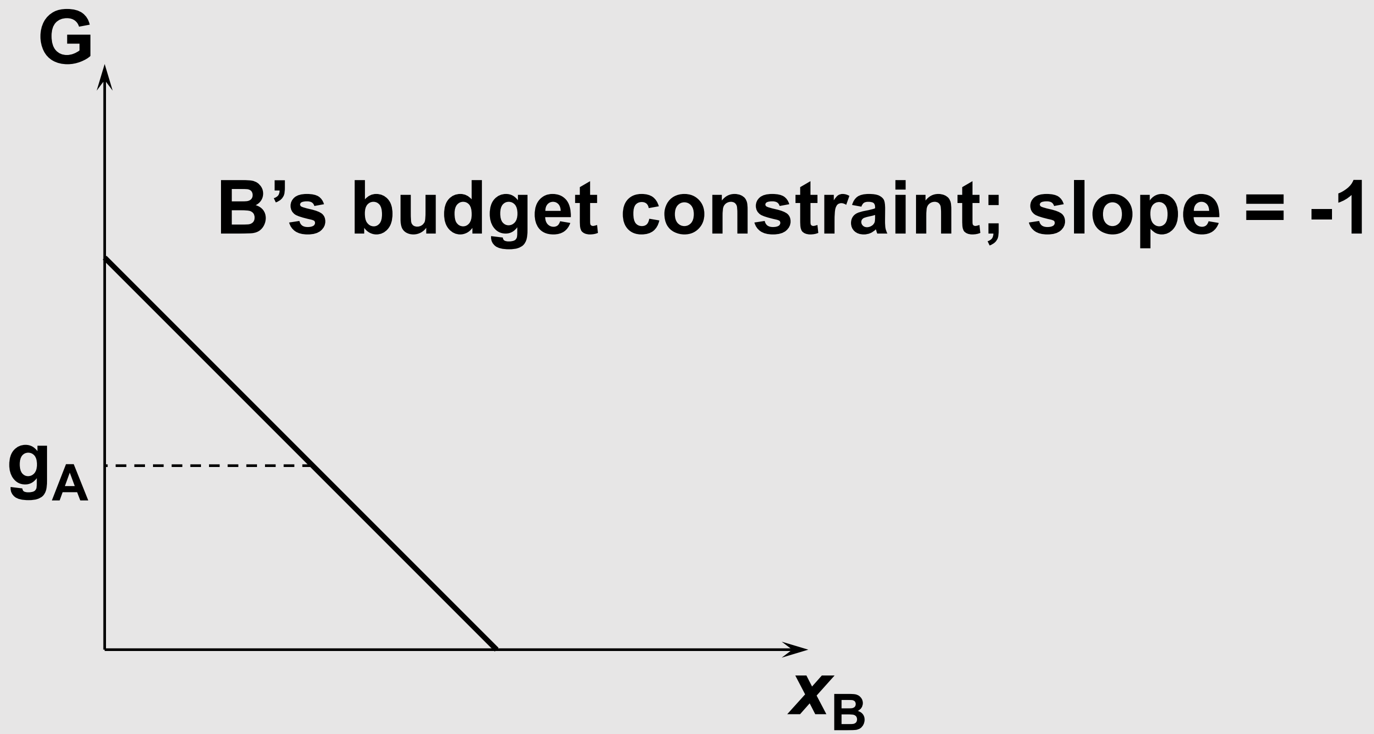
→ Individuals can contribute only positively to public good supply; nobody can lower the supply level. $g_1 \geq 0$

Given that A contributes g_A units of public good, B's problem is

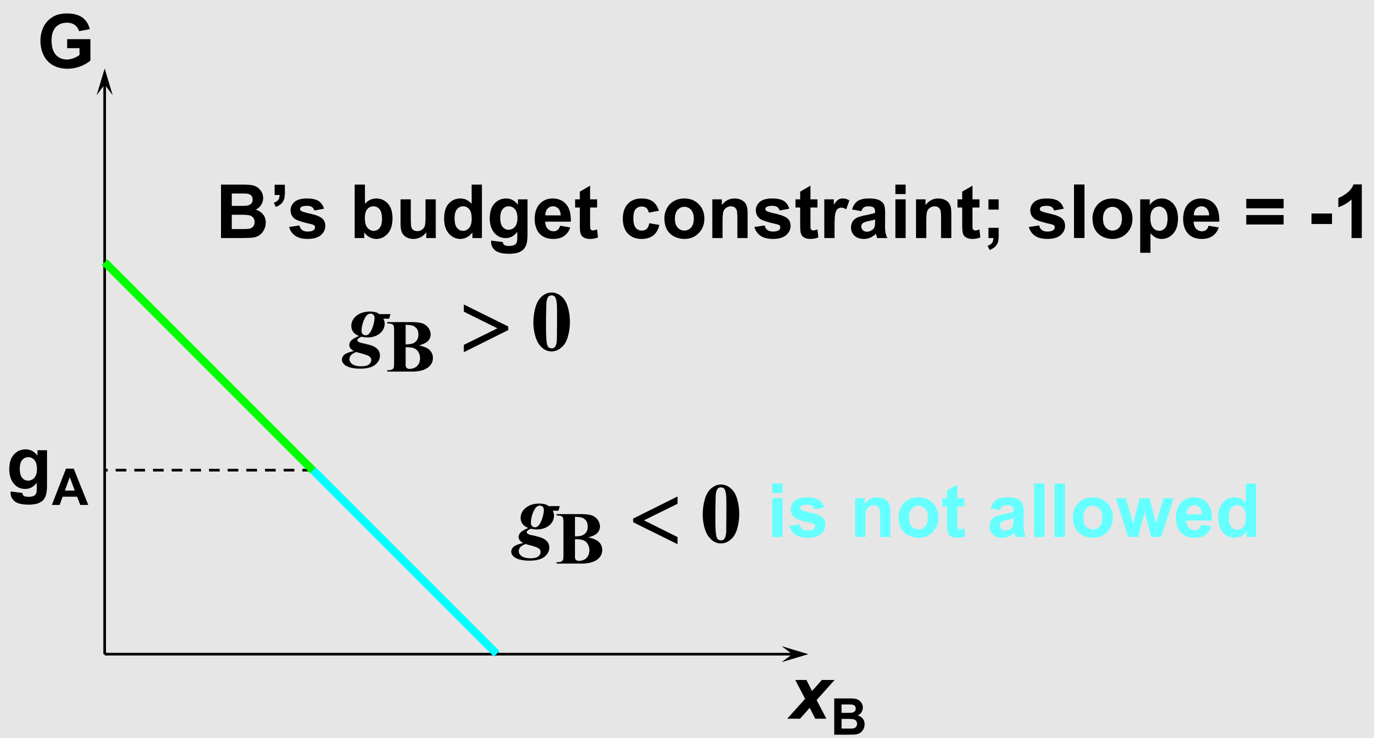
$$\max_{x_B, g_B} U_B(x_B, g_A + g_B)$$

Subject to $x_B + g_B = w_B$ and $g_B \geq 0$

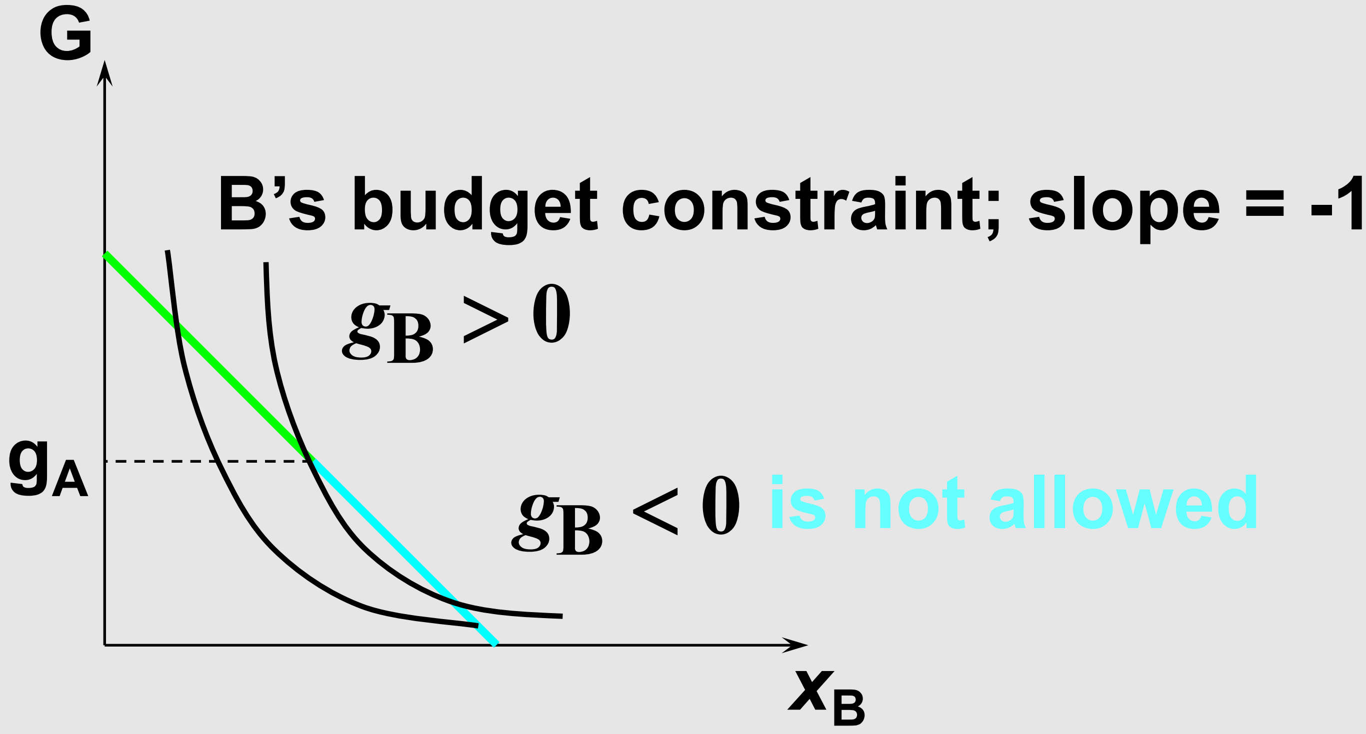
Free-Riding Revisited



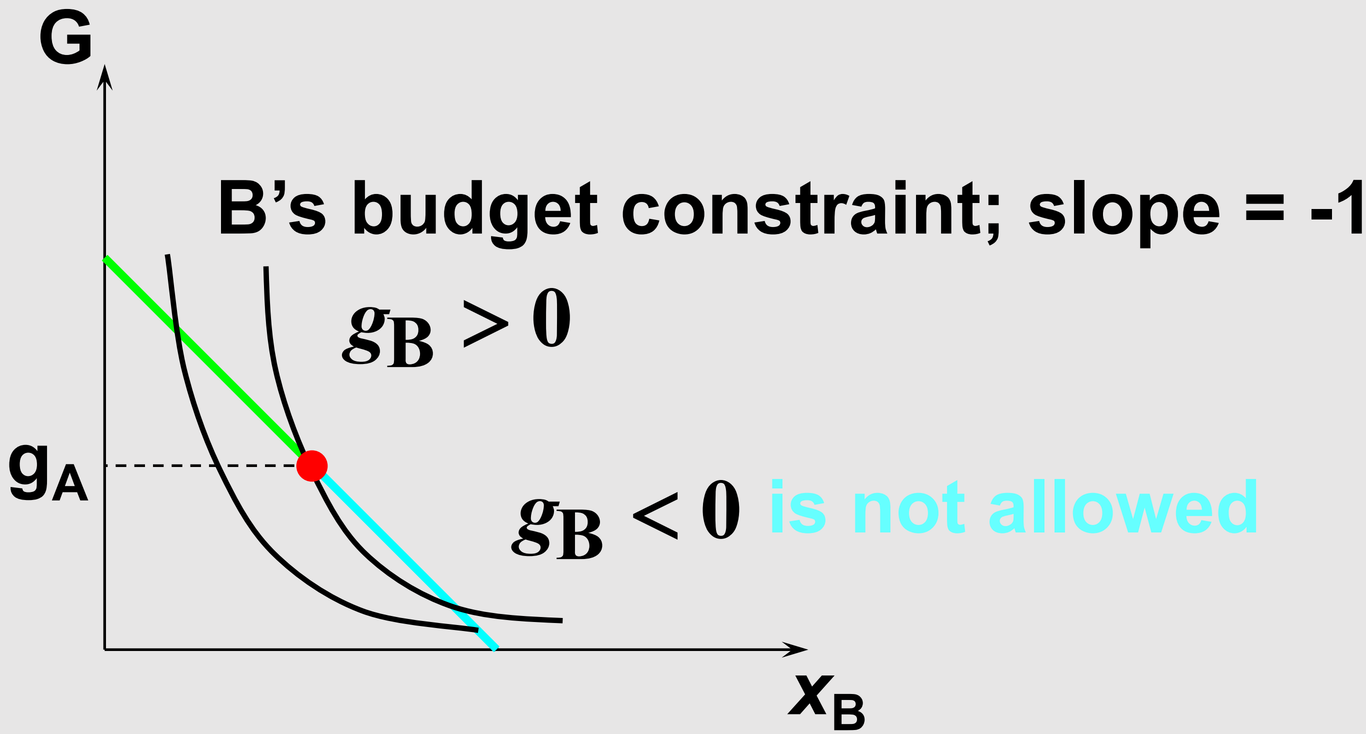
Free-Riding Revisited



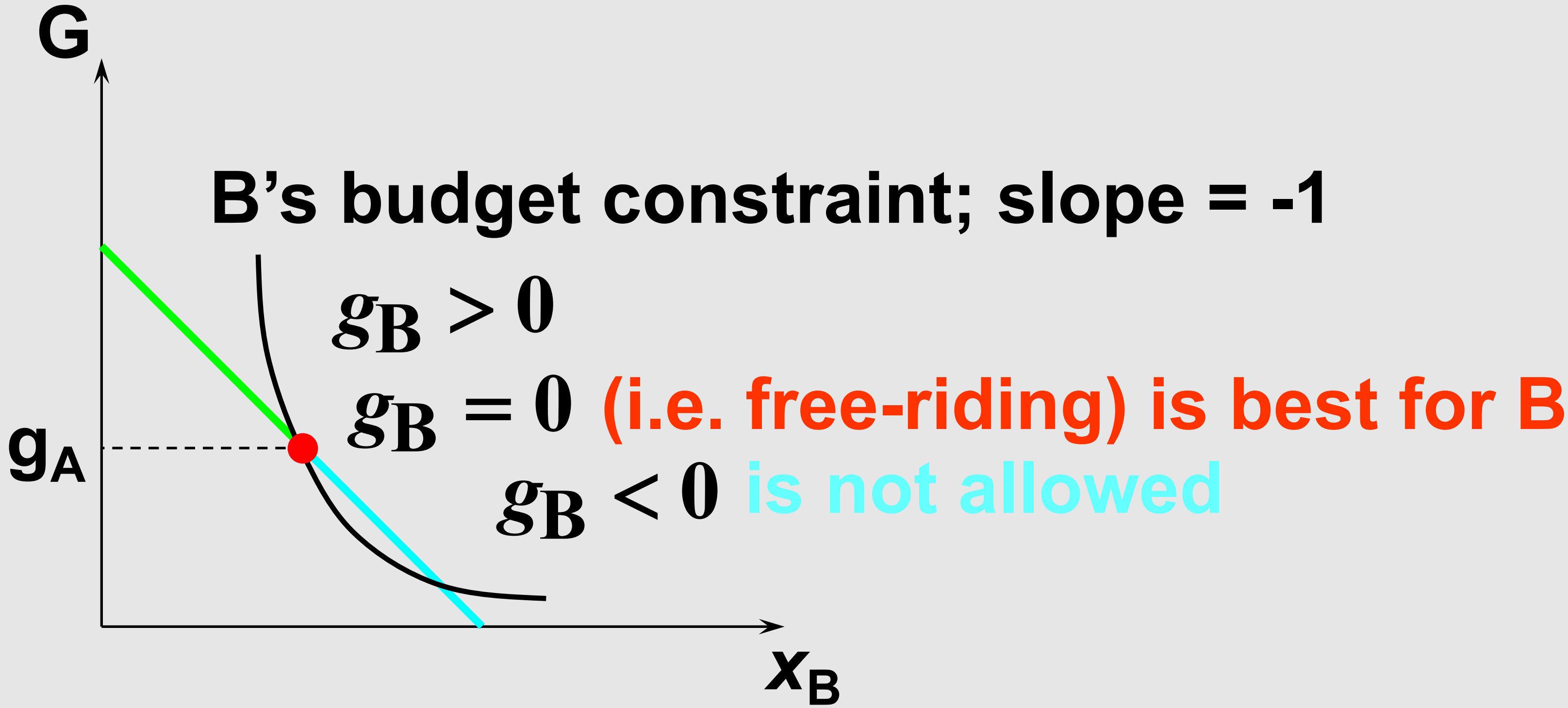
Free-Riding Revisited



Free-Riding Revisited



Free-Riding Revisited



Free-riding is rational in such cases.

Free-Riding Intuition

- Free rider problem: When an investment has a personal cost but a common benefit, individuals will underinvest.

- Because of the free rider problem, the private market undersupplies public goods

Another way to see it:

- Private provision of a public good creates a positive externality (as everybody else benefits)
  - Goods with positive externalities are under-supplied by the market

Free-Riding Intuition

- 2 individuals with identical utility functions defined on x private good (cookies) and G public good (fireworks)

$$G = g_A + g_A$$

- Utility of individual i is $\mathrm{U_i} = 2\log(\mathrm{x_i}) + \log(g_A + g_B)$
- Budget $x_i + g_i = 100$
- Individual A chooses $\mathrm{g_A}$ to maximize $2\log(100 - \mathrm{g_A}) + \log(g_A + g_B)$ taking g_B as given
- First order condition:
- $-\frac{2}{(100-\mathrm{g_A})} + \frac{1}{g_A+g_B} = 0 \qquad \mathrm{g_A} = \frac{100-2g_B}{3}$
- Note that g_A goes down with g_A due to the free rider problem (called the reaction curve)
- Symmetrically, we have $\mathrm{g}_B = \frac{100-2g_A}{3}$

Can Private Provision Overcome Free Rider Problem?

→ The free rider problem does not lead to a complete absence of private provision of public goods. Private provision works better when:

1. Some Individuals Care More than Others:
   – Private provision is particularly likely to surmount the free rider problem when individuals are not identical, and when some individuals have an especially high demand for the public good.
2. Altruism:
   – When individuals value the benefits and costs to others in making their consumption choices.

6. Collective decision mechanisms

For private goods

→ A competitive market mechanisms will achieve a Pareto-optimal allocation

- Important assumption: an individual's consumption did not affect other people's utility

For public goods

→ Utilities of the individuals are linked since everyone consumes the same amount of the public good

- Market provision will not be Pareto efficient
- Alternatives:
  - Command mechanism
  - Voting system

Voting system

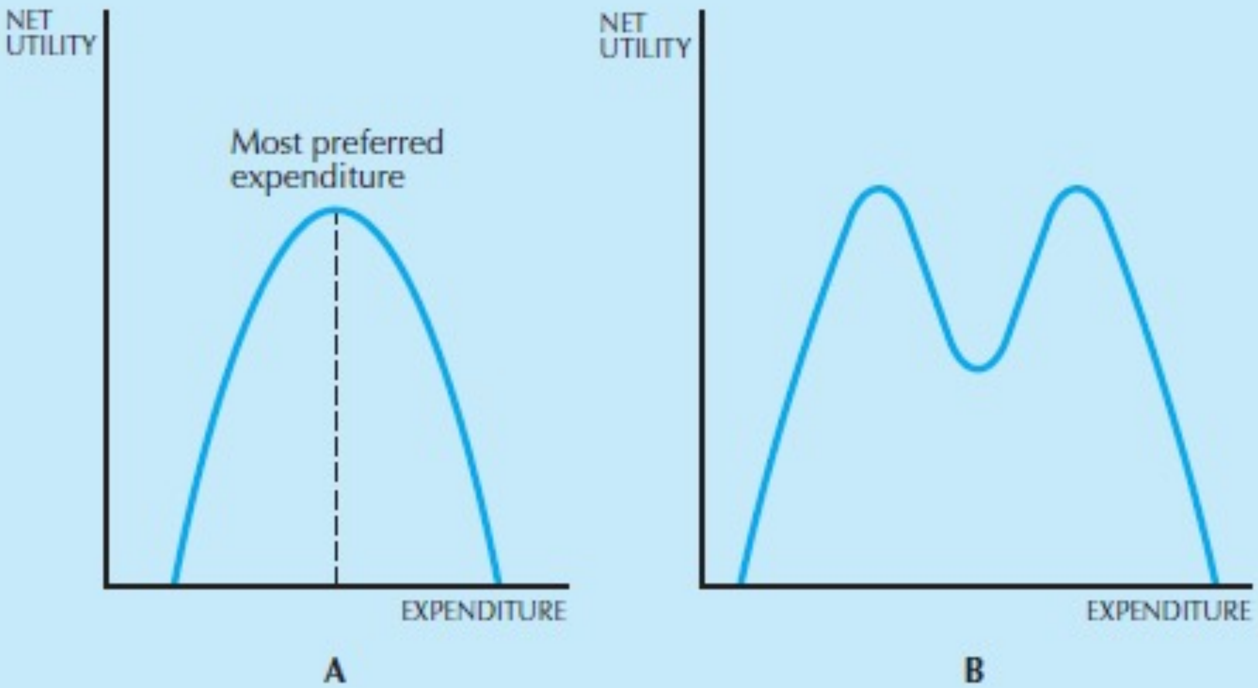
→ Let's imagine that the consumers are voting about the size of some public good

- Same problems with voting than in Chapter 3…
- Non transitivity
- Sensitive to manipulation

→ What restrictions on preferences will allow us to rule them out?

- **Single-peaked preferences:** net utility of expenditure on the public good rises at first due to the benefits of the public good but then eventually falls, due to the costs of providing it

Single peaked preferences



Shapes of preferences. Single-peaked preferences are shown in panel A and multiple peaked preferences in panel B.

Voting system

→ With single-peaked preferences, there will never be intransitivity

→ The chosen result is the median expenditure: one-half of the population wants to spend more, and one-half wants to spend less

Is it Pareto efficient?

→ In general, No. Since it doesn't say anything about *how much more* they want of the public good.

Voting system

Problem

→ Consumers may not have good incentives to report true utility values

→ Challenge: determine TRUE individual utility functions

→ A scheme that makes it rational for individuals to reveal truthfully their private valuations of a public good is a **revelation mechanism**.

→ E.g. the Groves-Clarke taxation scheme

7. Lindahl prices

→ With Public goods, everyone must consume the same quantity of G

→ Idea: can we come up with "individualized prices" that vary across individuals reflecting their willingness to pay? → **Lindahl Prices**

→ With ordinary private goods: people all face the same price; they choose the quantity they wish to consume.

→ With a pure public good (opposite situation): everyone consumes the same quantity of the good, but people's prices are different.

→ Lindahl price is a **hypothetical** price that a person would be willing to pay for a little more of the good

Lindahl prices

→ Rule (with 2 consumers):

$$p_G^A(G, R_A) + p_G^B(G, R_B) = c$$

Conditions for Lindahl equilibrium (general)

- Public good must be fully financed $\sum p_G^i = c$
- All individuals must demand the same quantity of G

→ Definition of equilibrium

- Equilibrium is a set of prices such that all persons demand the same level of the public good.

Lindahl prices - example

- 2 consumers

$$U^A(x_A, G) = 2\ln(x_A) + \ln(G)$$

$$U^B(x_B, G) = \ln(x_B) + 2\ln(G)$$

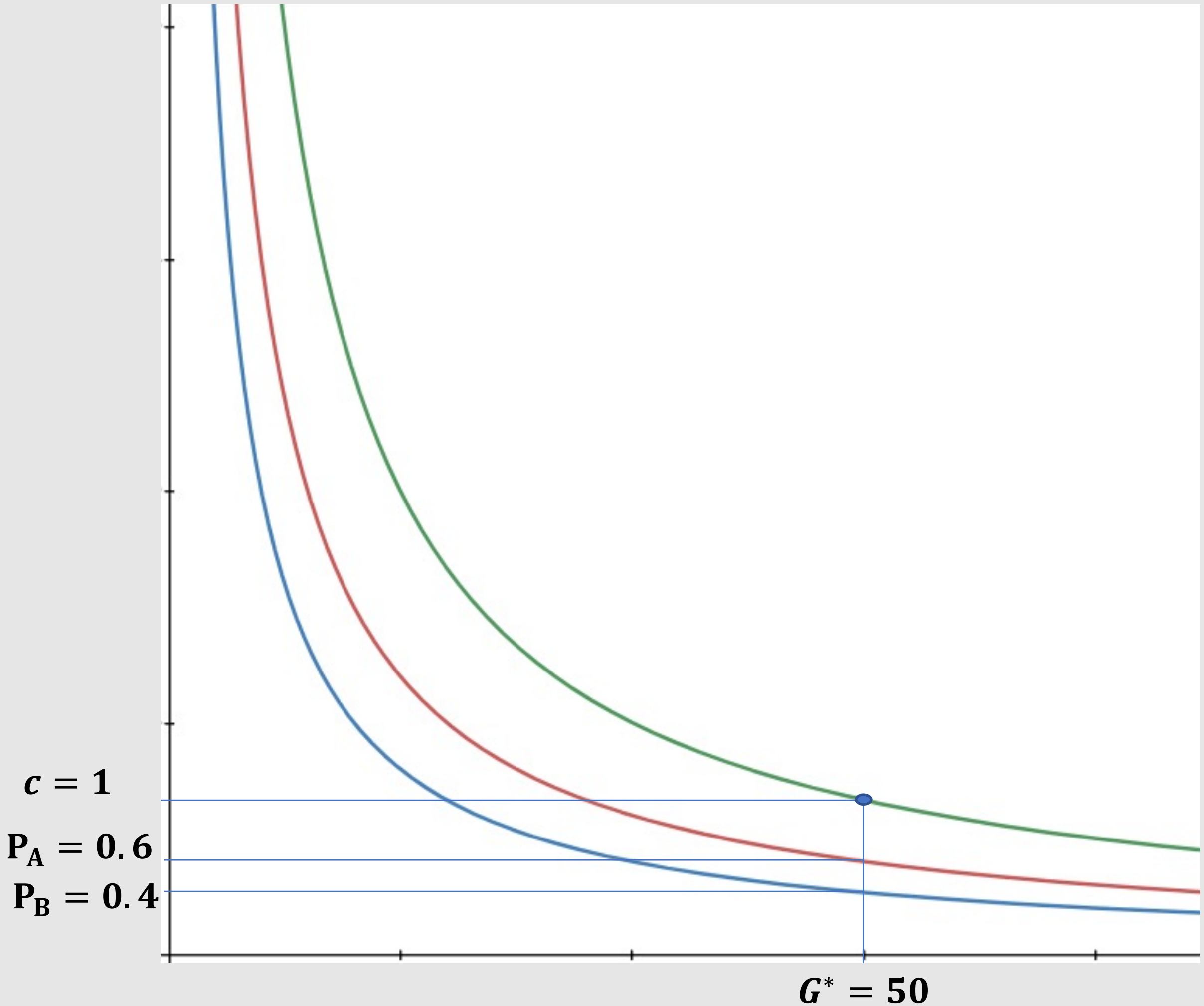
- Budget constraint is $\mathrm{x_i} + \mathrm{p_i G} = R_i$
- Find inverse demand functions using $\mathrm{MRS}^i_{Gx} = p_i$
- We obtain

$p_A = \frac{1}{3}\frac{1}{G}R_A$ and $p_B = \frac{2}{3}\frac{1}{G}R_B$

Lindahl prices - example

- Let's asume c=1. Then, we need to find G such that $\mathrm{p_A} + \mathrm{p_B} = 1$
- We obtain $G = \frac{R_A}{3} + \frac{2}{3} R_B$
- If $\mathrm{R_A} = 90$ and $\mathrm{R_B} = 30$
- Then $\mathrm{G} = 50$ and $\mathrm{p_A} = 0.6$ and $\mathrm{p_B} = 0.4$

Lindahl prices - example



$$p_A = \frac{30}{G}$$

$$p_B = \frac{20}{G}$$

$$c = \frac{30}{G} + \frac{20}{G}$$

Lindahl prices - intuition

→ Putting together the optimal condition and the Lindahl prices condition:

$$\mathrm{MRS}^{i}_{Gx} = p_i \qquad \text{and} \qquad p^{A}_{G}(G, R_A) + p(G, R_B) = c$$

→ We obtain that

$$\sum \mathrm{MRS}^{i}_{Gx} = \sum p_i = c$$

→ Hence, the Lindahl equilibrium satisfies the Samuelson Rule and outcome is Pareto efficient.

→ Intuition: each individual bears only a fraction of the cost of the Public Good, so people end up picking the right amount of the PG

→ General Conclusion: efficiency can be attained with public goods by the use of personalized prices.