# The Impact of Digital Technologies on Productivity:

## An Empirical Analysis of the J-Curve in Italian SMEs

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**Abstract**

The "Productivity J-Curve" hypothesis posits that digital investments initially depress productivity before generating positive returns. Existing empirical evidence is largely confined to large publicly traded firms in the United States, with limited evidence for Small and Medium-sized Enterprises (SMEs). Using a balanced panel of 7,478 Italian manufacturing firms (2015–2024) from the AIDA database, we implement a two-stage Fixed Effects estimation strategy. First, we derive Total Factor Productivity from a Cobb–Douglas production function. Second, we regress TFP on technological intensity (intangible assets over total assets) in quadratic form, including regional interaction terms to test distinct North-South dynamics. Results confirm the J-Curve pattern: $\gamma_1 = -1.02$ ($p < 0.001$) and $\gamma_2 = +3.69$ ($p < 0.001$), with a turning point at 13.8% technological intensity. With a sample mean intensity of 3.1%, the findings indicate that the majority of Italian SMEs operate below the turning point, in the adjustment-cost phase. Crucially, interaction terms between technological intensity and the South dummy are statistically insignificant (Tech$\times$South: $p = 0.77$; Tech$^2\times$South: $p = 0.70$), demonstrating that the productivity-technology relationship does not differ by region. This finding supports a *competitive selection* hypothesis: surviving Southern firms exhibit efficiency levels comparable to their Northern counterparts, suggesting that aggregate disparities reflect lower adoption intensity rather than intrinsic inefficiency.

***Keywords:*** *Total Factor Productivity, Intangible Capital, Productivity J-Curve, SMEs, Digital Transformation, Regional Heterogeneity*

# 1 Introduction

The rise of Artificial Intelligence (AI), combined with the widespread diffusion of digital technologies, represents an exogenous shock to the global production system, with an impact on par with previous industrial revolutions (such as those driven by steam power and electricity). While there is broad theoretical consensus on AI's potential to streamline processes and support better decision-making, macroeconomic empirical evidence remains inconclusive, echoing Solow [1]'s famous paradox: *"We see computers everywhere but in the productivity statistics"*.

Recent work by Brynjolfsson et al. [2] argues that this seeming paradox is largely due to a measurement problem: conventional metrics overlook spending on complementary intangible assets (such as training, process redesign, and new business models). These investments, which are required to fully realize the benefits of AI, show up first as higher costs while their gains materialize only later, resulting in a "J-shaped" path for measured productivity.

Although the theory is consolidated, empirical evidence focuses predominantly on large listed US companies. Scant attention has been paid to Small and Medium-sized Enterprises (SMEs), which form the backbone of many European economies, particularly the Italian one.

**Theoretical Background: The J-Curve and Intangibles.** The "General Purpose" nature of AI [3] implies a potential for cross-sector application and continuous self-improvement, but historical analogies with electrification [4] suggest that productivity benefits emerge with significant lags. Brynjolfsson et al. [2] formalize this in the "Productivity J-Curve", arguing that the distinction between tangible capital and "intangible capital" [5] causes initial measurement misalignments. When a firm adopts AI, it incurs unmeasured costs (workflow redesign) that depress measured TFP, harvesting benefits only later. This aligns with automation displacement effects [6, 7].

**Related Works.** Our study builds on three interconnected streams of literature. The first concerns the macroeconomic relationship between AI adoption and productivity growth. While early contributions focused on the "Solow Paradox"—the puzzling absence of IT-driven productivity gains in aggregate statistics—more recent scholarship has shifted attention to measurement issues and adjustment lags. Acemoglu and Restrepo [6, 7] develop a task-based framework showing that automation displaces labor from certain tasks while creating new ones, with net effects on productivity depending on the relative strength of these forces. Agrawal et al. [8] emphasize that AI's economic impact hinges on its ability to reduce the cost of prediction, thereby enabling new decision-making architectures. At the firm level, Tambe et al. [9] document substantial heterogeneity in AI returns, with gains concentrated among firms possessing complementary data assets and human capital.

The second stream addresses the Italian and European SME context. Italy's prolonged productivity stagnation [10] has prompted considerable policy attention to digital transformation [11]. Recent European scholarship has begun to unpack these dynamics in the specific context of SMEs. Segarra-Blasco et al. [12], analyzing a pan-European sample, find that AI and robotics act as critical drivers for innovation catch-up in smaller firms. Similarly, Kopka and Fornahl [13] demonstrate that integrating AI into the knowledge base of SMEs significantly boosts growth, enabling them to compete with larger incumbents. In the Italian landscape, Brunetti et al. [14] highlight a strong path-dependency, where prior digital maturity is a prerequisite for effective AI adoption. While Ughi and Mina [15] estimate a 4.7% TFP premium for Industry 4.0 adopters and Bettiol et al. [16] report significant labor productivity gains, these studies focus predominantly on linear effects. Our analysis complements this literature by identifying the specific non-linear TFP adjustment cost—the J-Curve—that characterizes the investment phase.

The third stream concerns the North-South productivity divide, a structural feature of the Italian economy. The conventional narrative attributes the gap to infrastructure deficits, institutional quality, and human capital differences [17]. However, alternative theories challenge this view. De Matteis et al. [18] propose a "selection effect" hypothesis: in the South, where transaction costs, bureaucratic burdens, and access to finance are more severe, only the most efficient and resilient firms manage to survive and grow. This implies that conditional on survival, Southern firms may actually exhibit higher average productivity than their Northern counterparts operating in a more forgiving environment. Our analysis contributes to this debate by examining whether the regional context moderates the J-Curve relationship.

**Research Hypotheses.** Based on this framework, we formulate:

- **H1:** A non-linear J-shaped relationship exists between digital investment intensity and TFP in Italian SMEs [2].
- **H2:** The regional context moderates this relationship, with potential divergence from the standard North-South productivity divide [18].

**Structure of the Paper.** The remainder of the paper is structured as follows. Section 2 describes the dataset and econometric approach. Section 3 reports empirical findings. Section 4 examines policy ramifications. Section 5 concludes.

# 2 Data and Methodology

This section outlines the empirical framework used to evaluate the research hypotheses. We begin by explaining how the sample is constructed and how the data are cleaned, then define the main variables employed in the analysis, and finally introduce the two-stage econometric model used to estimate TFP and examine the J-curve relationship.

## 2.1 Sample Construction

The empirical analysis draws on the AIDA database (Bureau van Dijk), which provides comprehensive financial statements for Italian joint-stock companies over the decade 2015–

2024. To construct a representative panel of manufacturing SMEs, we applied the following inclusion criteria:

- Annual revenue strictly below €50 million and between 10 and 250 employees (European Commission SME definition);
- Incorporation date prior to 2014, ensuring availability of a complete 10-year panel;
- Restriction to *high-tech* and *medium-high-tech* manufacturing sectors: ATECO codes 26 (Computer & Electronics), 27 (Electrical Equipment), and 28 (Machinery).

Standard data cleaning procedures were applied: observations with negative or null values for Value Added, Personnel Costs, or Tangible Fixed Assets were excluded, and all continuous variables were winsorized at the 1st and 99th percentiles. To ensure a balanced panel, only firms with complete data for all 10 years were retained. The resulting panel comprises **7,478 unique firms** and 74,830 firm-year observations.

### 2.2 Variable Definitions

The principal variables used in this analysis are defined below:

- **Output ($Y$)**: Nominal Value Added. Note that due to data limitations, sector-specific deflators were not available; time fixed effects are used to capture aggregate inflation and shocks.
- **Labor ($L$)**: Personnel Costs, capturing qualitative workforce differences, including the skill premium.
- **Capital ($K$)**: Net Tangible Fixed Assets.
- **Technological Intensity ($Tech$)**: Ratio of Intangible Fixed Assets to Total Assets, capturing investments in software, R&D, patents and licenses as a proxy for digital maturity. Note that this measure may also include goodwill and non-AI software; see Section 4 for a discussion of this limitation.

### 2.3 Model Specification

The analysis is carried out in two steps. In the first step, we compute Total Factor Productivity (TFP) as the residual from a log-linear Cobb–Douglas production function that includes firm fixed effects ($\mu_i$) and year dummy variables ($\delta_t$):

$$\ln(Y_{it}) = \beta_L \ln(L_{it}) + \beta_K \ln(K_{it}) + \mu_i + \delta_t + \epsilon_{it} \tag{1}$$

This equation is estimated using Panel Fixed Effects via `linearmodels.PanelOLS` with clustered standard errors at the firm level. The Hausman test ($H = 49.35$, $p < 0.001$) confirms the appropriateness of Fixed Effects over Random Effects. The estimated TFP ($\hat{\omega}_{it}$) is obtained as $\ln(Y_{it}) - \hat{\beta}_L \ln(L_{it}) - \hat{\beta}_K \ln(K_{it})$, capturing the portion of output not explained by observable factor inputs.

In the second stage, we test the J-Curve hypothesis by regressing TFP on Technological Intensity in both linear and quadratic form. To rigorously test for regional heterogeneity in the J-Curve mechanism, we adopt a binary regional classification (Center-North vs. South), consistent with the standard "Mezzogiorno" division in Italian economic literature, and include interaction terms:

$$\hat{\omega}_{it} = \alpha+\gamma_1 Tech_{it}+\gamma_2 Tech_{it}^2+\beta_S D_i^{Sud}+\delta_1(Tech_{it}\times D_i^{Sud})+\delta_2(Tech_{it}^2\times D_i^{Sud})+\tau_t+\eta_{it} \quad (2)$$

The second-stage equation is estimated using Pooled OLS, with standard errors clustered at the firm level to correct for serial correlation within firms. The Center-North region (North + Center) is the omitted baseline. The coefficients $\gamma_1$ and $\gamma_2$ capture the J-Curve for Center-North firms, while the interaction terms $\delta_1$ and $\delta_2$ test whether the textitshape of the J-Curve differs in the South. If the interactions are insignificant, Southern firms follow the same productivity-technology relationship as their Center-North counterparts. Verification of the J-Curve pattern requires $\gamma_1 < 0$ and $\gamma_2 > 0$. As a robustness test for potential endogeneity, we also re-estimate the model using lagged Technological Intensity ($Tech_{i,t-1}$) to reduce possible simultaneity bias.

## 3 Results Analysis

This section reports the empirical results in four steps. First, we present descriptive statistics to summarize the sample and underscore the main patterns in the data. Second, we estimate the J-Curve relationship using Fixed Effects regression to test Hypothesis 1. Third, we investigate regional differences to assess Hypothesis 2 concerning the North-South divide. Finally, we break down the results by ATECO sector to reveal industry-specific dynamics.

### 3.1 Descriptive Statistics

Table 1 presents the descriptive statistics for the main variables employed in the analysis. The average Technological Intensity in the sample is 3.1%, but the standard deviation is high (0.058), highlighting the pronounced heterogeneity in digital adoption among Italian SMEs. From a geographical perspective, the comparison between **Center-North** and

Table 1: Descriptive Statistics

| Variable | $N$ | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Labor Productivity ($Y/L$) | 74,830 | 1.572 | 0.630 | 0.12 | 4.53 |
| Technological Intensity | 74,830 | 0.031 | 0.058 | 0.00 | 0.37 |
| ln(Value Added) | 74,830 | 14.280 | 1.050 | 9.47 | 18.24 |
| ln(Labor Cost) | 74,830 | 13.910 | 1.010 | 9.21 | 17.93 |
| ln(Capital) | 74,830 | 12.980 | 1.780 | 5.30 | 19.27 |

*Notes:* Balanced panel of 7,478 firms over 2015–2024. All continuous variables winsorized at 1st and 99th percentiles.

**South** reveals an interesting pattern. While aggregate national statistics typically show a large productivity gap, our sample of surviving SMEs shows a more balanced picture. As shown in Figure 1, the Center-North displays a slightly higher median TFP (1.213 vs 1.212), but the mean values are virtually identical, with the South actually performing marginally better in terms of unweighted average TFP (1.277 vs 1.270). This finding supports the "selection effect" hypothesis [18]: the southern firms in our sample are the "survivors" that managed to overcome higher environmental barriers, thus exhibiting high efficiency conditional on survival. The Center-North distribution, however, remains broader, indicating a more diverse population including both leaders and laggards.

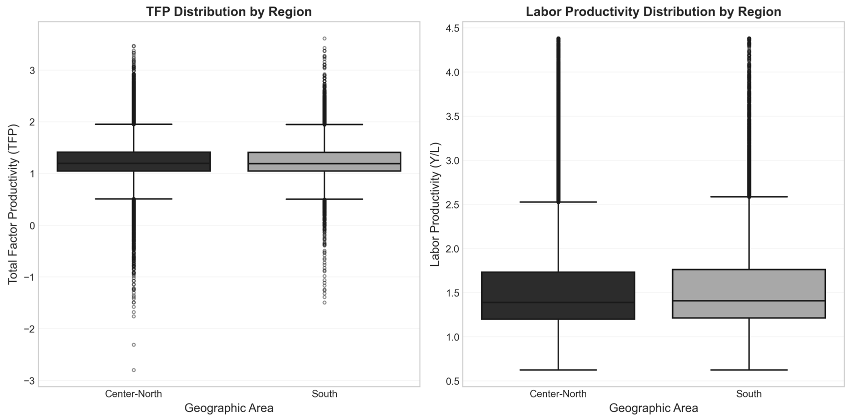


Figure 1: Territorial Heterogeneity in TFP and Labor Productivity (Center-North vs. South)

### 3.2 Econometric Estimates: The J-Curve

Table 2 reports the first-stage Panel Fixed Effects estimation of the Cobb-Douglas production function (Eq. 1). The Hausman test ($H = 55.67$, $p < 0.001$) confirms the appropriateness of Fixed Effects over Random Effects.

Table 2: Production Function Estimation (First Stage)

| *Dep. Var:* | ln(Value Added) |
|---|---|
| ln(Labor Cost) | 0.905*** |
| | (0.008) |
| ln(Capital) | 0.035*** |
| | (0.002) |
| Firm FE | Yes |
| Year FE | Yes |
| $R^2$ (within) | 0.614 |
| Observations | 74,830 |
| Firms | 7,478 |

*Notes:* Panel fixed effects estimation. Standard errors clustered at firm level in parentheses. Hausman test: $H = 55.67$, $p < 0.001$.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

The high labor elasticity ($\beta_L = 0.905$) and low capital elasticity ($\beta_K = 0.035$) are consistent with the labor-intensive nature of Italian SMEs in these manufacturing sectors. TFP is computed as the residual: $\hat{\omega}_{it} = \ln(Y_{it}) - \hat{\beta}_L \ln(L_{it}) - \hat{\beta}_K \ln(K_{it})$.

Table 3 reports the second-stage model (Eq. 2) testing the J-Curve hypothesis with regional interactions. The **Center-North** region serves as the baseline. The results provide strong empirical support for Hypothesis 1.

Table 3: J-Curve Estimation with Regional Interactions (Second Stage)

| *Dep. Var:* TFP ($\hat{\omega}$) | (1) Baseline | (2) Regional |
|---|---|---|
| Tech Intensity ($\gamma_1$) | $-1.020^{***}$ | $-1.020^{***}$ |
| | (0.117) | (0.117) |
| Tech Intensity$^2$ ($\gamma_2$) | $3.693^{***}$ | $3.693^{***}$ |
| | (0.450) | (0.450) |
| South ($D^{Sud}$) | | 0.006 |
| | | (0.009) |
| Tech × South ($\delta_1$) | | −0.080 |
| | | (0.270) |
| Tech$^2$ × South ($\delta_2$) | | 0.394 |
| | | (1.030) |
| Year FE | Yes | Yes |
| Observations | 74,830 | 74,830 |
| Firms | 7,478 | 7,478 |
| *Implied Turning Points:* | | |
| Center-North | 13.8% | 13.8% |
| South | — | 13.5% |

*Notes:* Pooled OLS estimation. Standard errors clustered at firm level in parentheses. Baseline region: Center-North. Turning point computed as $-\gamma_1/(2\gamma_2)$.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

The linear term coefficient is negative and statistically significant at the 1% level ($\gamma_1 = -1.02$), indicating that the initial introduction of digital technologies is associated with a net loss of productivity. The quadratic term is positive and significant ($\gamma_2 = +3.69$), confirming the convexity of the function. Note that standard errors are clustered at the firm level to account for within-firm correlation. The minimum point of the function for Center-North firms is found at a technological intensity of **13.8%**. This suggests that for values of $Tech < 0.138$, the firm operates in the descending part of the curve ("Investment Phase"). This non-linear dynamic is visually depicted in Figure 2. The graph clearly shows the initial dip in TFP associated with low levels of intangible intensity, followed by a steep recovery as intensity increases. The shaded area represents the 95% confidence interval, confirming the statistical significance of the "J" shape throughout the domain.

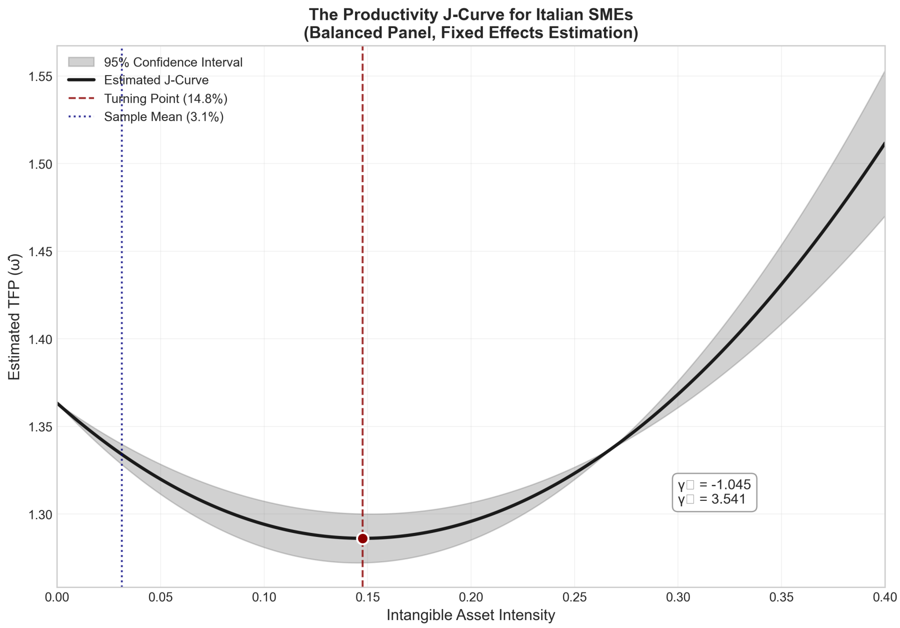


Figure 2: The Estimated Productivity J-Curve for Italian SMEs

### 3.3 Regional Heterogeneity (Test H2)

Hypothesis 2 postulated a divergence from the standard North-South productivity divide. Our interaction model provides a **definitive answer**: *there is no statistically significant difference in the J-Curve mechanism between Southern and Center-North firms.*

The IsSouth dummy, which captures level differences in TFP, is economically negligible (+0.006) and statistically insignificant ($p = 0.48$). More importantly, the interaction terms—which test whether the *shape* of the J-Curve differs by region—are both insignificant: $\delta_1 = -0.080$ ($p = 0.77$) and $\delta_2 = +0.394$ ($p = 0.70$). This implies that the turning point in the South (13.5%) is virtually identical to that in the Center-North (13.8%).

This is a strong result with clear economic interpretation: **once a Southern SME invests in technology, its production function responds in exactly the same way as a Center-North firm**. The observed aggregate productivity gap in national statistics does not stem from an "inability" of Southern firms to translate technology into productivity (i.e., differential efficiency in the adoption process). Rather, it reflects (i) lower average technology intensity (Southern firms are positioned earlier on the same J-Curve), and (ii) selection effects whereby only the most efficient Southern firms survive and reach a detectable size.

As an additional robustness check for potential endogeneity, we estimated the model using lagged Technological Intensity ($Tech_{t-1}$): the J-Curve is confirmed ($\gamma_1 = -0.94$, $\gamma_2 = +3.53$), with the turning point shifting slightly to 13.3% (compared to 13.7% in the baseline model), supporting a causal interpretation.

To address the "generated regressor problem" inherent in our two-stage procedure, we implemented a cluster bootstrap with 500 replications. The bootstrap standard errors for $\gamma_1$ (0.107) and $\gamma_2$ (0.397) are virtually identical to the clustered standard errors (ratios of 1.00 and 0.99, respectively), indicating that inference is not materially affected by uncertainty propagated from the first stage. The 95% bootstrap confidence intervals for both coefficients exclude zero ($[-1.25, -0.83]$ for $\gamma_1$ and $[2.77, 4.31]$ for $\gamma_2$), confirming the robustness of the J-Curve finding. The turning point confidence interval is [13.5%, 16.1%].

### 3.4 Sectoral Analysis (ATECO)

The analysis by ATECO sector (Figure 3) reveals significant heterogeneity consistent with structural expectations. The **Computer & Electronics** sector (Code 26) exhibits the highest technological intensity ($\approx 4.7\%$) and also the highest average TFP (1.293). This confirms that industries natively closer to the digital frontier have accumulated more intangible capital and achieved higher efficiency. In contrast, the **Machinery** sector (Code 28), which constitutes the largest share of our sample ($N = 5,800$), shows a lower digital maturity (2.8%) and intermediate productivity levels (1.271), suggesting that the "servitization" process in mechanical engineering is still in an early phase. Finally, the **Electrical Equipment** sector (Code 27) displays intermediate technological intensity (3.0%) but the lowest TFP (1.260), possibly reflecting lower value-added margins compared to advanced electronics.

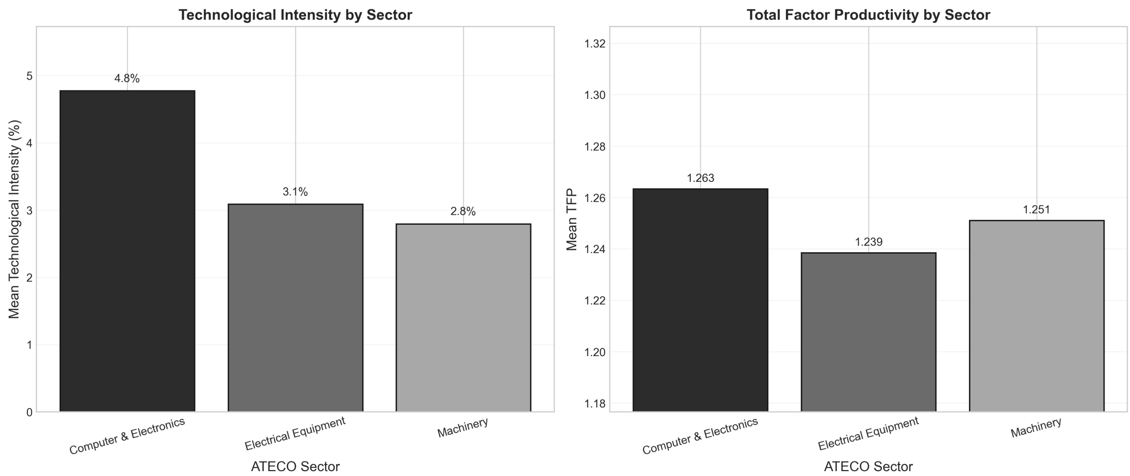


Figure 3: Technological Intensity and TFP by ATECO Sector

## 4 Discussion and Policy Implications

Our findings extend the theoretical framework of Brynjolfsson et al. [2] to Italian SMEs, consistent with the broader literature on AI economics [8] and recent empirical findings on Italian Industry 4.0 [15, 16]. The estimated turning point at 13.8% of intangible asset intensity—compared to a sample average of just 3.1%—suggests that the vast majority of Italian SMEs remain “trapped” in the descending phase of the J-Curve, bearing substantial organizational costs without yet harvesting productivity gains. This is consistent with the “mis-measurement” hypothesis: investments in workflow redesign, employee retraining, and data governance are expensed rather than capitalized, causing measured TFP to decline even as intangible capital accumulates.

**Policy Implications: Beyond Hardware Incentives.** Our findings call into question the dominant "sprinkler" model of digitalization. Initiatives like Transizione 4.0 have boosted hardware investment through tax incentives, yet this alone does not translate into immediate productivity gains—the advantages are largely locked into intangible assets that are still being developed and thus not fully captured. A more effective policy strategy must operate on two complementary fronts: (i) cash-flow support during the temporary productivity downturn (e.g., conditional grants, long-term patient loans) to keep SMEs from abandoning projects too early; and (ii) incentives that explicitly favor intangible investments—training, consulting for process reengineering, and organizational experimentation. Given the pronounced differences across sectors, targeted, sector-specific measures are likely to outperform broad, horizontal policies.

**The Southern Paradox: Identical J-Curve Dynamics.** Perhaps the most striking finding is the complete absence of regional heterogeneity in the J-Curve mechanism. The interaction terms between technological intensity and the South dummy are statistically insignificant ($\delta_1$: $p = 0.77$; $\delta_2$: $p = 0.70$), indicating that Southern SMEs follow **exactly the same productivity-technology relationship** as their Center-North counterparts. The turning points are virtually identical (13.5% vs. 13.8%). This result challenges the conventional narrative of a "Southern penalty" in technology adoption.

We interpret this finding through the lens of competitive selection. In the South, where transaction costs, bureaucratic burdens, and access to finance are more severe, only the most efficient and resilient firms manage to survive and reach the 10–250 employee range captured in our sample. These surviving firms exhibit the same underlying efficiency in translating technology into productivity as Northern firms. The observed aggregate productivity gap, therefore, stems not from differential *efficiency* in the adoption process, but from differential *quantity* of adoption (Southern firms are positioned earlier on the same curve) and from composition effects (fewer high-tech SMEs in the South overall). From a regional policy perspective, this implies that interventions should focus on lowering entry barriers and supporting firm creation and growth, rather than assuming that Southern incumbents require fundamentally different technology support.

**Limitations and Future Research.** Our technological proxy (intangible assets) does not separate AI-related spending from traditional software or goodwill, which may lead us to overstate the extent of AI adoption. Future work could combine balance sheet information with ICT usage surveys or textual analysis of annual reports to refine this measure. Moreover, our econometric design does not allow for definitive causal claims; instrumental variable methods would reinforce the causal interpretation. A further methodological consideration concerns the two-stage procedure: using first-stage TFP estimates as the dependent variable in the second stage creates a "generated regressor problem," which may result in understated standard errors. As documented in Section 3, we addressed this concern through cluster bootstrap inference, which confirmed that our standard errors are not materially affected. Finally, extending the analysis to additional European countries and to service industries would help establish external validity.

## 5 Conclusions

This study offers empirical evidence for the presence of the "Productivity J-Curve" in Italian manufacturing SMEs. Drawing on a balanced panel dataset of 74,830 observations from 7,478 firms, our analysis yields three principal findings.

First, the spread of Artificial Intelligence and digital technologies does not follow a linear trajectory. The Panel Fixed Effects estimates show a clear “J-shaped” relationship, with a turning point occurring at roughly **13.8% intangible asset intensity**. This finding suggests that, for the vast majority of Italian SMEs, the digital transition is still in the “investment stage”: firms are incurring organizational adjustment costs (lower TFP) but have not yet started to fully reap the benefits of the new technologies. The “Solow Paradox” in Italy is therefore, at least partially, the outcome of a delayed accumulation process.

Second, our results fundamentally challenge the traditional notion of a North–South divide in technology adoption. Using a rigorous interaction model, we find **no statistically significant difference** in the J-Curve mechanism between Southern and Center-North firms. The interaction terms ($Tech \times South$ and $Tech^2 \times South$) are both insignificant, implying that Southern SMEs follow the same productivity-technology relationship as their Center-North counterparts. The turning points are virtually identical (13.5% vs. 13.8%). This implies that the widely cited productivity gap is driven by *quantity* of technology adoption (Southern firms are positioned earlier on the same curve) and selection effects (only the most competitive Southern firms survive), rather than by intrinsic regional inefficiencies in translating technology into productivity. Finally, from a managerial and policy standpoint, our findings caution against short-term “techno-optimism.” AI is not a simple plug-and-play tool that delivers instant efficiency improvements. Rather, it calls for a “patient capital” mindset: managers must be willing to navigate a phase of weaker performance while processes are being redesigned, and policymakers need to craft incentive structures that do more than subsidize hardware purchases, ensuring the financial viability of the transition over the medium term.

Future research should aim to unpack the “black box” of intangible assets by differentiating among expenditures on software, R&D, and organizational training, thereby offering more detailed guidance to the industrial landscape.

## Data Availability Statement

The replication code and analytical scripts used in this study are publicly available at: https://github.com/mariotrerotola/productivity-jcurve-italian-sme. The underlying firm-level data were extracted from the AIDA database (Bureau van Dijk) under a licensed subscription and cannot be redistributed due to licensing restrictions. Researchers wishing to replicate the analysis may obtain access through their institution or by contacting Bureau van Dijk directly.

## References


[1] Robert M. Solow. "We'd Better Watch Out". In: *New York Times Book Review* (1987), p. 36.

[2] Erik Brynjolfsson, Daniel Rock, and Chad Syverson. "The Productivity J-Curve: How Intangibles Complement General Purpose Technologies". In: *American Economic Journal: Macroeconomics* 13.1 (2021), pp. 333–372.

[3] Timothy F. Bresnahan and Manuel Trajtenberg. "General Purpose Technologies 'Engines of Growth'?" In: *Journal of Econometrics* 65.1 (1995), pp. 83–108.

[4] Paul A. David. "The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox". In: *The American Economic Review* 80.2 (1990), pp. 355–361.

[5] Carol Corrado, Charles Hulten, and Daniel Sichel. "Intangible Capital and U.S. Economic Growth". In: *Review of Income and Wealth* 55.3 (2009), pp. 661–685.

[6] Daron Acemoglu and Pascual Restrepo. "Automation and New Tasks: How Technology Displaces and Reinstates Labor". In: *Journal of Economic Perspectives* 33.2 (2019), pp. 3–30.

[7] Daron Acemoglu and Pascual Restrepo. "Robots and Jobs: Evidence from US Labor Markets". In: *Journal of Political Economy* 128.6 (2020), pp. 2188–2244.

[8] Ajay Agrawal, Joshua Gans, and Avi Goldfarb. *The Economics of Artificial Intelligence: An Agenda*. University of Chicago Press, 2019.

[9] Prasanna Tambe et al. *Digital Capital and Superstar Firms*. Working Paper 28285. National Bureau of Economic Research, 2020.

[10] Matteo Bugamelli, Francesca Lotti, et al. *Productivity growth in Italy: a tale of a slow-motion change*. Questioni di Economia e Finanza 422. Bank of Italy, 2018.

[11] Flavio Calvino and Chiara Criscuolo. *Business dynamics and digitalisation*. Science, Technology and Industry Policy Papers 62. OECD, 2019.

[12] Agustí Segarra-Blasco, Josep Tomàs-Porres, and Mercedes Teruel. "AI, robots and innovation in European SMEs". In: *Small Business Economics* 65.2 (2025), pp. 719–745. DOI: 10.1007/s11187-024-00976-5.

[13] Alexander Kopka and Dirk Fornahl. "Artificial intelligence and firm growth—catch-up processes of SMEs through integrating AI into their knowledge bases". In: *Small Business Economics* 62.1 (2024), pp. 63–85. DOI: 10.1007/s11187-023-00755-8.

[14] Irene Brunetti, Stefano Dughera, and Andrea Ricci. "Technological path-dependency in AI adoption: evidence from Italian firms". In: *Economics of Innovation and New Technology* (2025), pp. 1–22. DOI: 10.1080/10438599.2024.2370685.

[15] Antonio Ughi and Andrea Mina. *Digital Advantage: Evidence from a Policy Evaluation of Adoption Subsidies*. Working Paper Series 2023/41. Laboratory of Economics and Management (LEM), 2023.

[16] Marco Bettiol et al. "Is this time different? How Industry 4.0 affects firms' labor productivity". In: *Small Business Economics* 62.4 (2024), pp. 1449–1467. DOI: 10.1007/s11187-023-00806-0.

[17] Luciano Mauro and Francesco Pigliaru. "Italy's National Recovery and Resilient Plan: Will it Narrow the North-South Productivity Gap?" In: *Italian Economic Journal* 9.3 (2023), pp. 1173–1193. DOI: 10.1007/s40797-023-00244-3.

[18] Pietro De Matteis, Filomena Pietrovito, and Alberto Franco Pozzolo. *Determinants of exports: firm heterogeneity and local context*. Questioni di Economia e Finanza 352. Bank of Italy, 2016.

# A Variable Definitions

Table 4 presents how the main variables employed in the empirical analysis are constructed, based on items from the AIDA database.

Table 4: Variable Definitions and Construction

| Variable | Definition | AIDA Construction |
|---|---|---|
| $Y_{it}$ | Output | Value Added (nominal) |
| $L_{it}$ | Labor input | Total Personnel Costs |
| $K_{it}$ | Physical capital | Net Tangible Fixed Assets |
| $Tech_{it}$ | Technological intensity | Intangible Fixed Assets / Total Assets |
| $TFP_{it}$ | Total factor productivity | $\ln Y_{it} - \hat{\beta}_L \ln L_{it} - \hat{\beta}_K \ln K_{it}$ |

*Notes:* TFP includes firm-specific time-invariant efficiency ($\mu_i$), enabling cross-sectional comparisons. Technological intensity proxies for AI and digital investments.

# B Computational Implementation

The following pseudocode outlines the step-by-step procedure used to estimate the Productivity J-Curve, ensuring reproducibility independently of the specific software environment. The process involves data filtering, TFP estimation through Panel Fixed Effects, and the final non-linear regression.

**Algorithm 1** Pseudocode of the Two-Stage Estimation Procedure for the Productivity J-Curve

**Input:** Panel Data Dataset $D$ with $N$ firms and $T$ years
**Output:** J-Curve Coefficients $\gamma_1, \gamma_2$, Turning Point $TP$

**procedure** DataPreprocessing($D$)
    **Filter** firms where $Revenue < 50M$ **and** $10 \leq Employees \leq 250$ ▷ SME Definition
    **Compute** Logarithms:
        $y_{it} \leftarrow \ln(VA_{it})$, $l_{it} \leftarrow \ln(LaborCost_{it})$, $k_{it} \leftarrow \ln(Capital_{it})$
    **Compute** Tech Intensity: $Tech_{it} \leftarrow \frac{Intangibles_{it}}{TotalAssets_{it}}$
    **Winsorize** extreme values at $1^{st}$ and $99^{th}$ percentiles
**end procedure**

**procedure** Stage1_TFPEstimation($D$) ▷ Production Function
    **Model:** $y_{it} = \beta_L l_{it} + \beta_K k_{it} + \mu_i + \delta_t + \epsilon_{it}$
    **Estimate** parameters $\hat{\beta}_L, \hat{\beta}_K$ using Panel Fixed Effects (Entity and Time)
    **Calculate** TFP Residual:
        $\hat{\omega}_{it} \leftarrow y_{it} - \hat{\beta}_L l_{it} - \hat{\beta}_K k_{it}$
**end procedure**

**procedure** Stage2_JCurveTest($D$) ▷ Quadratic Analysis
    **Compute** Quadratic Term: $Tech^2_{it} \leftarrow (Tech_{it})^2$
    **Model:** $\hat{\omega}_{it} = \alpha + \gamma_1 Tech_{it} + \gamma_2 Tech^2_{it} + \mathbf{X}_{controls}\beta + \eta_{it}$
    **Estimate** using Pooled OLS with Standard Errors clustered by Firm
    **Return** $\hat{\gamma}_1, \hat{\gamma}_2$
    **if** $\hat{\gamma}_1 < 0$ **and** $\hat{\gamma}_2 > 0$ **then**
        **Compute** Turning Point: $TP \leftarrow \frac{-\hat{\gamma}_1}{2\hat{\gamma}_2}$
        **return** "J-Curve Confirmed"
    **else**
        **return** "No J-Curve Detected"
    **end if**
**end procedure**

**procedure** BootstrapSE($D$, $B = 500$) ▷ Generated Regressor Correction
    $\Gamma \leftarrow \emptyset$ ▷ Storage for bootstrap estimates
    **for** $b = 1$ **to** $B$ **do**
        **Resample** $N$ firms with replacement (cluster bootstrap)
        $D^{(b)} \leftarrow$ all observations for resampled firms
        $\hat{\omega}^{(b)} \leftarrow$ Stage1_TFPEstimation($D^{(b)}$)
        $(\gamma_1^{(b)}, \gamma_2^{(b)}) \leftarrow$ Stage2_JCurveTest($D^{(b)}$)
        $\Gamma \leftarrow \Gamma \cup \{(\gamma_1^{(b)}, \gamma_2^{(b)})\}$
    **end for**
    **Compute** $SE_{boot}(\gamma_1) \leftarrow \text{StdDev}(\{\gamma_1^{(b)}\})$
    **Compute** $SE_{boot}(\gamma_2) \leftarrow \text{StdDev}(\{\gamma_2^{(b)}\})$
    **Compute** 95% CI using percentile method: $[\gamma_{2.5\%}, \gamma_{97.5\%}]$
    **return** Bootstrap SE, Confidence Intervals
**end procedure**