Total Causal Effect Calculation for Fuzzy Cognitive Maps (TCEC-FCM) - Pseudocodes

Marios Tyrovolas[®], Nikolaos D. Kallimanis, and Chrysostomos Stylios[®]

In this document, we present the pseudocode for two variants of the TCEC-FCM algorithm, as proposed in our recent research work: 'Tyrovolas, M., Kallimanis, N. D., & Stylios, C. (2024). Causal Effect Analysis in Large-Scale Fuzzy Cognitive Maps for Explainable Artificial Intelligence (XAI).' Specifically, we detail the implementation of the original TCEC-FCM algorithm and a simplified variant, termed TCEC-FCM-LS, that uses linear search.

```
Algorithm 1: "Total Causal Effect Calculation for Fuzzy Cognitive
  Maps" Algorithm with Linear Search (TCEC-FCM-LS)
    Input: n, number of concepts; \mathbf{W}_{\mathbf{n}\times\mathbf{n}}, weight matrix.
    Output: T_{\rm eff}, vector of total causal effects.
    /* Extract and sort non-zero weights and their indices
                                                                                                            */
 1 W_{nz} \leftarrow \{(i, j, w_{ij}) \mid w_{ij} \in W, w_{ij} \neq 0\}
 2 W_{sorted} \leftarrow \text{sortDescending}(\{w_{ij} \mid (i, j, w_{ij}) \in W_{nz}\}) // W_{sorted} \in \mathbb{R}^{e \times 1}
      where e is the number of non-zero weights
    /* Initialize T_{\text{eff}}
                                                                                                            */
 3 \mathbf{T}_{\text{eff}} \leftarrow [0]_{n \times 1}
 4 for C_i \leftarrow 1 to n-1 do
         /* Initialize a copy of the FCM with isolated concepts
         \mathbf{W}_{\text{copy}} \leftarrow [0]_{n \times n}
 5
         foreach (i, j, w_{ij}) \in \mathcal{W}_{sorted} do
 6
               /* Update \mathbf{W}_{	exttt{copy}} for the current weight
               \mathbf{W}_{\text{copy}}[i][j] \leftarrow w_{ij}
 7
               /* BFS for reachability to C_n
               reachableConcepts \leftarrow BFS(\mathbf{W}_{copy}, C_i, C_n)
               if C_n \in reachableConcepts then
 9
                    \mathbf{T}_{\text{eff}}[C_i] \leftarrow w_{ij}
10
                    break
11
         if \mathbf{T}_{eff}[C_i] = 0 then
12
              \mathbf{T}_{\mathrm{eff}}[C_i] \leftarrow 0 \text{ // No path found}
14 return T_{\rm eff}
```

Algorithm 2: Total Causal Effect Calculation for Fuzzy Cognitive Maps (TCEC-FCM)

```
Input: n, number of concepts; \mathbf{W}_{n \times n}, weight matrix.
     Output: T_{\rm eff}, vector of total causal effects.
     // Extract and sort non-zero weights and their indices
 1 W_{nz} \leftarrow \{(i, j, w_{ij}) \mid w_{ij} \in W, w_{ij} \neq 0\}
 2 W_{sorted} \leftarrow \text{sortDescending}(\{w_{ij} \mid (i, j, w_{ij}) \in W_{nz}\}) // W_{sorted} \in \mathbb{R}^{e \times 1}
      where e is the number of non-zero weights
     // Initialize T_{	ext{eff}}
 3 \mathbf{T}_{\text{eff}} \leftarrow [0]_{n \times 1}
 4 for C_i \leftarrow 1 to n-1 do
          /* Init binary search vars
                                                                                                                   */
          exIdxs \leftarrow \emptyset, pathFound \leftarrow false
 5
          upperIndex \leftarrow 1, midIndex \leftarrow 1
 6
          lowerIndex \leftarrow length(\mathcal{W}_{sorted})
 7
          while upperIndex - lowerIndex \ge 1 do
 8
                exIdxs \leftarrow exIdxs \cup \{midIndex\}
                /* FCM copy for weights subset
10
                \mathbf{W}_{\text{copy}} \leftarrow [0]_{n \times n}
                for idx \leftarrow 1 to midIndex do
11
                     (i, j, w_{ij}) \leftarrow \mathcal{W}_{\text{sorted}}[idx]
12
                     \mathbf{W}_{\text{copy}}[i][j] \leftarrow w_{ij}
13
                /* BFS for reachability to C_n
                                                                                                                   */
                reachableConcepts \leftarrow BFS(\mathbf{W}_{copy}, C_i, C_n)
14
                if C_n \in reachableConcepts then
15
                     \mathbf{T}_{\text{eff}}[C_i] \leftarrow \mathcal{W}_{\text{sorted}}[\text{midIndex}]
16
17
                     lowerIndex \leftarrow midIndex
18
                     pathFound \leftarrow true
                else
19
                    upperIndex \leftarrow midIndex
20
                \operatorname{midIndex} \leftarrow \operatorname{Round}\left(\tfrac{\operatorname{upperIndex} + \operatorname{lowerIndex}}{2}\right)
21
                /* Check for convergence
               if (upperIndex - lowerIndex) = 1 and midIndex \in exIdxs then
22
23
                     break
          if not pathFound then
\mathbf{24}
               \mathbf{T}_{\mathrm{eff}}[C_i] \leftarrow 0 \text{ // No path found}
25
26 return T_{\rm eff}
```