Solució al problema 28 b) I

$$U^T U = \begin{pmatrix} u_{11} & & & \\ u_{12} & u_{22} & & \\ \vdots & \vdots & \ddots & \\ u_{1n} & u_{2n} & \dots & u_{nn} \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ & u_{22} & \dots & u_{2n} \\ & & \ddots & \vdots \\ & & & u_{nn} \end{pmatrix}$$

$$= \begin{pmatrix} u_{11}^2 & u_{11}u_{12} & \dots & \dots & \dots & u_{11}u_{1n} \\ u_{11}u_{12} & u_{12}^2+u_{22}^2 & u_{12}u_{13}+u_{22}u_{23} & & \dots & u_{12}u_{1n}+u_{22}u_{2n} \\ u_{11}u_{13} & u_{12}u_{13}+u_{22}u_{23} & \sum_{k=1}^3 u_{k3}^2 & \sum_{k=1}^3 u_{k3}u_{k4} & \dots & \sum_{k=1}^3 u_{k3}u_{kn} \\ \vdots & & & & & \\ u_{11}u_{1n} & u_{12}u_{1n}+u_{22}u_{2n} & \dots & & & \sum_{k=1}^n u_{kn}^2 \end{pmatrix}$$

Com que U és triangular superior, tenim

$$a_{ij} = \sum_{k=1}^{\min\{i,j\}} u_{ki}u_{kj}\ , \ 1 \le i,j \le n.$$

Igualem els elements de A i de $U^T U$, fila a fila i calculem primer l'element de la diagonal i després els altres $(u_{11}, u_{12}, \ldots, u_{1n}, u_{22}, u_{23}, \ldots, u_{2n}, u_{33}, \ldots, u_{nn})$. Tenim:

fila i ($i = 1, \ldots, n$)

$$a_{ii} = \sum_{k=1}^{i} u_{ki}^2 \quad \text{i} \quad a_{ij} = \sum_{k=1}^{i} u_{ki} u_{kj}\,,\ j > i$$

$$u_{ii} = \left(a_{ii} - \sum_{k=1}^{i-1} u_{ki}^2 \right)^{1/2} \quad \text{i} \quad u_{ij} = \left(a_{ij} - \sum_{k=1}^{i-1} u_{ki} u_{kj} \right) / u_{ii}, j = i+1, \ldots, n$$

Solució al problema 28 b) III

Oper.	Pas $i-$è	Total d'Oper.
$\sqrt{\ }$	1	$\sum_{i=1}^{n} 1 = n$
$*$	$\overbrace{i-1}^{u_{ii}} + \overbrace{(i-1)(n-i)}^{u_{ij}, j>i}$	$\displaystyle\sum_{i=1}^{n}(i-1) + \sum_{i=1}^{n}(i-1)(n-i) = \frac{n(n^2-1)}{6}$
$+,-$	$\overbrace{i-1}^{u_{ii}} + \overbrace{(i-1)(n-i)}^{u_{ij}, j>i}$	$\displaystyle\frac{n(n^2-1)}{6}$
$/$	$n-i$	$\sum_{i=1}^{n}(n-i) = (n-1+0)n/2 = \frac{n(n-1)}{2}$

Nombre de productes:

$$\sum_{i=1}^{n}(i-1)+\sum_{i=1}^{n}(i-1)(n-i)=\frac{n(n-1)}{2}+n\sum_{i=1}^{n}(i-1)-\sum_{i=1}^{n}(i-1)i$$

$$=\quad\frac{n(n-1)}{2}+\frac{n^2(n-1)}{2}-\sum_{i=1}^{n}i^2+\sum_{i=1}^{n}i$$

$$=\quad\frac{n(n-1)}{2}+\frac{n^2(n-1)}{2}-\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}=\frac{n(n^2-1)}{6}$$