Useful Formulas

Set Operations

$$A \cup B, \quad A \cap B, \quad A^C$$
$$(A \cup B)^C = A^C \cap B^C,$$
$$(A \cap B)^C = A^C \cup B^C$$

Basic Probability

$$P\Big(\bigcup_i A_i\Big) = \sum_i P(A_i) \text{ (disjoint } A_i)$$

$$P(A) + P(A^C) = 1$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Law of Total Probability :

$$P(A) = \sum_i P(A|B_i)P(B_i)$$

Bayes' Theorem :

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_j P(B|A_j)P(A_j)}$$

Counting

$$P_{N,k} = \frac{N!}{(N-k)!}, \quad C_{N,k} = \binom{N}{k} = \frac{N!}{k!(N-k)!}$$

Expected Value & Variance

$$E(Y) = \sum_y y p(y)$$

$$V(Y) = \sum_y (y-\mu)^2 p(y) = E[Y^2] - \mu^2$$

$$\sigma = \sqrt{V(Y)}$$

Binomial Distribution

$$Y \sim B(n,p)$$
$$P(Y=y) = \binom{n}{y} p^y (1-p)^{n-y}, \ y = 0,1,\ldots,n$$
$$E[Y] = np, \quad V(Y) = np(1-p)$$

Poisson Distribution

$$Y \sim \text{Poisson}(\lambda)$$
$$P(Y=y) = \frac{\lambda^y}{y!} e^{-\lambda}, \ y = 0,1,2,\ldots$$
$$E[Y] = \lambda, \quad V(Y) = \lambda$$

Approximation : $B(n,p) \approx \text{Poisson}(\lambda = np)$ for n large, p small.

Tchebysheff's Theorem

$$P(|Y-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$
$$P(|Y-\mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Marginal Distributions

$$p_X(x) = \sum_y p(x,y), \quad p_Y(y) = \sum_x p(x,y)$$

$$f_X(x) = \int_y f(x,y)\,dy, \quad f_Y(y) = \int_x f(x,y)\,dx$$

Conditional Distributions

$$p(y|x) = \frac{p(x,y)}{p_X(x)}, \quad f(y|x) = \frac{f(x,y)}{f_X(x)}$$
$$p(x,y) = p_X(x)p(y|x) = p_Y(y)p(x|y)$$

Expected Value (Joint)

Discrete :

$$E[g(X,Y)] = \sum_x \sum_y g(x,y)p(x,y)$$

Continuous :

$$E[g(X,Y)] = \int_y \int_x g(x,y)f(x,y)\,dx\,dy$$

$$E[XY] = \sum_x \sum_y xy\,p(x,y)$$

$$E[X] = \sum_x xp_X(x), \quad E[Y] = \sum_y yp_Y(y)$$

Covariance and Correlation

$$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = E[XY]-E[X]E[Y]$$

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}, \quad -1 \le \rho_{XY} \le 1$$

Linear Combinations

If $U = aX + bY$,

$$E[U] = aE[X] + bE[Y]$$

$$Var(U) = a^2 Var(X) + b^2 Var(Y) + 2ab\,Cov(X,Y)$$

Markov's Inequality

$$P(X \ge a) \le \frac{E[X]}{a}.$$