Problem Set 4: Externalities

Exercise 1: Beekeeper and farmer

A honey producer is located near an apple plantation. The honey producer and the apple producer are competing firms. The cost function of the apple producer is $C_A(a) = a^2/100 - h$. The cost function of the honey producer is $C_H(h) = h^2/100$. The quantities h and a represent the production levels of honey and apples. The selling prices of honey and apples are $p_h = 2$ and $p_a = 3$.

1. Calculate the equilibrium quantities in the two markets, when the two firms act as competitors.
2. Assume that the two firms merge, then calculate the equilibrium quantities in the two markets.
3. What is the socially optimal honey production? Why?
4. We now assume that the two producers remain independent. We propose to subsidize the honey production by a unitary subsidy mechanism of amount s. What is the value of s that restores the social optimum?

Exercise 2: Pollution

A company produces batteries at a constant marginal private cost of 4 euros. The market demand of this product is given by the equation: $P = 22 - Q$.

1. What is the firm's level of production under pure and perfect competition?
2. What are the consumer and producer surpluses?
3. In the rest of the exercise, it is assumed that this activity generates air pollution. Its environmental costs are represented by the external marginal cost function $MD = 0.2Q$. In terms of efficiency, how many batteries must be produced?
4. The environmental protection agency requires this company to adopt a new, less polluting production technology, which increases the marginal cost of production to $MC = 10$ euros. What is the production level in this case?
5. What are the consumer and producer surpluses with the new technology?
6. Does this legislation improve the situation?

Exercise 3: The airport and the real estate developer

An airport is located next to a piece of land that has just been acquired by a real estate developer. The developer would like to build houses on this land, but the passage of airplanes

reduces the value of these houses. With X the number of airplanes that pass through the airport each day and Y the number of houses that the developer builds, the profit function of the airport owner A is $\Pi_A = 48X - X^2$ while that of the property developer D is $\Pi_D = 60Y - Y^2 - XY$. Let us consider different scenarios.

1. A and D are simply not in contact. Determine the number of airplanes that A will fly, the number of houses that D will build, and the total amount of profit they will make from their respective activities.

2. A loses the right to fly any plane through its airport. Determine the number of houses that D will build, and the profits he will earn from them.

3. A regains the right to fly planes through his airport, but is obliged to compensate D for the negative impact of his activity on the value of his houses. Their respective product functions thus become $\Pi'_A = 48X - X^2 - XY$ and $\Pi'_D = 60Y - Y^2$. Determine the number of houses that D will build, the number of airplanes that A will fly, and the total sum of the profits they will make from their respective activities.

4. D buys back A's airport. Determine the number of airplanes and the number of houses that maximize his profit, and the total amount of profit he will make from these two activities. Compare your answers to the previous questions and comment.

Exercise 4: The Carcity

Carcity, Mexico has 1001 inhabitants. There is not much to do in Carcity except drive around the city. Although everyone likes to drive, the residents complain about the traffic jams, noise and pollution caused by the traffic. All residents have the same utility function defined by $U(f, d, t) = f + 16d - d^2 - 6t/1000$ where f is the daily consumption of the citizen in tacos, d is the number of hours per day that this citizen drives and t is the total number of hours of driving (per day) by all the other drivers of Carcity. The price of a taco is 1 peso. Each person in Carcity has an income of 40 pesos per day. To keep the calculations simple, let's assume that driving a car costs nothing.

1. If an individual thinks that the time he drives does not affect the time driven by other people, how many hours a day does he choose to drive?

2. If everyone chooses their preferred driving time, what is the total driving time of other people?

3. What is the utility level of each individual?

4. If everyone drives 6 hours a day, what is the utility level of a resident of Carcity?

5. Suppose the residents decide to pass a law that reduces the total number of hours each person can drive. How many hours is each person allowed to drive per day if the goal of the restriction is to maximize the utility of a resident?

6. Could the same objective be achieved by imposing a tax on each hour of driving? How much should this tax be?

Exercise 5: Production Externalities

We consider two farms. Farm 1 produces honey, farm 2 produces apples. The two farms act as competing firms. To produce honey, there are two technologies. Technology t1 consists in making 1 kilo of honey from 1 liter of cane sugar syrup and providing 1 unit of work. The t2 technology, ancestral, consists in keeping bees and also working a little: 1 kilo of honey is obtained with b bees and l work units. Whatever the technology, the farm 1 can only produce H kilos of honey. The second farm produces apples using work (1 unit per kilo), knowing that e bees do one unit of work for free. It can only produce A kilos of apples.

We note p_b the price of maintenance per bee, p_s the price per liter of sugar syrup, w the wage per unit of work, p_h the price per kilo of honey and p_a the price per kilo of apples.

1. Exploring all possible scenarios, calculate the production and profits of farms 1 and 2.

2. Suppose that $p_s + w < bp_b + wL < p_h$ and $w < p_a$. Under what conditions is the market efficient? Explain intuitively why it is or is not efficient.

3. Assume that the conditions for inefficiency are met. How much would Farm 2 be willing to pay Farm 1 to use bees? What can the government do about it in terms of taxes?