Article

Optimal Fuzzy Controller Design for Autonomous Robot Path Tracking using Genetic Algorithms

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† This paper is an extended version of our paper published in INFUS-21.

Abstract: In this work, we propose, through the use of a population-based metaheuristic, an optimization method to solve the problem of autonomous path tracking using a rear-wheel controller in the framework of fuzzy logic. This approach enables the design of controllers using rules that are linguistically familiar to human users. Moreover, a new validation technique is presented by using three different paths to validate the performance of each candidate configuration. We extend on our previous work by adding two more membership functions to the previous fuzzy model, intending to have a finer grain of adjustment. Experiments show that compared to a published control law, the proposed fuzzy controller has a better performance when compared with the RMSE but also has limitations with respect to undesired yaw oscillation. Nevertheless, the experiments also highlight problems with the common practice of evaluating the performance of fuzzy controllers with a single problem case and performance metric, resulting in controllers that tend to be overtrained.

Keywords: Fuzzy Systems; Fuzzy Control; Bioinspired Algorithms.

Citation: Mancilla, A.; García-Valdez, M.; Castillo, O.; Merelo-Guervós, J.J. Title. *Symmetry* **2021**, *1*, 0. https://doi.org/

Received:
Accepted:
Published:

1. Introduction

Proposed by Lotfi Zadeh [1], fuzzy logic introduces the concepts of fuzzy sets and fuzzy logic operators [2]. Contrary to boolean logic, in which an element is a member of a set or is not, now elements have a degree of membership to many sets. Fuzzy logic uses so-called membership functions (MFs) to assign a numerical value to each set member, indicating their degree of membership. MFs must be defined for each linguistic variable, then these variables can be used in a rule-based system to express knowledge in a way similar to natural language, for instance, the rule if distance is near, uses the linguistic variable distance, with a fuzzy MF assigning a degree of membership to the set near to each element of the distance domain, for instance for 2mm and 50mm the function will assign the following degrees of membership: near(2)=.92 and near(50)=.40. These rule base systems can express complex relationships well suited for control applications.

That is why, since the earlier years of fuzzy logic theory, fuzzy inference systems [3] have been applied to control problems [3–6], in many research projects [7,8] and commercial systems.

An essential caveat of applying fuzzy control strategies to real-world problems is that we require the use of optimization or adaptive techniques to tune some aspects of the fuzzy inference system. From the membership functions (MFs) that define the linguistic variables to the rule definition of the rule-based system, including the defuzzification method [9,10].

Tuning is needed because the fuzzy controllers' performance is highly dependent on the parameters of the fuzzy system used. Moreover, the option of making a manual selection of these parameters is difficult because the search space is of considerable size and requires the validation of establishing the controller's performance by running time-consuming simulations. Evolutionary algorithms (EAs) and other metaheuristics are often employed in tuning FISs [11].

EAs are a kind of optimization metaheuristic used for solving complex combinatorial problems [12] with applications in many engineering areas [13]. One of the most common EAs are Genetic Algorithms (GAs) [14] which are population-based search methods that are inspired in natural selection. In this model, surviving individuals reproduce through genetic crossover and mutation operators to generate offspring, and there is a replacement mechanism to select the most adapted offspring [15]. GAs work on a set of potential solutions called a population that is evolved for a certain number of generations until a suitable or optimal solution is found. Even if this combination of techniques is the basis of computational intelligence [16,17], and there are many contributions on the subject, there are still many application areas that have specific requirements and challenges that have not been addressed or considered by earlier works.

Many papers address the problem of path tracking control with fuzzy logic, the earlier works used simple fuzzy rules to make adjustments to linear controllers [18], or integrated fuzzy logic to a complex adaptive controller [19]. Meng [20] proposes a Fuzzy PID algorithm to a two-degree-of-freedom arm robot. Antonin et al. [21] propose a set of rules to emulate the way humans drive, using as inputs of curve and distance to the system. We have also reviewed works where authors use controllers for the follow-up and planning of routes. For example, using a simplified kinematic model of the bicycle type using the control law for the navigation and follow-up of a route [22], in work presented by Beleño et al. [23] they propose the planning and tracking of the trajectories of a land vehicle based on the steering control in a natural environment. Guerrero-Castelanos et al. [24] addresses the path following problem for the robot (3, 0) based on its kinematic model and proposes a solution by designing a control strategy that mainly considers the maximum permitted levels of the control signal.

We have found in the literature various studies that optimize the parameters of a fuzzy controller applied to mobile autonomous robots using different bio-inspired metaheuristics [25,26]. Wagner & Hagras [27] propose a GA to evolve the architecture of a type-2 fuzzy controller in robot navigation for real environments; they optimize the standard deviation of Gaussian type-2 MFs. Again a GA is presented by Wu & Wan Tan [28] for evolving the parameters of all the MFs of a coupled-tank liquid-level control system. Astudillo et al. [29] propose a new metaheuristic based on chemical reactions to tune the parameters of a fuzzy controller for a uni-cycle robot. There is also work focused on the metaheuristic optimization of fuzzy type-2 controllers, the main works are reviewed by Castillo [30], and the main reason for using this type of controller is to model the uncertainty of the sensor data or the fuzzy model itself.

We have observed that most of these studies optimize the parameters of MFs directly. For instance, if we have a triangular function for a fuzzy set A, defined by a lower limit a, and upper limit of b and a value m where $a < m < b$ as

$$\mu_{trian}(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{m-a}, & a < x \leq m \\ \frac{b-x}{b-m}, & m < x \leq b \\ 0, & x \geq b \end{cases}, \tag{1}$$

each value is optimized independently, sometimes validating only the restriction $a < m < b$. In this work, we propose a method to include further restrictions, including symmetric definition and limiting the range of values of each parameter. In our previous work, we compared symmetrical and asymmetrical definitions and found that symmetrical restrictions give better results for rear-weel-based controllers.

The main contribution of this work is to propose, through the use of a population-based metaheuristic, an optimization method to solve the problem of autonomous path tracking using a rear-wheel controller in the framework of fuzzy logic. This approach enables the design of controllers using rules that are linguistically familiar to human users.

Therefore, following the preliminary work in [31,32], in this paper, we propose the addition of a more granular definition of fuzzy variables by including two more membership functions. Also, we propose a new parameterization technique that enables the definition of symmetric MFs, by using an aperture factor instead of the previous method that used a delta from a fixed point. These additions greatly improve the controller's evolutionary design by significantly decreasing the tracking error (RMSE) compared to our previous work. We also propose a new evaluation method for establishing the fitness of candidate solutions by using three different trajectories in each simulation. Experimental results show that this method reduces the risk of generating an overtrained controller with low error for a specific track but cannot function in other paths. To demonstrate the application of the method, we report an experimental case study using a bicycle-like mobile robot with nonholonomic constraints.

In this work, we choose the problem of trajectory tracking since it has the particularity of being naturally symmetric since the error is measured by moving away either to the left or right of the desired path. Furthermore, in the literature, we have not found applications of fuzzy systems to follow the trajectory of a route so that this research could solve similar problems.

We structure this document as follows: in Section 2, we present the proposed method, configurations, and we describe the experimental setup. In Section 3 we present the results achieved, and finally, in Section 4 we discuss the results and highlight future research directions.

2. Materials and Methods

2.1. Rear-Wheel Feedback and Kinematic Model

In this work, we use a simplified model of a bicycle-type kinematic robot consisting of two wheels connected by a rigid link of size l with nonholonomic restrictions [33,34]. The front-wheel can steer in the axis normal to the plane of motion, the steering angle is δ (see Figure 1), The position of the midpoint of the rear-wheel is given by the coordinates x_r and y_r. The heading θ is the angle of the link between the two wheels and the x axis. We follow the model describe in [35], with the differential constraint:

$$\begin{aligned} \dot{x}_r &= v_r \cos(\theta), \\ \dot{y}_r &= v_r \sin(\theta), \\ \dot{\theta} &= \tfrac{v_r}{l} \tan(\delta). \end{aligned} \tag{2}$$

The controller selects the steering angle δ with a value between the limits of the vehicle $\delta \in [\delta_{min}, \delta_{max}]$ and a desired velocity v_r again limited by $v \in [v_{min}, vmax]$. The heading rate ω is related to the steering angle by

$$\delta = \arctan\left(\frac{l\omega}{v_r}\right), \tag{3}$$

and we can simplify the heading dynamics to

$$\dot{\theta} = \omega, \quad \omega \in \left[\frac{v_r}{l}\tan(\delta_{min}), \frac{v_r}{l}\tan(\delta_{max})\right]. \tag{4}$$

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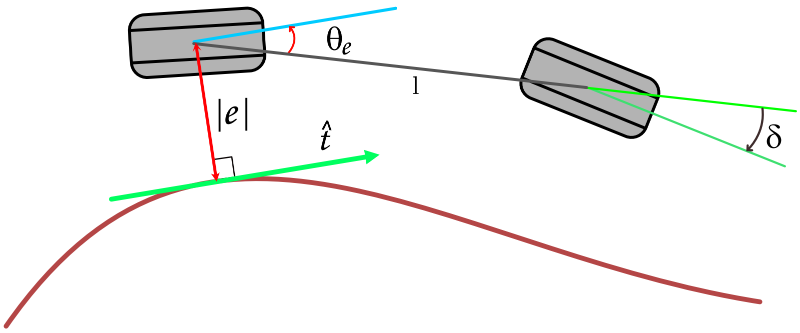

Figure 1. Feedback and actuator variables for the rear-wheel-based control. The magnitude of e illustrated in red, is the error measured from the rear wheel to the nearest point on the path. When e > 0, the wheel is at the right of the path, and when e < 0 is at the left. $teta_e$ is the difference between the tangent at the nearest point in the path and the the heading θ. The output of the controller is the heading rate ω, from which we calculate the steering angle δ of the front wheel.

Now we explain the path tracking method described by Paden et al. [35]. This controller takes the feedback from the rear-wheel position as Figure 1 illustrates. The path (shown in red in the figure) is a continuous function with properties described in [36], and the feedback is a function of the nearest point on the reference path given by

$$s(t) = \arg\min_{\gamma} \|(x_r(t), y_r(t)) - (x_{ref}(\gamma), y_{ref}(\gamma))\|. \tag{5}$$

and the tracking error vector is

$$d(t) = (x_r(t), y_r(t)) - (x_{ref}(s(t)), y_{ref}(s(t))) \tag{6}$$

The heading error is based on a unit vector $\hat{t}$ (shown in green on Figure 1) tanget to the path at $s(t)$ given by

$$\hat{t} = \frac{\left(\frac{\partial x_{ref}}{\partial s}\Big|_{s(t)}, \frac{\partial y_{ref}}{\partial s}\Big|_{s(t)} \right)}{\left\| \left(\frac{\partial x_{ref}(s(t))}{\partial s}, \frac{\partial y_{ref}(s(t))}{\partial s} \right) \right\|}, \tag{7}$$

The error e is the cross product of the two vectors

$$e = d_x \hat{t}_y - d_y \hat{t}_x \tag{8}$$

The heading error uses the angle θ_e between the robot's heading vector and $\dot{(t)}$

$$\theta_e(t) = \theta - arctan_2\left(\frac{\partial x_{ref}(s(t))}{\partial s}, \frac{\partial y_{ref}(s(t))}{\partial s} \right) \tag{9}$$

2.2. Fuzzy Controllers

To design a fuzzy controller from the model we just described, we must consider the magnitude of the scalar value e, the distance from the rear wheel to the route's closest point. This error is positive if it is on the right and negative on the left of the route. Another input variable is the angle θ_e defined between the heading vector and the tangent vector of the route. This angle can also be negative or positive depending on the vehicle's position concerning the route; the controller output is the heading rate ω, which allows us to calculate the steering angle δ. The target velocity of the robot will be constant constat, so we will not control the velocity v using the fuzzy controller we will use the following simple proportional controller instead

$$a = K_p(v_{ref} - v_r). \tag{10}$$

In this work, we will compare two fuzzy models, one with three MFs, which was proposed in our previous work [32], and a new model using five MFs. We describe both fuzzy systems next.

2.2.1. Fuzzy Model with Three Membership Functions

We now model the same input and output variables mentioned above as fuzzy variables using three membership functions. Each variable has the same fuzzy values: high negative (`hi_neg`), low (`low`), and high positive (`hi_pos`). Depending on the case, the sign indicates whether it is to the right or left of the path. We use trapezoidal functions for the "high" fuzzy term (positive and negative) and a triangular function for the "low" value.

We consider a total membership in extreme values (depending on each variable's domain). Because of this, we ignore the trapezoidal parameters' values outside the value domain. In total, we could modify up to 33 parameters for all membership functions. We propose keeping the extreme parameters outside the domain and the central point of the triangular function fixed. The center of the triangular functions will always be at 0 since it is the minimum error possible. We kept the fuzzy rules symmetrical, representing how a human driver would adjust the steering wheel when driving. The rules are presented in Table 1. We have nine rules, to establish a relation between all the possible inputs and output values.

Table 1. Proposed fuzzy rules for the basic controller with three membership functions.

Rule 1:	If θ_e is	`hi_neg`	and e is	`low`	then ω is	`hi_pos`
Rule 2:	If θ_e is	`hi_pos`	and e is	`low`	then ω is	`hi_neg`
Rule 3:	If θ_e is	`low`	and e is	`low`	then ω is	`low`
Rule 4:	If θ_e is	`hi_neg`	and e is	`hi_neg`	then ω is	`hi_pos`
Rule 5:	If θ_e is	`hi_pos`	and e is	`hi_pos`	then ω is	`hi_neg`
Rule 6:	If θ_e is	`hi_pos`	and e is	`hi_neg`	then ω is	`low`
Rule 7:	If θ_e is	`hi_neg`	and e is	`hi_pos`	then ω is	`low`
Rule 8:	If θ_e is	`low`	and e is	`hi_pos`	then ω is	`hi_neg`
Rule 9:	If θ_e is	`low`	and e is	`hi_neg`	then ω is	`hi_pos`

2.2.2. Fuzzy Model with Five Membership Functions

We now add two more membership functions to the previous fuzzy model, intending to have a finer grain of adjustment. Adding more functions also adds more complexity to the rules, and now we have even more parameters to tune. Instead of having just two values for each type of error `hi` and `low` we add a middle value, represented by the `medium` membership function. The knowledge in the fuzzy rule base is now more complex than before, with 25 rules. In this case, defining the rules was not done by simply specifying a driver's knowledge. We needed to adjust the rules by running several simulations. The rules are presented in Table 2.

Table 2. Proposed fuzzy rules for the basic controller with three membership functions.

Rule 1:	If θ_e is	`hi_neg`	and e is	`hi_neg`	then ω is	`hi_pos`
Rule 2:	If θ_e is	`hi_neg`	and e is	`med_neg`	then ω is	`hi_pos`
Rule 3:	If θ_e is	`hi_neg`	and e is	`low`	then ω is	`hi_pos`
Rule 4:	If θ_e is	`hi_neg`	and e is	`med_pos`	then ω is	`med_pos`
Rule 5:	If θ_e is	`hi_neg`	and e is	`hi_pos`	then ω is	`low`
Rule 6:	If θ_e is	`med_neg`	and e is	`hi_neg`	then ω is	`med_pos`
Rule 7:	If θ_e is	`med_neg`	and e is	`med_neg`	then ω is	`med_pos`
Rule 8:	If θ_e is	`med_neg`	and e is	`low`	then ω is	`med_pos`
Rule 9:	If θ_e is	`med_neg`	and e is	`med_pos`	then ω is	`med_pos`
Rule 10:	If θ_e is	`med_neg`	and e is	`hi_pos`	then ω is	`low`
Rule 11:	If θ_e is	`low`	and e is	`hi_neg`	then ω is	`hi_pos`
Rule 12:	If θ_e is	`low`	and e is	`med_neg`	then ω is	`low`
Rule 13:	If θ_e is	`low`	and e is	`low`	then ω is	`low`
Rule 14:	If θ_e is	`low`	and e is	`med_pos`	then ω is	`low`
Rule 15:	If θ_e is	`low`	and e is	`hi_pos`	then ω is	`hi_neg`
Rule 16:	If θ_e is	`med_pos`	and e is	`hi_neg`	then ω is	`low`
Rule 17:	If θ_e is	`med_pos`	and e is	`med_neg`	then ω is	`med_neg`
Rule 18:	If θ_e is	`med_pos`	and e is	`low`	then ω is	`med_neg`
Rule 19:	If θ_e is	`med_pos`	and e is	`med_pos`	then ω is	`med_neg`
Rule 20:	If θ_e is	`med_pos`	and e is	`hi_pos`	then ω is	`med_neg`
Rule 21:	If θ_e is	`hi_pos`	and e is	`hi_neg`	then ω is	`low`
Rule 22:	If θ_e is	`hi_pos`	and e is	`med_neg`	then ω is	`med_neg`
Rule 23:	If θ_e is	`hi_pos`	and e is	`low`	then ω is	`hi_neg`
Rule 24:	If θ_e is	`hi_pos`	and e is	`med_pos`	then ω is	`hi_neg`
Rule 25:	If θ_e is	`hi_pos`	and e is	`hi_pos`	then ω is	`hi_neg`

2.3. Tuning of Parameters Using a Genetic Algorithm

As we mentioned earlier, the proposed fuzzy controllers we proposed in the previous Section 2.2 need to be tunned to give acceptable results on a simulation. In this section, we describe our proposed method for MFs parameter optimization. First, we need to establish which parameters we are going to keep fixed and which parameters we are going to tune. The following two sections explain the parameter selection and the overall technique.

2.3.1. Parameter Tuning for Three MFs configuration

In this configuration, we modified the input and output variables' parameters symmetrically, varying the minimum extremes of the trapezoidal functions

$$\mu_{trap}(x) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) \tag{11}$$

where the trapezoidal begins to descend/ascend and the point where it reaches zero.

We varied these parameters symmetrically (a, b, d, e, g, h); in the case of the triangular function (1), the aperture we symmetrically optimized (c, f, i) the parameters as shown in Table 3. To keep the symmetry, we just repeat the same parameter value on both sides of a triangular function or on the values of the trapezoidal. Some values are used only to make the function broader or narrower at the bottom. In this case, we parameterize the controller with just nine variables, and these are the parameters that the GA will optimize. That we will explain in Section 2.3.

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2.3.2. Parameter Tuning for Five MFs configuration

In a similar fashion as before, we need to define which parameters will be tuned for the five MFs controller. Again we keep the MFs symmetrical around zero. In this case,

Table 3. Nine parameter configuration for three MFs fuzzy controller.

Variable	Linguistic Value	MF	Parameters
θ_e	high negative	μ_{trap}	$[-100, -100, -a, -a+b]$
θ_e	low	μ_{tria}	$[-c, 0, c]$
θ_e	high positive	μ_{trap}	$[a+b, a, 100, 100]$
error	high negative	μ_{trap}	$[-100, -100, -d, -d+e]$
error	low	μ_{tria}	$[-f, 0, f]$
error	high positive	μ_{trap}	$[d+e, d, 100, 100]$
omega	high negative	μ_{trap}	$[-8, -8, -g, -g+h]$
omega	low	μ_{tria}	$[-i, 0, i]$
omega	high positive	μ_{trap}	$[g+h, g, 8, 8]$

we just needed ten variables to parameterize the controller. To achieve this, we kept the parameters of ω fixed. The parameters are illustrated in Figure 4.

Table 4. Ten parameter configuration for five MFs fuzzy controller.

Variable	Linguistic Value	MF	Parameters
θ_e	high negative	μ_{trap}	$[-50, -5, -b, -b+c]$
θ_e	medium negative	μ_{tria}	$[-d-e, -d, -d+e]$
θ_e	low	μ_{tria}	$[-a, 0, a]$
θ_e	medium positive	μ_{tria}	$[-d-e, d, d+e]$
θ_e	high positive	μ_{trap}	$[b-c, b, 5, 50]$
error	high negative	μ_{trap}	$[-50, -5, -g, -g+h]$
error	medium negative	μ_{tria}	$[-i-j, -i, -i+j]$
error	low	μ_{tria}	$[-f, 0, f]$
error	medium positive	μ_{tria}	$[-i-j, i, i+j]$
error	high positive	μ_{trap}	$[g-h, g, 5, 50]$
ω	high negative	μ_{trap}	$[-50, -5, -1, -0.5]$
ω	medium negative	μ_{tria}	$[-1, -0.5, 0]$
ω	low	μ_{tria}	$[-0.5, 0, 0.5]$
ω	medium positive	μ_{tria}	$[0, 0.5, 1]$
ω	high positive	μ_{trap}	$[0.5, 1, 5, 50]$

2.3.3. Parameter Tuning for Three MFs configuration

Another essential aspect to consider when tuning the above parameters is the range of values that each parameter can have. Normally, we keep all the parameters in the same range when using a population-based metaheuristic. In our previous work, we compared two ranges of values for the first controller, with ranges $[0, 1]$ and $[0, 2]$. Our experiments showed better results with the narrower range, so we selected the same configuration for the three MFs controllers for this work. In this work, we propose a simple technique to change the tuning ranges for the MFs while keeping the adjustable parameters in the same range of values $[0, 1]$. We define different ranges for the input variables and normalize the values before they are passed to the membership functions. These values are shown in Table 5.

Table 5. Ranges defined for each pararamter for the 5MF controller.

Paramerer	Range	Parameter	Range
a	[0,1]	f	[0, 1]
b	[0.5,2]	g	[0.5, 2]
c	[0,2]	h	[0, 2]
d	[0.5,1.5]	i	[0.5,1.5]
e	[0,1]	j	[0, 1]

2.3.4. Genetic Algorthm Optimization

As a population-based metaheuristic, the GA needs to evaluate the fitness of each candidate solution (also called individuals) in its population. This process is illustrated in Figure 2. Each candidate solution is represented by a chromosome, here implemented as a list of objects of type `float` . To evaluate each solution, we need to generate an instance of the fuzzy controller. The membership functions are defined using the parameters included in the chromosome. Once created, the fuzzy controller is passed as a parameter, together with the tracks the mobile robot will follow. The output of the simulations is the RMSE of the accumulated errors e obtained during the simulations. We consider this measure as the fitness of the candidate solution. The GA will not select the candidate solutions with the worst fitness for reproduction.

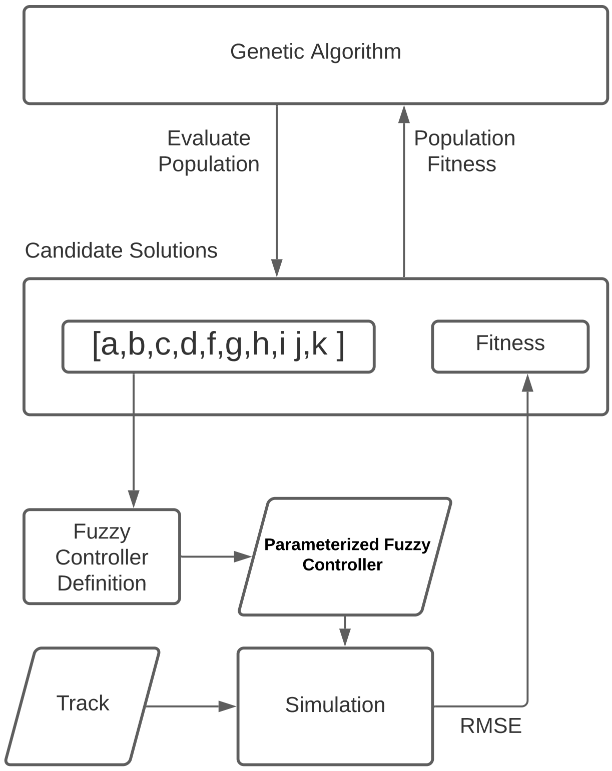
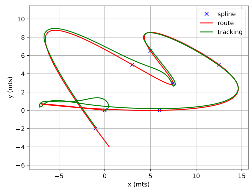
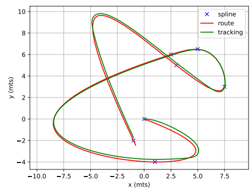
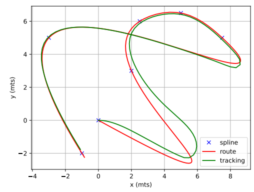


Figure 2. For each individual in the population, a controller is created with the parameters, this controller is then tested by running one or more simulations. The RMSE of the tracking is considered as the fitness for that particular candidate solution. This process is repeated for each member of the population.

In our previous work [32] we evolved the controllers with a single path; we noticed that this could lead to overtraining. Similar to what happens in supervised learning algorithms, the controller found by the GA could be specialized only to the track used for its evolution. We found that the controller with only three MFs could follow new unseen tracks, but this was not the case for the five MFs controller. It is well known that in rule-based learning, adding more rules can harm the capacity to generalize to unseen problems [37]. Noticing this, we added a list of three tracks to evaluate the fitness for the experiments, which is the average RMSE of the three simulations. We ran experiments with one and three tracks. We defined each track using cubic splines over a list of coordinates; this is a common approach in the literature [38]. The tracks and the parameters to define them are shown in Table 6. The first track was used on prevous work and was taken from the library of [39], this track starts with a very narrow curve to the left that is difficult for controllers to follow, but it remains differentiable throughout the path, we call this track "M". The other tracks are called "A" and "S" and have smoother curves, with long straight segments and different curvatures. All tracks have seven anchor points for the cubic spline.

Table 6. Tracks used for fitness evaluation, they are defined by cubic splines with the parameters shown below each plot.

Track M	Track A	Track S
		
$ax = [0, 6, 12, 5, 7.5, 3, -1]$ $ay = [0, 0, 5, 6.5, 3, 5, -2]$	$ax = [0, 1, 2.5, 5, 7.5, 3, -1]$ $ay = [0, -4, 6, 6.5, 3, 5, -2]$	$ax = [0, 2, 2.5, 5, 7.5, -3, -1]$ $ay = [0, 3, 6, 6.5, 5, 5, -2]$

2.3.5. Experimental Setup

We implemented the algorithm in Python using the DEAP [40] library for the GA implementation. We used the scikit-fuzzy [1] library to code the fuzzy inference systems. Finally, we modified a fork of the PythonRobotics repository [39], adding the odeint library from the SciPy package [2] to integrate the system of ordinary equations.

All experiments ran with the same parameters for the GA. The only obvious difference was the size of the chromosomes because this is the same as the number of parameters we need to optimize. As we mentioned earlier, we proposed two controllers, one with three membership functions; we are going to call this controller 3MF, and the other controller 5MF because it has five membership functions. For details on these controllers, see Section 2.2. After several preliminary experiments, we settled for a configuration with a population size of 50 with 20 generations. We used single-point crossover with 0.3 probability. We opted for tournament selection with a tournament size of three. For mutation, and because we are using continuous values, we selected a gaussian mutation with $\mu = 0.0$ and $\sigma = 0.2$. Finally, the probability for each gene to be mutated was 0.2. Each individual's fitness was the RMSE mentioned earlier, but to economize the computational resources, while the imulations were running, we interrupted those simulations where the robot was clearly out of the track or did not finish near the final point of the track. In these cases, we assigned a very low fitness (we want to minimize the error) of 5000 to the first case and 2000 to the second.

As the basis for comparison, we compare or results against the controller in [35], with the following control law defined as

$$\omega = \frac{v_r \mathcal{K}(s)\cos(\theta_e)}{1 - \mathcal{K}(s)e} - (k_\theta |v_r|)\theta_e - \left(k_e v_r \frac{\sin(\theta_e)}{\theta_e}\right)e, \tag{12}$$

the parameters of the simulations and the canonical controller we are comparing against are summarized in Table 7.

[1] https://github.com/scikit-fuzzy/scikit-fuzzy
[2] https://github.com/scipy/scipy

Table 7. Simulation and controller parameters.

Paramerer	Value
Wheel-base	$l = 2.5$
Steering limit	$\|\delta\| \leq \frac{\pi}{4}$
Initial configuration	$x_r(0), y_r(0), \theta(0) = (0, 0, 0)$
Velocity controller configuration	$K_p = 1, v(0) = 0, a(0) = 0$
Target velocity	$v_r = \frac{10}{3}$
Maximum time	50
Control law parameters	$k_e = 0.3, k_\theta = 1.0$

We ran the experiments on a Desktop PC with AMD Ryzen 9 3900x 12-core processor with 24 threads and 48 GB RAM with Ubuntu Linux 21.04, and Python 3.7.5 code. Code and data can be found in the following GitHub repository https://github.com/mariosky/fuzzy-control.

3. Results

This section shows the results of running the GA optimization of parameters for the two controllers described in the previous sections. We have the 3MF and 5MF controllers by running the GA optimization 30 times, using the average of the three tracks to establish the fitness. Moreover, we also ran a GA for both controllers using only the most challenging track (Track M) for the evaluation. The descriptive statistics of these results are shown in Table 8. As expected, and because the optimization is non-deterministic, there are a few outliers at both extremes of the performance. However, the deviation is higher when using only one track for the GAs evaluation. Nevertheless, the 5MF controller has the lower RMSE in both experiments. The problem we found and mentioned earlier is that the 5MF controller trained with a single track can not follow the path of the other tracks; this is not the case with the 3MF controller. For this reason, we will only consider the controller optimized with three tracks for further comparison. In the last column, we have the average execution time in seconds, for as expected, evaluating with three tracks is more time-consuming. However, we can also observe that it takes more time to evolve the 5MF controller.

Table 8. Descriptive statistics for 30 executions of the GA with an evaluation of one and three tracks.

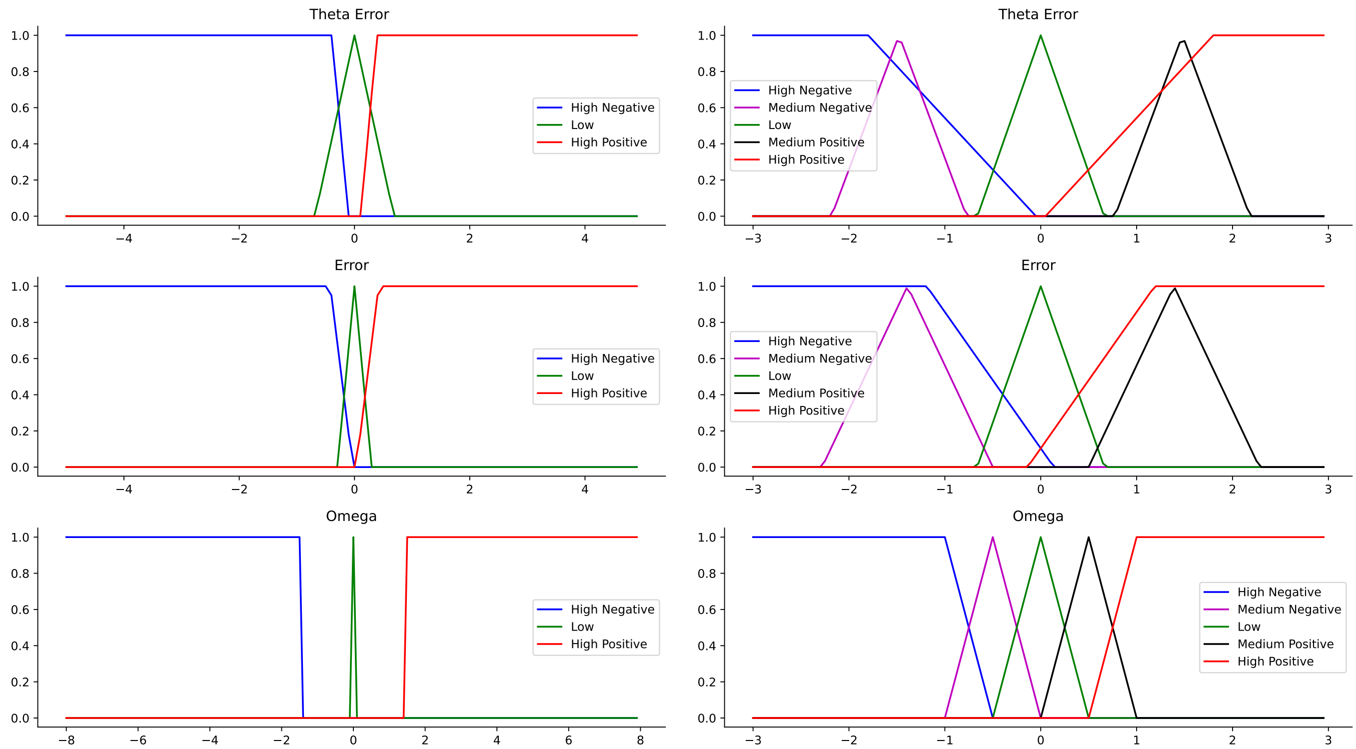
Evaluation	Controller	Average RMSE	Standard Deviation	Median	Average Exec. Time (s)
Three tracks	MF3	0.4321	0.071	0.4136	1283s
Three tracks	MF5	**0.0156**	0.031	0.0091	2664s
Single track	MF3	0.6831	0.1096	0.6866	533s
Single track	MF5	**0.0330**	0.1030	0.0055	640s

We selected the best controllers found in the 30 experiments and compared them against a controller using the control law in Equation 12. This results are presented in Table 9. Again the controller 5FM has the lower RMSE on all individual tracks. The 3MF controller has a similar RMSE when we compare it against the control law.

Table 9. RMSE of the best controllers in all three tracks.

Track	Control law	3MF	5MF
Route M	2.003	0.820	**0.003**
Route A	0.014	0.016	**0.005**
Route S	0.521	0.162	**0.008**

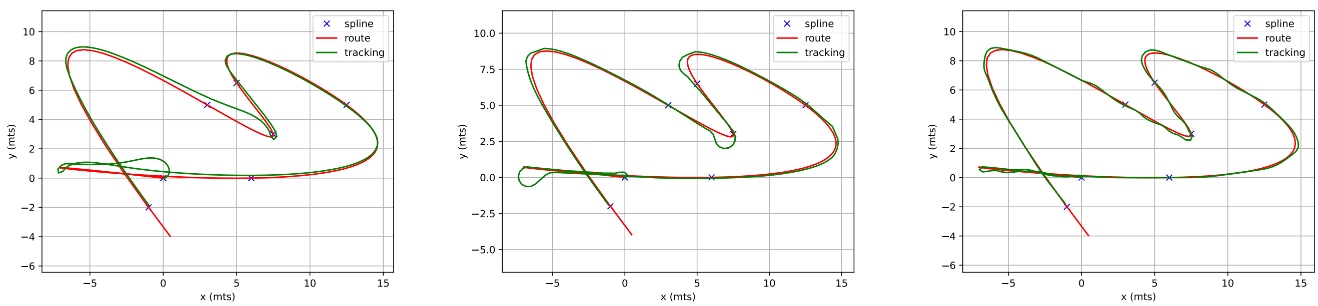
When we observe the optimized MFs for both controllers, 3MF in Figure 3(**a**) and 5MF in Figure 3(**b**), we can see that they are similar on the `low` and `high` MFS, for both the θ_r and e, moreover, in each of them both variables are very similar. With only omega having very different parameters. We can also see that the values of ω remain fixed on 5MF, because we kept those parameters fixed.



(**a**) Optimized membership functions for the 3MF fuzzy controller. (**b**) Optimized membership functions for the 5MF fuzzy controller.

Figure 3. GA optimized membership functions for both controllers.

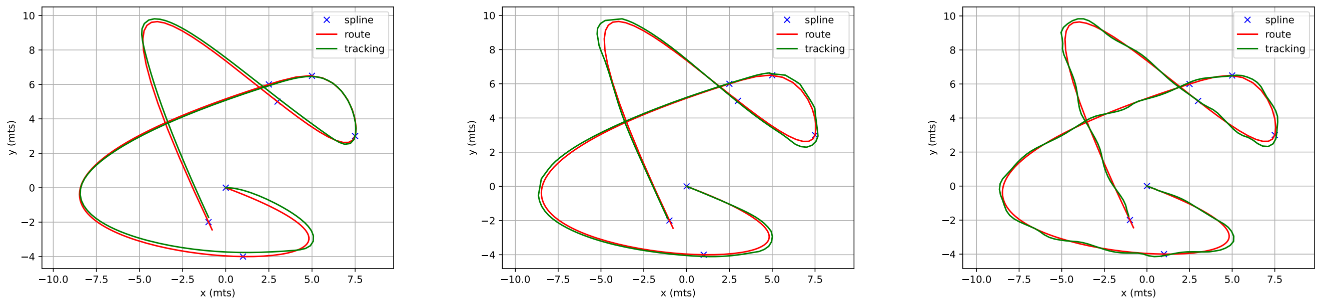
We now show a plot of the tracks and the robot's movement following the track. First, we have the track "M" (see Figure 4), in which all controllers have problems at the beginning with a curve that is very sharp to the left and then a sharp U-turn to go back to the start. We can see that the 3MF controller follows the path with less zig-zag than controller 5MF, which keeps the tracking closer to the path but with noticeable zig-zagging.



(**a**) Control law (**b**) 3MF (**c**) 5FM

Figure 4. Plot for the best simulations on Track M

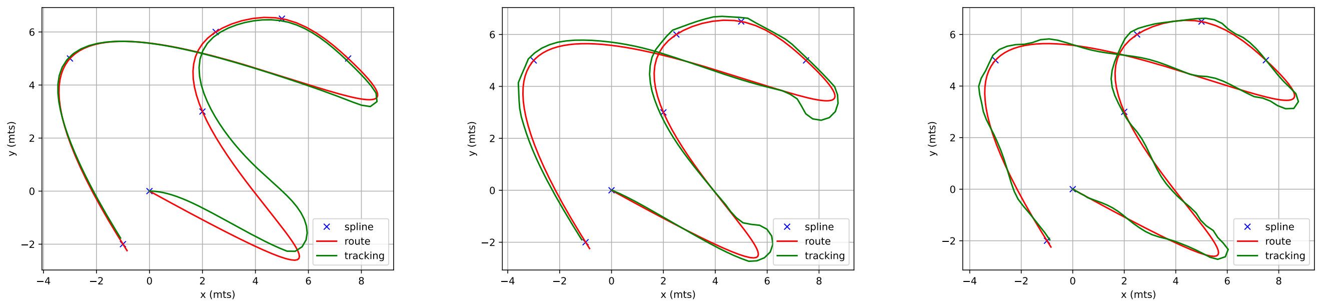
The plots in Figure 5 show the "A" track. This time all controllers closely follow the path. Again controller 5MF has a noticeable zig-zagging but manages to have the lower RMSE of the three.



(**a**) Control law (**b**) 3MF (**c**) 5FM

Figure 5. Plot for the best simulations on Track A

The results of Figure 6 highlight the overall behavior of the three controllers. The control law takes more time to reach the path when it deviates from the reference because it has smoother steering, but once it is over the track, e does not increase. That is by design because one of the conditions is that a small initial tracking error will remain small. On the other hand controller 3MF, does not deviate much from the reference, has a smoother control than the 5MF, but it has a higher RMSE.



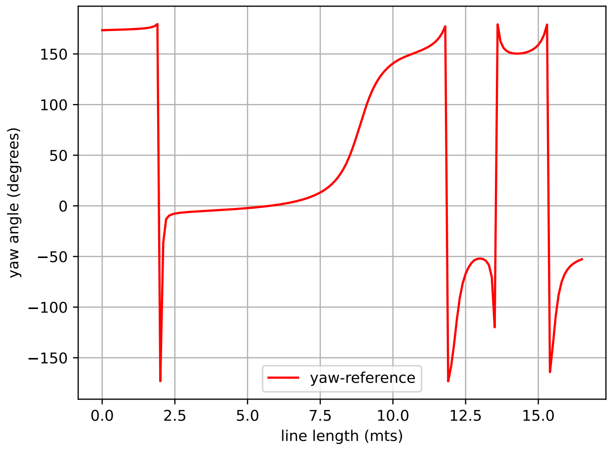
(**a**) Control law (**b**) 3MF (**c**) 5FM

Figure 6. Plot for the best simulations on Track S

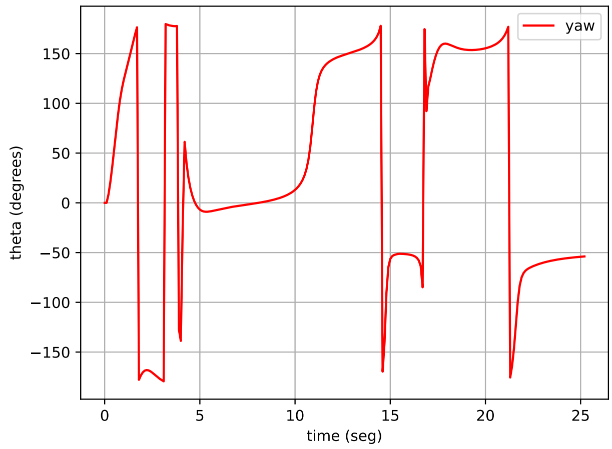
Finally, in Figure 7 we can have a better look at the steering motion of each of the controllers concerning the yaw of the "M" path. Here we can see the advantage of the control law. In this case, the 3MF controller has a very high oscillation, which is not desirable in this type of controller. The same is observed in the 5MF controller but lesser; this could be related to how these controllers evolved, giving only weight to e. This is a crucial aspect to be considered in future work. We can consider including a damping module or an evaluation metric that negatively weights this kind of oscillation. We can even include a fuzzy variable to the controller; in comparison, the control law also uses the curvature of the path $\mathcal{K}(s)$ as a variable for determining ω.

4. Discussion

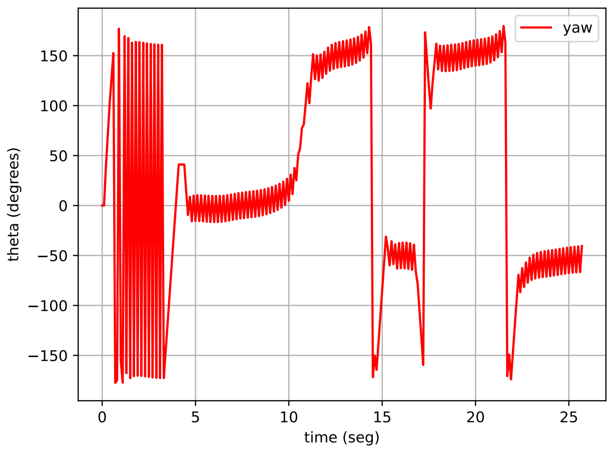
The optimization of fuzzy controllers for real-world applications requires a broader perspective. We need to treat the problem as a supervised learning task. The quality of solutions needs to consider other metrics part of the desired control properties. In this work, experiments show that the overall result improved by adding a robust validation



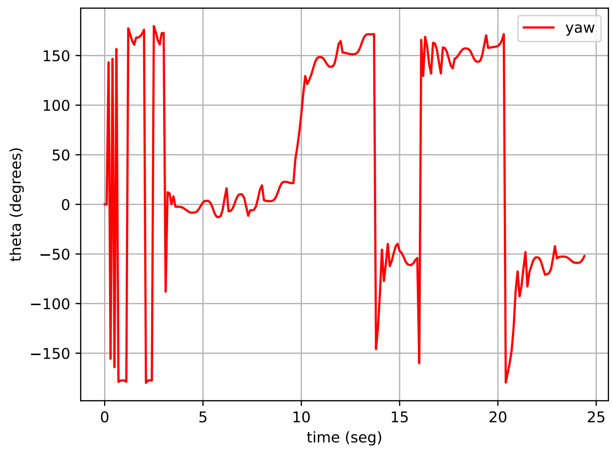
(**a**) Yaw along the refernce path $s(\gamma)$ in meters.



(**b**) Yaw of the control law as it was tracking the refernce path, in seconds.



(**c**) Yaw of the 3MF controller.



(**d**) Yaw of the 5MF controller.

Figure 7. Comparation of the yaw (steering) for each of the controllers as it was tracking the reference path "M", in seconds.

of candidate solutions. Moreover, adding more complexity to the knowledge base also improved the control in terms of RMSE, although needing more computational resources. Our literature review shows that using only the RMSE is a common practice but could not be enough for this type of application.

Furthermore, because of the computational cost as future work, we will propose a technique to execute these experiments in a distributed way. Later we could try other bio-inspired methods and compare the results. We could also add more membership functions and test other functions to improve the inference system.

Author Contributions: Conceptualization, A.M., M.G, and O.C.; methodology, A.M.; software, M.G.; validation, O.C. and J.M.; data curation, A.M.; writing—original draft preparation, A.M.; writing—review and editing, M.G. and J.M ; visualization, A.M.; supervision, O.C.; All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by projects TecNM-5654.19-P and DeepBio (TIN2017-85727-C4-2-P)

Institutional Review Board Statement: Not applicable

Informed Consent Statement: Not applicable

Data Availability Statement: All data and code is available with an open source license from https://github.com/mariosky/fuzzy-control

Conflicts of Interest: "The authors declare no conflict of interest."

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