Solució al problema 44

1. Fem desenvolupaments de Taylor:

$$f(a+h) = f(a) + f'(a)h + \frac{1}{2}f''(a)h^2 + \frac{1}{6}f'''(a)h^3 + \frac{1}{24}f^{(4)}(a)h^4 + \dots$$

$$f(a+3h) = f(a) + 3f'(a)h + \frac{9}{2}f''(a)h^2 + \frac{27}{6}f'''(a)h^3 + \frac{81}{24}f^{(4)}(a)h^4 + \dots$$

Llavors

$$\frac{Af(a) + Bf(a+h) + Cf(a+3h)}{h}$$

$$= \frac{1}{h}[A+B+C]f(a) + [B+3C]f'(a) + [B+9C]\frac{1}{2}f''(a)h +$$

$$+[B+27C]\frac{1}{6}f'''(a)h^2 + [B+81C]\frac{1}{24}f^{(4)}(a)h^3 + \cdots$$

Imposem

$$\begin{aligned} A + B + C &= 0 \\ B + 3C &= 1 \\ B + 9C &= 0 \end{aligned}$$

Resolent el sistema resulta

$$C = -\frac{1}{6}, \quad B = \frac{3}{2}, \quad A = -\frac{4}{3}$$

i

$$B + 27C \neq 0$$

Per tant,

$$F(h) = \frac{-8f(a) + 9f(a+h) - f(a+3h)}{6h} = f'(a) + Kh^2 + O(h^3),$$

on $K \neq 0$ si $f'''(a) \neq 0$.

2. Podem agafar els passos $h_1 = 0.3$ i $h_2 = 0.1$, i tenim com a aproximació de $f'(0)$ els valors

$$F(0.3) = 0.4888888\ldots$$

i

$$F(0.1) = 0.5166666\ldots$$

Usant extrapolació de Richardson, amb $q = 3$ i $p_1 = 2$, tenim

$$F_2(h) = \frac{9F(h) - F(3h)}{8} = f'(a) + O(h^3).$$

Per tant,

$$F_2(0.1) = 0.52013888\ldots.$$