Problem Set 1: Basics of Probability

Exercise 1: A Cergy delegation
A delegation of three is to be chosen from the faculty of the Thema Department in CY Cergy Paris (numbering ten) to represent the department in an Cergy-wide committee. In how many ways...

(a) can the delegation be chosen?

(b) can it be chosen, if two people refuse to go together?

(c) can it be chosen, if two particular members insist on either both going or neither going?

(d) can it be chosen, if two people must be chosen from Thema assistant faculty (6 professors) and one person must be chosen from visiting assistant faculty (4 professors)?

Exercise 2: Galileo's gamblers' problem
In the seventeenth century, Italian gamblers used to bet on the total number of spots rolled with three dice. They believed that the chance of rolling a total of 9 ought to equal the chance of rolling a total of 10. They noted that altogether there are six combinations to make 9: $(1,2,6)$, $(1,3,5)$, $(1,4,4)$, $(2,3,4)$, $(2,2,5)$, and $(3,3,3)$. Similarly, there are six combinations for 10: $(1,4,5)$, $(1,3,6)$, $(2,2,6)$, $(2,3,5)$, $(2,4,4)$, and $(3,3,4)$. Thus, argued the gamblers, 9 and 10 should have the same chance. Empirically, they found this not to be true, however. Galileo solved the gamblers' problem. How?

(a) How many permutations of three dice are there that sum to 9?

(b) How many permutations of three dice are there that sum to 10?

(c) How many total permutations of three dice are there? What was Galileo's solution? Explain.

Exercise 3: Venn diagrams
Venn diagrams or set diagrams are diagrams that show all hypothetically possible logical relations between a finite collection of sets (groups of things). Venn diagrams were invented around 1880 by John Venn. They are used in many fields, including set theory, probability, logic, statistics, and computer science.

(a) Draw a Venn diagram for the three events A, B, and C contained in the sample space S and properly label all possible union and intersections of events.

(b) Draw a Venn diagram for the three events A, B, and C contained in the sample space S where all combinations of events are possible except the triple interaction, i.e. where $A \cap B \cap C = \varnothing$.

(c) Try (but don't spend too much time–it's just for fun) to draw a complete Venn diagram for the four events A, B, C, and D contained in the sample space S where you include all possible unions and intersections of events. How many mutually exclusive regions should such a diagram include?

Exercise 4: Partitions
In Chapter 1, you learned about event partitions. Give three different examples of partitions of a single dray from a deck of playing cards.

Exercise 5: Football team
The Cergy football team plays 12 games in a season. In each game they have 1/3 probability of winning, 1/2 probability of losing, and 1/6 probability of tying. Games are independent. What is the probability that the team has 8-3-1 record? (8 wins, 3 losses, and 1 ties)

Exercise 6: Lemon cars
You and your friends just rented a car from CarRent for an 8,000km cross- country road trip to see all of the sights across Europe (don't do that, take the train instead!). Your rental car may be of three different types: brand new, nearly 1 year old, or a lemon (bound to break down).

That many km can be demanding on a rental car. If the car you receive is brand new (New), it will break down with probability 0.05. If it is one year old (One), it will break down with probability 0.1. If it is just a lemon (Lemon), it will break down with probability 0.9. The probability that the car CarRent gives you a car that is New, One, or Lemon is 0.8, 0.1, and 0.1, respectively. Compute the probability that your car is going to break down on your road trip.

Exercise 7: Bayes
Bayes' formula is really important. Write down Bayes' formula and describe it in words. Further, here are a couple of common applications.

1. Suppose that five percent of men and 0.25 percent of women are color blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. What if there were twice as many males as females?

2. Suppose that there exists an imperfect test for Tuberculosis (TB). If some- one has TB, there is a ninety-five percent chance that the test will come up "red" If someone does not have TB, there is only a two percent chance that the test will come up red. Finally, the chance that anyone has TB is, say, five percent (in the United States; in other countries Tuberculosis is endemic). Once someone takes the test and it comes up red, what is the probability that they have TB?