

Chapter 2: Exchange with production

Ch 32 in H. Varian 8th Ed.

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Inspired by [Michael D. Robinson](#), Mount Holyoke College

Exchange Economies (revisited)

So far...

- No production, only endowments, so no description of how resources are converted to consumables.
- General equilibrium: all markets clear simultaneously.
- 1st and 2nd Fundamental Theorems of Welfare Economics.

Now

- Add input markets, output markets, describe firms' technologies, the distributions of firms' outputs and profits ... **That's not easy!**

Outline

1. Robinson Crusoe's Economy
2. Robinson Crusoe as a Firm
3. Fundamental Theorems of Welfare Economics
4. Production Possibilities
5. Coordination of Production & Consumption

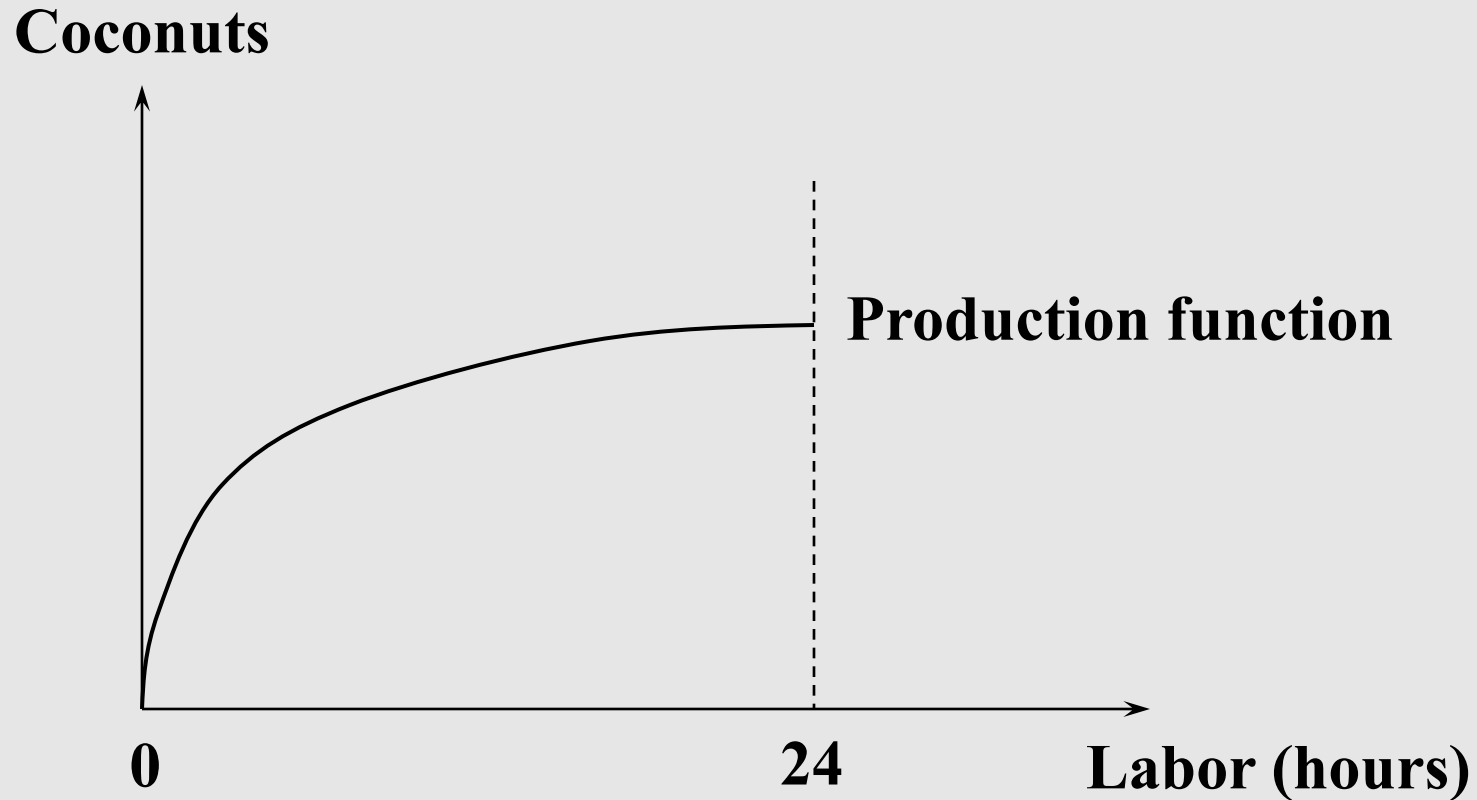
1. Robinson Crusoe's Economy

- One agent, RC.
- Endowed with a fixed quantity of one resource → 24 hours.
- Use time for labor (production) or leisure (consumption).
- Labor time = L . Leisure time = $24 - L$.
- What will RC choose?

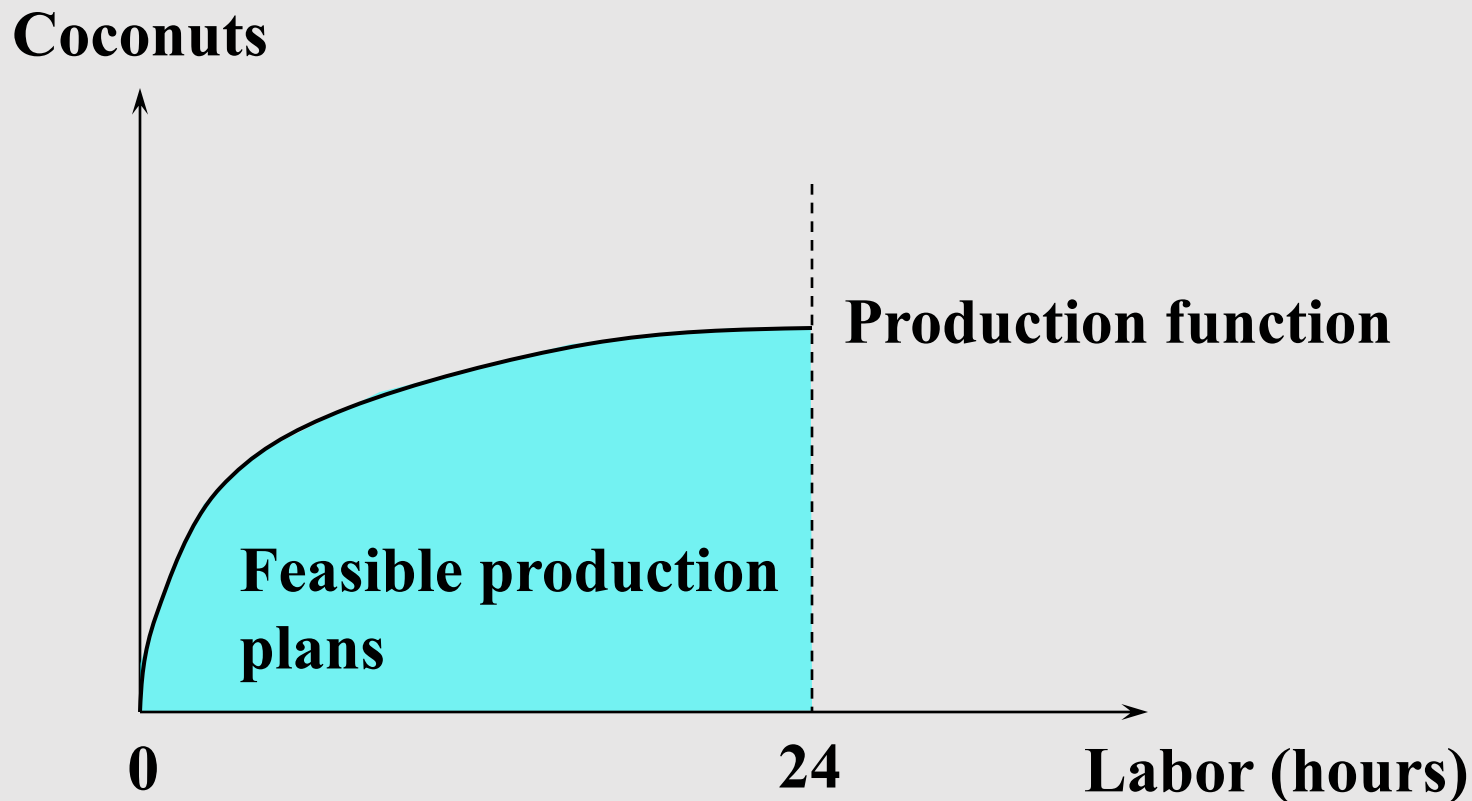
Robinson Crusoe's Technology

- Technology: Labor produces output (coconuts) according to a concave production function.

Robinson Crusoe's Technology



Robinson Crusoe's Technology

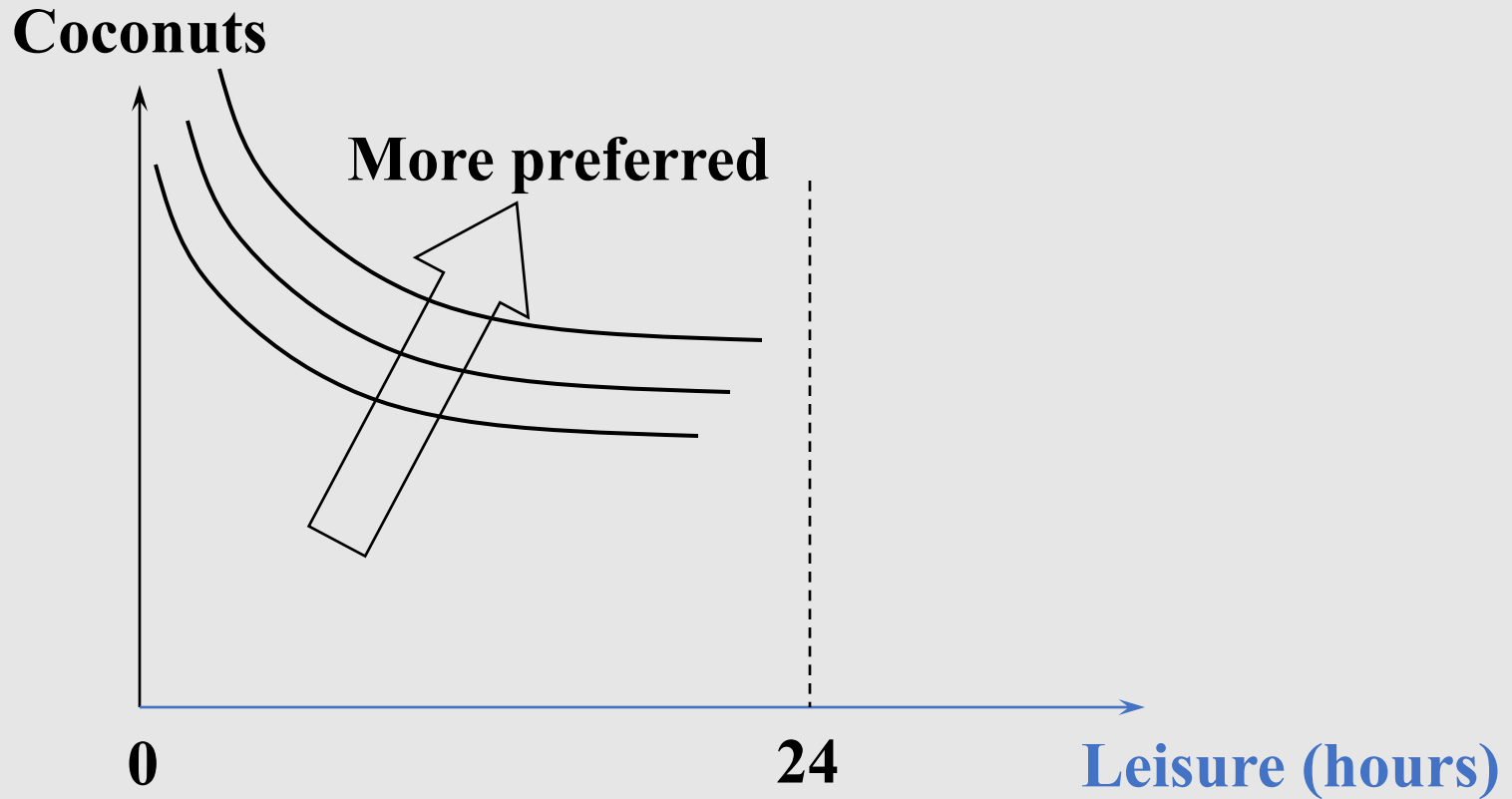


Robinson Crusoe's Preferences

→ RC's preferences:

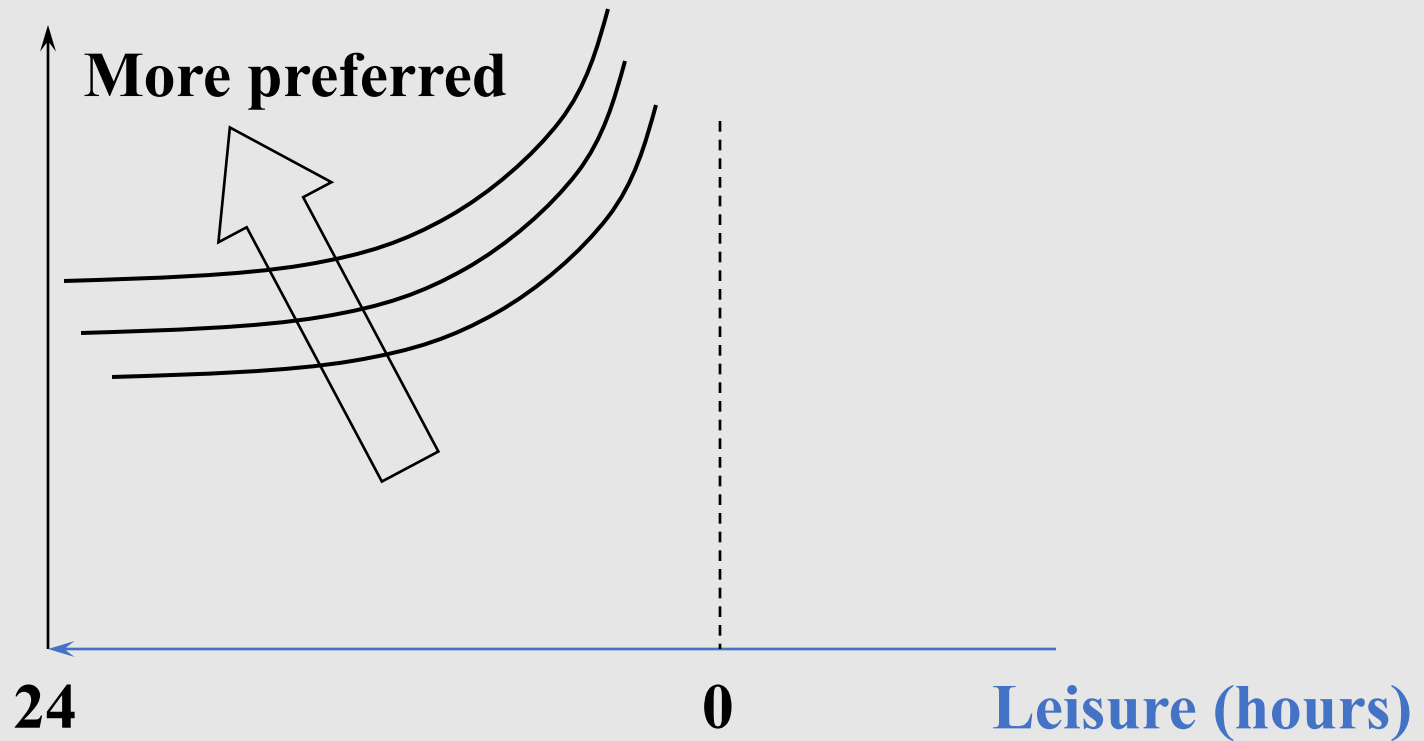
- coconut is a good
- leisure is a good

Robinson Crusoe's Preferences

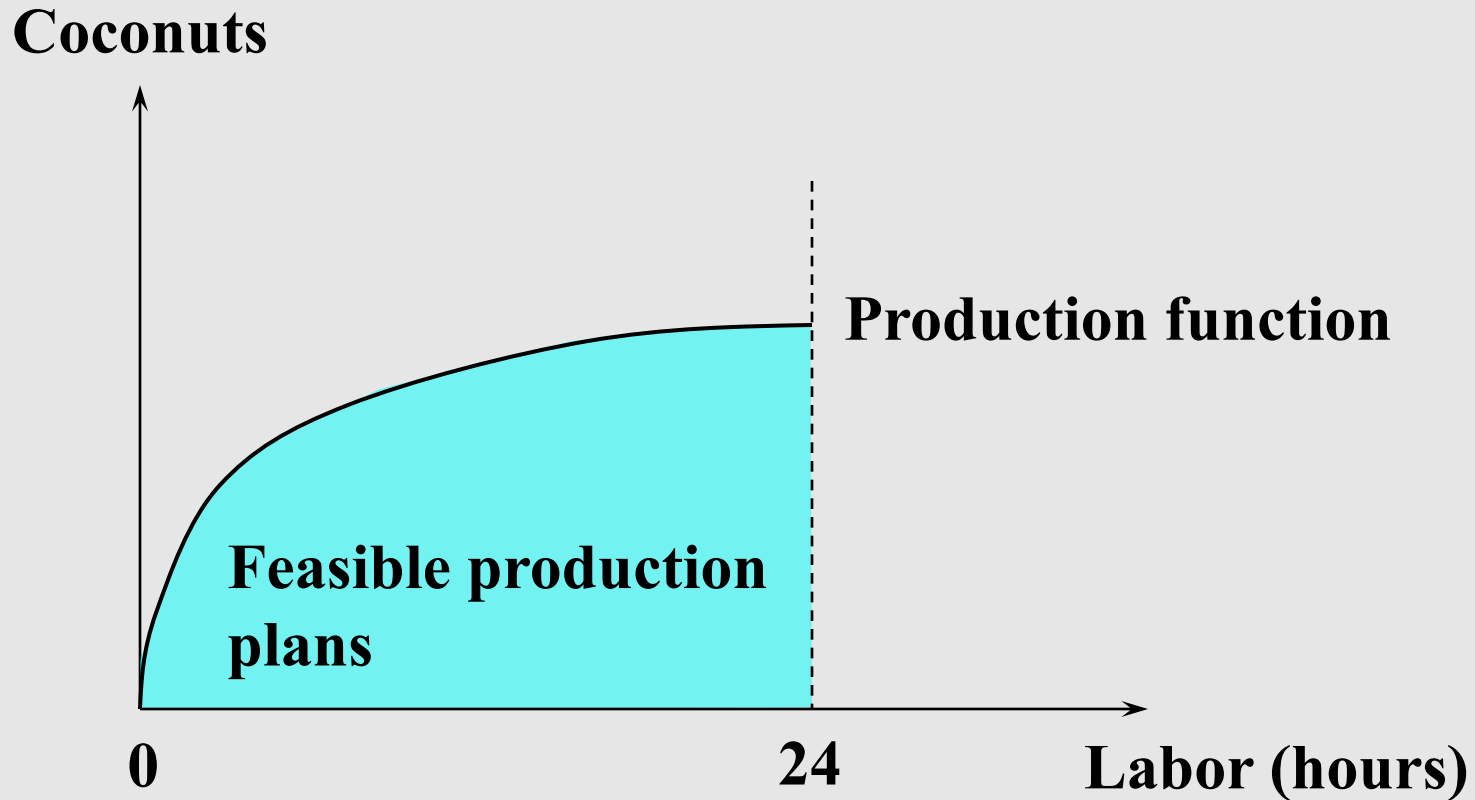


Robinson Crusoe's Preferences

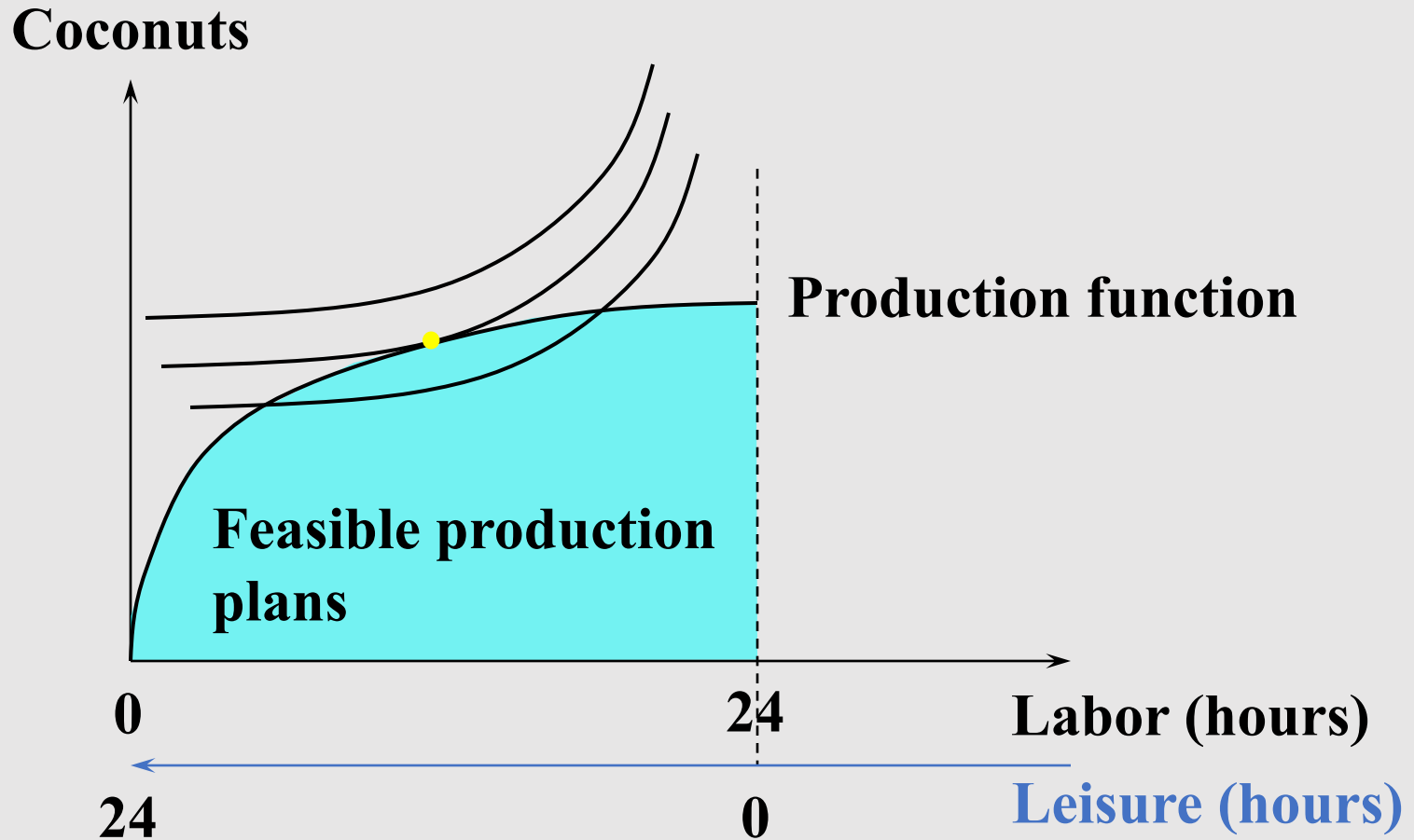
Coconuts



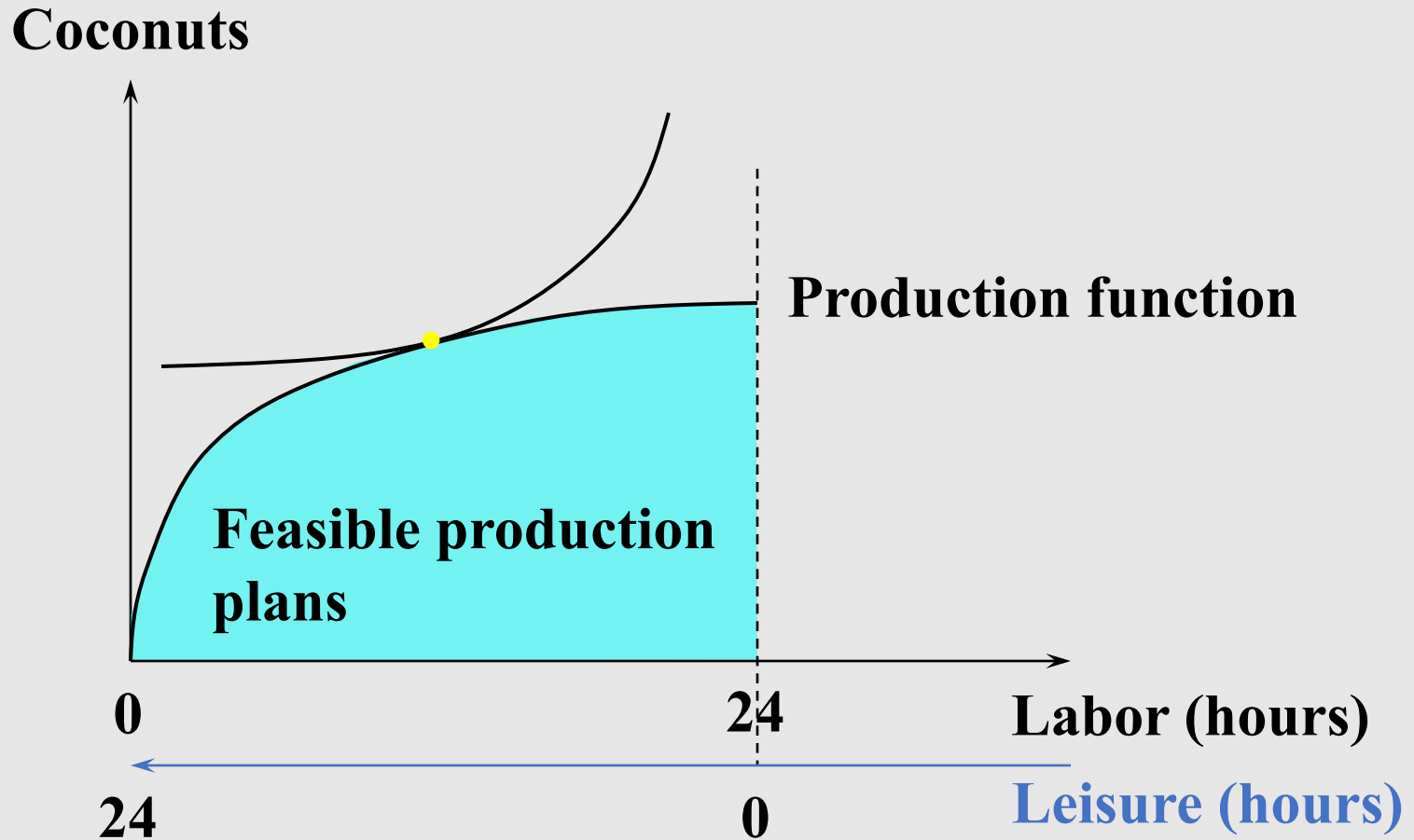
Robinson Crusoe's Choice



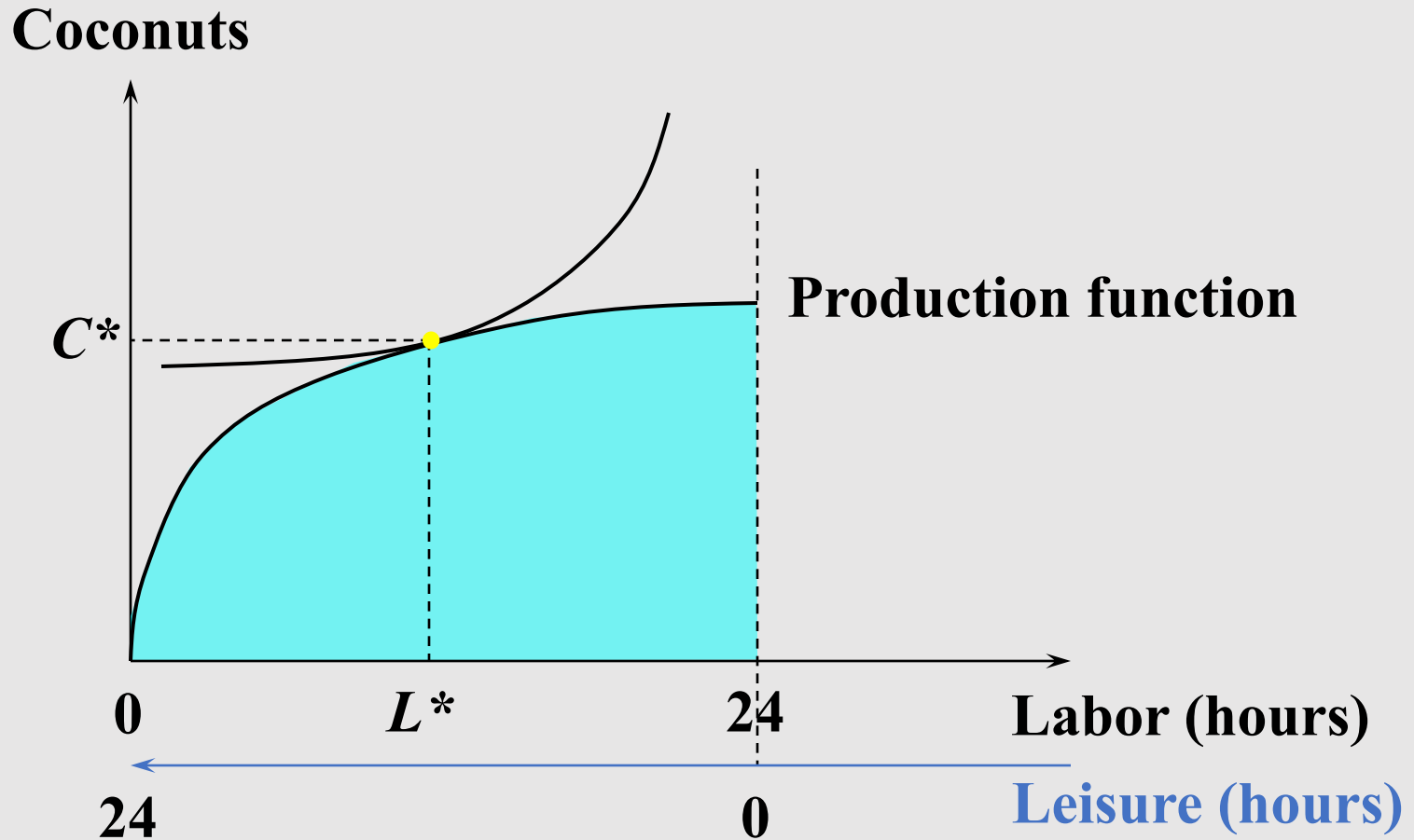
Robinson Crusoe's Choice



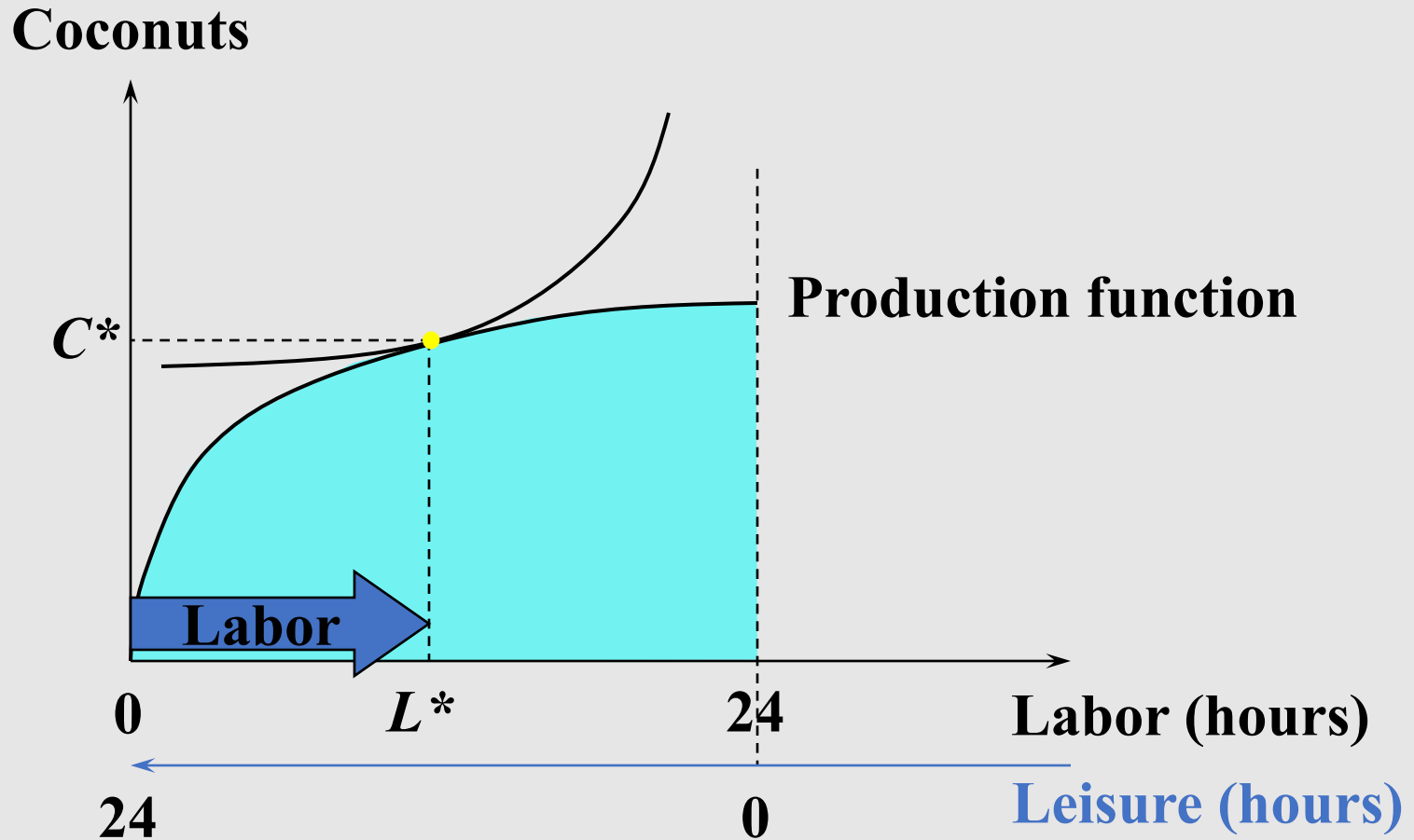
Robinson Crusoe's Choice



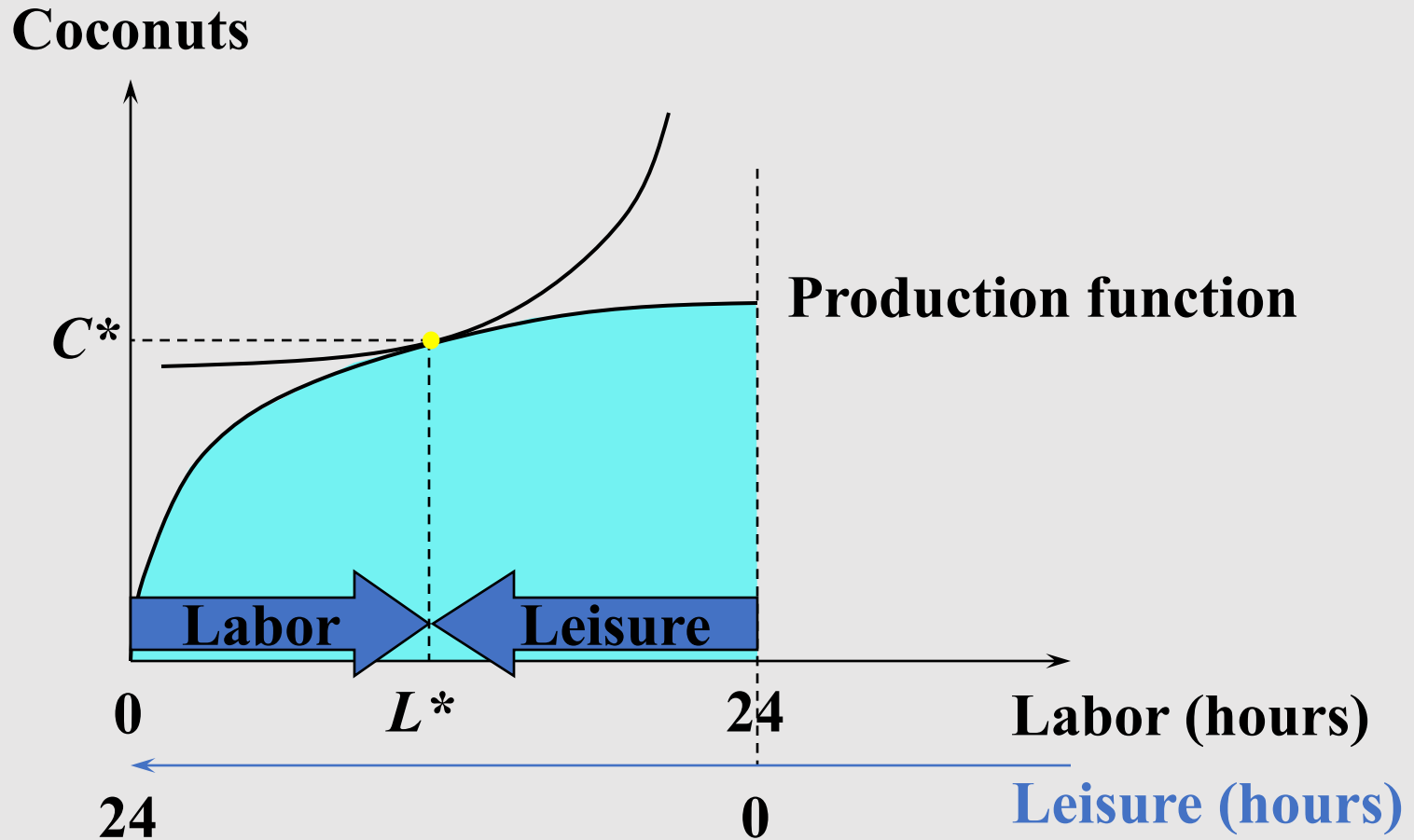
Robinson Crusoe's Choice



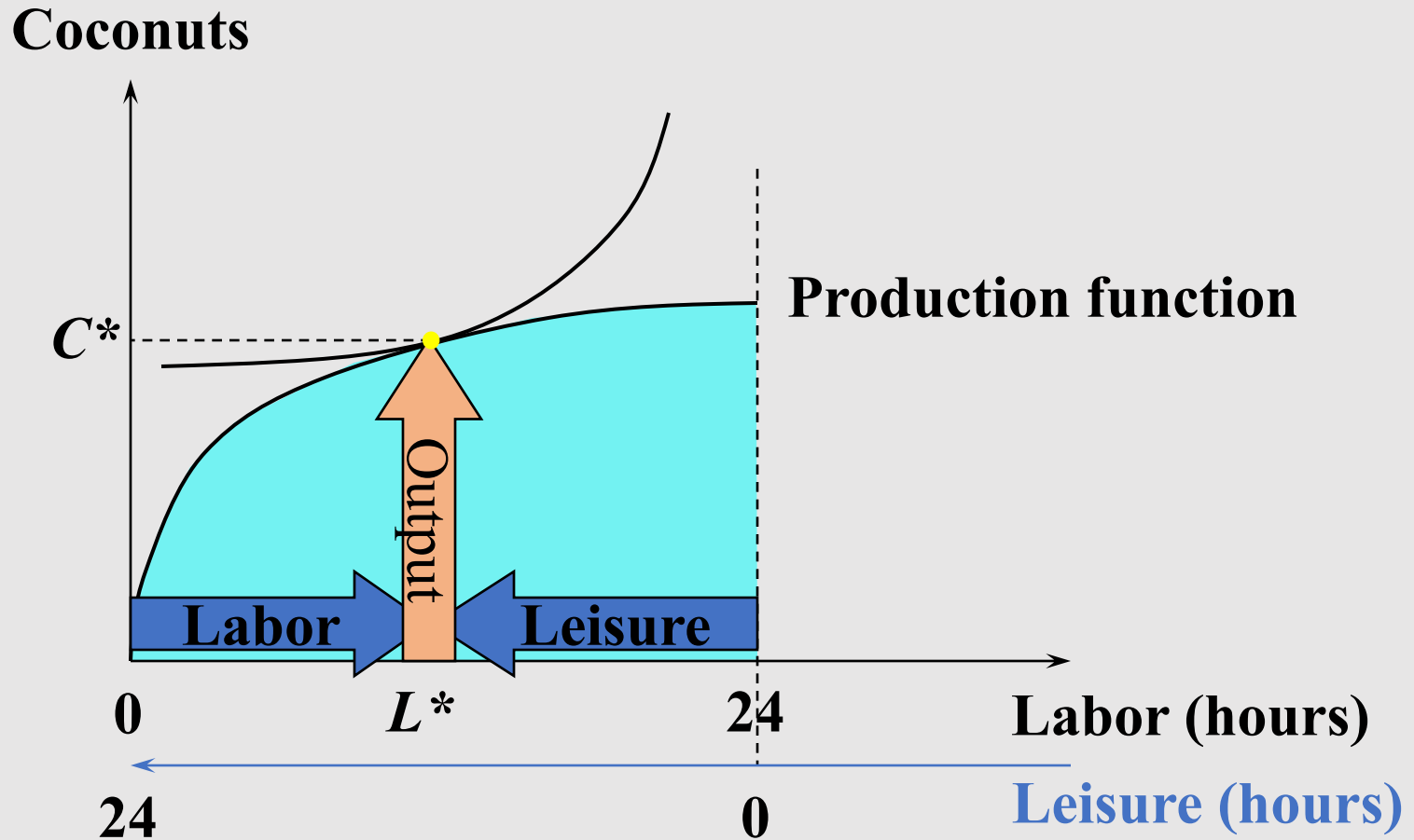
Robinson Crusoe's Choice



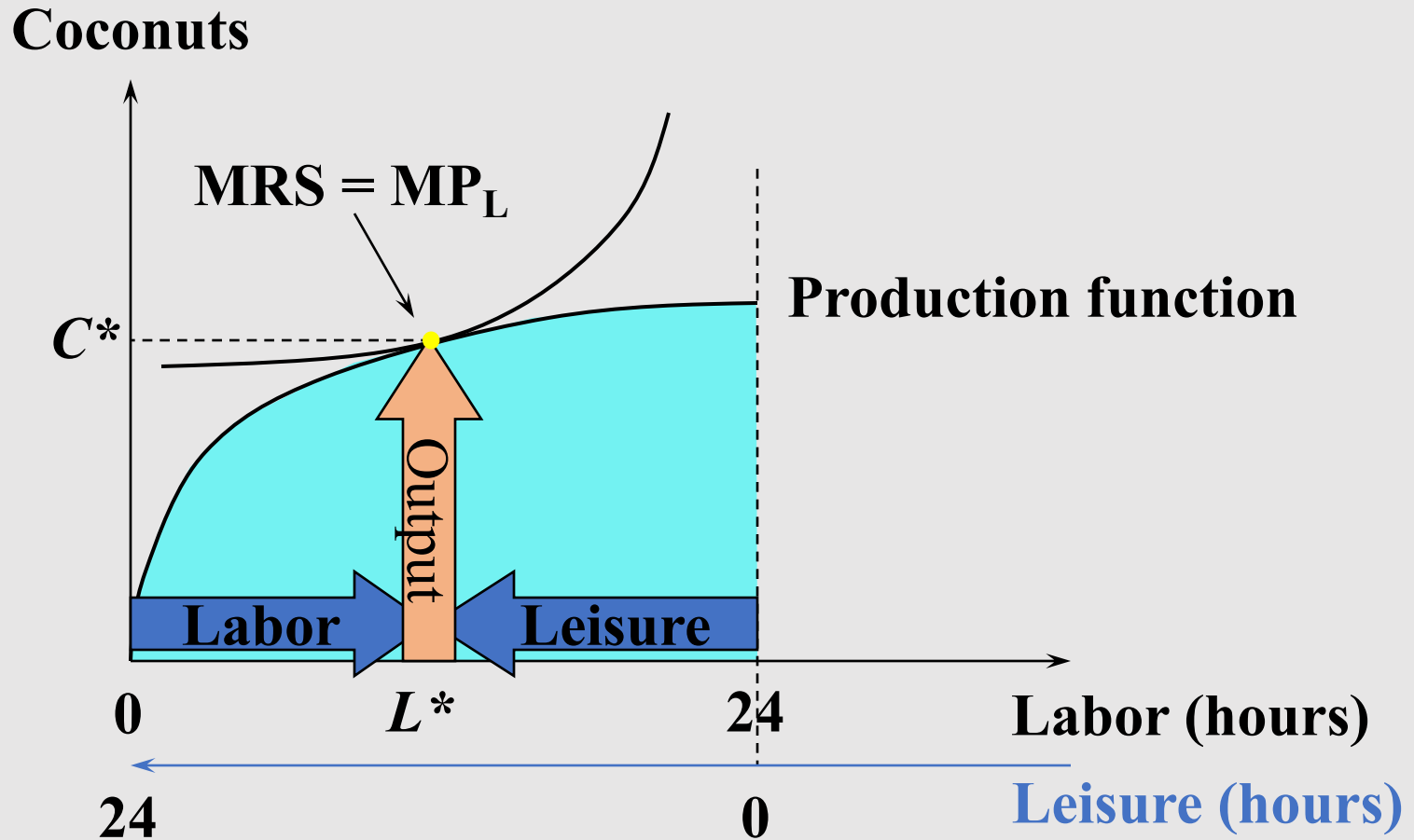
Robinson Crusoe's Choice



Robinson Crusoe's Choice



Robinson Crusoe's Choice



2. Robinson Crusoe as a Firm

- Now suppose RC is both a utility-maximizing consumer and a profit-maximizing firm.
- Use coconuts as the numeraire good; i.e. price of a coconut $p_c = \$1$.
- RC's wage rate is w .
- Coconut output level is C .

Robinson Crusoe as a Firm

→ RC's firm's profit is $\pi = p_c C - wL \rightarrow \pi = C - wL$.

→ $\pi = C - wL \Leftrightarrow C = \pi + wL$, the equation of an isoprofit line.

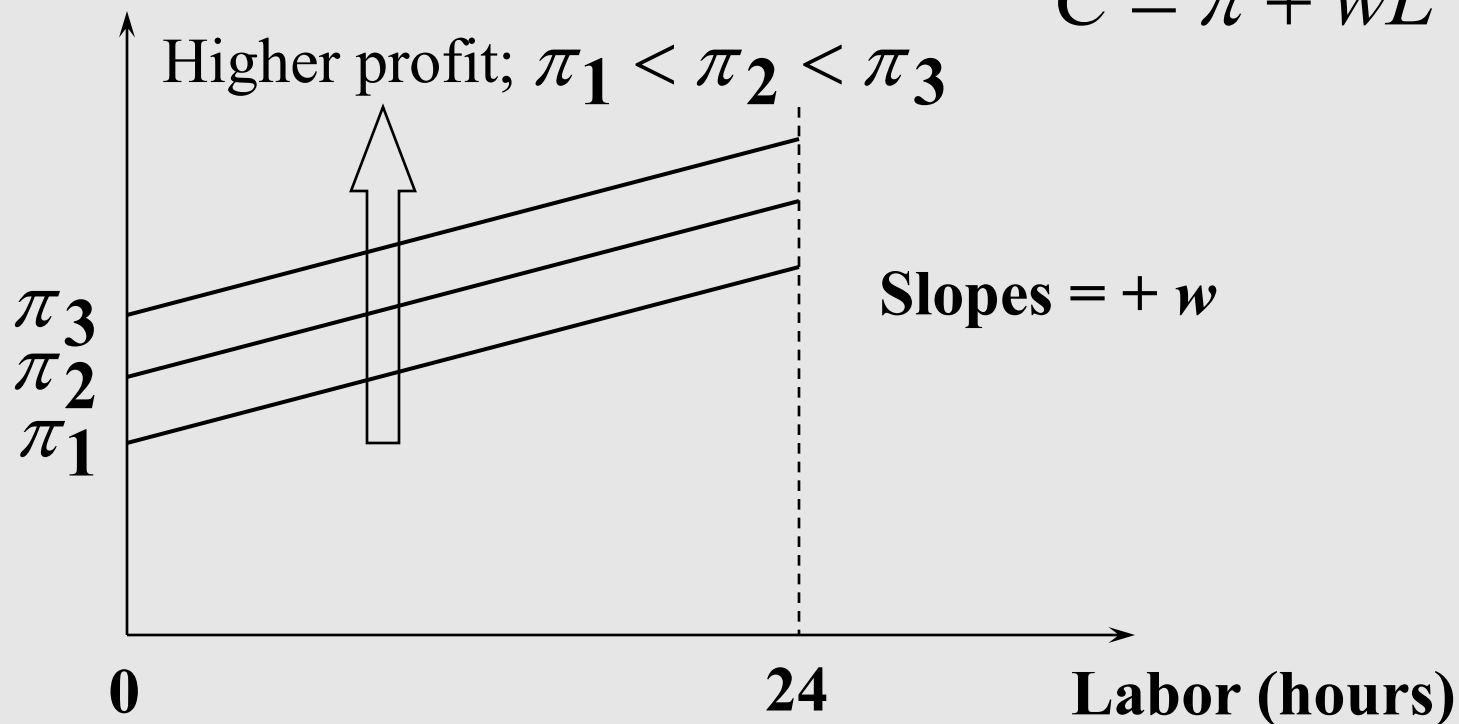
→ Slope = $+ w$.

→ Intercept = π .

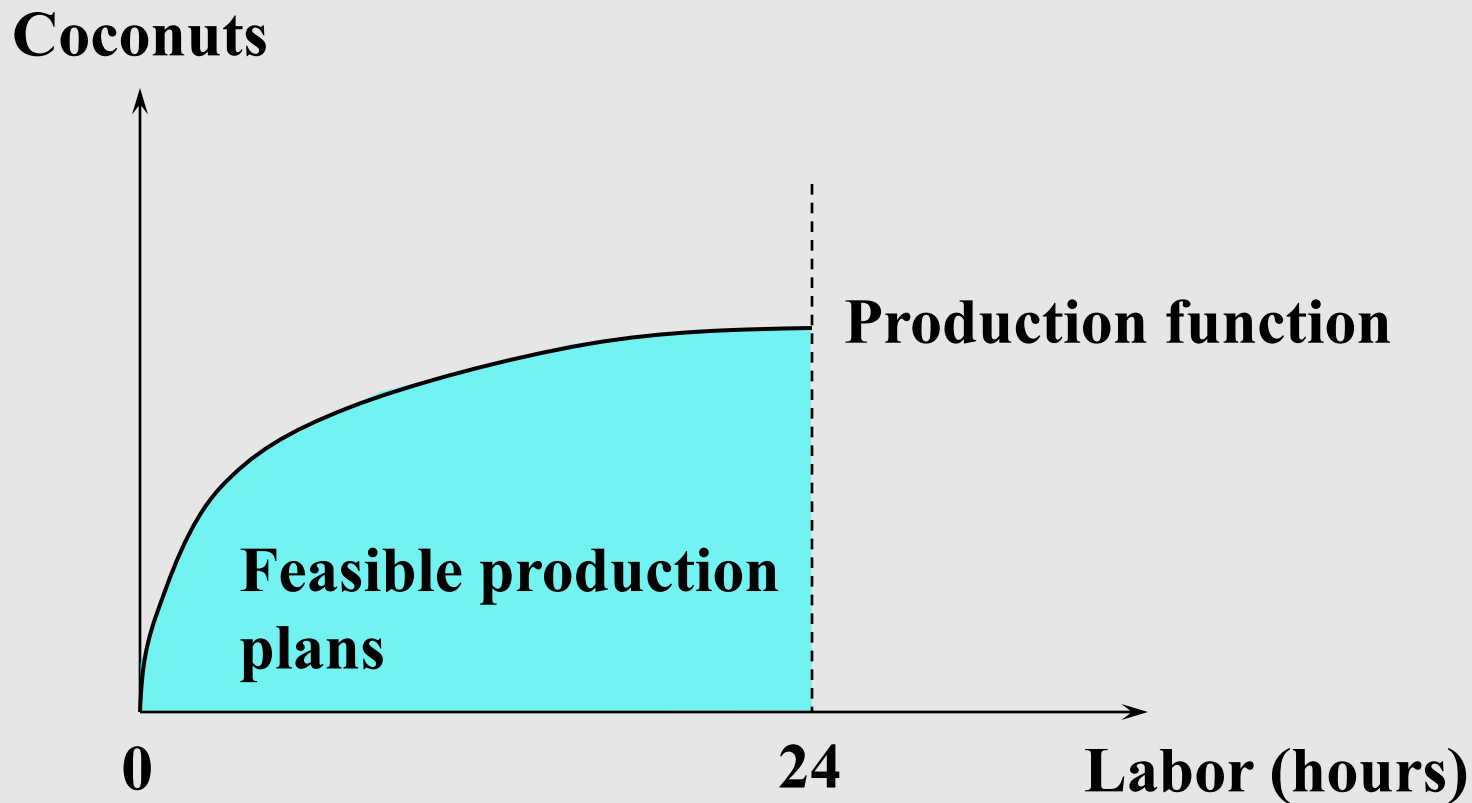
Isoprofit Lines

Coconuts

$$C = \pi + wL$$

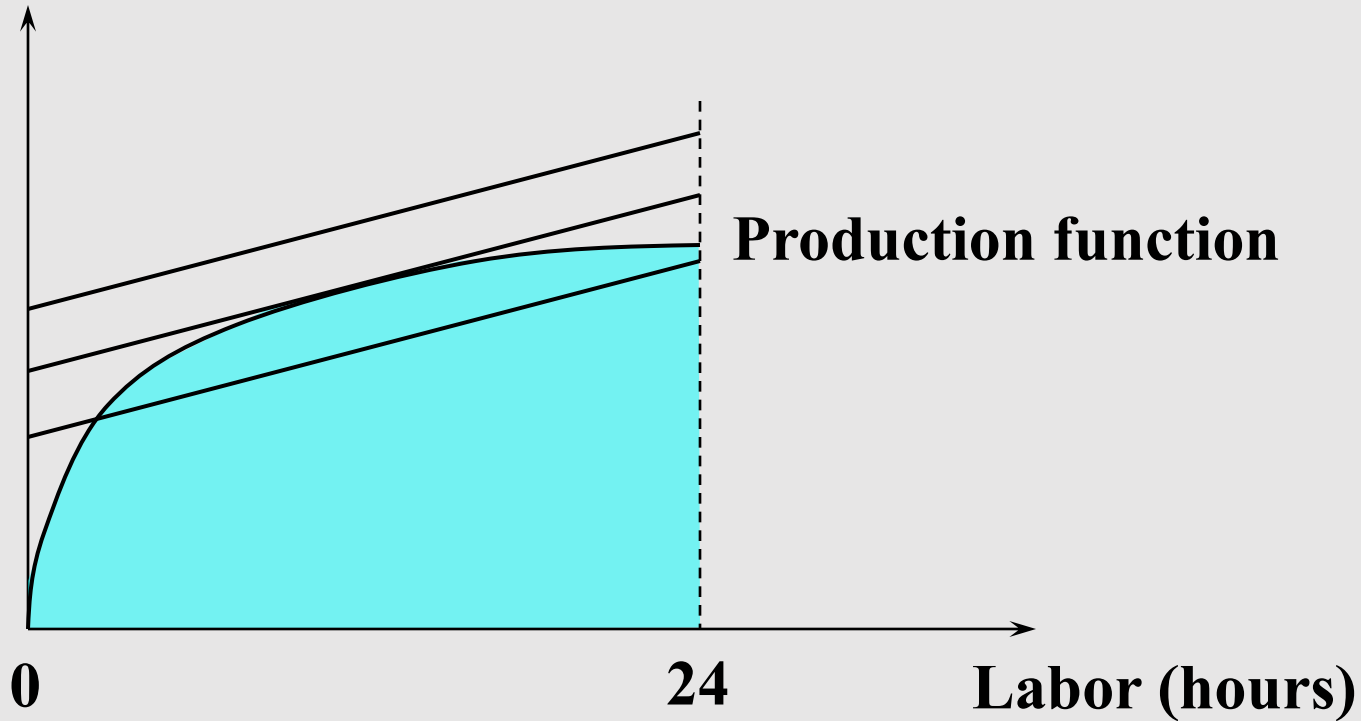


Profit-Maximization

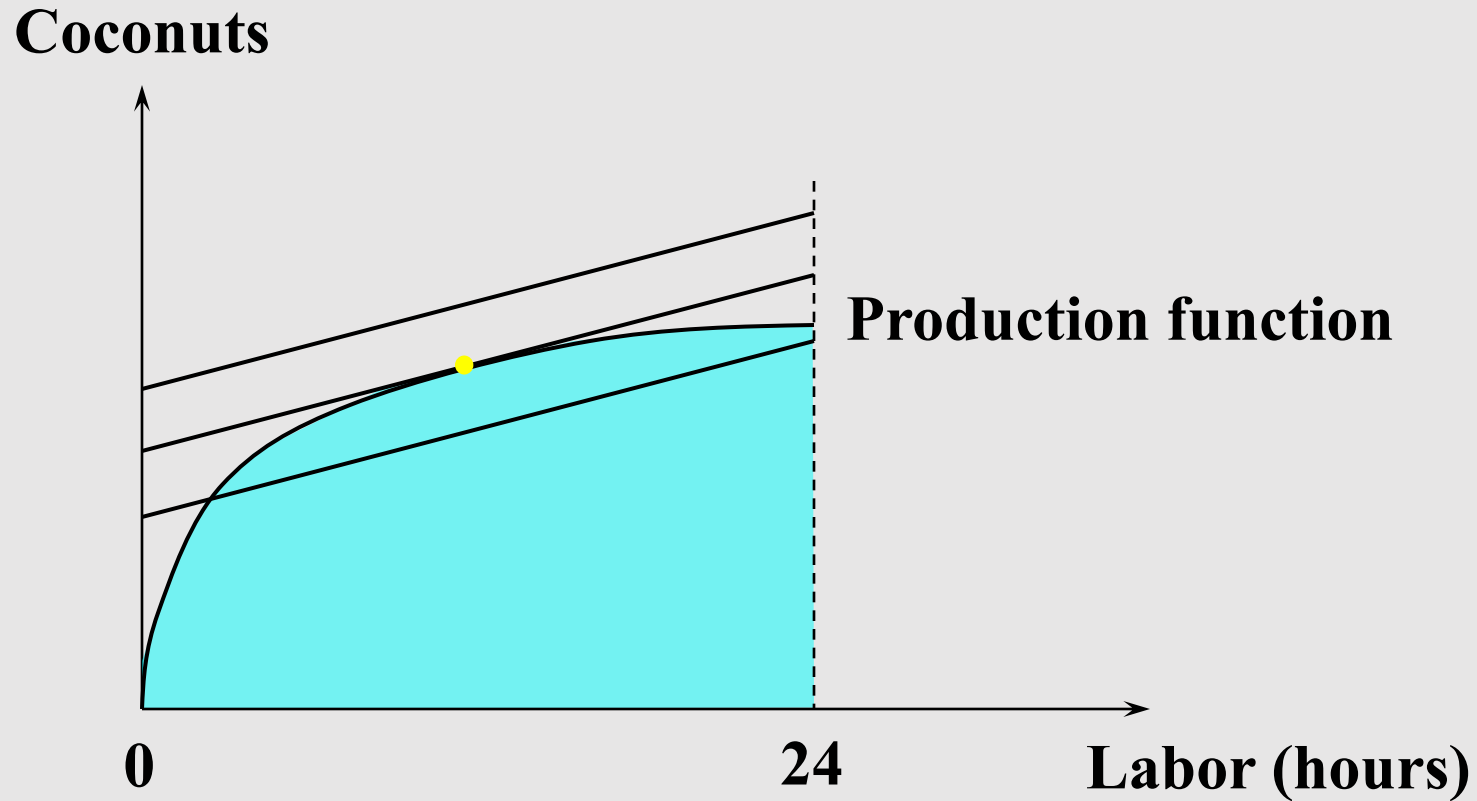


Profit-Maximization

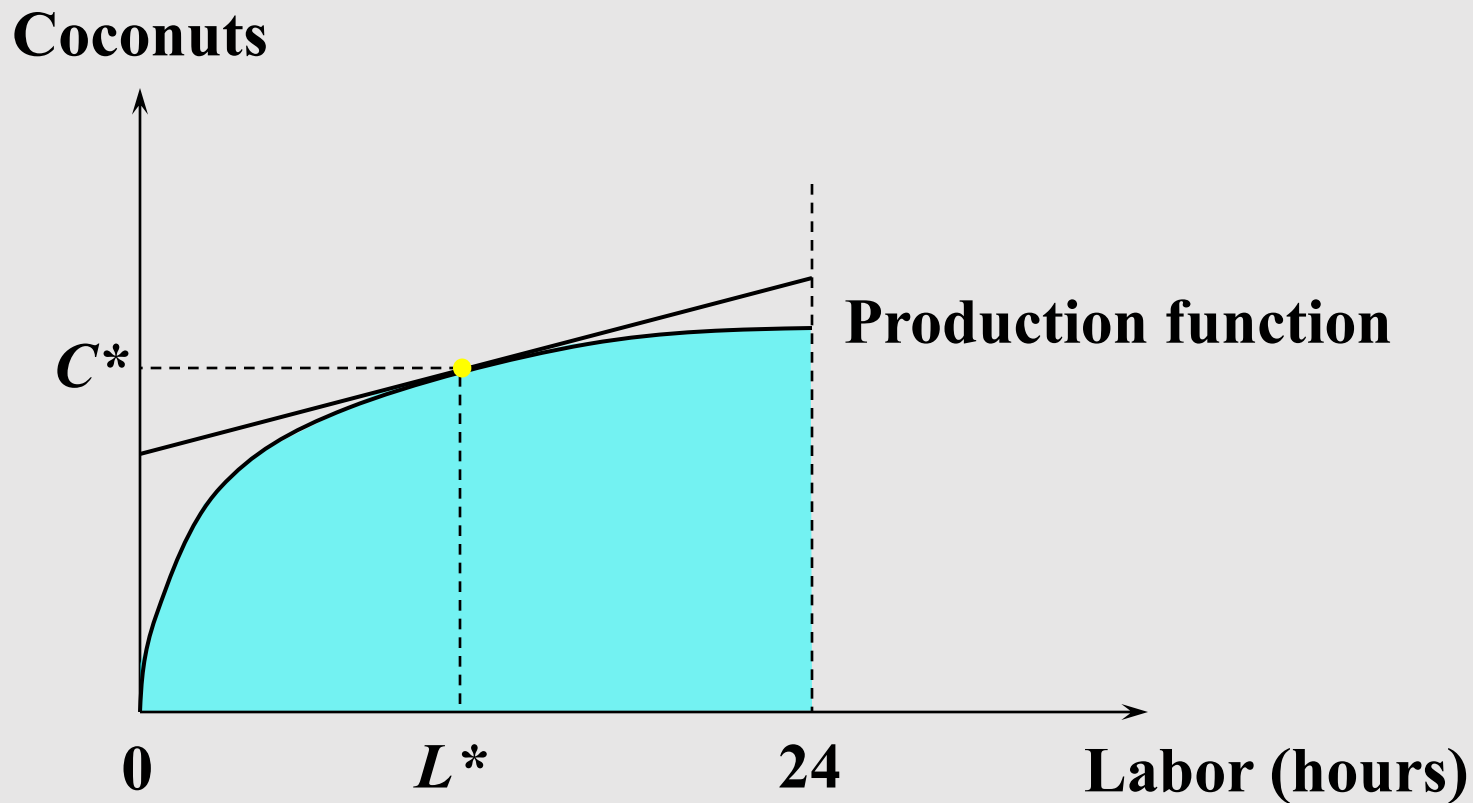
Coconuts



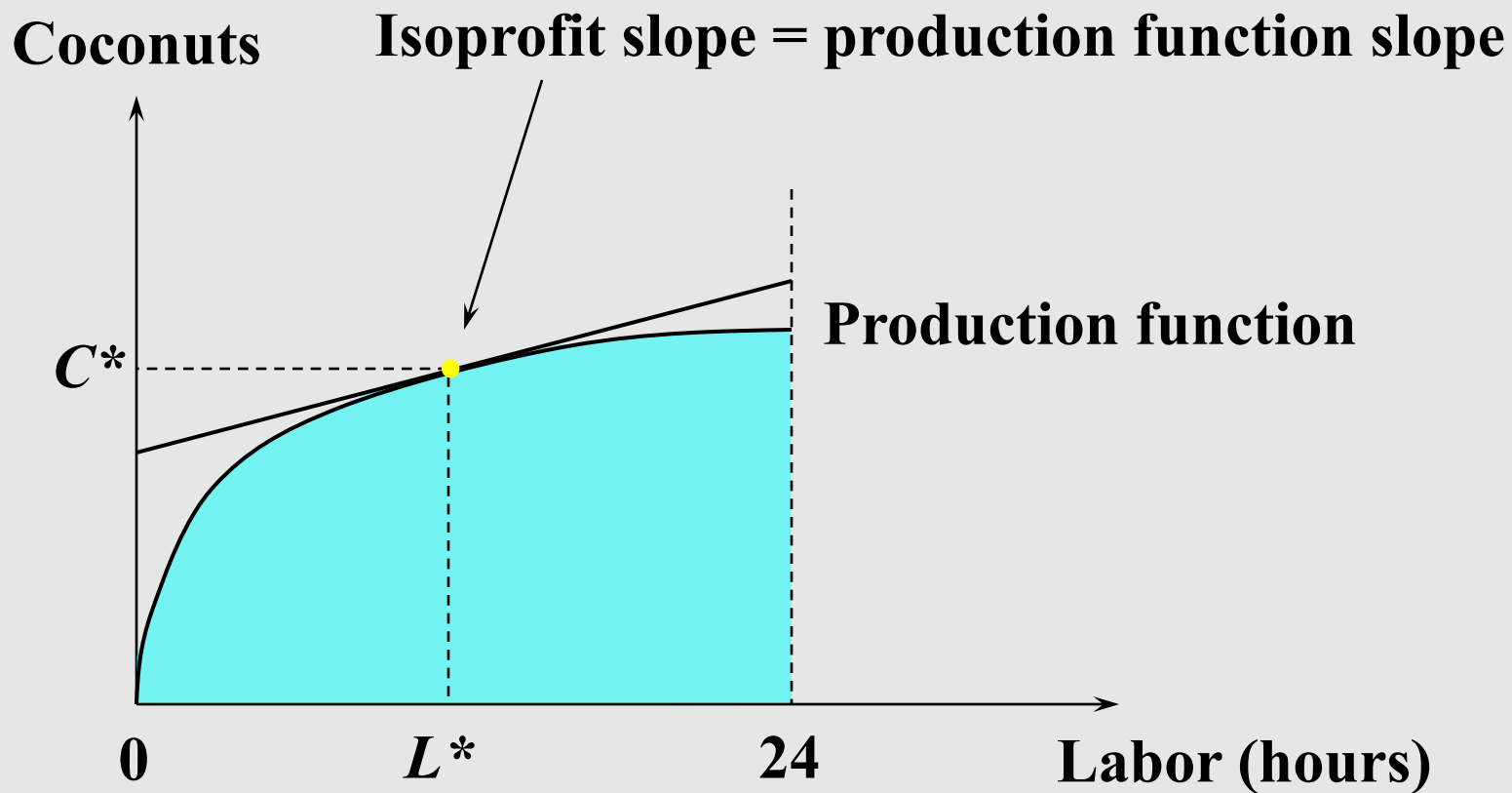
Profit-Maximization



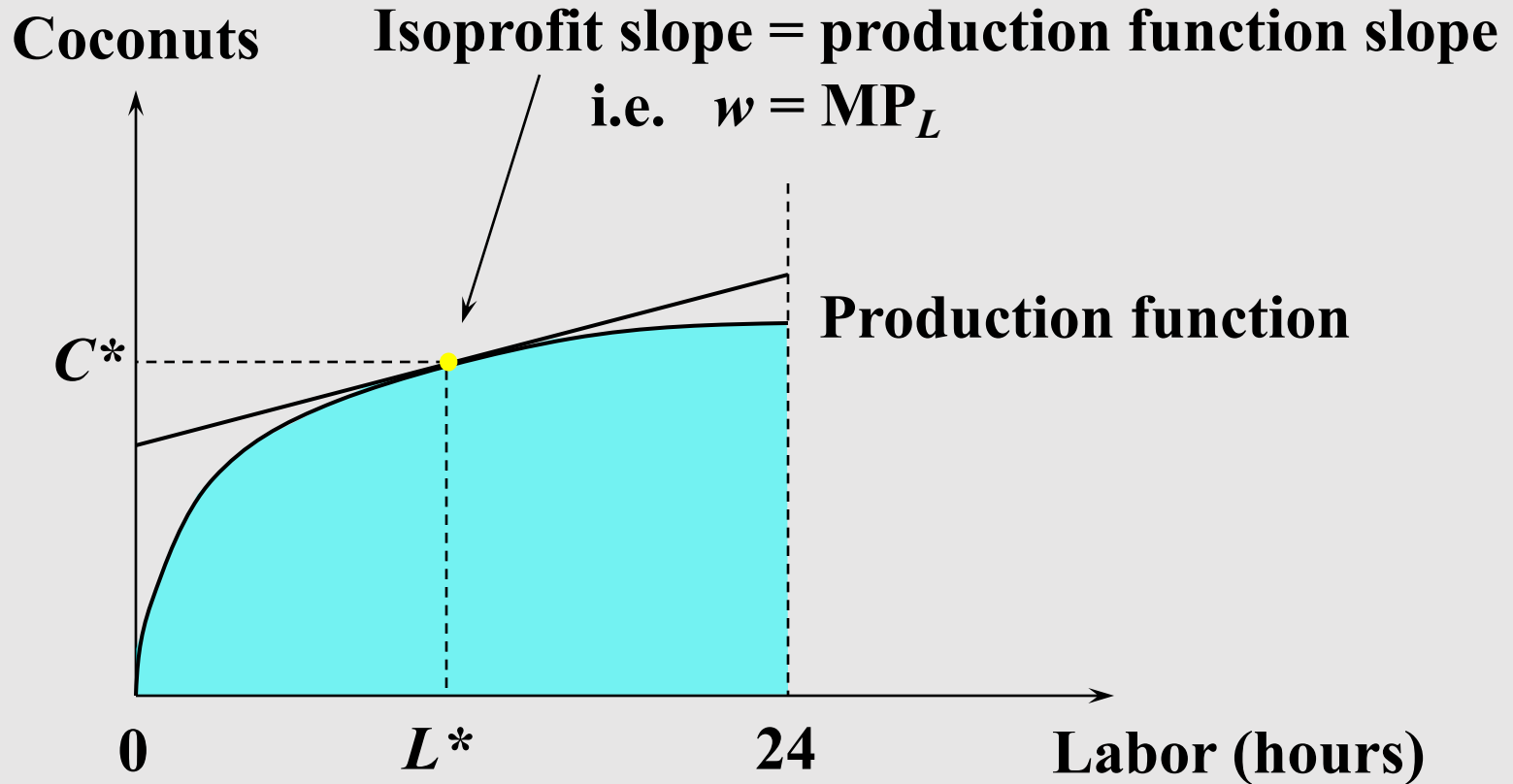
Profit-Maximization



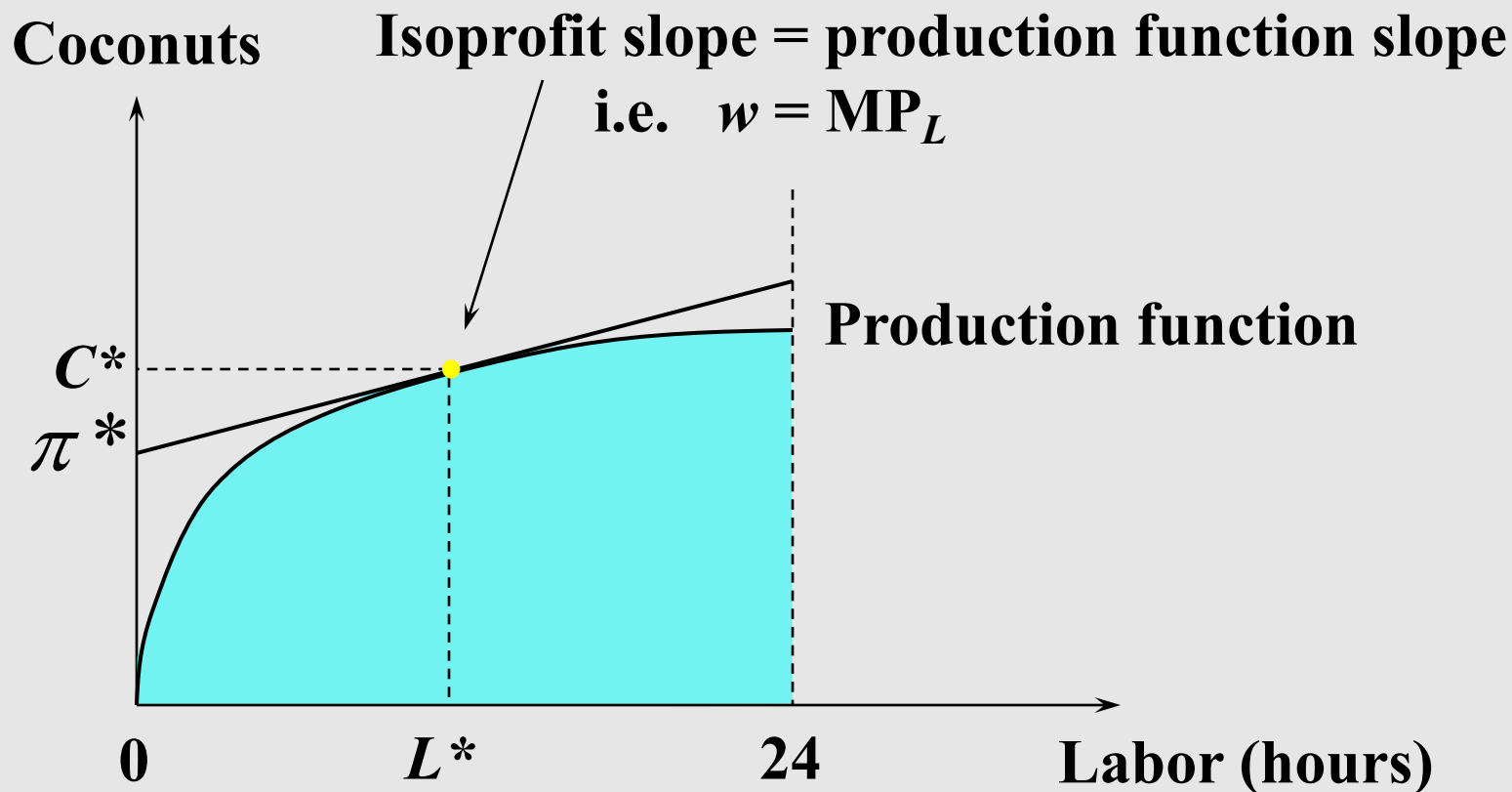
Profit-Maximization



Profit-Maximization

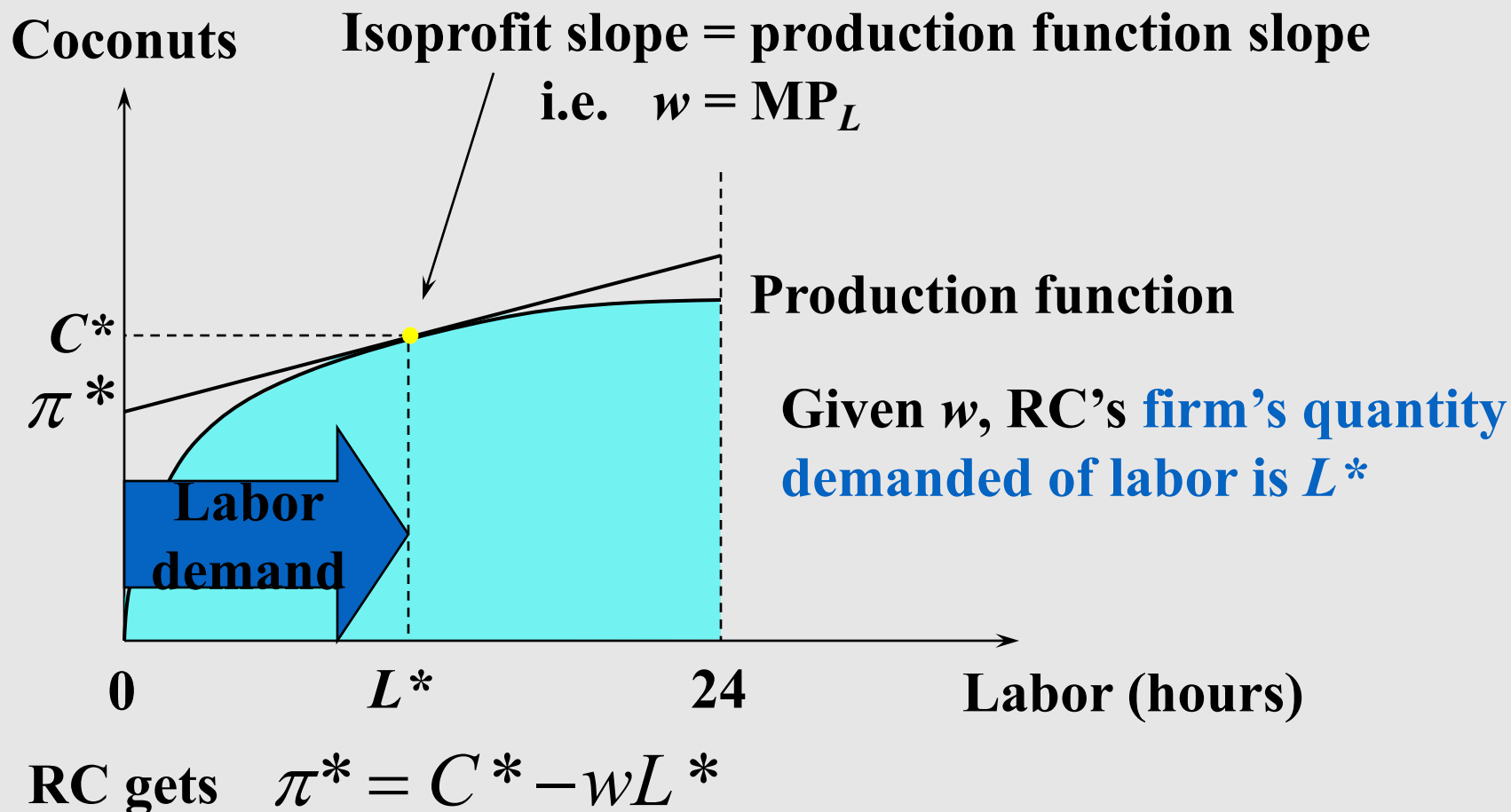


Profit-Maximization

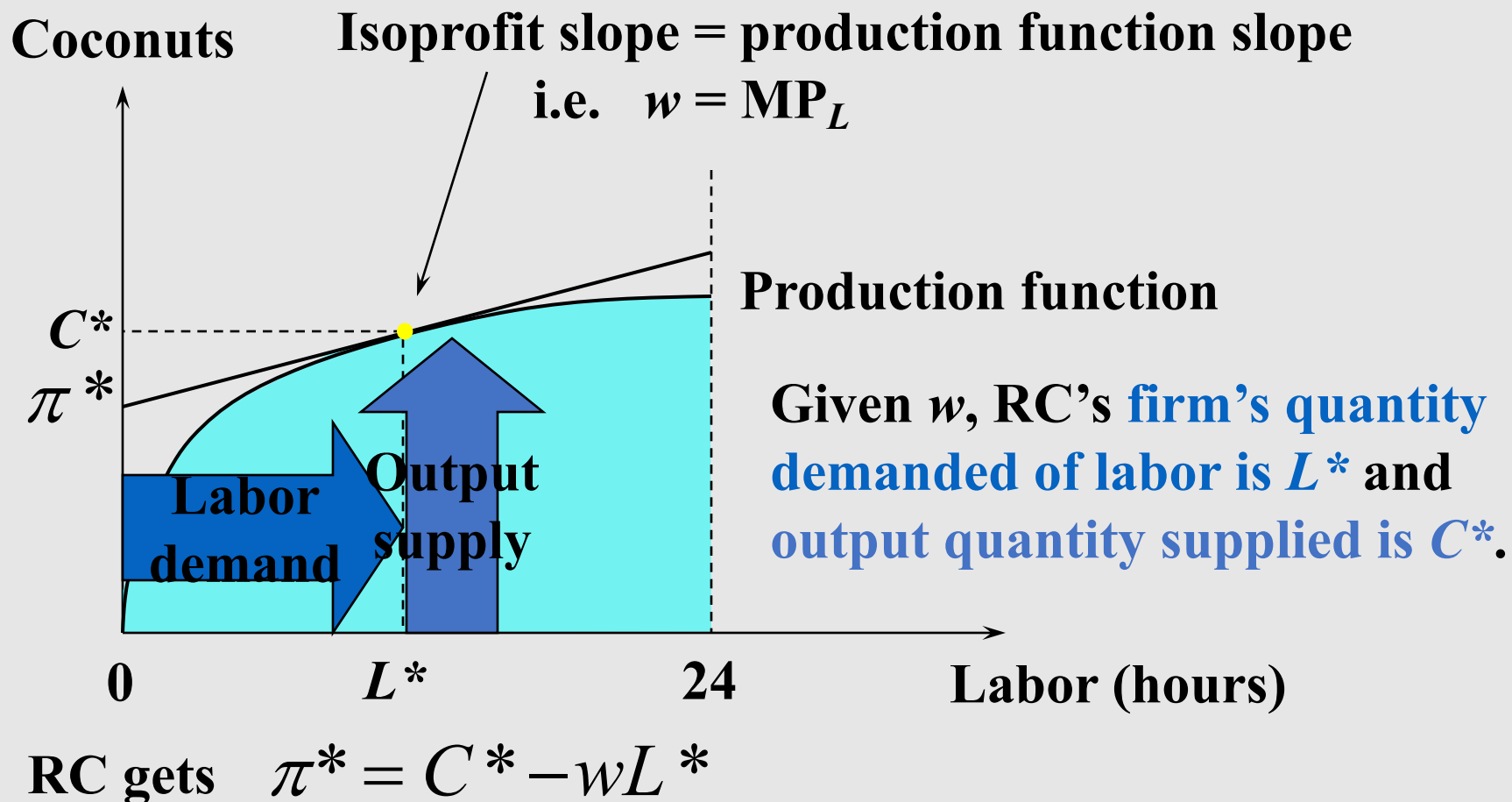


RC gets $\pi^* = C^* - wL^*$

Profit-Maximization



Profit-Maximization



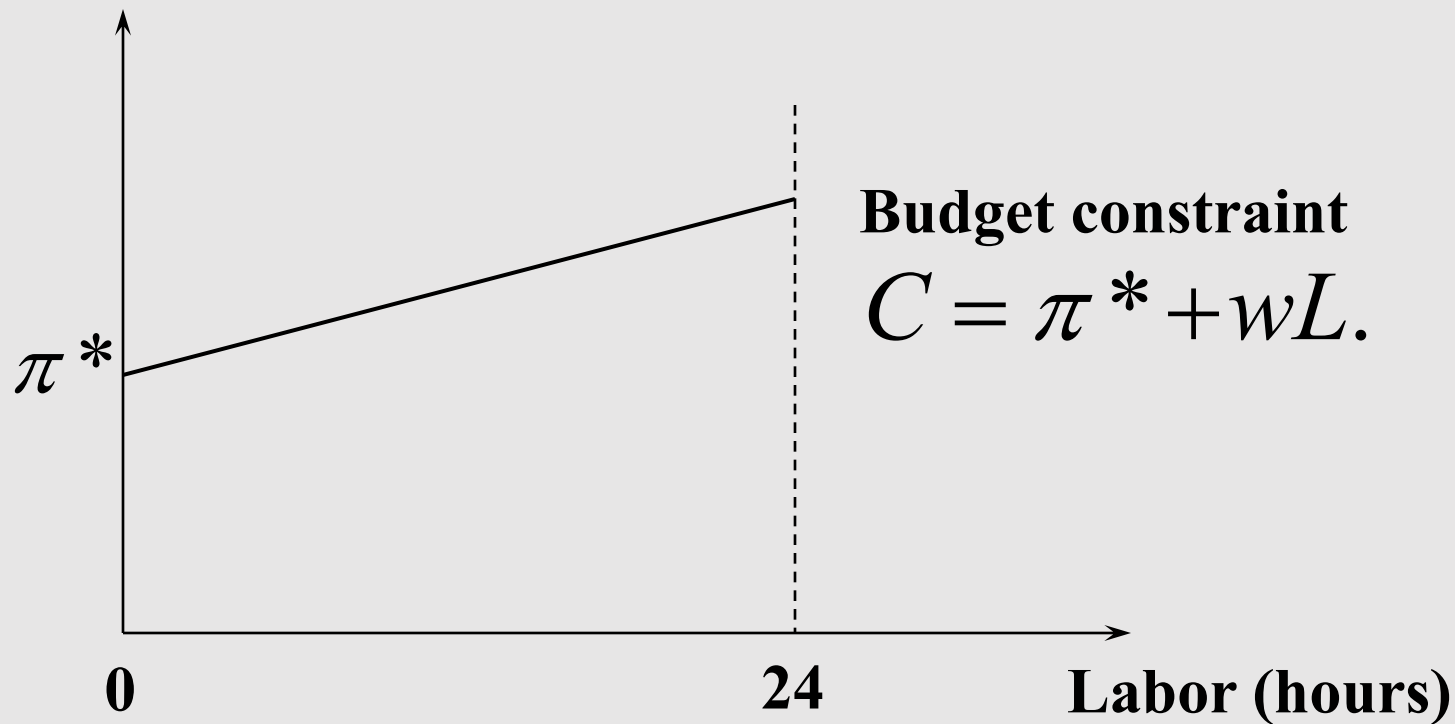
Utility-Maximization

- Now consider RC as a consumer endowed with π^* who can work for w per hour.
- What is RC's most preferred consumption bundle?
- Budget constraint is

$$p_c C = \pi^* + wL \rightarrow C = \frac{\pi^*}{p_c} + \frac{w}{p_c} L$$

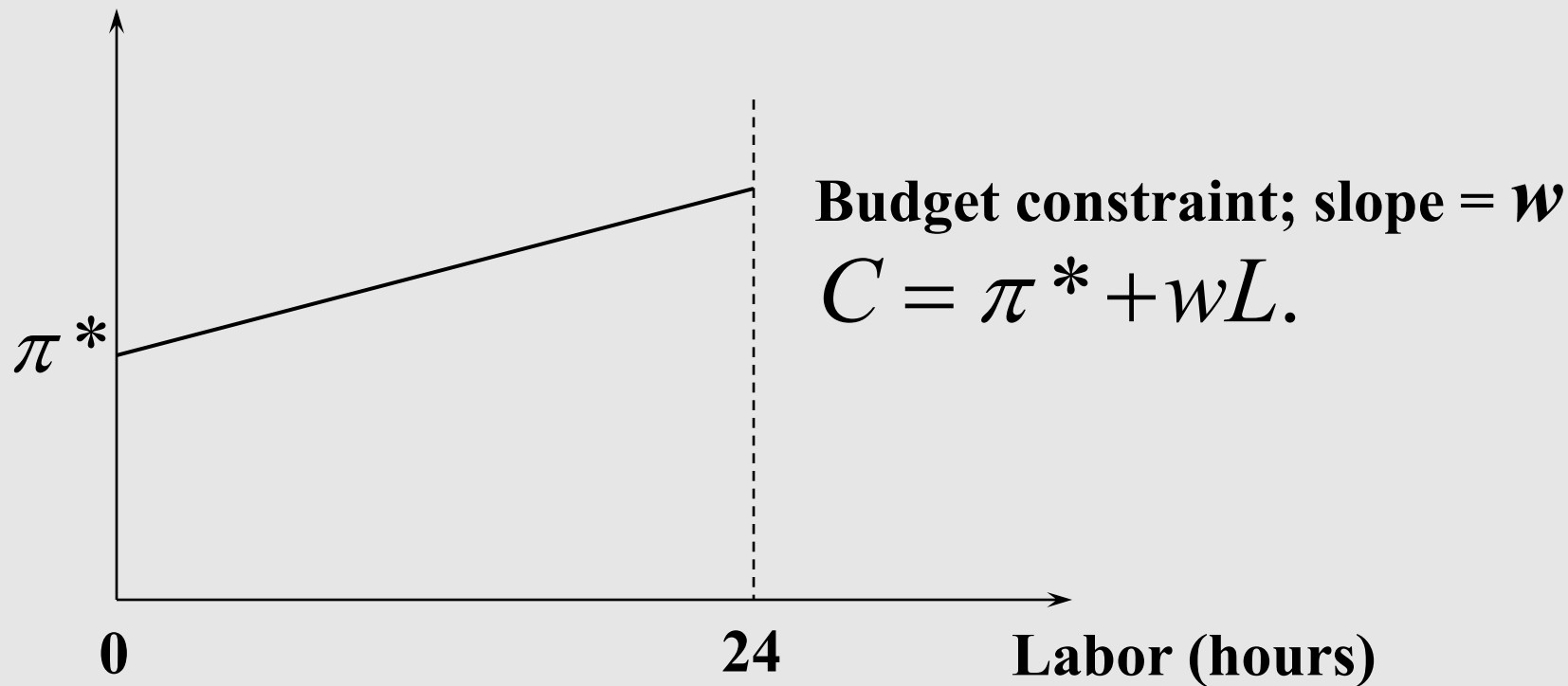
Utility-Maximization

Coconuts



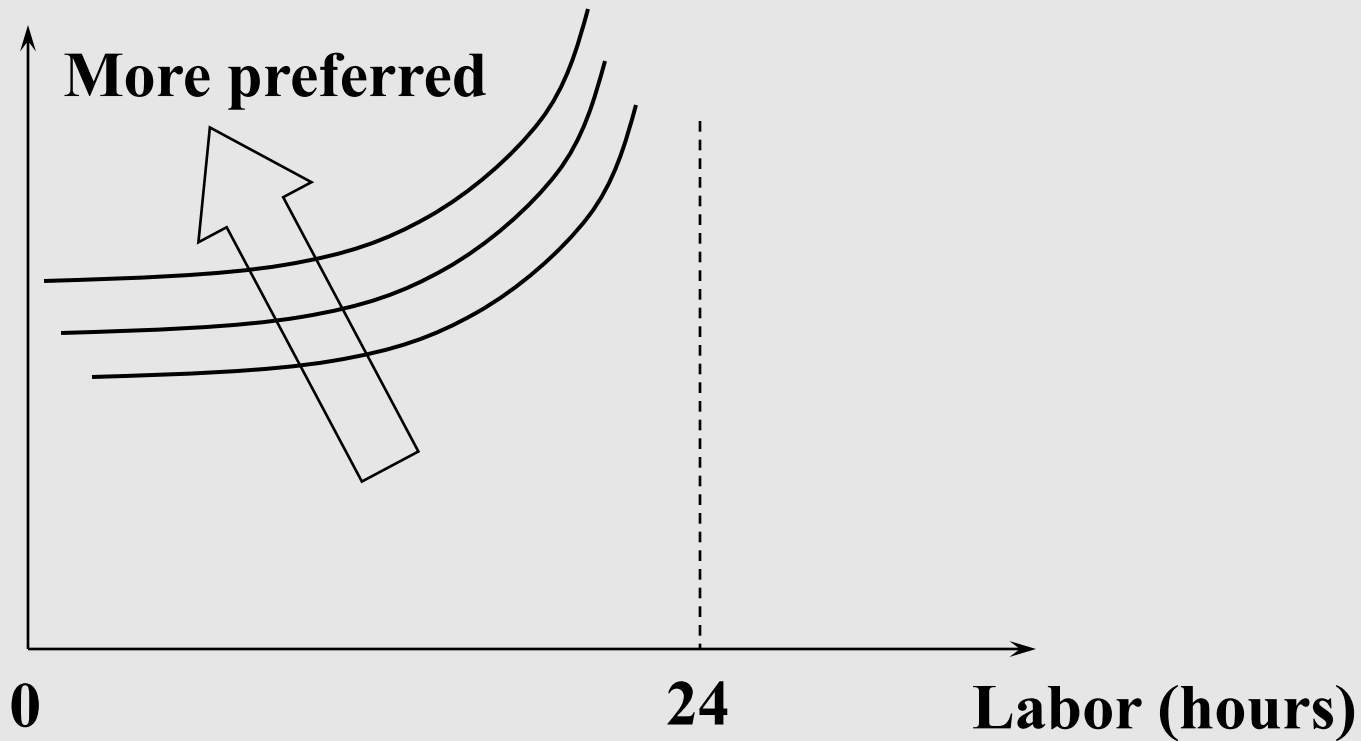
Utility-Maximization

Coconuts



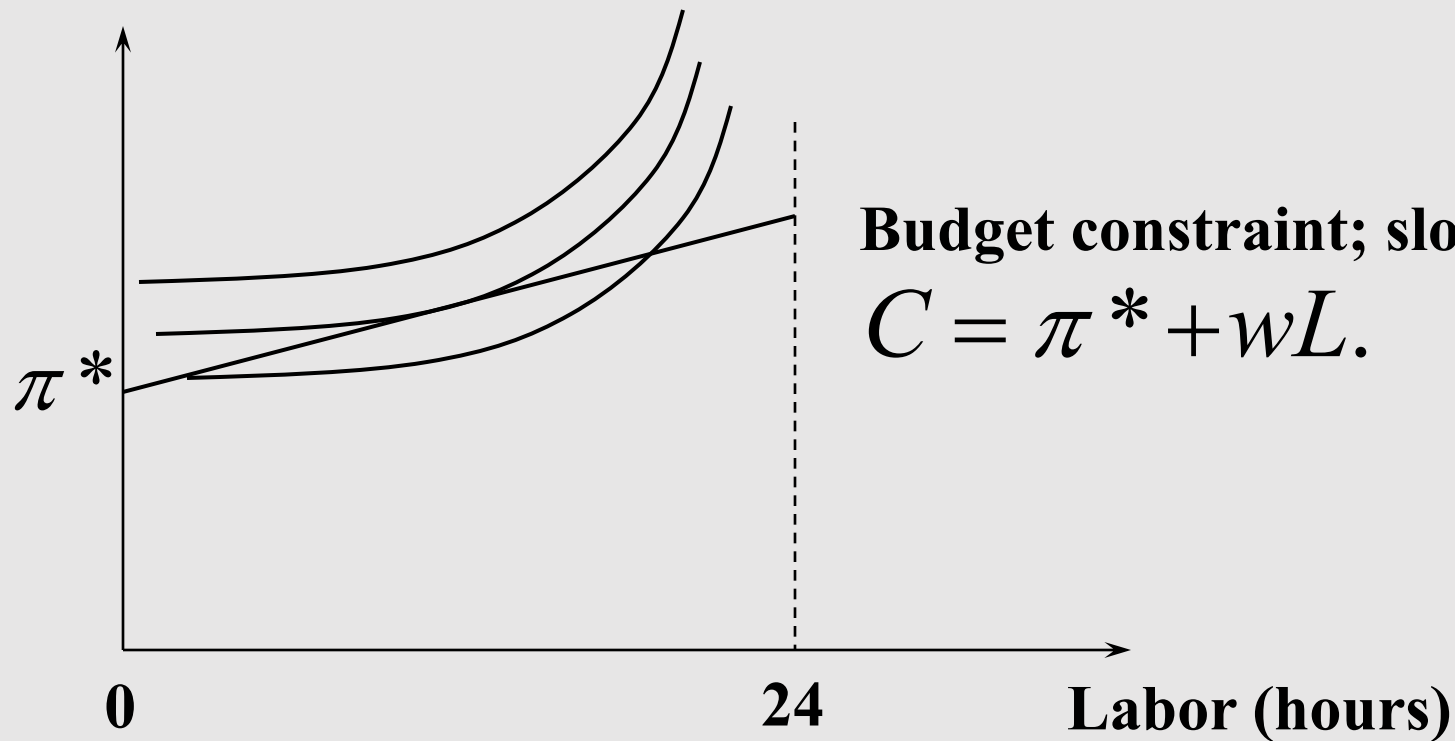
Utility-Maximization

Coconuts



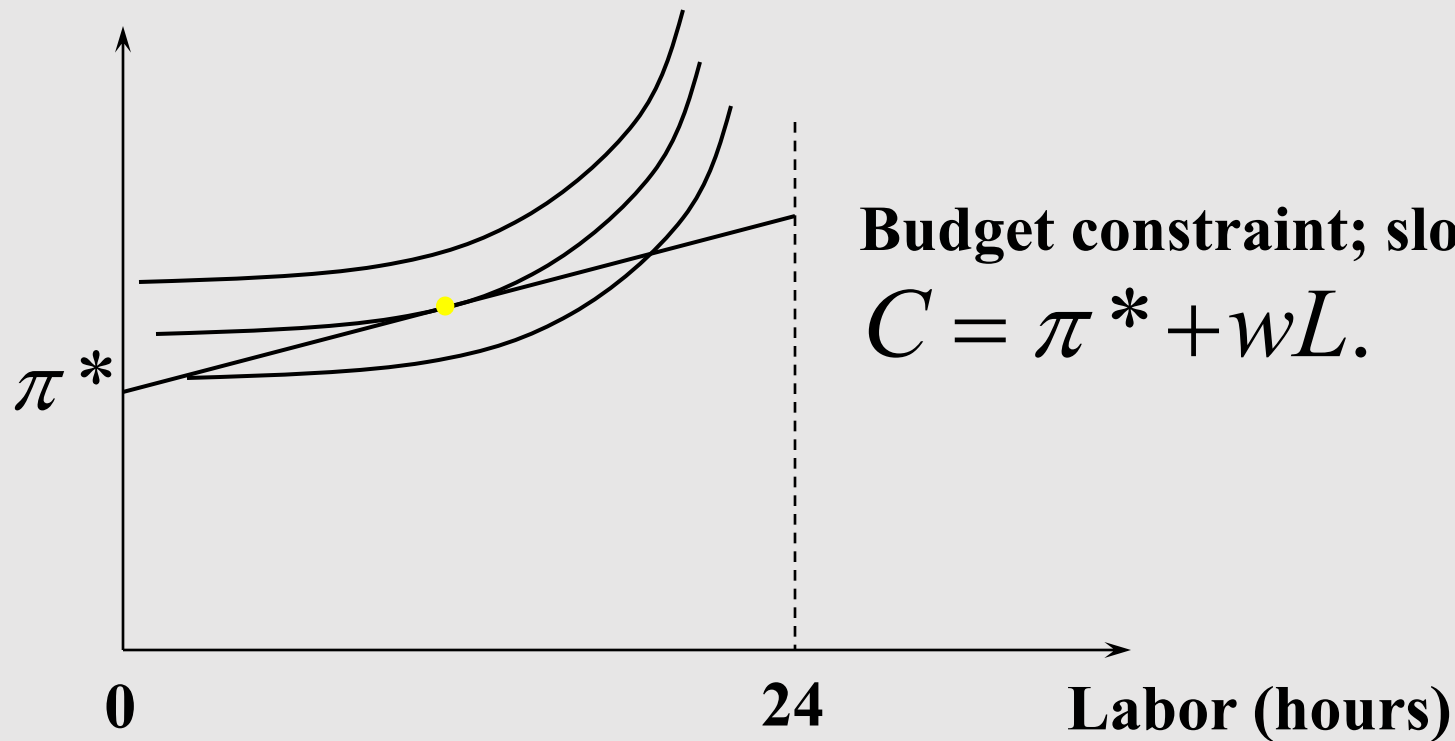
Utility-Maximization

Coconuts



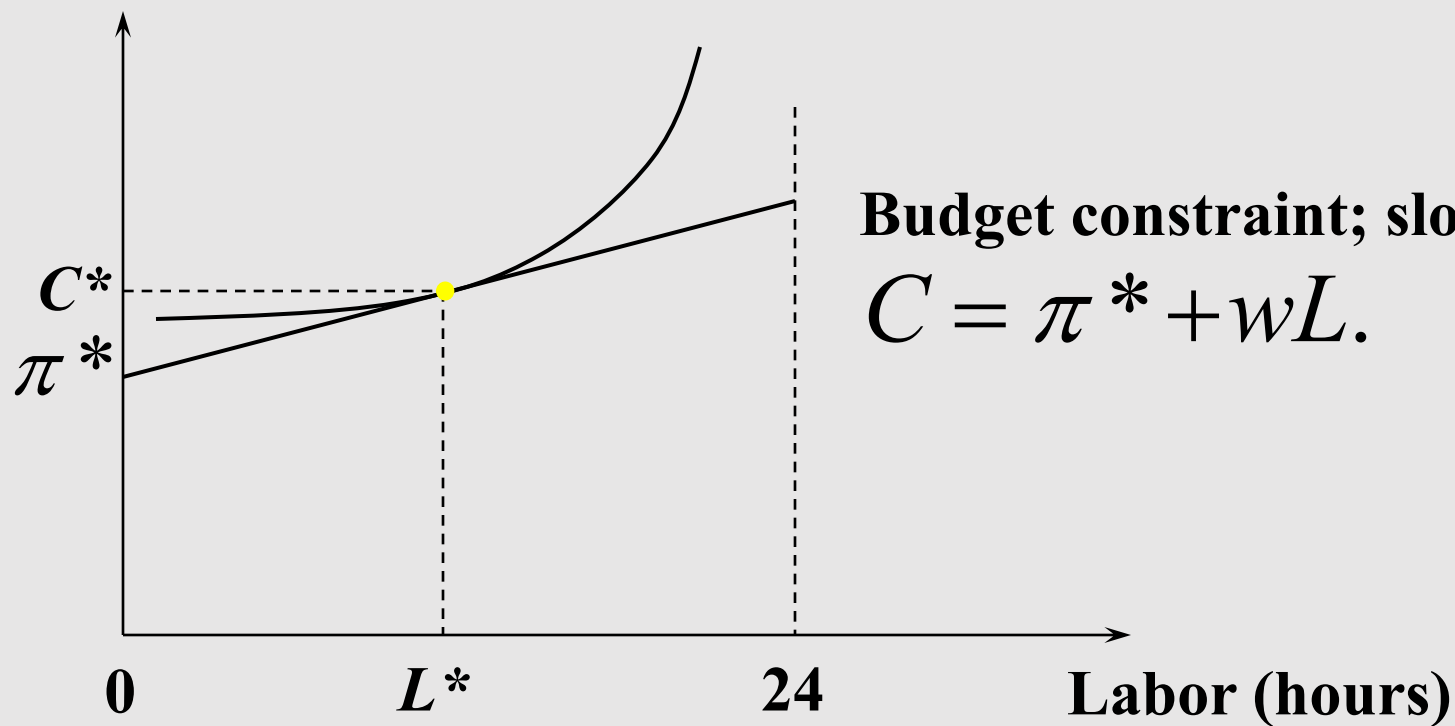
Utility-Maximization

Coconuts



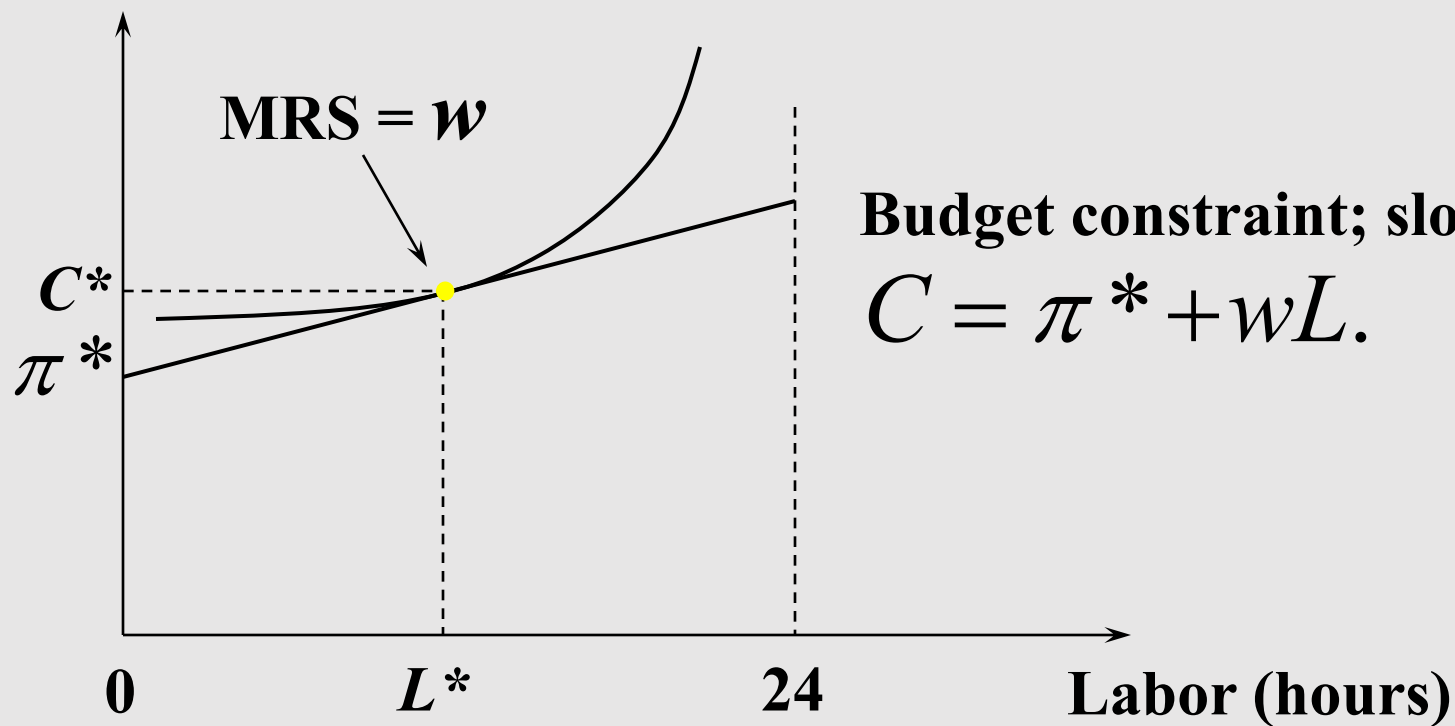
Utility-Maximization

Coconuts



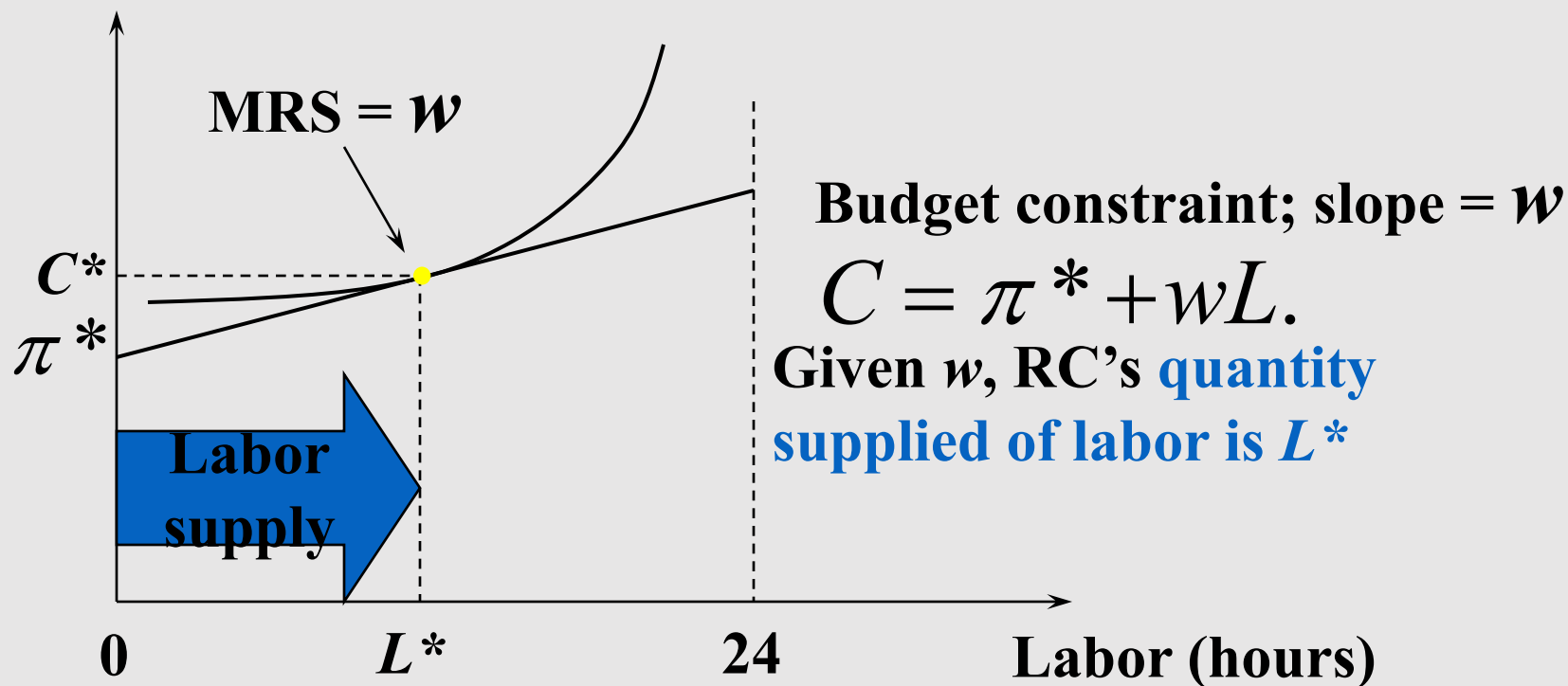
Utility-Maximization

Coconuts



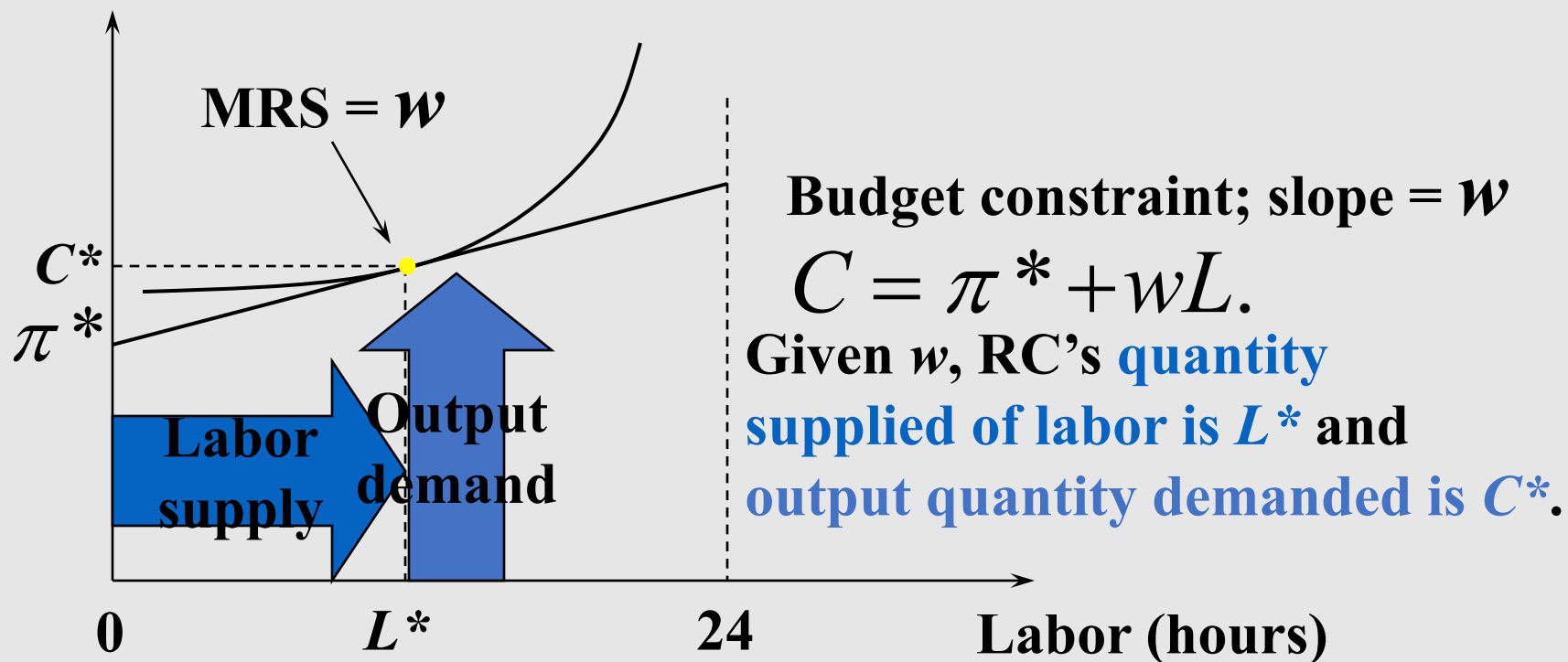
Utility-Maximization

Coconuts



Utility-Maximization

Coconuts



Utility-Maximization & Profit-Maximization

→ Profit-maximization:

- $w = MP_L$
- quantity of output supplied = C^*
- quantity of labor demanded = L^*

→ Utility-maximization:

- $w = MRS$
- quantity of output demanded = C^*
- quantity of labor supplied = L^*

Utility-Maximization & Profit-Maximization

→ Profit-maximization:

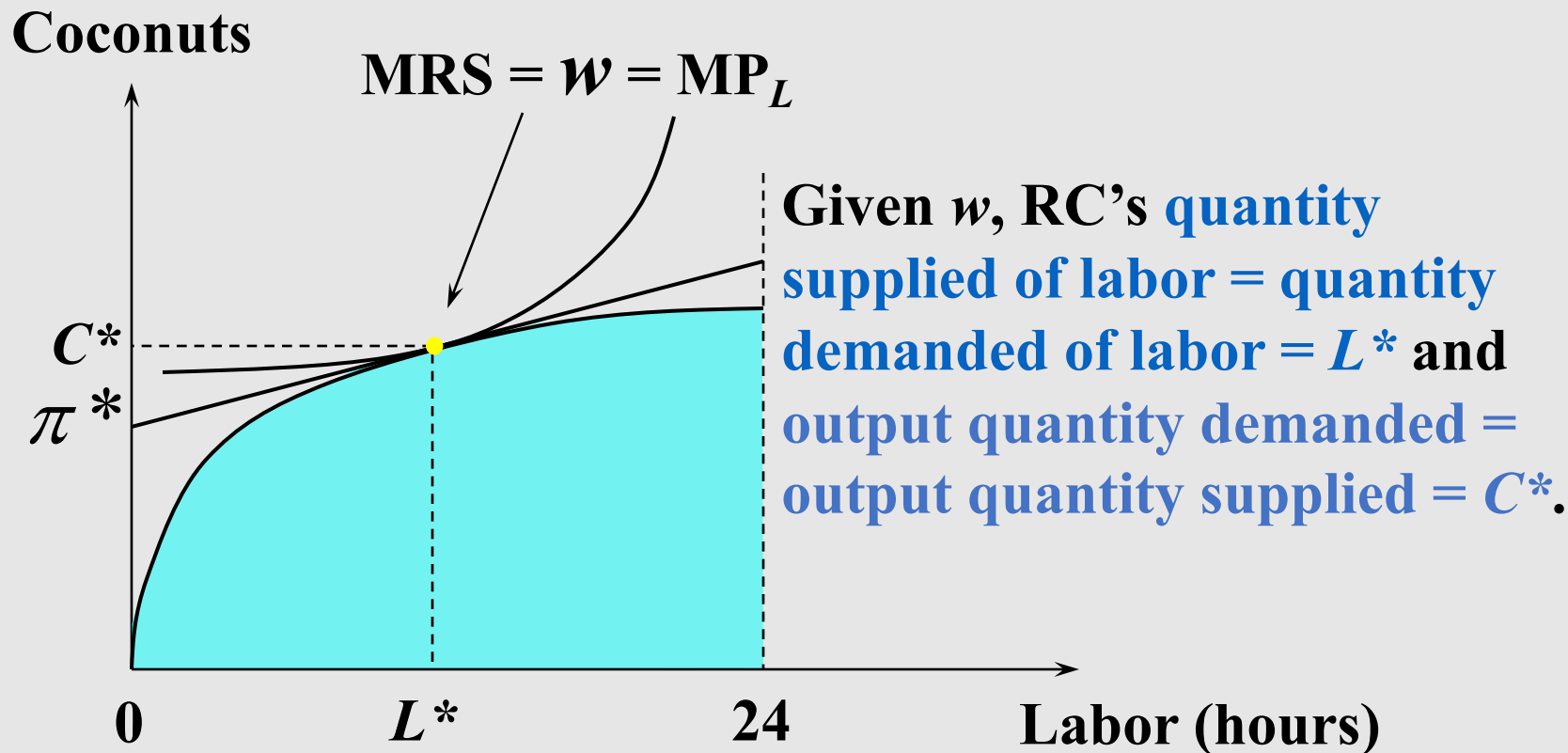
- $w = MP_L$
- quantity of output supplied = C^*
- quantity of labor demanded = L^*

→ Utility-maximization:

- $w = MRS$
- quantity of output demanded = C^*
- quantity of labor supplied = L^*

**Coconut and labor
markets both clear.**

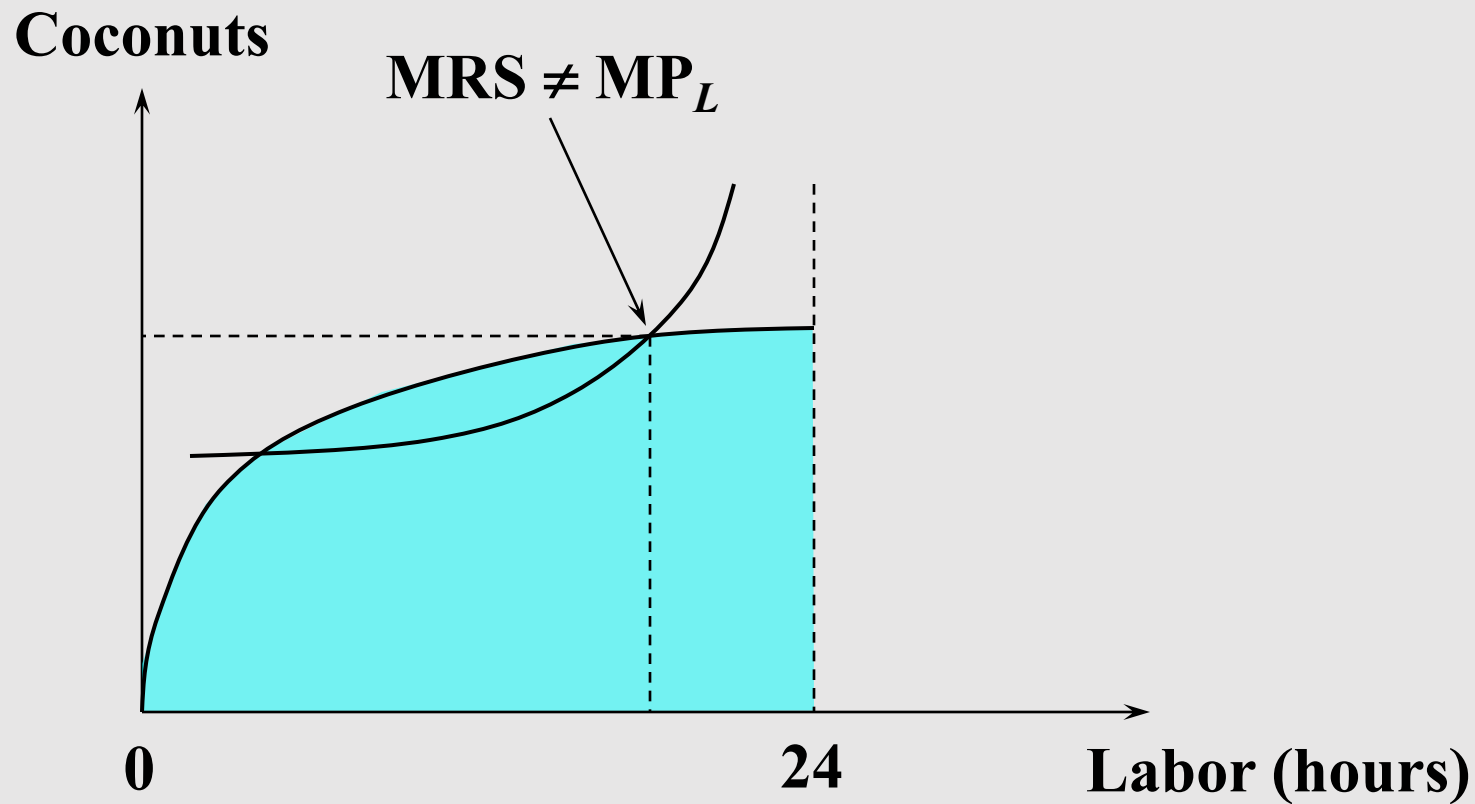
Utility-Maximization & Profit-Maximization



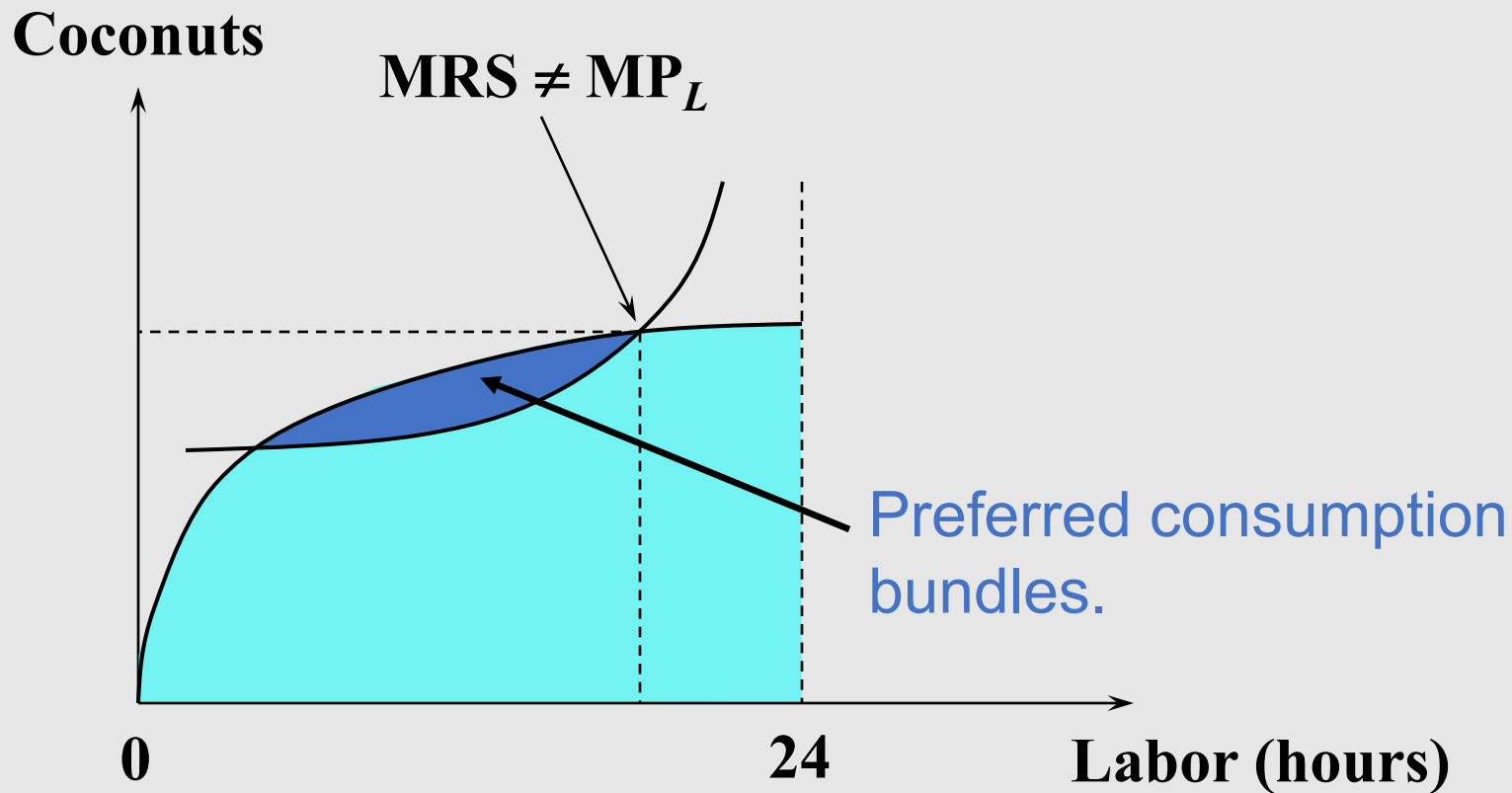
Pareto Efficiency

→ Must have $MRS = MP_L$.

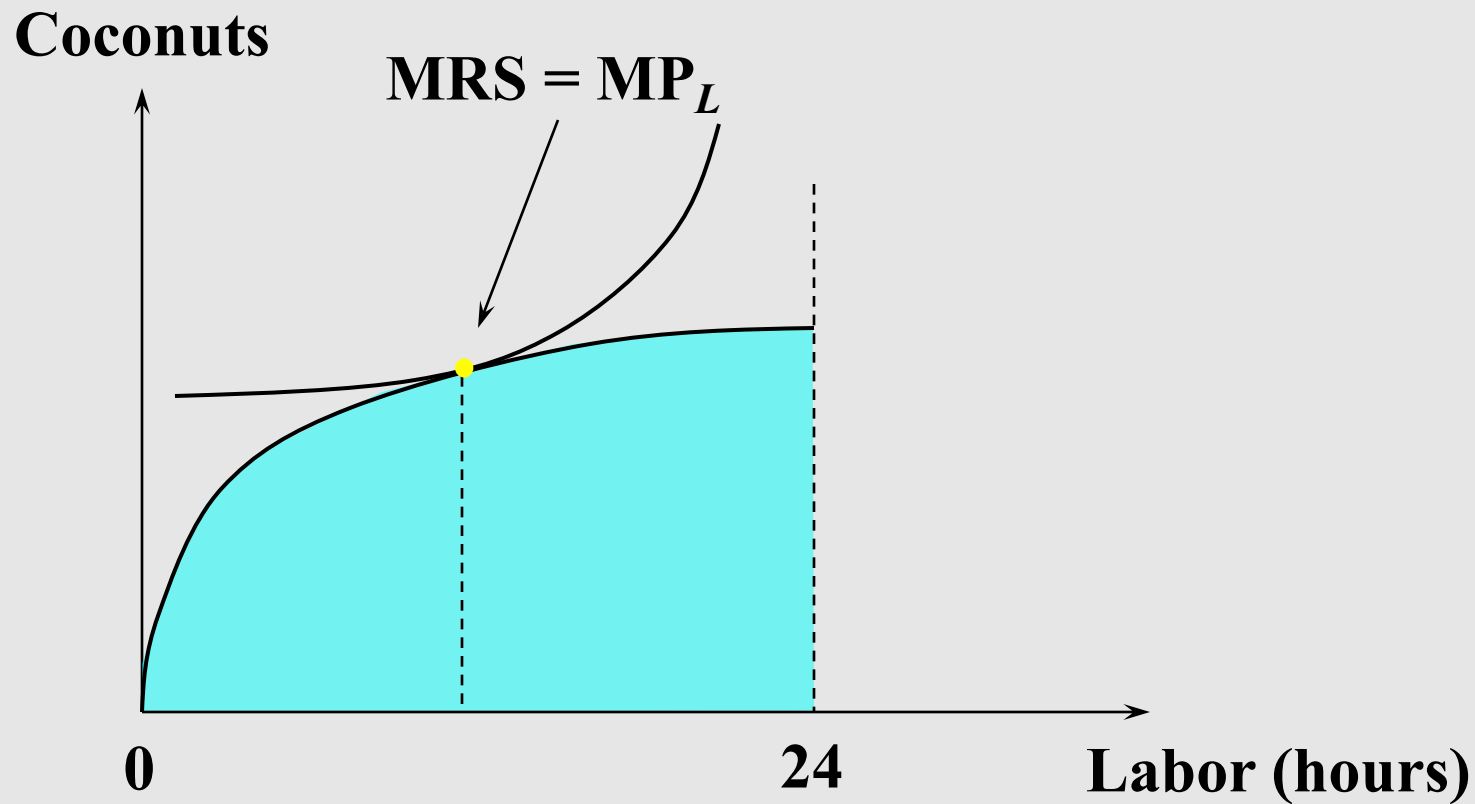
Pareto Efficiency



Pareto Efficiency



Pareto Efficiency



3. Fundamental Theorems of Welfare Economics

First Fundamental Theorem of Welfare Economics

- A competitive market equilibrium is Pareto efficient if
 - all firms act as competitive profit maximizers
 - there are **no externalities** in consumption or production.

Fundamental Theorems of Welfare Economics

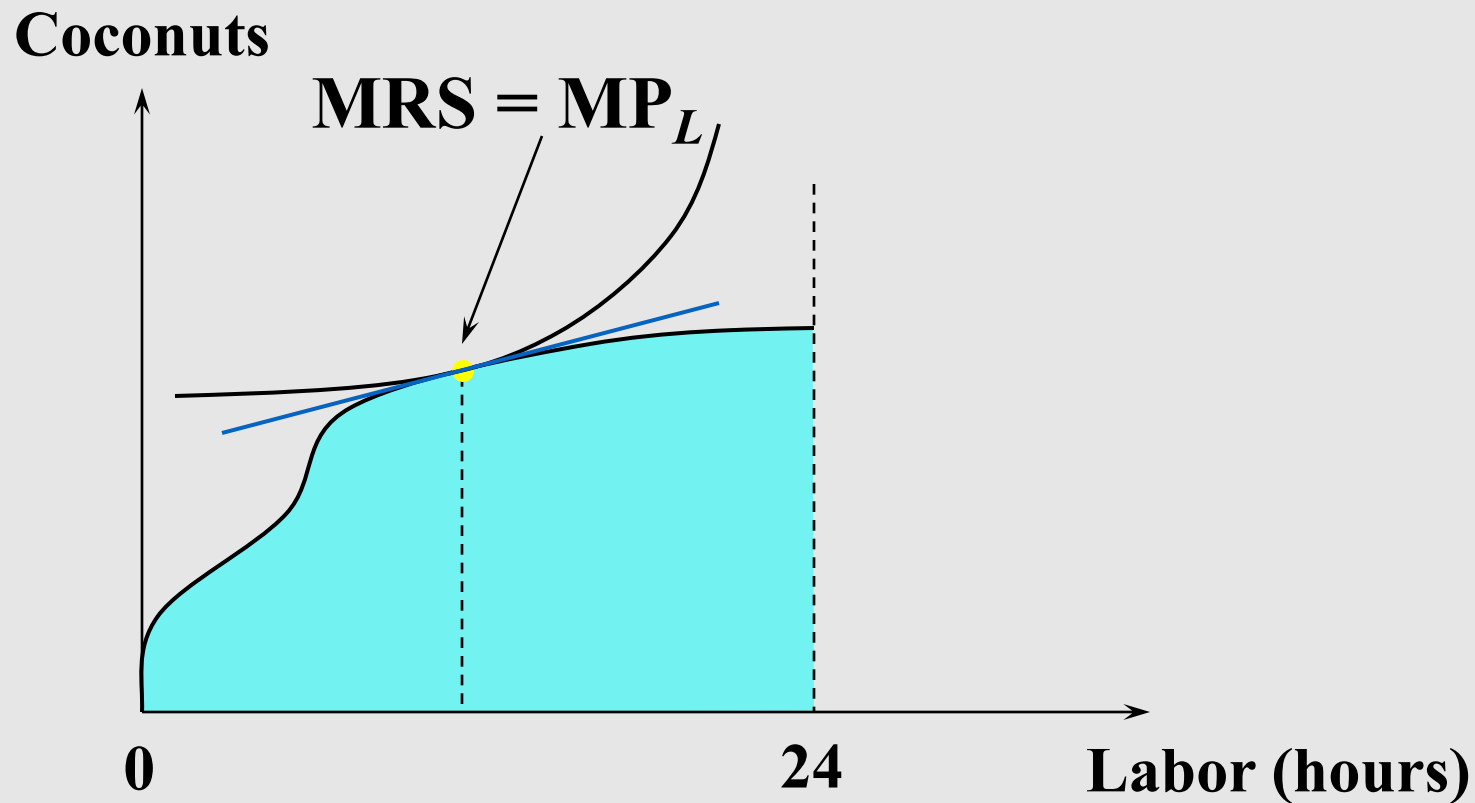
Second Fundamental Theorem of Welfare Economics

- Any Pareto efficient economic state can be achieved as a competitive market equilibrium if
- consumers' preferences are convex
 - firms' technologies are convex: feasible production plans are a convex set
 - For every two points in the set, the line that joins them belong to the set
 - there are no externalities in consumption or production.

Non-Convex Technologies

- Do the Welfare Theorems hold if firms have non-convex technologies?
- The 1st Theorem does not rely upon firms' technologies being convex.

Non-Convex Technologies

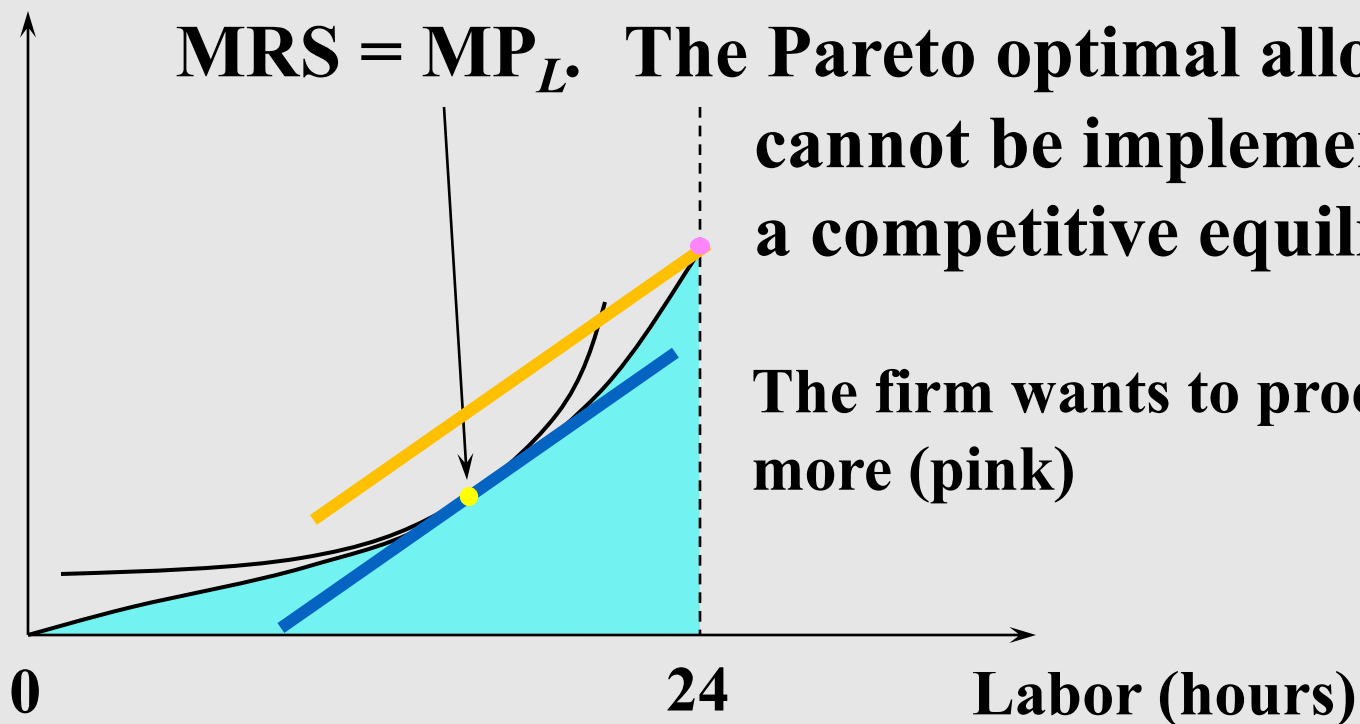


Non-Convex Technologies

- Do the Welfare Theorems hold if firms have non-convex technologies?
- The 2nd Theorem **does** require that firms' technologies be convex.
 - This is, the feasible production plans are a convex set
 - For every two points in the set, the line that joins them belong to the set
 - This means that all situations with **increasing returns to scale are ruled out.**

Non-Convex Technologies

Coconuts



4. Production Possibilities

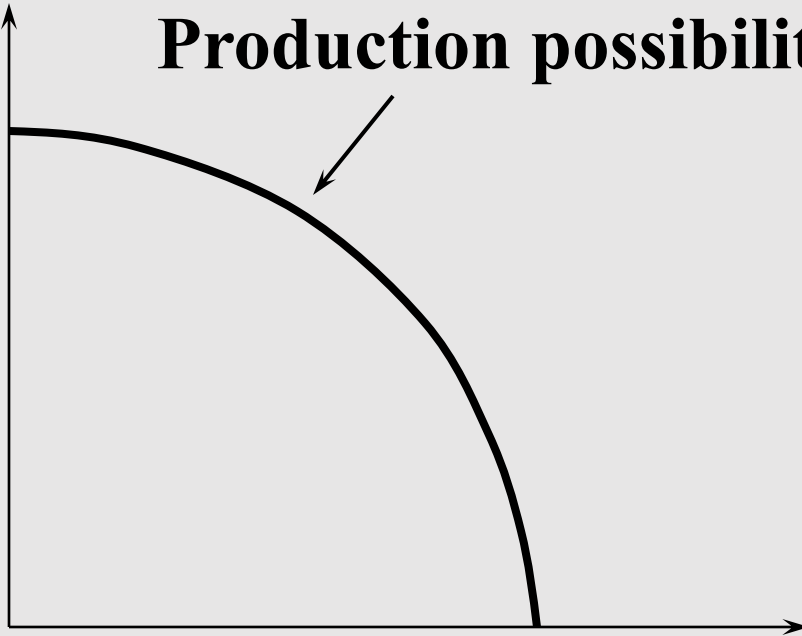
Let's generalize to two outputs

- Resource and technological limitations restrict what an economy can produce.
- The set of all feasible output bundles is the economy's **production possibility set**.
- The set's outer boundary is the **production possibility frontier**.
- Difference: a function of the two outputs, inputs are not in the graph anymore

Production Possibilities

Coconuts

Production possibility frontier (ppf)



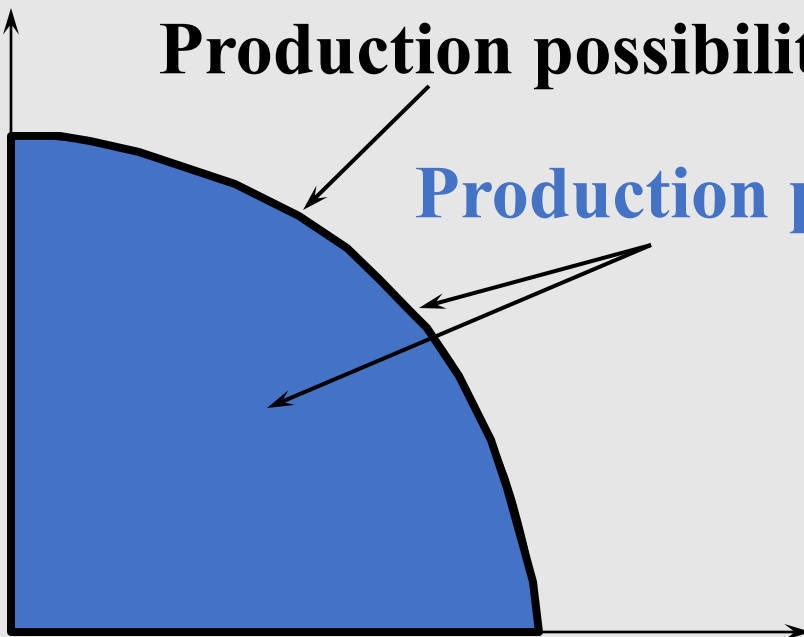
Fish

Production Possibilities

Coconuts

Production possibility frontier (ppf)

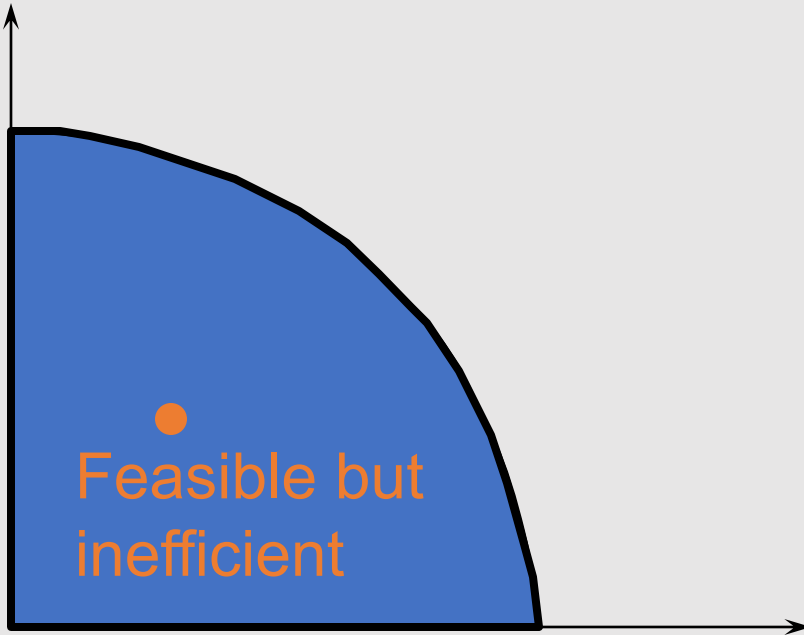
Production possibility set



Fish

Production Possibilities

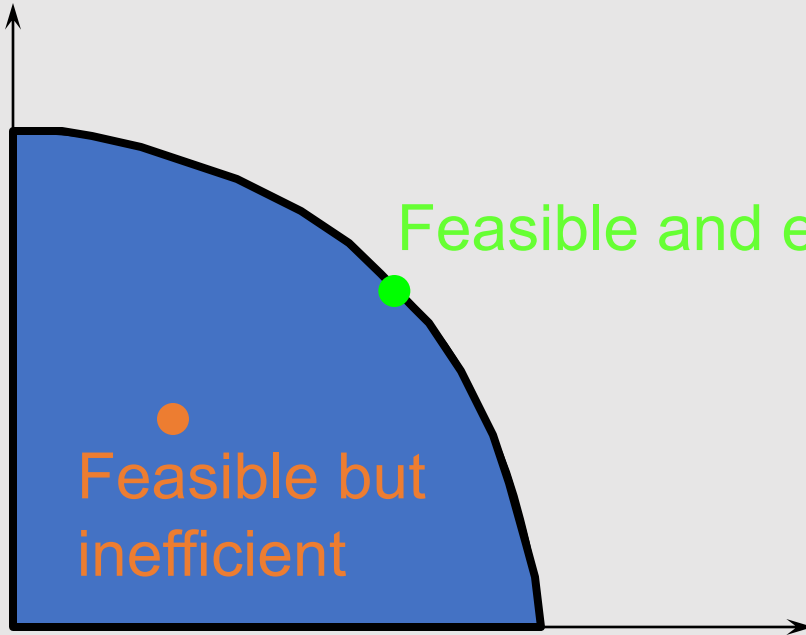
Coconuts



Fish

Production Possibilities

Coconuts



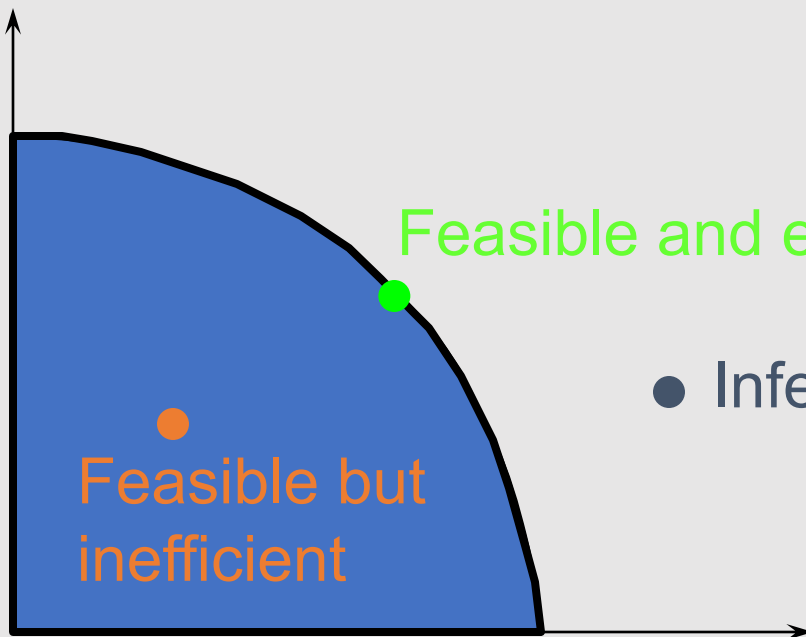
Feasible and efficient

Feasible but
inefficient

Fish

Production Possibilities

Coconuts



Feasible and efficient

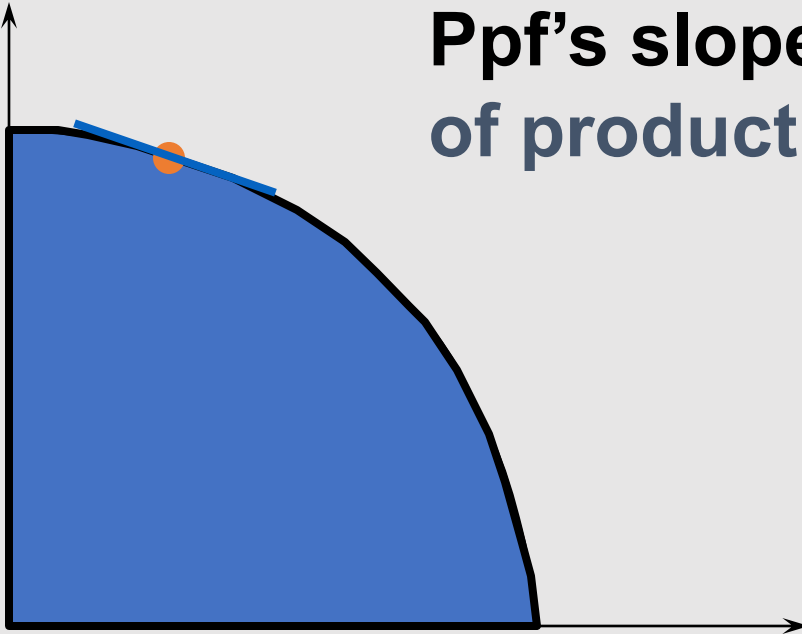
● Infeasible

Feasible but
inefficient

Fish

Production Possibilities

Coconuts

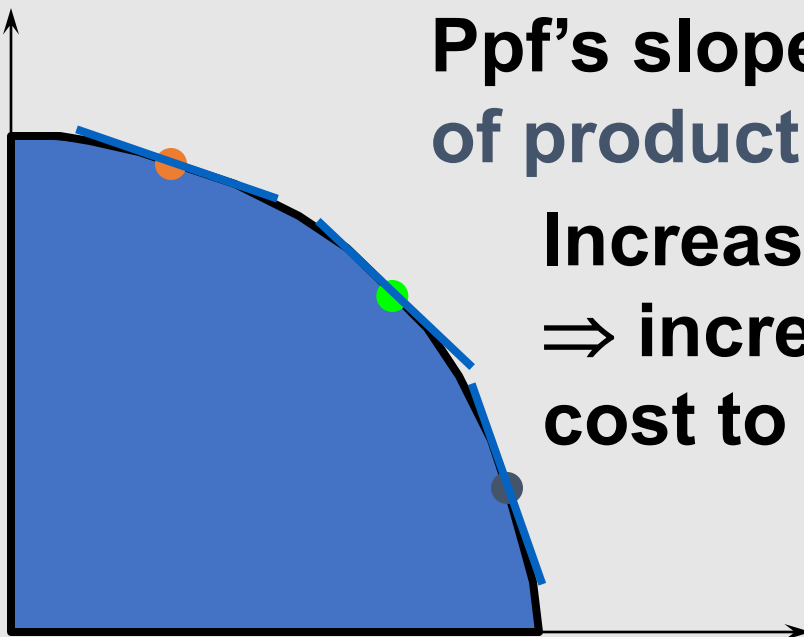


Ppf's slope is the marginal rate of product transformation.

Fish

Production Possibilities

Coconuts



Ppf's slope is the marginal rate of product transformation.

**Increasingly negative MRPT
⇒ increasing opportunity
cost to specialization.**

Fish

Production Possibilities

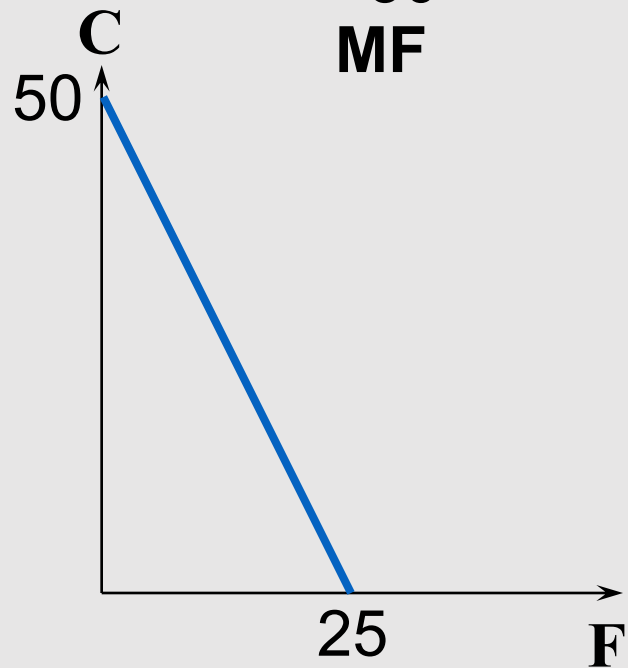
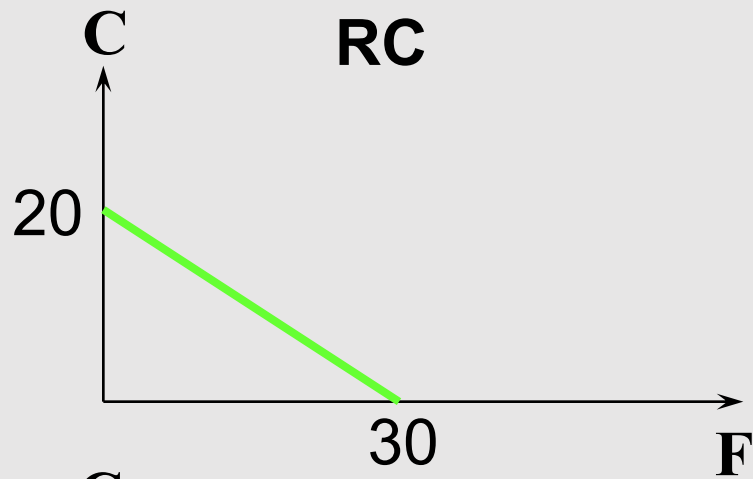
- The construction of the ppf was quite simple since there was only one way to produce fish and coconuts.
- What if there is more than one way to produce each good?
- Let's add another worker with different skills in producing fish and coconuts.

5. Coordinating Production & Consumption

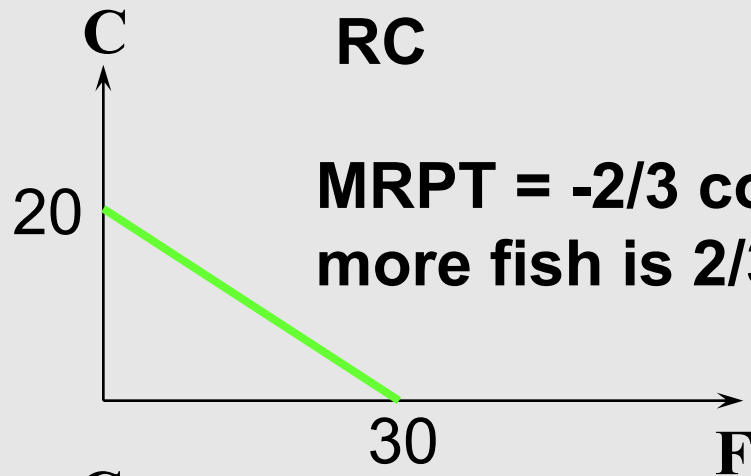
- Two agents, RC and Man Friday (MF).
- RC can produce at most 20 coconuts or 30 fish.
- MF can produce at most 50 coconuts or 25 fish.

Who produces what? → this will depend on their **comparative advantage**

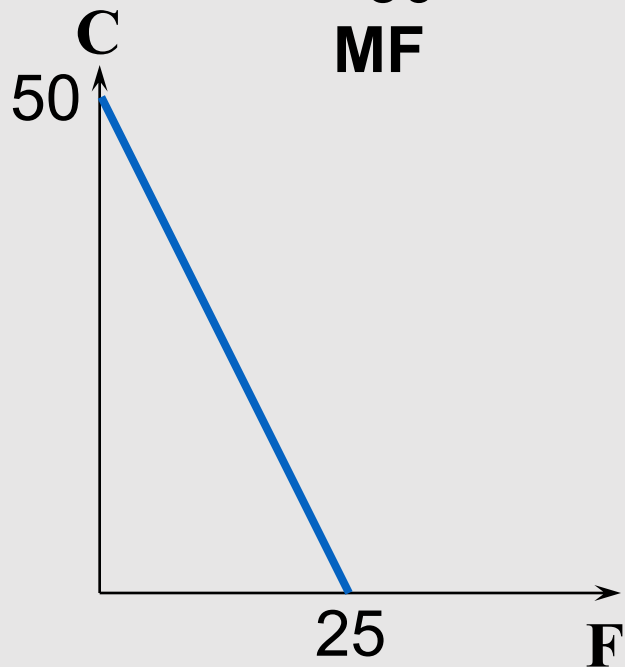
Comparative Advantage



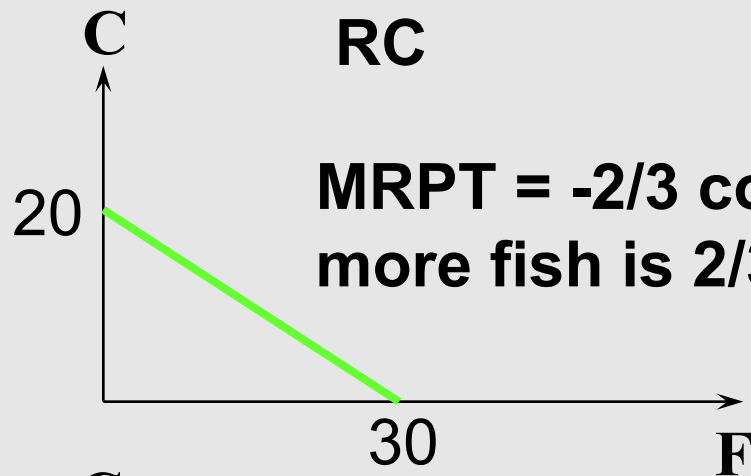
Comparative Advantage



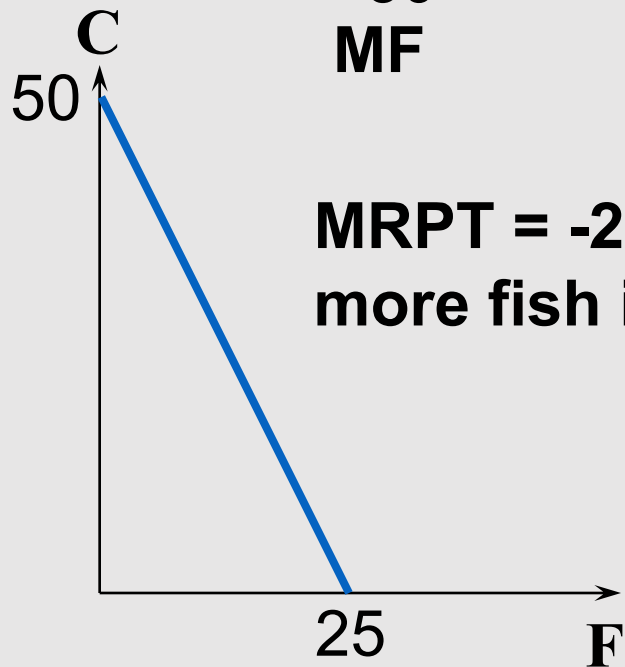
MRPT = $-2/3$ coconuts/fish so opp. cost of one more fish is $2/3$ foregone coconuts.



Comparative Advantage

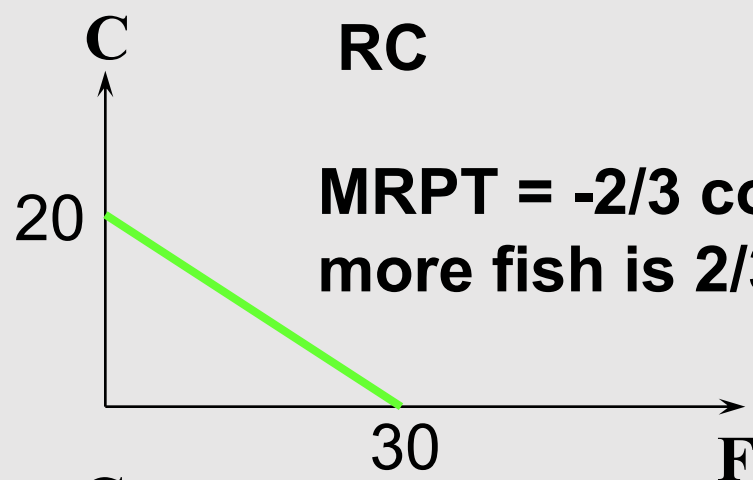


MRPT = $-\frac{2}{3}$ coconuts/fish so opp. cost of one more fish is $\frac{2}{3}$ foregone coconuts.



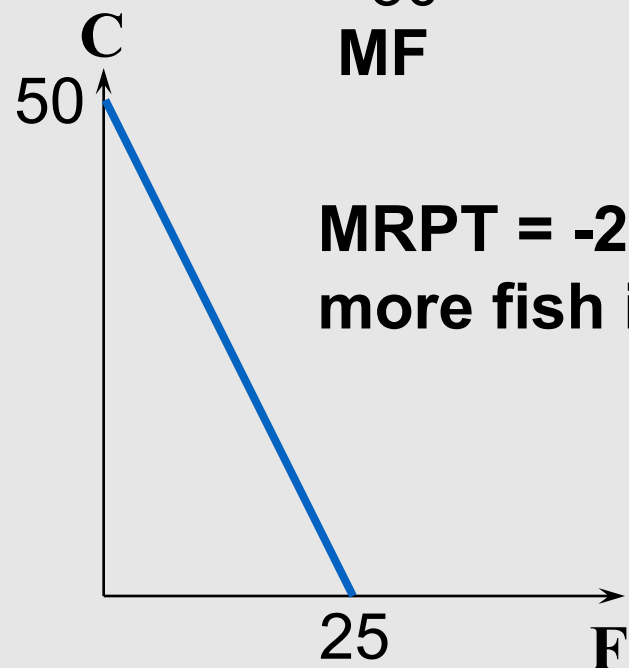
MRPT = -2 coconuts/fish so opp. cost of one more fish is 2 foregone coconuts.

Comparative Advantage



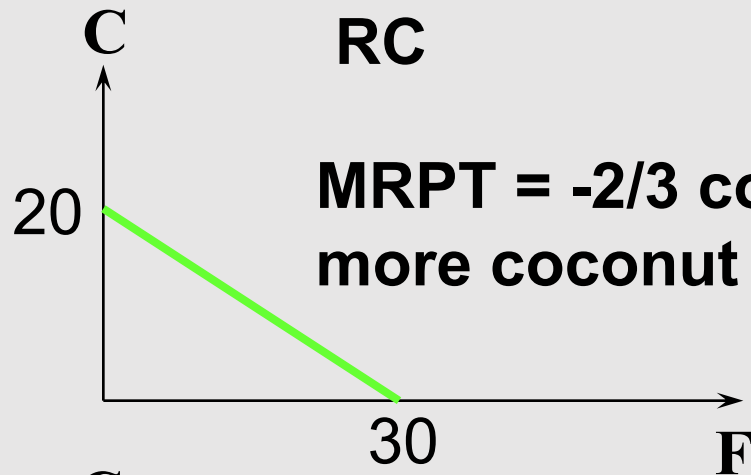
MRPT = $-2/3$ coconuts/fish so opp. cost of one more fish is $2/3$ foregone coconuts.

RC has the comparative opp. cost advantage in producing fish.

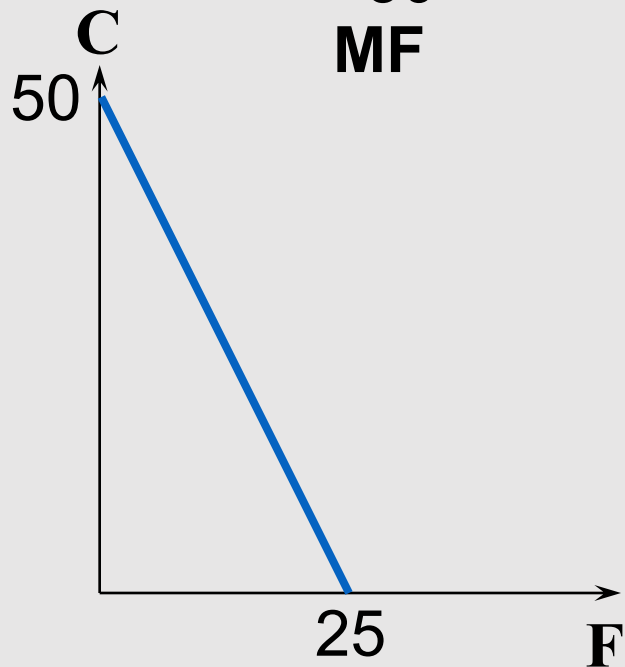


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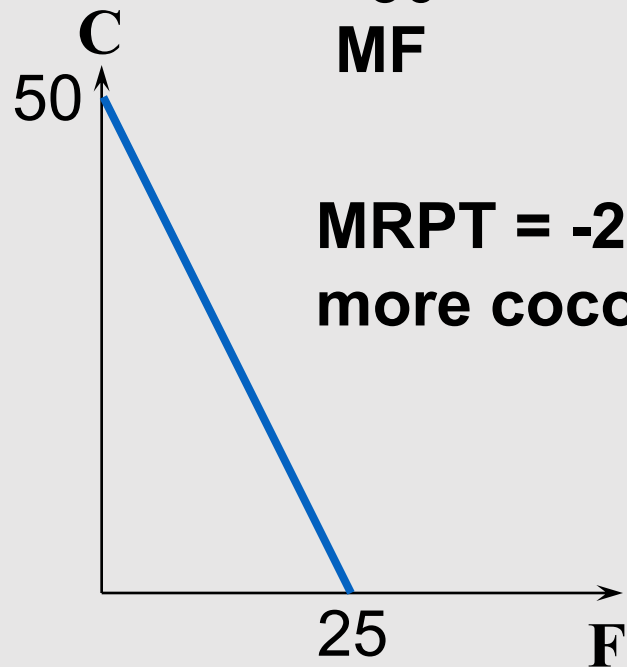
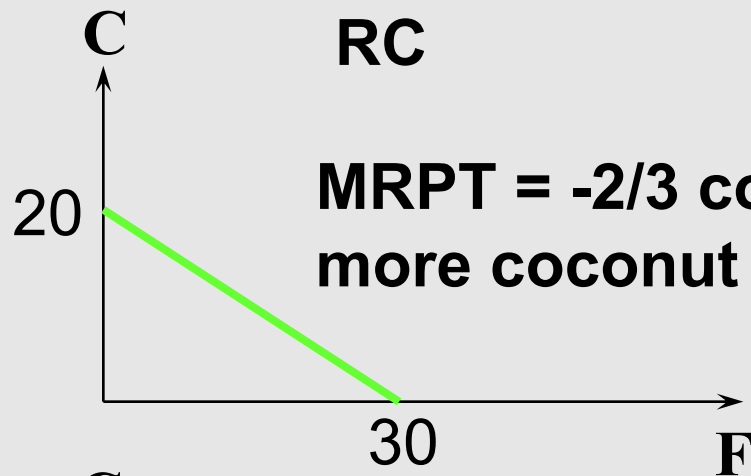
Comparative Advantage



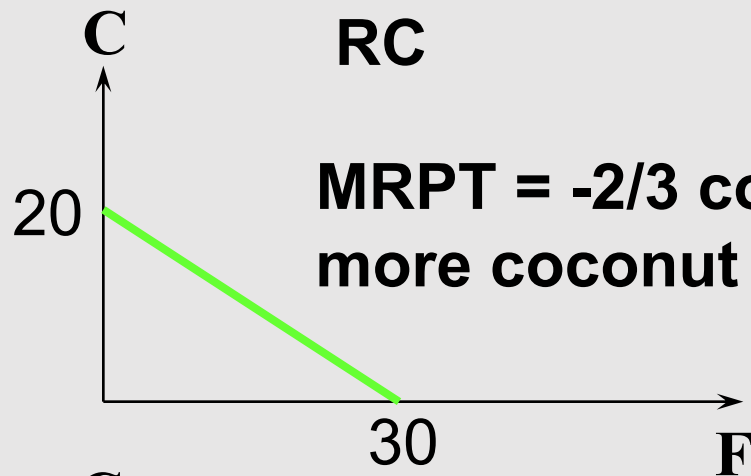
MRPT = $-2/3$ coconuts/fish so opp. cost of one more coconut is $3/2$ foregone fish.



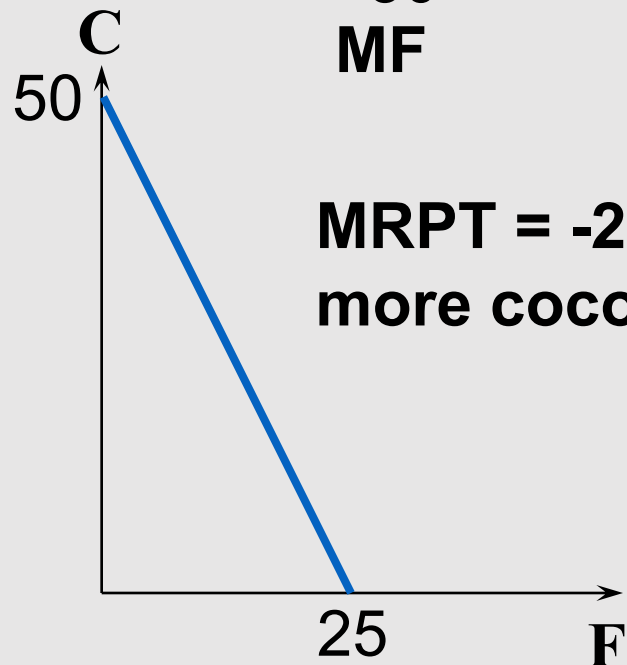
Comparative Advantage



Comparative Advantage



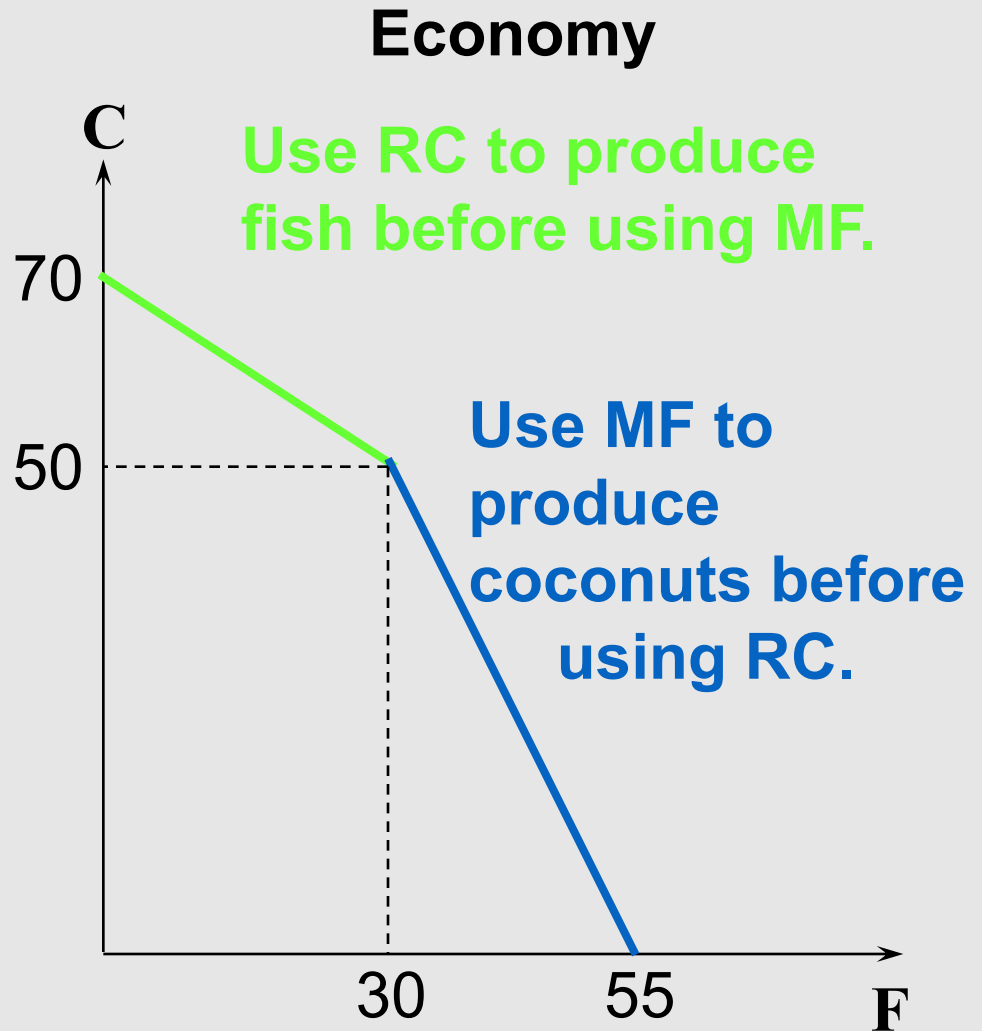
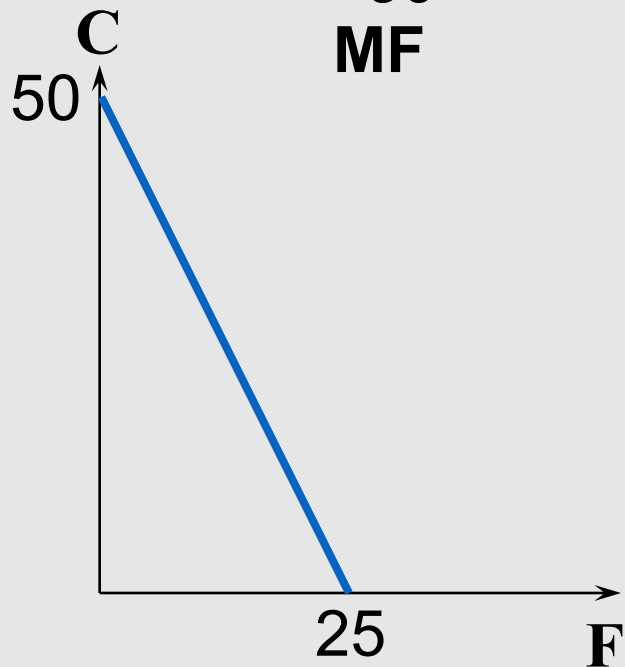
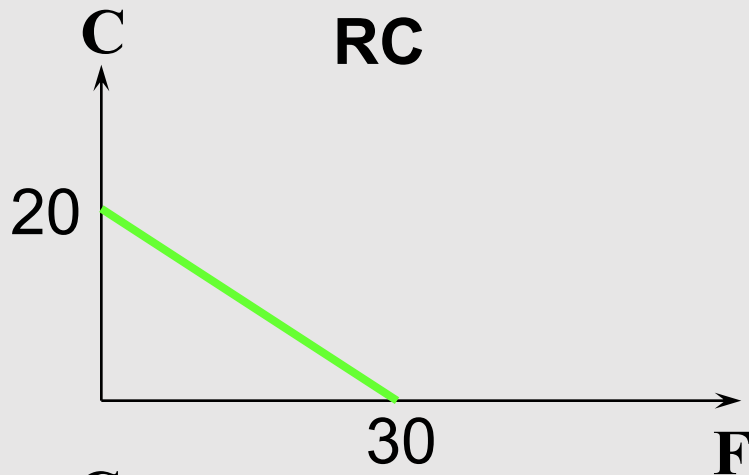
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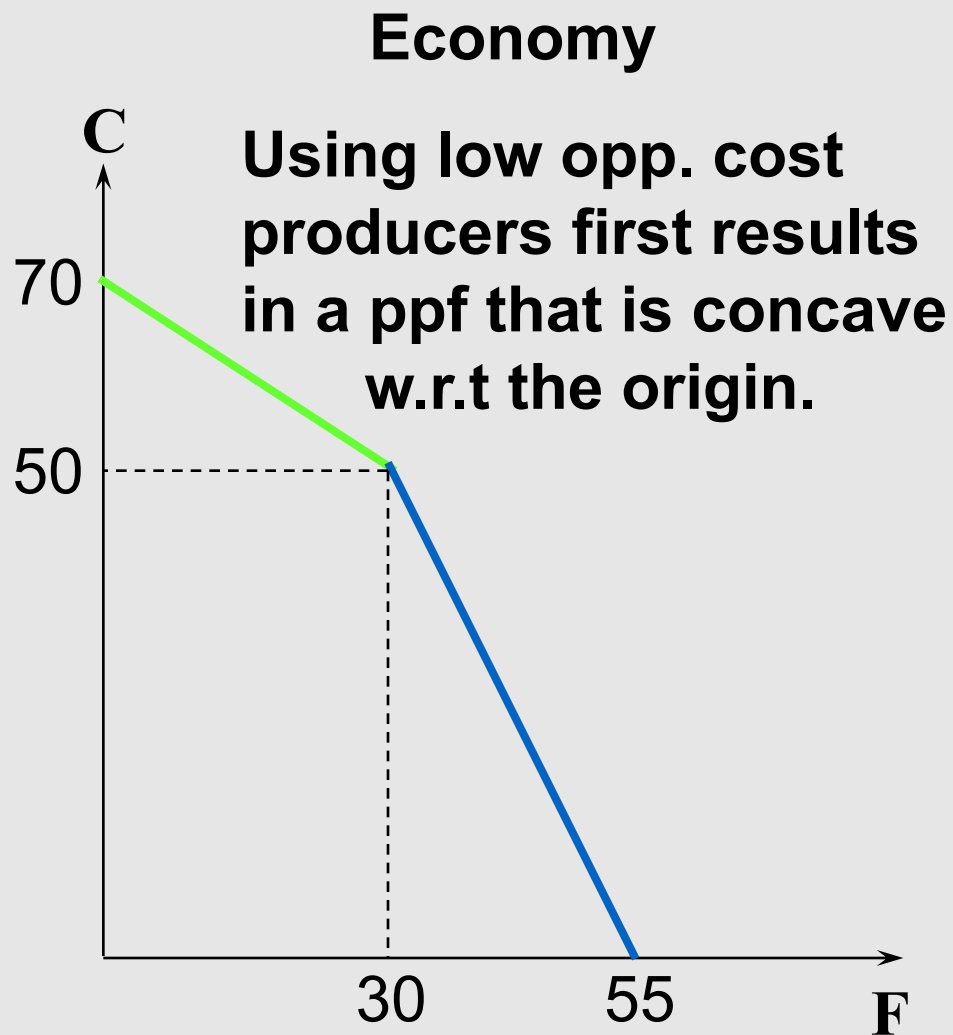
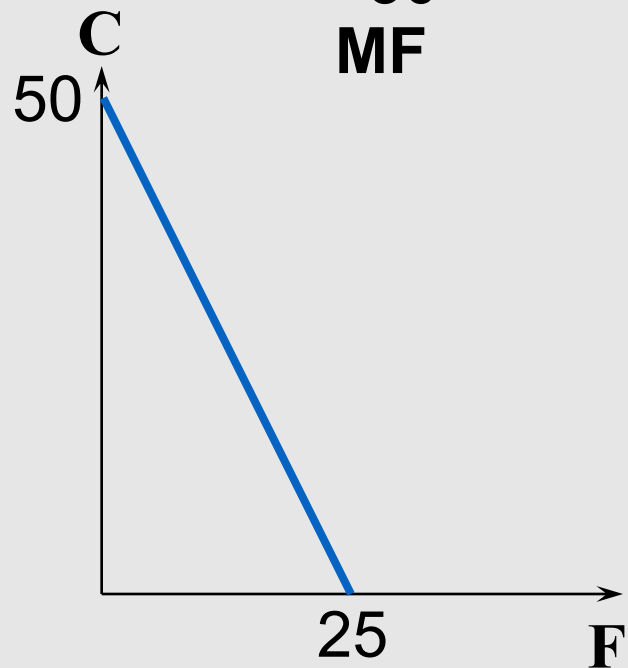
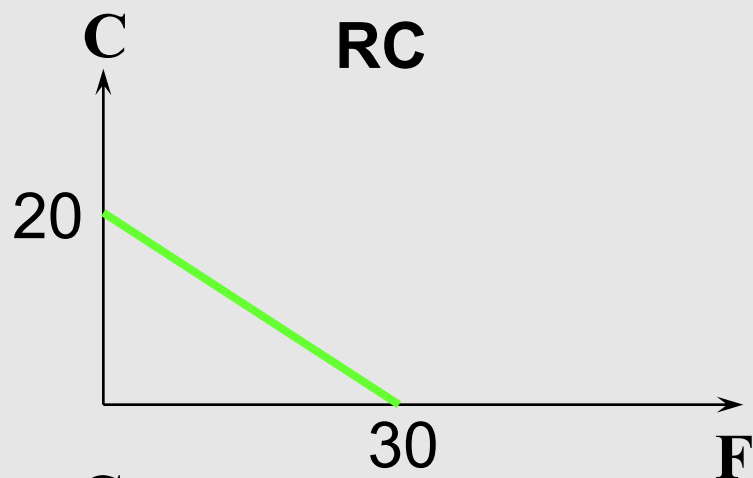
MRPT = -2 coconuts/fish so opp. cost of one more coconut is $1/2$ foregone fish.

MF has the comparative opp. cost advantage in producing coconuts.

Comparative Advantage



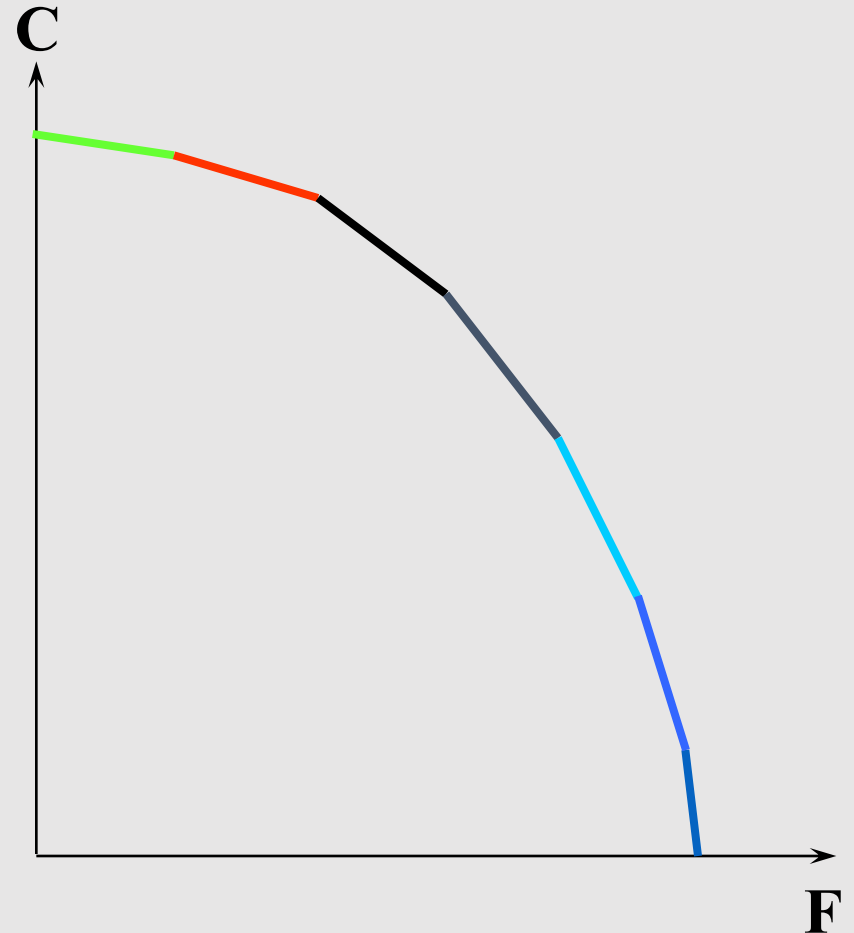
Comparative Advantage



Comparative Advantage

Economy

More producers with
different opp. costs
“smooth out” the ppf.

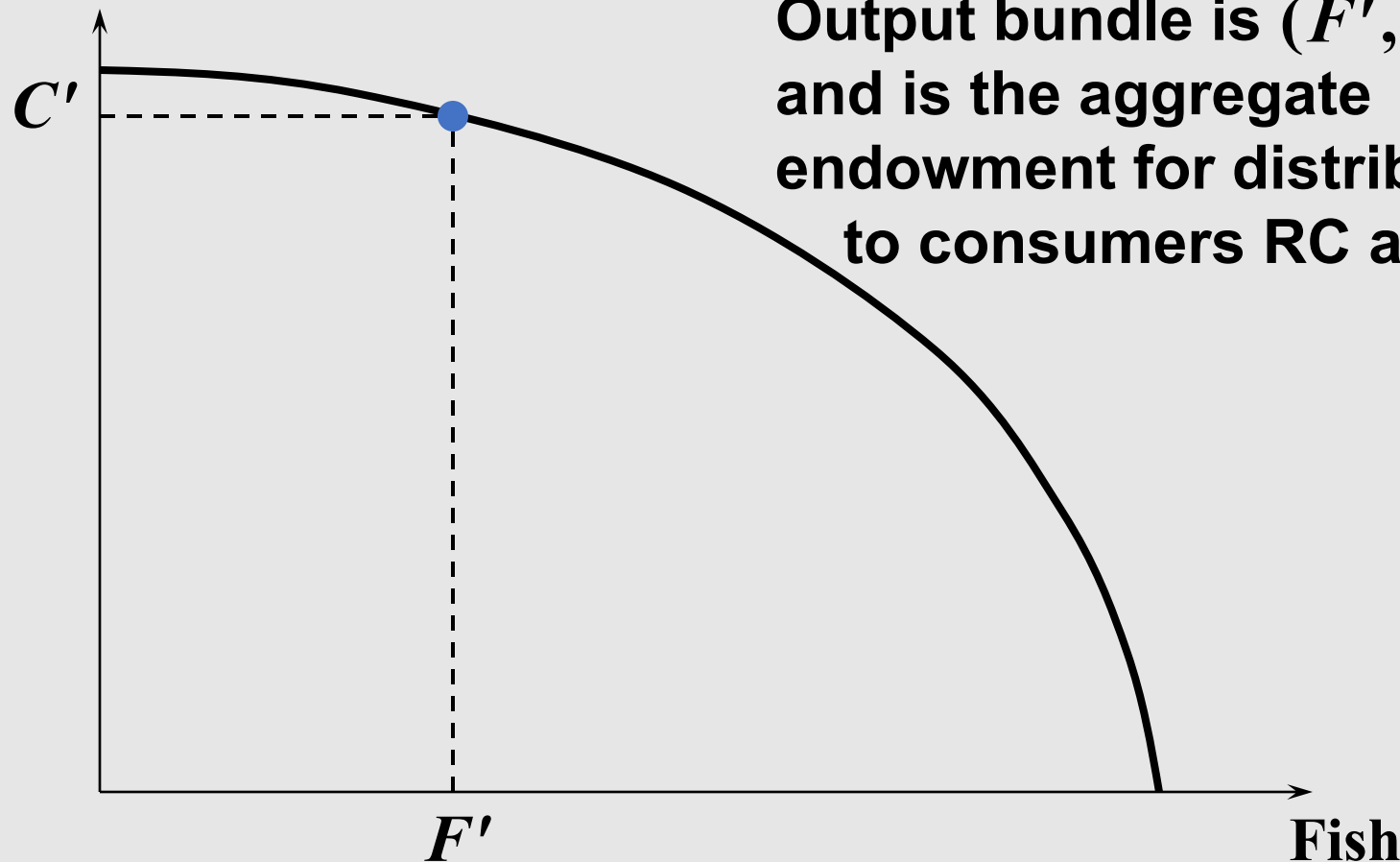


Coordinating Production & Consumption

- The ppf contains many technically efficient output bundles.
- Which are Pareto efficient for consumers?

Choosing the Pareto-efficient bundle

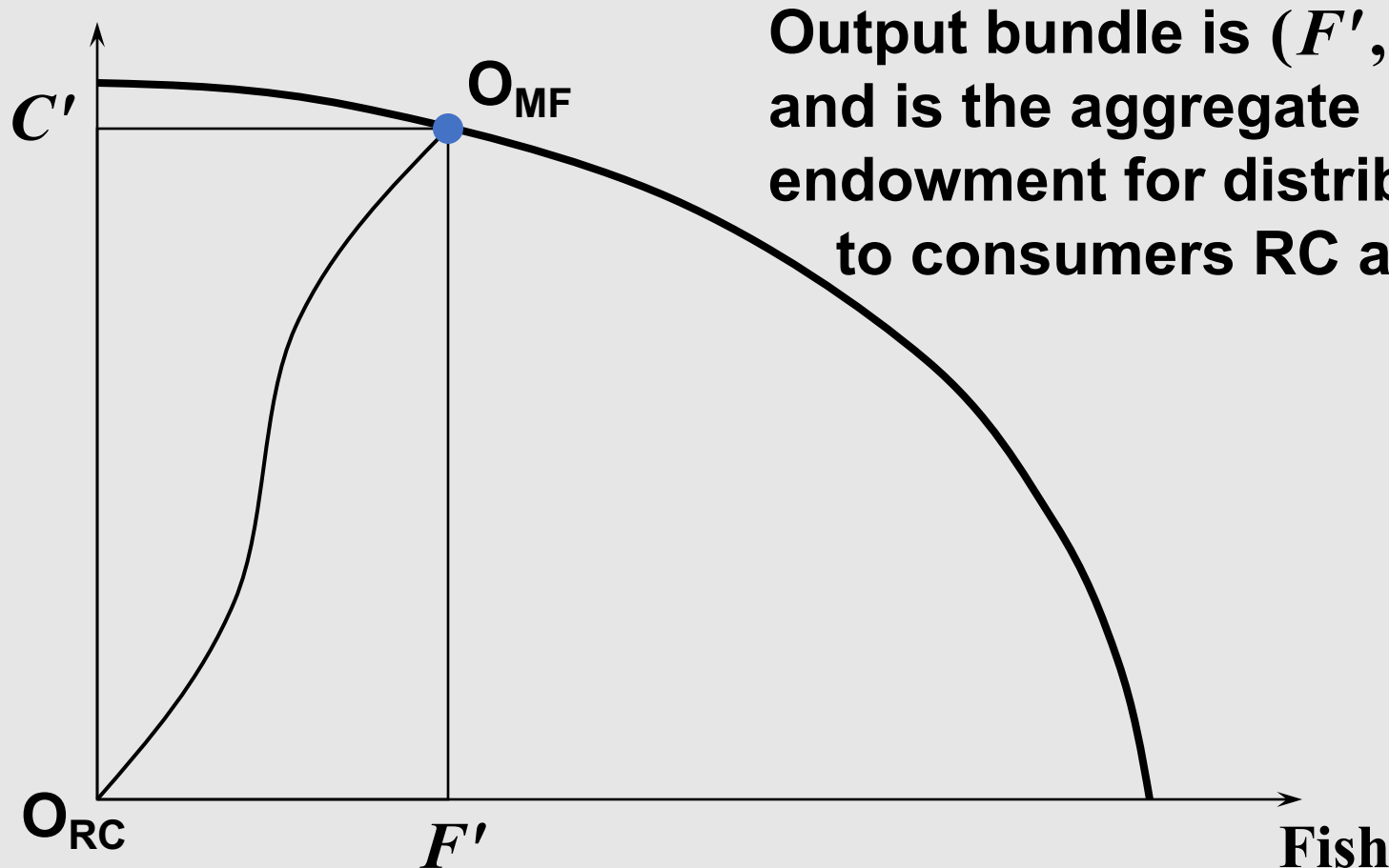
Coconuts



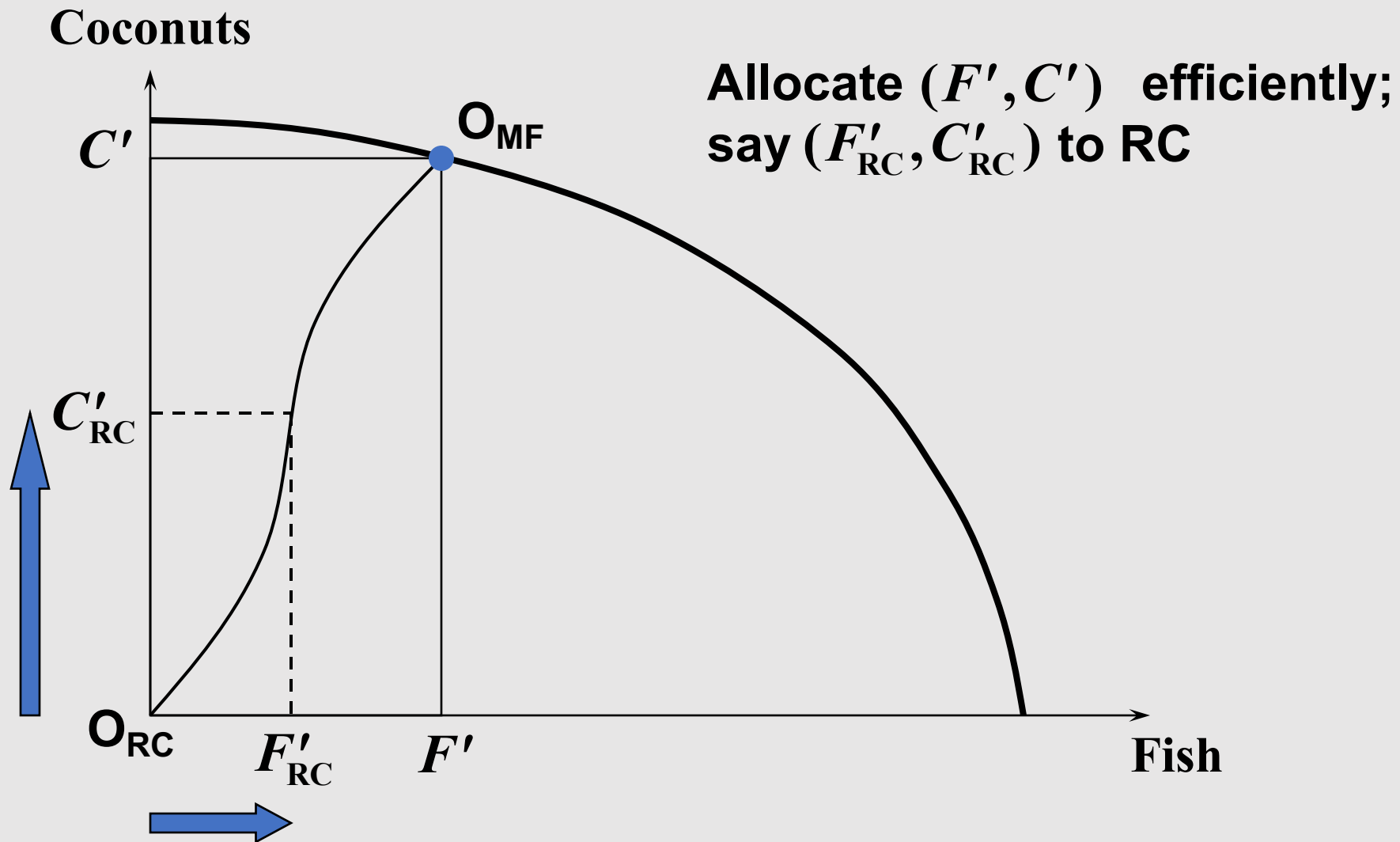
**Output bundle is (F', C')
and is the aggregate
endowment for distribution
to consumers RC and MF.**

Choosing the Pareto-efficient bundle

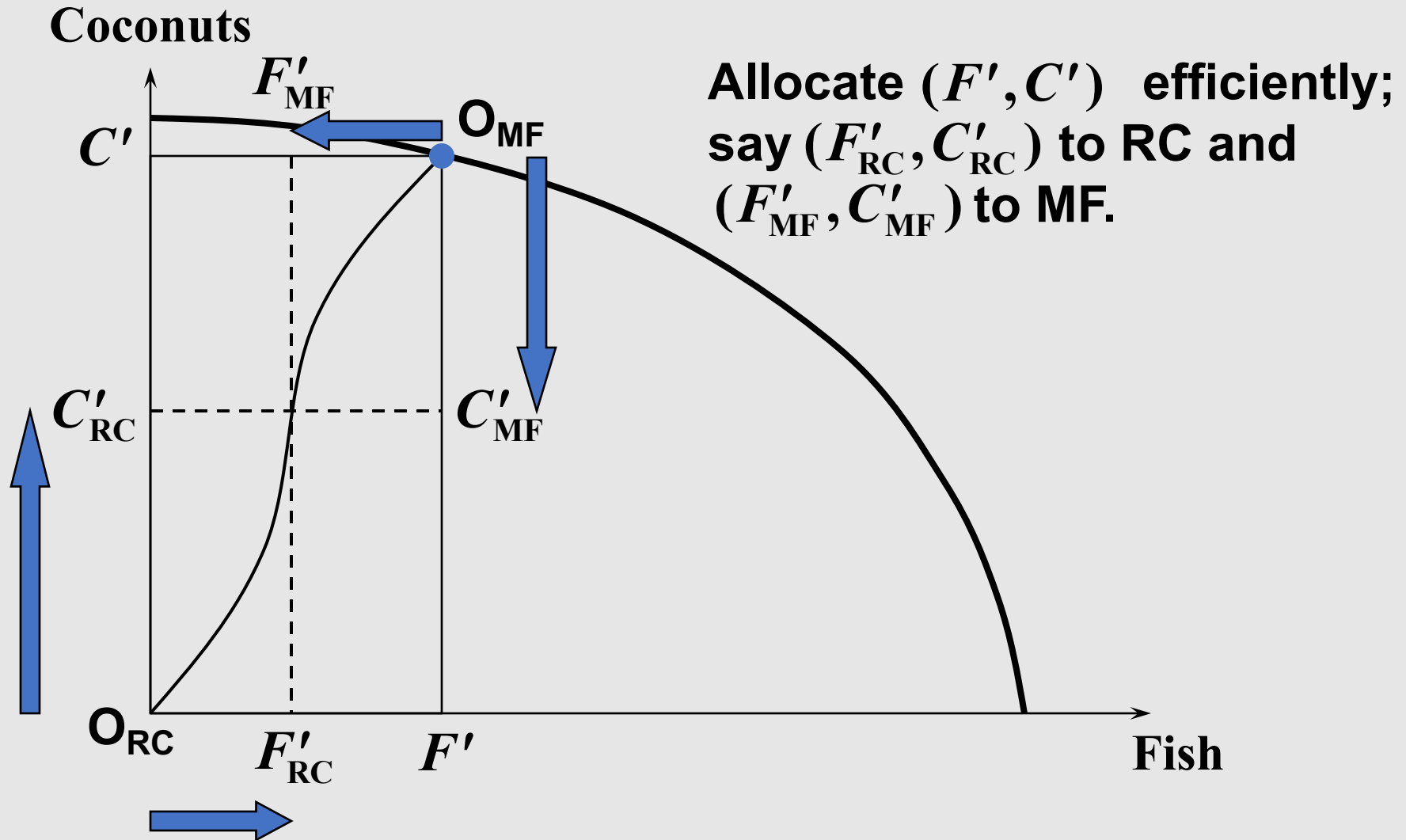
Coconuts



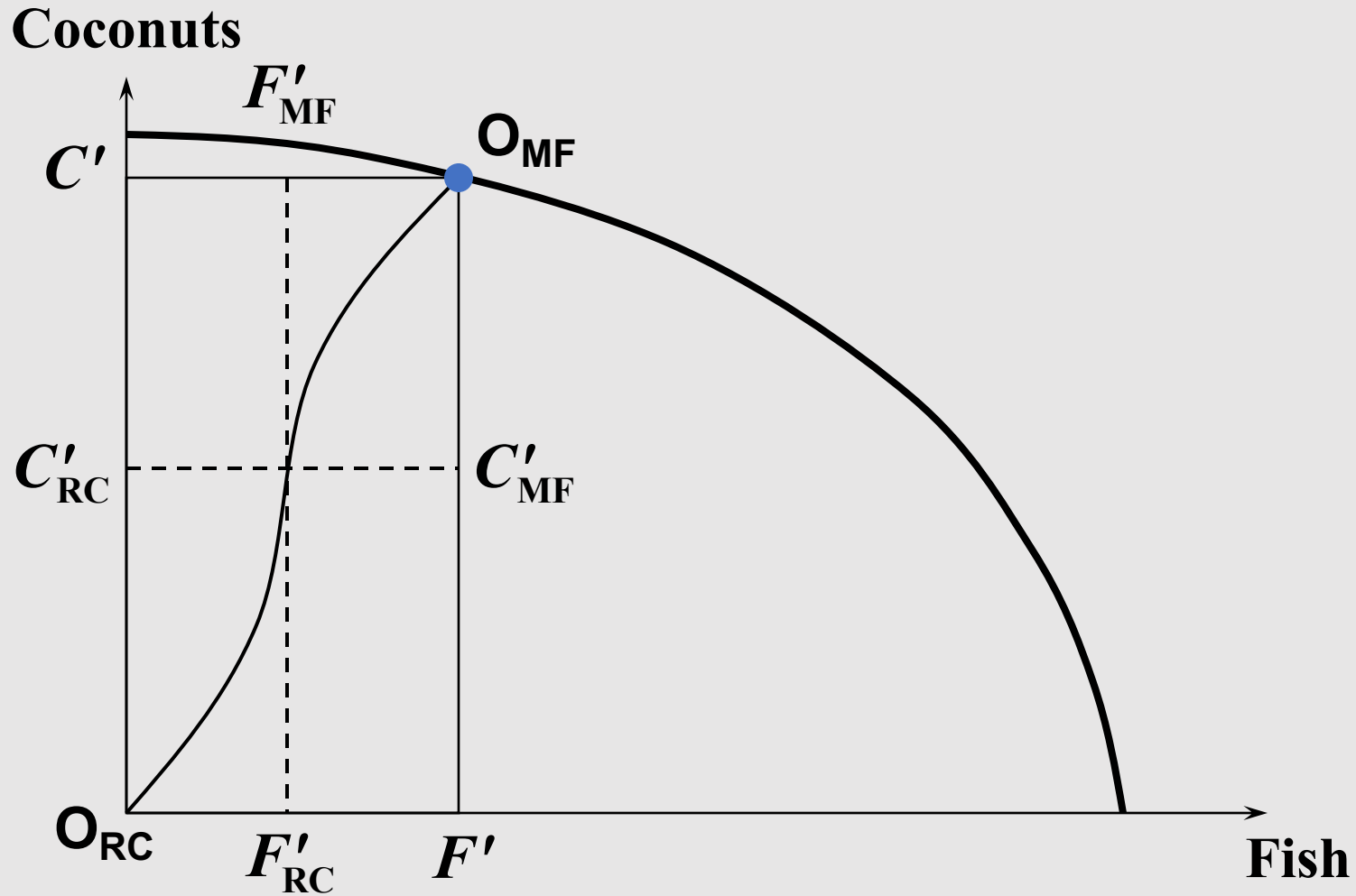
Choosing the Pareto-efficient bundle



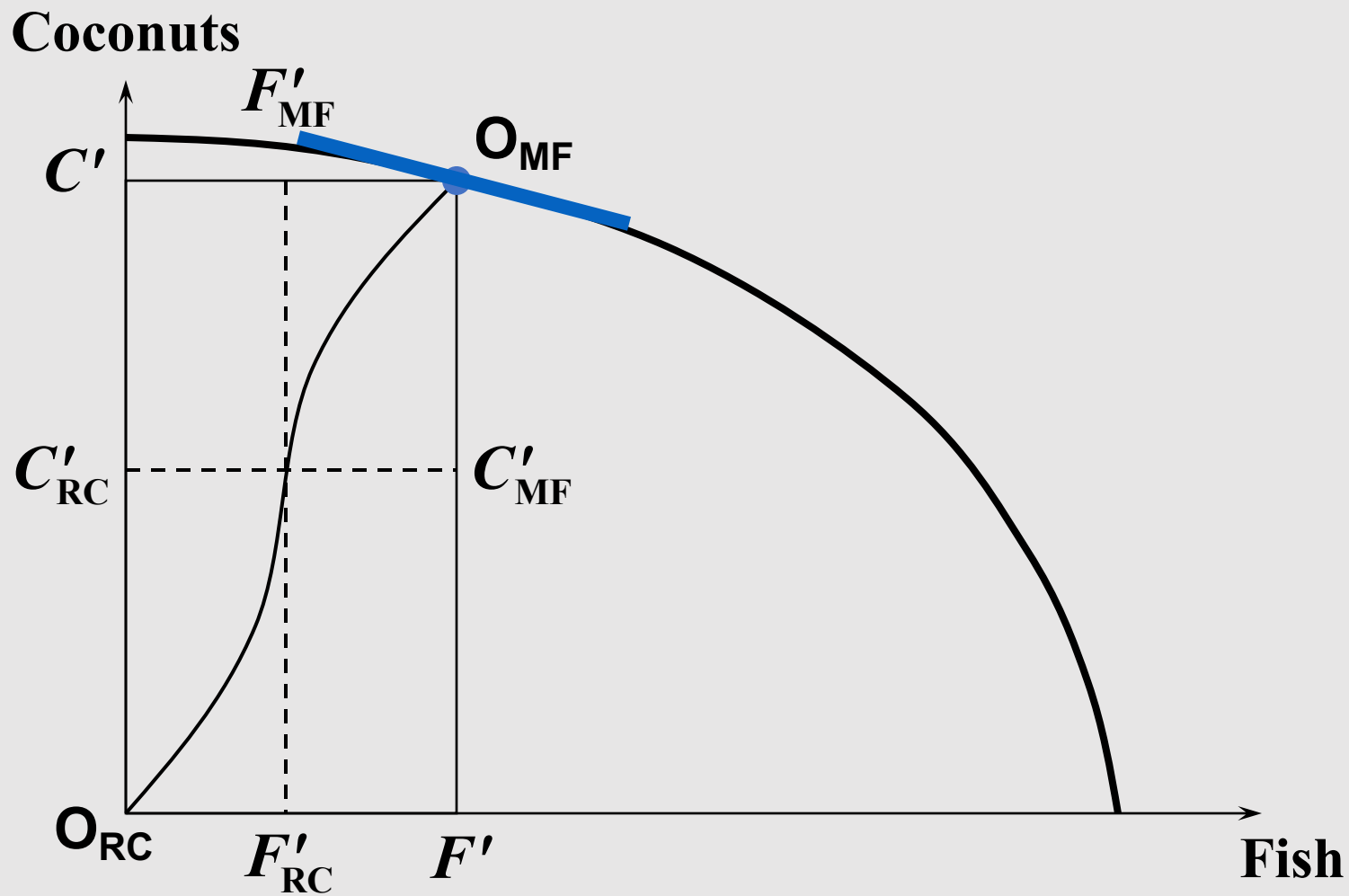
Choosing the Pareto-efficient bundle



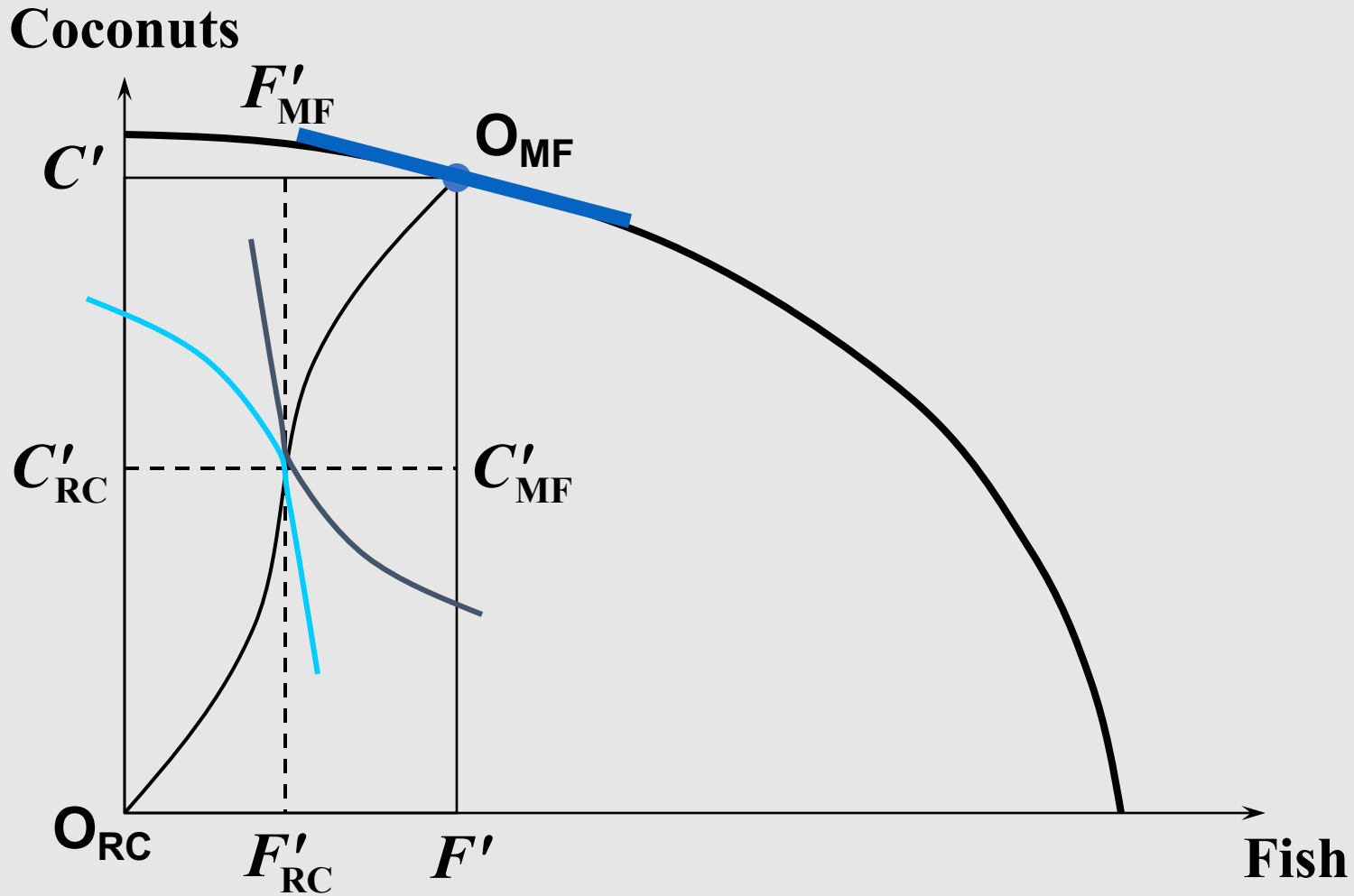
Choosing the Pareto-efficient bundle



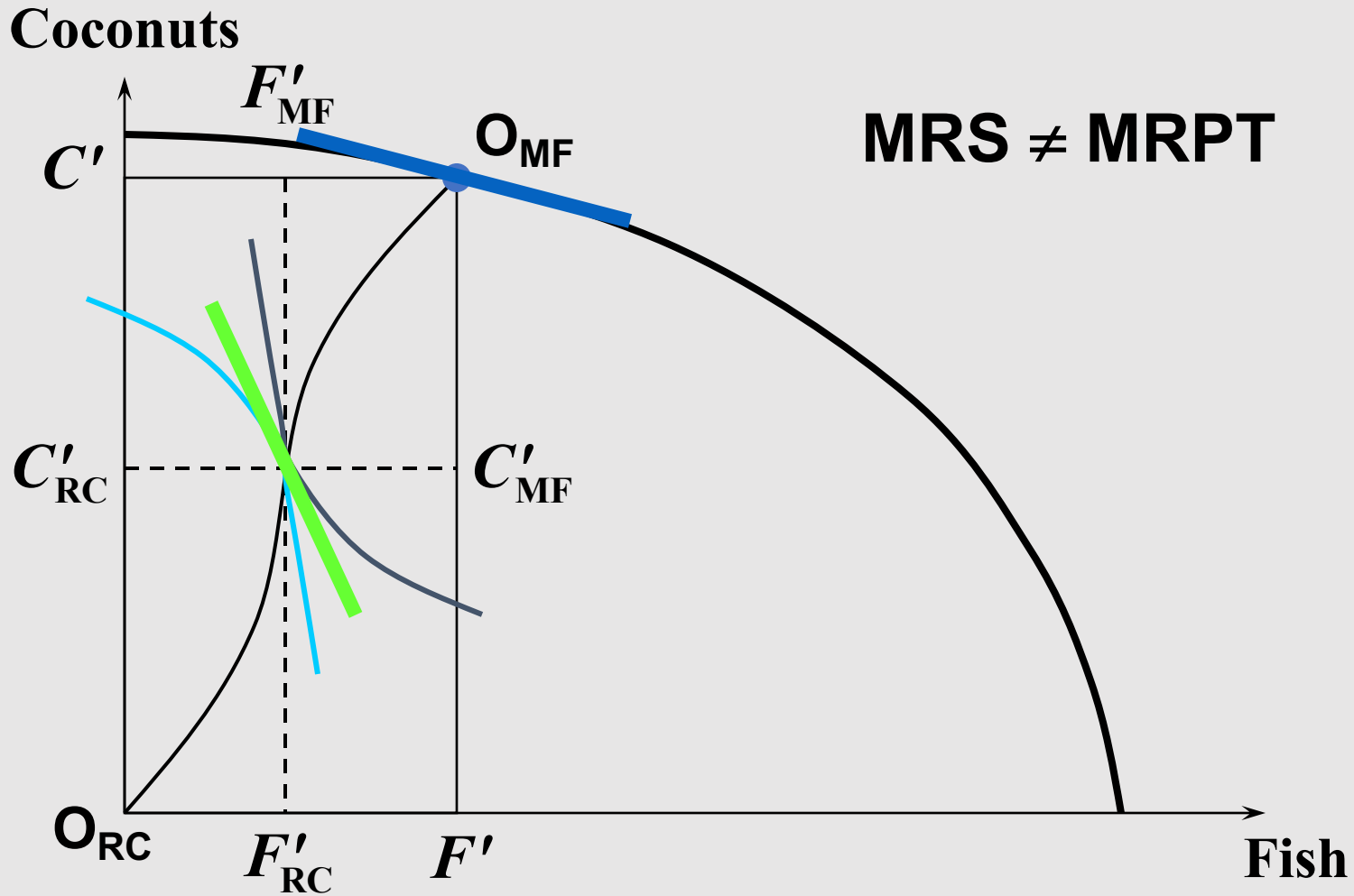
Choosing the Pareto-efficient bundle



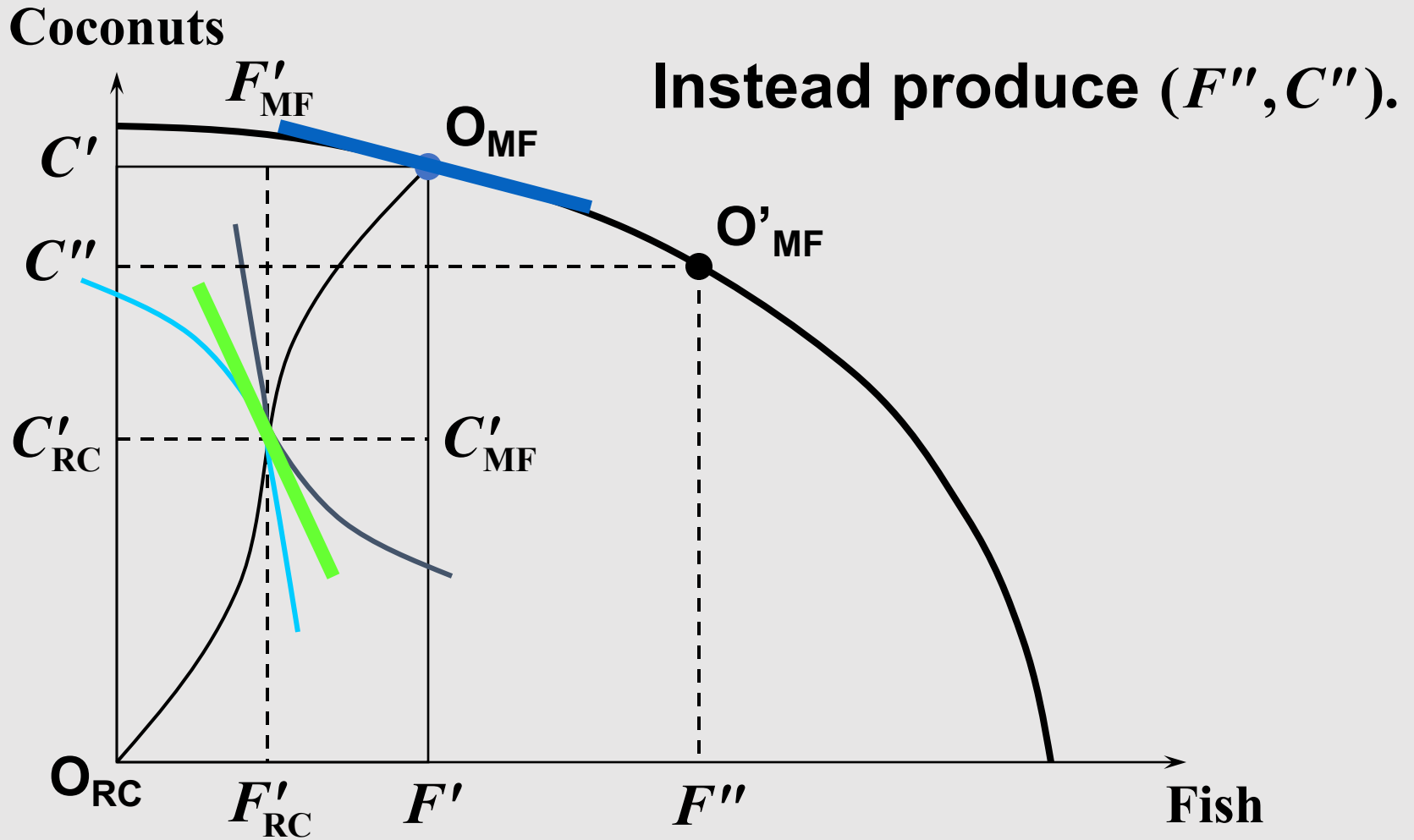
Choosing the Pareto-efficient bundle



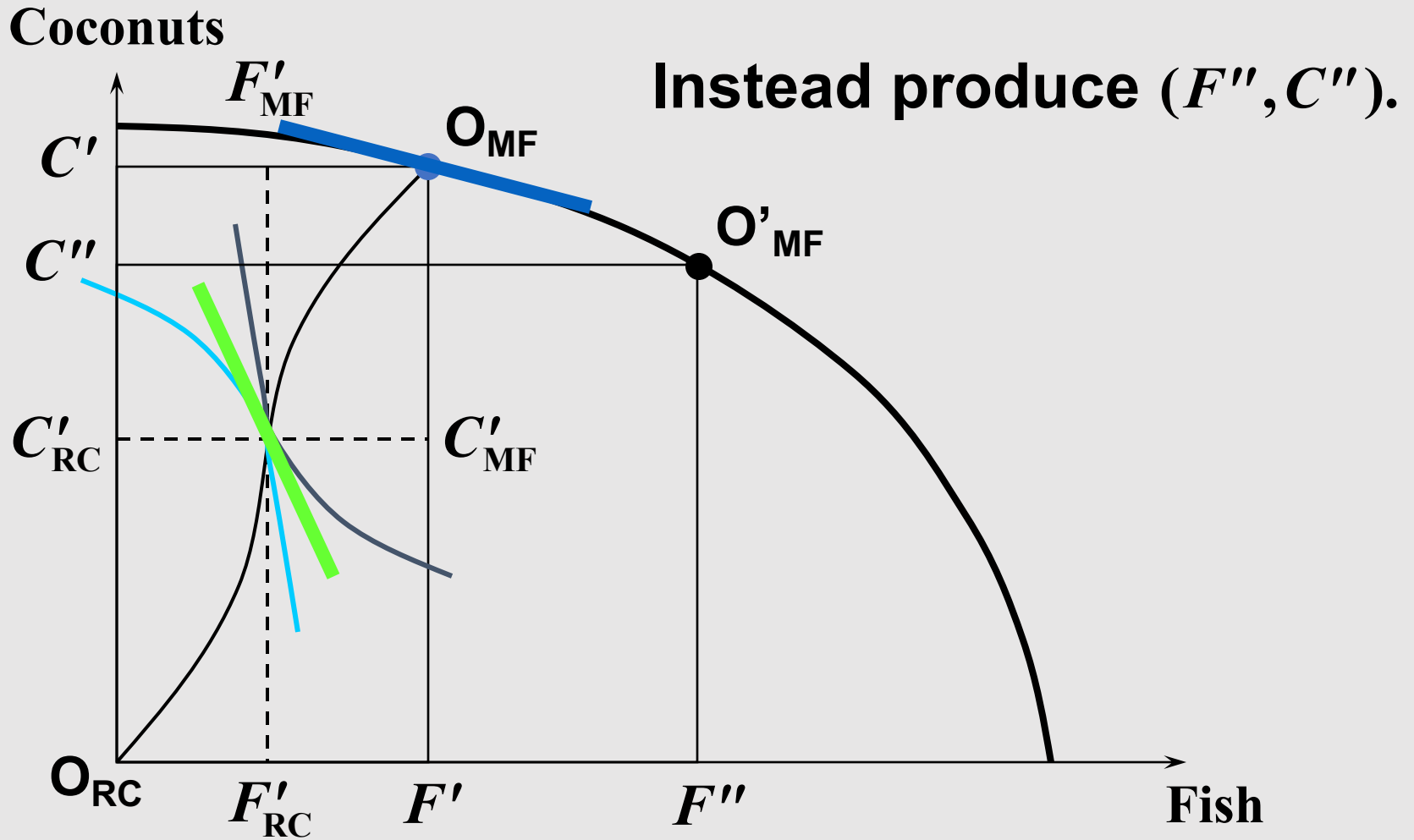
Choosing the Pareto-efficient bundle



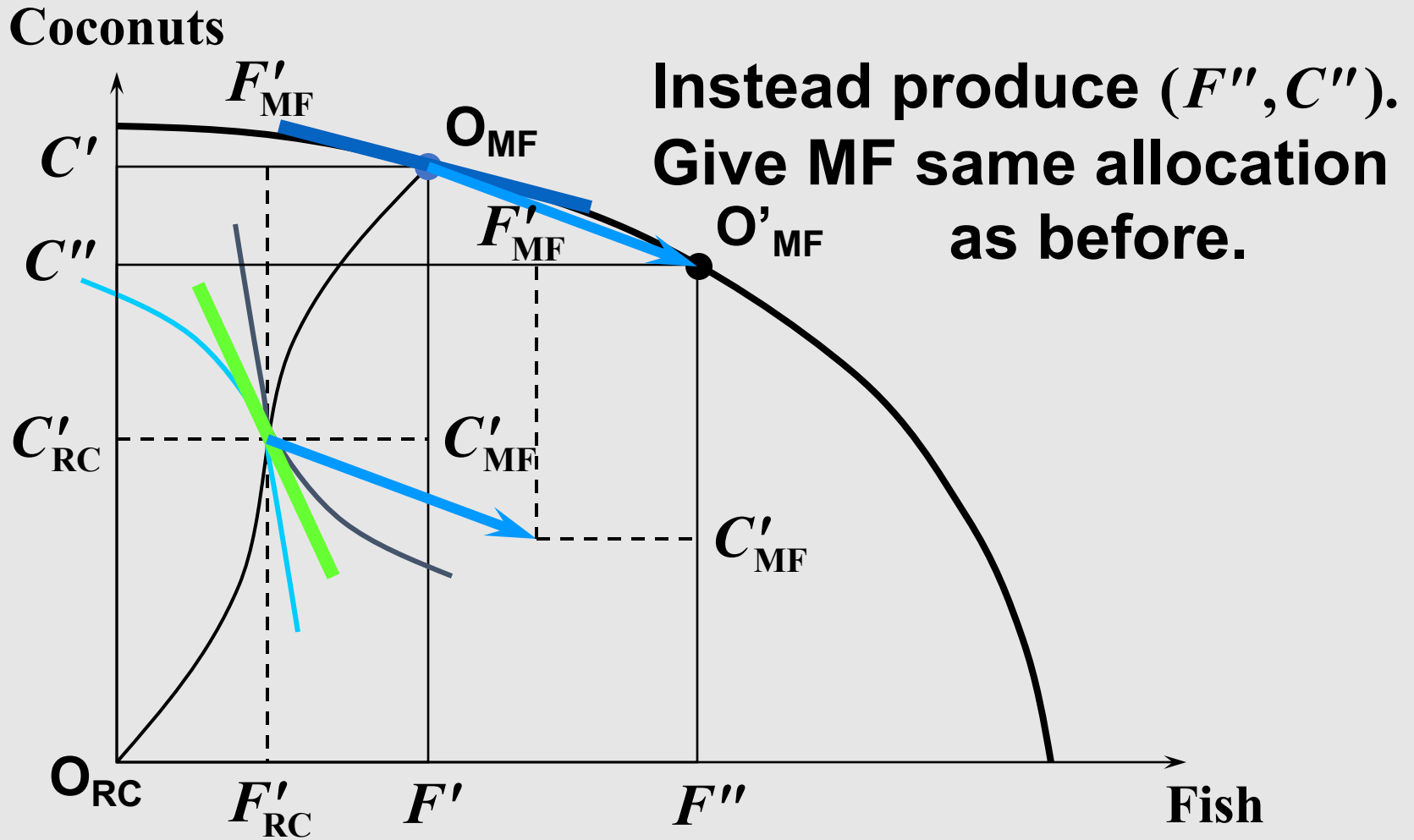
Choosing the Pareto-efficient bundle



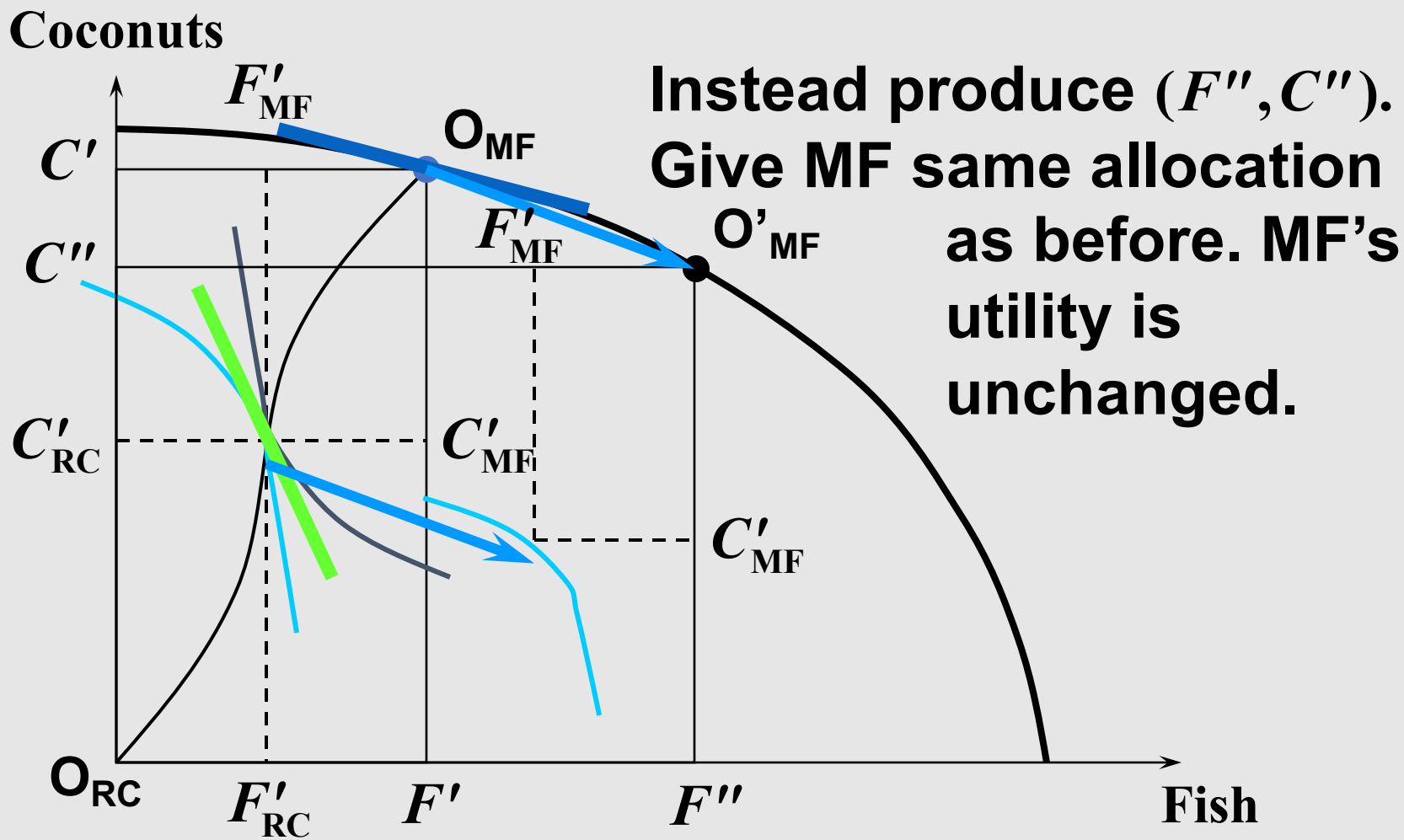
Choosing the Pareto-efficient bundle



Choosing the Pareto-efficient bundle

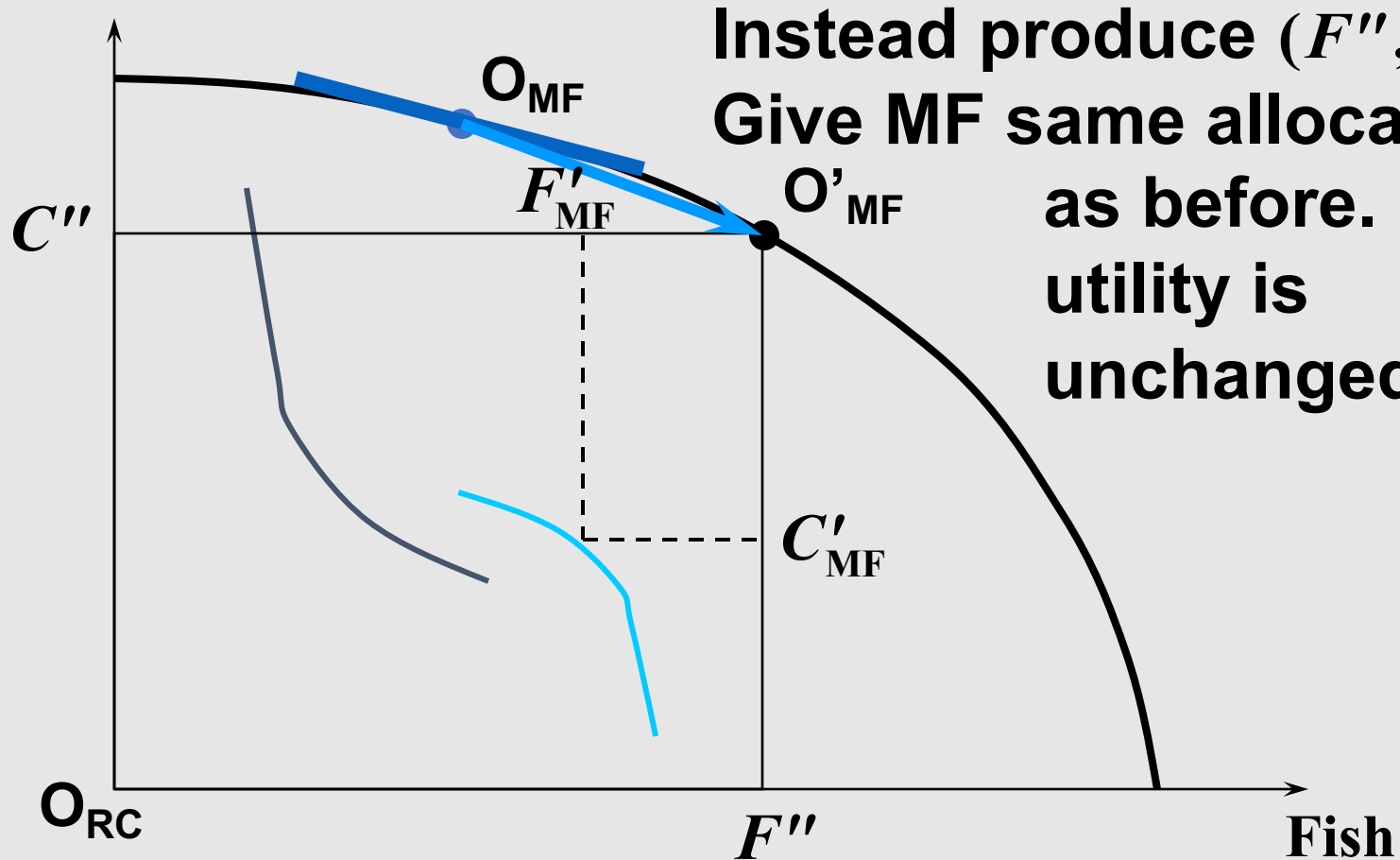


Choosing the Pareto-efficient bundle

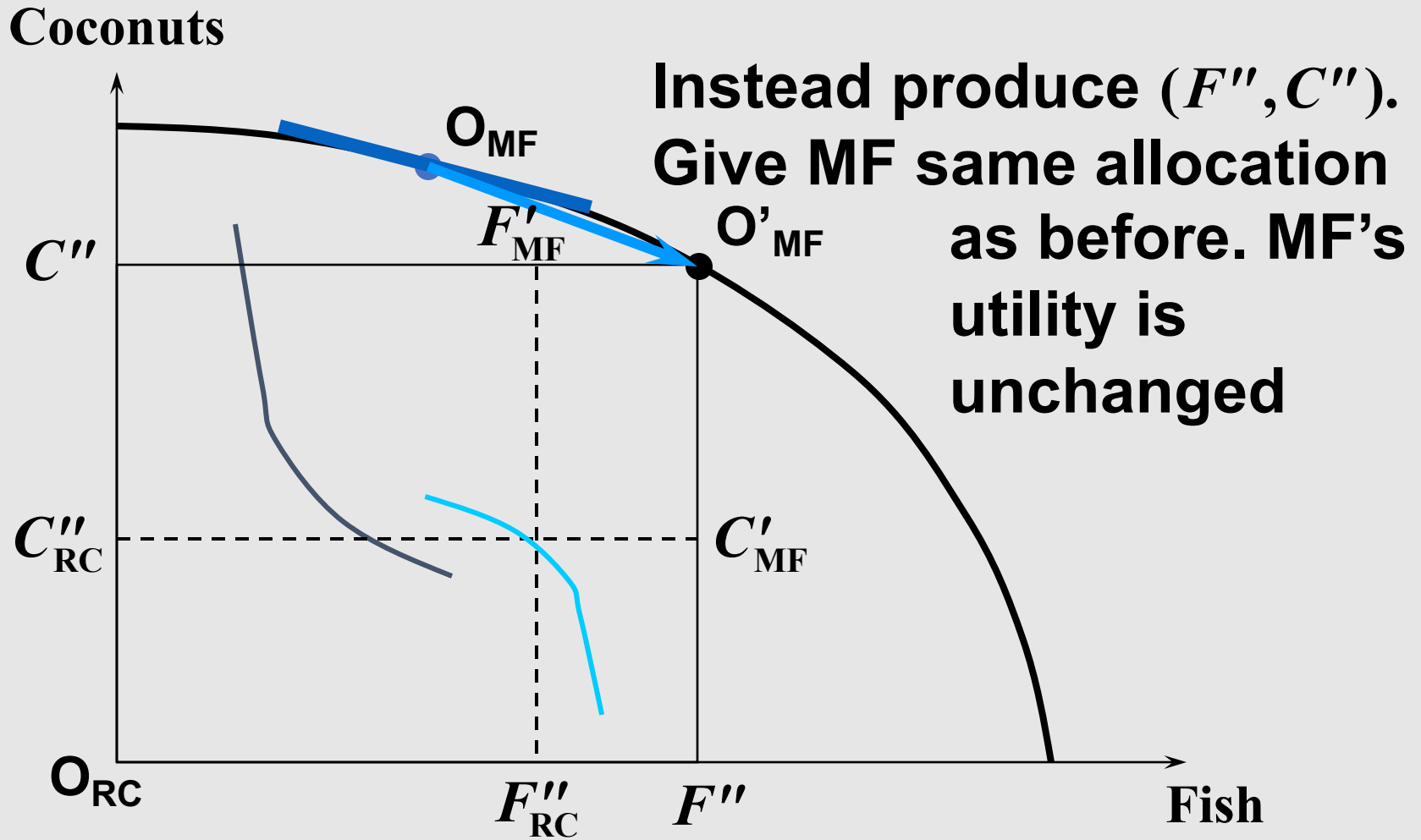


Choosing the Pareto-efficient bundle

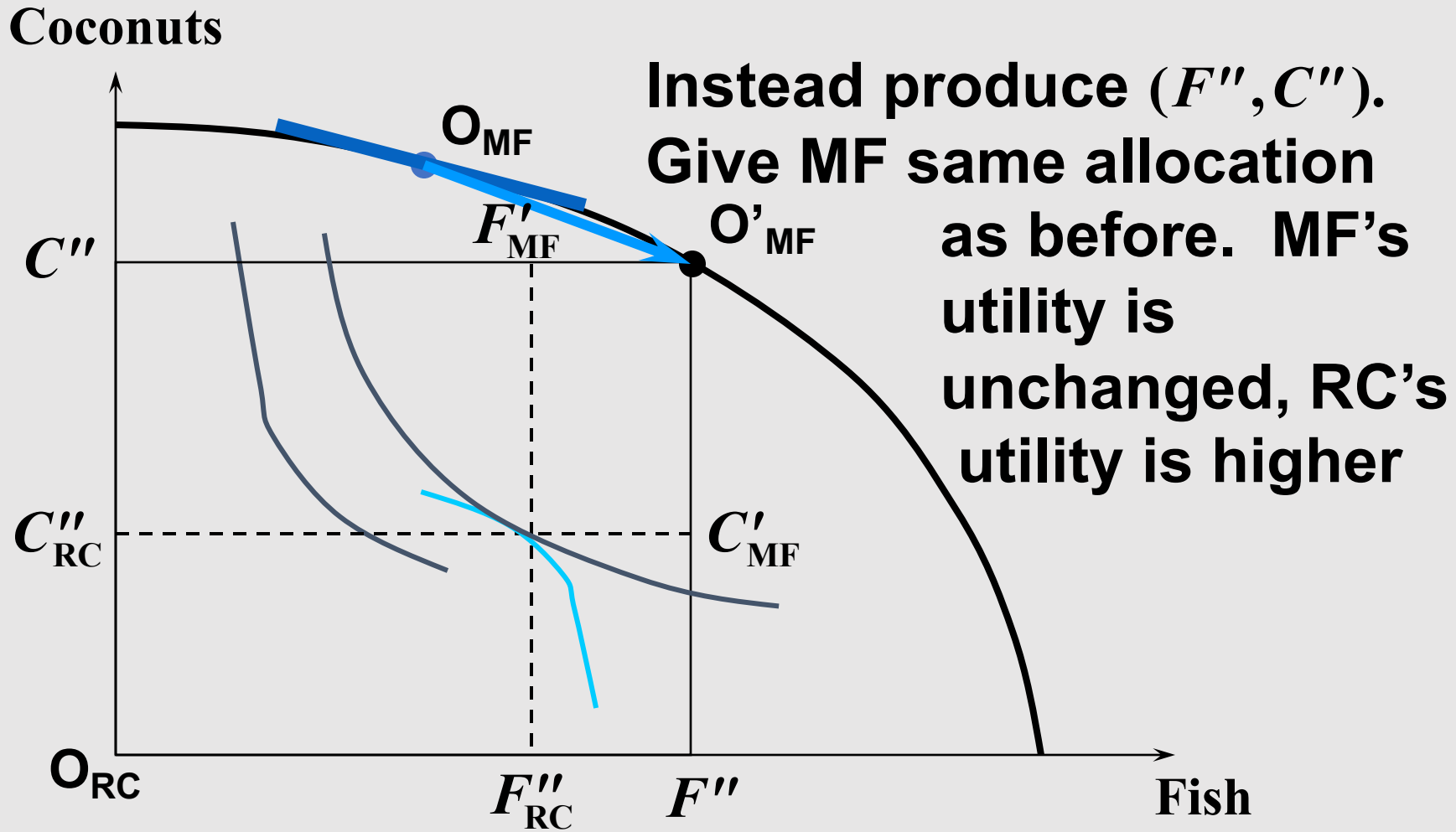
Coconuts



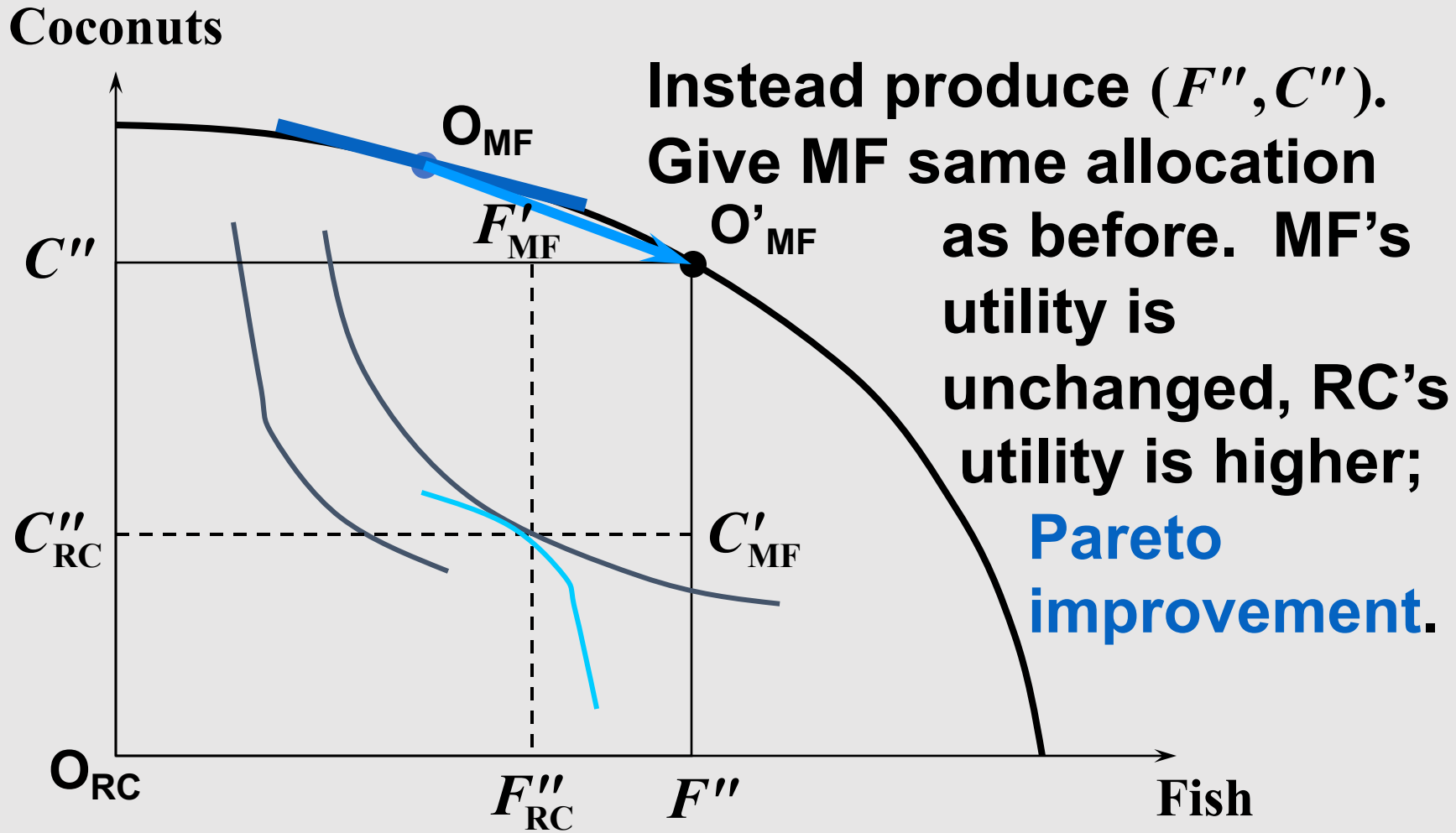
Choosing the Pareto-efficient bundle



Choosing the Pareto-efficient bundle



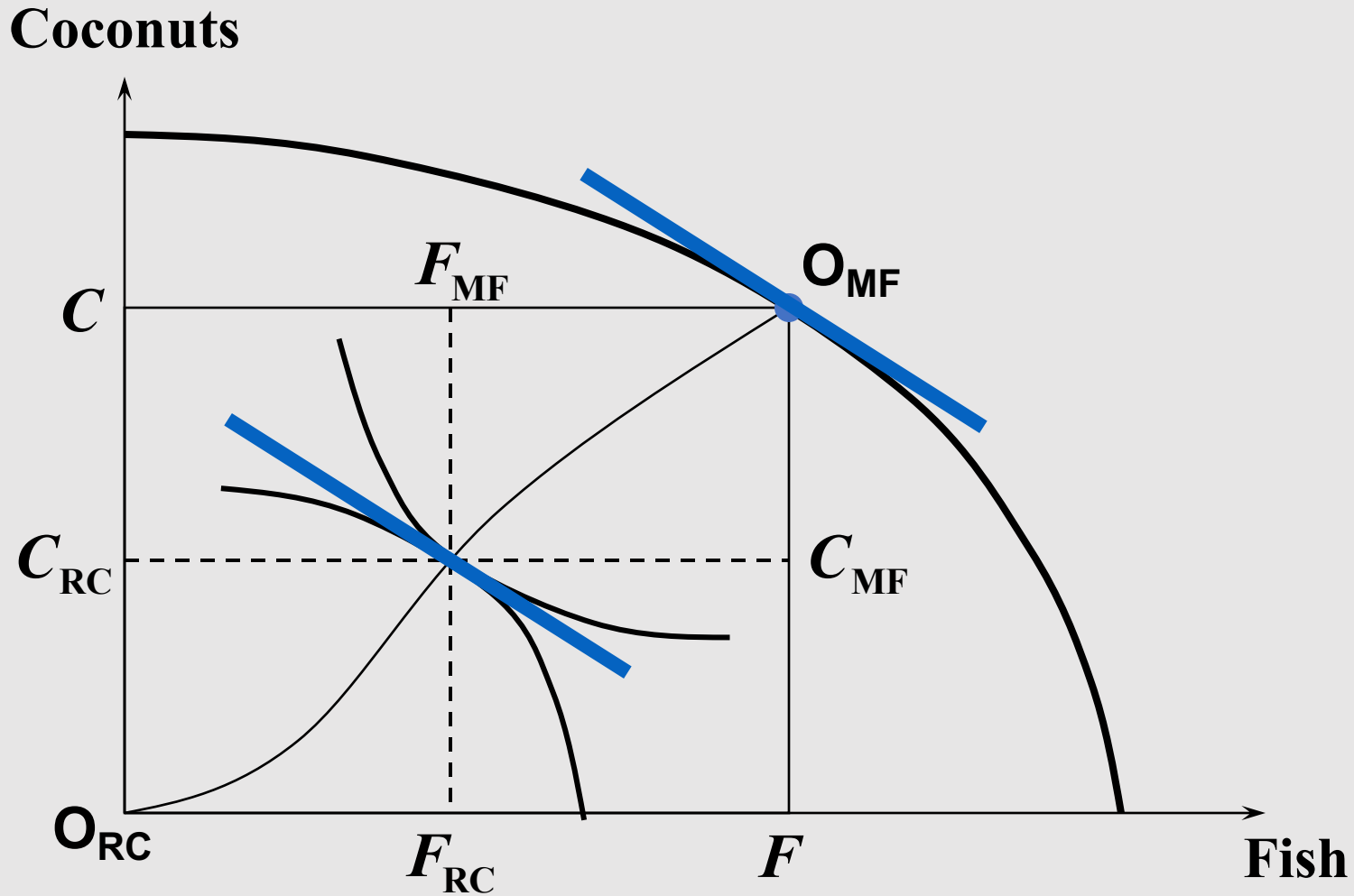
Choosing the Pareto-efficient bundle



Choosing the Pareto-efficient bundle

- $MRS \neq MRPT \Rightarrow$ inefficient coordination of production and consumption.
- Hence, $MRS = MRPT$ is necessary for a Pareto optimal economic state.

Choosing the Pareto-efficient bundle



Choosing the production distribution

- RC and MF jointly run a firm producing coconuts and fish.
- RC and MF are also consumers who can sell labor.
- Price of coconut = p_C .
- Price of fish = p_F .
- RC's wage rate = w_{RC} .
- MF's wage rate = w_{MF} .

Choosing the production distribution

- L_{RC} , L_{MF} are amounts of labor purchased from RC and MF.
- Firm's profit-maximization problem is choose C , F , L_{RC} and L_{MF} to

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

Choosing the production distribution

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

Isoprofit line equation is

$$\text{constant } \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}$$

Choosing the production distribution

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

Isoprofit line equation is

$$\text{constant } \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}$$

which rearranges to

$$C = \frac{\pi + w_{RC} L_{RC} + w_{MF} L_{MF}}{p_C} - \frac{p_F}{p_C} F.$$

Choosing the production distribution

$$\max \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}.$$

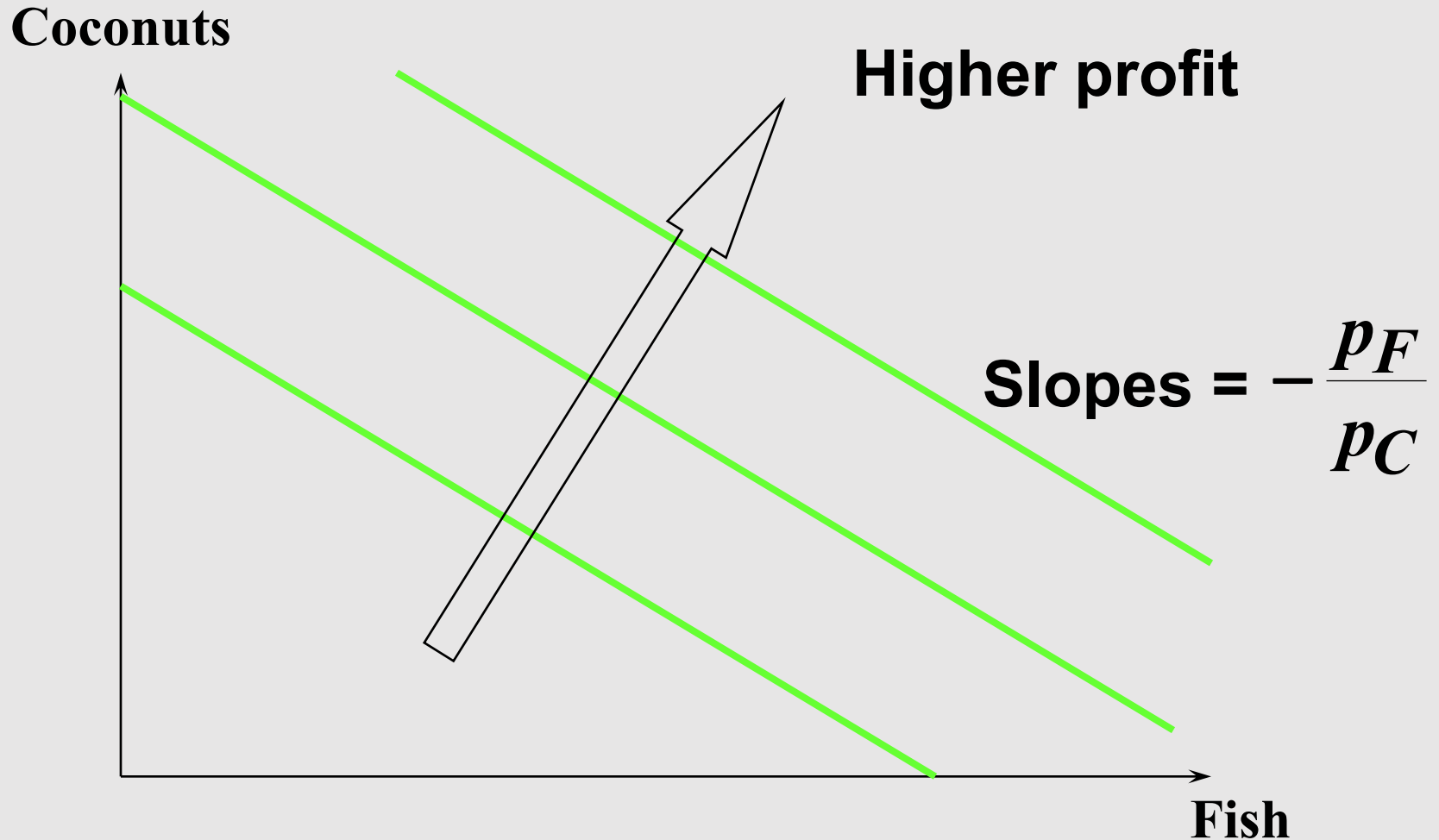
Isoprofit line equation is

$$\text{constant } \pi = p_C C + p_F F - w_{RC} L_{RC} - w_{MF} L_{MF}$$

which rearranges to

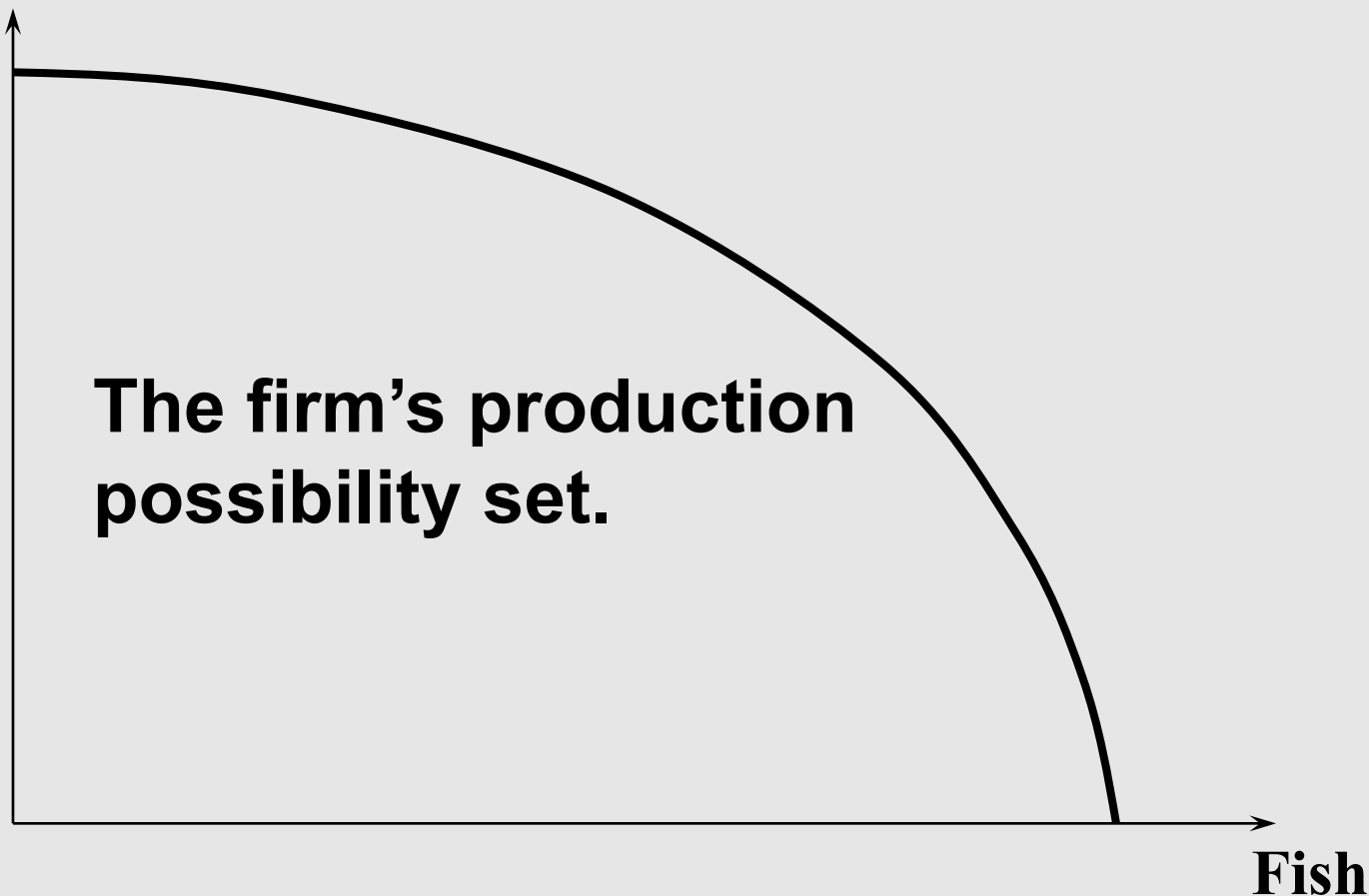
$$C = \underbrace{\frac{\pi + w_{RC} L_{RC} + w_{MF} L_{MF}}{p_C}}_{\text{Intercept}} - \underbrace{\frac{p_F}{p_C} F}_{\text{Slope}}.$$

Choosing the production distribution



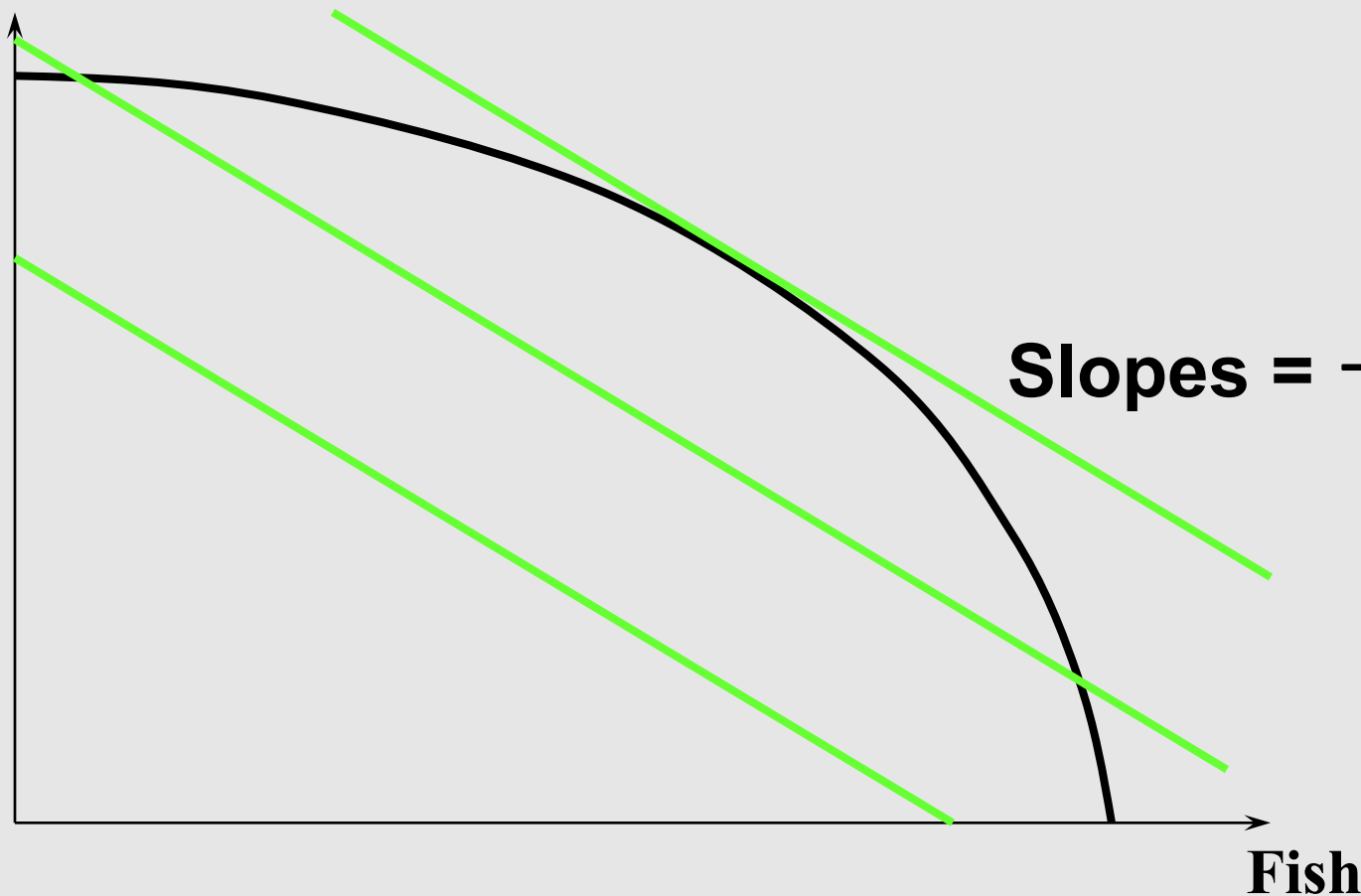
Choosing the production distribution

Coconuts

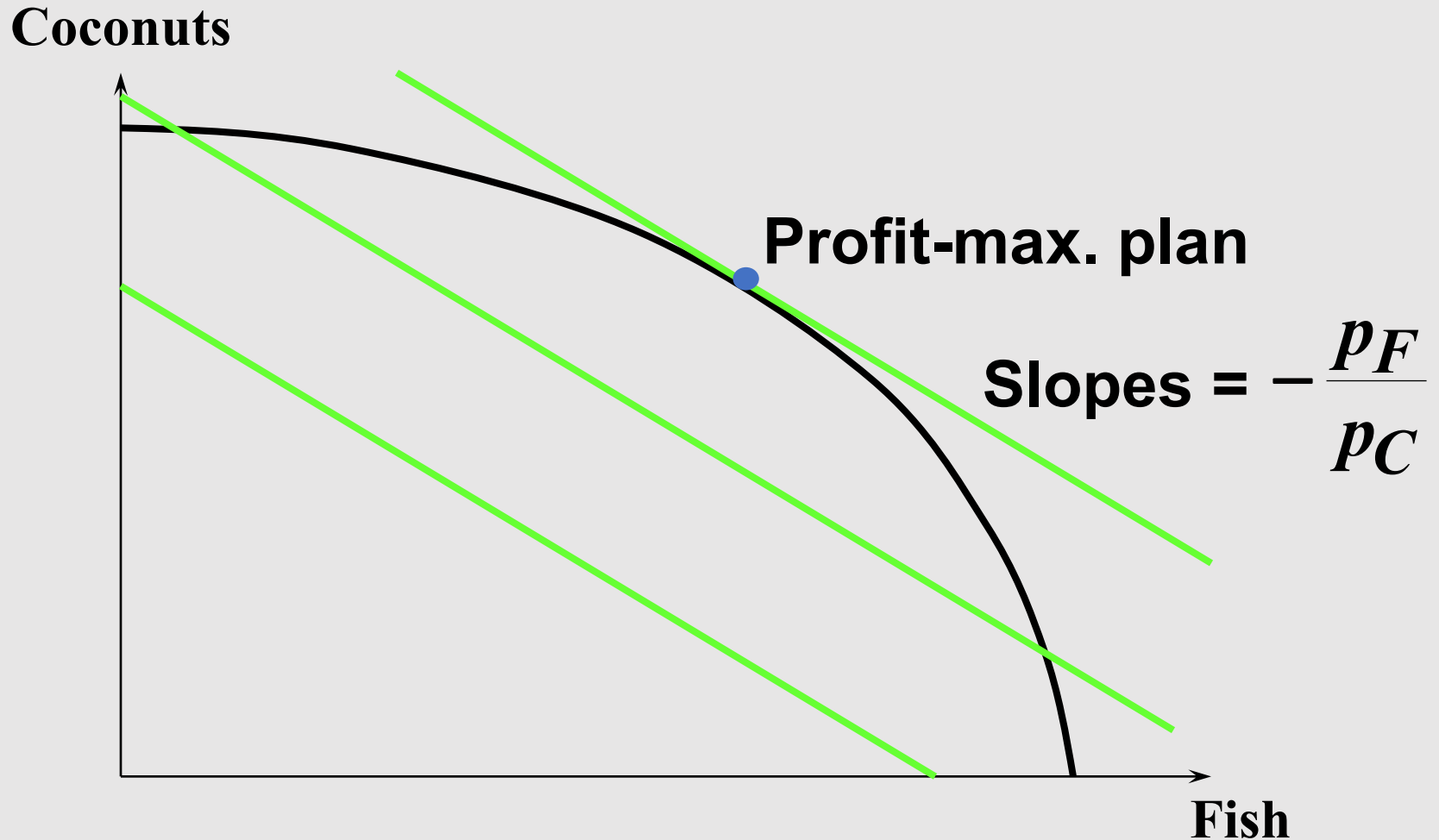


Choosing the production distribution

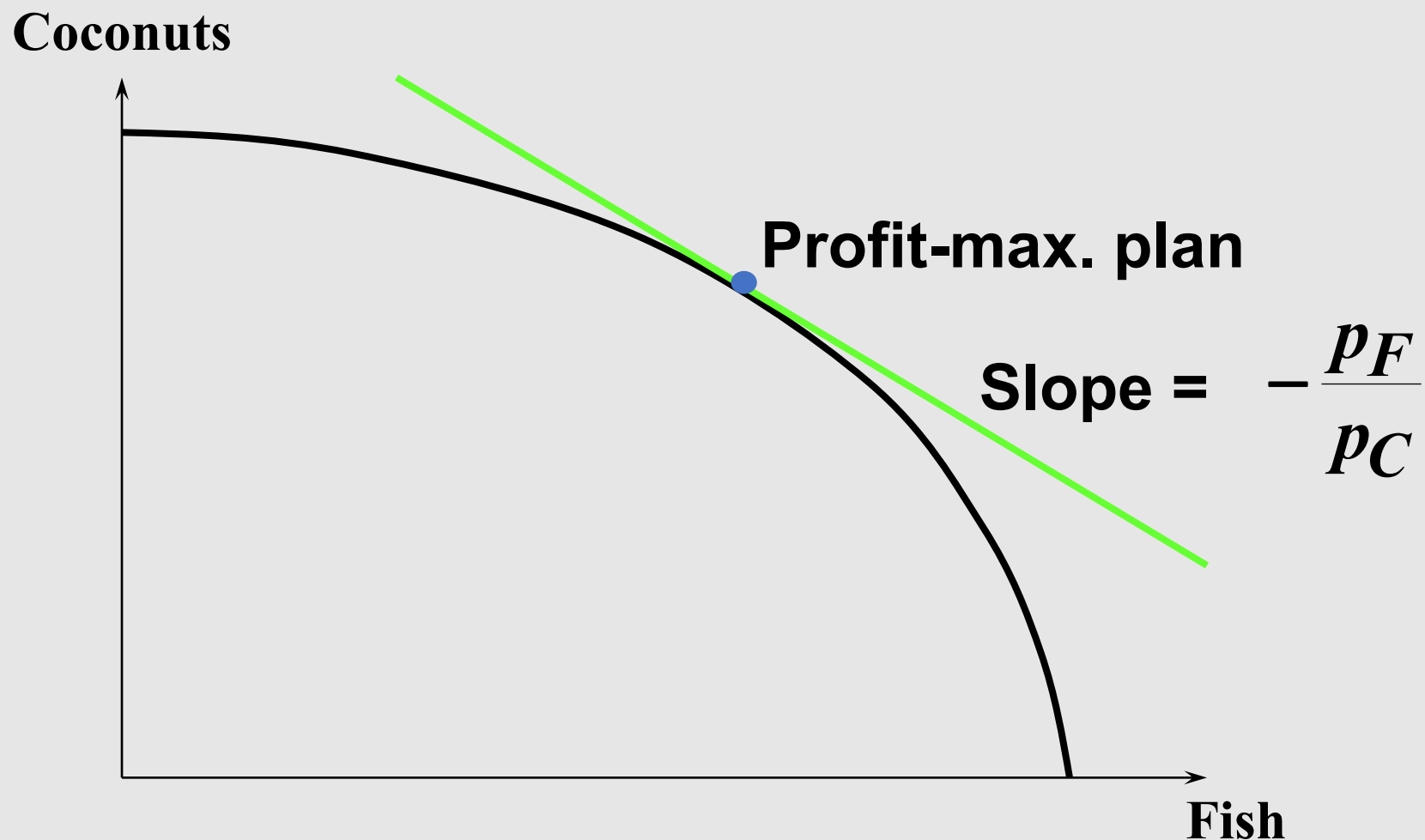
Coconuts



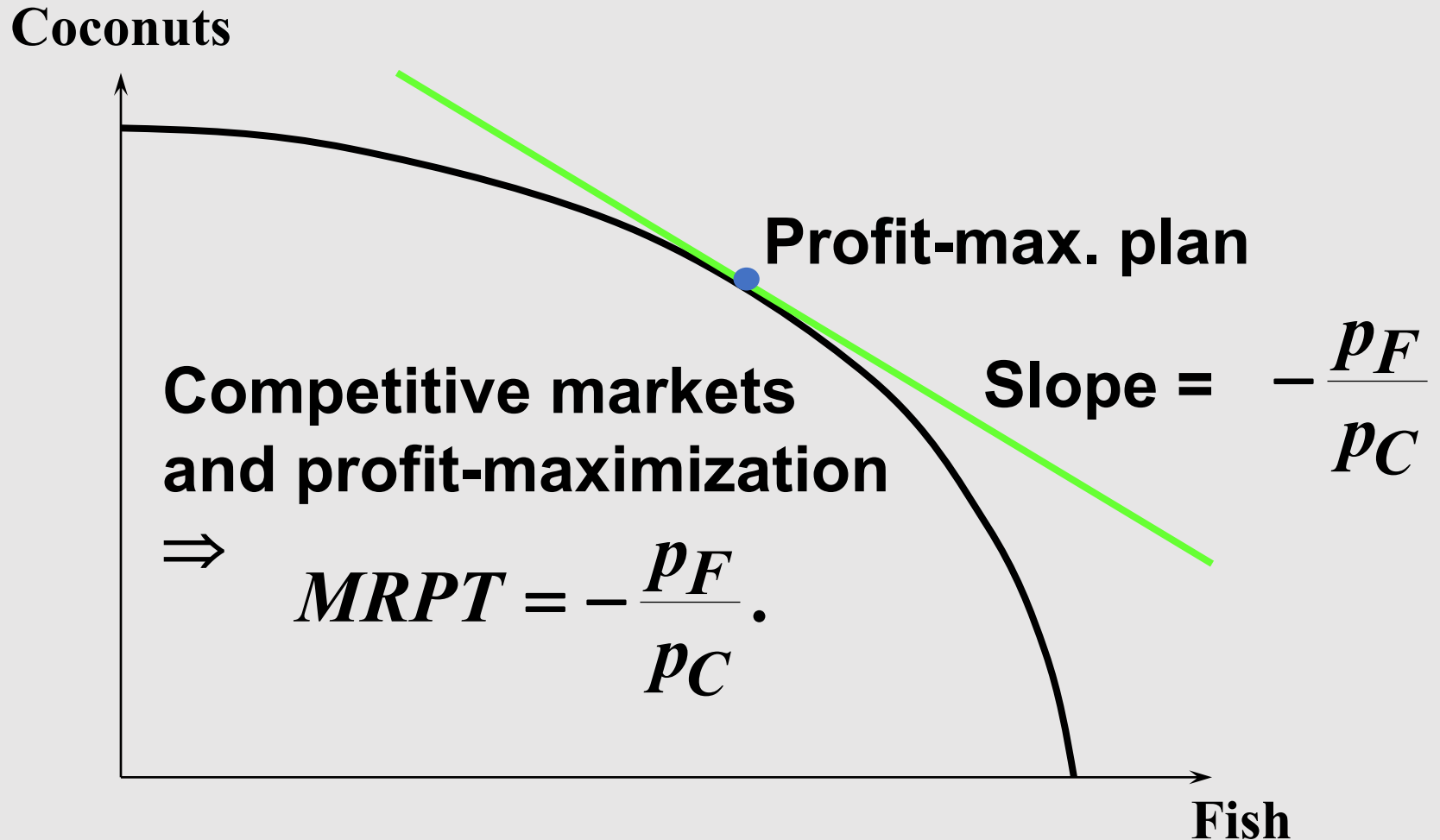
Choosing the production distribution



Choosing the production distribution



Choosing the production distribution



Choosing the production distribution

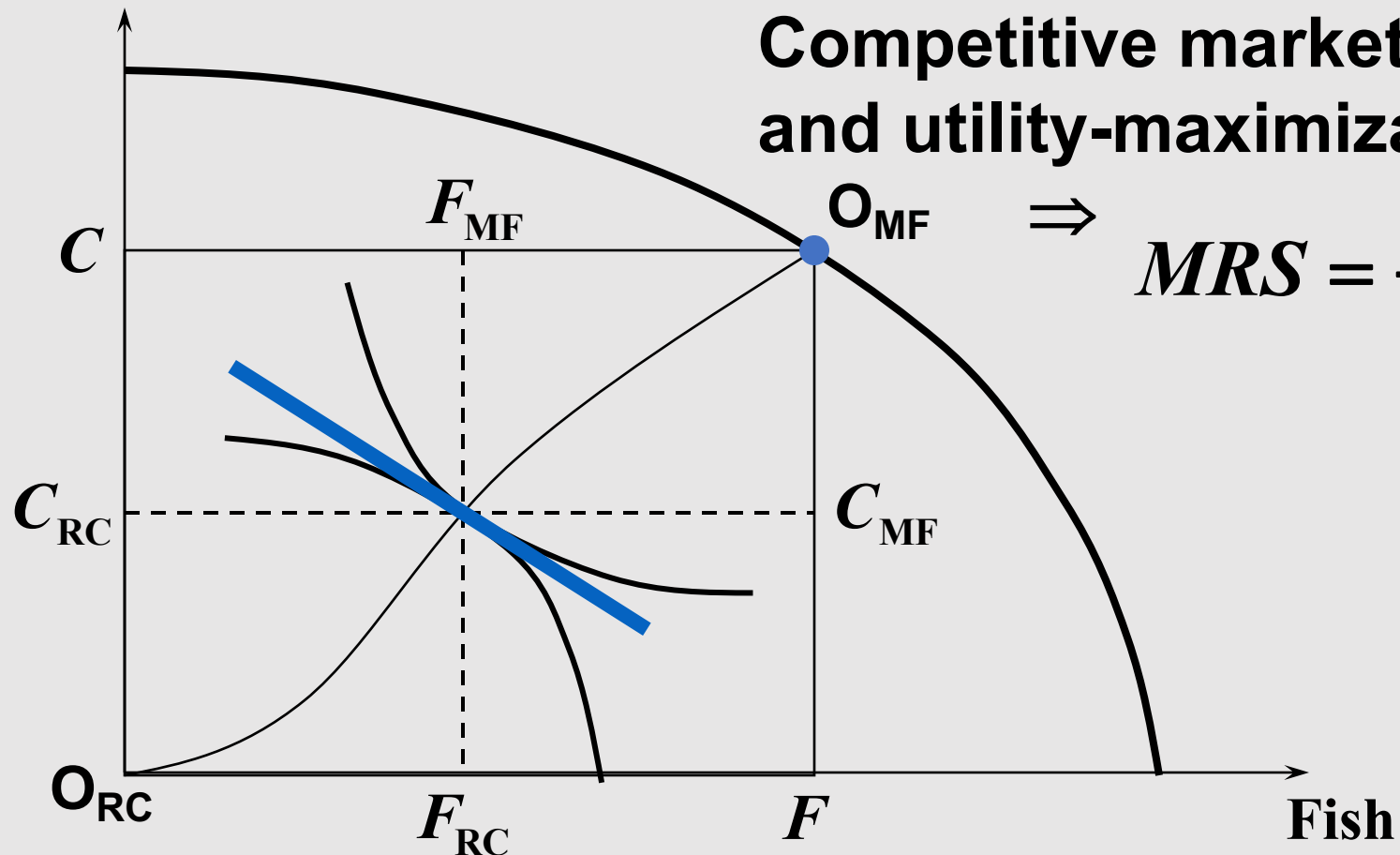
→ So competitive markets, profit-maximization, and utility maximization all together cause

$$MRPT = -\frac{p_F}{p_C} = MRS,$$

→ The condition necessary for a Pareto optimal economic state.

Coordination of Production & Consumption

Coconuts



Coordination of Production & Consumption

