Problem Set 5: Public Goods

Exercise 1: Classifying public goods

In the plan (ease of exclusion, degree of rivalry), indicate where the goods and services below are located.

1. Old age insurance
2. A vaccination campaign
3. Cable TV
4. A toll highway
5. A hiking trail
6. Theoretical research
7. Public parking
8. Street cleaning service
9. Electricity
10. The local police

Exercise 2: The roommates 1

Patrick and André are roommates. They plan to buy a sofa for their living room. Patrick's utility function is $U_P(S, M_P) = (1 + S)M_P$, André's utility function is $U_A(C, M_A) = (2 + S)M_A$, where M_P and M_A is the amount of money that Patrick and André spend on their consumption of private goods, and $S \in \{0, 1\}$ depending on whether they buy the sofa or not. Patrick has a total wealth of W_P and André a total wealth of W_A.

1. Express Patrick's and André's willingness to pay for the sofa as a function of their respective total wealth.
2. If $W_P = 100$ and $W_A = 75$, up to what price would the purchase of a sofa be a Pareto-improving decision for both Patrick and André?
3. Still under the same assumptions, what reasoning would André follow if the selling price of the sofa was 50 euros?

Exercise 3: The roommates 2

Lucie and Malia share an apartment. They spend part of their income on private goods such as food and clothing, which they consume separately, and part of their income on public goods such as a refrigerator, heating, and rent, which they share. Lucie's utility function is $2X_L + G$ and Malia's utility function is $X_M G$ where X_L and X_M is the amount of money spent on private goods for Lucie and for Malia and G is the amount of money spent on public goods. Lucie and Malia collectively have 8000 euros a year which they spend on their respective private and on public goods.

1. What is the absolute value of the marginal rate of substitution between private and public goods for Lucie? What is this value for Malia?
2. Write an equation to calculate the Pareto optimal quantity of public goods.
3. Suppose that Malia and Lucie each spend 2000 euros on private goods and that they spend the remaining 4000 euros on public goods. Is this result Pareto-optimal?
4. Give an example of another Pareto optimal result in which Malia spends more than 2,000 on private goods and Lucie spends less than 2,000.
5. Give an example of another Pareto optimum in which Lucie spends more than 2000.
6. In the Pareto optima that treat Lucie better and Malia worse, is there more, less or the same amount of public goods as the Pareto optimum that treats them the same way?
7. Assume that each roommate has a budget of 4000€. They want to optimally choose the amount of private money (X_L and X_M) and their contribution to the public good (G_L and G_M). We know that $G = G_L + G_M$. How much public good would result from a market equilibrium without coordination?

Exercise 4: Bonnie and Clyde

Bonnie and Clyde are work colleagues. When they work, they have to do it together. Their only source of income is the profit from their partnership. Their total profits per year is $50H$ where H is the number of hours they work in a year. Since they have to work together, they both have to work the same number of hours, and the variable "number of hours" is like a public "bad" (instead of a public good) for Clyde and Bonnie. Bonnie's utility function is $U_B(C_B, H) = C_B - 0.02H^2$ and Clyde's utility function is $U_C(C_C, H) = C_C - 0.005H^2$ where C_B and C_C are the annual amounts of money spent on Bonnie's and Clyde's consumption respectively and where H is the total number of hours worked.

1. If the number of hours they both work is H, what is the ratio for Bonnie between the marginal utility of hours worked and the marginal utility of private goods? What is this ratio for Clyde?

2. If Bonnie and Clyde both work H hours, the total amount of money needed to compensate for any additional hour is the sum of what is needed to compensate Bonnie and what is needed to compensate Clyde. This quantity is approximately equal to the sum of the absolute values of their marginal rates of substitution between labor and money. Write an expression for this quantity as a function of H. How much more money will they earn if they work one more hour?

3. Write an equation whose solution calculates the Pareto optimal number of hours Bonnie and Clyde work. Find this Pareto optimal amount H. (Hint: Notice that this model is formally the same as a model with a public good H and a private good, income).

Exercise 5: Fermont's ice rink

The population of Fermont, Quebec is 1000 people. The citizens of Fermont consume only a private good, maple syrup. There is one public good, the town's ice rink. Although they may differ on other aspects, the inhabitants have the same utility function $U_i(X_i, G) = X_i - 100/G$ where X_i is the number of bottles of maple syrup consumed by each citizen i and G is the size of the ice rink measured in square meters. The price of maple syrup is 1 euro per bottle and the price of the ice rink is 10 euros per square meter. Each inhabitant of Fermont has an income of 1000 euros per year.

1. Write an expression for the absolute value of the marginal rate of substitution between the skating rink and the maple syrup for a typical citizen. What is the marginal cost of a square meter of skating rink?

2. Write an equation whose solution allows you to calculate the Pareto optimal quantity of G. Find this Pareto optimal quantity.

3. Assume that everyone in town pays an equal share of the cost of the ice rink. The city's total expenditure on the ice rink will be $10G$ euros. The tax paid by each individual to contribute to the rink will be $G_i = 10G/1000 = G/100$ euros. Each year, the citizens of Fermont must vote to express their opinion on how big the ice rink should be. The citizens realize that they will have to pay their share of the cost of the rink. Write the budget constraint of a voter when the size of the rink is G.

4. In order to choose the size of the rink to vote for, an elector simply looks for the combination of X_i and G that maximizes his utility under his own budget constraint and he votes for the quantity of G that he finds optimal. What is this quantity G in our example?

5. If the city has an ice rink of the size requested by the voters, will it be larger, smaller or the same size as the Pareto optimal size of the rink?

6. What would be the size of the rink if there were no coordination mechanism in place? in place?