1. $z = \left[3\left(\frac{4}{3}-1\right)-1\right] = 0$ $\qquad fl(z) = ?$ $\left\{\begin{array}{l}\text{En base 10} \\ \text{En base 2}\end{array}\right\}$ usant t dígits significatius

(a) $\boxed{\text{Base 10}}$

$fl\left(\frac{4}{3}\right) = \underbrace{1.3\ldots3}_{t}$ → ($t-1$ decimals) (arrodoniment "per sota")

$fl\left(\frac{4}{3}-1\right) = 0.\underbrace{3\ldots3}_{t-1}$ (en aquesta resta es perd un dígit significatiu)

$fl\left(3\left(\frac{4}{3}-1\right)\right) = 0.\underbrace{9\ldots9}_{t-1}$

$fl\left[3\left(\frac{4}{3}-1\right)-1\right] = -0.0\ldots0\underset{\substack{\uparrow \\ \text{posició } t-1}}{1} = \boxed{(-1)\cdot 10^{-t+1}}$

(b) $\boxed{\text{Base 2}}$

Passem $\frac{4}{3} = 1.\hat{3}$ a base 2 : $\frac{4}{3} = 1.\widehat{01}_{(2}$

Cal distingir entre t parell i t senar, quan s'arrodoneix a t dígits binaris.

$\boxed{t \text{ parell}}$

$fl\left(\frac{4}{3}\right) = \underbrace{1.\underline{01}\,\underline{01}\ldots\underline{01}\,1}_{t}{}_{(2}$ (arrodoniment "per sobre")

$fl\left(\frac{4}{3}-1\right) = 0.\underbrace{\underline{01}\,\underline{01}\ldots\underline{01}\,1}_{t-1}{}_{(2}$ (restant, es perden 2 dígits binaris)

$fl\left[3\left(\frac{4}{3}-1\right)\right] = 1.\underbrace{00\ldots001}_{t-1}{}_{(2}$

$3_{(10} = 11_{(2} \Rightarrow$

$$\begin{array}{r} 0.0101\ldots011 \\ \times \quad 11 \\ \hline 0.0101\ldots011 \\ +\ 0.101\ldots011 \\ \hline 1.000\ldots0001_{(2} \end{array}$$

$fl\left[3\left(\frac{4}{3}-1\right)-1\right] = 0.\underbrace{00\ldots001}_{t-1}{}_{(2} = \boxed{(+1)\cdot 2^{-t+1}}$

$\boxed{t \text{ senar}}$

$fl\left(\frac{4}{3}\right) = \underbrace{1.\underline{01}\,\underline{01}\ldots\underline{01}\,\underline{01}}_{t}{}_{(2}$ (arrodoniment "per sota")

$fl\left(\frac{4}{3}-1\right) = 0.\underbrace{\underline{01}\,\underline{01}\ldots\underline{01}\,\underline{01}}_{t-1}{}_{(2)}$ (en la resta es perden 2 dígits binaris)

$$\begin{array}{r} 0.0101\ldots0101 \\ \times \quad 11 \\ \hline 0.0101\ldots0101 \\ +\ 0.1010\ldots101 \\ \hline 0.1111\ldots1111 \end{array}$$

$fl\left[3\left(\frac{4}{3}-1\right)\right] = 0.\underbrace{11\ldots11}_{t-1}{}_{(2}$

$fl\left[3\left(\frac{4}{3}-1\right)-1\right] = -0.\underbrace{00\ldots01}_{t-1}{}_{(2} = \boxed{(-1)\cdot 2^{-t+1}}$

(1) continuació

$\boxed{\text{Comentari}}$. Què dóna el càlcul $3\left(\frac{4}{3}-1\right)-1$ en el sistema IEEE?

- S'usa base 2
- En notació normalitzada, sempre hi ha un 1 davant el punt fraccionari, i no es guarda.
- En float, la mantissa té 23 dígits

  En double, la mantissa té 52 dígits

$\boxed{\text{float}}$ (correspon a base 2 : $t = 23+1 = 24$ parell)

$$\frac{4}{3} \sim 1.\underbrace{01}\underbrace{01}\cdots\underbrace{01}\,\underset{\substack{\uparrow\\(\text{posició } 23)}}{1}$$

$$\frac{4}{3}-1 \sim 0.\underbrace{01}\underbrace{01}\cdots\underbrace{01}\,\underset{\substack{\uparrow\\(23)}}{1}$$

$$3\left(\frac{4}{3}-1\right) \sim \begin{array}{r} 0.\underbrace{01}\underbrace{01}\cdots\underbrace{01}1 + \\ +0.1010\cdots 11 \\ \hline \end{array} = 1.0000\cdots 00\underset{\substack{\uparrow\\(23)}}{1}$$

$$3\left(\frac{4}{3}-1\right)-1 \sim 0.0\cdots 0\underset{(23}{1} = \boxed{(+1)\cdot 2^{-23}} = 1.1920928955078125\times 10^{-7}$$

$\boxed{\text{double}}$ (correspon a base 2 : $t = 52+1 = 53$ senar)

$$\frac{4}{3} \sim 1.\underbrace{01}\underbrace{01}\cdots\underbrace{01}\,\underset{\substack{\uparrow\\(52)}}{\underbrace{01}}$$

$$\frac{4}{3}-1 \sim 0.\underbrace{01}\underbrace{01}\cdots\underbrace{01}\,\underset{(52)}{\underbrace{01}}$$

$$3\left(\frac{4}{3}-1\right) \sim \begin{array}{r} 0.\underbrace{01}\underbrace{01}\cdots\underbrace{01}\underbrace{01} \\ +\ 0.1010\cdots 101 \\ \hline \end{array} = 0.1111\cdots 111\underset{(52)}{1}$$

$$3\left(\frac{4}{3}-1\right)-1 \sim -0.00\cdots 0\underset{\substack{\uparrow\\(52)}}{1} = \boxed{(-1)\cdot 2^{-52}} = 2.22044604925\ldots\times 10^{-16}$$

② $f(x,y) = x^y \quad (= \exp(y \cdot \ln x))$ domini: $0 < x, y$

(a) Propagació de l'error <u>relatiu</u> a 1er. ordre

$$\frac{\partial f}{\partial x} = y \cdot x^{y-1}$$

$$\frac{\partial f}{\partial y} = x^y \cdot \ln x$$

Fórmula general de propag. de l'error a 1er. ordre: (relaciona errors <u>absoluts</u>)

$$\Delta f \approx \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y \quad \Rightarrow \quad \Delta f \approx (y \cdot x^{y-1}) \Delta x + (x^y \cdot \ln x) \Delta y$$

Per a obtenir una relació entre errors relatius, cal introduir valors f, x, y:

$$\frac{\Delta f}{f} \approx \frac{y \cdot x^{y-1}}{f} \frac{\Delta x}{x} x + \frac{x^y \ln x}{f} \frac{\Delta y}{y} y \underset{f = x^y}{\Rightarrow} \boxed{\frac{\Delta f}{f} \approx (y) \frac{\Delta x}{x} + (y \cdot \ln x) \frac{\Delta y}{y}}$$

Els factors de propagació de l'error relatiu són: y i $(y \ln x)$

En valor absolut, aquests valors són <u>molt grans</u> quan $\begin{cases} \text{o bé } y \gg 0 \\ \text{o bé } x \approx 0 \end{cases}$

(b) $\left.\begin{array}{l} x = 0.11\,(1 \pm \varepsilon) \\ y = 10\,(1 \pm \varepsilon) \\ \varepsilon = 10^{-2} \end{array}\right\}$ ¿ Fita (a 1r ordre en ε) de l'error relatiu en $f = x^y$?

Aplicant (a): $$\left|\frac{\Delta f}{f}\right| \lesssim |y| \underbrace{\left|\frac{\Delta x}{x}\right|}_{\leq \varepsilon} + |y \cdot \ln x| \underbrace{\left|\frac{\Delta y}{y}\right|}_{\leq \varepsilon} \leq (|y| + |y \cdot \ln x|)\, \varepsilon$$

$$= (10 + |10 \cdot \underbrace{\ln 0.11}_{<0}|) \cdot 10^{-2} \approx \boxed{32.07 \times 10^{-2}}$$

Resol. sist. heptadiagonal mitjançant $A=LU$, $Ly=h$, $Ux=y$

3)

$$\begin{pmatrix} a_1 & b_1 & d_1 & f_1 & & & & & \\ c_2 & a_2 & b_2 & d_2 & f_2 & & & & \\ e_3 & c_3 & a_3 & b_3 & d_3 & f_3 & & & \\ g_4 & e_4 & c_4 & a_4 & b_4 & d_4 & f_4 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & g_{n-3} & e_{n-3} & c_{n-3} & a_{n-3} & b_{n-3} & d_{n-3} & f_{n-3} \\ & & g_{n-2} & e_{n-2} & c_{n-2} & a_{n-2} & b_{n-2} & d_{n-2} \\ & & & g_{n-1} & e_{n-1} & c_{n-1} & a_{n-1} & b_{n-1} \\ & & & & g_n & e_n & c_n & a_n \end{pmatrix} \begin{pmatrix} \\ \\ x \\ \\ \\ \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ \vdots \\ h_{n-3} \\ h_{n-2} \\ h_{n-1} \\ h_n \end{pmatrix}$$

El "caràcter banda" es conserva quan es fa eliminació gaussiana $\Rightarrow$

$\Rightarrow$ L i U tindran 4 diagonals essencials (la resta d'elements són 0)

Factorització mitjançant eliminació gaussiana

$\forall i = 1, 2, \ldots, n-3$

$$\left.\begin{array}{l} c_{i+1} \leftarrow c_{i+1}/a_i \\ a_{i+1} \leftarrow a_{i+1} - c_{i+1} b_i \\ b_{i+1} \leftarrow b_{i+1} - c_{i+1} d_i \\ d_{i+1} \leftarrow d_{i+1} - c_{i+1} f_i \end{array}\right\} 7$$

$$\left.\begin{array}{l} e_{i+2} \leftarrow e_{i+2}/a_i \\ c_{i+2} \leftarrow c_{i+2} - e_{i+2} \cdot b_i \\ a_{i+2} \leftarrow a_{i+2} - e_{i+2} d_i \\ b_{i+2} \leftarrow b_{i+2} - e_{i+2} f_i \end{array}\right\} 7$$

$$\left.\begin{array}{l} g_{i+3} \leftarrow g_{i+3}/a_i \\ e_{i+3} \leftarrow e_{i+3} - g_{i+3} \cdot b_i \\ c_{i+3} \leftarrow c_{i+3} - g_{i+3} \cdot d_i \\ a_{i+3} \leftarrow a_{i+3} - g_{i+3} f_i \end{array}\right\} 7$$

$21(n-3)$

$i = n-2$

$$\left.\begin{array}{l} c_{n-1} \leftarrow c_{n-1}/a_{n-2} \\ a_{n-1} \leftarrow a_{n-1} - c_{n-1} b_{n-2} \\ b_{n-1} \leftarrow b_{n-1} - c_{n-1} d_{n-2} \end{array}\right\} 5$$

$$\left.\begin{array}{l} e_n \leftarrow e_n/a_{n-2} \\ c_n \leftarrow c_n - e_n \cdot b_{n-2} \\ a_n \leftarrow a_n - e_n \cdot d_{n-2} \end{array}\right\} 5$$

10

$i = n-1$

$$\left.\begin{array}{l} c_n \leftarrow c_n/a_{n-1} \\ a_n \leftarrow a_n - c_n b_{n-1} \end{array}\right\} 3$$

$\boxed{21n - 50}$

Resolució $Ly=h$

$$\begin{pmatrix} 1 & & & & & & \\ c_2 & 1 & & & & & \\ e_3 & c_3 & 1 & & & & \\ g_4 & e_3 & c_4 & 1 & & & \\ & \ddots & \ddots & \ddots & \ddots & & \\ & & & g_n & e_n & c_n & 1 \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ \vdots \\ h_n \end{pmatrix}$$

h_1

$$\left.\begin{array}{l} h_2 \leftarrow h_2 - c_2 h_1 \\ h_3 \leftarrow h_3 - e_3 h_1 - c_3 h_2 \end{array}\right\} 6$$

$\forall i = 4, \ldots, n$

$$h_i \leftarrow h_i - g_i h_{i-3} - e_i h_{i-2} - c_i h_{i-1} \Big\} \; 6(n-3)$$

$\boxed{6n-12}$

Resolució $Ux=h$

$$\begin{pmatrix} a_1 & b_1 & d_1 & f_1 & & \\ & a_2 & b_2 & d_2 & f_2 & \\ & & \ddots & \ddots & \ddots & f_{n-3} \\ & & & \ddots & \ddots & d_{n-2} \\ & & & & \ddots & b_{n-1} \\ & & & & & a_n \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_{n-3} \\ h_{n-2} \\ h_{n-1} \\ h_n \end{pmatrix}$$

$$\left.\begin{array}{l} h_n \leftarrow h_n/a_n \\ h_{n-1} \leftarrow (h_{n-1} - b_{n-1} h_n)/a_{n-1} \\ h_{n-2} \leftarrow (h_{n-2} - b_{n-2} h_{n-1} - d_{n-2} h_n)/a_{n-2} \end{array}\right\} 9$$

$\forall i = n-3, n-2, \ldots, 1$

$$h_i \leftarrow (h_i - b_i h_{i+1} - d_i h_{i+2} - f_i h_{i+3})/a_i \Big\} \; 7(n-3)$$

$\boxed{7n-12}$

$\boxed{\text{Total d'operacions}}$ $\boxed{34n - 74}$

Temps (gener 2017)

$n = 10^7 \rightarrow 0{,}44$ s.

$n = 10^8 \rightarrow 4{,}45$ s

El nombre d'operacions és essencialment lineal respecte n $\Rightarrow$

$\Rightarrow$ si ($n = 10^7 \Rightarrow$ 2 segons)

(4)

x	0	1	2
$f(x)$	1	3	2
$f'(x)$	3	(?)	-2

$\max\{|f^{(k)}(x)| : 0 \le x \le 2\} \equiv M_k \quad \forall k \geq 0 \qquad f'(1) = ?$

(a) $p \in \mathcal{P}_4$ pol. interp. per les 5 cond. conegudes

$$\begin{array}{cccccc}
0 & 1 & & & & \\
 & & 3 & & & \\
0 & 1 & & -1 & & \\
 & & 2 & & -1/4 & \\
1 & 3 & & -3/2 & & 1/4 \\
 & & -1 & & 1/4 & \\
2 & 2 & & -1 & & \\
 & & -2 & & & \\
2 & 2 & & & &
\end{array}$$

$\Rightarrow p(x) = 1+3x-x^2-\frac{1}{4}\overbrace{x^2(x-1)}^{x^3-x^2}+\frac{1}{4}\overbrace{x^2(x-1)(x-2)}^{x^4-3x^3+2x^2} = 1+3x-\frac{1}{4}x^2-x^3+\frac{1}{4}x^4$

$p'(x) = 3-2x-\frac{1}{4}(3x^2-2x)+\frac{1}{4}(4x^3-9x^2+4x)$

$p'(1) = 3-2-\frac{1}{4}(1)+\frac{1}{4}(4-9+4) = 1-\frac{1}{2} = \boxed{\frac{1}{2}}$

$f(x)-p(x) = \dfrac{f^{(5)}(\xi(x))}{5!}\,\omega(x) \qquad \omega(x) = x^2(x-1)(x-2)^2$

$f'(x)-p'(x) = \left[\dfrac{f^{(5)}(\xi(x))}{5!}\right]'\omega(x) + \dfrac{f^{(5)}(\xi(x))}{5!}\,\omega'(x)$

En $x=+1 \Rightarrow \omega(+1)=0,\ \omega'(+1) = 1^2(1-2)^2 = 1 \quad \Rightarrow \boxed{|f'(1)-p'(1)| \le \frac{M_5}{5!}}$

(b) $S_0(x)$ interpola

x	0	1
f	1	3
f'	3	m

$$\begin{array}{ccccc}
0 & 1 & & & \\
 & & 3 & & \\
0 & 1 & & -1 & \\
 & & 2 & & m-1 \\
1 & 3 & & m-2 & \\
 & & m & & \\
1 & 3 & & &
\end{array}$$

$\Rightarrow S_0(x) = 1+3x-x^2+(m-1)\overbrace{x^2(x-1)}^{x^3-x^2} = 1+3x-mx^2+(m-1)x^3$

$S_0'(x) = 3-2x+(m-1)(3x^2-2x) \qquad S_0'(1) = 3-2+(m-1)(1) =$

$S_0''(x) = -2+(m-1)(6x-2) \qquad S_0''(x) = -2+(m-1)\cdot 4 = \underline{4m-6}$

$S_1(x)$ interpola

x	1	2
f	3	2
f'	m	-2

$$\begin{array}{ccccc}
1 & 3 & & & \\
 & & m & & \\
1 & 3 & & -1-m & \\
 & & -1 & & m \\
2 & 2 & & -1 & \\
 & & -2 & & \\
2 & 2 & & &
\end{array}$$

$\Rightarrow S_1(x) = 3+m(x-1)-(1+m)\overbrace{(x-1)^2}^{x^2-2x+1}+m\overbrace{(x-1)^2(x-2)}^{x^3-4x^2+5x-2} = \begin{cases}(2-4m)+(2+8m)x+\\ -(1+5m)x^2+mx^3\end{cases}$

$S_1'(x) = m-(1+m)(2x-2)+m(3x^2-8x+5) \qquad S_1'(1) = m$

$S_1''(x) = -(1+m)(2)+m(6x-8) \qquad S_1''(1) = -2-2m+m(-2) = \underline{-4m-2}$

$S_0''(1) = S_1''(1) \Leftrightarrow 4m-6 = -4m-2 \Leftrightarrow 8m = 4 \Leftrightarrow m = \frac{1}{2}$

$\Rightarrow S_0''(1) = S_1''(1) = m = \boxed{\frac{1}{2}}$

<u>Nota</u>. Que les dues aproximacions de $f'(1)$ (la de (a) i la de (b)) donin el mateix, és una <u>casualitat</u>: no hi ha cap motiu teòric que ho justifiqui.

<u>Exercici</u>. Canvieu (-2) per (-4) a la taula i vegeu que (a) i (b) donen diferent

$\left[p'(1) = \frac{1}{4}\ ;\ m = 1\right]$

(5) Recordem el mètodo d'extrapolació per a aquesta fórmula aproximada.

Se sap

$$\underbrace{\frac{f(a+h)-f(a)}{h}}_{\substack{||| \\ D_0(h) \\ \text{valors que} \\ \text{es calculen} \\ \text{per a diverses } h\neq 0}} = \underbrace{f'(a)}_{\substack{\uparrow \\ \text{Valor} \\ \text{buscat}}} + \underbrace{\frac{1}{2}f''(a)h^1 + \frac{1}{3!}f'''(a)h^2 + \frac{1}{4!}f^{iv}(a)h^3 + \dots}_{\substack{\text{error absolut.} \\ \boxed{\text{potències de } h: 1,2,3,4,\dots}}}$$

Aquesta és la informació necessària essencial!

S'usen passos $h = 0.08, 0.04, 0.02, 0.01$. Relacionats pel $\boxed{\text{factor } q = 2}$

Primera etapa d'extrapolació

$$\left.\begin{array}{l} D_0(h) = f'(a) + a_1 h^1 + a_2 h^2 + \dots \\ D_0(h/2) = f'(a) + a_1(h/2)^1 + a_2(h/2)^2 + \dots \end{array}\right\} \Rightarrow \underbrace{\frac{2^1 \cdot D_0(h/2) - D_0(h)}{2^1 - 1}}_{\substack{||| \\ D_1(h)}} = f'(a) + b_2 h^2 + b_3 h^3 + \dots$$

eliminem aquest terme $O(h^1)$

Segona etapa: s'elimina el terme $O(h^2)$:

$$\underbrace{\frac{2^2 \cdot D_1(h/2) - D_1(h)}{2^2 - 1}}_{\substack{||| \\ D_2(h)}} = f'(a) + c_3 h^3 + c_4 h^4 + \dots$$

Tercera etapa: s'elimina $O(h^3)$:

$$\underbrace{\frac{2^3 \cdot D_2(h/2) - D_2(h)}{2^3 - 1}}_{D_3(h)} = f'(a) + d_4 h^4 + O(h^5)$$

Fem els números: $a = 0$

$f(0) = 1.105897$

h	$f(a+h)$	$D_0(h)$	$D_1(h)$	$D_2(h)$	$D_3(h)$
0.08	1.138693	0.40995	0.29725	0.31905	$\boxed{0.3183071429}$
0.04	1.120041	0.3536	0.3136	0.3184	
0.02	1.112569	0.3336	0.3172		
0.01	1.109151	0.3254			

Nota. Observeu:

1) El mètodo numèric $D_0(h)$ té un error $O(h)$
2) L'extrapolació (3 etapes) permet obtenir un mètode $D_3(h)$ amb error $O(h^4)$
3) També hi ha l'error d'arrodoniment en els càlculs. El tret més característic aquí és: encara que es coneix $f(x)$ amb 7 dígits significatius, la fórmula de $D_0(h)$ fa que només en quedin 4 o 5.
4) Les dades donades corresponen a la funció $f(x) = \frac{1}{7}e^{6x} + \cos\left(2x + \frac{3}{11}\right)$

$f'(0) \approx 0.318425$

(6) $f(x) = x - e\cdot\sin(x) - M = 0 \qquad e\in(0,1),\ M\in\mathbb{R}$

(a)

$$\left.\begin{array}{l} f'(x) = 1 - e\cdot\cos(x) \geq 1-e > 0 \Rightarrow f(x) \text{ estrict. creixent} \\ \lim\limits_{x\to+\infty} f(x) = +\infty \\ \lim\limits_{x\to-\infty} f(x) = -\infty \end{array}\right\} \Rightarrow f(x) \text{ té una única arrel real } \alpha$$

$$\left.\begin{array}{l} f(M-e) = -e - e\cdot\sin(M-e) = -e\,(\underbrace{1+\sin(M-e)}_{\geq 0}) \leq 0 \\ f(M+e) = e - e\cdot\sin(M+e) = e\,(\underbrace{1-\sin(M+e)}_{\geq 0}) \geq 0 \end{array}\right\} \Rightarrow \boxed{\alpha\in[M-e, M+e]}$$

(b) $x_0 = M,\quad x_{k+1} = e\cdot\sin(x_k) + M \quad \forall k\geq 0$

Sigui $g(x) = e\cdot\sin(x) + M$. Com que $f(\alpha)=0 \to g(\alpha)=\alpha$.

$$\left.\begin{array}{l} \text{Llavors, } x_{k+1}-\alpha = g(x_k) - g(\alpha) = g'(\xi_k)\,(x_k-\alpha) \\ |g'(x)| = |e\cdot\cos(x)| \leq e \end{array}\right\} \Rightarrow \boxed{|x_{k+1}-\alpha| \leq e\cdot|x_k-\alpha|}$$

Usant aquesta desigualtat recurrentment, s'obté: $|x_n-\alpha| \leq e^n\,|x_0-\alpha| \xrightarrow{(n\to\infty)} 0$

Per tant $(x_n) \xrightarrow[n\to\infty]{} \alpha$

(c) Notació: $\varepsilon_k \equiv |x_k-\alpha| \quad \forall k\geq 0$. Per tant sabem $\varepsilon_n \leq e^n\cdot\varepsilon_0 \quad \forall n\geq 0$

$$\left.\begin{array}{l} \alpha\in[M-e, M+e] \\ x_0 = M \end{array}\right\} \Rightarrow \varepsilon_0 \equiv |x_0-\alpha| \leq e \quad \Rightarrow \boxed{\varepsilon_n \leq e^{n+1} \quad \forall n\geq 0}$$

Si es vol tenir $\varepsilon_n \leq \text{prec}$, és suficient $e^{n+1} \leq \text{prec} \Leftrightarrow (n+1)\log_{10} e \leq \log_{10}\text{prec}$

$$\Leftrightarrow (n+1) \geq \frac{\log_{10}\text{prec}}{\log_{10} e}$$

sabem ~~suposem~~ $0 < \cancel{\text{prec}}\ e < 1$, de manera que $\log_{10} e < 0$

$$\Leftrightarrow \boxed{n \geq \frac{\log_{10}\text{prec}}{\log_{10} e} - 1}$$

Si $\text{prec} = 10^{-20}$
$$\left\{\begin{array}{l} \text{i } e = 0.1 \Rightarrow n \geq \dfrac{-20}{\log_{10} 0.1} - 1 = \dfrac{-20}{-1} - 1 = 19 \Rightarrow \boxed{n\geq 19} \\ \text{i } e = 0.9 \Rightarrow n \geq \dfrac{-20}{\log_{10} 0.9} - 1 \approx 436.087 \Rightarrow \boxed{n\geq 437} \end{array}\right.$$

(d) <u>Newton-Raphson</u>: $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$

Deduïm una relació entre $\varepsilon_{k+1} \equiv x_{k+1}-\alpha$ i $\varepsilon_k \equiv x_k - \alpha$

$$\left.\begin{array}{l} 0 = f(x_k) + f'(x_k)(x_{k+1}-x_k) \\ 0 = f(\alpha) = f(x_k) + f'(x_k)(\alpha - x_k) + \frac{1}{2} f''(\xi_k)(\alpha-x_k)^2 \end{array}\right\} \text{Restant } 0 = f'(x_k)(\alpha - x_{k+1}) + \tfrac{1}{2} f''(\xi_k)(\alpha-x_k)^2$$

O sigui $\boxed{\varepsilon_{k+1} = \frac{1}{2}\frac{f''(\xi_k)}{f'(x_k)}\varepsilon_k^2,\ \xi_k \in \mathrm{Int}(x_k,}$

$$\left.\begin{array}{l} \text{I ara fitem: } f'(x) = 1 - e\cos(x) \Rightarrow |f'(x)| \geq 1-e \\ \phantom{\text{I ara fitem: }} f''(x) = e\cdot\sin(x) \Rightarrow |f''(x)| \leq e \end{array}\right\} \text{Per tant, } \boxed{\varepsilon_{k+1} \leq \left(\frac{1}{2}\frac{e}{1-e}\right)\varepsilon_k^2} \ \forall k\geq 0$$

Cas $e=0.1$ $\Rightarrow$ $\frac{1}{2}\frac{e}{1-e} = \frac{1}{2}\frac{1/10}{9/10} = 1/18$ $\Rightarrow$ $\boxed{\varepsilon_{k+1} \leq \frac{1}{18}\varepsilon_k^2 \quad \forall k \geq 0}$

Llavors,

$$\varepsilon_0 \leq e = 10^{-1}$$

$$\varepsilon_1 \leq \frac{1}{18}\cdot\varepsilon_0^2 = \frac{1}{18}\cdot 10^{-2} \qquad \approx 5.6\times10^{-4}$$

$$\varepsilon_2 \leq \frac{1}{18}\left(\frac{1}{18}\cdot10^{-2}\right)^2 = \frac{1}{18^3}\cdot10^{-4} \qquad \approx 1.7\times10^{-8}$$

$$\varepsilon_3 \leq \frac{1}{18}\cdot\left(\frac{1}{18^3}\cdot10^{-4}\right)^2 = \frac{1}{18^7}\cdot10^{-8} \qquad \approx 1.6\times10^{-17}$$

$$\varepsilon_4 \leq \frac{1}{18^{15}}\cdot10^{-16} \qquad \approx 1.5\times10^{-36}$$

$\Rightarrow$ $\boxed{n=4}$ és suficient

Cas $e=0.9$ $\Rightarrow$ $\frac{1}{2}\frac{e}{1-e} = \frac{1}{2}\frac{9/10}{1/10} = \frac{9}{2} = 4.5$ $\Rightarrow$ $\boxed{\varepsilon_{k+1} \leq \frac{9}{2}\cdot\varepsilon_k^2 \quad \forall k\geq 0}$

Llavors,

$$\varepsilon_0 \leq e = 9/10$$

$$\varepsilon_1 \leq \frac{9}{2}\left(\frac{9}{10}\right)^2 = \frac{9^3}{200} = \frac{729}{200} > 1 \ !$$

i les fites dels successius ε_k van creixent.

$\boxed{\text{No podem assegurar convergència}}$