

# Chapter 4: Externalities

Ch 34 in H. Varian 8<sup>th</sup> Ed.

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# Introduction

- An **externality** is a cost or a benefit imposed upon someone by actions taken by others. The cost or benefit is thus generated externally to that somebody.
- **Consumption externality**: when a consumer cares directly about consumption or production of others.
- An externally imposed benefit is a **positive externality**. Examples?
- An externally imposed cost is a **negative externality**. Examples?
- **Production externality**: when the production of one firm is influenced by the choices of another firm or consumer

# Examples of Negative Externalities

- Air pollution.
- Water pollution.
- Loud parties next door.
- Traffic congestion.
- Second-hand cigarette smoke.
- Hospital collapse due to alcohol or tobacco consumption (or Covid-sick non-vaccinated people).

# Examples of Positive Externalities

- A well-maintained property next door that raises the market value of your property.
- A pleasant cologne or scent worn by the person seated next to you.
- Improved driving habits that reduce accident risks.
- A scientific advance.

# Externalities and Efficiency

→ Crucially, an externality impacts a **third party**; i.e. somebody who is not a participant in the activity that produces the external cost or benefit.

Up until now...

→ A market mechanism can achieve a Pareto efficient allocation IF  
THERE ARE NO EXTERNALITIES

→ Externalities cause Pareto inefficiency; typically

- Too much scarce resource is allocated to an activity which causes a negative externality
- Too little resource is allocated to an activity which causes a positive externality.

# Externalities as public commodities

- An externality will be viewed as a purely public commodity.
- A commodity is purely public if
  - It is consumed by everyone (nonexcludability), and
  - Everybody consumes the entire amount of the commodity (nonrivalry in consumption).
- E.g. a broadcast television program.

# Outline

1. Example of a negative externality
2. Externalities and Property Rights
3. Plotting externalities
4. Numerical example of production externalities
5. The Coarse solution (Solution 1)
6. Merger and Internalization (Solution 2)
7. Public Sector Remedies (Solution 3)
8. The Tragedy of the Commons

# 1. Example of a Negative Externality

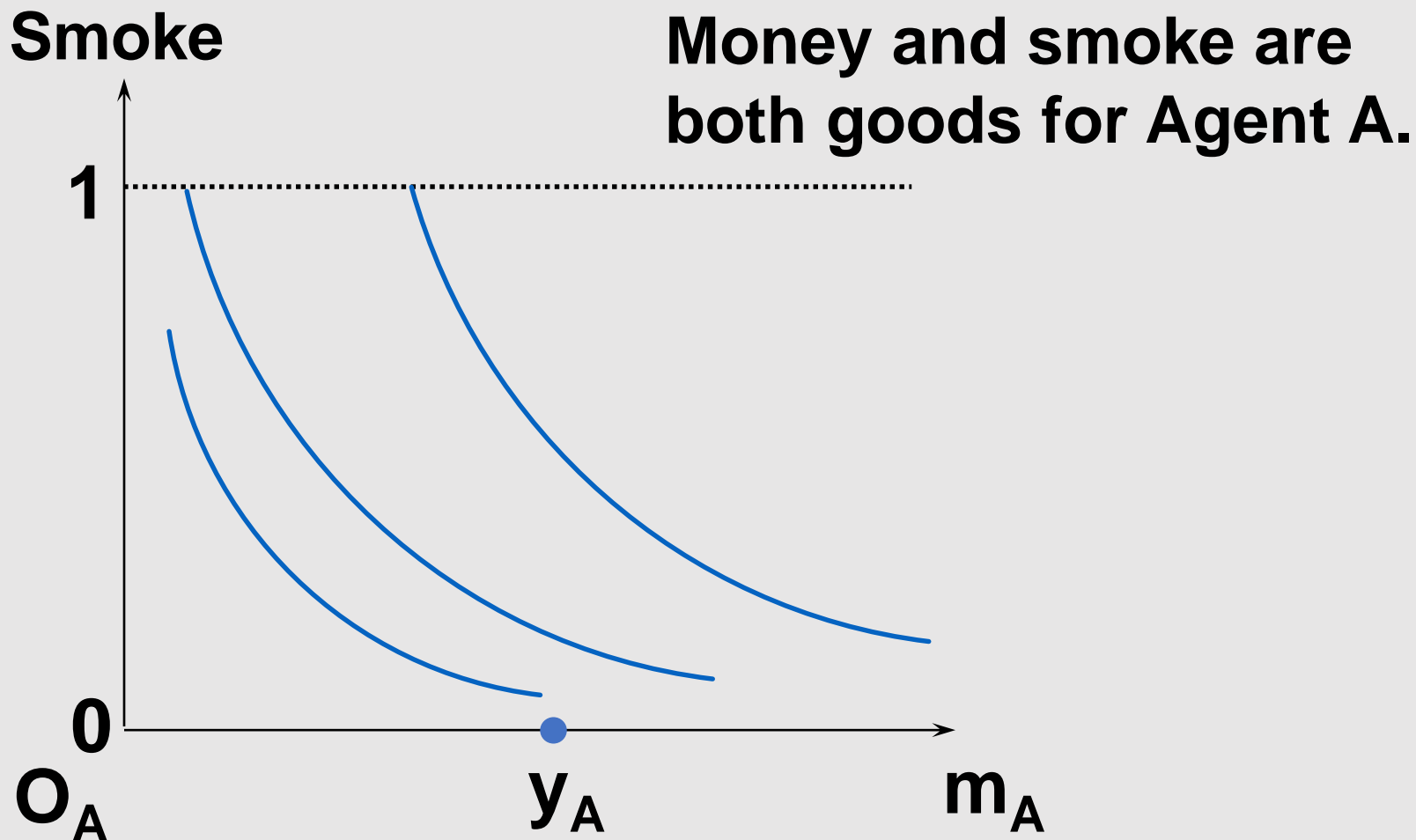
- Consider two agents, A and B, and two commodities, money and smoke.
- Both smoke and money are goods for Agent A.
- Money is a good and smoke is a bad for Agent B.
- Smoke is a purely public commodity.
- Smoke can be traded since property rights over smoke are clear
  - This is, smoke or clear air can be scarified in exchange of money



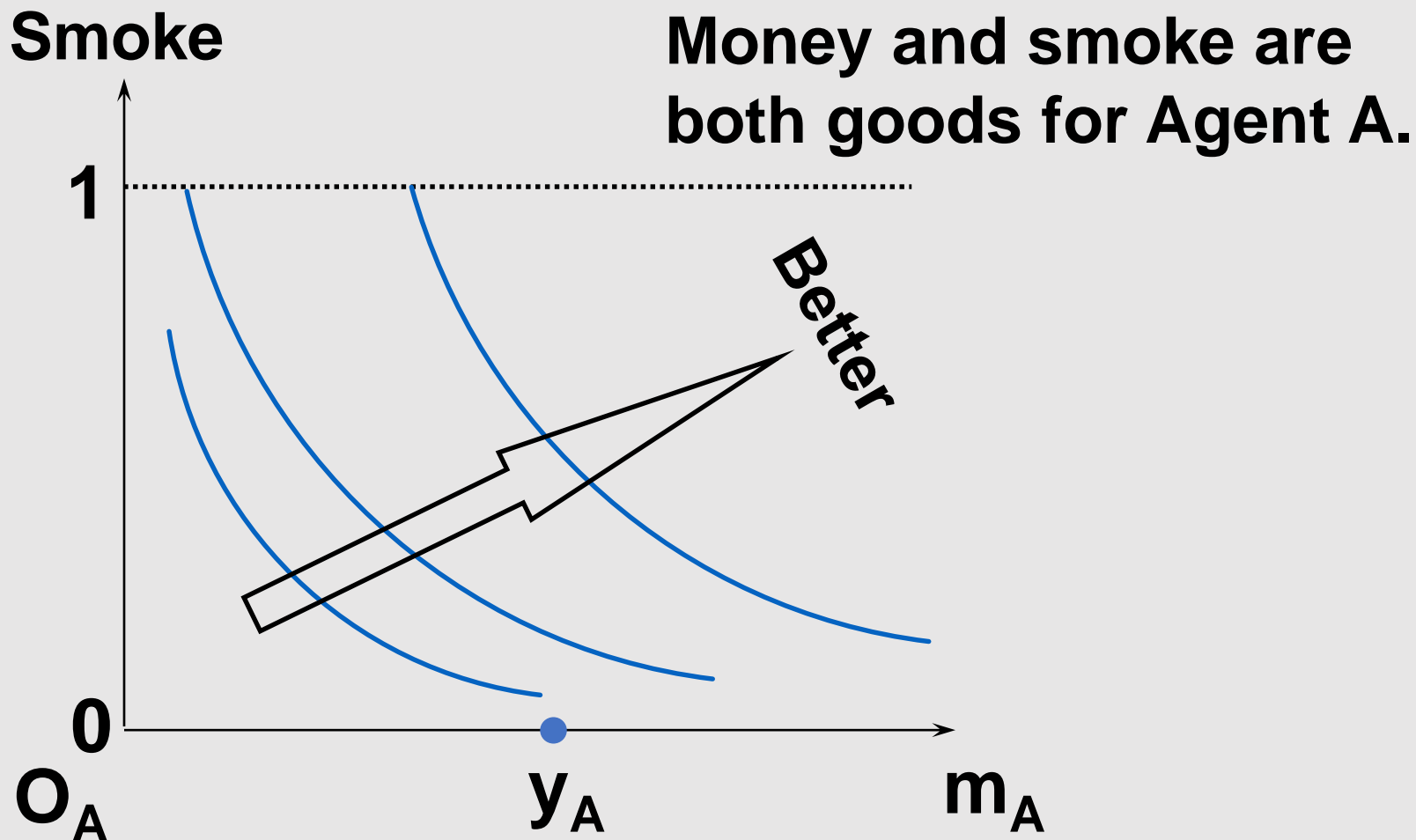
# Example of a Negative Externality

- Agent A is endowed with  $y_A$ .
- Agent B is endowed with  $y_B$ .
- Smoke intensity is measured on a scale from 0 (no smoke) to 1 (maximum concentration).

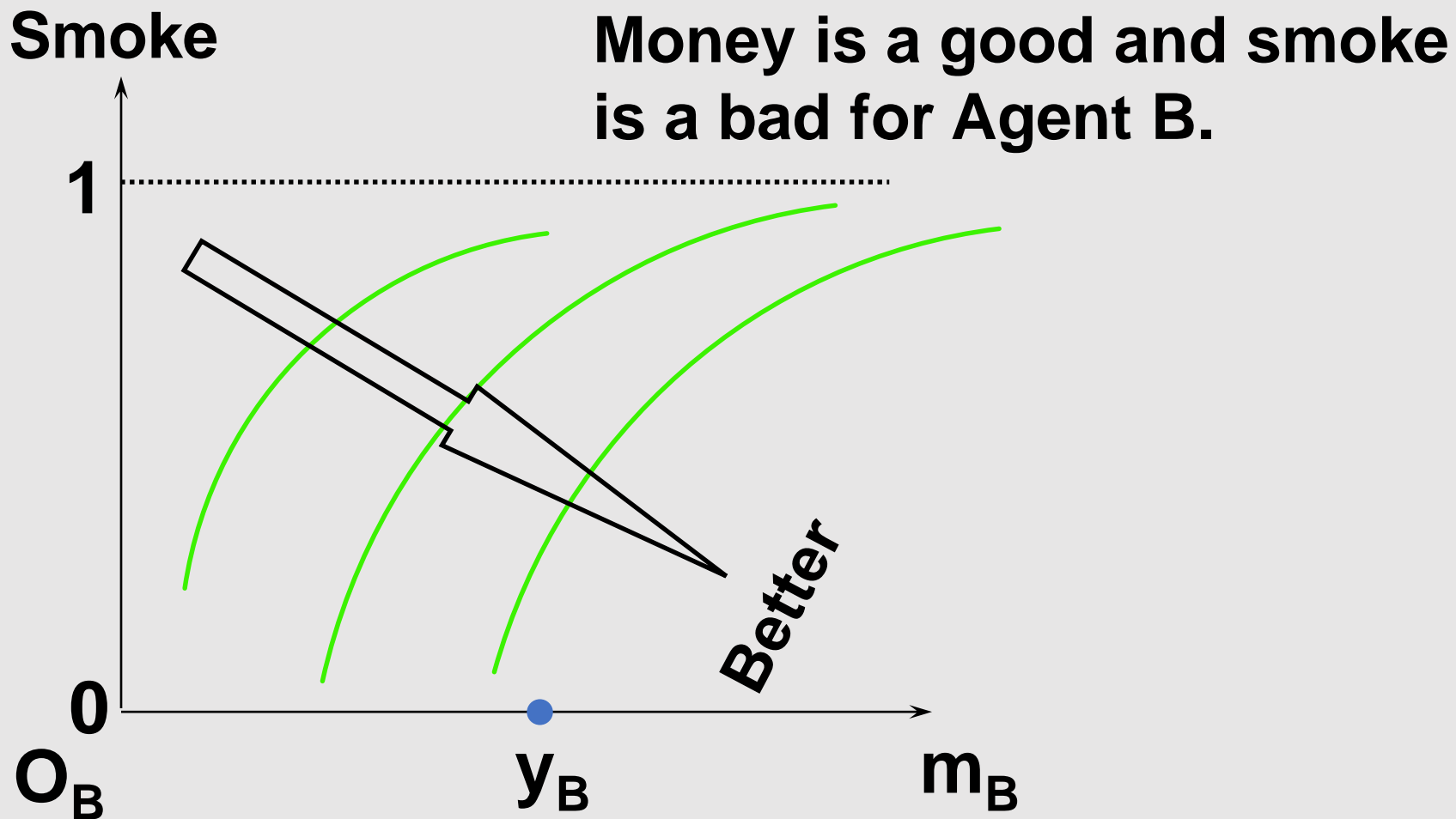
## Example of a Negative Externality



## Example of a Negative Externality

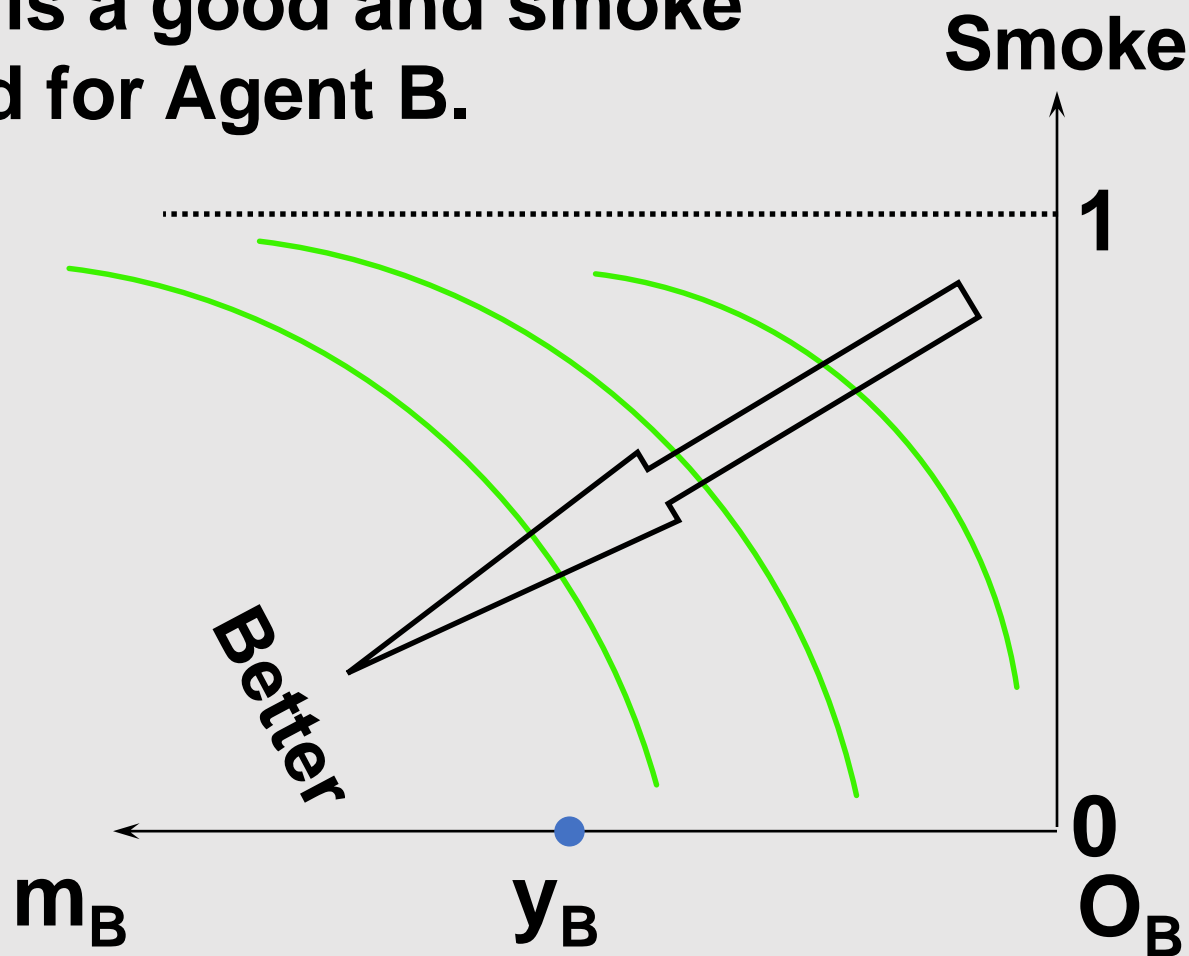


## Example of a Negative Externality



## Example of a Negative Externality

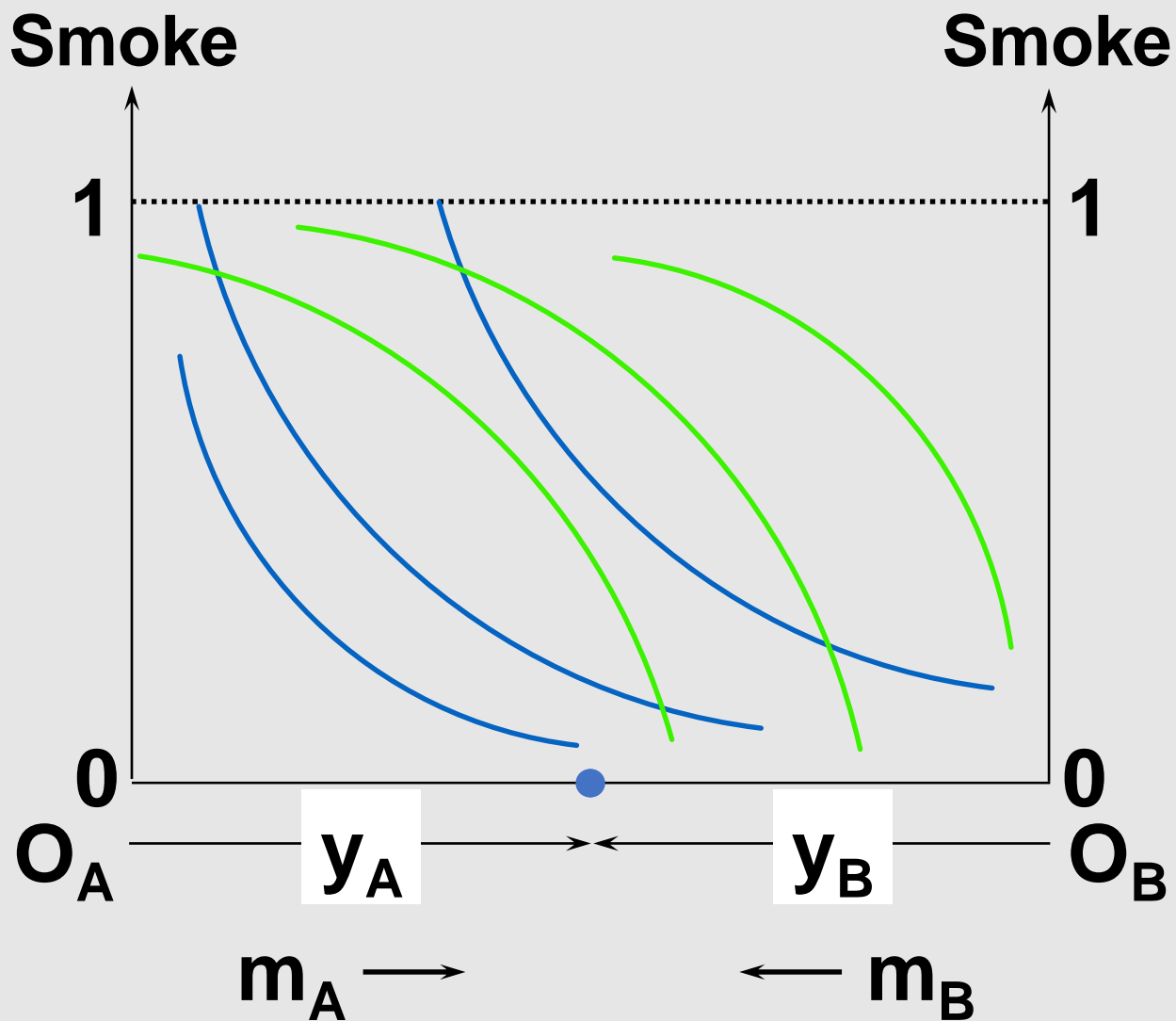
Money is a good and smoke is a bad for Agent B.



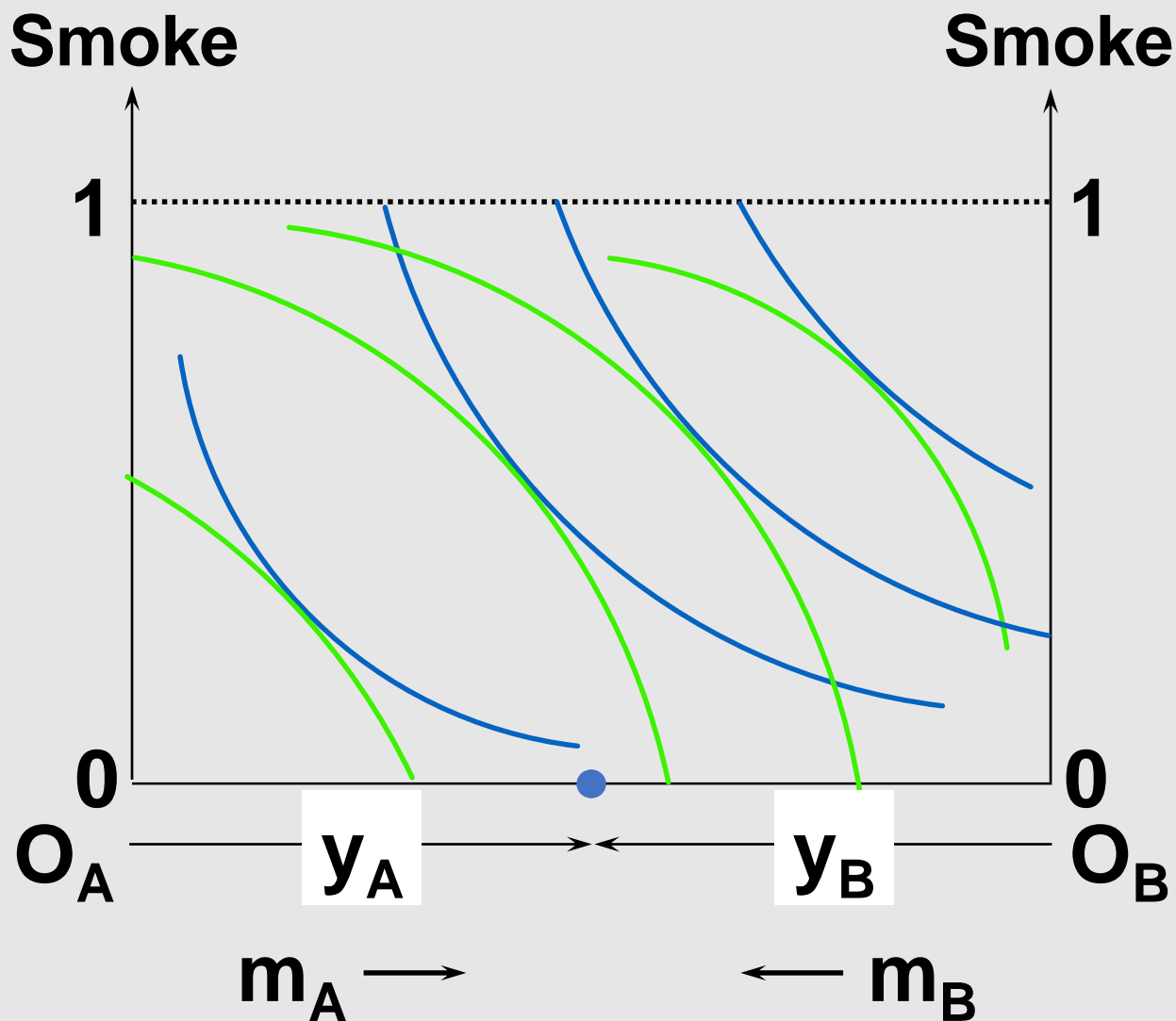
# Example of a Negative Externality

→ What are the efficient allocations of smoke and money?

# Example of a Negative Externality

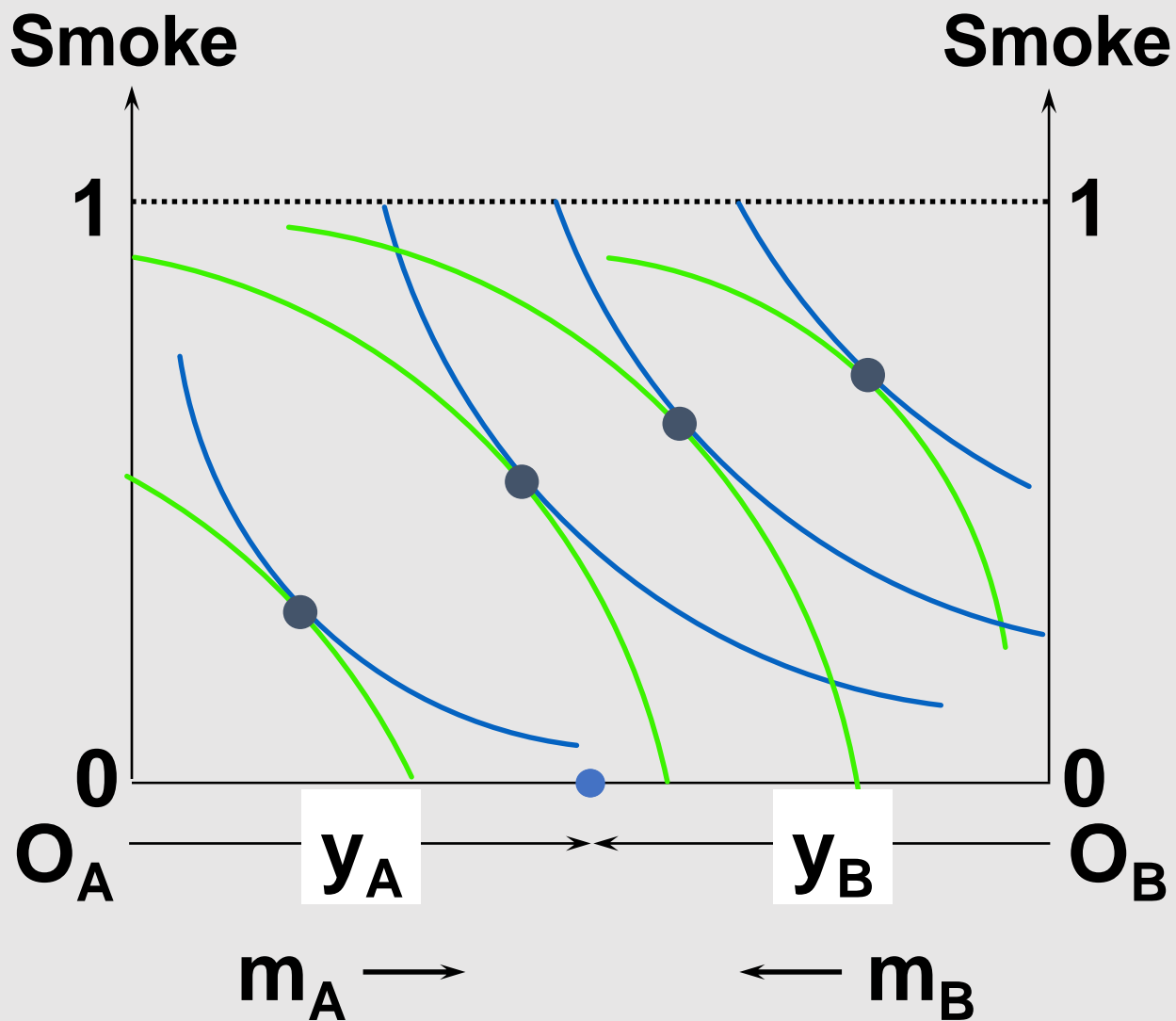


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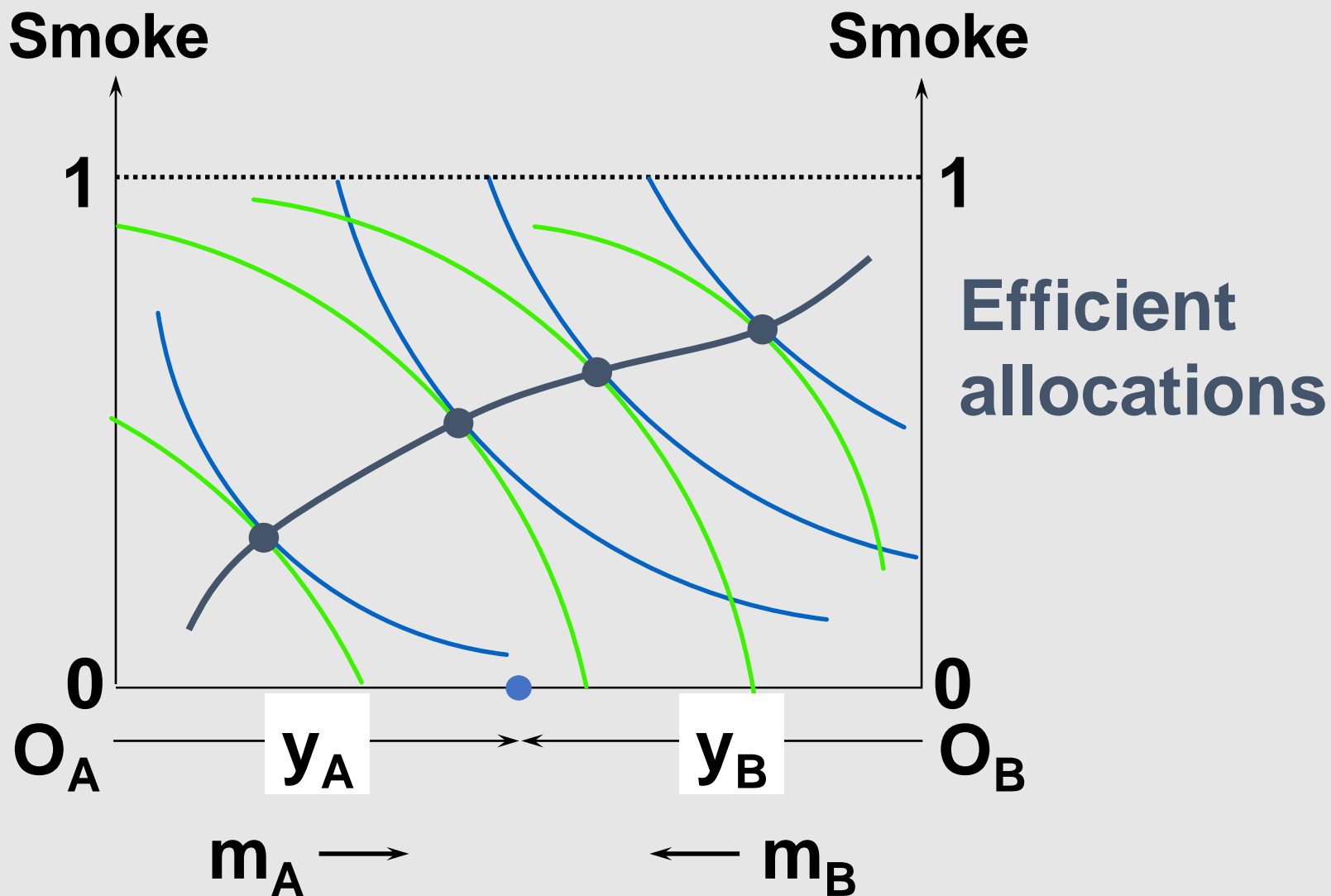




# Example of a Negative Externality



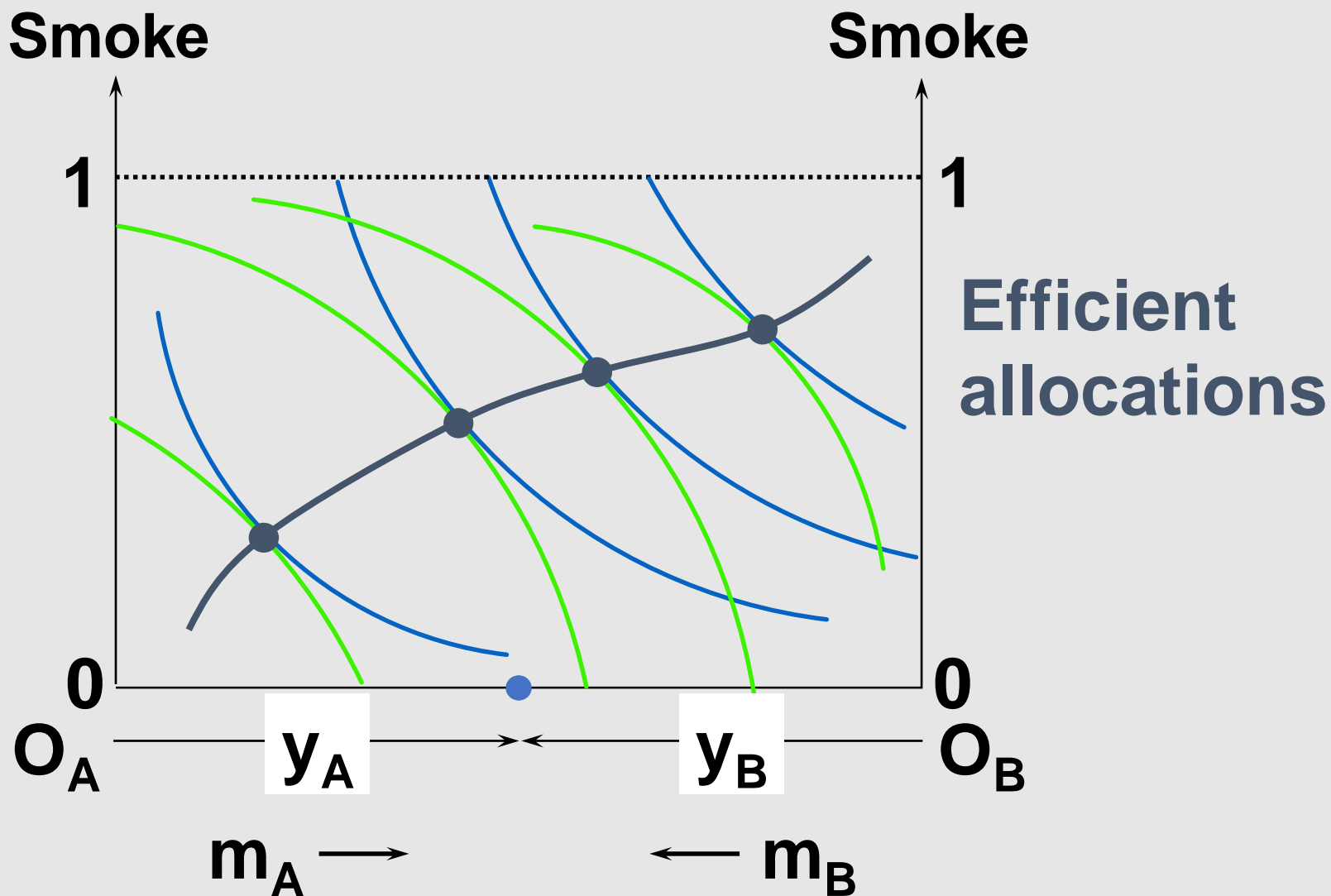
# Example of a Negative Externality



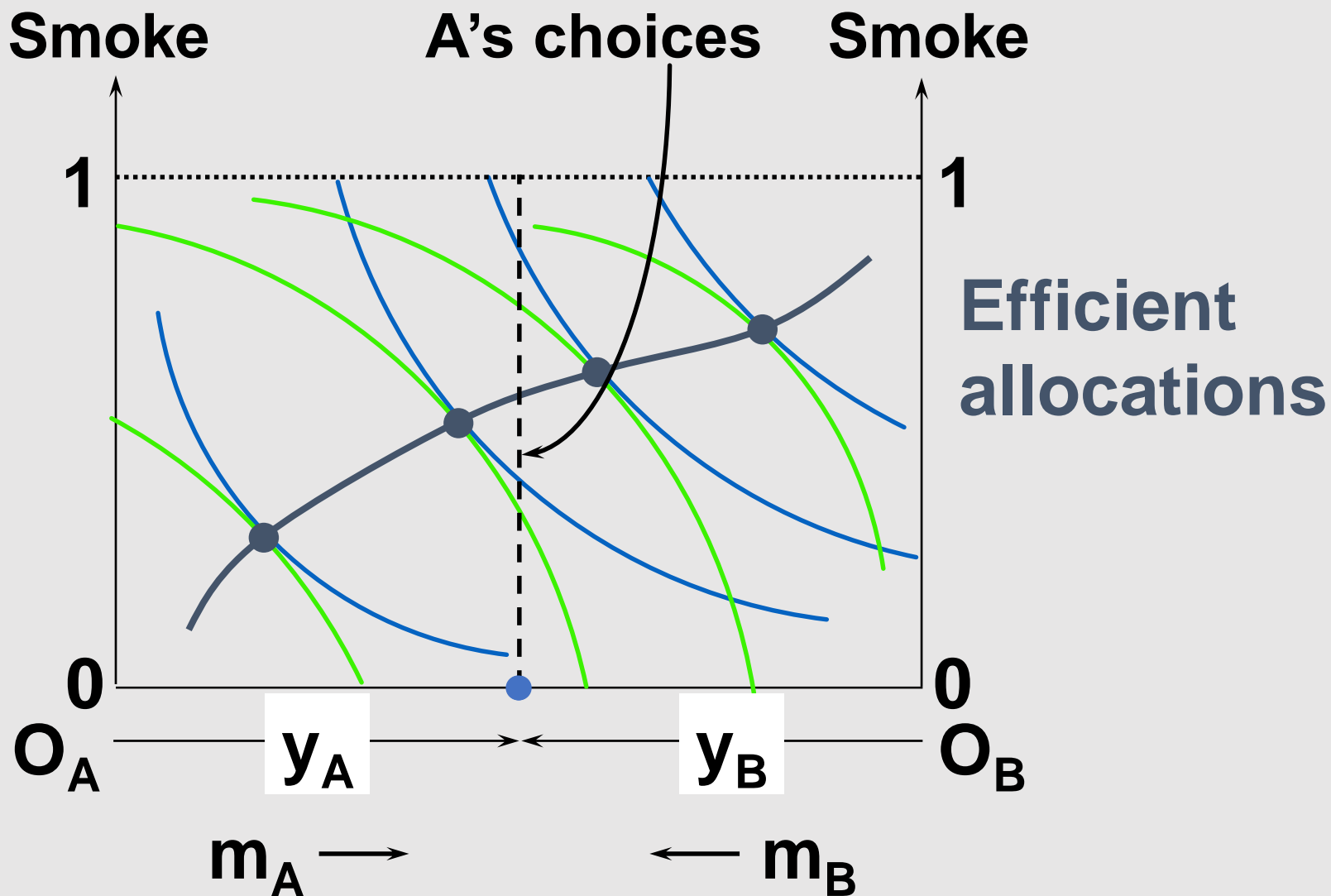
# Example of a Negative Externality

- Suppose now that there is no means by which money can be exchanged for smoke.
- What then is Agent A's most preferred allocation?
- Is this allocation efficient?

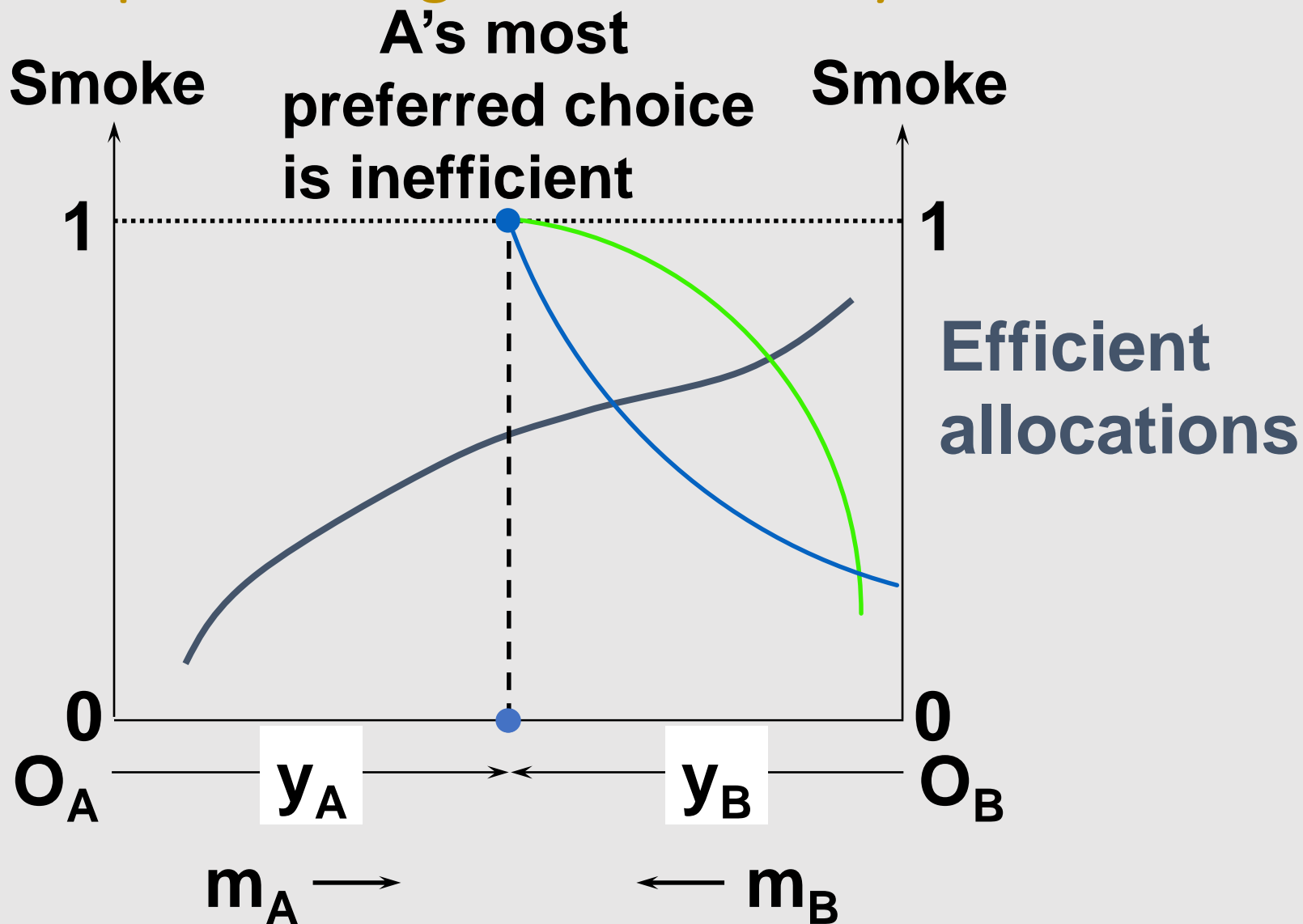
# Example of a Negative Externality



## Example of a Negative Externality



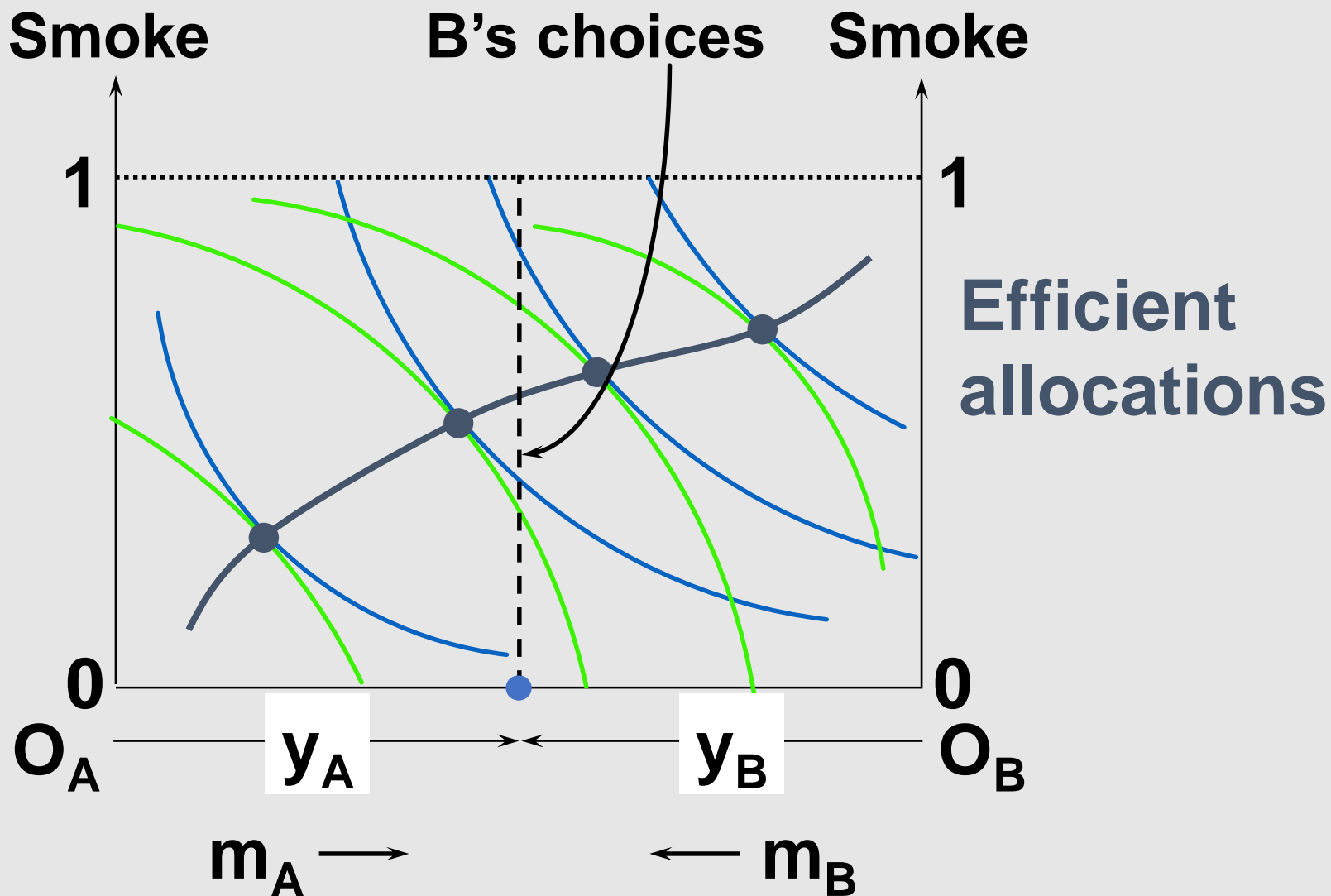
## Example of a Negative Externality



# Example of a Negative Externality

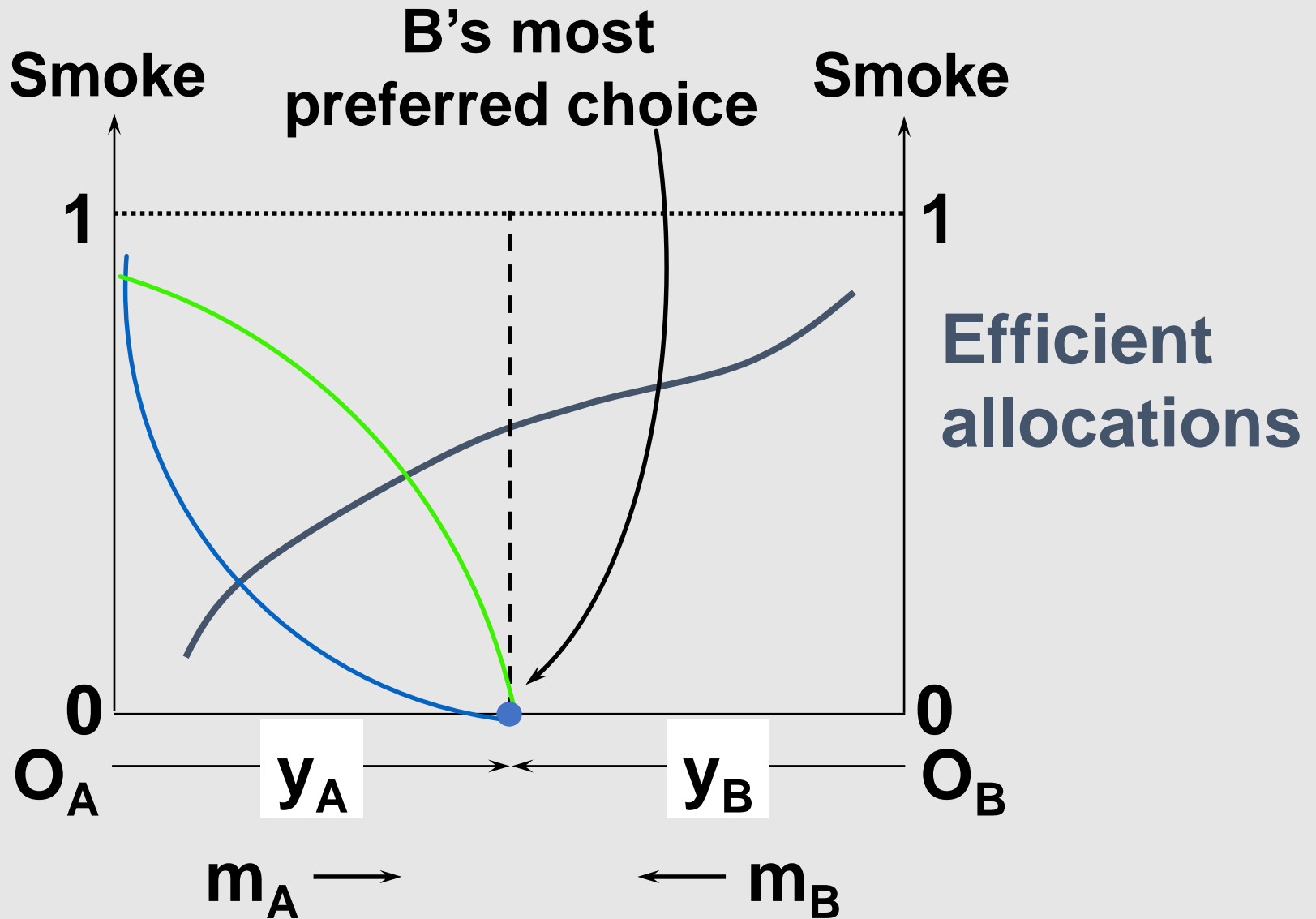
- Continue to suppose there is no means by which money can be exchanged for changes in smoke level.
- What is Agent B's most preferred allocation?
- Is this allocation efficient?

## Example of a Negative Externality

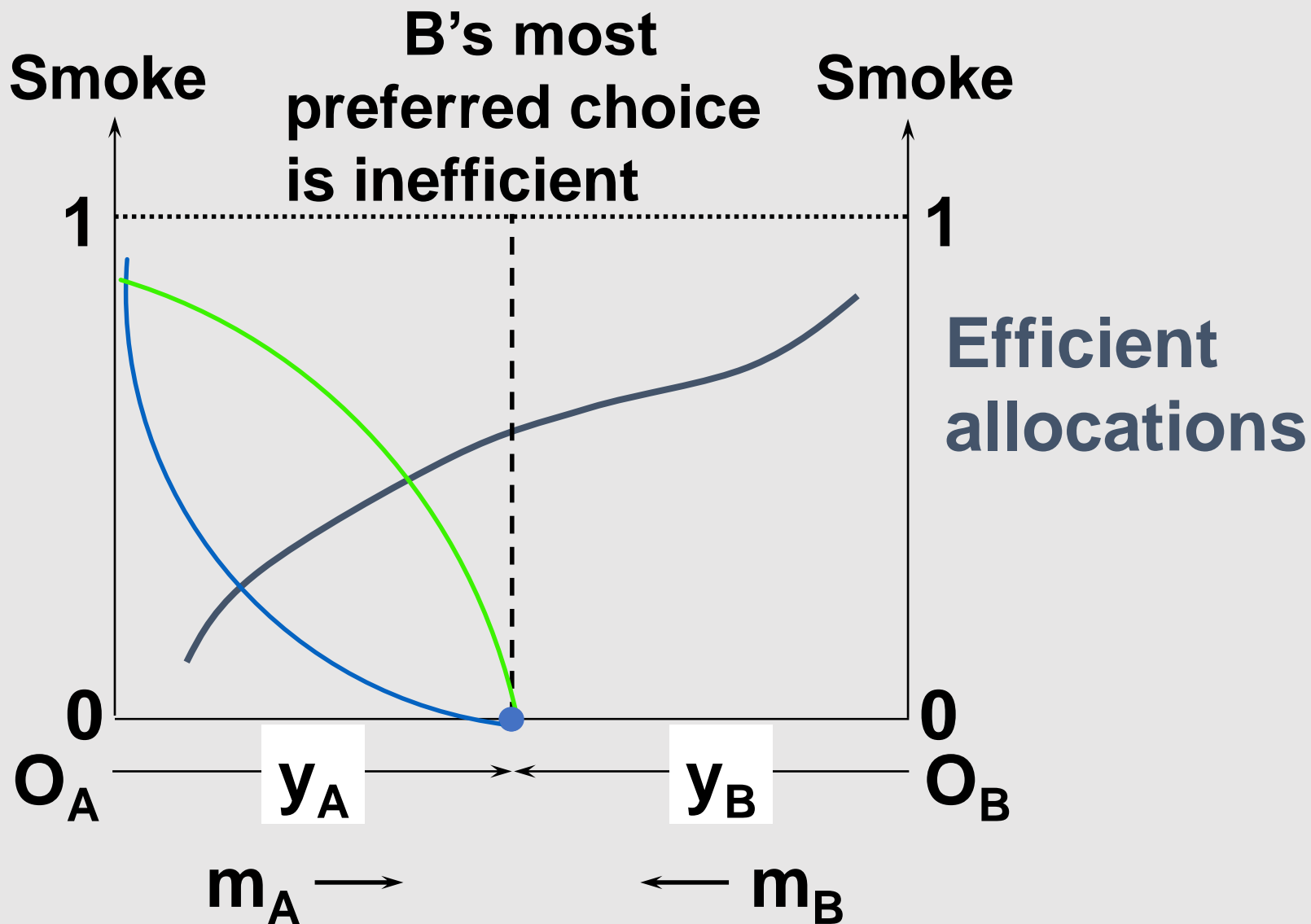




## Example of a Negative Externality



## Example of a Negative Externality



# Example of a Negative Externality

- So if A and B cannot trade money for changes in smoke intensity, then the outcome is inefficient.
- Either there is too much smoke (A's most preferred choice) or there is too little smoke (B's choice).
- If A believes that he has the right to smoke and B believes that he has the right to clean air, we have difficulties.
- The practical problems with externalities generally arise because of poorly defined property rights.

# Example of a Negative Externality

**Externalities + well defined property rights =**

→ A competitive trade mechanism can lead to a Pareto efficient allocation

**Externalities + poorly defined property rights =**

→ A competitive trade is insufficient to obtain a Pareto efficient allocation, since trade is not possible.

## 2. Externalities and Property Rights

### Private-Sector Solutions to Negative Externalities

- Key question raised by **Ronald Coase** (famous Nobel Prize winner):
  - Are externalities really outside the market mechanism?
  - Can externalities be broad back to the market? Can they be internalized?
- **Internalizing the externality**: When either private negotiations or government action lead the price of the good to fully reflect the external costs or benefits of the actions.

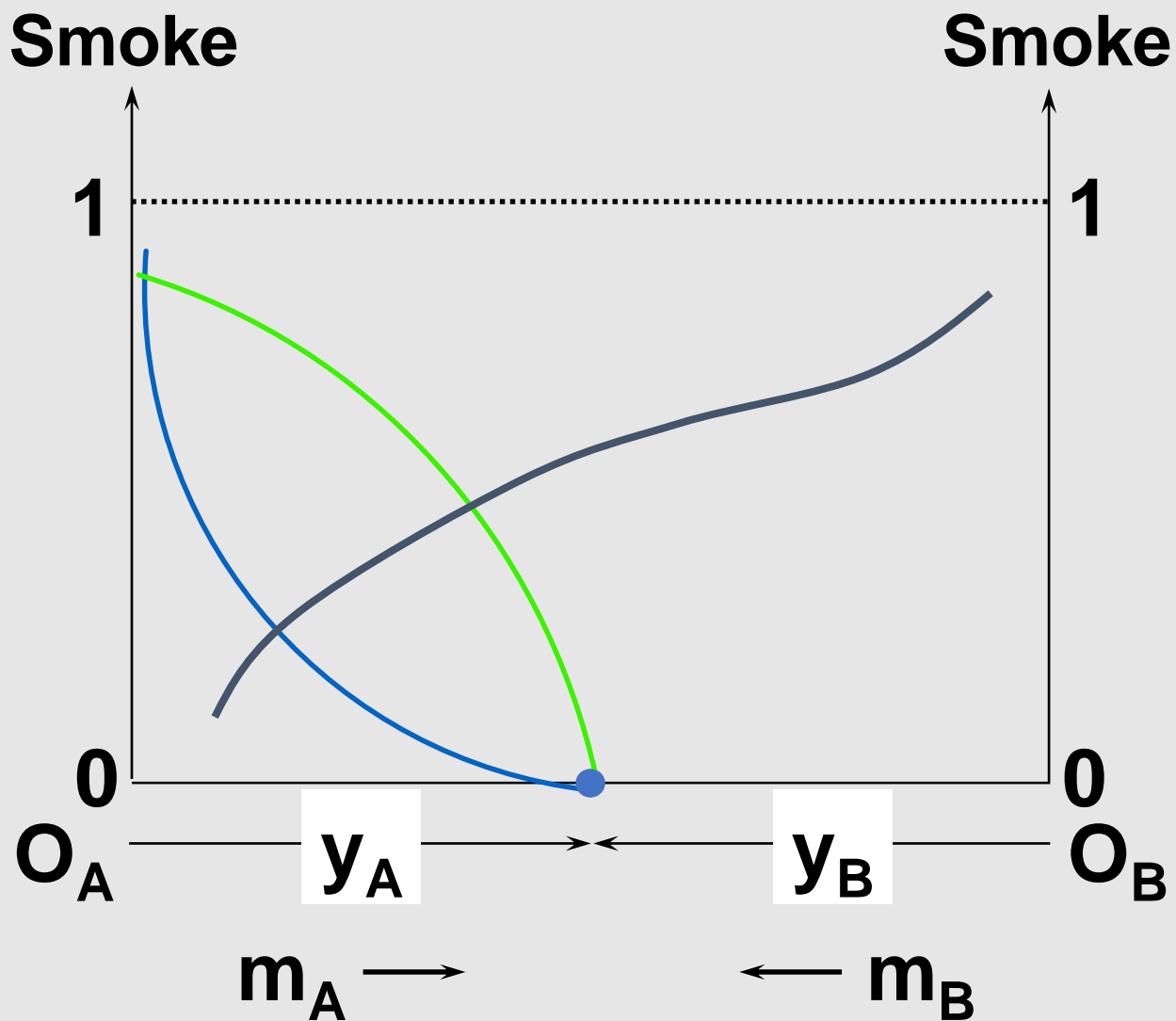
# Externalities and Property Rights

- Ronald Coase's insight is that most externality problems are due to an inadequate specification of property rights and, consequently, an absence of markets in which trade can be used to internalize external costs or benefits.
- Causing a producer of an externality to bear the full external cost or to enjoy the full external benefit is called **internalizing the externality**.
- Neither Agent A nor Agent B owns the air in their room.
- What happens if this property right is created and is assigned to one of them?

# Externalities and Property Rights

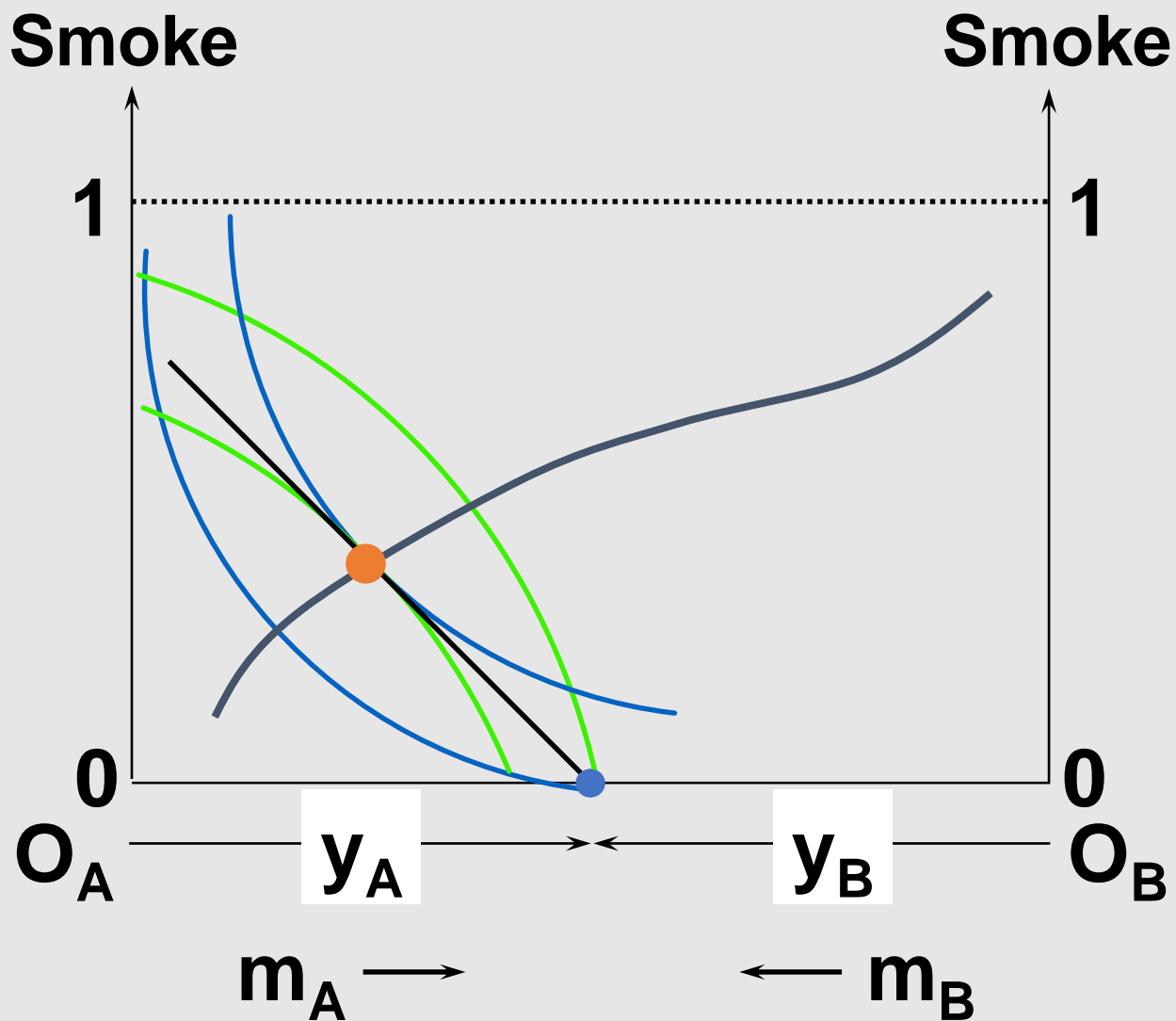
- Suppose Agent B is assigned ownership of the air in the room.
- Agent B can now sell “rights to smoke”.
- Will there be any smoking?
- If so, how much smoking and what will be the price for this amount of smoke?
- Let  $p(s_A)$  be the price paid by Agent A to Agent B in order to create a smoke intensity of  $s_A$ .

# Externalities and Property Rights

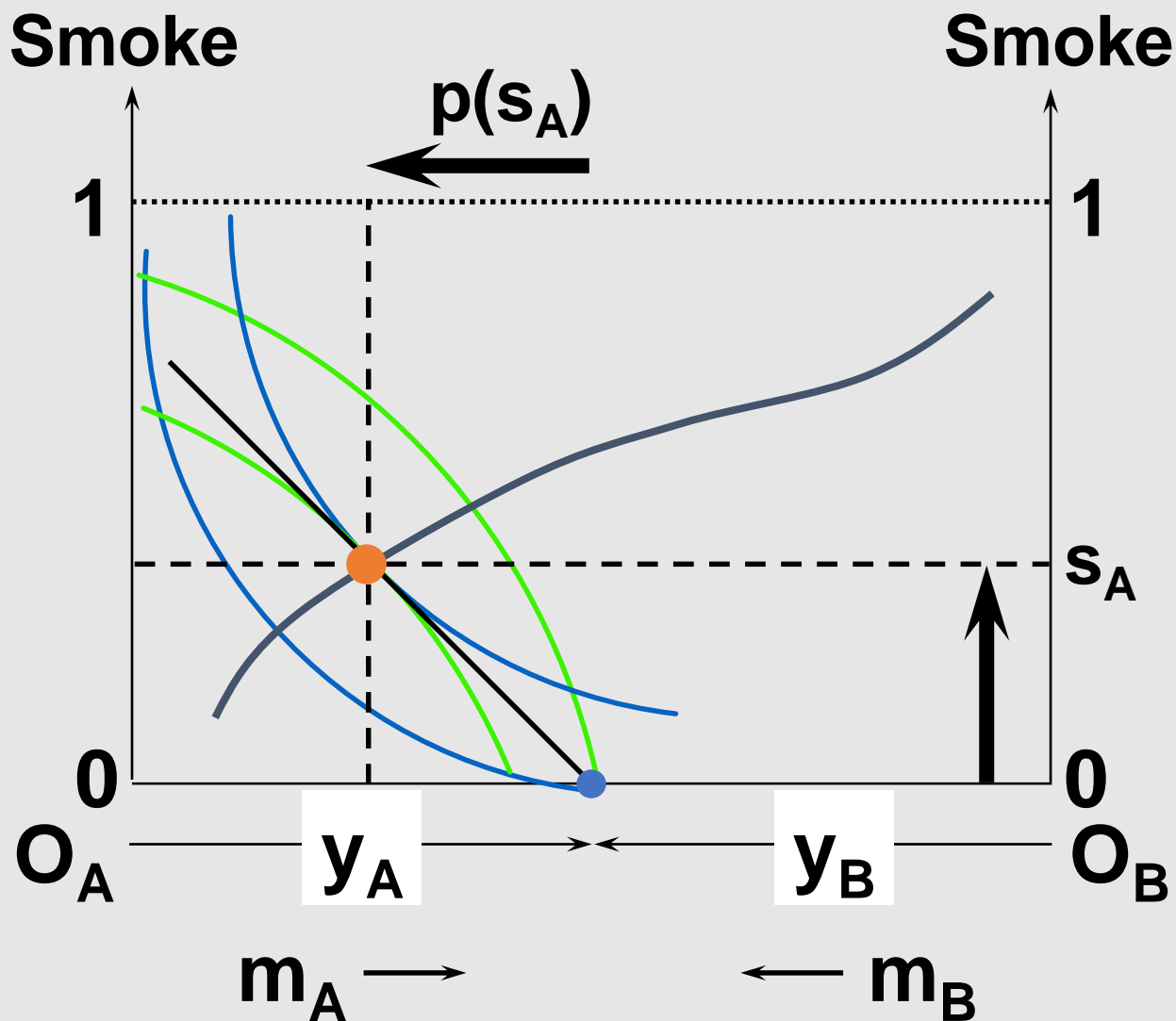




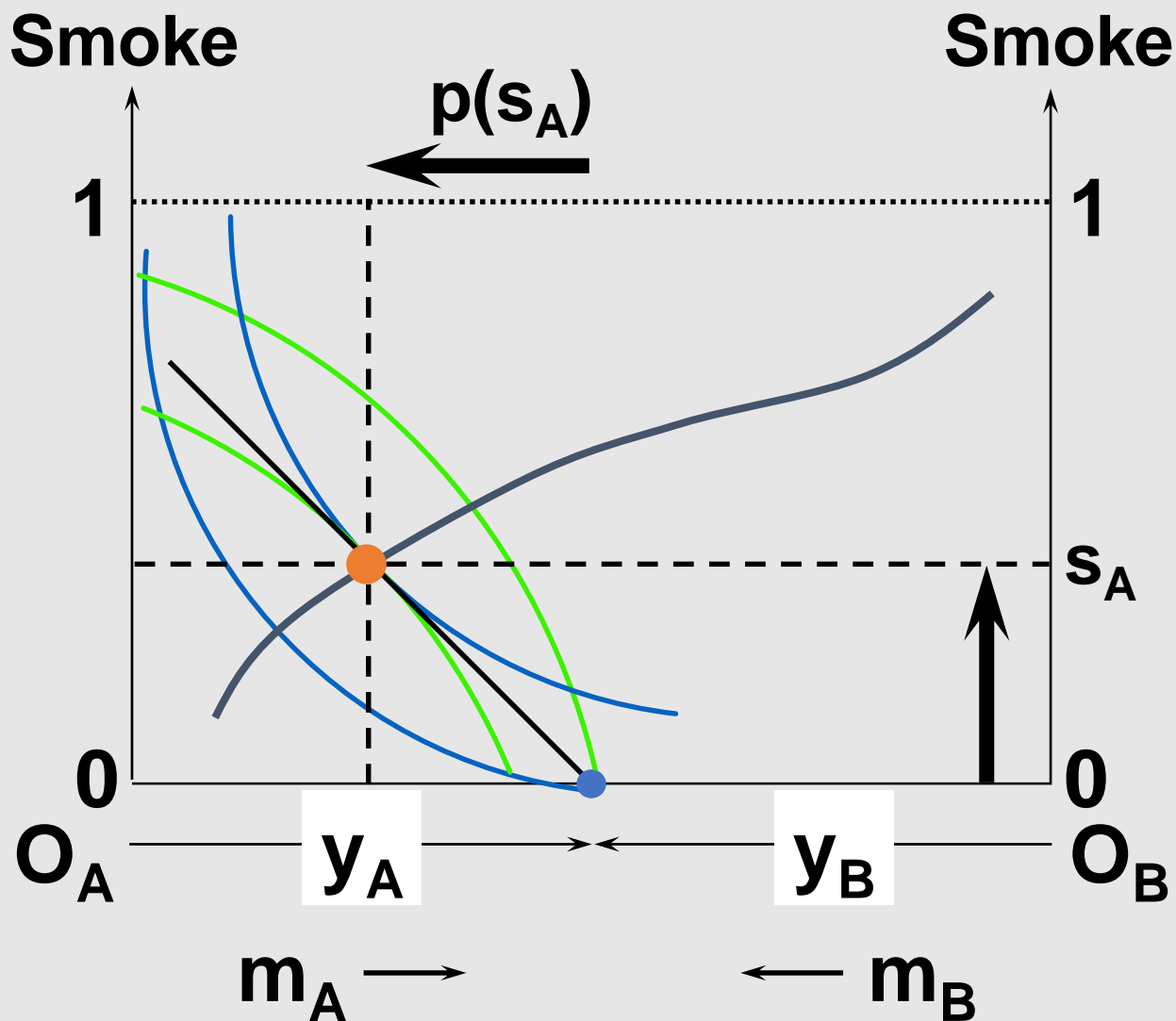
# Externalities and Property Rights



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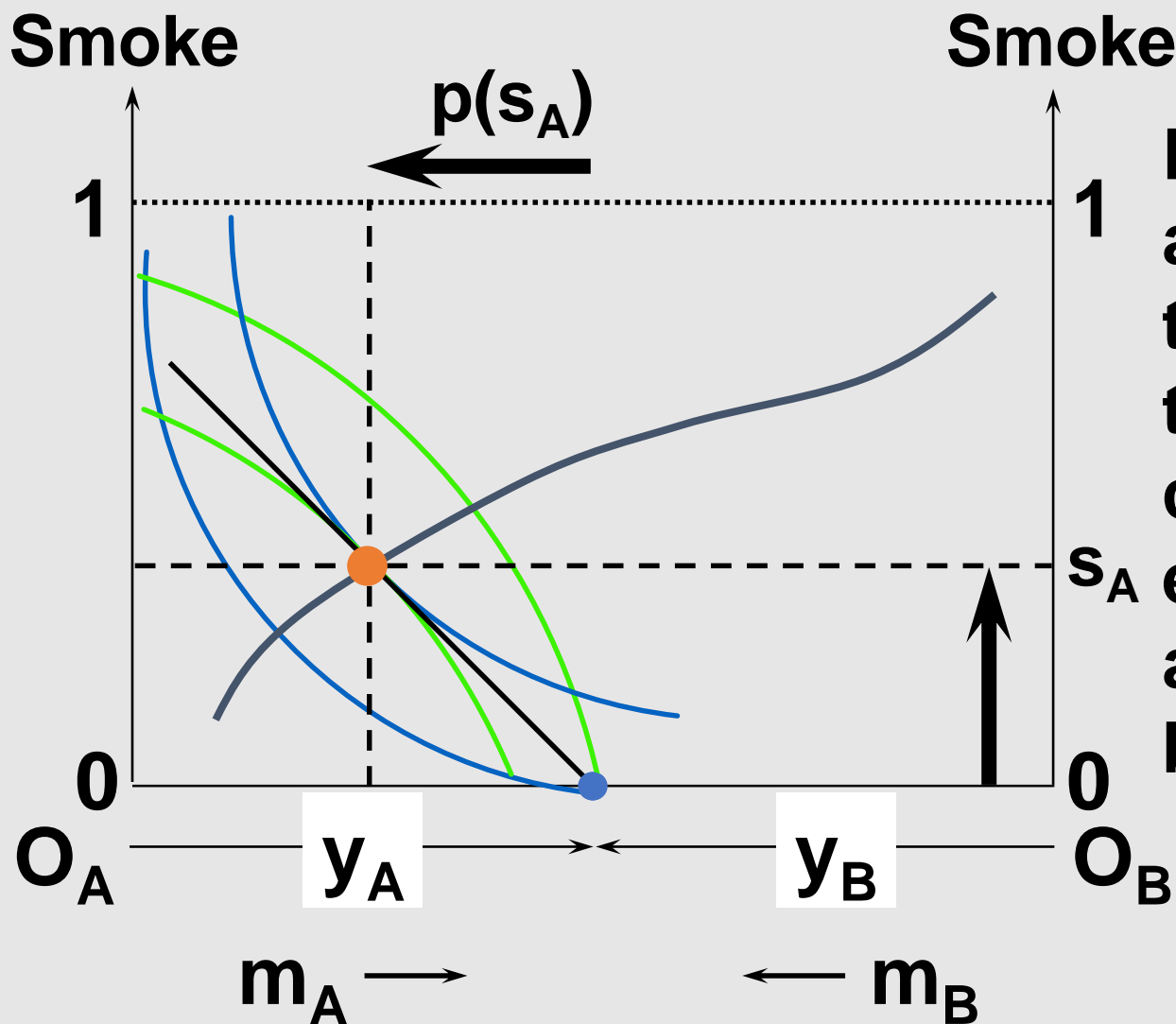


# Externalities and Property Rights



**Both agents gain and there is a positive amount of smoking.**

# Externalities and Property Rights

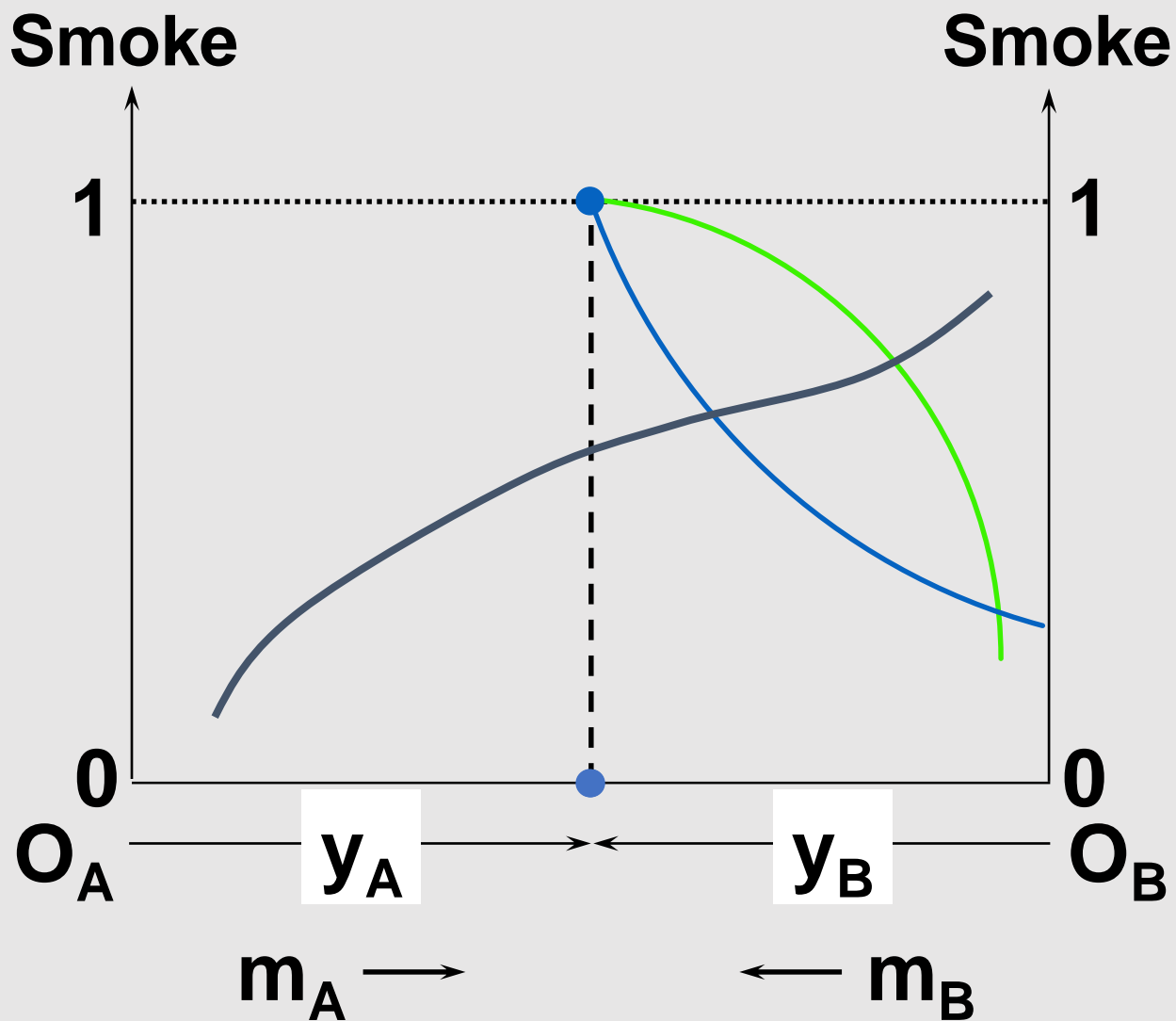


**Establishing a market for trading rights to smoke causes an efficient allocation to be achieved.**

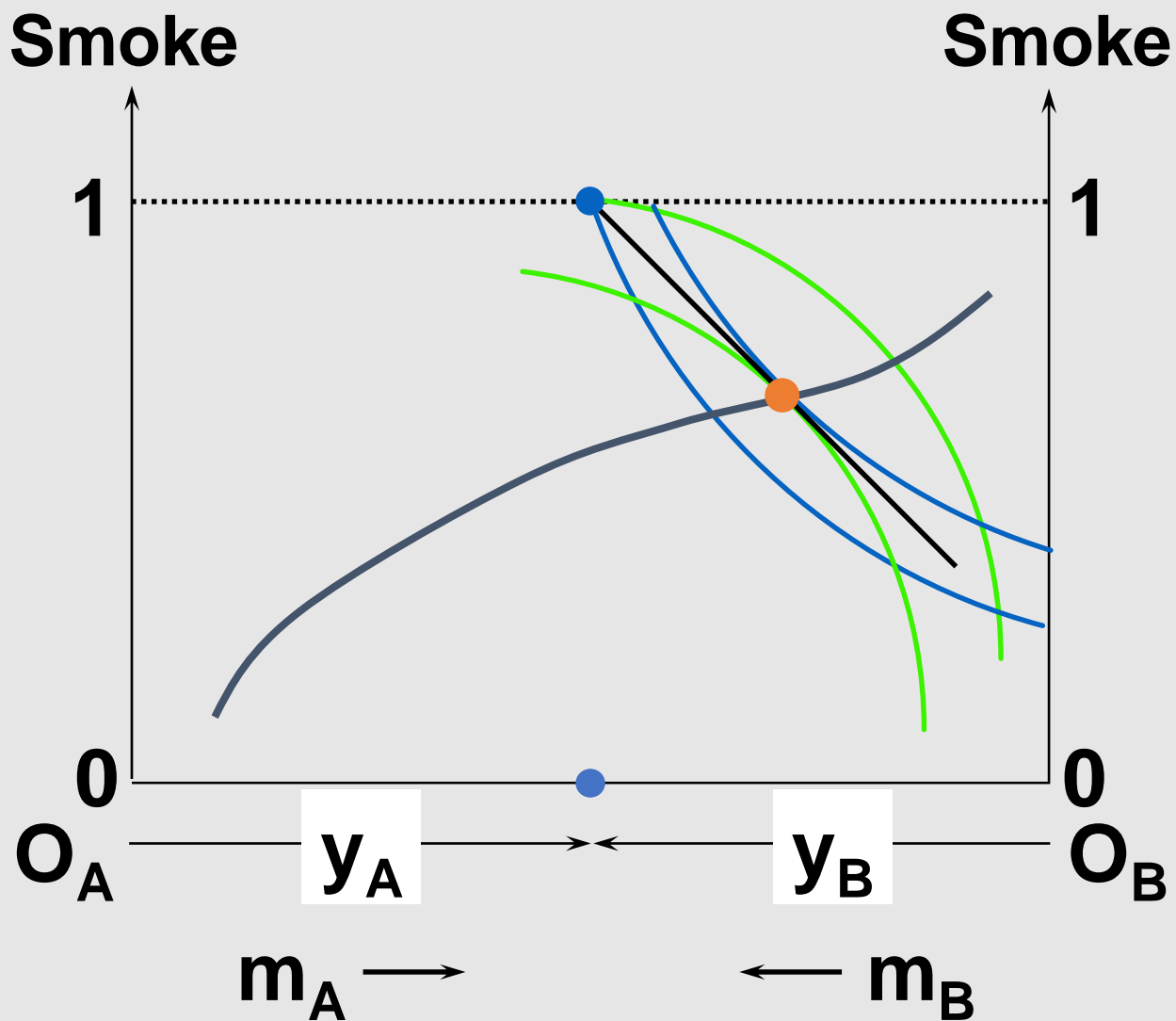
# Externalities and Property Rights

- Suppose instead that Agent A is assigned the ownership of the air in the room.
- Agent B can now pay Agent A to reduce the smoke intensity.
- How much smoking will there be?
- How much money will Agent B pay to Agent A?

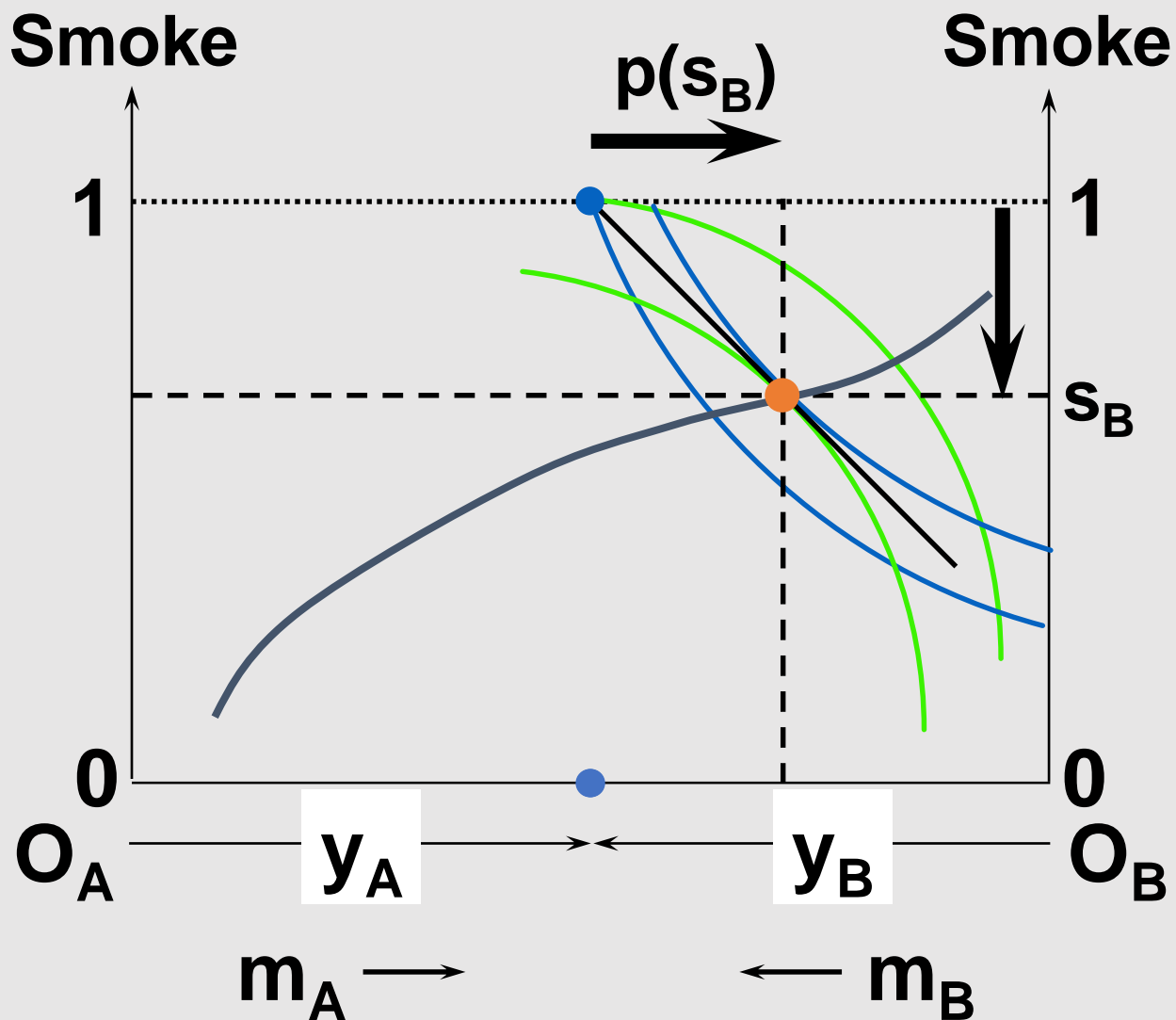
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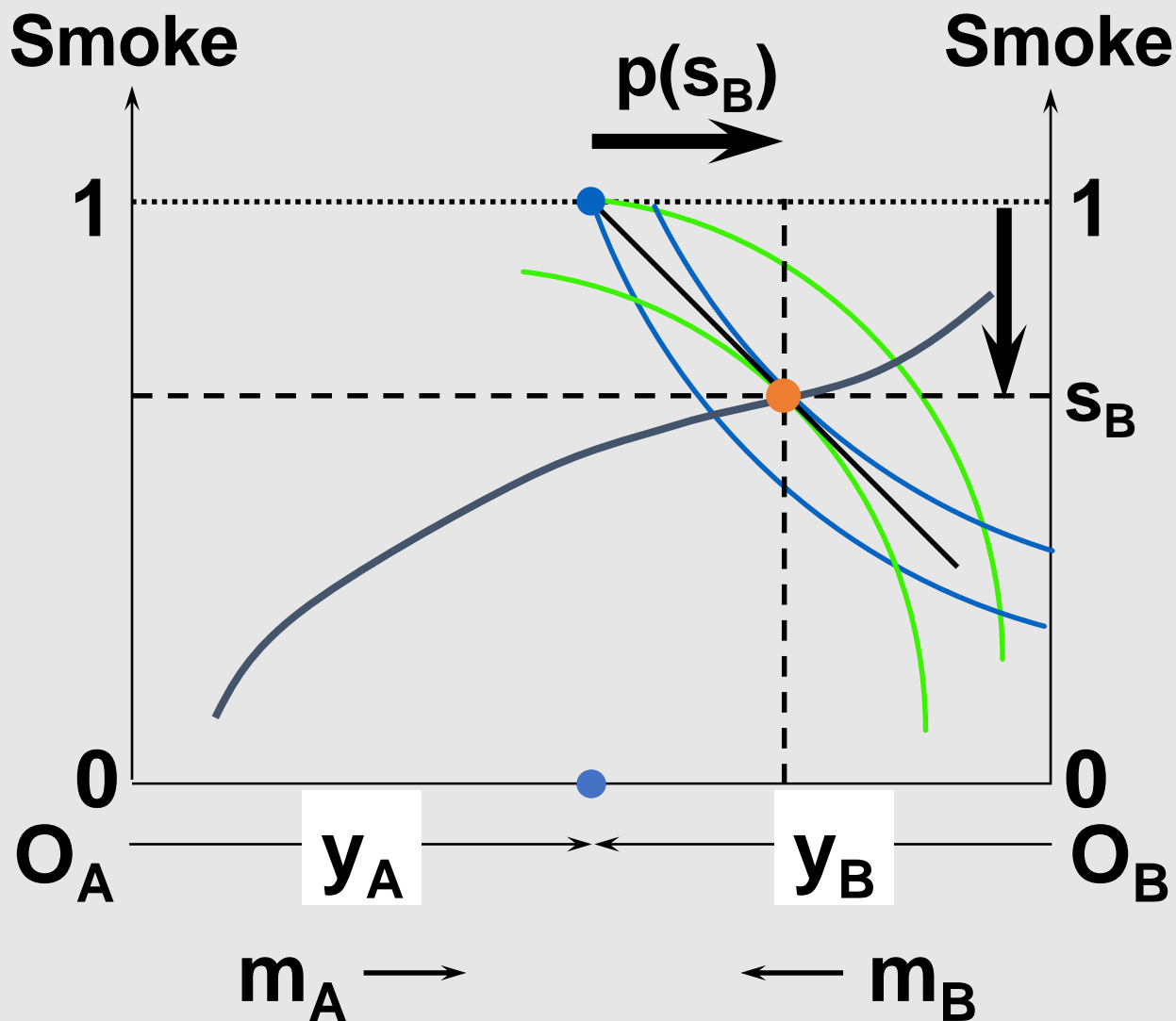


# Externalities and Property Rights



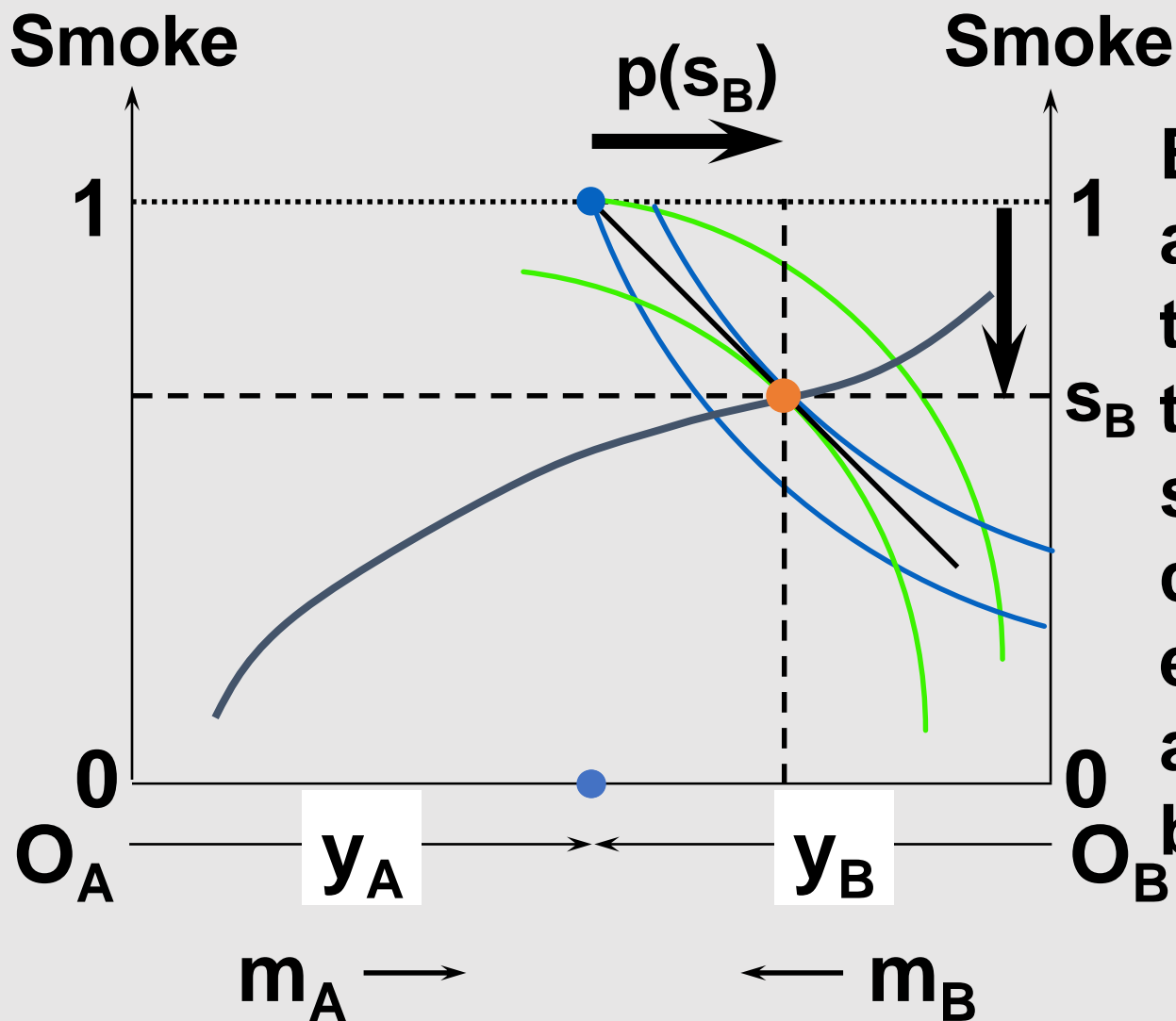


# Externalities and Property Rights



**Both agents gain and there is a reduced amount of smoking.**

# Externalities and Property Rights



**Establishing a market for trading rights to reduce smoke causes an efficient allocation to be achieved.**

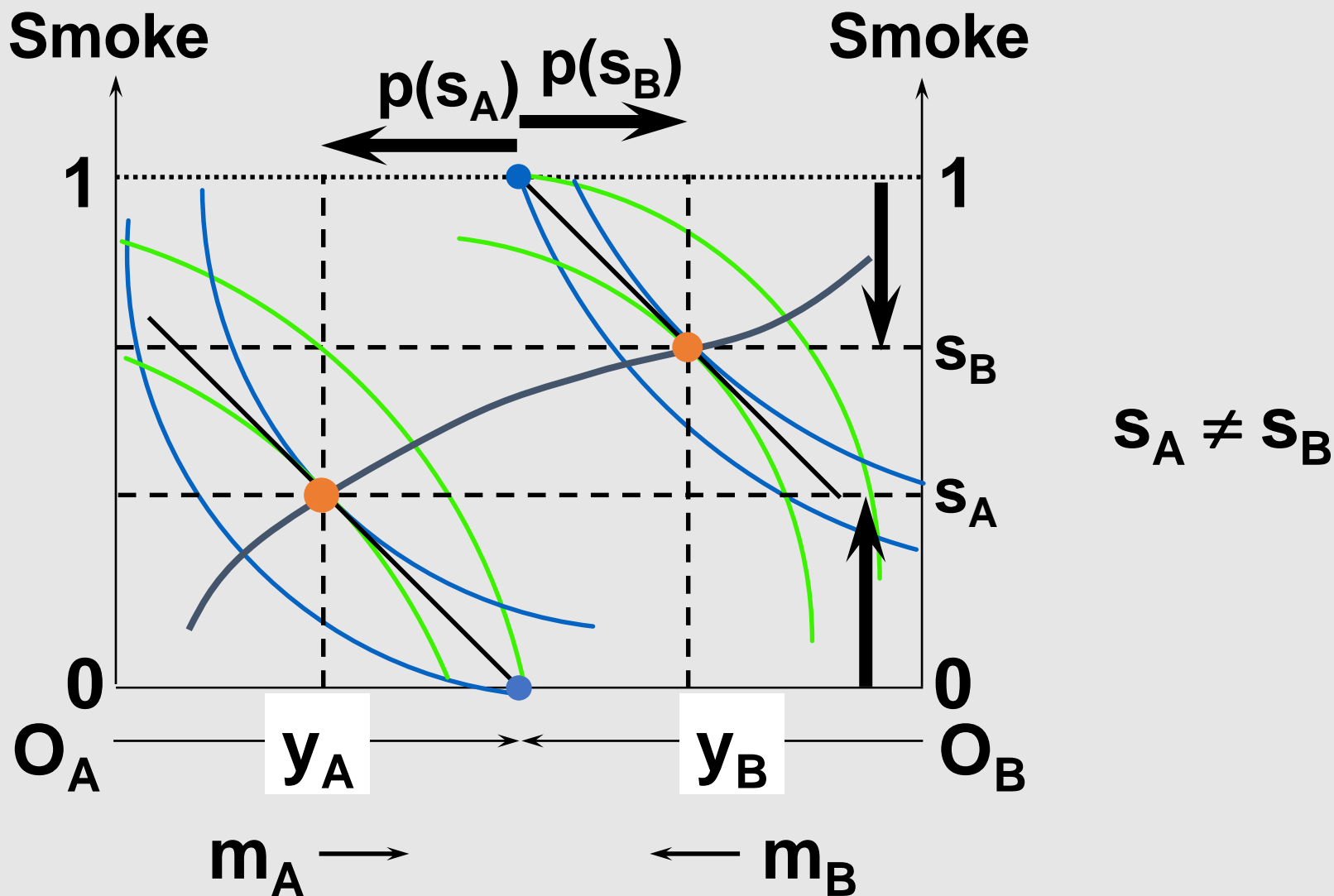
# Externalities and Property Rights

→ Notice that

- The agent given the property right (asset) is better off than at her own most preferred allocation in the absence of the property right.
- The amount of smoking that occurs in equilibrium depends upon which agent is assigned the property right.

→ Hence, the amount of smoke generated will depend on whether the smoker has the property rights or the nonsmoker has them.

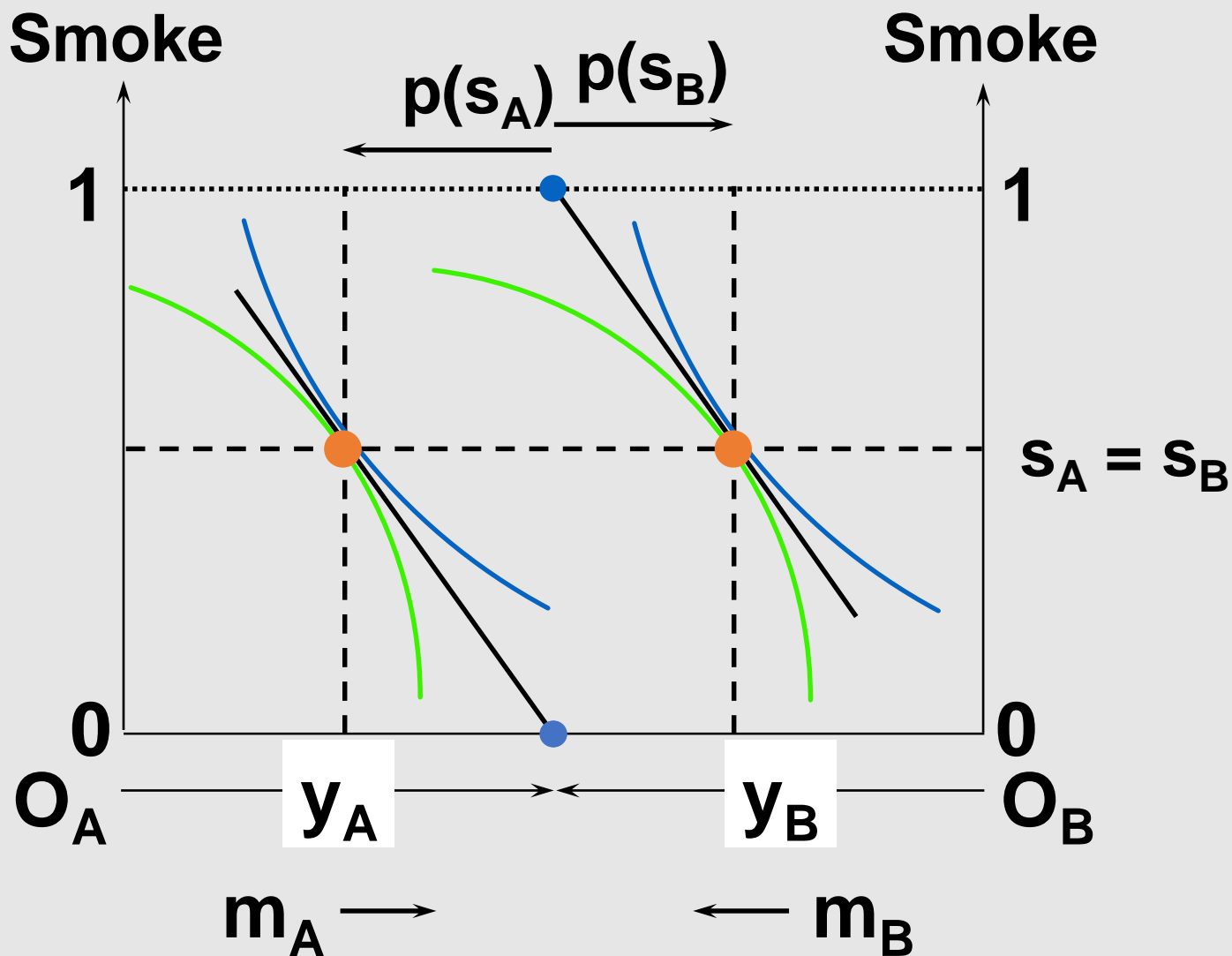
# Externalities and Property Rights



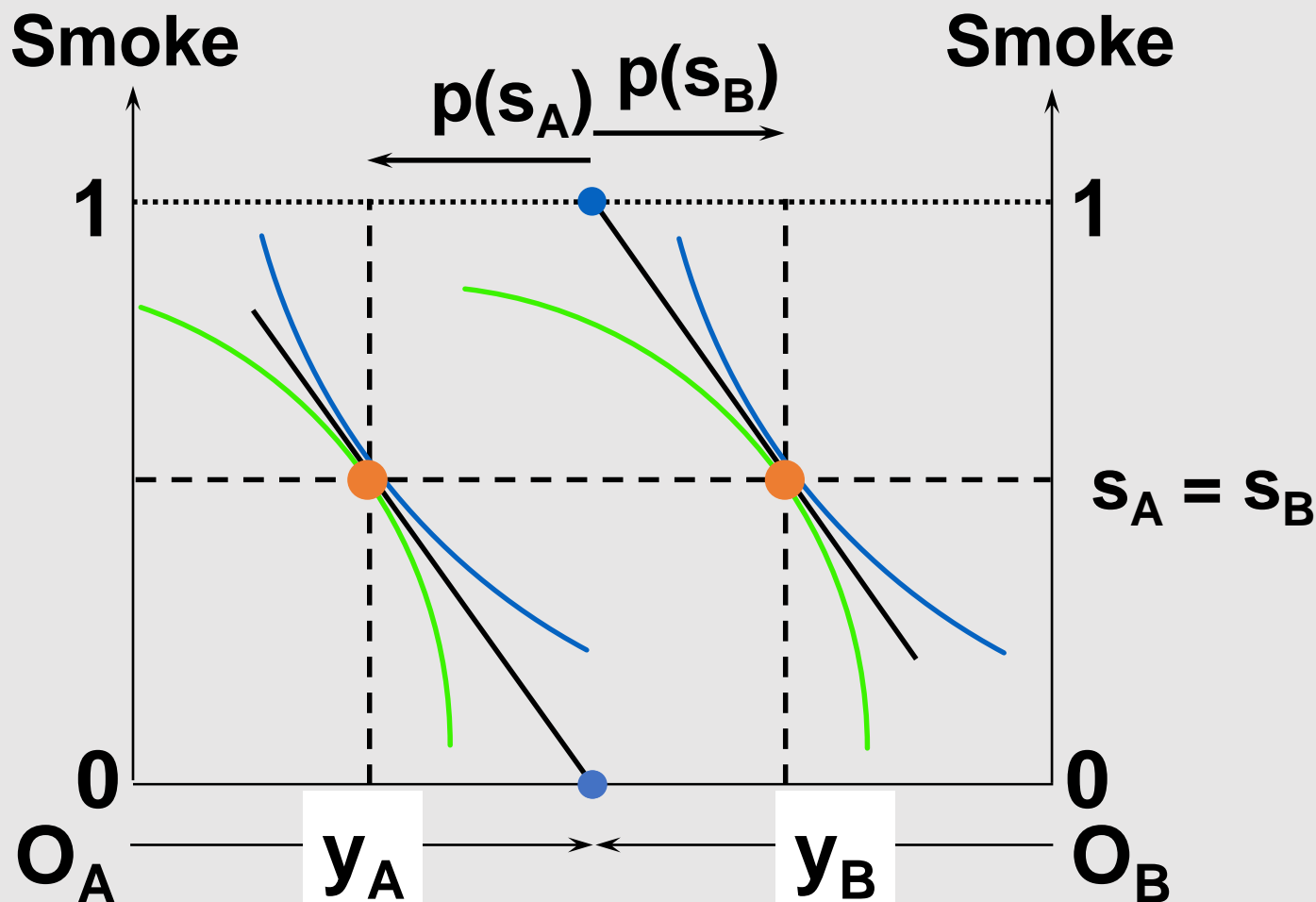
# Externalities and Property Rights

- Is there a case in which the same amount of smoking occurs in equilibrium no matter which agent is assigned ownership of the air in the room?
- Yes, if preferences are **quasilinear in money** (if utility is a linear function of money)

# Externalities and Property Rights

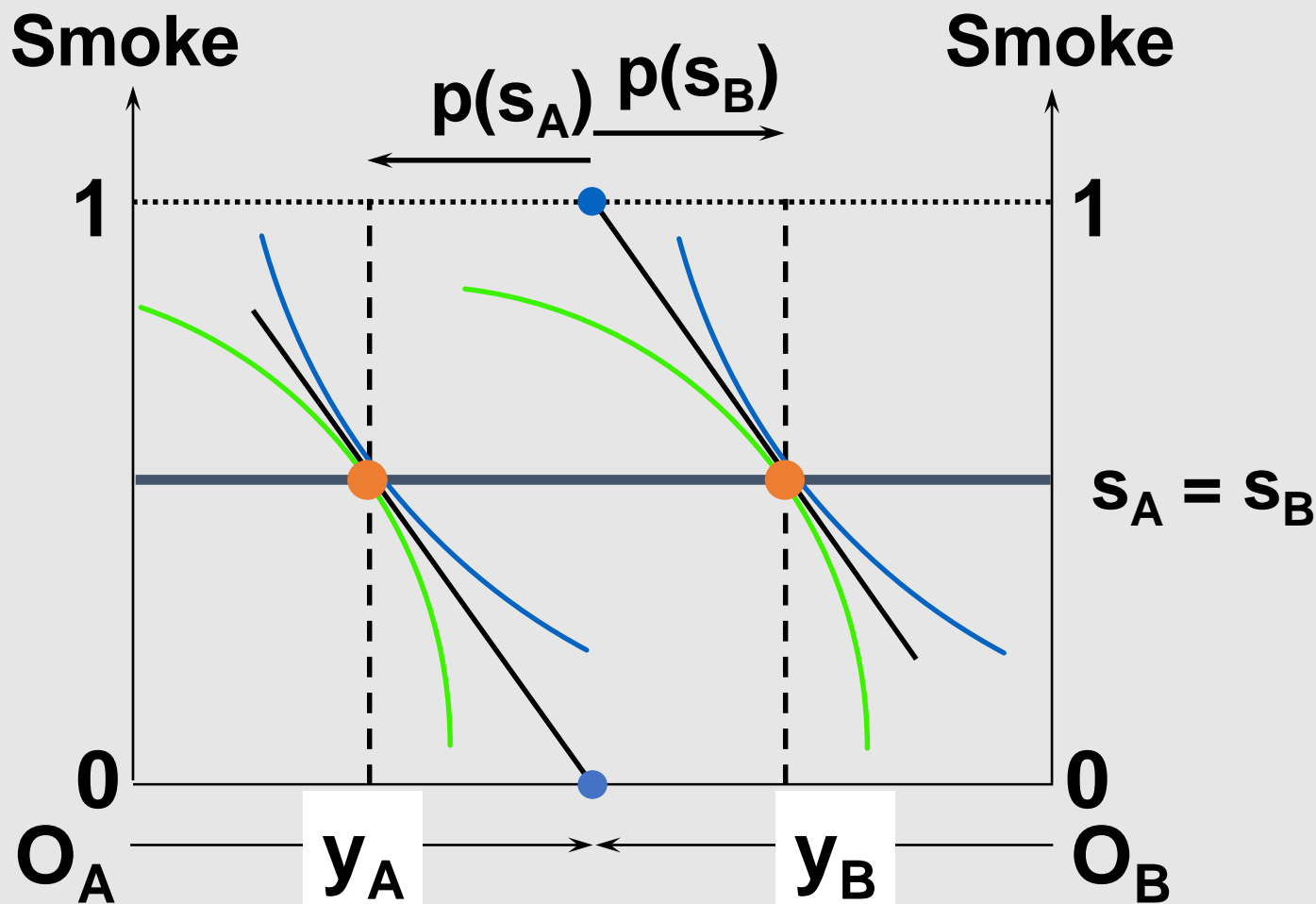


# Externalities and Property Rights



For both agents, the MRS is constant as money changes, for given smoke intensity.

# Externalities and Property Rights



So, for both agents, preferences must be quasilinear in money;  
 $U(m, s) = m + f(s)$ .



# Coase's Theorem

- Coase Theorem (Part I): When there are well-defined property rights and costless bargaining, then negotiations between the party creating the externality and the party affected by the externality can bring about the socially optimal market quantity.
- Coase Theorem (Part II): If preferences are quasilinear, the efficient quantity for a good producing an externality does not depend on which party is assigned the property rights, as long as someone is assigned those rights.

# Coase's Theorem

## Attention

- Quasilinear preferences are a strong assumption: demands for the good causing the externality doesn't depend on the distribution of income
- Implication: the amount of the externality—the amount of smoke—will be independent of the distribution of wealth.

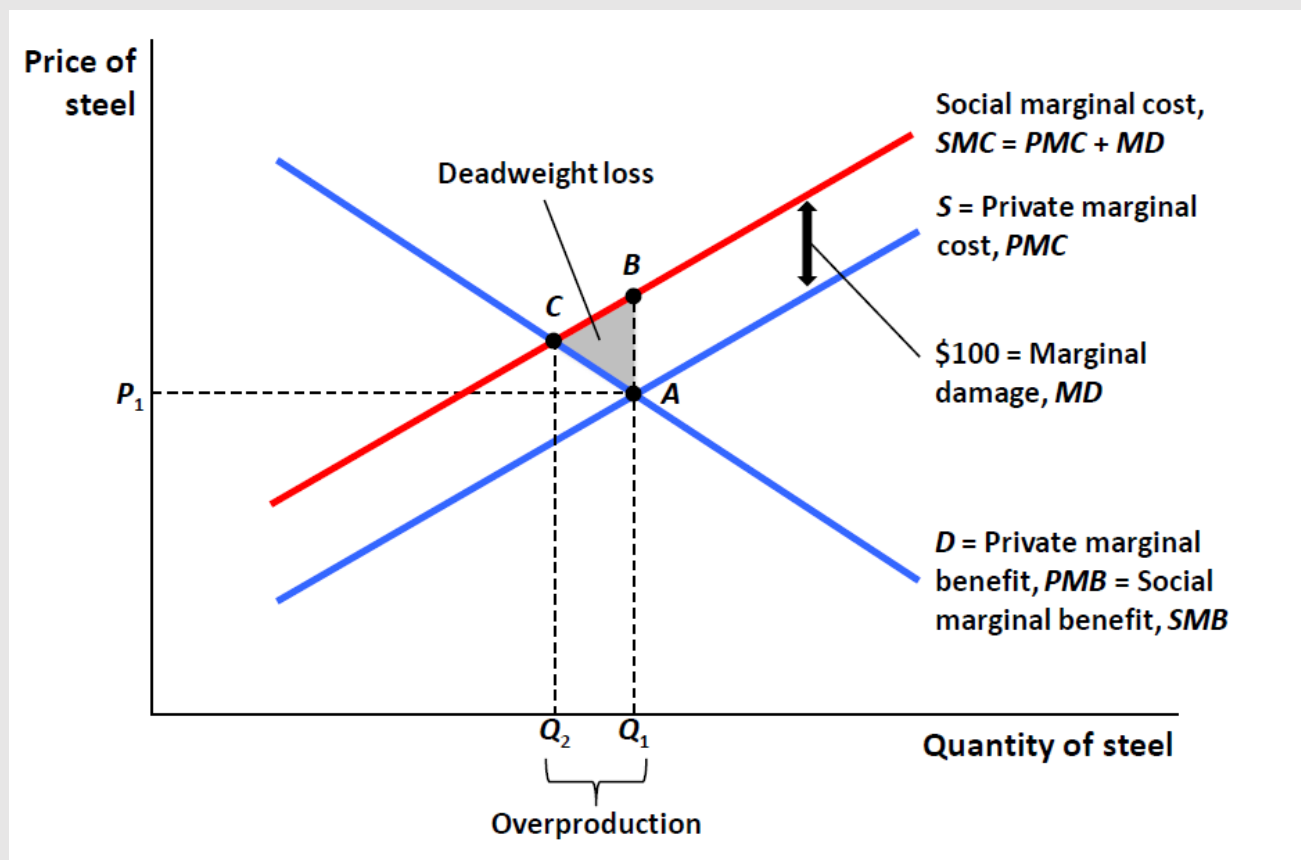
### 3. Plotting externalities

#### Negative production externality

- Private marginal cost (PMC): The direct cost to producers of producing an additional unit of a good. Before it was just MC.
- Marginal Damage (MD): Any additional costs associated with the production of the good that are imposed on others but that producers do not pay
- Social marginal cost ( $SMC = PMC + MD$ ): The private marginal cost to producers plus marginal damage
- Example: steel plant pollutes a river but plant does not face any pollution regulation (and hence ignores pollution when deciding how much to produce)

# Plotting externalities

## Negative production externality



At social optimum,  $SMC = SMB = PMB$

In free market,  $PMC = PMB$

# Plotting externalities

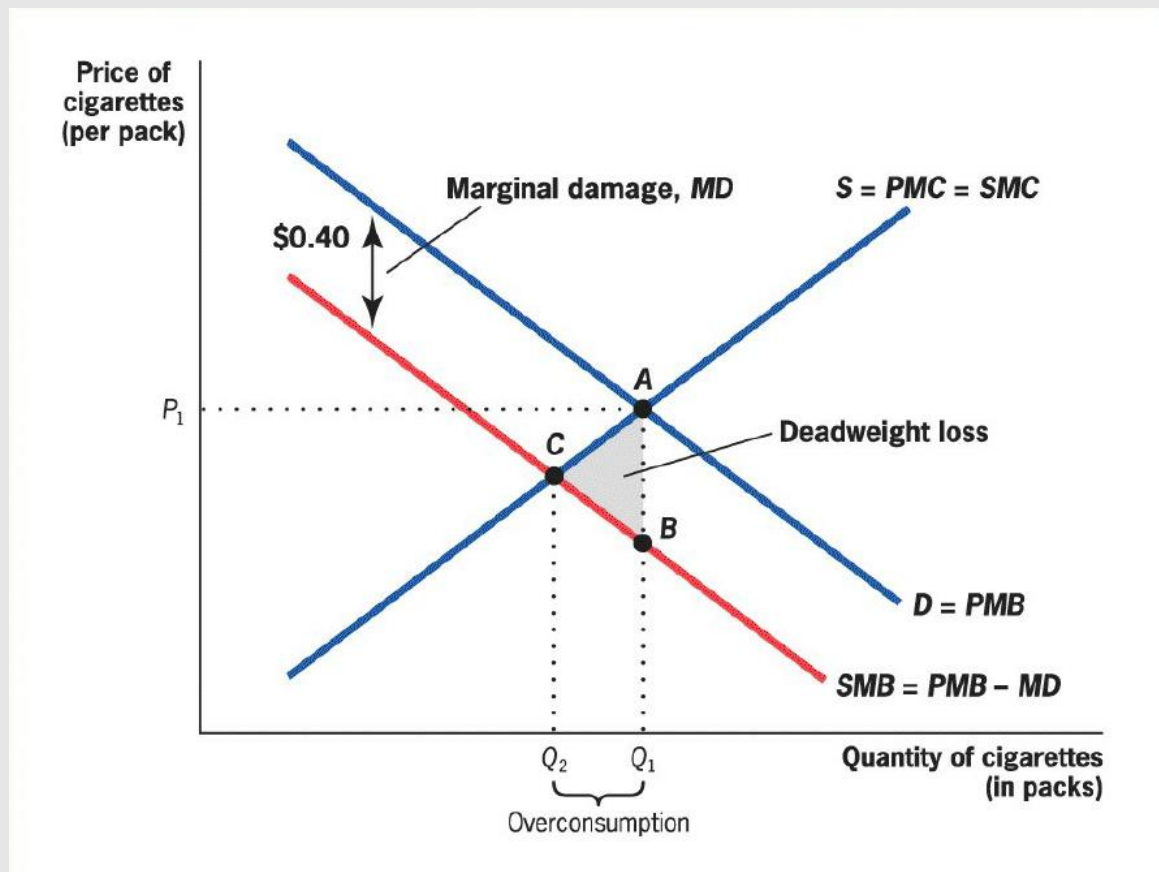
## **Negative consumption externality:**

When an individual's consumption reduces the well-being of others who are not compensated by the individual.

- Private marginal benefit (PMB): The direct benefit to consumers of consuming an additional unit of a good by the consumer.
- Social marginal benefit (SMB): The private marginal benefit to consumers plus any costs associated with the consumption of the good that are imposed on others
- Example: Smoking a cigarette

# Plotting externalities

## Negative consumption externality



At social optimum,  $PMC = SMC = SMB$

In free market,  $PMC = PMB$

# Plotting externalities

## **Positive production externality:**

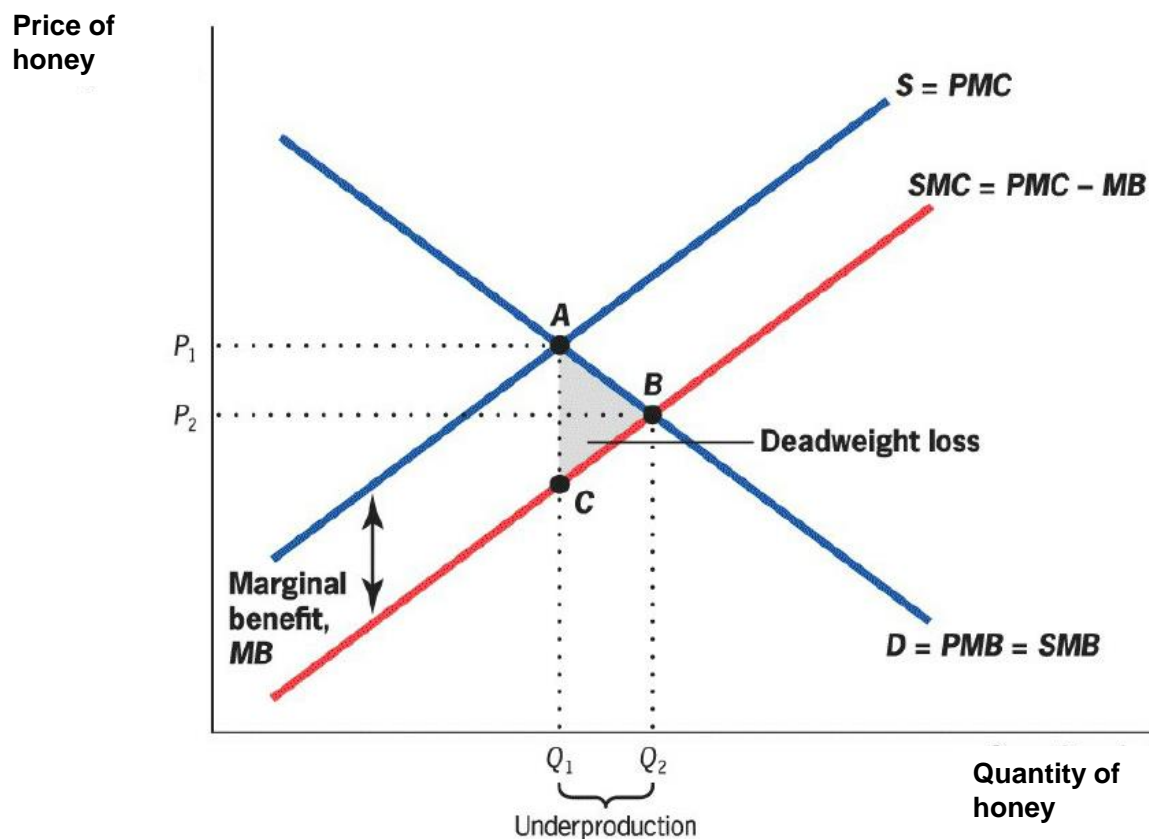
- When a firm's production increases the well-being of others, but the firm is not compensated by those others.
- Example: Beehives of honey producers have a positive impact on pollination and agricultural output

## **Positive consumption externality:**

- When an individual's consumption increases the well-being of others, but the individual is not compensated by those others.
- Example: Beautiful private garden that passers-by enjoy seeing

# Plotting externalities

## Positive production externality



At social optimum,  $SMC = SMB = PMB$

In free market,  $PMC = PMB$



# Plotting externalities

With a free market, quantity and price are such that  $PMB = PMC$

Social optimum is such that  $SMB = SMC$

- Private market leads to an inefficient outcome (1st welfare theorem does not work)
- Negative production externalities → over production (SMC curve above PMC curve)
- Positive production externalities → under production (SMC curve below PMC curve)
- Negative consumption externalities → over consumption (SMB curve lies below PMB curve)
- Positive consumption externalities: → under consumption (SMB curve lies above PMB curve)

## 4. Numerical example of production externalities

- A steel mill produces jointly steel and pollution.
- The pollution adversely affects a nearby fishery.
- Both firms are price-takers.
- $p_s$  is the market price of steel.
- $p_f$  is the market price of fish.

# Numerical example of production externalities

→  $c_s(s, x)$  is the steel firm's cost of producing  $s$  units of steel jointly with  $x$  units of pollution.

→ With  $\frac{\Delta c_s}{\Delta x} \leq 0$ , more pollution decreases the production cost.

→ If the steel firm does not face any of the external costs of its pollution production then its profit function is

$$\Pi_s(s, x) = p_s s - c_s(s, x)$$

and the firm's problem is

$$\max_{s, x} \Pi_s(s, x) = p_s s - c_s(s, x)$$

# Numerical example of production externalities

→ The first-order profit-maximization conditions are

$$p_s = \frac{\partial c_s(s,x)}{\partial s}$$

Meaning: firm should produce  $s$  such as price equals marginal cost of production

$$\text{and } 0 = \frac{\partial c_s(s,x)}{\partial x}$$

$\frac{\partial c_s(s,x)}{\partial x}$  is the rate at which production cost decreases when pollution increases

$-\frac{\partial c_s(s,x)}{\partial x}$  is the marginal cost of pollution reduction for the firm

# Numerical example of production externalities

- What is the marginal benefit to the steel firm from reducing pollution?
- Zero, since the firm does not face its external cost.
- Hence the steel firm chooses the pollution level for which

$$0 = \frac{\partial c_s(s,x)}{\partial x}$$

# Numerical example of production externalities

Suppose  $c_s(s, x) = s^2 + (x - 4)^2$  and  $p_s = 12$ .

→ Then  $\Pi_s(s, x) = 12s - s^2 - (x - 4)^2$

→ and the first-order profit-maximization conditions are

$$12 = 2s \text{ and } 0 = -2(x - 4)$$

# Numerical example of production externalities

→  $p_s = 12 = 2s$  determines the optimal quantity of steal  $s^* = 6$

→  $0 = -2(x - 4)$  determines the amount of pollution that will be produced  $x^* = 4$

→ The steel firm's maximum profit level is thus

$$\Pi_s(s^*, x^*) = 12s^* - s^{*2} - (x^* - 4)^2 =$$

$$12 * 6 - 6^2 - (4 - 4)^2 =$$

$$\mathbf{\Pi_s(s^*, x^*) = 36}$$

# Numerical example of production externalities

→ The cost to the fishery of catching  $f$  units of fish when the steel mill emits  $x$  units of pollution is  $c_F(f, x)$ .

→ With  $\frac{\Delta c_f}{\Delta x} \geq 0$ , the steel firm inflicts a negative externality on the fishery since the more pollution the more costly it is to produce  $f$ .

The fishery's profit function is

$$\Pi_F(f, x) = p_f f - c_F(f, x)$$

and the fishery's problem is

$$\max_f \Pi_F(f, x) = p_f f - c_F(f, x)$$

Note that the fishery CANNOT choose the level of  $x$



# Numerical example of production externalities

→ The first-order profit-maximization condition is

$$p_f = \frac{\partial c_F(f, x)}{\partial f}$$

→ Higher pollution raises the fishery's marginal production cost and lowers both its output level and its profit. This is the external cost of the pollution.

# Numerical example of production externalities

Suppose  $c_F(f,x) = f^2 + xf$  and  $p_S = 10$ .

The external cost inflicted on the fishery by the steel firm is  $xf$ . Since the fishery has no control over  $x$  it must take the steel firm's choice of  $x$  as a given. The fishery's profit function is thus

→ Then  $\Pi_F(f, x) = 10f - f^2 - xf$

→ and the first-order profit-maximization condition is

$$10 = 2f + x$$

# Numerical example of production externalities

→ So, given a pollution level  $x$  inflicted upon it, the fishery's profit-maximizing output level is

$$f^* = 5 - \frac{x}{2}$$

→ Notice that the fishery produces less, and earns less profit, as the steel firm's pollution level increases.

→ If  $x^* = 4$  then  $f^* = 3$

→ The fishery's maximum profit level is thus

$$\begin{aligned}\Pi_F(f^*, x^*) &= 10f^* - f^{*2} - x^*f^* \\ 10 * 3 - 3^2 - 4 * 3 &= \end{aligned}$$

$$\Pi_F(f^*, x^*) = 9 \quad \text{the externality cost is 12€}$$

# Numerical example of production externalities

- Are these choices by the two firms efficient?
- When the steel firm ignores the external costs of its choices, the sum of the two firm's profits is  $\$36 + \$9 = \$45$ .
- Is \$45 the largest possible total profit that can be achieved?
- How might internalization be caused so that efficiency can be achieved?

## 5. The Coarse solution (Solution 1)

Assign property rights!

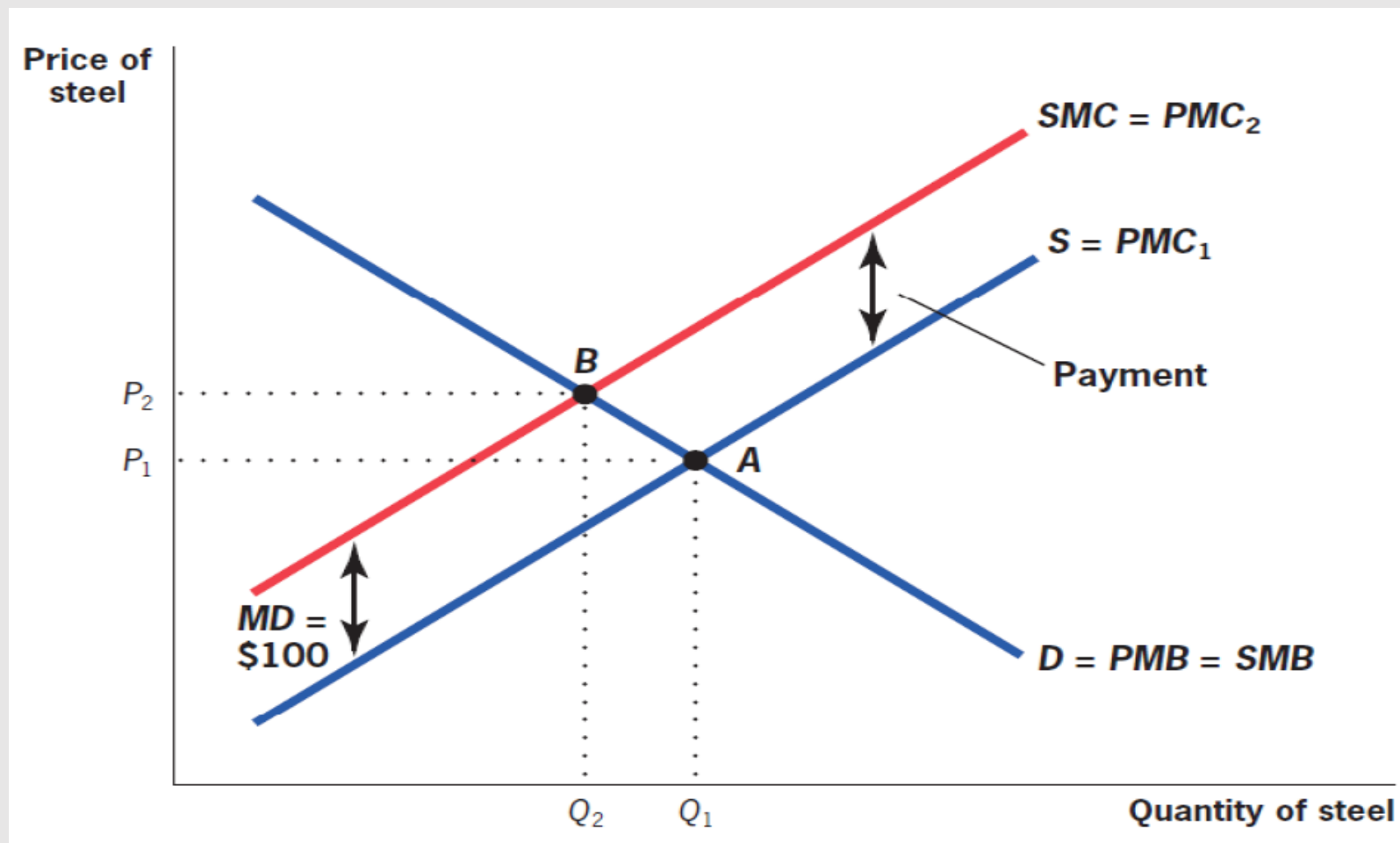
EXAMPLE: Firms producing steel pollute a river where fishers get their fish. If the firms ignore fishers, there is too much pollution

1. Fishers own river: If river is owned by fishers, then fishers can charge firms for polluting the river. They will charge firms the marginal damage (MD) per unit of pollution. (Shifts up the PMC of the firm to the level of SMC).
  - Why price pollution at MD? If price is above MD, fishers would want to sell an extra unit of pollution and get hit by pollution damage MD, so price must fall. MD is the equilibrium efficient price in the newly created pollution market.

## The Coarse solution

2. Firms own river: If river is owned by firms, then fishers are willing to pay firms MD for each unit of steel it does NOT produce. This increases the firms' cost of producing each unit of steel. Their cost shifts from PMC to  $SMC = PMC + MD$  for each quantity of steel produced.
- Final level of pollution will be the same in 1) and 2)

# The Coarse solution



# Coase and Production Externalities

- Suppose the fishery owns the water.
- Then it can sell pollution rights, in a competitive market, at  $\$p_x$  each.
- The fishery's profit function becomes

$$\Pi_F(f, x) = p_f f - f^2 - x f + p_x * x$$

- Given  $p_f$  and  $p_x$ , how many fish and how many rights does the fishery wish to produce? (Notice that  $x$  is now a choice variable for the fishery.)



# Coase and Production Externalities

→ The profit-maximum conditions are

$$\frac{\partial \Pi_F}{\partial f} = p_f - 2f - x = 0$$

Pollution right supply

$$x_S^* = p_f - 2f^*$$

$$\frac{\partial \Pi_F}{\partial x} = -f - p_x = 0$$

Fish supply

$$f^* = p_x$$

$$\text{Hence } x_S^* = p_f - 2p_x$$

# Coase and Production Externalities

→ The steel firm must buy one right for every unit of pollution it emits so its profit function becomes

$$\Pi_S(s, x) = p_s s - s^2 - (x - 4)^2 - p_x x$$

→ Given  $p_f$  and  $p_x$ , how much steel does the steel firm want to produce and how many rights does it wish to buy?

→ The profit-maximum conditions are

$$\frac{\partial \Pi_S}{\partial s} = p_s - 2s = 0$$

Steel supply

$$s^* = \frac{p_s}{2}$$

$$\frac{\partial \Pi_S}{\partial x} = -2(x - 4) - p_x = 0$$

Pollution rights demand

$$x_D^* = \frac{-p_x}{2} + 4$$

# Coase and Production Externalities

→ In a competitive market for pollution rights the price  $p_x$  must adjust to clear the market so, at equilibrium:

→ Supply  $x_S^* = p_f - 2p_x$  must equal demand  $x_D^* = \frac{-p_x}{2} + 4$

→ The market-clearing price for pollution rights is thus

$$p_x = \frac{2p_f - 8}{3}$$

→ And the equilibrium quantity of rights traded is

$$x_D^* = x_S^* = \frac{16 - p_f}{3}$$

# Coase and Production Externalities

→ Hence, in equilibrium:

$$s^* = \frac{p_s}{2} \quad f^* = p_x \quad x_D^* = x_S^* = \frac{16 - p_f}{3} \quad \text{and} \quad p_x = \frac{2p_f - 8}{3}$$

→ So if  $p_s = 12$  and  $p_f = 10$  then

$$s^* = 6 \quad f^* = 4 \quad x_D^* = x_S^* = 2 \quad \text{and} \quad p_x = 4$$

This is the efficient outcome (Same as with the merge!)

# Coase and Production Externalities

- Q: Would it matter if the property right to the water had instead been assigned to the steel firm?
- A: No. Profit is linear, and therefore quasi-linear, in money so Coase's Theorem states that the same efficient allocation is achieved whichever of the firms was assigned the property right. (And the asset owner gets richer.)

# Problems with the Coarse solution

In practice, the Coase theorem is unlikely to solve many of the types of externalities that cause market failures.

## 1. The assignment problem:

1. Can you assign blame to one single entity (e.g., a long river with many polluting firms);
2. can you assign the exact damage (how is MD really measured?);
3. who gets the property rights?

→ In cases where externalities are caused by and affect many agents (e.g. global warming), assigning property rights is difficult

→ Coasian solutions are likely to be more effective for small, localized externalities than for larger, more global externalities involving large number of people and firms

# Problems with the Coarse solution

2. **The holdout problem:** when the property rights are held by more than one party: the shared property rights give each owner power over all others (because joint owners have to all agree to the Coasian solution).
    - Imagine the swimmers who own property rights for a clean river. After 99 swimmers have agreed to receive their compensation from the firm, the 100th swimmer has an incentive to ask for more (to hold out).
    - Anticipating this, all swimmers should try to hold out.
- As with the assignment problem, the holdout problem would be amplified with an externality involving many parties.

# Problems with the Coarse solution

- 3. **The Free Rider Problem:** When an investment has a personal cost but a common benefit, individuals will underinvest.
  - In the swimmers' example, if property rights are assigned to the firm, the 100th swimmer has no incentive to pay for their share of pollution reduction, as the pollution is almost at socially optimal level and the damage caused by the last unit of pollution that they have to pay for is shared among all swimmers.



# Problems with the Coarse solution

- 4. **Transaction Costs and Negotiating Problems:** The Coasian approach ignores the fundamental problem that it is hard to negotiate when there are large numbers of individuals on one or both sides of the negotiation.
  - This problem is amplified for an externality such as global warming, where the potentially divergent interests of billions of parties on one side must be somehow aggregated for a negotiation...

## 6. Merger and Internalization (Solution 2)

- How else might internalization be caused so that efficiency can be achieved?
- Suppose the two firms merge to become one. What is the highest profit this new firm can achieve?
- What choices of  $s$ ,  $f$  and  $x$  maximize the new firm's profit?

$$\Pi^m(s, f, x) = 12s + 10f - s^2 - (x - 4)^2 - f^2 - xf$$

- The first-order profit-maximization conditions are

$$\frac{\partial \Pi^m}{\partial s} = 12 - 2s = 0 \qquad s^* = 6$$

$$\frac{\partial \Pi^m}{\partial f} = 10 - 2f - x = 0 \qquad f = 5 - \frac{x}{2} \qquad f^* = 4$$

$$\frac{\partial \Pi^m}{\partial x} = -2(x - 4) - f = 0 \qquad f = -2(x - 4) \qquad x^* = 2$$

# Merger and Internalization

→ And the merged firm's maximum profit level is

$$\begin{aligned}\Pi^m(s^*, f^*, x^*) &= 12s^* + 10f^* - s^{*2} - (x^* - 4)^2 - f^{*2} - x^*f^* \\ &= 12 * 6 + 10 * 4 - 6^2 - (2 - 4)^2 - 4^2 - 2 * 4 \\ \Pi^m(s^*, f^*, x^*) &= 48\end{aligned}$$

→ This exceeds \$45, the sum of the non-merged firms.

# Merger and Internalization

- Merger has improved efficiency.
- On its own, the steel firm produced  $x^* = 4$  units of pollution.
- Within the merged firm, pollution production is only  $x^m = 2$  units.
- So merger has caused both an improvement in efficiency and less pollution production. Why?

# Merger and Internalization

- Compare the marginal cost of pollution between non-merged and merged

$$MC_s(x) = 2(x - 4) \quad \text{vs} \quad MC^m(x) = 2(x - 4) + f$$

- When it does not have to face the external costs of its pollution, the steel firm increases pollution until this marginal cost is zero
- The merged firm's marginal pollution cost is larger because it faces the full cost of its own pollution through increased costs of production in the fishery, so less pollution is produced by the merged firm.

# Merger and Internalization

→ But why is the merged firm's pollution level of  $x^m = 2$  efficient?

→ The external cost inflicted on the fishery is  $xf$ , so the marginal external pollution cost is

$$MC_x^E = f$$

→ The steel firm's cost of **reducing** pollution is

$$MC_s(x) = 2(x - 4)$$

Now the firm produces pollution until...

$$MC_x^E = MC_s(x) \Rightarrow f = 2(x - 4)$$

→ Merger therefore internalizes an externality and induces economic efficiency.

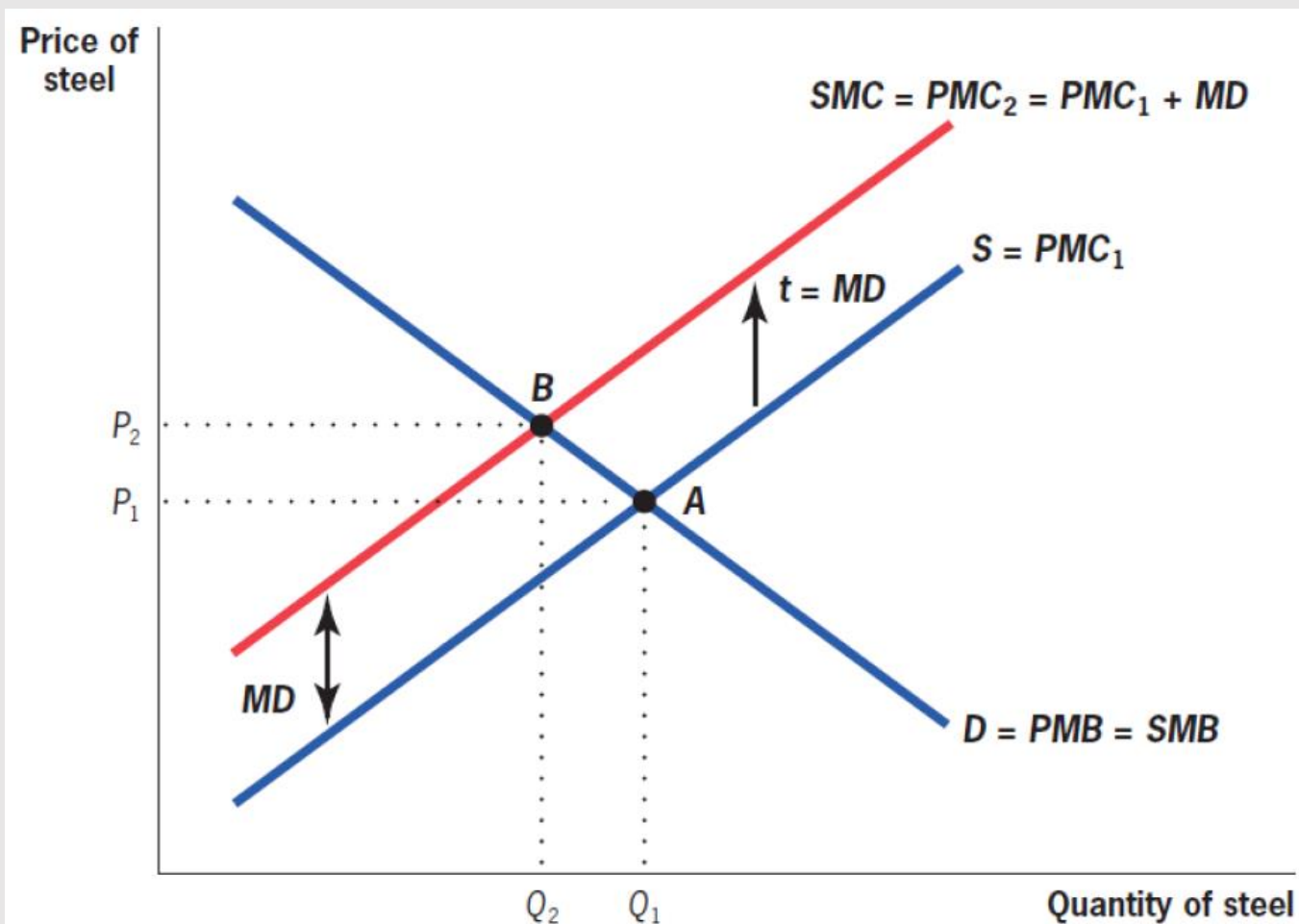
## 7. Public Sector Remedies (Solution 3)

→ Public policy makers employ two types of remedies to resolve negative externalities:

1. Quantity regulation: government limits use of externality producing chemicals. Example CFCs [chlorofluorocarbons] that deplete ozone layer
2. Corrective taxation: corrective tax or subsidy equal to marginal damage per unit. Example: Carbon tax to fight global warming due to CO<sub>2</sub> emissions

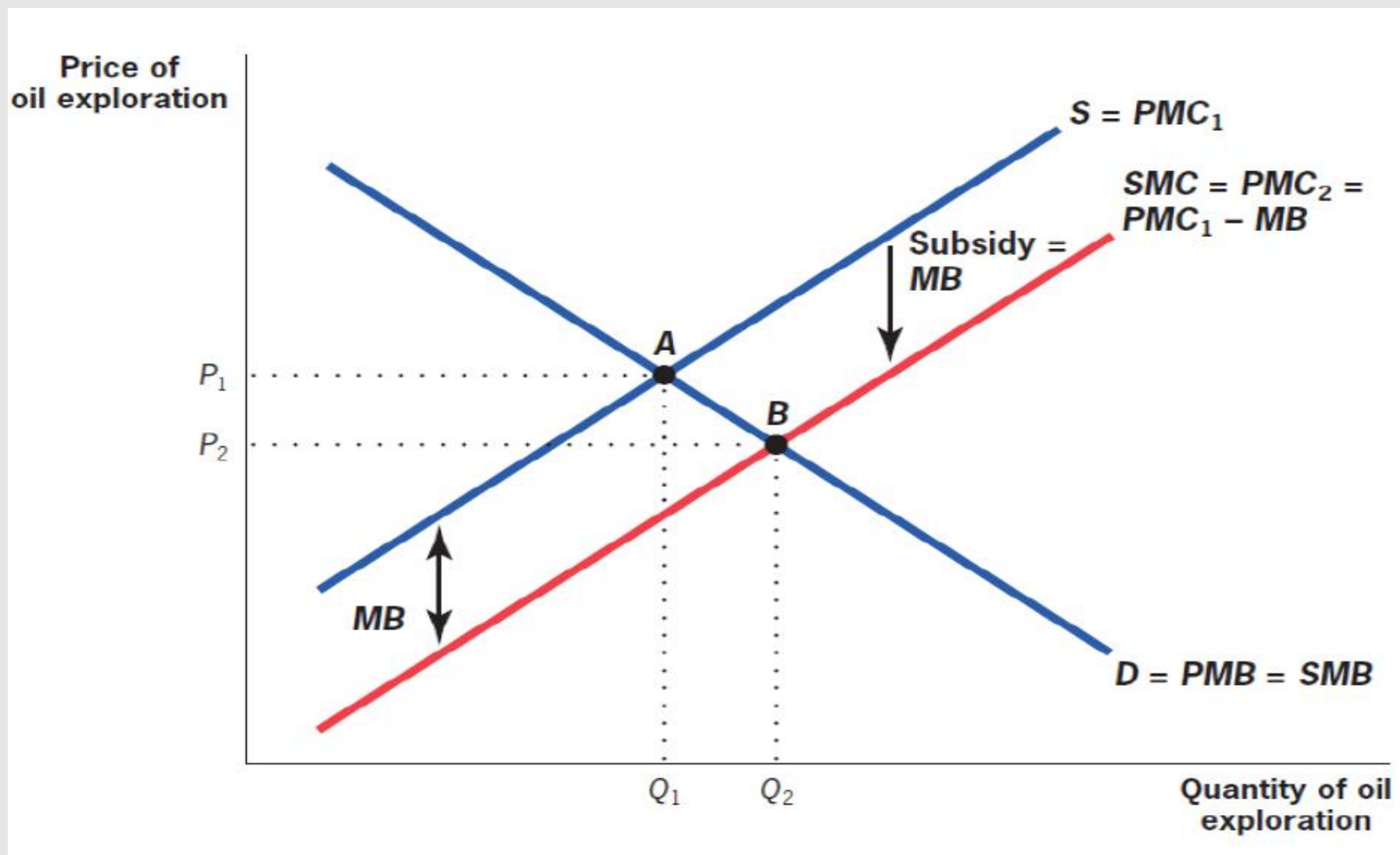
→ 1) and 2) can be combined with tradable emissions permits to firms that can then be traded (cap-and-trade for carbon emissions)

# Corrective Tax





# Corrective Subsidy

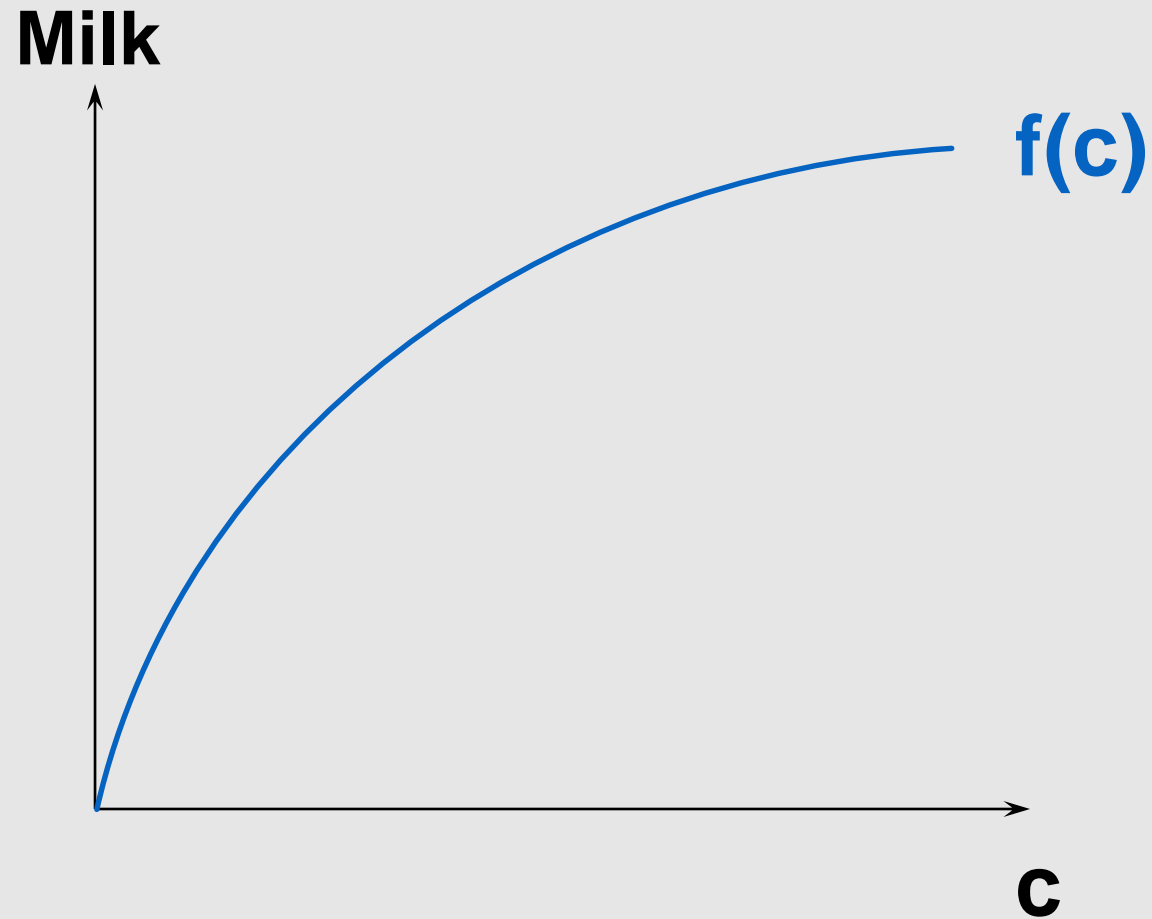


## 8. The Tragedy of the Commons

A very well-known inefficiency

- Consider a grazing area owned “in common” by all members of a village.
- Villagers graze cows on the common.
- When  $c$  cows are grazed, total milk production is  $m=f(c)$ , where  $f'>0$  and  $f''<0$ .
- How should the villagers graze their cows so as to maximize their overall income?

# The Tragedy of the Commons



# The Tragedy of the Commons

→ Make the price of milk \$1 and let the relative cost of grazing a cow be  $p_c$ . Then the profit function for the entire village is

$$\Pi(c) = f(c) - p_c c$$

and the village's problem is to

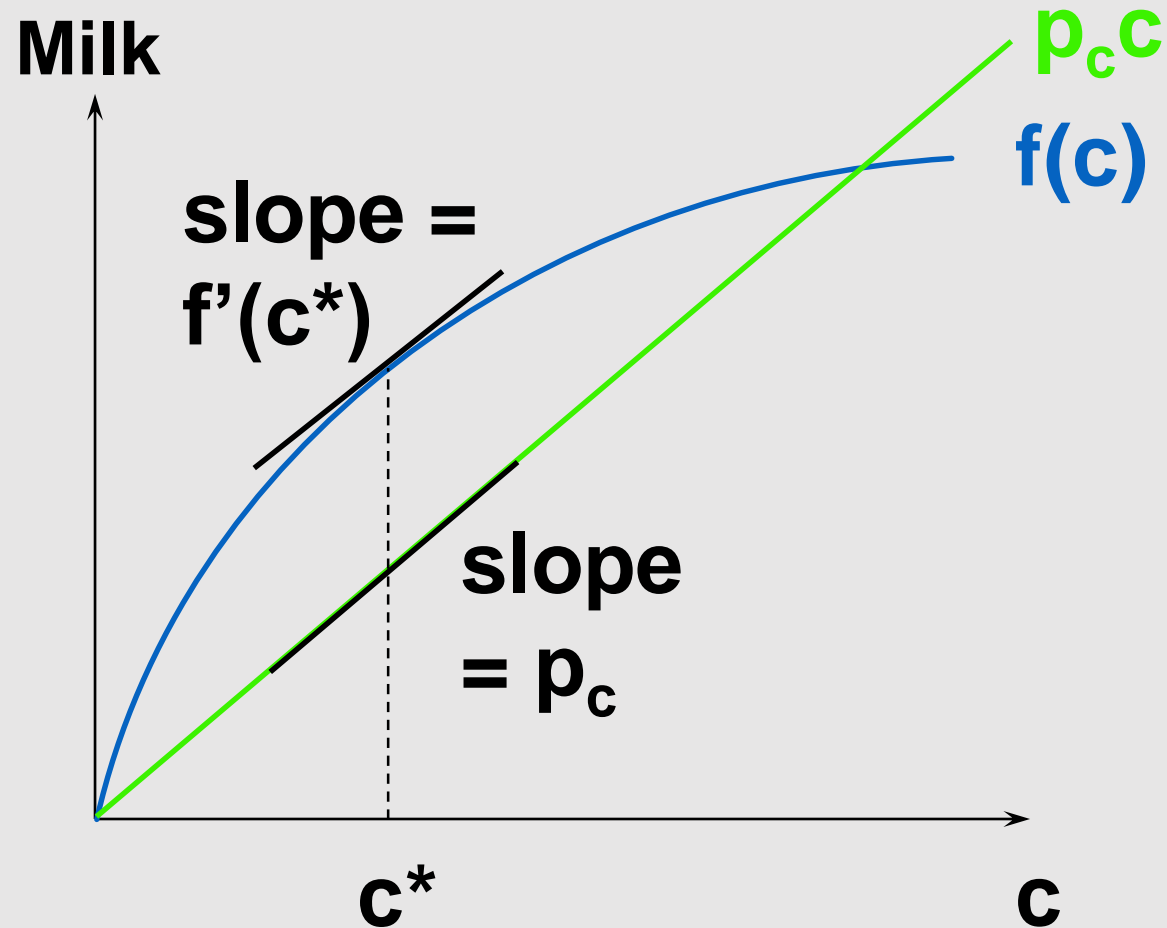
$$\max_{c \geq 0} \Pi(c) = f(c) - p_c c$$

→ The income-maximizing number of cows to graze,  $c^*$ , satisfies

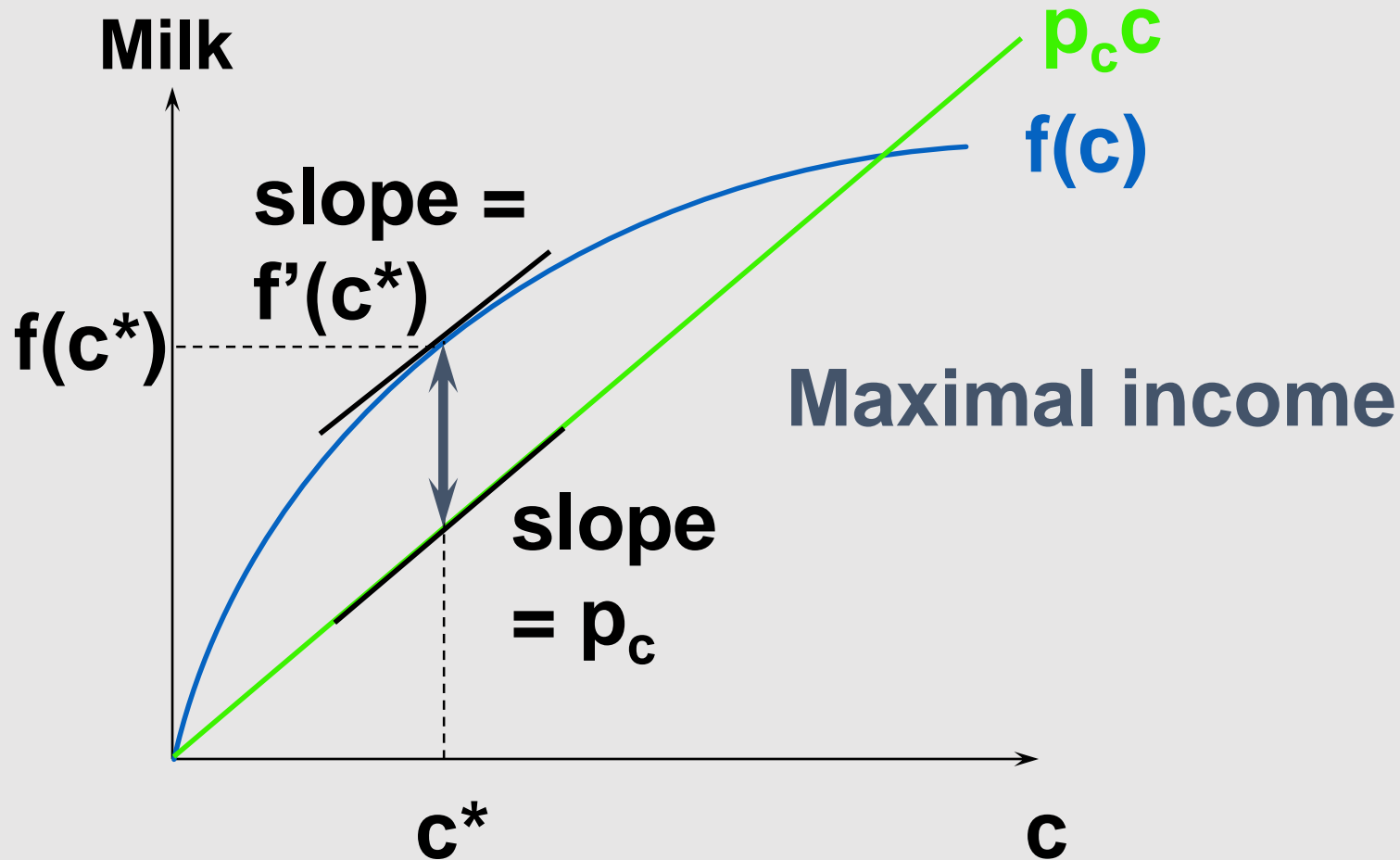
$$f'(c) = p_c$$

→ i.e. the **marginal** income gain from the last cow grazed must equal the **marginal** cost of grazing it.

# The Tragedy of the Commons



# The Tragedy of the Commons



# The Tragedy of the Commons

→ For  $c = c^*$ , the average gain per cow grazed is

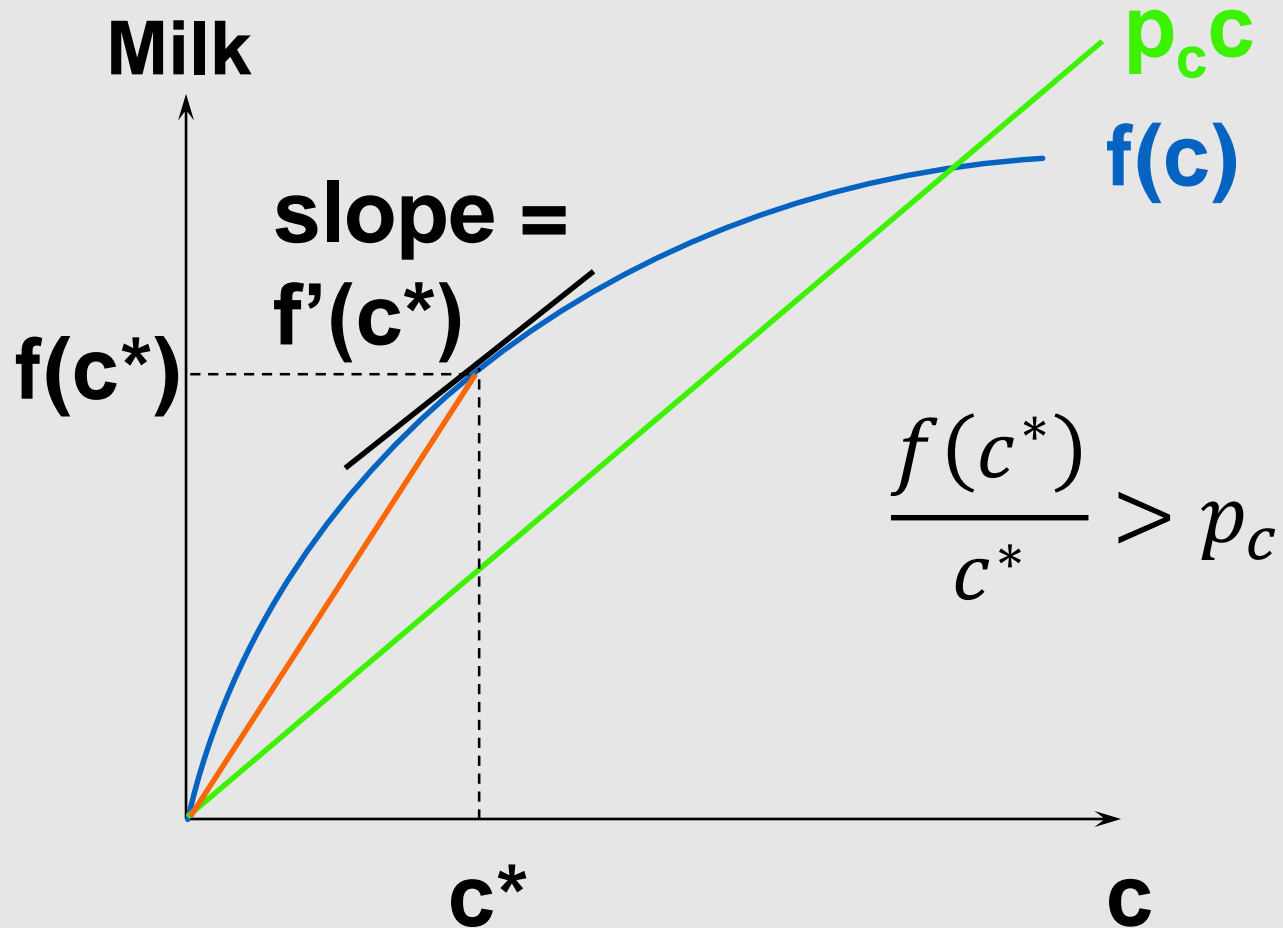
$$\frac{\Pi(c^*)}{c^*} = \frac{f(c^*) - p_c c^*}{c^*} = \frac{f(c^*)}{c^*} - p_c > 0$$

→ because  $f' > 0$  and  $f'' < 0$

→ So the economic profit from introducing one more cow is positive.

→ Since nobody owns the common, entry is not restricted.

# The Tragedy of the Commons



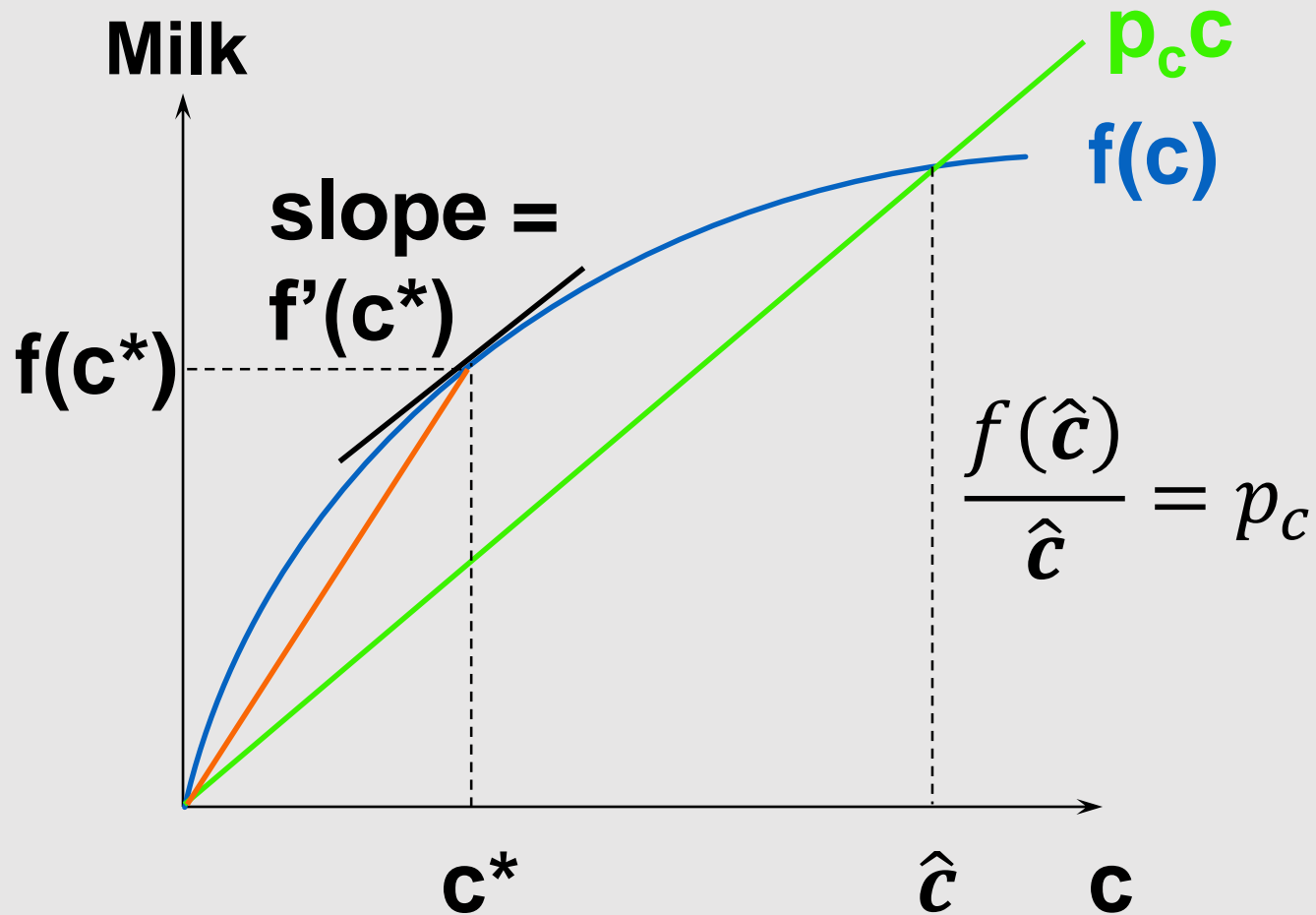


# The Tragedy of the Commons

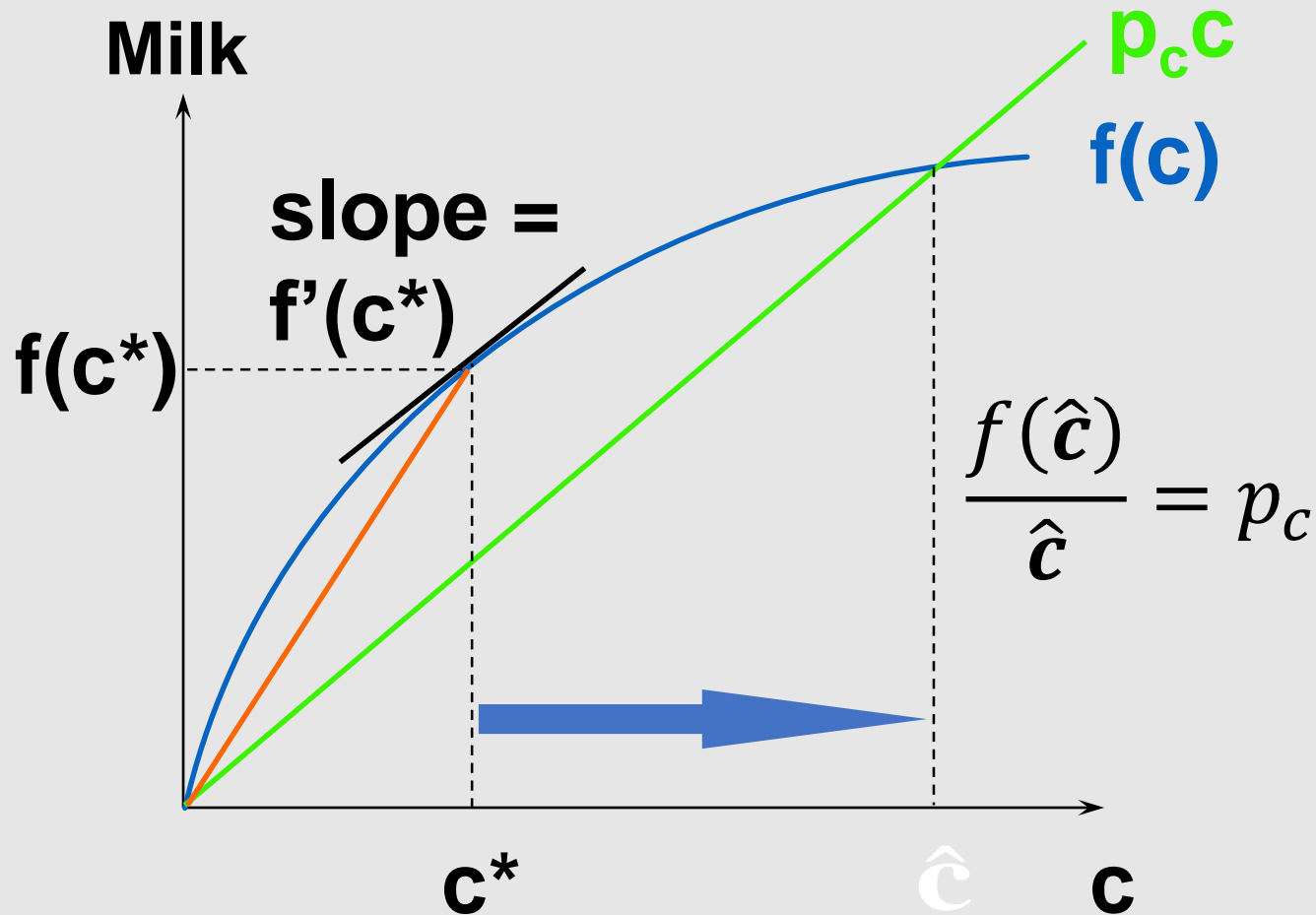
→ Entry continues until the economic profit of grazing another cow is zero; that is, until

$$\frac{\Pi(c)}{c} = \frac{f(c) - p_c c}{c} = \frac{f(c)}{c} - p_c = 0$$

# The Tragedy of the Commons



# The Tragedy of the Commons



**The commons are over-grazed, tragically.**

# The Tragedy of the Commons

- The reason for the tragedy is that when a villager adds one more cow his income rises (by  $f(c)/c - p_c$ ) but every other villager's income falls.
- The villager who adds the extra cow takes no account of the cost inflicted upon the rest of the village. → externality

# The Tragedy of the Commons

- Modern-day “tragedies of the commons” include
  - over-fishing the high seas
  - over-logging forests on public lands
  - over-intensive use of public parks; e.g. Yellowstone.
  - urban traffic congestion.