Chapter 3: Welfare

Ch 33 in H. Varian 8th Ed.

Slides by Mariona Segú, CYU Cergy Paris Université

Inspired by Michael D. Robinson, Mount Holyoke College

Introduction

Until now…

→ Allocations were evaluated according to efficiency

→ BUT, there are other characteristics of the allocations that might be important.
Ex: welfare distribution, fairness

→ THIS CHAPTER: technics used to formalize the idea of welfare distribution

→ How can a society choose among different Pareto efficient allocations?

→ How can individual preferences be "aggregated" into a social preference over all possible economic states?

Introduction

Main focus of this chapter: the welfare function

→ A way to “add together” different consumers’ utility

→ A way to rank different distributions of utility among consumers

To do so, first we need to

→ Go from **individual** preferences to **social** preferences

Outline

1. Aggregating Preferences
2. Social Welfare Functions
3. Welfare Maximization
4. Fair Allocations

1. Aggregating Preferences

Individual preferences

→ Assume they are transitive, as always. We have 3 agents; Bill, Bertha and Bob.

→ Instead of goods, the preferences are now over allocations. Ex: x, y, z denote different allocations.

Social preference

→ Aggregate individual preferences into a social preference

→ Use simple **majority** voting to decide a state? Use a **rank-order** vote?

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

More preferred

Less preferred

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

<u>Majority Vote Results</u>

x beats y

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

Majority Vote Results

x beats y
y beats z

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

Majority Vote Results

x beats y
y beats z
z beats x

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

<u>Majority Vote Results</u>

x beats y
y beats z
z beats x

No socially best alternative!

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

<u>Majority Vote Results</u>

x beats y
y beats z
z beats x

No socially best alternative!

Majority voting does not always aggregate transitive individual preferences into a transitive social preference.

Aggregating Preferences

Bill	Bertha	Bob
x	y	z
y	z	x
z	x	y

Majority Vote Results

Solution: vote 2 by 2

1) Vote on x vs y → x wins
2) Vote on x vs z → z wins

BUT, the order matters!

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results
(low score wins).

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results (low score wins).

x-score = 6

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

<u>Rank-order vote results</u>
(low score wins).

x-score = 6
y-score = 6

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results (low score wins).

x-score = 6
y-score = 6
z-score = 6

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results
(low score wins).

x-score = 6
y-score = 6
z-score = 6

No
state is
selected!

Aggregating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Rank-order vote results (low score wins).

x-score = 6
y-score = 6
z-score = 6

No state is selected!

Rank-order voting is indecisive in this case.

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Possible solution: add an additional option

These are truthful preferences.

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	x(3)	y(3)

Possible solution: add an additional option

These are truthful preferences.

Bob introduces a new alternative

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	α(3)	y(3)
α(4)	x(4)	α(4)

Possible solution: add an additional option

These are truthful preferences.

Bob introduces a new alternative

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	x(2)
z(3)	α(3)	y(3)
α(4)	x(4)	α(4)

Possible solution: add an additional option

These are truthful preferences.

Bob introduces a new alternative **then lies**

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	**α(2)**
z(3)	α(3)	x(3)
α(4)	x(4)	y(4)

Possible solution: add an additional option

These are truthful preferences.

Bob introduces a new alternative

Rank-order vote results
x-score = 8

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	**α(2)**
z(3)	α(3)	x(3)
α(4)	x(4)	y(4)

Possible solution: add an additional option

These are truthful preferences.

Bob introduces a new alternative

Rank-order vote results
x-score = 8
y-score = 7

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	**α(2)**
z(3)	α(3)	x(3)
α(4)	x(4)	y(4)

Possible solution: add an additional option

These are truthful preferences.

Bob introduces a new alternative

<u>Rank-order vote results</u>

x-score = 8
y-score = 7
Z-score = 6

Manipulating Preferences

Bill	Bertha	Bob
x(1)	y(1)	z(1)
y(2)	z(2)	**α(2)**
z(3)	α(3)	x(3)
α(4)	x(4)	y(4)

Possible solution: add an additional option

These are truthful preferences.

Bob introduces a new alternative

Rank-order vote results

x-score = 8

y-score = 7

z-score = 6

α-score = 9

z wins!!

Manipulating Preferences

- These 2 voting schemes can be **manipulated**:
- Majority voting: manipulated by changing the order on which things are voted
- Rank-order voting: manipulated by introducing new alternatives that change the final ranks
  - I.e. one individual can cast an “untruthful” vote to improve the social outcome for himself.

Desirable Voting Rule Properties

What social decision mechanisms are immune to manipulation?

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.

2. If all individuals rank x before y then so should the voting rule.

3. Social preference between x and y should depend on individuals' preferences between x and y only.

→ Kenneth Arrow's Impossibility Theorem: The only voting rule with all of properties 1, 2 and 3 is **dictatorial**.

→ Implication is that a nondictatorial voting rule requires giving up at least one of properties 1, 2 or 3.

Desirable Voting Rule Properties

But, it is hard to find a system that satisfies all the conditions.

In fact…

Kenneth Arrow's Impossibility Theorem:

→ The only voting rule with all of properties 1, 2 and 3 is **dictatorial**.

→ Implication is that a nondictatorial voting rule requires giving up at least one of properties 1, 2 or 3.

2. Social Welfare Functions

1. If all individuals’ preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.

2. If all individuals rank x before y then so should the voting rule.

3. Social preference between x and y should depend on individuals’ preferences between x and y only.

Give up which one of these?

Social Welfare Functions

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.

2. If all individuals rank x before y then so should the voting rule.

3. ~~Social preference between x and y should depend on individuals' preferences between x and y only.~~

Give up which one of these?

Social Welfare Functions

1. If all individuals’ preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.

2. If all individuals rank x before y then so should the voting rule.

There is a variety of voting procedures with both properties 1 and 2.

Social Welfare Functions

→ $u_i(x)$ is individual i's utility from **overall** allocation x.

– A welfare function is an increasing function of each agent's utility

1. Utilitarian: add up individual utilities $$W = \sum_{i=1}^{n} u_i(x).$$
2. Weighted-sum: $$W = \sum_{i=1}^{n} a_i u_i(x) \text{ with each } a_i > 0.$$

   – Each weight indicates how important each individual is to the overall social welfare.
3. Minimax or Rawlsian: $$W = \min\{u_1(x), \ldots, u_n(x)\}.$$

   – The social welfare depends only on the utility of the worst off agent

Each welfare function represents a different ethical judgement

3. Welfare maximization

→ Find the feasible allocation that maximizes social welfare

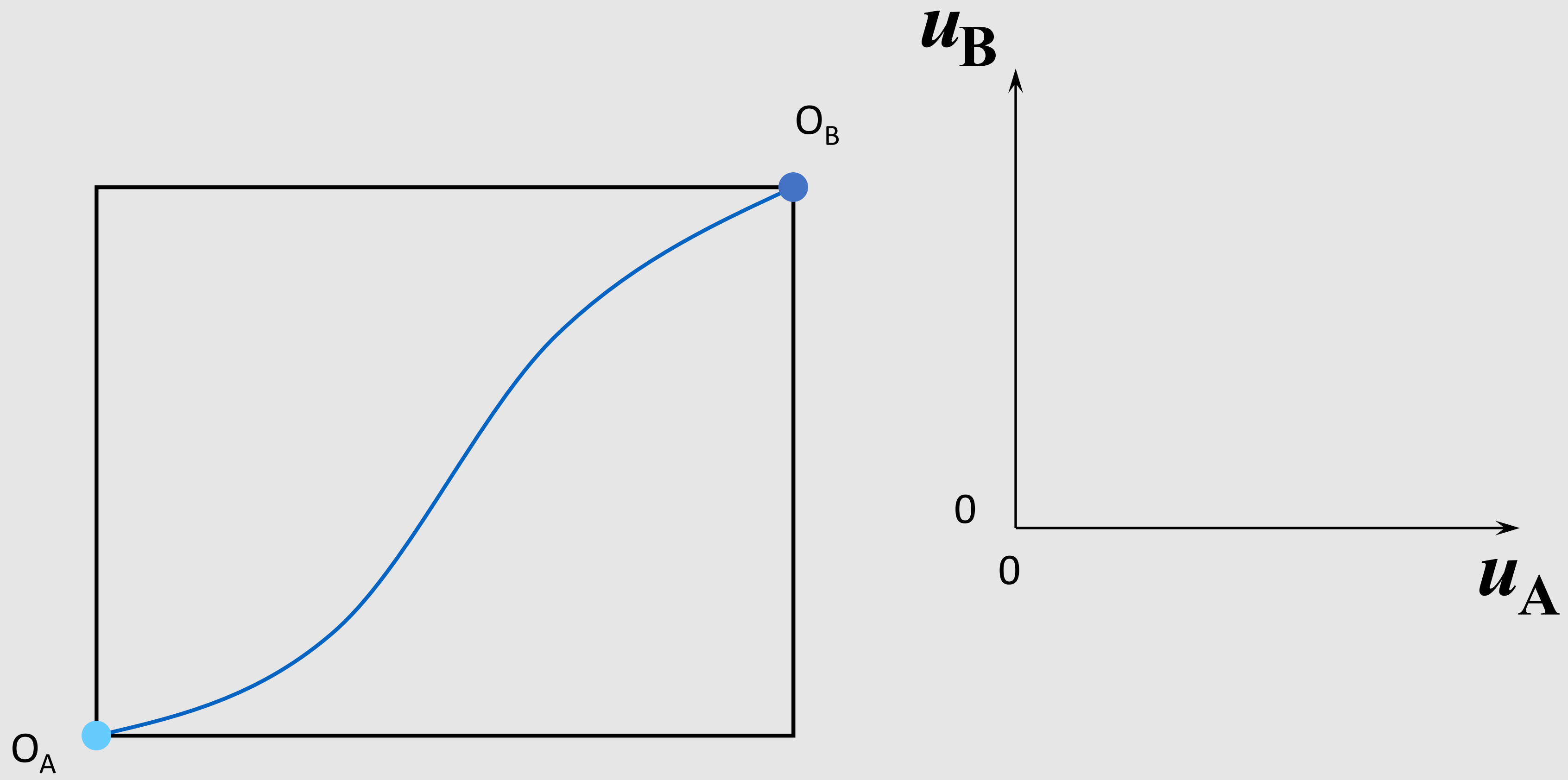
Properties of the optimal allocation

1. Any social optimal allocation must be Pareto optimal.

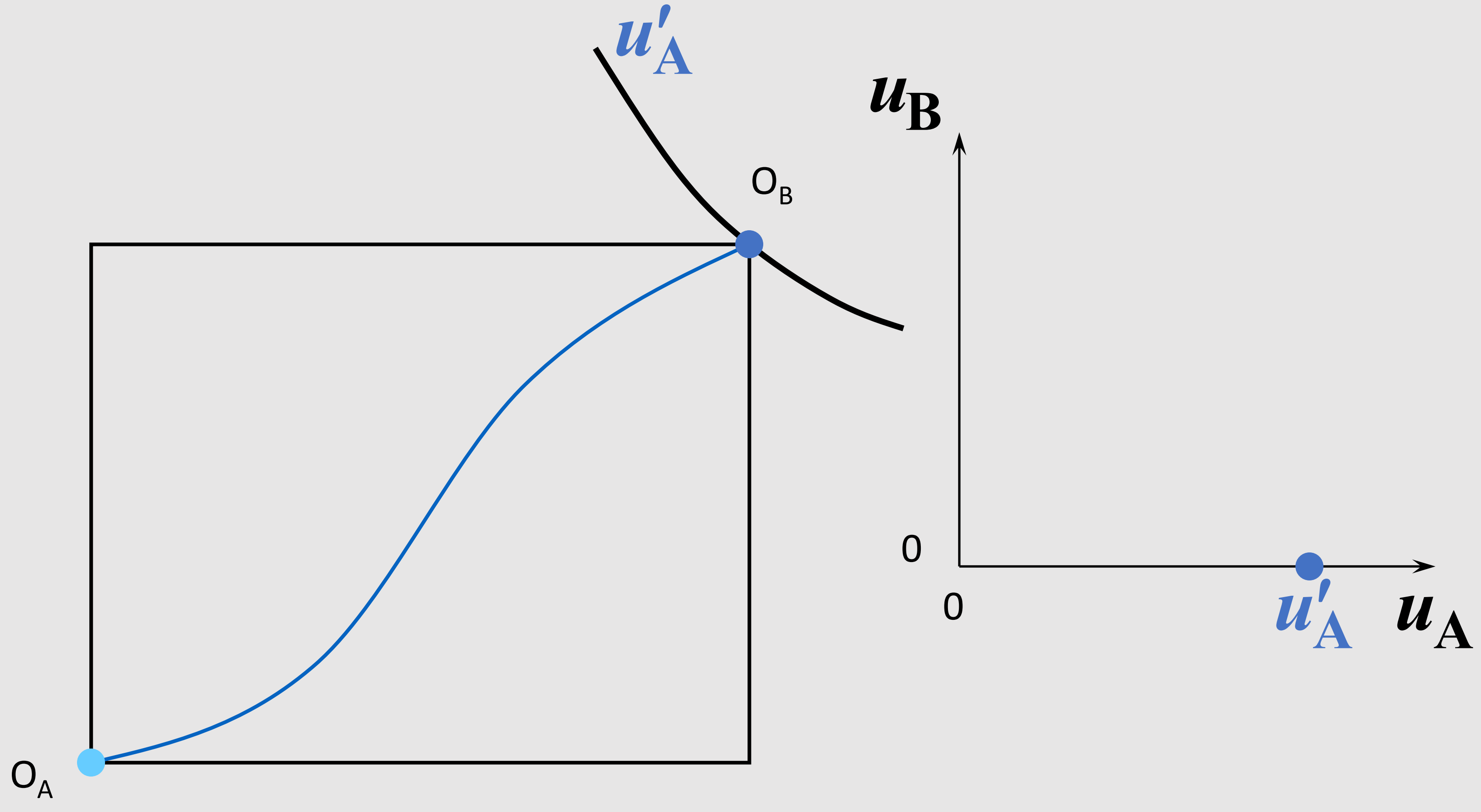
   - Why?
   - If not, then somebody's utility can be increased without reducing anyone else's utility;

     social suboptimality $\Rightarrow$ inefficiency.

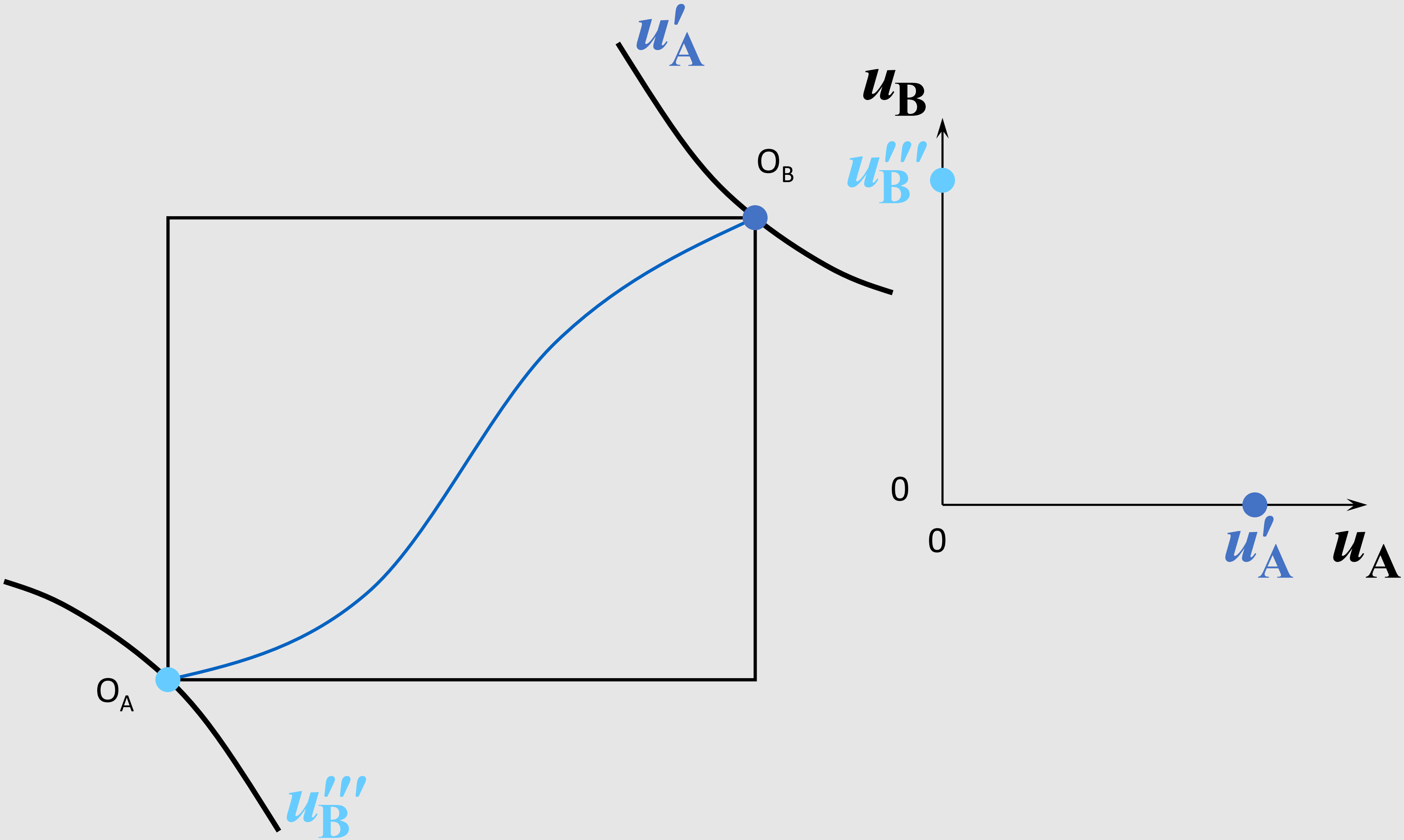
Utility Possibilities



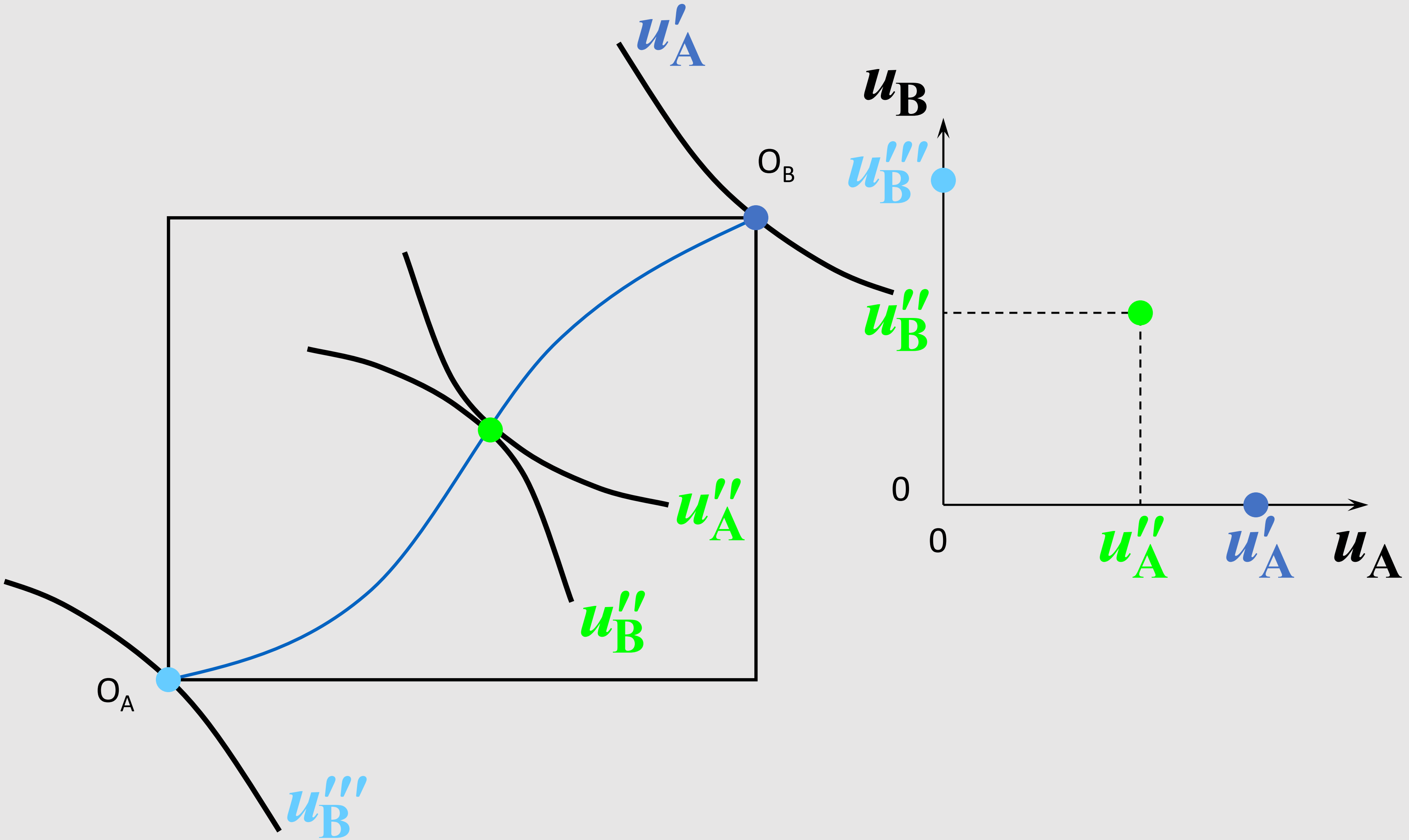
Utility Possibilities



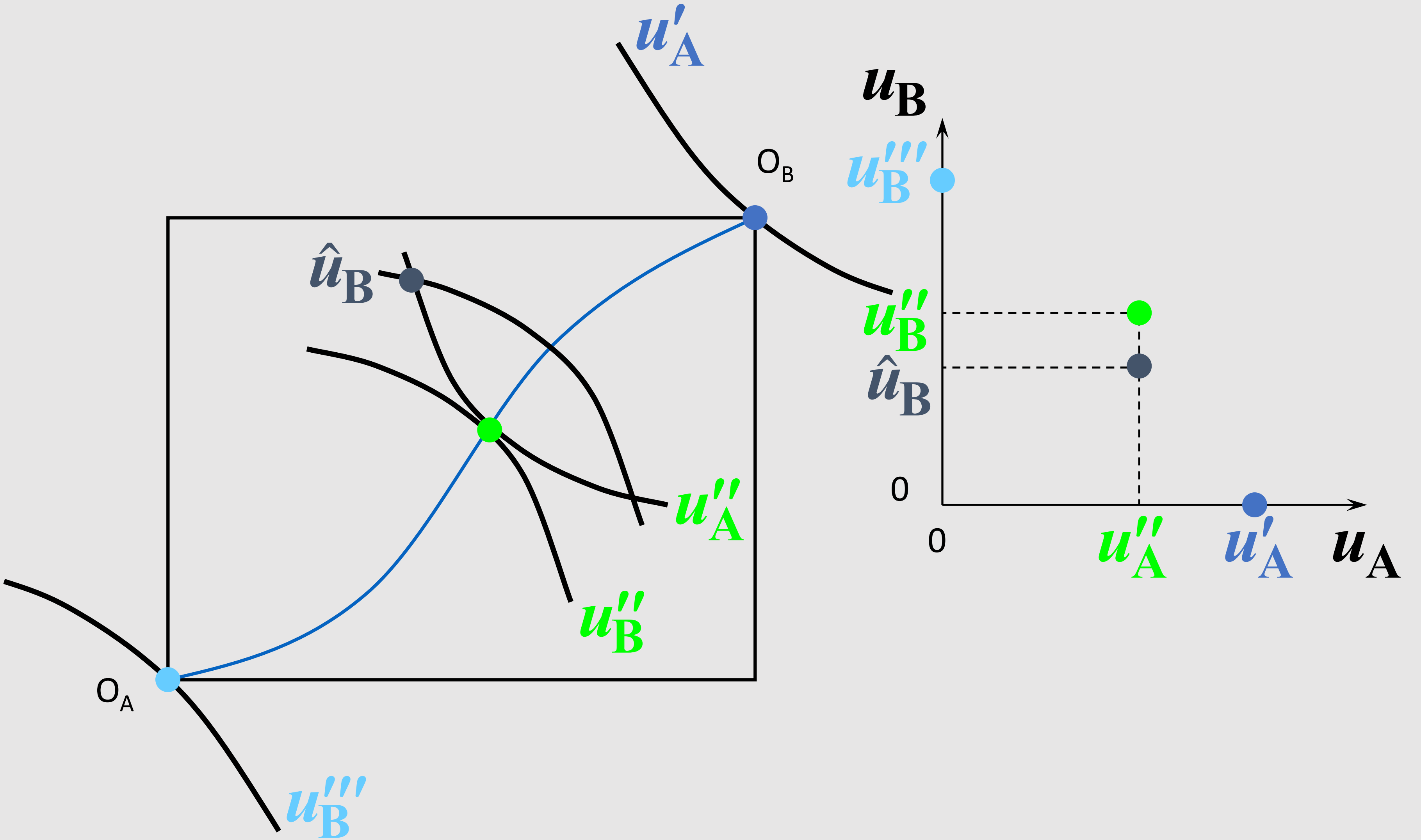
Utility Possibilities



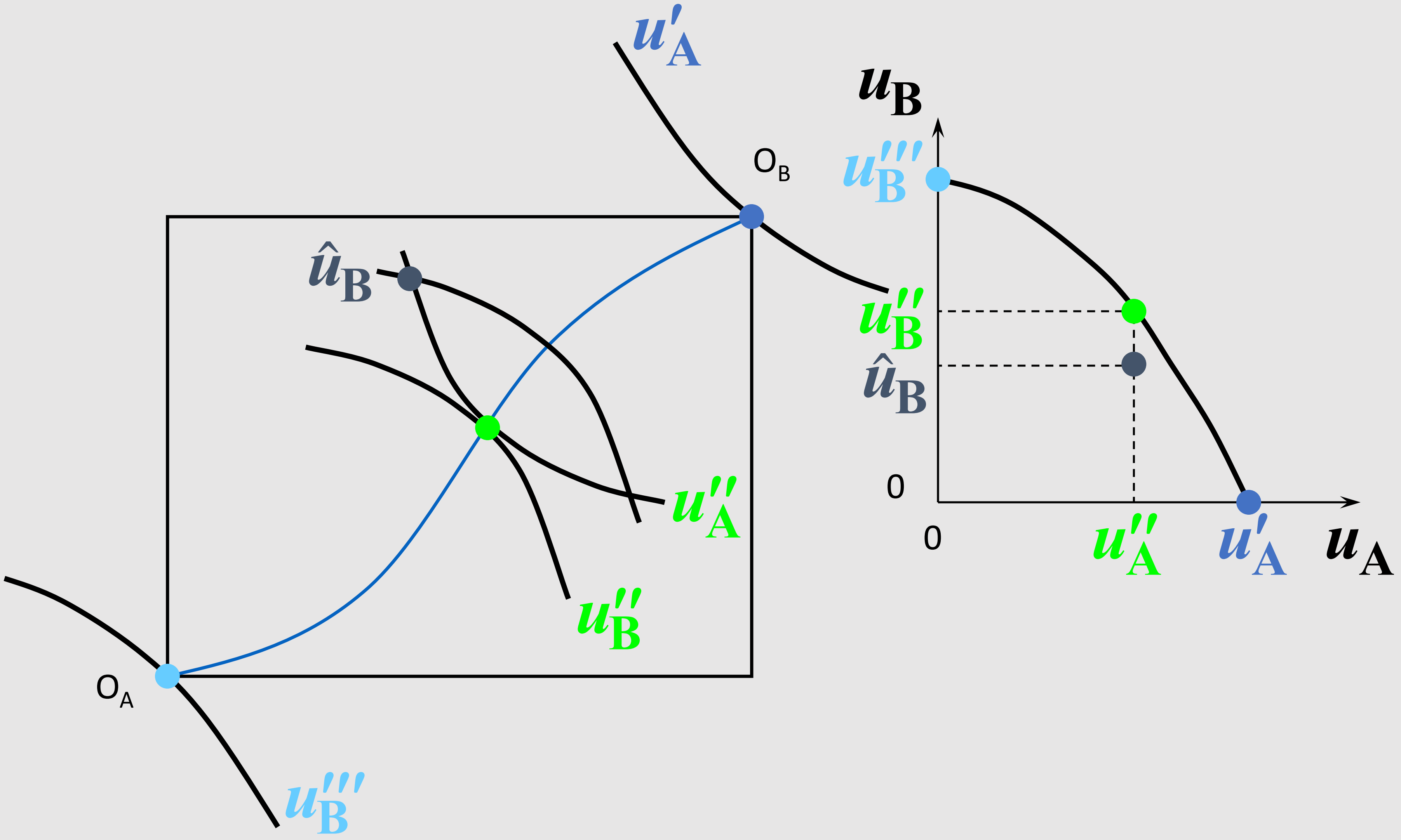
Utility Possibilities



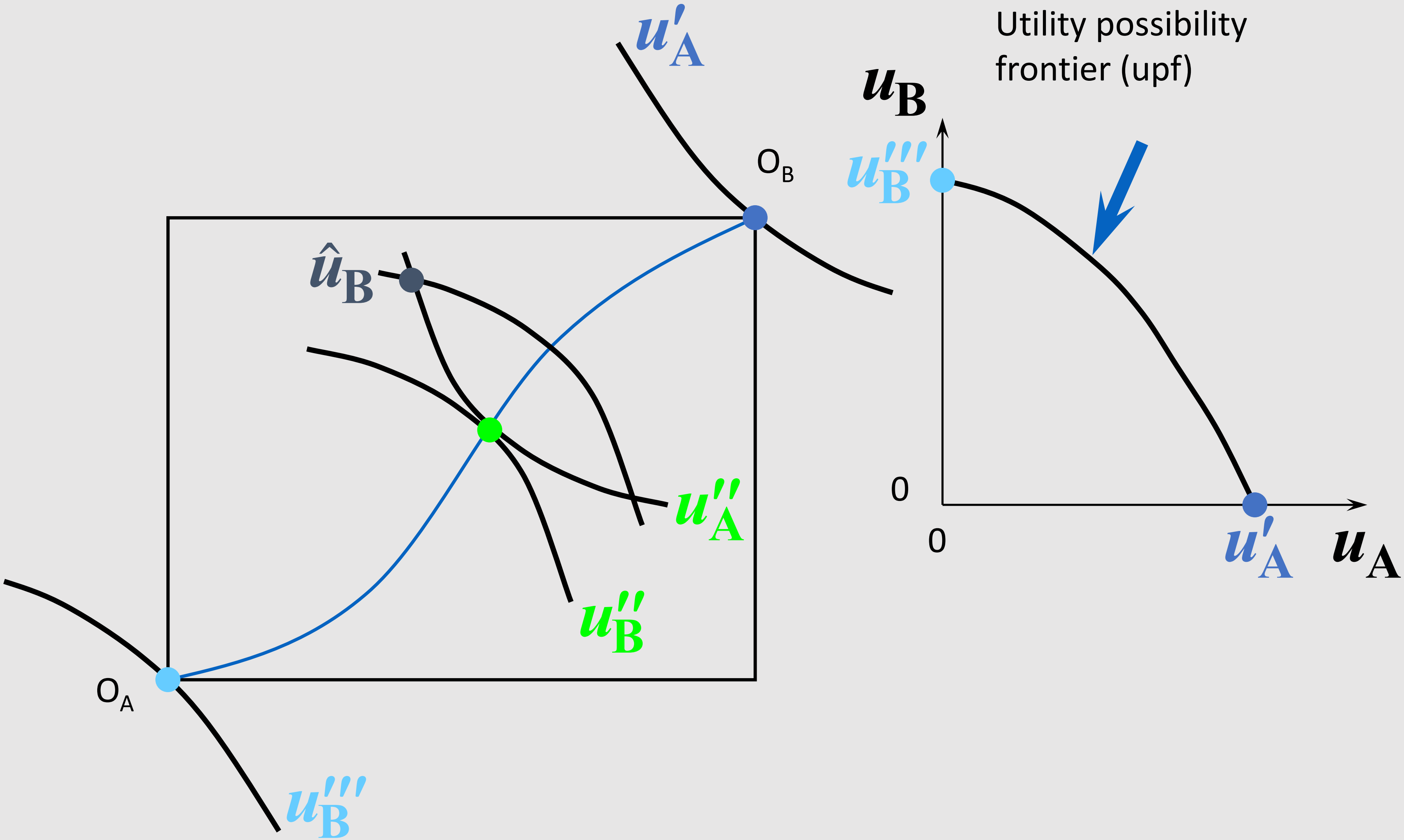
Utility Possibilities



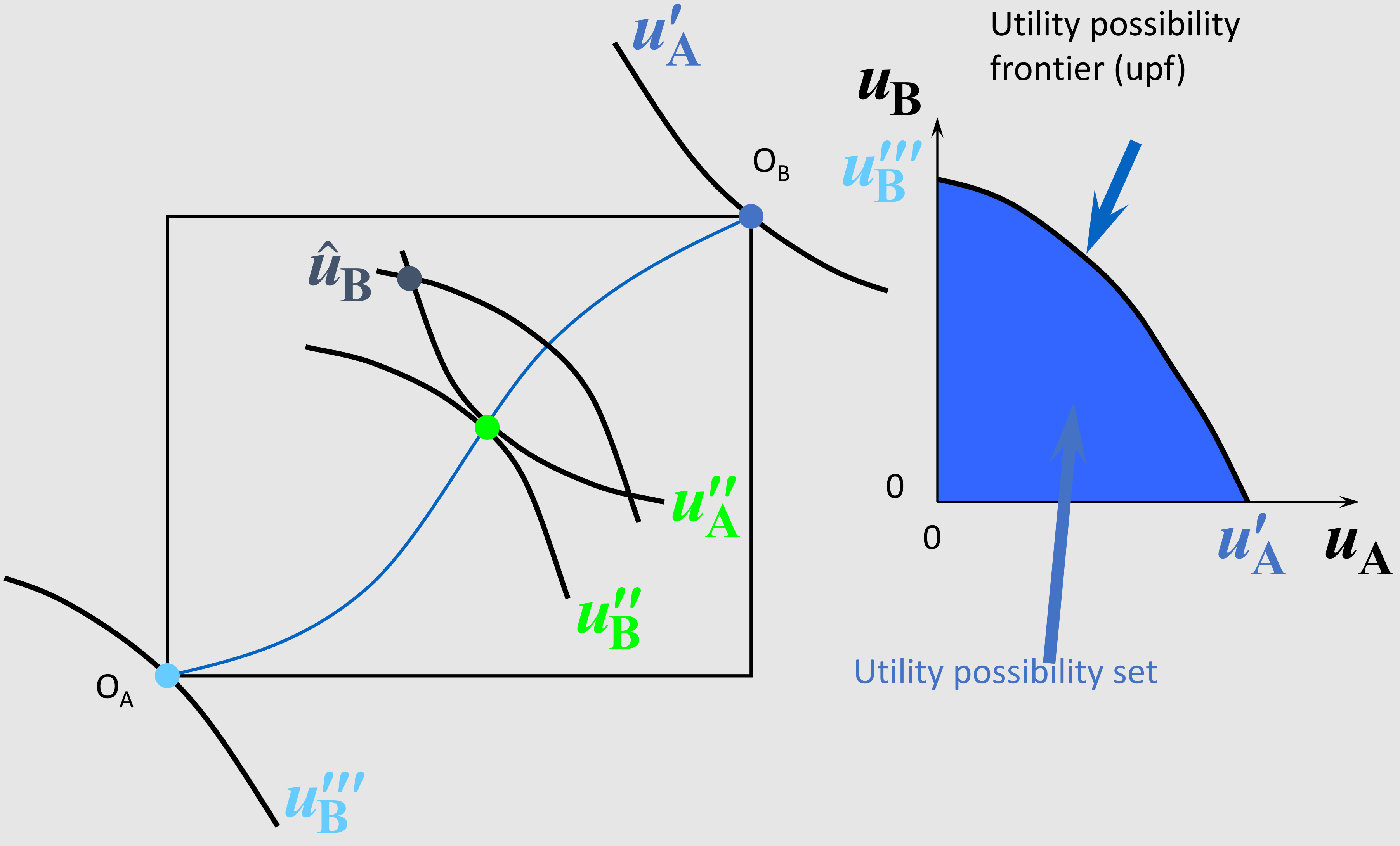
Utility Possibilities



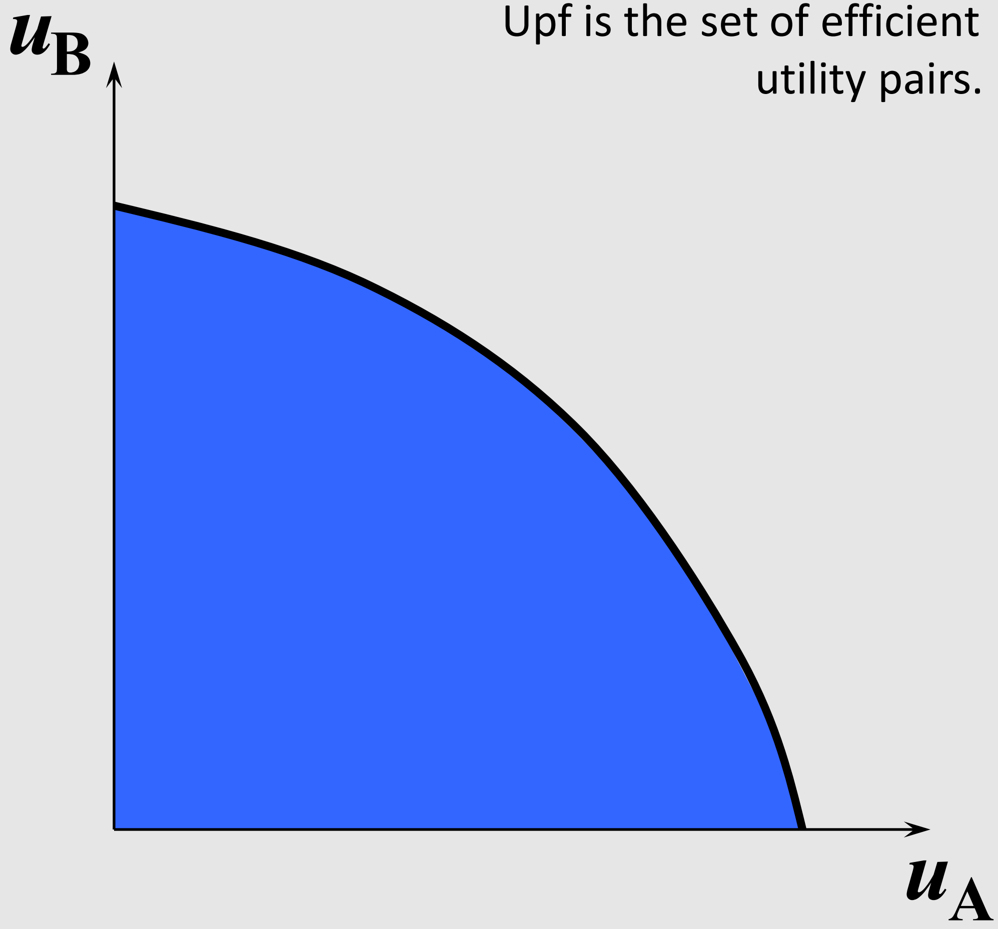
Utility Possibilities



Utility Possibilities

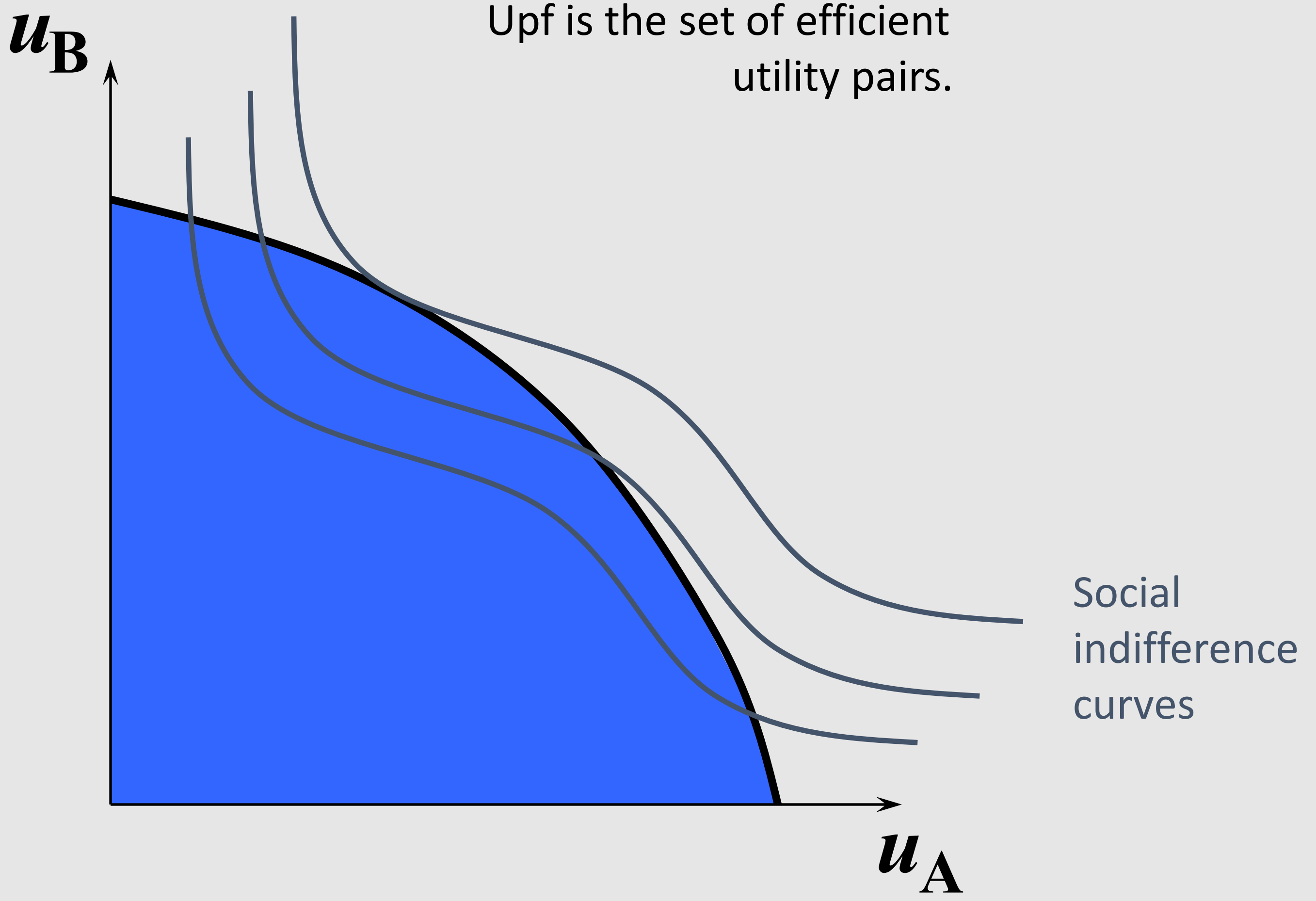


Social Optima & Efficiency

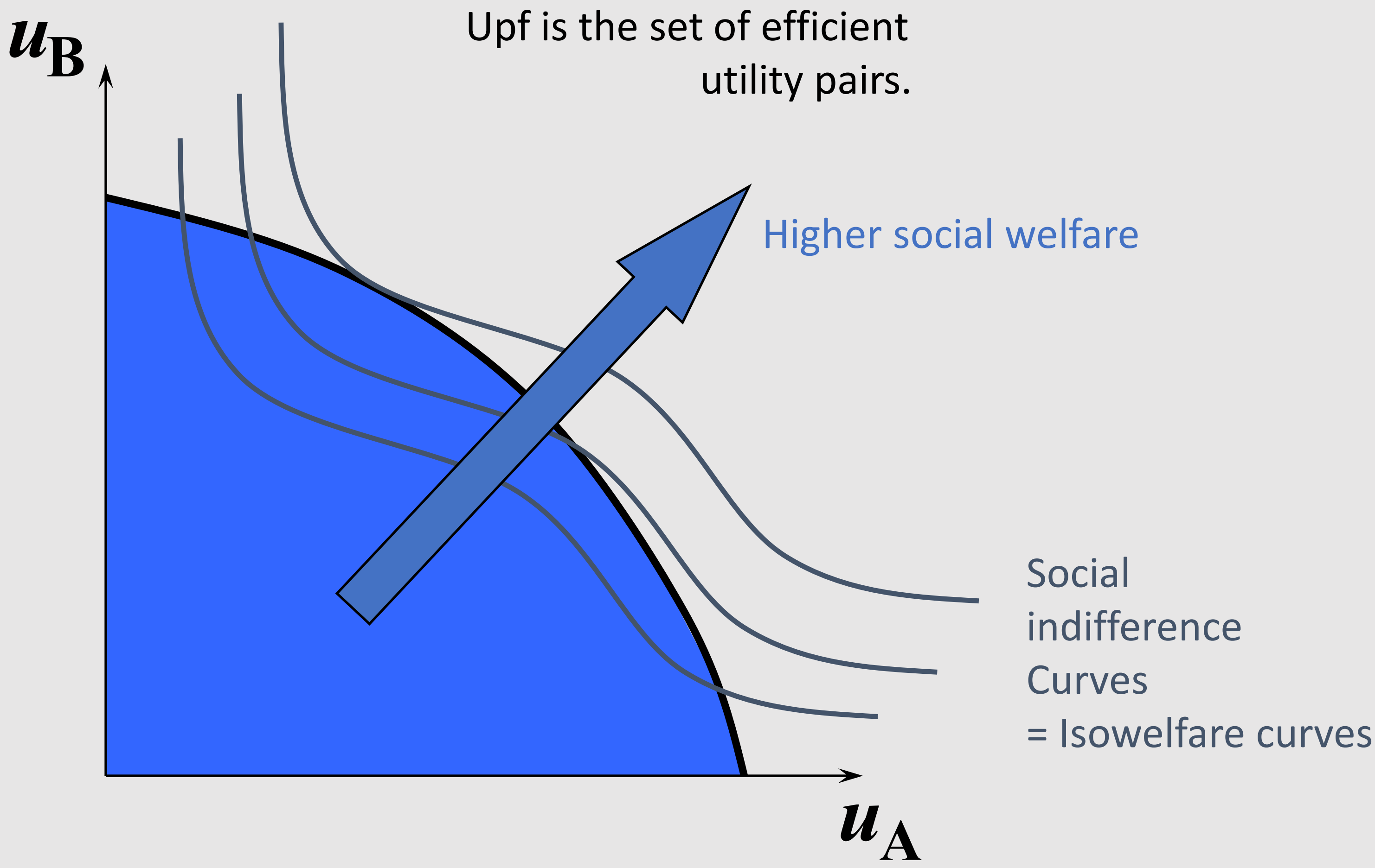


Upf is the set of efficient utility pairs.

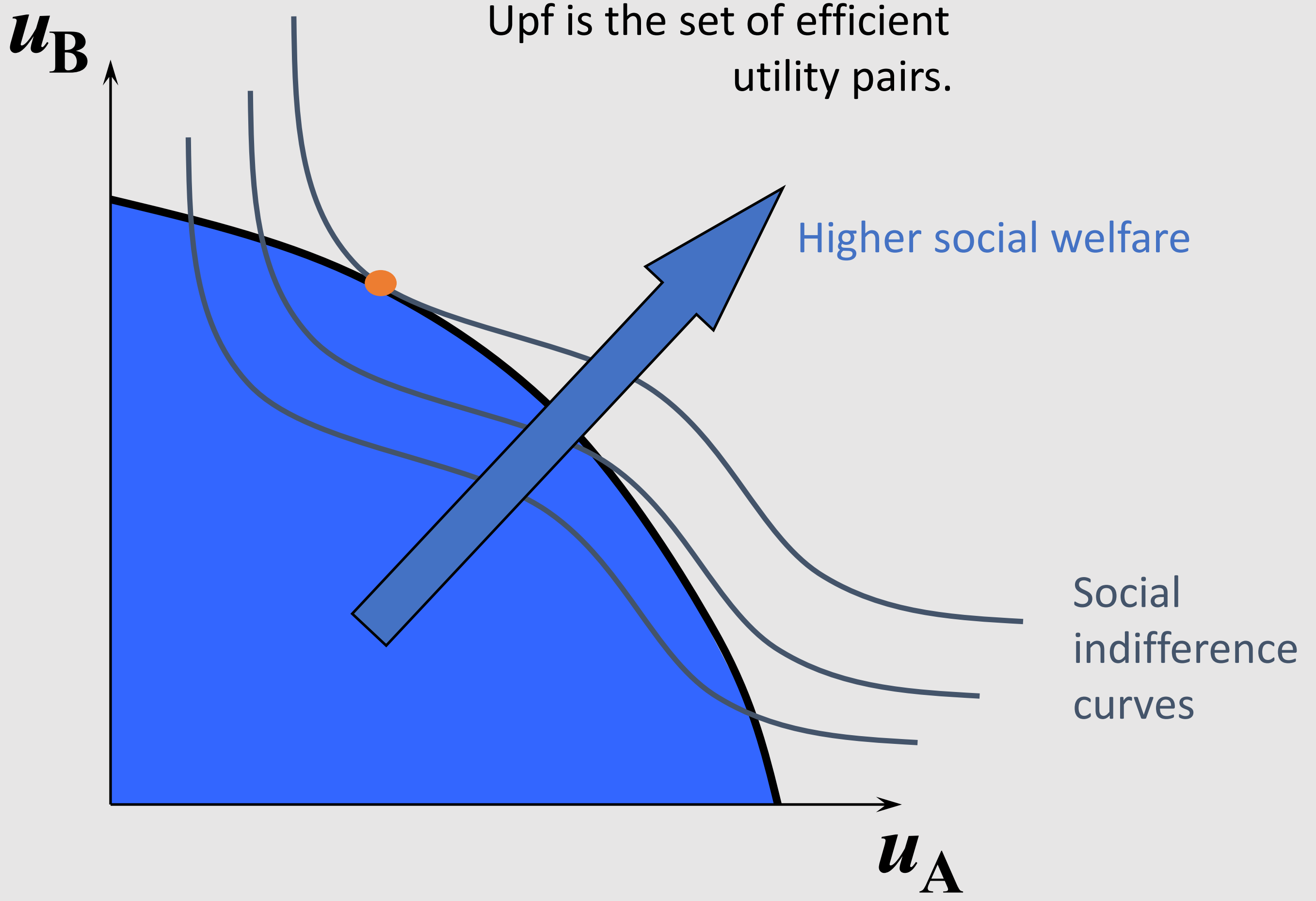
Social Optima & Efficiency



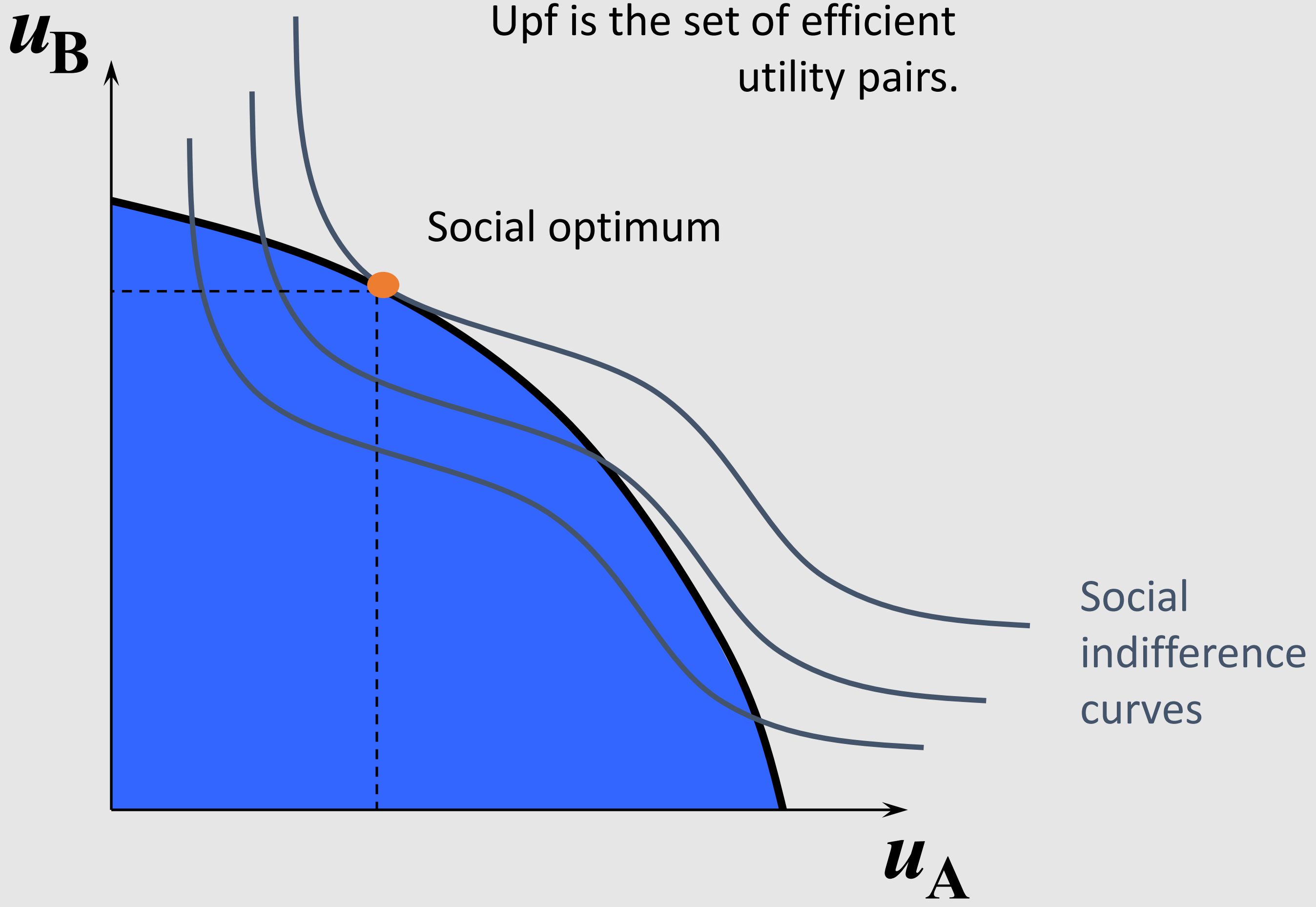
Social Optima & Efficiency



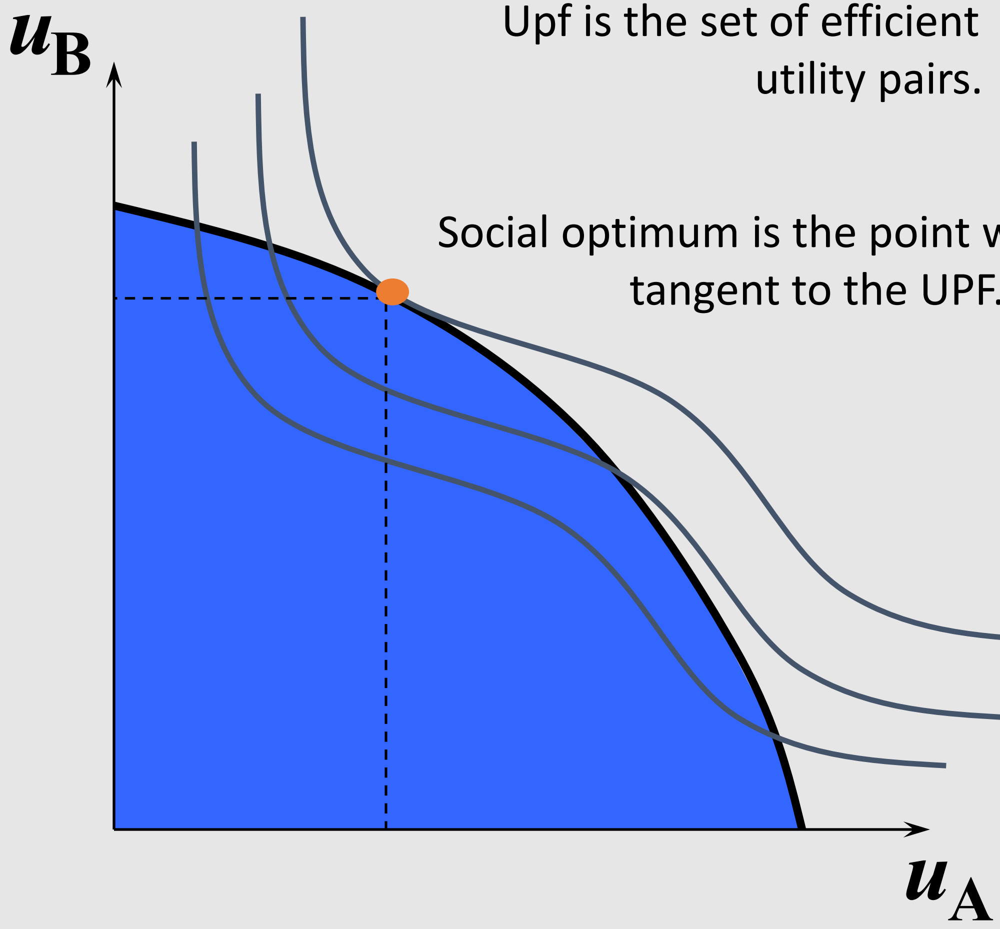
Social Optima & Efficiency



Social Optima & Efficiency



Social Optima & Efficiency

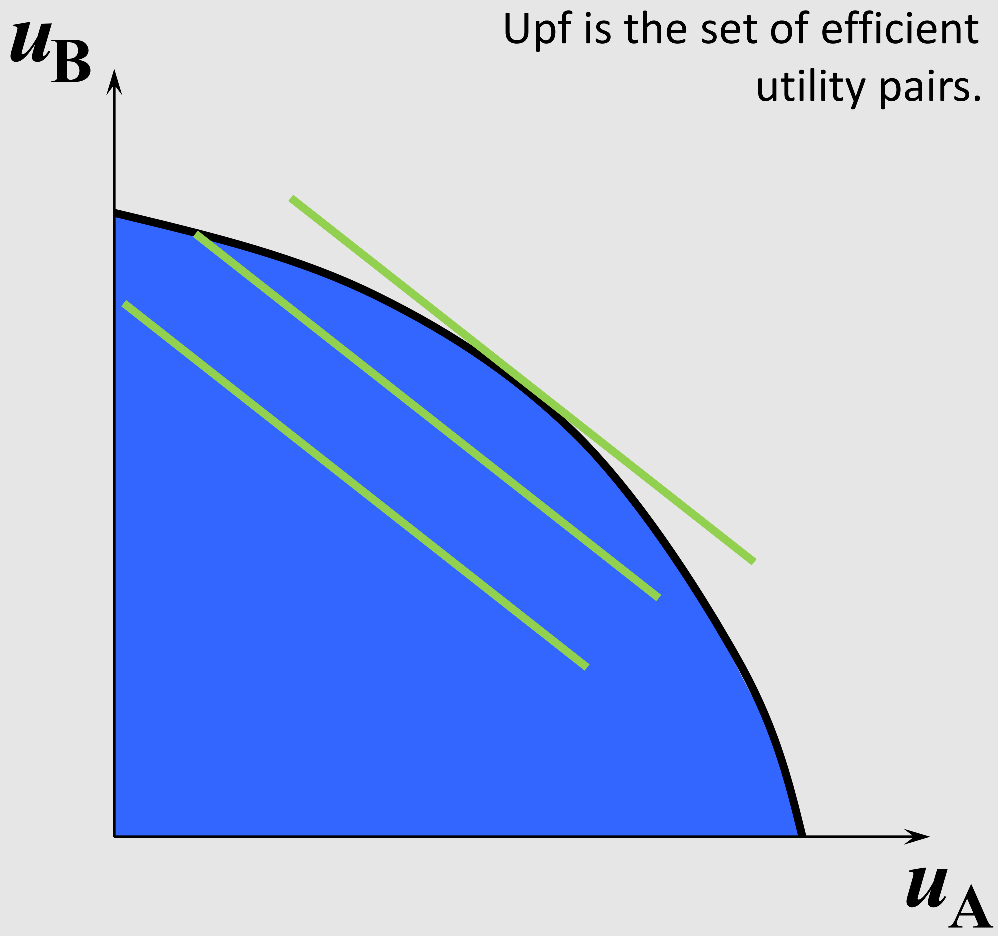


Upf is the set of efficient utility pairs.

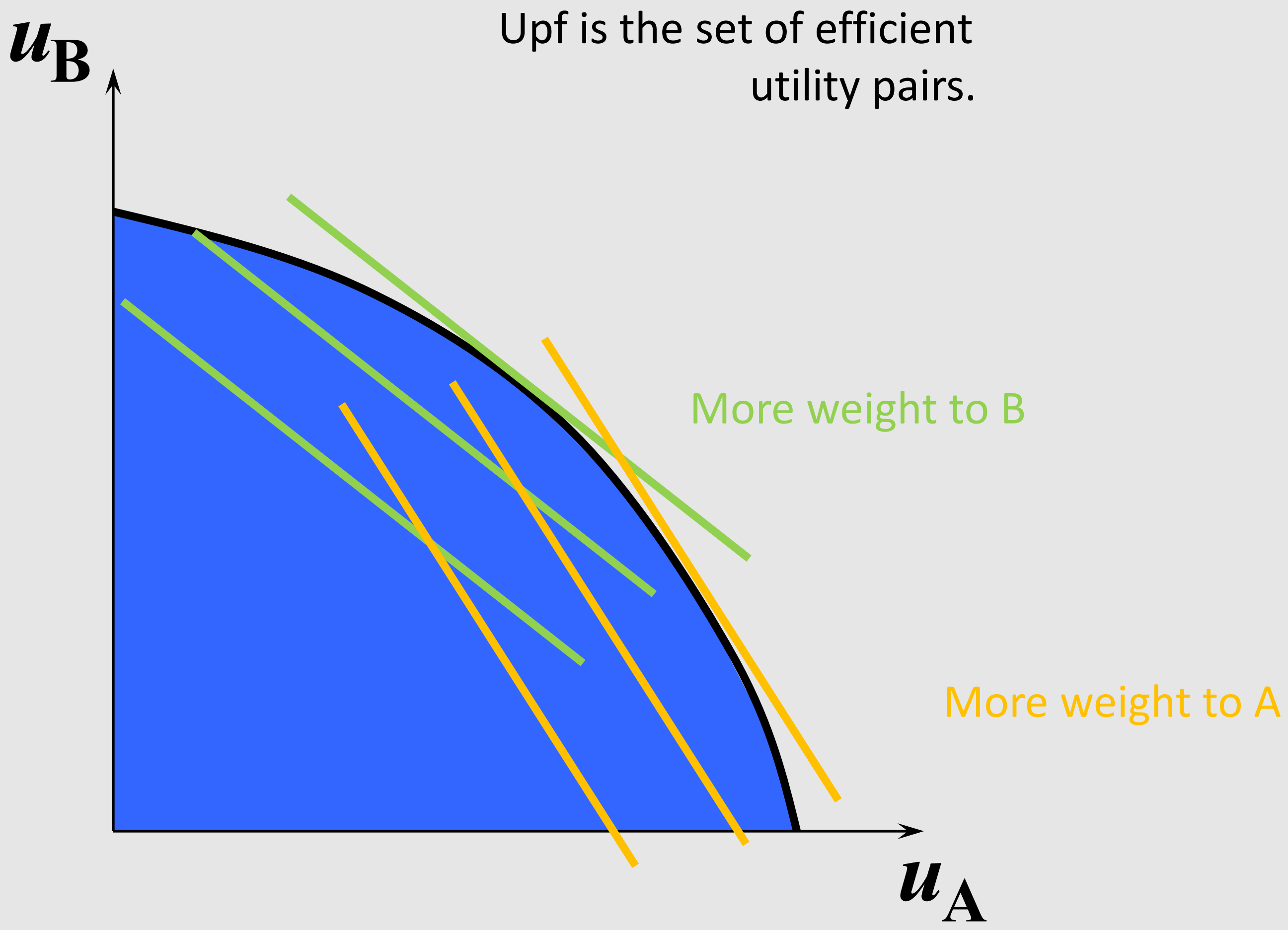
Social optimum is the point where the isowelfare curve is tangent to the UPF. It is Pareto efficient.

Social indifference curves

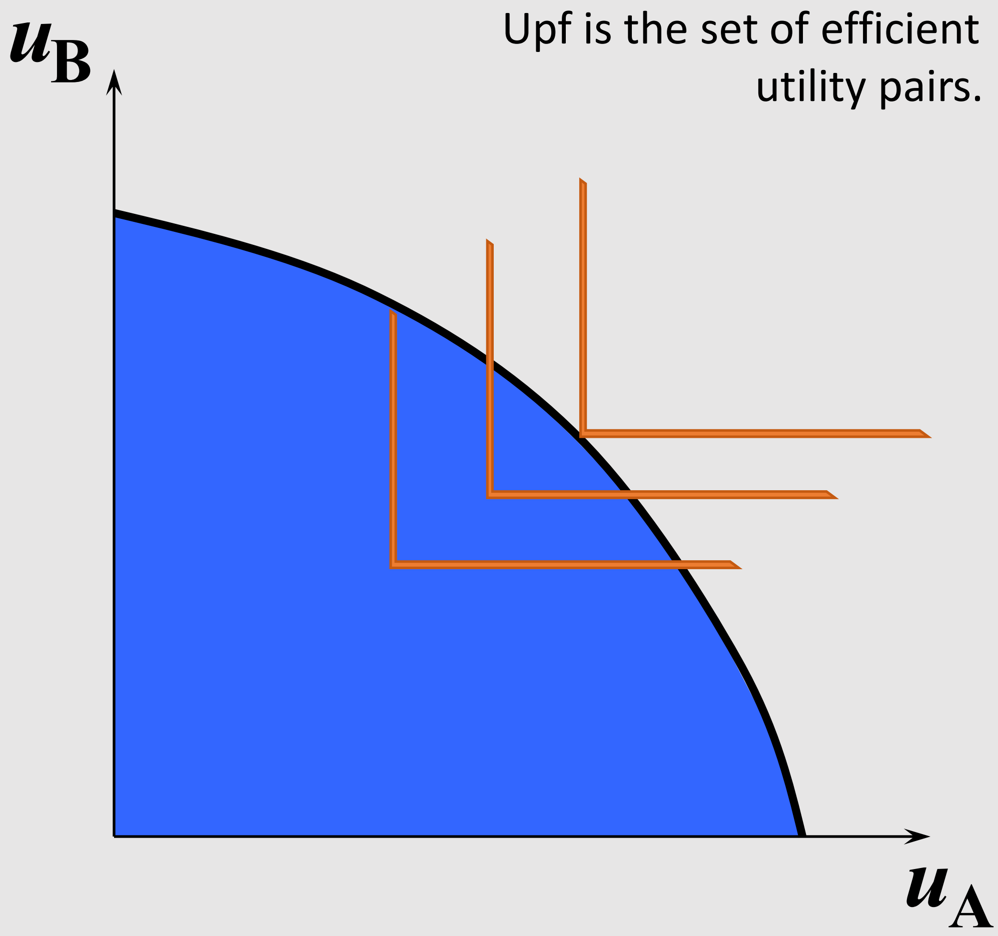
Utilitarian



Weighted sum of utilities



Rawlsian



Social Optima & Efficiency

→ If the Utility Possibility Frontier is convex, then every point of the frontier is a Pareto efficient allocation for a weighted-sum of utilities welfare function.

→ Every welfare maximum is a Pareto efficient allocation, and every Pareto efficient allocation is a welfare maximum

4. Fair Allocations

Welfare function is not useful in deciding what ethical judgments are more reasonable

→ Some Pareto efficient allocations are "unfair".

- E.g. one consumer eats everything is efficient, but "unfair".

Definition of fair allocation

→ An allocation is fair if it is

- Pareto efficient
- Envy free (equitable): no agent prefers any other agent's bundle of goods to his or her own

Can competitive markets guarantee that a "fair" allocation can be achieved?

Fair Allocations

- Must equal endowments create fair allocations?
  - No. Why not?

Example 1

- 3 agents, same endowments.
- Agents A and B have the same preferences. Agent C does not.
- Agents B and C trade $\Rightarrow$ agent B achieves a more preferred bundle.
- Therefore agent A must envy agent B $\Rightarrow$ **unfair allocation**.

Fair Allocations

Example 2

→ 2 agents, equal division endowments.

→ Now trade is conducted in competitive markets (according to 1st Welfare Theorem, this leads to a Pareto efficient allocation.

→ Must the post-trade allocation be fair?

- Why?

Fair Allocations

- Endowment of each is (ω_1, ω_2).
- Post-trade bundles are

$$(x_1^A, x_2^A) \quad \text{and} \quad (x_1^B, x_2^B).$$

Then $p_1 x_1^A + p_2 x_2^A = p_1 \omega_1 + p_2 \omega_2$

and $p_1 x_1^B + p_2 x_2^B = p_1 \omega_1 + p_2 \omega_2$.

With $\omega_1 = \omega_1^A = \omega_1^B = \frac{\bar{\omega}}{2}$

Fair Allocations

Let’s suppose that the result of trade is not equitable. This means:

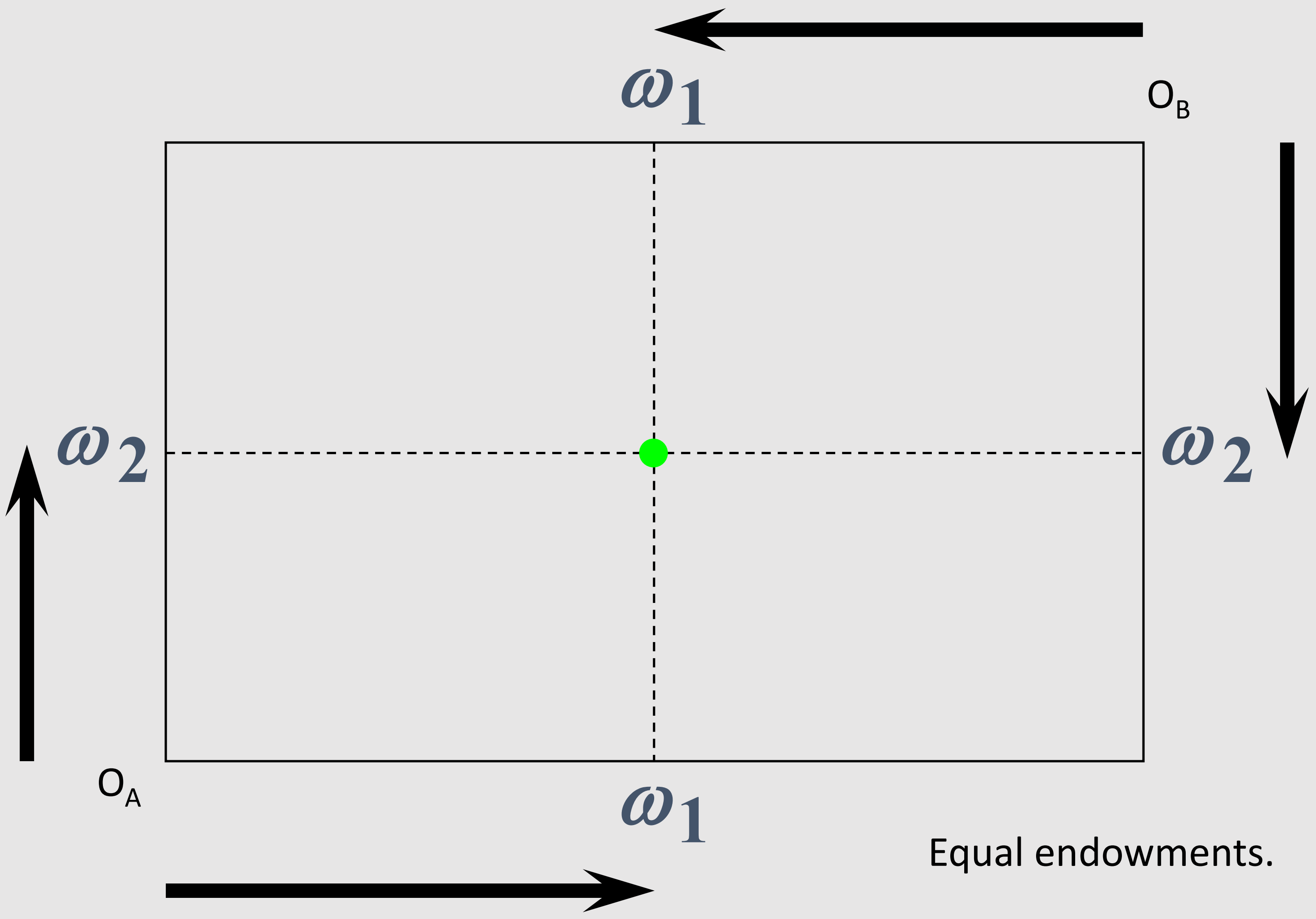
- → Suppose agent A envies agent B.
- → I.e. $(x_1^B, x_2^B) \succ_A (x_1^A, x_2^A)$.
- → Then, for agent A, $$p_1x_1^B + p_2x_2^B > p_1x_1^A + p_2x_2^A$$ $$= p_1\omega_1 + p_2\omega_2.$$
- → Contradiction (x_1^B, x_2^B) is not affordable for agent A.
  - – A and B started with exactly the same bundle, since they started from an equal division. If A can’t afford B’s bundle, then B can’t afford it either!

Fair Allocations

→ This proves that:

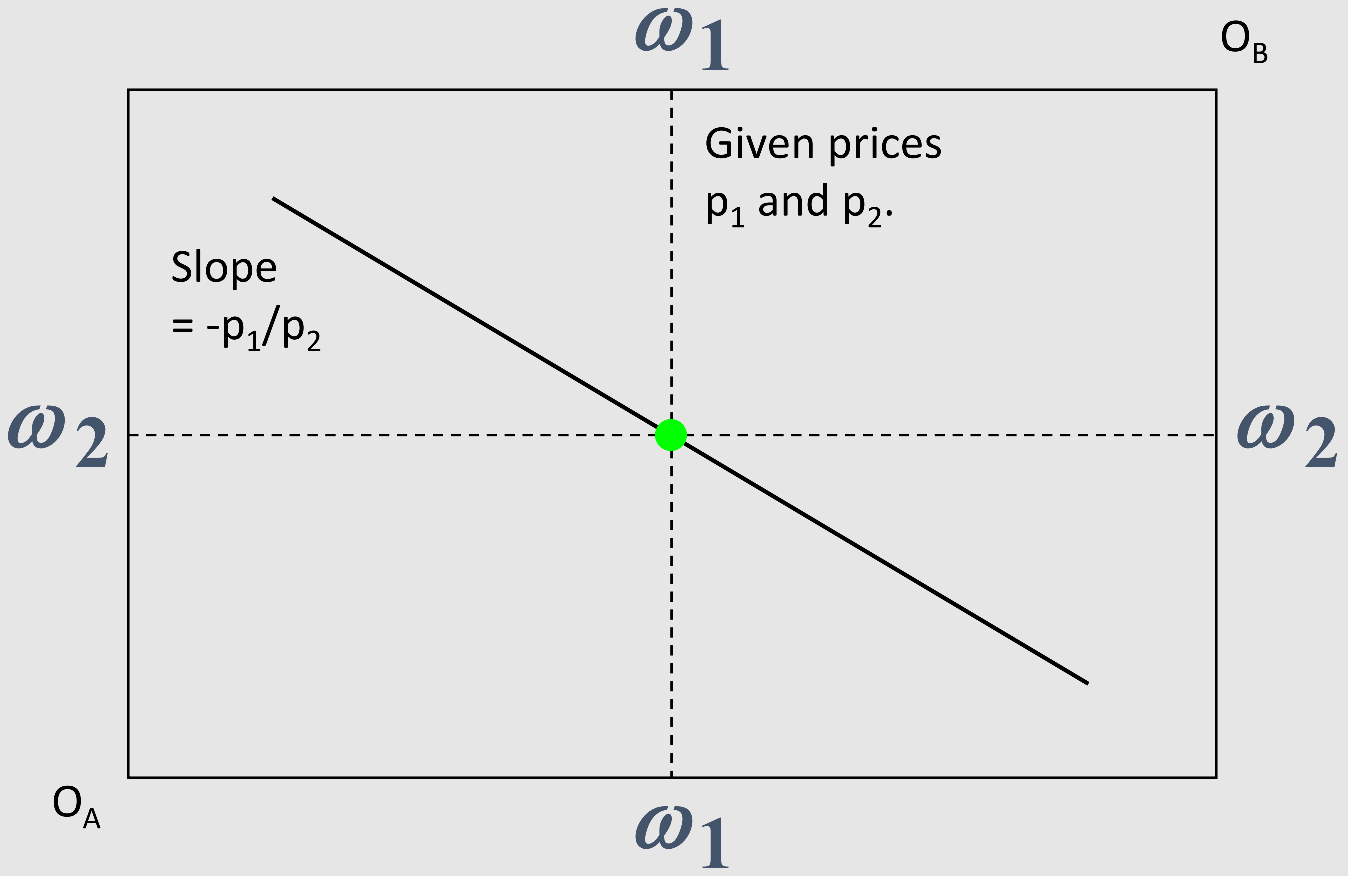
If every agent's endowment is identical, then trading in competitive markets results in a fair allocation.

Fair Allocations

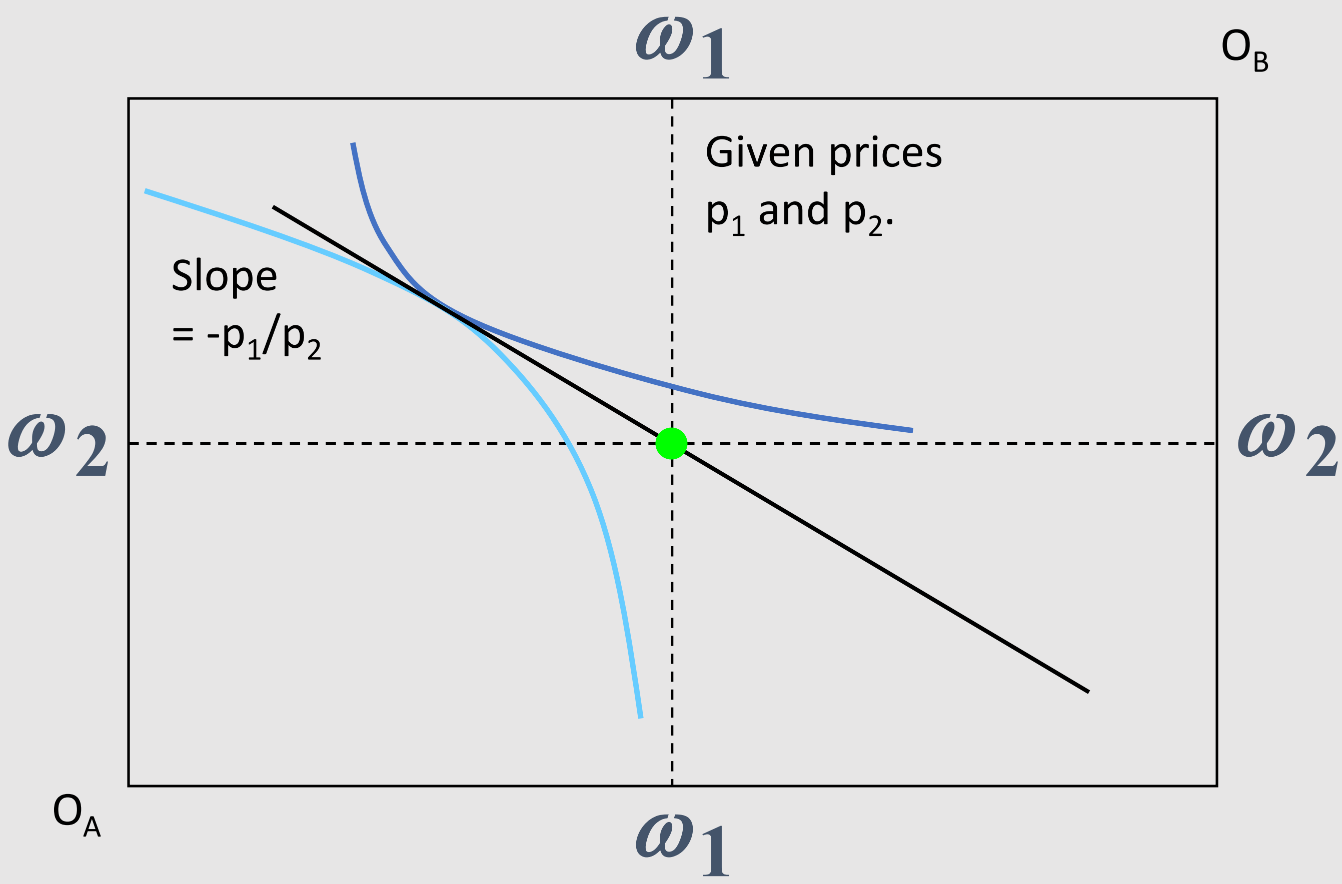


Equal endowments.

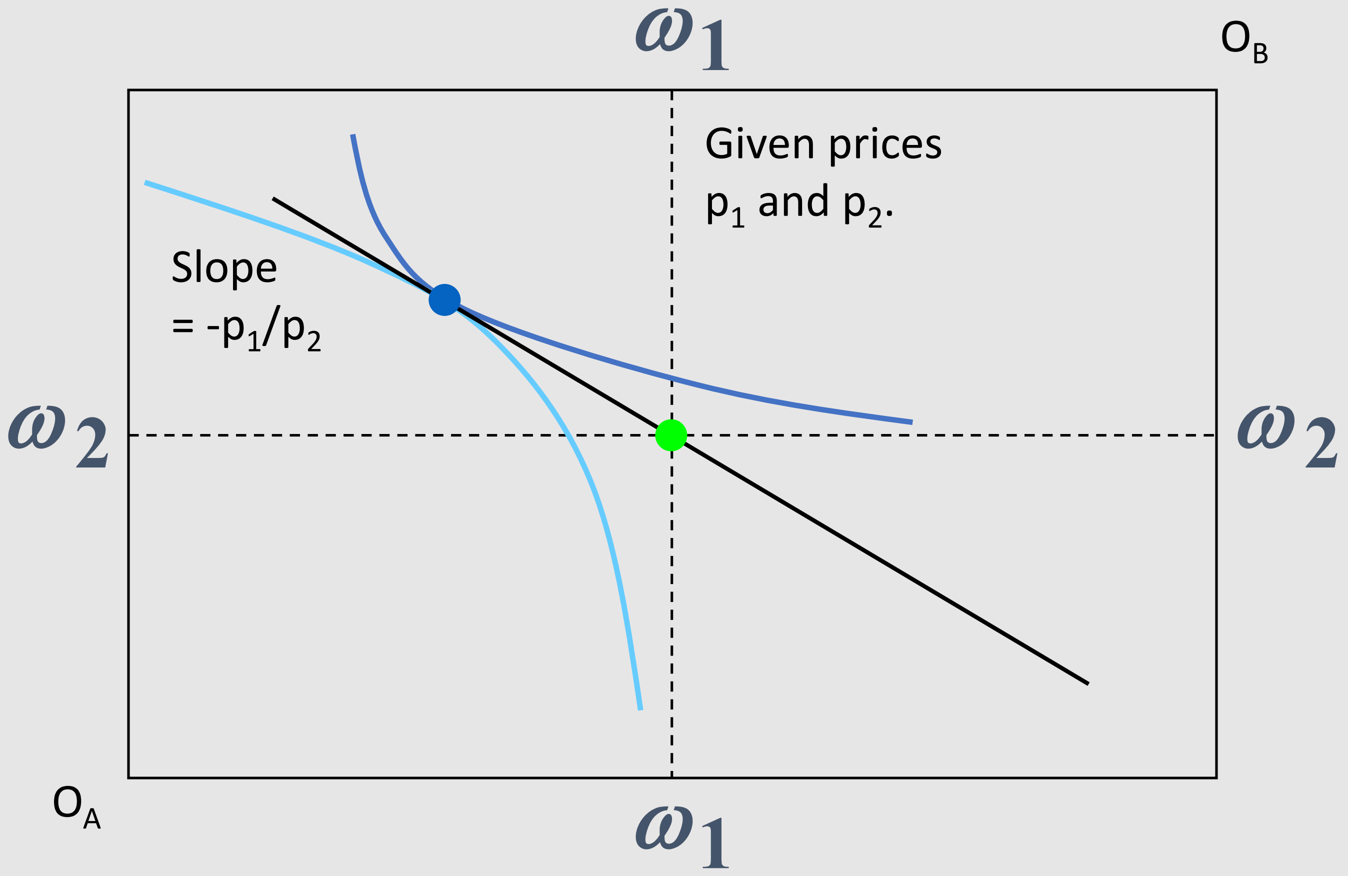
Fair Allocations



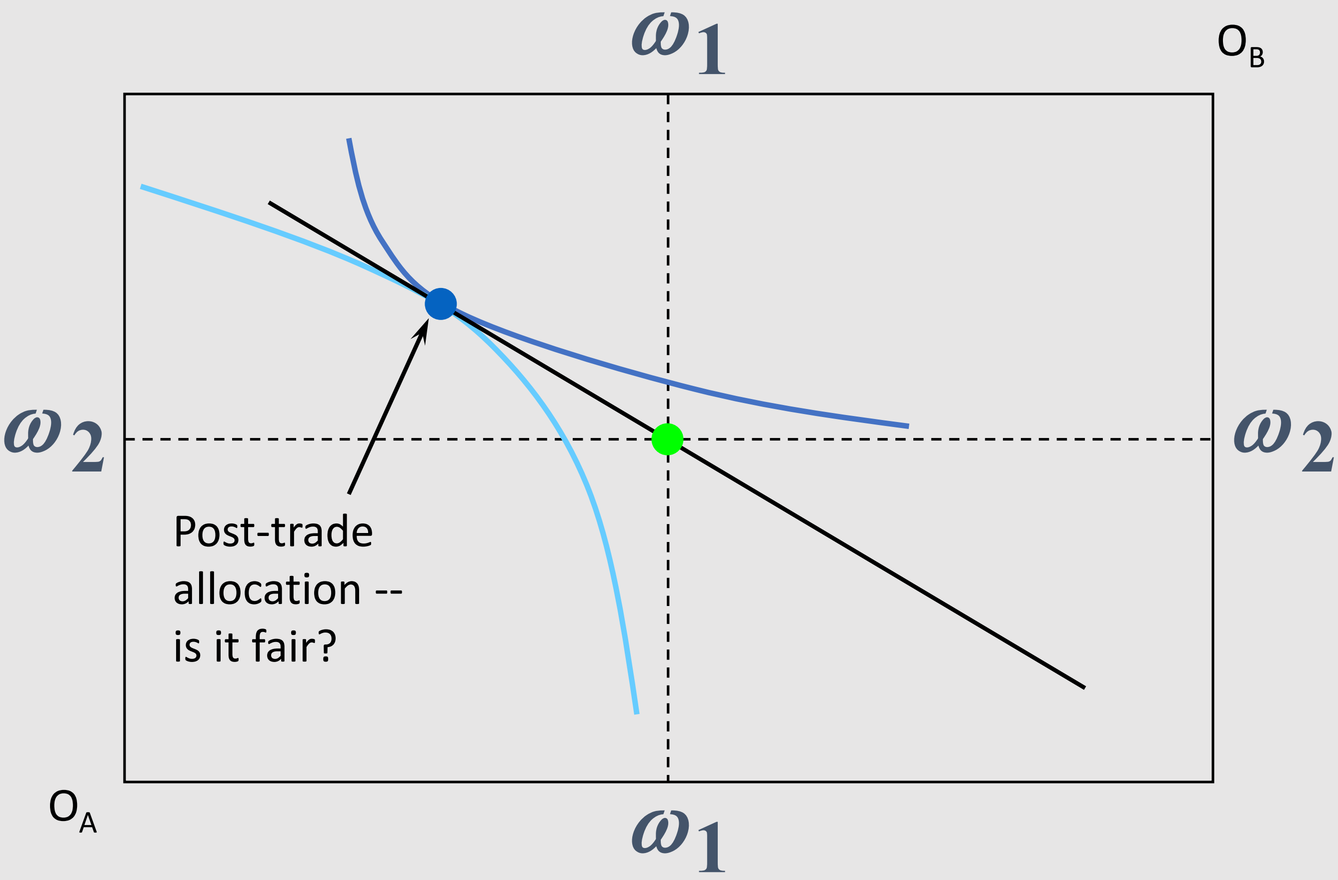
Fair Allocations



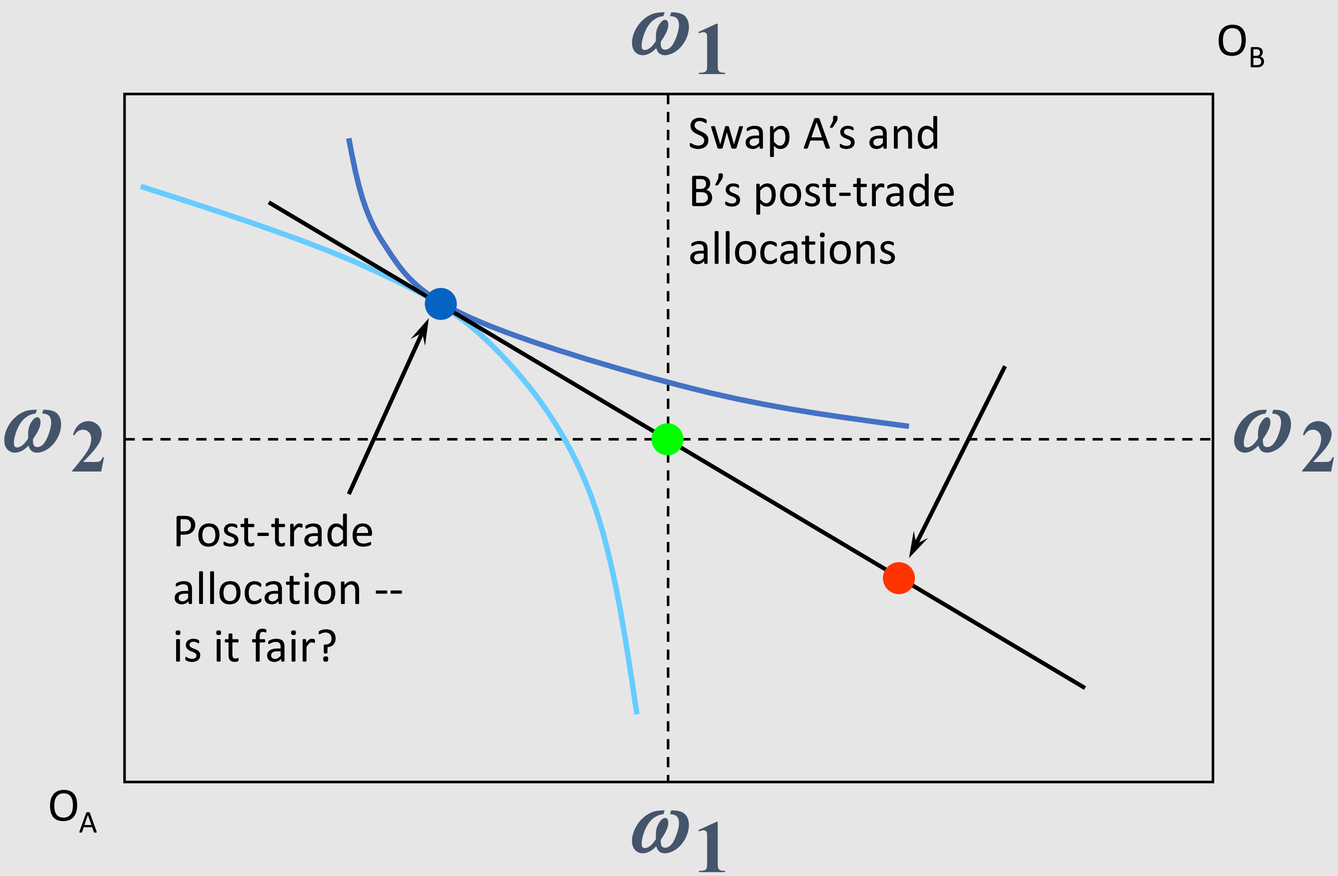
Fair Allocations



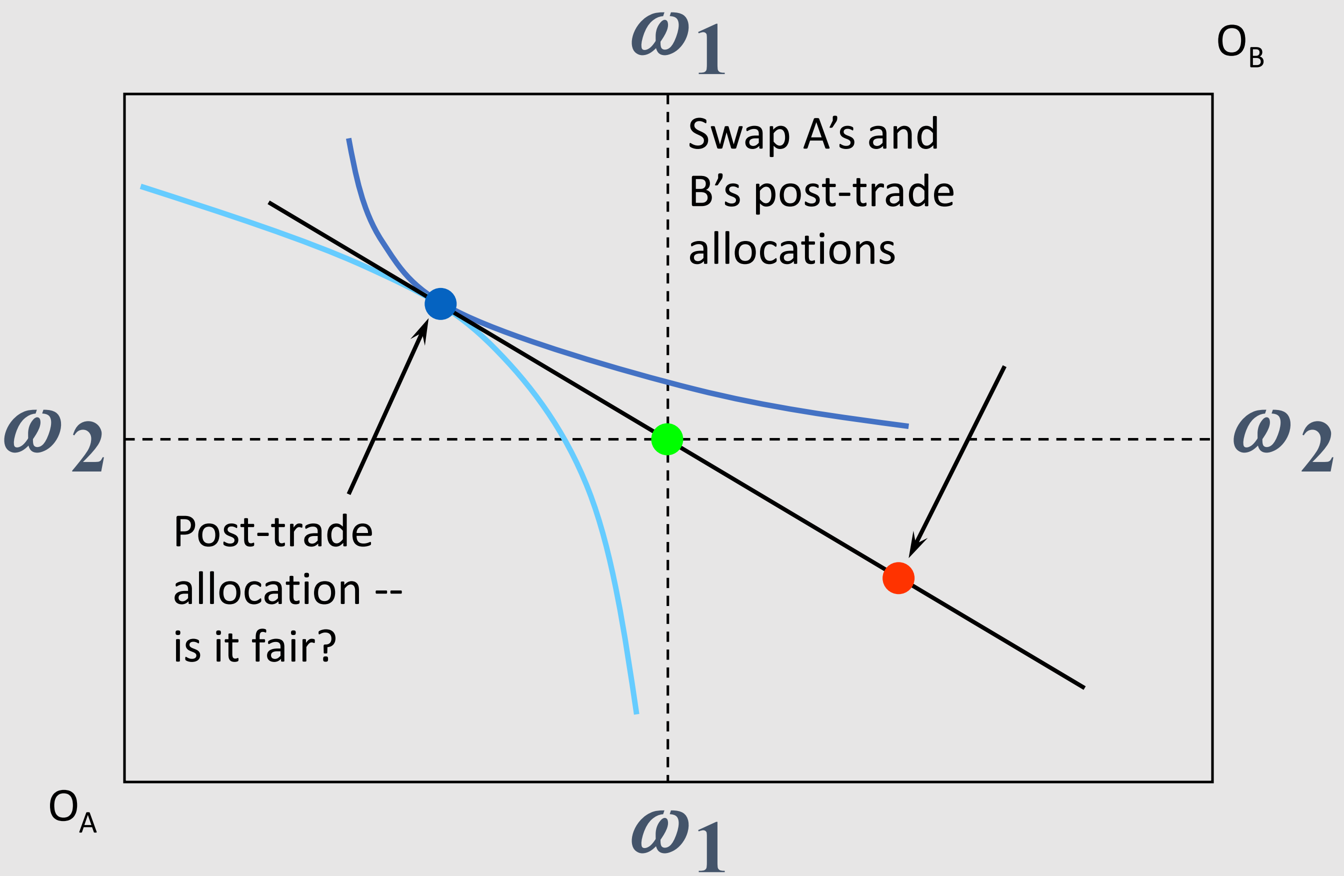
Fair Allocations



Fair Allocations

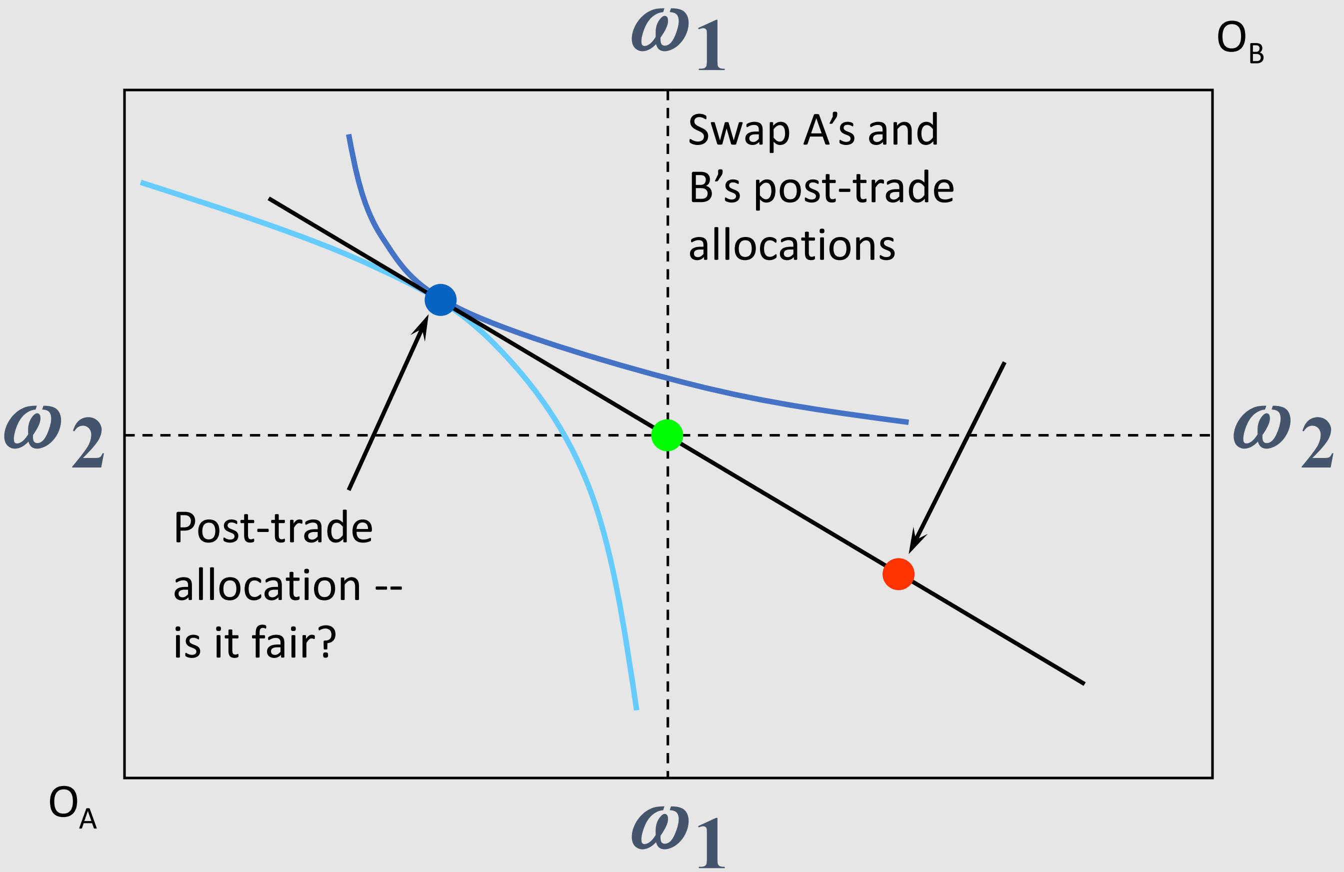


Fair Allocations



A does not envy B's post-trade allocation.
B does not envy A's post-trade allocation.

Fair Allocations



Post-trade allocation is Pareto-efficient and envy-free; hence it is fair.