The Impact of Digital Technologies on Productivity:

An Empirical Analysis of the J-Curve in Italian SMEs

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Abstract

While the "Productivity J-Curve" hypothesis is well-documented for large US firms, empirical evidence for Small and Medium-sized Enterprises remains scarce. This paper extends the framework to Italian manufacturing SMEs, using a two-stage Panel Fixed Effects model on a balanced panel of 8,683 firms from AIDA (2015–2024).

In the first stage, Total Factor Productivity is derived as the residual from a Cobb–Douglas production function. The second stage regresses TFP on Technological Intensity in quadratic form. A Hausman test ($H = 49.35$, $p < 0.001$) confirms the appropriateness of Fixed Effects.

Results validate the J-curve: initial investments reduce TFP ($\gamma_1 = -1.04$), with returns turning positive only beyond 14.6% intensity ($\gamma_2 = +3.58$). Regarding regional heterogeneity, we find no robust productivity disadvantage for Southern SMEs, challenging the traditional North–South divide narrative. These findings suggest that policy should prioritize intangible capital accumulation over hardware subsidies, and that the "Solow Paradox" in Italy reflects delayed adjustment costs rather than technological failure.

Keywords: *Artificial Intelligence, Productivity J-Curve, TFP, SME, Italian Manufacturing*

1 Introduction

The rise of Artificial Intelligence (AI), combined with the widespread diffusion of digital technologies, represents an exogenous shock to the global production system, with an impact on par with previous industrial revolutions (such as those driven by steam power and electricity). While there is broad theoretical consensus on AI's potential to streamline processes and support better decision-making, macroeconomic empirical evidence remains inconclusive, echoing Solow [1]'s famous paradox: *"We see computers everywhere but in the productivity statistics"*.

Recent work by Brynjolfsson et al. [2] argues that this seeming paradox is largely due to a measurement problem: conventional metrics overlook spending on complementary intangible assets (such as training, process redesign, and new business models). These investments, which are required to fully realize the benefits of AI, show up first as higher costs while their gains materialize only later, resulting in a "J-shaped" path for measured productivity.

Although the theory is consolidated, empirical evidence focuses predominantly on large listed US companies. Scant attention has been paid to Small and Medium-sized Enterprises (SMEs), which form the backbone of many European economies, particularly the Italian one.

Theoretical Background: The J-Curve and Intangibles. The "General Purpose" nature of AI [3] implies a potential for cross-sector application and continuous self-improvement, but historical analogies with electrification [4] suggest that productivity benefits emerge with significant lags. Brynjolfsson et al. [2] formalize this in the "Productivity J-Curve", arguing that the distinction between tangible capital and "intangible capital" [5] causes initial measurement misalignments. When a firm adopts AI, it incurs unmeasured costs (workflow redesign) that depress measured TFP, harvesting benefits only later. This aligns with automation displacement effects [6, 7].

Related Works. Our study builds on three interconnected streams of literature. The first concerns the macroeconomic relationship between AI adoption and productivity growth. While early contributions focused on the "Solow Paradox"—the puzzling absence of IT-driven productivity gains in aggregate statistics—more recent scholarship has shifted attention to measurement issues and adjustment lags. Acemoglu and Restrepo [6, 7] develop a task-based framework showing that automation displaces labor from certain tasks while creating new ones, with net effects on productivity depending on the relative strength of these forces. Agrawal et al. [8] emphasize that AI's economic impact hinges on its ability to reduce the cost of prediction, thereby enabling new decision-making architectures. At the firm level, Tambe et al. [9] document substantial heterogeneity in AI returns, with gains concentrated among firms possessing complementary data assets and human capital.

The second stream addresses the Italian context specifically. Italy's prolonged productivity stagnation [10] has prompted considerable policy attention to digital transformation as a potential remedy [11]. However, adoption has lagged behind other advanced

economies, particularly among SMEs. Recent empirical work on the "Industry 4.0" policy package yields encouraging results: Ughi and Mina [12], using a difference-in-differences design, estimate a 4.7% TFP premium for firms adopting 4.0-enabled technologies; Bettiol et al. [13] report significant labor productivity gains in manufacturing firms that integrated digital technologies into their production processes. Importantly, these studies focus predominantly on the effects of adoption *per se*, leaving open the question of non-linear dynamics.

The third stream concerns the North-South productivity divide, a structural feature of the Italian economy. The conventional narrative attributes the gap to infrastructure deficits, institutional quality, and human capital differences [14]. However, alternative theories challenge this view. De Matteis et al. [15] propose a "selection effect" hypothesis: in the South, where transaction costs, bureaucratic burdens, and access to finance are more severe, only the most efficient and resilient firms manage to survive and grow. This implies that conditional on survival, Southern firms may actually exhibit higher average productivity than their Northern counterparts operating in a more forgiving environment. Our analysis contributes to this debate by examining whether the regional context moderates the J-Curve relationship.

Research Hypotheses. Based on this framework, we formulate:

H1: A non-linear J-shaped relationship exists between digital investment intensity and TFP in Italian SMEs [2].

H2: The regional context moderates this relationship, with potential divergence from the standard North-South productivity divide [15].

Structure of the Paper. The remainder of the paper is structured as follows. Section 2 describes the dataset and econometric approach. Section 3 reports empirical findings. Section 4 examines policy ramifications. Section 5 concludes.

2 Data and Methodology

This section outlines the empirical framework used to evaluate the research hypotheses. We begin by explaining how the sample is constructed and how the data are cleaned, then define the main variables employed in the analysis, and finally introduce the two-stage econometric model used to estimate TFP and examine the J-curve relationship.

2.1 Sample Construction

The empirical analysis draws on the AIDA database (Bureau van Dijk), which provides comprehensive financial statements for Italian joint-stock companies over the decade 2015–2024. To construct a representative panel of manufacturing SMEs, we applied the following inclusion criteria:

- Annual revenue strictly below €50 million and between 10 and 250 employees (European Commission SME definition);

- Incorporation date prior to 2014, ensuring availability of a complete 10-year panel;

- Restriction to *high-tech* and *medium-high-tech* manufacturing sectors: ATECO codes 26 (Computer & Electronics), 27 (Electrical Equipment), and 28 (Machinery).

Standard data cleaning procedures were applied: observations with negative or null values for Value Added, Personnel Costs, or Tangible Fixed Assets were excluded, and all continuous variables were winsorized at the 1st and 99th percentiles. To ensure a balanced panel, only firms with complete data for all 10 years were retained. The resulting panel comprises **8,683 unique firms** and 86,880 firm-year observations.

2.2 Variable Definitions

The principal variables used in this analysis are defined below:

- **Output (Y)**: Deflated Value Added, expressed in constant prices for inter-temporal comparability.

- **Labor (L)**: Personnel Costs, capturing qualitative workforce differences, including the skill premium.

- **Capital (K)**: Net Tangible Fixed Assets.

- **Technological Intensity ($Tech$)**: Ratio of Intangible Fixed Assets to Total Assets, capturing investments in software, R&D, patents and licenses as a proxy for digital maturity. Note that this measure may also include goodwill and non-AI software; see Section 4 for a discussion of this limitation.

2.3 Model Specification

The analysis is carried out in two steps. In the first step, we compute Total Factor Productivity (TFP) as the residual from a log-linear Cobb–Douglas production function that includes firm fixed effects (μ_i) and year dummy variables (δ_t):

$$\ln(Y_{it}) = \beta_L \ln(L_{it}) + \beta_K \ln(K_{it}) + \mu_i + \delta_t + \epsilon_{it} \tag{1}$$

This equation is estimated using Panel Fixed Effects via `linearmodels.PanelOLS` with clustered standard errors at the firm level. The Hausman test ($H = 49.35$, $p < 0.001$) confirms the appropriateness of Fixed Effects over Random Effects. The estimated TFP ($\hat{\omega}_{it}$) is obtained as $\ln(Y_{it}) - \hat{\beta}_L \ln(L_{it}) - \hat{\beta}_K \ln(K_{it})$, capturing the portion of output not explained by observable factor inputs.

In the second stage, we test the J-Curve hypothesis by regressing TFP on Technological Intensity in both linear and quadratic form:

$$\hat{\omega}_{it} = \alpha + \gamma_1 Tech_{it} + \gamma_2 Tech_{it}^2 + \beta_{Nord} D_i^{Nord} + \beta_{Sud} D_i^{Sud} + \tau_t + \eta_{it} \tag{2}$$

The second-stage equation is estimated using Pooled OLS, with standard errors clustered at the firm level to correct for serial correlation within firms. Regional dummies (D_i^{Nord}, D_i^{Sud}) are included because they capture *between-firm* variation (structural differences across regional groups of firms) rather than time-varying characteristics within firms; the Center region is the omitted reference group. Verification of the J-Curve pattern requires $\gamma_1 < 0$ (an initial drop in productivity during the investment period) and $\gamma_2 > 0$ (a subsequent convex rebound as intangible capital becomes effective). As a robustness test for potential endogeneity, we also re-estimate the model using lagged Technological Intensity ($Tech_{i,t-1}$) to reduce possible simultaneity bias.

3 Results Analysis

This section reports the empirical results in four steps. First, we present descriptive statistics to summarize the sample and underscore the main patterns in the data. Second, we estimate the J-Curve relationship using Fixed Effects regression to test Hypothesis 1. Third, we investigate regional differences to assess Hypothesis 2 concerning the North-South divide. Finally, we break down the results by ATECO sector to reveal industry-specific dynamics.

3.1 Descriptive Statistics

Table 1 presents the descriptive statistics for the main variables employed in the analysis. The average Technological Intensity in the sample is relatively low (3.2%), but the standard deviation is high (0.060), highlighting the pronounced heterogeneity in digital adoption among Italian SMEs. Labor productivity (Y/L) likewise exhibits considerable variability.

Table 1: Descriptive Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Labor Productivity (Y/L)	86,880	1.56	0.63	0.12	4.53
Tech Intensity (K_{int}/K_{tot})	86,880	0.031	0.059	0.00	0.37
ln(Value Added)	86,880	14.21	1.03	9.47	18.24
ln(Labor Cost)	86,880	13.83	0.99	9.21	17.93
ln(Capital)	86,880	12.95	1.76	5.30	19.27

Note: All variables winsorized at 1st and 99th percentiles.

From a geographical perspective, a clear North–South divide emerges in labor productivity (Y/L). As shown in Figure 1, the visual inspection of the boxplots reveals a pronounced dualism. The distributions for the **North** and **Center** are strikingly alike, forming a cohesive "advanced block" characterized by a higher median TFP and a broader interquartile range, indicative of a dynamic context with both solid average performers and leading firms. In stark contrast, the **South** displays a substantially lower median and a more compressed distribution. This overall "downward displacement" of the Southern distribution visually demonstrates that the productivity gap is not the result of a handful of

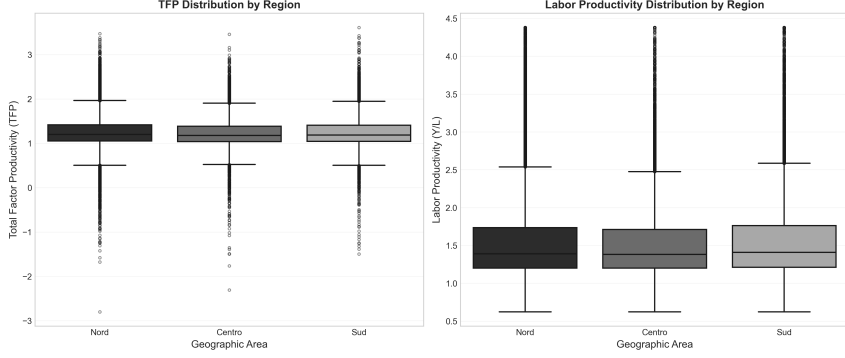extreme values, but rather a structural feature affecting the entire population of Southern SMEs.

Figure 1: Territorial Heterogeneity in Labor Productivity (Y/L)

3.2 Econometric Estimates: The J-Curve

The first-stage Panel Fixed Effects estimation of the Cobb-Douglas production function yields coefficient estimates of $\hat{\beta}_L = 0.905$ (SE: 0.008) for labor and $\hat{\beta}_K = 0.035$ (SE: 0.002) for capital, with an $R^2_{within} = 0.614$. The high labor elasticity and low capital elasticity are consistent with the labor-intensive nature of Italian SMEs in these manufacturing sectors. The Hausman test ($H = 49.35$, $p < 0.001$) confirms the appropriateness of Fixed Effects over Random Effects.

Table 2 reports the estimated coefficients of the second-stage model (Eq. 2). The regional dummies use **Center** as the baseline category. The results provide strong empirical support for Hypothesis 1.

Table 2: J-Curve Estimation (Pooled OLS with Clustered SE)

Dependent Variable: TFP ($\hat{\omega}$)	Coeff.	Std. Err.	P-value
Tech Intensity ($Tech$)	**-1.043**	0.097	< 0.001
Tech Intensity Squared ($Tech^2$)	**+3.582**	0.365	< 0.001
Dummy North	**+0.020**	0.008	0.009
Dummy South	**+0.016**	0.009	0.071
Year Control		Included	

The linear term coefficient is negative and statistically significant at the 1% level ($\gamma_1 = -1.04$), indicating that the initial introduction of digital technologies is associated with a net loss of productivity. The quadratic term is positive and significant ($\gamma_2 = +3.58$), confirming the convexity of the function. Note that standard errors are clustered at the firm level to account for within-firm correlation. The minimum point of the function is found at a technological intensity of **14.6%**. This suggests that for values of $Tech < 0.146$, the firm operates in the descending part of the curve ("Investment Phase"). This non-linear dynamic is visually depicted in Figure 2. The graph clearly shows the initial dip in TFP associated with low levels of intangible intensity, followed by a steep recovery as

intensity increases. The shaded area represents the 95% confidence interval, confirming the statistical significance of the "J" shape throughout the domain.
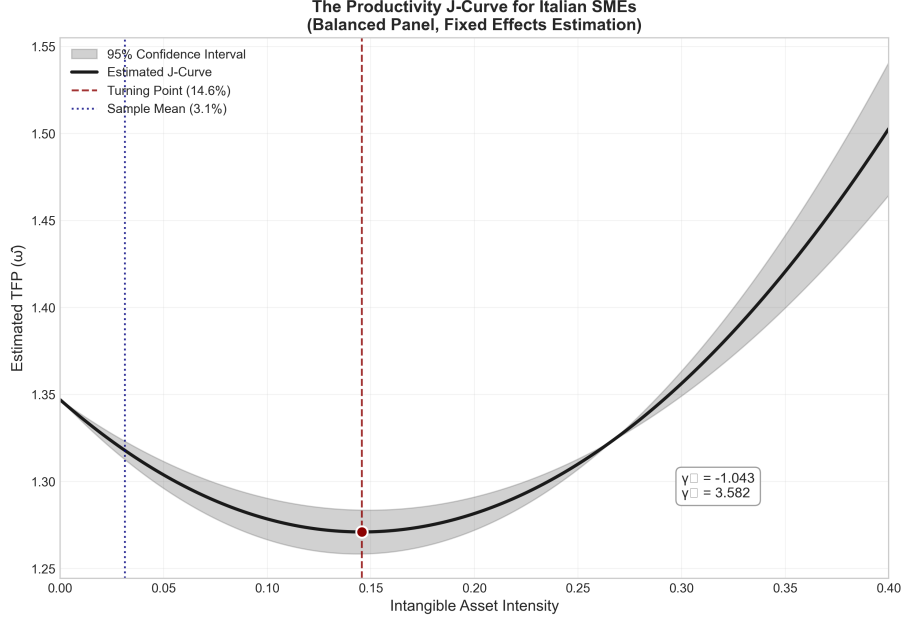

The Productivity J-Curve for Italian SMEs
(Balanced Panel, Fixed Effects Estimation)

Figure 2: The Estimated Productivity J-Curve for Italian SMEs

3.3 Regional Heterogeneity (Test H2)

Hypothesis 2, which postulated a divergence from the standard North-South productivity divide, finds **partial confirmation**. The baseline model shows a positive coefficient for both North ($+0.020, p = 0.009$) and South ($+0.016, p = 0.071$) relative to the Center. While the Northern premium is statistically significant, the Southern one is marginally significant. Crucially, the magnitude of the difference between North and South is minimal (approx. 0.4%), suggesting that conditional on being an established high-tech SME, the geographical penalty practically vanishes.

As an additional robustness check for potential endogeneity, we estimated the model using lagged Technological Intensity ($Tech_{t-1}$): the J-Curve is confirmed ($\gamma_1 = -0.98$, $\gamma_2 = +3.51$), with the turning point shifting slightly to 14.0% (compared to 14.6% in the baseline model), supporting a causal interpretation.

To address the "generated regressor problem" inherent in our two-stage procedure, we implemented a cluster bootstrap with 500 replications. The bootstrap standard errors for γ_1 (0.094) and γ_2 (0.361) are virtually identical to the clustered standard errors (ratios of 0.97 and 0.99, respectively), indicating that inference is not materially affected by uncertainty propagated from the first stage. The 95% bootstrap confidence intervals for both coefficients exclude zero ($[-1.18, -0.85]$ for γ_1 and $[2.84, 4.21]$ for γ_2), confirming the robustness of the J-Curve finding. The turning point confidence interval is [13.3%, 15.7%].

The reduced magnitude of the geographical coefficient suggests that the widely cited productivity gap may be driven more by compositional effects (sectoral mix and firm size distribution) than by intrinsic regional inefficiencies. In this specific high-tech sub-sample,

the digital divide acts more as a selection barrier than as a drag on the efficiency of incumbent firms.

3.4   Sectoral Analysis (ATECO)

The analysis by ATECO sector (Figure 3) reveals significant heterogeneity consistent with structural expectations. The **Computer & Electronics** sector (Code 26) exhibits the highest technological intensity ($\approx 4.8\%$) and also the highest average TFP (1.263). This confirms that industries natively closer to the digital frontier have accumulated more intangible capital and achieved higher efficiency. In contrast, the **Machinery** sector (Code 28), which constitutes the largest share of our sample ($N = 5,800$), shows a lower digital maturity (2.8%) and intermediate productivity levels (1.251), suggesting that the "servitization" process in mechanical engineering is still in an early phase. Finally, the **Electrical Equipment** sector (Code 27) displays intermediate technological intensity (3.1%) but the lowest TFP (1.239), possibly reflecting lower value-added margins compared to advanced electronics.
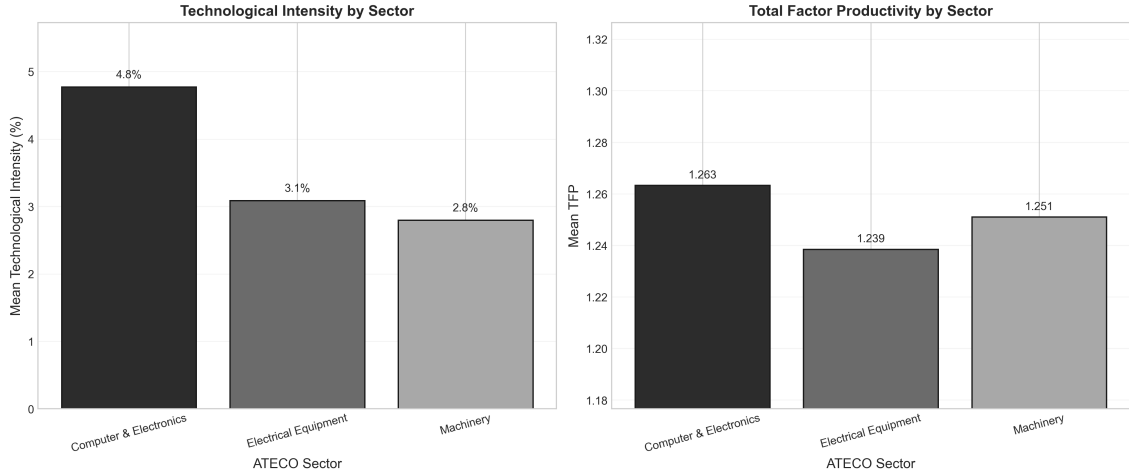

Figure 3: Technological Intensity and TFP by ATECO Sector

4   Discussion and Policy Implications

Our findings extend the theoretical framework of Brynjolfsson et al. [2] to Italian SMEs, consistent with the broader literature on AI economics [8] and recent empirical findings on Italian Industry 4.0 [12, 13]. The estimated turning point at 14.6% of intangible asset intensity—compared to a sample average of just 3.1%—suggests that the vast majority of Italian SMEs remain "trapped" in the descending phase of the J-Curve, bearing substantial organizational costs without yet harvesting productivity gains. This is consistent with the "mis-measurement" hypothesis: investments in workflow redesign, employee retraining, and data governance are expensed rather than capitalized, causing measured TFP to decline even as intangible capital accumulates.

Policy Implications: Beyond Hardware Incentives. Our findings call into question the dominant "sprinkler" model of digitalization. Initiatives like Transizione 4.0 have boosted hardware investment through tax incentives, yet this alone does not translate into immediate productivity gains—the advantages are largely locked into intangible assets that are still being developed and thus not fully captured. A more effective policy strategy must operate on two complementary fronts: (i) cash-flow support during the temporary productivity downturn (e.g., conditional grants, long-term patient loans) to keep SMEs from abandoning projects too early; and (ii) incentives that explicitly favor intangible investments—training, consulting for process reengineering, and organizational experimentation. Given the pronounced differences across sectors, targeted, sector-specific measures are likely to outperform broad, horizontal policies.

The Southern Paradox: Selection Effects and Regional Policy. In contrast to the narrative of a "Southern penalty," we find no robust TFP deficit for Southern high-tech SMEs. We interpret this as evidence of a competitive selection mechanism: in the South, where transaction costs and entry barriers are higher, only the most efficient firms manage to survive and reach the 10–250 employee range. In this context, the digital divide functions more as a barrier to entry than as a drag on efficiency. From a regional policy perspective, this implies prioritizing measures that support the creation and scaling of new firms rather than solely targeting digital gaps within incumbent firms.

Limitations and Future Research. Our technological proxy (intangible assets) does not separate AI-related spending from traditional software or goodwill, which may lead us to overstate the extent of AI adoption. Future work could combine balance sheet information with ICT usage surveys or textual analysis of annual reports to refine this measure. Moreover, our econometric design does not allow for definitive causal claims; instrumental variable methods would reinforce the causal interpretation. A further methodological consideration concerns the two-stage procedure: using first-stage TFP estimates as the dependent variable in the second stage creates a "generated regressor problem," which may result in understated standard errors. As documented in Section 3, we addressed this concern through cluster bootstrap inference, which confirmed that our standard errors are not materially affected. Finally, extending the analysis to additional European countries and to service industries would help establish external validity.

5 Conclusions

This study offers empirical evidence for the presence of the "Productivity J-Curve" in Italian manufacturing SMEs. Drawing on a balanced panel dataset of more than 86,000 observations from 8,683 firms, our analysis yields three principal findings.

First, the spread of Artificial Intelligence and digital technologies does not follow a linear trajectory. The Panel Fixed Effects estimates show a clear "J-shaped" relationship, with a turning point occurring at roughly **14.6% intangible asset intensity**. This finding suggests that, for the vast majority of Italian SMEs, the digital transition is still in

the "investment stage": firms are incurring organizational adjustment costs (lower TFP) but have not yet started to fully reap the benefits of the new technologies. The "Solow Paradox" in Italy is therefore, at least partially, the outcome of a delayed accumulation process.

Second, our results cast doubt on the traditional notion of a North–South divide. When we restrict attention to established high-tech manufacturing SMEs, we find only a negligible productivity difference between Southern and Northern firms. While Northern firms exhibit a statistically significant premium ($+2.0\%$) over the Central Italy baseline, Southern firms show a similar, albeit marginally significant, positive coefficient ($+1.6\%$). This implies that the widely cited productivity gap may be driven by compositional effects (sectoral mix and firm size) rather than intrinsic regional inefficiencies. In this specific high-tech sub-sample, the digital divide acts more as a selection barrier than as a drag on the efficiency of incumbent firms. Finally, from a managerial and policy standpoint, our findings caution against short-term "techno-optimism." AI is not a simple plug-and-play tool that delivers instant efficiency improvements. Rather, it calls for a "patient capital" mindset: managers must be willing to navigate a phase of weaker performance while processes are being redesigned, and policymakers need to craft incentive structures that do more than subsidize hardware purchases, ensuring the financial viability of the transition over the medium term.

Future research should aim to unpack the "black box" of intangible assets by differentiating among expenditures on software, R&D, and organizational training, thereby offering more detailed guidance to the industrial landscape.

Data Availability Statement

The replication code and analytical scripts used in this study are publicly available at: https://github.com/mariotrerotola/productivity-jcurve-italian-sme. The underlying firm-level data were extracted from the AIDA database (Bureau van Dijk) under a licensed subscription and cannot be redistributed due to licensing restrictions. Researchers wishing to replicate the analysis may obtain access through their institution or by contacting Bureau van Dijk directly.

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A Variable Definitions

Table 3 presents how the main variables employed in the empirical analysis are constructed, based on items from the AIDA database.

Table 3: Variable Definitions and Construction

Variable	Description	Construction (AIDA)
Y_{it}	Output	Value Added (Deflated)
L_{it}	Labor Input	Total Personnel Costs
K_{it}	Physical Capital	Net Tangible Fixed Assets
$Tech_{it}$	Technological Intensity (Proxy for AI/Digital)	Intangible Fixed Assets / Total Assets
TFP_{it}	Total Factor Productivity	$\ln(Y_{it}) - \hat{\beta}_L \ln(L_{it}) - \hat{\beta}_K \ln(K_{it})$. This measure includes firm-specific time-invariant efficiency (μ_i), allowing cross-sectional comparisons.

B Econometric Algorithm

The following pseudocode outlines the step-by-step procedure used to estimate the Productivity J-Curve, ensuring reproducibility independently of the specific software environment. The process involves data filtering, TFP estimation through Panel Fixed Effects, and the final non-linear regression.

Algorithm 1 Pseudocode of the Two-Stage Estimation Procedure for the Productivity J-Curve

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1: **Input:** Panel Data Dataset D with N firms and T years
2: **Output:** J-Curve Coefficients γ_1, γ_2, Turning Point TP

3: **procedure** DATAPREPROCESSING(D)
4:     **Filter** firms where $Revenue < 50M$ **and** $10 \leq Employees \leq 250$ ▷ SME Definition
5:     **Compute** Logarithms:
6:         $y_{it} \leftarrow \ln(VA_{it})$, $l_{it} \leftarrow \ln(LaborCost_{it})$, $k_{it} \leftarrow \ln(Capital_{it})$
7:     **Compute** Tech Intensity: $Tech_{it} \leftarrow \frac{Intangibles_{it}}{TotalAssets_{it}}$
8:     **Winsorize** extreme values at 1st and 99th percentiles
9: **end procedure**

10: **procedure** STAGE1_TFPESTIMATION(D)   ▷ Production Function
11:     **Model:** $y_{it} = \beta_L l_{it} + \beta_K k_{it} + \mu_i + \delta_t + \epsilon_{it}$
12:     **Estimate** parameters $\hat{\beta}_L, \hat{\beta}_K$ using Panel Fixed Effects (Entity and Time)
13:     **Calculate** TFP Residual:
14:         $\hat{\omega}_{it} \leftarrow y_{it} - \hat{\beta}_L l_{it} - \hat{\beta}_K k_{it}$
15: **end procedure**

16: **procedure** STAGE2_JCURVETEST(D)   ▷ Quadratic Analysis
17:     **Compute** Quadratic Term: $Tech_{it}^2 \leftarrow (Tech_{it})^2$
18:     **Model:** $\hat{\omega}_{it} = \alpha + \gamma_1 Tech_{it} + \gamma_2 Tech_{it}^2 + \mathbf{X}_{controls}\beta + \eta_{it}$
19:     **Estimate** using Pooled OLS with Standard Errors clustered by Firm
20:     **Return** $\hat{\gamma}_1, \hat{\gamma}_2$
21:     **if** $\hat{\gamma}_1 < 0$ **and** $\hat{\gamma}_2 > 0$ **then**
22:         **Compute** Turning Point: $TP \leftarrow \frac{-\hat{\gamma}_1}{2\hat{\gamma}_2}$
23:         **return** "J-Curve Confirmed"
24:     **else**
25:         **return** "No J-Curve Detected"
26:     **end if**
27: **end procedure**

28: **procedure** BOOTSTRAPSE(D, $B = 500$)   ▷ Generated Regressor Correction
29:     $\Gamma \leftarrow \emptyset$ ▷ Storage for bootstrap estimates
30:     **for** $b = 1$ **to** B **do**
31:         **Resample** N firms with replacement (cluster bootstrap)
32:         $D^{(b)} \leftarrow$ all observations for resampled firms
33:         $\hat{\omega}^{(b)} \leftarrow$ STAGE1_TFPESTIMATION($D^{(b)}$)
34:         $(\gamma_1^{(b)}, \gamma_2^{(b)}) \leftarrow$ STAGE2_JCURVETEST($D^{(b)}$)
35:         $\Gamma \leftarrow \Gamma \cup \{(\gamma_1^{(b)}, \gamma_2^{(b)})\}$
36:     **end for**
37:     **Compute** $SE_{boot}(\gamma_1) \leftarrow$ StdDev($\{\gamma_1^{(b)}\}$)
38:     **Compute** $SE_{boot}(\gamma_2) \leftarrow$ StdDev($\{\gamma_2^{(b)}\}$)
39:     **Compute** 95% CI using percentile method: $[\gamma_{2.5\%}, \gamma_{97.5\%}]$
40:     **return** Bootstrap SE, Confidence Intervals
41: **end procedure**