

# Chapter 5: Public Goods

Ch 36 in H. Varian 8<sup>th</sup> Ed.

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# Introduction

So far...

- Externalities can be solved if property right are properly assigned
- But... not all externalities are solved like that, especially when there are more than 2 agents.
  - Smoke example with 2 nonsmokers: Now, agents need to coordinate on how much clean air to sell.
  - This can lead to coordination problems... (pollution in a whole country)
- Public goods are one example of consumption externality
  - Everyone must consume the same amount of the good
  - Even if they have different preferences about it
  - What is the optimal amount of public good?
  - How can we socially decide about public goods?

# Public Goods – Definition

- A good is **purely public** if it is both **nonexcludable** and **nonrival** in consumption.
  - Nonexcludable: all consumers can consume the good. Individuals cannot deny each other the opportunity to consume a good.
  - Nonrival: each consumer can consume all of the good. One individual's consumption of a good does not affect another's opportunity to consume the good
- **Impure public goods**: Goods that satisfy the two public good conditions (non-rival in consumption and non-excludable) to some extent, but **not fully**.

# Public Goods – Examples

- Broadcast radio and TV programs.
- National defense.
- Public highways.
- Reductions in air pollution.
- National parks.

# Reservation Prices

- A consumer's reservation price for a unit of a good is his maximum willingness-to-pay for it.
- Consumer's wealth is  $w$
- Utility of not having the good is  $U(w, 0)$
- Utility of paying  $p$  for the good is  $U(w - p, 1)$
- Reservation price is  $r$  such that  $U(w, 0) = U(w - r, 1)$ 
  - Meaning, the price  $r$  such that consumer is indifferent between paying  $r$  and having the good and not having the good

# Reservation Prices: An Example

→ Consumer utility is  $U(x_1, x_2) = x_1(x_2 + 1)$

→ Utility of not buying a unit of good 2 is

$$V(w, 0) = \frac{w}{p_1} (0 + 1) = \frac{w}{p_1}$$

→ Utility of buying one unit of good 2 at price  $p$  is

$$V(w - p, 1) = \frac{w - p}{p_1} (1 + 1) = \frac{2(w - p)}{p_1}$$

# Reservation Prices: An Example

→ Reservation price  $r$  is such that  $V(w, 0) = V(w - r, 1)$

→ This is

$$\frac{w}{p_1} = \frac{2(w-r)}{p_1} \quad \text{hence} \quad r = \frac{w}{2}$$

→ Reservation price of each person will depend on that person's wealth:

- the maximum amount that an individual will be willing to pay will depend to some degree on how much that individual is able to pay.

# Outline

1. When Should a Public Good Be Provided?
2. Free-Riding
3. Variable Public Good Quantities
4. Quasilinear Preferences and Public Goods
5. Free-Riding Revisited
6. Collective decision mechanisms



# 1. When Should a Public Good Be Provided?

Example: two roommates A and B decide whether or not to purchase a TV. TV will be in the living room and enjoyed by both of them

- The TV is a public good that costs  $c$ .
- Individual wealth is  $w_A$  and  $w_B$
- Individual payments for providing the public good are  $g_A$  and  $g_B$ .
- Each person's money left to spend on private consumption is  $x_A$  and  $x_B$
- Budget constraint is then  $x_A + g_A = w_A$  and  $x_B + g_B = w_B$
- The TV is purchased if  $g_A + g_B \geq c$

# When Should a Public Good Be Provided?

→ Utilities depend on private good and on public good

$$U_A(x_A, G) \text{ and } U_B(x_B, G)$$

→ 2 possible allocations:

- No TV:  $(w_A, w_B, 0)$
- TV is bought:  $(x_A, x_B, 1)$  with  $x_i = w_i - g_i$  for  $i=A,B$

→ TV should be bought when both people are better off having the TV and paying their share than not having the TV

→ This is: payments must be individually rational, i.e.

$$U_A(w_A, 0) < U_A(w_A - g_A, 1)$$

$$U_B(w_B, 0) < U_B(w_B - g_B, 1)$$

# When Should a Public Good Be Provided?

→ Using the reservation wage definition...

$$U_A(w_A, 0) = U_A(w_A - r_A, 1) < U_A(w_A - g_A, 1)$$

$$U_B(w_B, 0) = U_B(w_B - r_B, 1) < U_B(w_B - g_B, 1)$$

→ Hence,  $w_A - r_A < w_A - g_A$  and  $w_B - r_B < w_B - g_B$

→ Which implies  $r_A > g_A$  and  $r_B > g_B$

1. The contribution to the public good needs to be smaller than the reservation price → **necessary condition**
2. The sum of willingnesses to pay must be greater than the cost of the TV → **sufficient condition**

$$r_A + r_B > g_B + g_A = c$$

# When Should a Public Good Be Provided?

If  $r_A + r_B > g_B + g_A = c$ , then it is Pareto-efficient to provide the public good.

- Provision of public good only depends on individual's willingness to pay and on the total cost of the good
- Provision of public good generally depends on each individuals' wealth (since  $r_i$  depends generally on wealth)
  - One exception is with quasilinear preferences

# Private Provision of a Public Good?

- Suppose  $r_A > c$  and  $r_B < c$
- Then A would supply the good even if B made no contribution.
- B then enjoys the good for free; **free-riding**.

# Private Provision of a Public Good?

- Suppose  $r_A < c$  and  $r_B < c$
- Then neither A nor B will supply the good alone.
- Yet, if  $r_A + r_B > c$  also, then it is Pareto-improving for the good to be supplied.
- Still, A and B may try to free-ride on each other, causing no good to be supplied.

## 2. Free-Riding

- Suppose A and B each have just two actions – individually supply a public good, or not.
- Cost of supply  $c = \$100$ .
- Payoff to A from the good = \$80.
- Payoff to B from the good = \$65.
- $\$80 + \$65 > \$100$ , so supplying the good is Pareto-improving.

# Free-Riding

Suppose only one player can buy the good...

**Player B**

**Don't**

**Buy**

**Buy**

**Buy**

**-\$20, -\$35**

**-\$20, \$65**

**Player A**

**Don't**

**Buy**

**\$80, -\$35**

**\$0, \$0**



# Free-Riding

**Player B**

**Don't  
Buy**

**Buy**

**Player A**

**Buy**

**-\$20, -\$35**

**-\$20, \$65**

**Don't  
Buy**

**\$80, -\$35**

**\$0, \$0**

**(Don't Buy, Don't Buy) is the unique NE.**

# Free-Riding

**Player B**

**Buy**

**Don't  
Buy**

**Buy**

**-\$20, -\$35**

**-\$20, \$65**

**Player A**

**Don't  
Buy**

**\$80, -\$35**

**\$0, \$0**

**But (Don't' Buy, Don't Buy) is inefficient.**

# Free-Riding

- Now allow A and B to make contributions to supplying the good.
- E.g. A contributes \$60 and B contributes \$40.
- Payoff to A from the good =  $80 - 60 = \$20 > \$0$ .
- Payoff to B from the good =  $65 - 40 = \$25 > \$0$ .

# Free-Riding

**Player B**

**Don't**

**Contribute Contribute**

**Contribute**

**\$20, \$25**

**-\$60, \$0**

**Player A**

**Don't**

**Contribute**

**\$0, -\$40**

**\$0, \$0**

## Free-Riding

**Player B**

**Don't**

**Contribute Contribute**

---

**Contribute**

**\$20, \$25**

**-\$60, \$0**

**Player A**

**Don't**

**Contribute**

**\$0, -\$40**

**\$0, \$0**

**Two NE: (Contribute, Contribute) and  
(Don't Contribute, Don't Contribute).**

# Free-Riding

- So allowing contributions makes possible supply of a public good when no individual will supply the good alone.
- But what contribution scheme is best?
- And free-riding can persist even with contributions.
  - Ex: incentive to lie about willingness to pay

### 3. Variable Public Good Quantities

- E.g. how many broadcast TV programs, or how much land to include into a national park.
- $c(G)$  is the production cost of  $G$  units of public good.
- Two individuals,  $A$  and  $B$ .
- Private consumptions are  $x_A, x_B$ .

# Variable Public Good Quantities

→ Budget allocations must satisfy

$$x_A + x_B + c(G) = w_A + w_B$$

→ For consumer A, the maximization problem is:

$$\max_{x_A, x_B, G} U_A(x_A, G)$$

Subject to  $x_A + x_B + c(G) = w_A + w_B$

And fixing  $U_B$  to  $\overline{u}_B$ , we add that  $U_B(x_B, G) = \overline{u}_B$

The solution to this problem is such that

$$|MRS_A| + |MRS_B| = MC(G)$$

This is the **Samuelson rule**



# Variable Public Good Quantities

$$|MRS_A| + |MRS_B| = \left| \frac{\Delta x_A}{\Delta G} \right| + \left| \frac{\Delta x_B}{\Delta G} \right| = \frac{MU_G}{MU_{x_A}} + \frac{MU_G}{MU_{x_B}} = MC(G)$$

- $MRS_A$  &  $MRS_B$  are A & B's marg. rates of substitution between the private and public goods.
- $MC(G)$  is the marginal cost of providing an extra unit of  $G$
- This is the Pareto efficiency condition for public good supply
- Remember: the public good is nonrival in consumption, so 1 extra unit of public good is fully consumed by both A and B.
- Why?

# Variable Public Good Quantities

→ To understand why, let's suppose  $|MRS_A| + |MRS_B| < MC(G)$

→ Ex:  $|MRS_A| = \frac{1}{4}$        $|MRS_B| = \frac{1}{2}$       and       $MC = 1$

→ If the public good is reduced by 1 unit, A needs to be compensated with  $\frac{1}{4}$  units of private good and B by  $\frac{1}{2}$ . Hence, there is still  $\frac{1}{4}$  of a dollar left over.

→ Making 1 less public good unit releases more private good than the compensation payment requires  $\Rightarrow$  Pareto-improvement from reduced G.

→ Meaning that both consumers will be better off if G is reduced

# Variable Public Good Quantities

→ Now, let's suppose  $|MRS_A| + |MRS_B| > MC(G)$

→ Ex:  $|MRS_A| = \frac{2}{3}$        $|MRS_B| = \frac{1}{2}$       and       $MC = 1$

→ A would give up  $\frac{2}{3}$  units of private good to get 1 unit of G and B would give up  $\frac{1}{2}$ . If they give up more  $\frac{2}{3}$  and  $\frac{1}{2}$  we have more than enough to produce 1 more G and we could give back some of the extra money.

→ Making 1 more public good unit is Pareto-improving

→ Meaning that both consumers will be better off if G is increased

# Variable Public Good Quantities

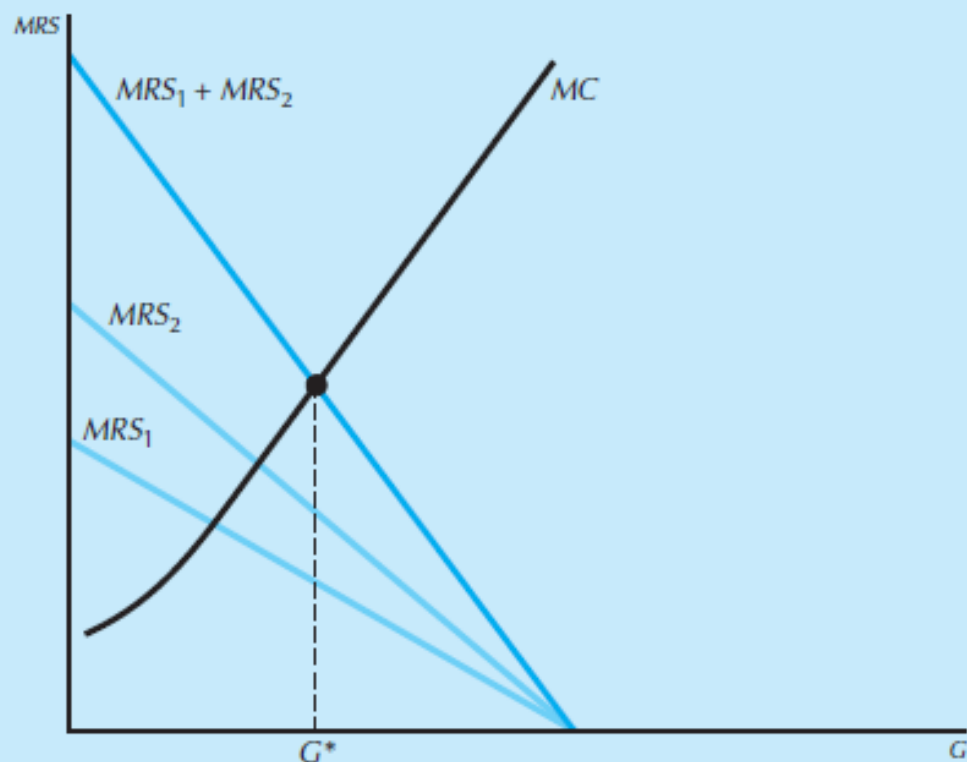
→ Hence, necessarily, efficient public good production requires

$$|MRS_A| + |MRS_B| = MC(G)$$

→ Suppose there are  $n$  consumers;  $i = 1, \dots, n$ . Then efficient public good production requires

$$\sum_{i=1}^n |MRS_i| = MC(G)$$

# Variable Public Good Quantities



**Determining the efficient amount of a public good.** The sum of the marginal rates of substitution must equal the marginal cost.

## 4. Quasilinear Preferences and Public Goods

→ Two consumers, A and B.

$$U_i(x_i, G) = x_i + f_i(G) \quad \text{for } i=A, B$$

→ Marginal utility of the private good is always 1

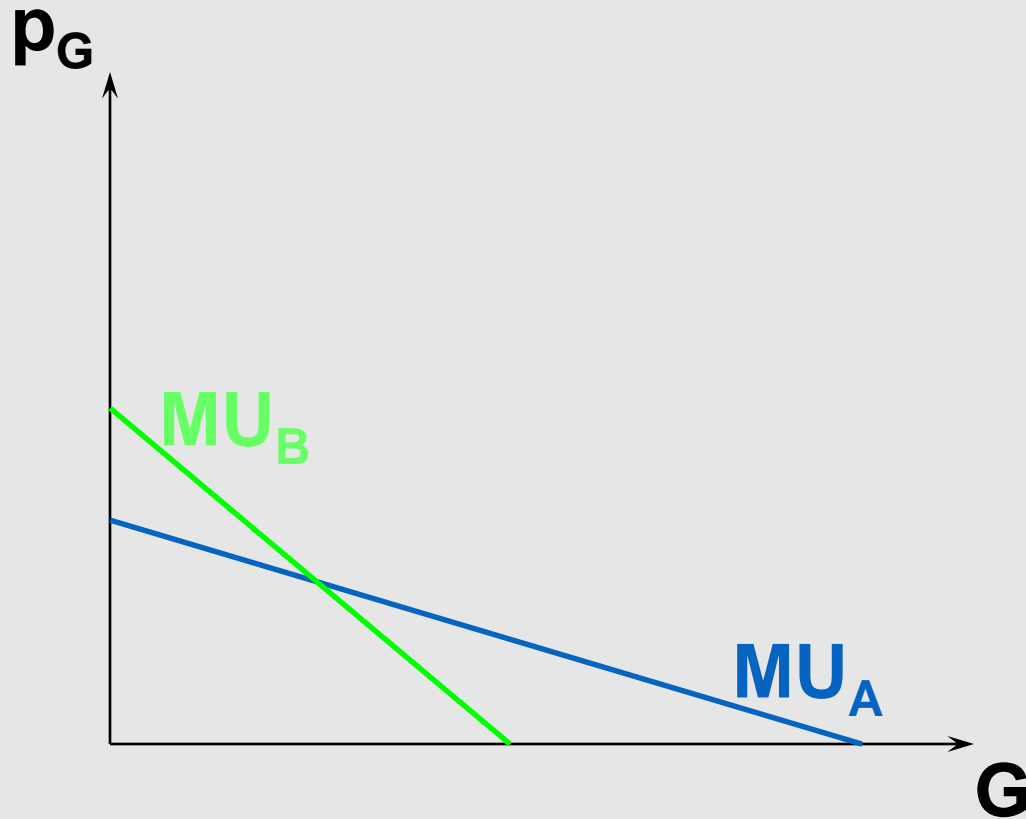
→ Utility-maximization requires

$$|MRS_i| = \frac{\frac{\Delta U_i(x_1, G)}{\Delta G}}{\frac{\Delta U_i}{\Delta x_1}} = \frac{\Delta U_i(x_1, G)}{\Delta G} = f'_i(G)$$

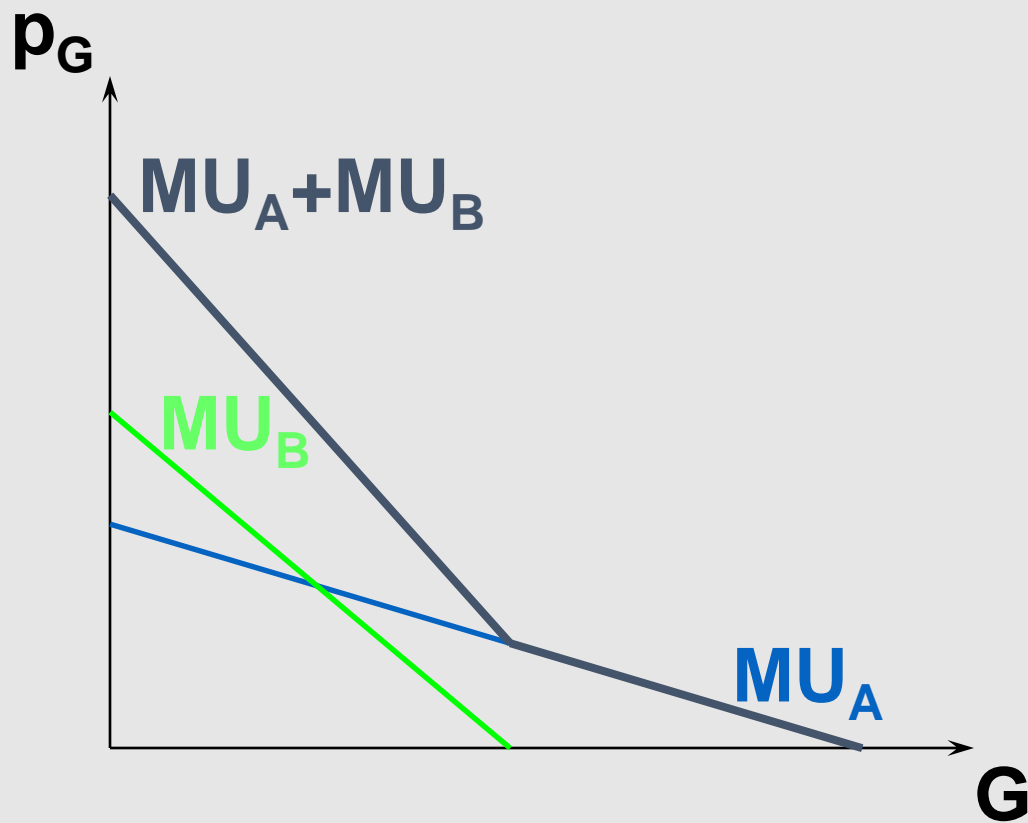
$$\text{Since } |MRS_A| + |MRS_B| = MC(G)$$

$$\text{Then } f'_A(G) + f'_B(G) = MC(G)$$

# Quasilinear Preferences and Public Goods

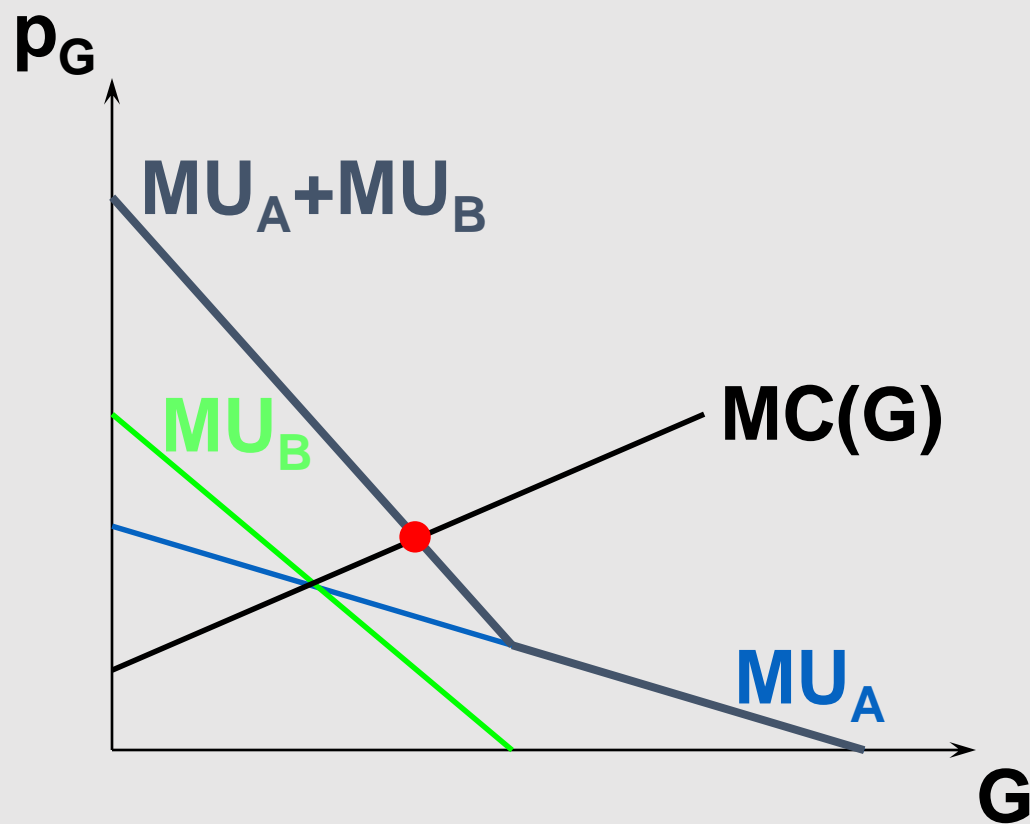


# Quasilinear Preferences and Public Goods

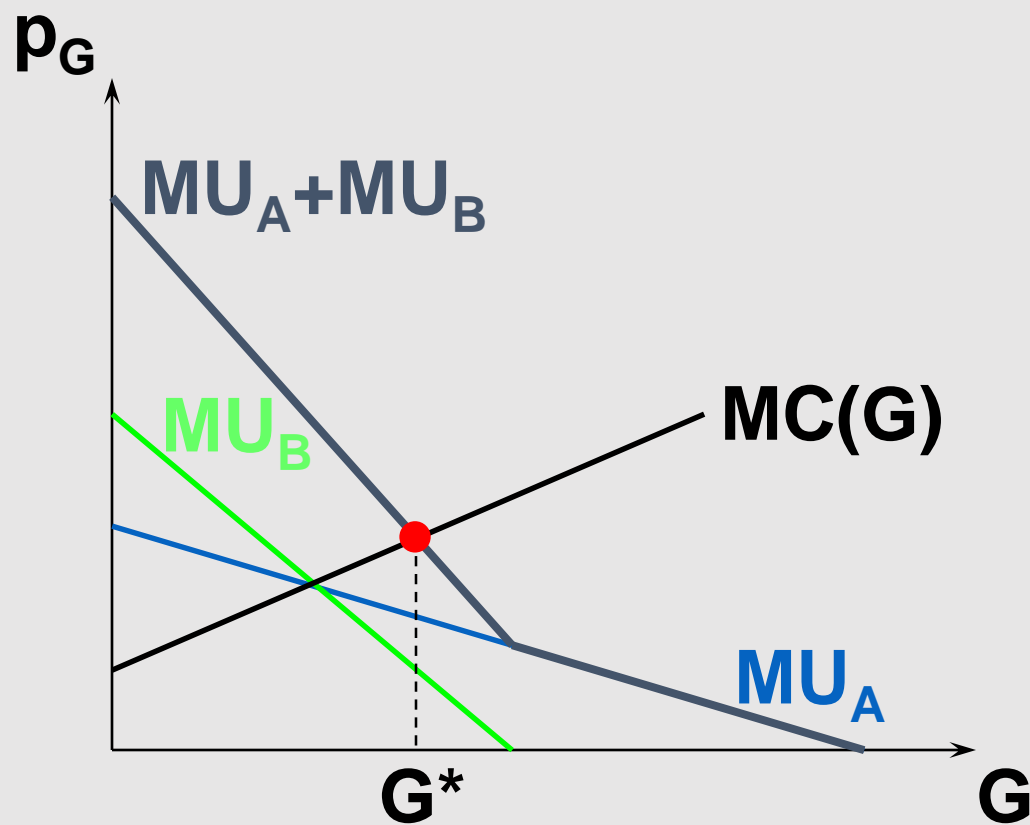




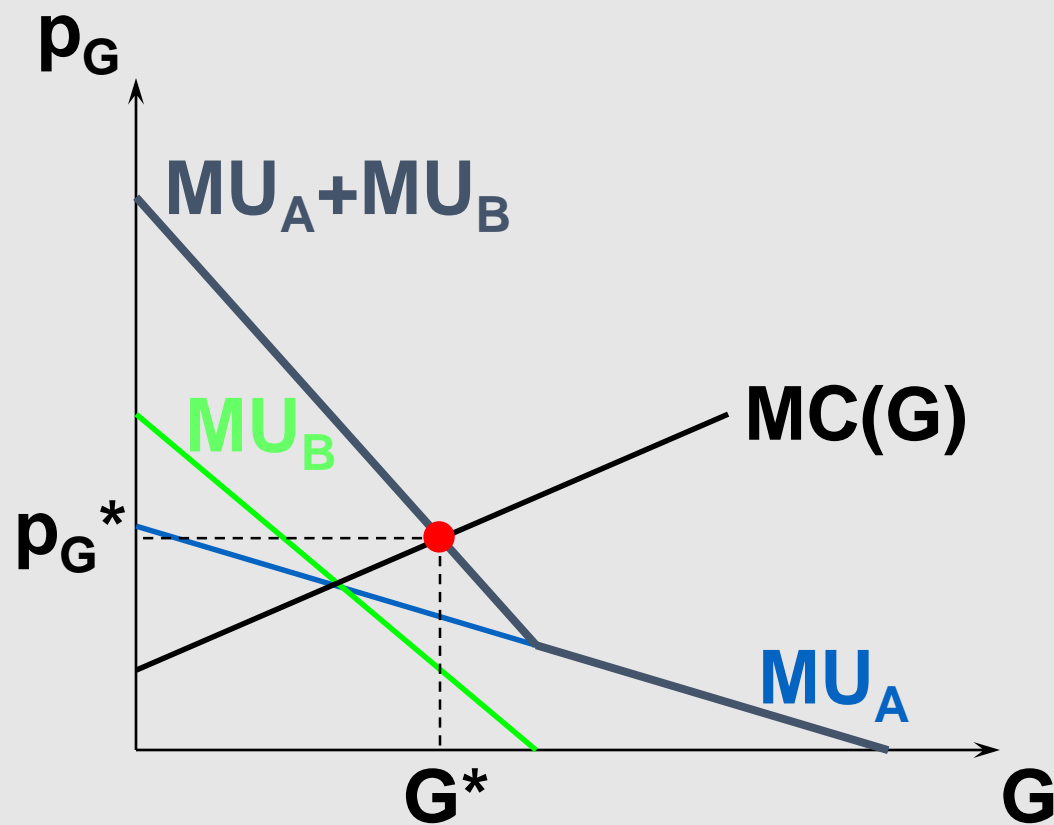
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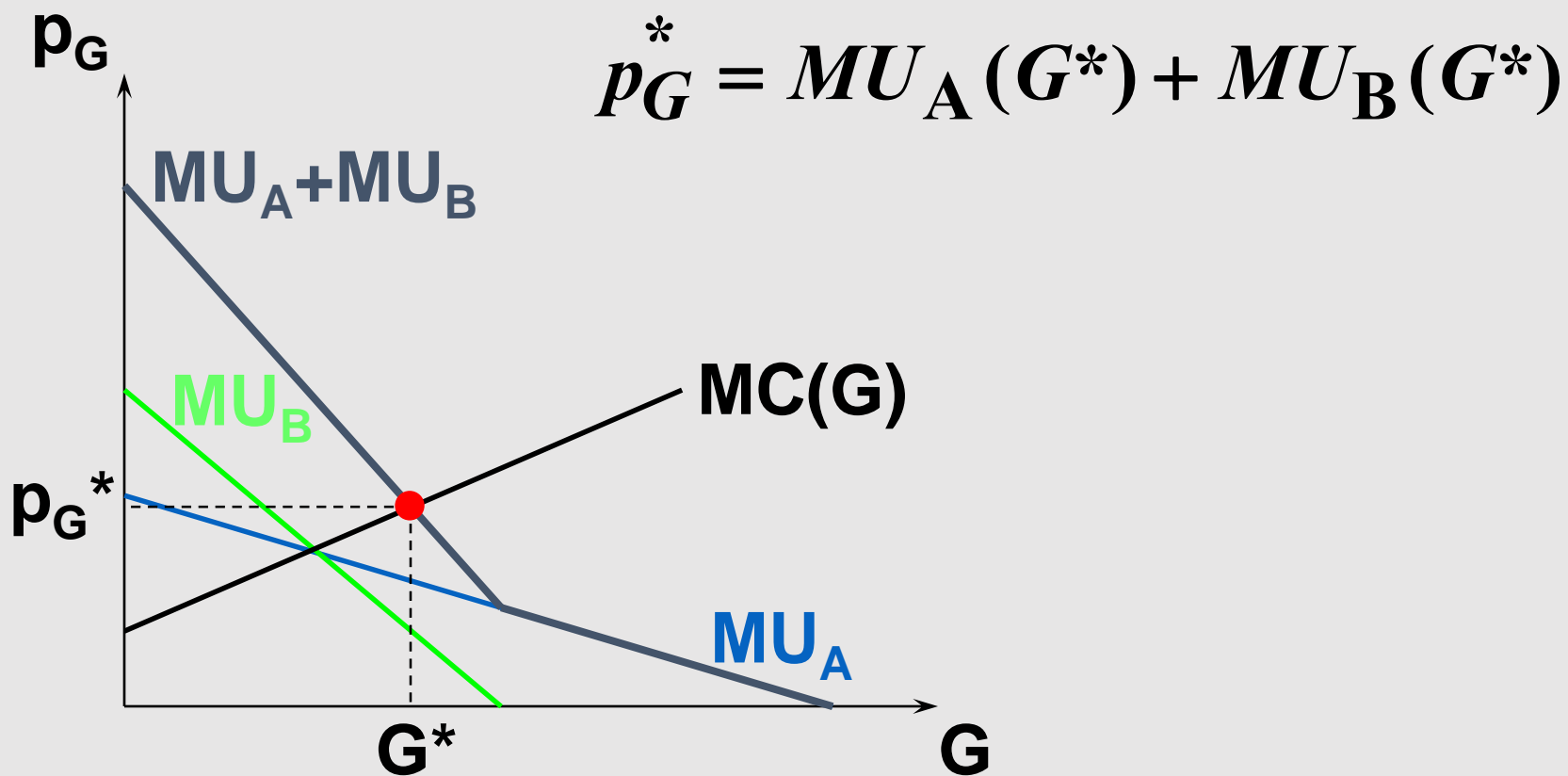
# Quasilinear Preferences and Public Goods



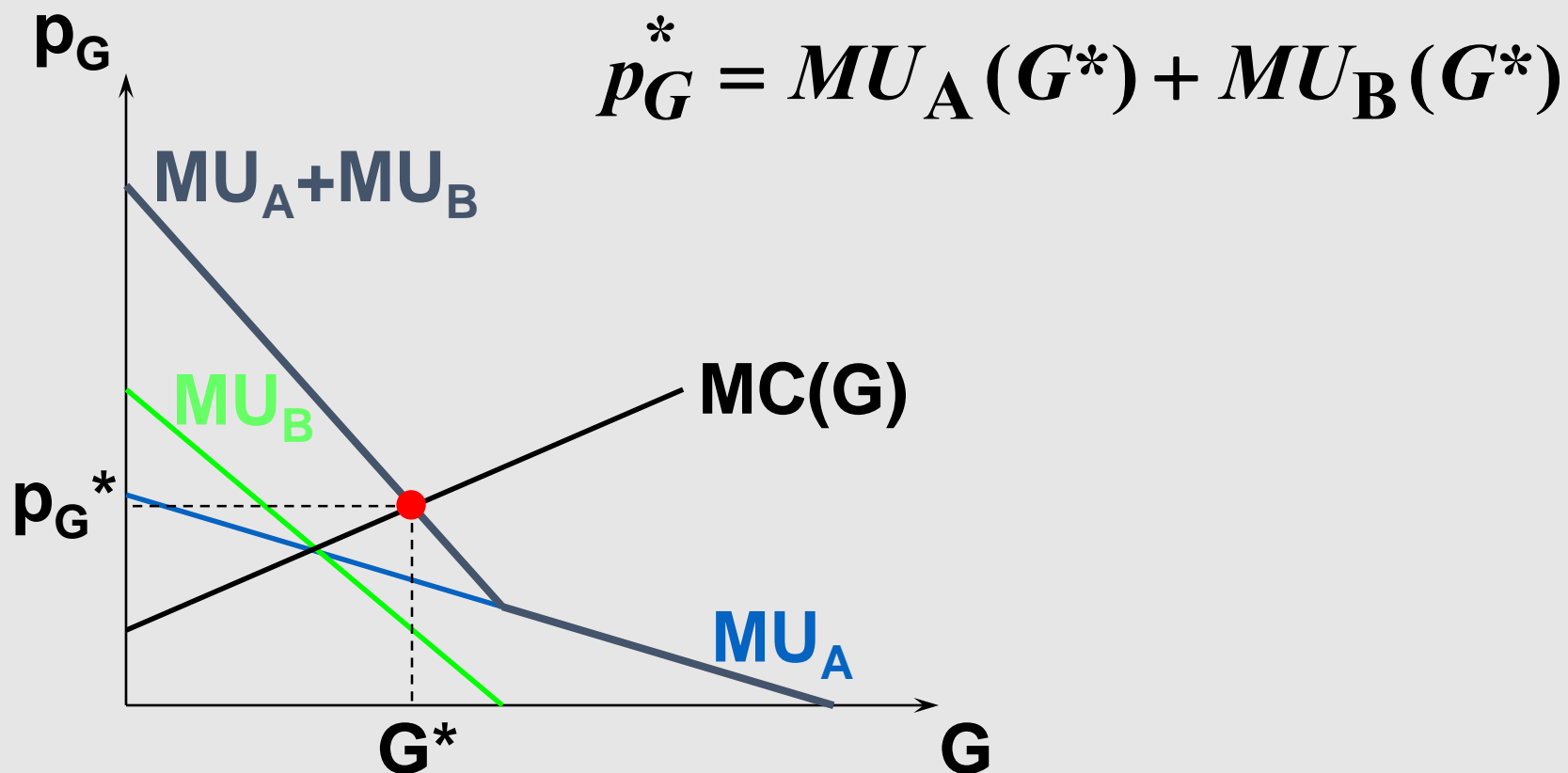
# Quasilinear Preferences and Public Goods



# Quasilinear Preferences and Public Goods



# Quasilinear Preferences and Public Goods



Efficient public good supply requires A & B to state **truthfully** their marginal valuations.

## 5. Free-Riding Revisited

→ When is free-riding individually rational?

Suppose that

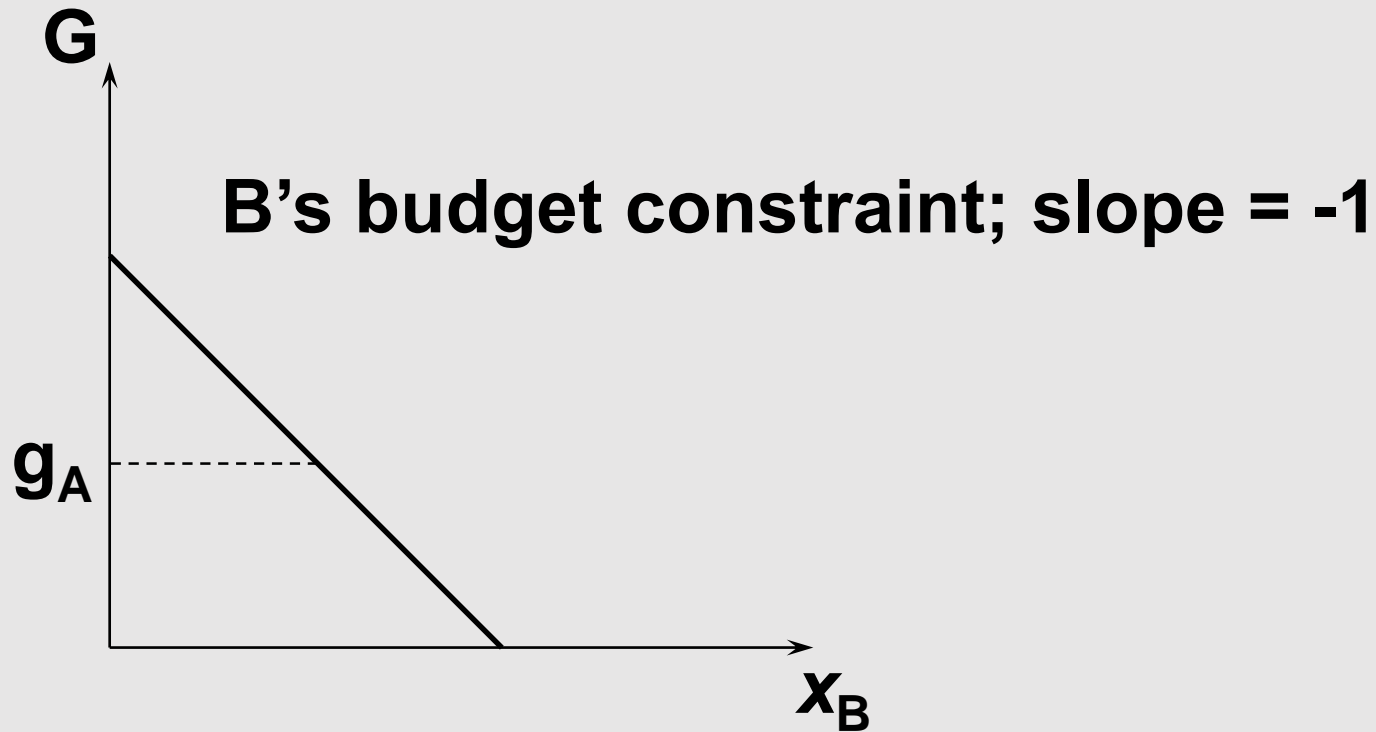
→ Individuals can contribute only positively to public good supply; nobody can lower the supply level.  $g_1 \geq 0$

Given that A contributes  $g_A$  units of public good, B's problem is

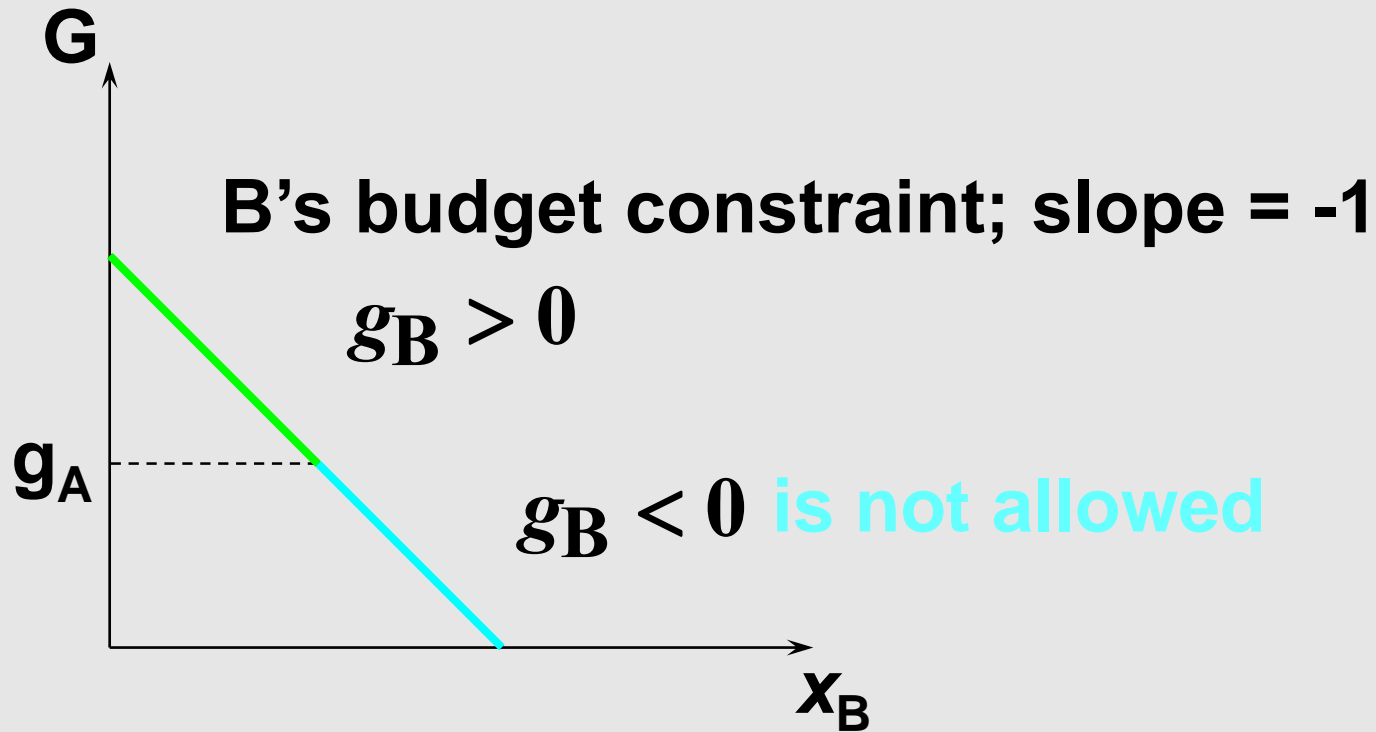
$$\max_{x_B, g_B} U_B(x_B, g_A + g_B)$$

Subject to  $x_B + g_B = w_B$  and  $g_B \geq 0$

# Free-Riding Revisited

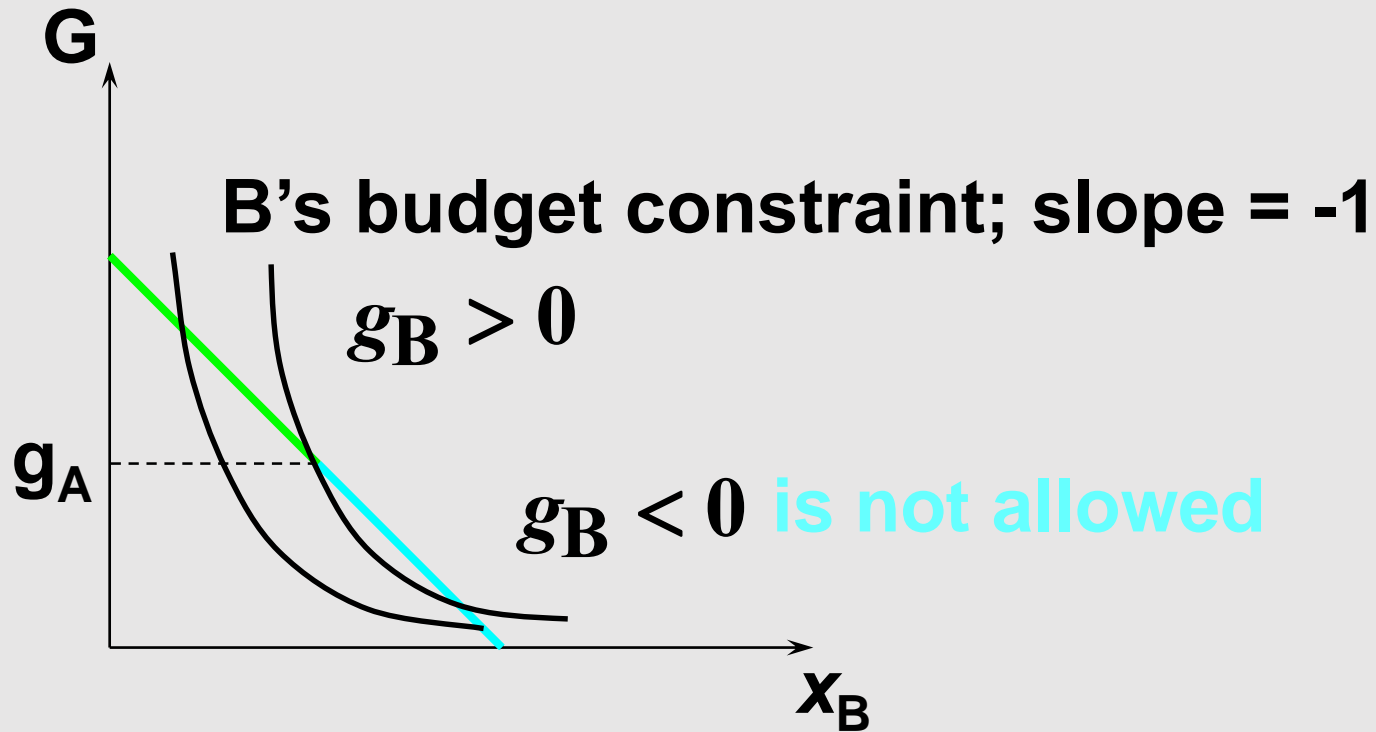


# Free-Riding Revisited

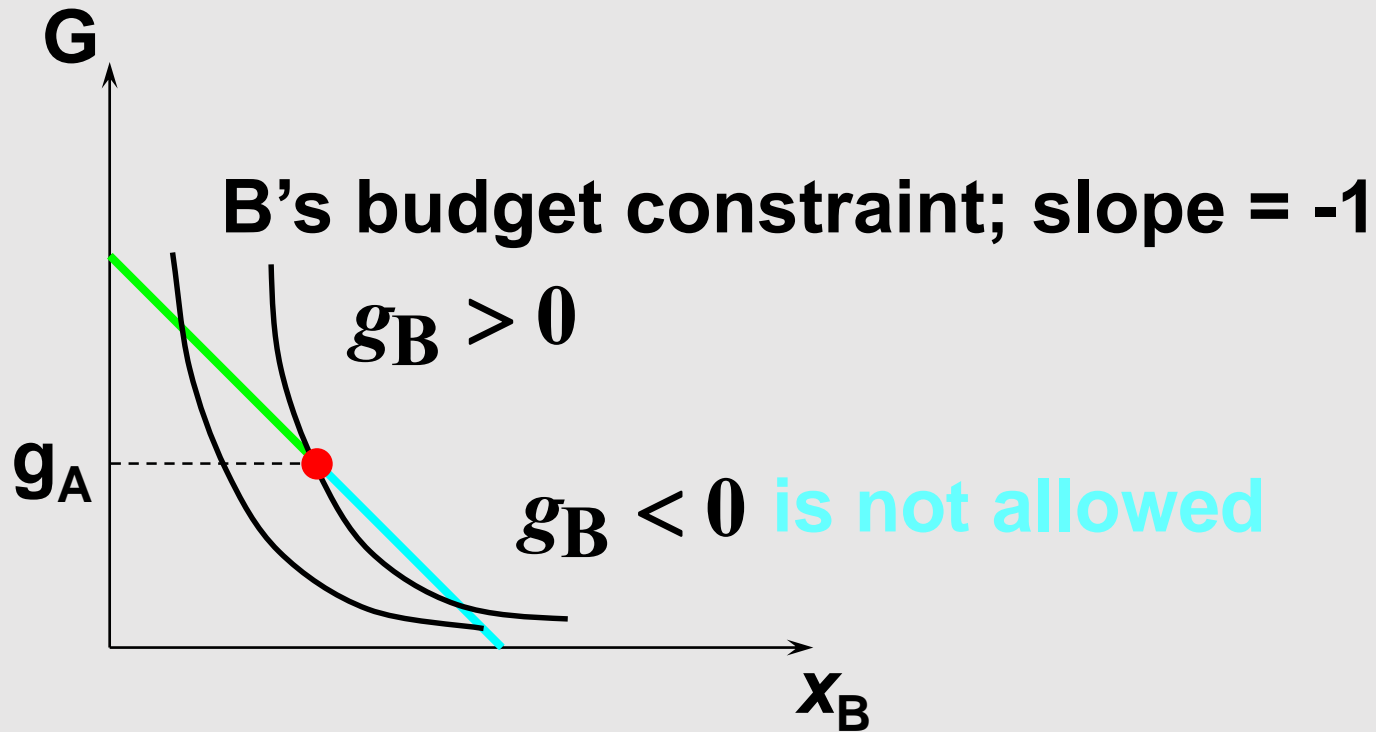




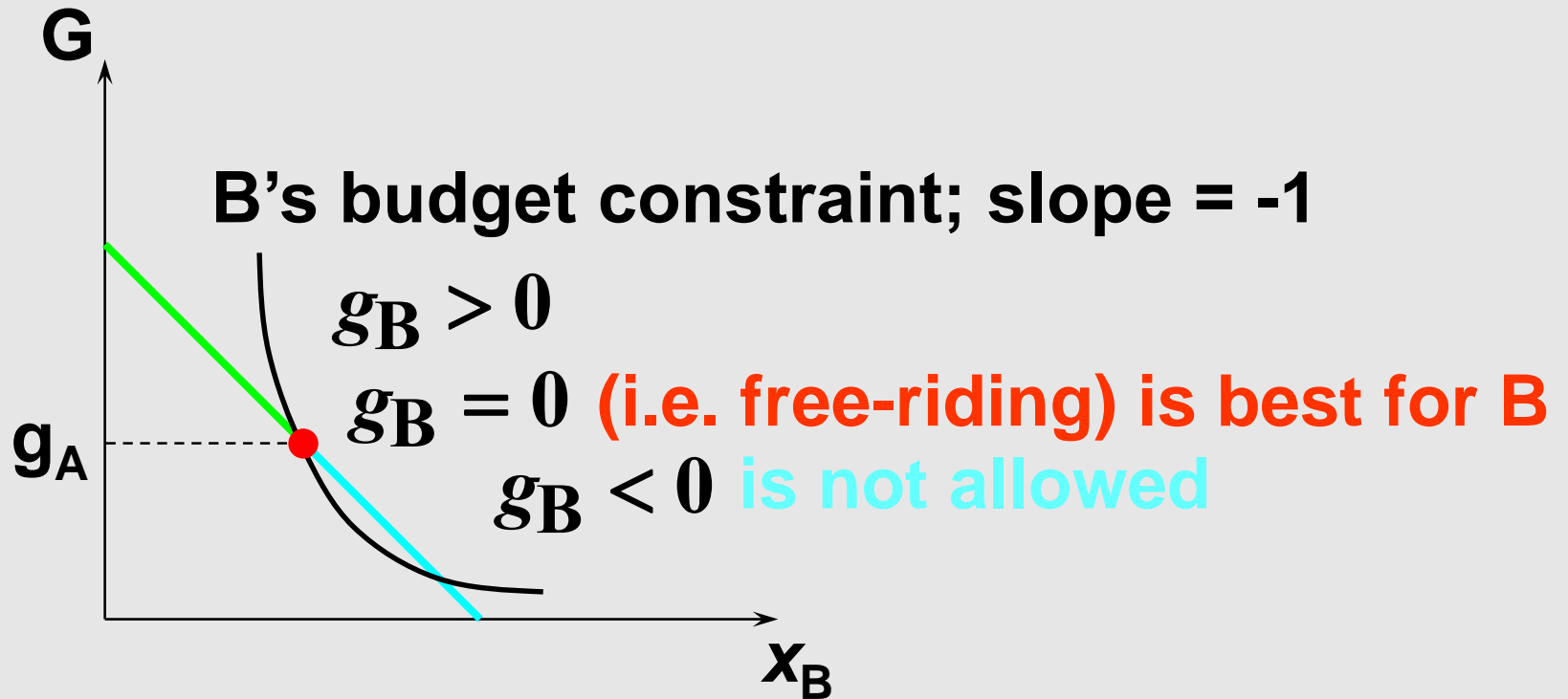
# Free-Riding Revisited



# Free-Riding Revisited



# Free-Riding Revisited



**Free-riding is rational in such cases.**

# Free-Riding Intuition

- Free rider problem: When an investment has a personal cost but a common benefit, individuals will underinvest.
- Because of the free rider problem, the private market undersupplies public goods

Another way to see it:

- Private provision of a public good creates a positive externality (as everybody else benefits)
  - Goods with positive externalities are under-supplied by the market

# Free-Riding Intuition

- 2 individuals with identical utility functions defined on  $x$  private good (cookies) and  $G$  public good (fireworks)

$$G = g_A + g_B$$

- Utility of individual  $i$  is  $U_i = 2\log(x_i) + \log(g_A + g_B)$

- Budget  $x_i + g_i = 100$

- Individual A chooses  $x_A$  to maximize  $2\log(100 - g_A) + \log(g_A + g_B)$  taking  $g_B$  as given

- First order condition:

$$\rightarrow -\frac{2}{(100-g_A)} + \frac{1}{g_A+g_B} = 0 \qquad g_A = \frac{100-2g_B}{3}$$

- Note that  $g_A$  goes down with  $g_B$  due to the free rider problem (called the reaction curve)

- Symmetrically, we have  $g_B = \frac{100-2g_A}{3}$

# Can Private Provision Overcome Free Rider Problem?

→ The free rider problem does not lead to a complete absence of private provision of public goods. Private provision works better when:

## 1. Some Individuals Care More than Others:

- Private provision is particularly likely to surmount the free rider problem when individuals are not identical, and when some individuals have an especially high demand for the public good.

## 2. Altruism:

- When individuals value the benefits and costs to others in making their consumption choices.

## 3. Warm Glow:

- Model of public goods provision in which individuals care about both the total amount of the public good and their particular contributions as well.

## 6. Collective decision mechanisms

For private goods

- A competitive market mechanisms will achieve a Pareto-optimal allocation
  - Important assumption: an individual's consumption did not affect other people's utility

For public goods

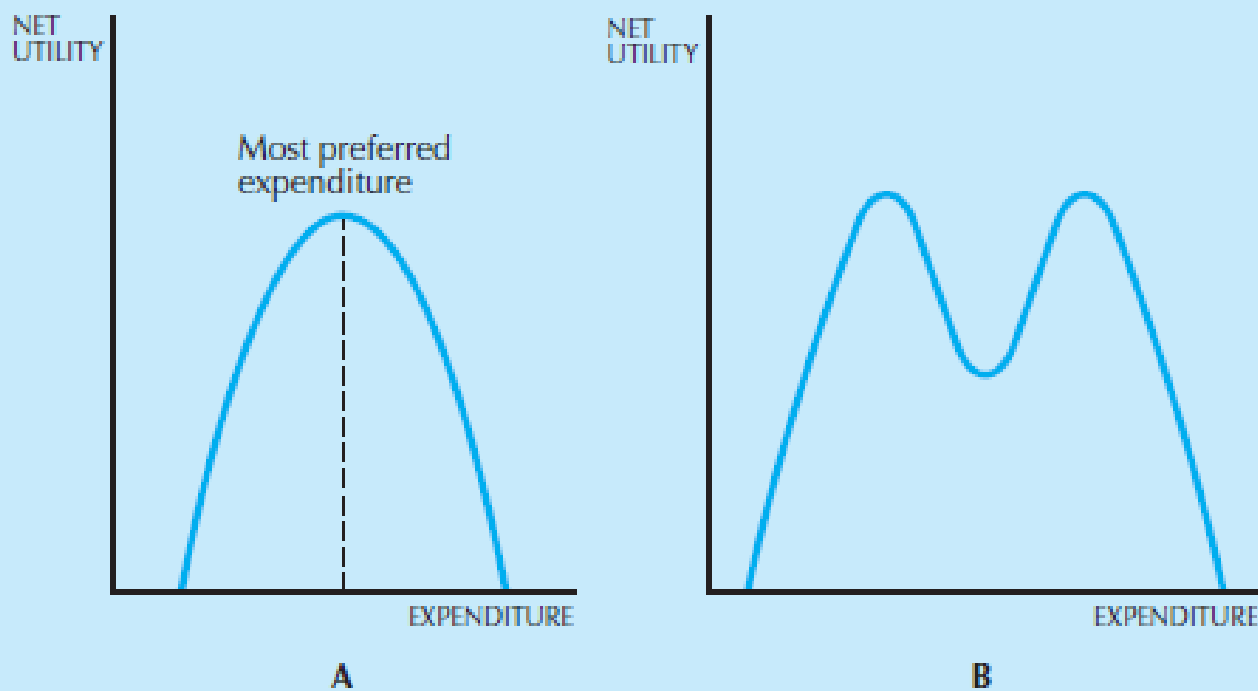
- Utilities of the individuals are linked since everyone consumes the same amount of the public good
  - Market provision will not be Pareto efficient
  - Alternatives:
    - Command mechanism
    - Voting system

# Voting system

- Let's imagine that the consumers are voting about the size of some public good
  - Same problems with voting than in Chapter 3...
  - Non transitivity
  - Sensitive to manipulation
- What restrictions on preferences will allow us to rule them out?
  - **Single-peaked preferences:** net utility of expenditure on the public good rises at first due to the benefits of the public good but then eventually falls, due to the costs of providing it
  - With single-peaked preferences, there will never be intransitivity
  - The chosen result is the median expenditure: one-half of the population wants to spend more, and one-half wants to spend less



# Single peaked preferences



**Shapes of preferences.** Single-peaked preferences are shown in panel A and multiple peaked preferences in panel B.

# Voting system

- With single-peaked preferences, there will never be intransitivity
- The chosen result is the median expenditure: one-half of the population wants to spend more, and one-half wants to spend less

Is it Pareto efficient?

- In general, No. Since it doesn't say anything about *how much more* they want of the public good.

# Voting system

## Problem

- Consumers may not have good incentives to report true utility values
- Challenge: determine TRUE individual utility functions
- A scheme that makes it rational for individuals to reveal truthfully their private valuations of a public good is a **revelation mechanism**.
- E.g. the Groves-Clarke taxation scheme