Draft version May 7, 2020
Preprint typeset using LaTeX style emulateapj v. 11/26/04

THE MODELLING OF THE DARK MATTER AMOUNT AS A WIMP PARTICLE AND THE WIMP FREEZE OUT IN THE EARLY UNIVERSE

Marius Torsvoll

Draft version May 7, 2020

ABSTRACT

The WIMP freeze out and modelling dark matter as this phenomenon is very attractive as this produces the observed amounts of dark matter naturally. As this process needs no physically unreasonable assumptions it is a truly alluring alternative to most other dark matter models as all needed to prove the feasibility as a model is to detect a new massive weakly interacting particle, although this has not been done this is left as an exercise for the particle physicists out there. Thus it becomes clear that a WIMP darkmatter model is attractive, though the model has a serious problem as mentioned, no suitable particles has been detected to fill the WIMP void.

Subject headings:

1. INTRODUCTION

Thus the aim of this paper is to firstly describe the mathematical model that governs the numerical simulation that will be thoroughly described in the method section. The assumptions of this model is that there exists a particle in the $\sim$ 1000 GeV range of masses and that they are able to chemically decouple from the rest of the standard model, and that these particles are as the name suggests only interacting via the weak force. Thus we are led to the problem at hand, we propose then a supersymmetric extension to the standard model and thus the introduction of a particle fulfilling the requirements of a WIMP particle. This is rather useful, as it turns out WIMPs show up as natural parameter extensions in a supersymmetric extended standard model. This is an alluring alternative as this requires no more new physics than a few particles.

This then leads us to the decoupling, the only relevant process to this is the process of annihilation and can be described as the following: $p_1 + p_2 \to p_3 + p_4$ here p_1 and p_2 is a WIMP and WIMP anti particle, p_3 and p_4 is here represented as particles from the standard model. This process is entirely analogous to electron-positron annihilation and should behave identically except the end result particle production. This then gives us the basis for the modelling of the balancing of WIMP particles with standard model particles. As we are modelling the WIMPs as fermions that obey the Pauli exclusion principle the electron-positron annihilation is rather analagous as mentioned. As the decoupling is intimately linked to the decreasing temperature of the universe the decoupling time is then of interest. And to specify the decoupling is then when the process of annihilation of WIMPs to standard model particles stop.

2. METHOD

Electronic address: mariutel@astro.uio.no
[1] Institute of Theoretical Astrophysics, University of Oslo, P.O. Box 1029 Blindern, N-0315 Oslo, Norway

Thus we consider the the Boltzmann equation for the particles designated as p_3 and p_4 to be as the following:

$$\frac{dn_1}{dt} + 3Hn_1 = -\langle\sigma v\rangle[n_1 n_2 - n_1^{(0)} n_2^{(0)}] \tag{1}$$

For us to actually find this equation we must consider the full and general Boltzmann equation and thus we begin with the following:

$$a^{-3}\frac{d(n_1 a^3)}{dt} = -n_1^{(0)} n_2^{(0)}[\frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} - \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}}] \tag{2}$$

And then we have the following for n_2

$$a^{-3}\frac{d(n_2 a^3)}{dt} = -n_2^{(0)} n_1^{(0)}[\frac{n_2 n_1}{n_2^{(0)} n_1^{(0)}} - \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}}] \tag{3}$$

Thus when we consider this and that p_3 and p_4 are in thermodynamical equilibrium. We have that the second term in the brackets denoting the state of the WIMP particles becomes 1 when in thermodynamical equilibrium we are thus obviously left with equation 1.

Thus we move forward by considering the Fermi-Dirac distribution. As written here:

$$\bar{n_i} = \frac{1}{e^{\frac{(\epsilon_i - \mu)}{k_B T}} + 1} \tag{4}$$

We have that when the chemical potential becomes equal or vanish we are left with a distribution that only deals with the energy states of the particles and are dominated by the energy of the relativistic particles. Thus the non relativistic mass contribution then becomes small or vanish.

Then by conservation of entropy we will show the following:

$$\frac{dY}{dt} = -S\langle\sigma v\rangle(Y^2 - Y_{eq}^2) \tag{5}$$

Thus we begin with the following:

$$g(aT^3) = constant, ga^3 = constant \tag{6}$$

Then we move on:

$$\frac{dY}{dt} = \frac{d}{dt}(\frac{N_\chi}{S} = \frac{1}{S^2}(S\frac{dn_\chi}{dt} - N_\chi\frac{dS}{dt}) \quad (7)$$

$$\frac{dS}{dt} = \frac{d}{dt}(\frac{c}{a}) = c\frac{d}{dt}\frac{1}{a^3}$$

$$= c(-3\frac{1}{a^4}\frac{da}{dt})$$

$$= -3c\frac{\dot{a}}{a^4} = -3c\frac{H}{a^2} = -3HS \quad (8)$$

Then with this result we once again consider the entropy conservation and move on further:

$$\frac{dY}{dt} = \frac{1}{S^2}(s\frac{dn_\chi}{dt} - N_\chi(-3HS))$$

$$\frac{1}{S^2}(S\frac{dn_\chi}{dt} + 3HN_\chi S)$$

$$= \frac{1}{S}(-\langle\sigma v\rangle[n_\chi^2 - (n_\chi^{(0)})^2])$$

Then since $Y = \frac{n_\chi}{S}$

$$\frac{dY}{dt} = -S\langle\sigma v\rangle[Y^2 - Y_{eq}^2] \quad (9)$$

Then we move on by considering the chain rule and the Boltzmann equation thus we use the following:

$$\frac{dY}{dt} = \frac{dY}{dx}\frac{dx}{dT}\frac{dT}{dS}\frac{dS}{dt}$$

Thus we consider the following:

$$\frac{dx}{dT} = -\frac{m_\chi}{T^2}$$

$$\frac{dS}{dT} = \frac{6\pi^2}{45}g_{*s}T^2 \quad (10)$$

Then by moving around the chain rule, we have the following:

$$\frac{dY}{dx} = \frac{dY}{dt}\frac{dS}{dT}[\frac{dx}{dt}\frac{dS}{dt}]^{-1}$$

$$\frac{dY}{dx} = \frac{dY}{dt}[\frac{dx}{dt}]^{-1} \quad (11)$$

Then we move on further by the input of known sizes we have the following:

$$\frac{dY}{dx} - \frac{2\pi^2}{45}g_{*s}\frac{T^4}{Hm_\chi}\langle\sigma v\rangle(Y^2 - Y_{eq}^2)$$

$$\Rightarrow -\frac{S}{xH(x)}\langle\sigma v\rangle(Y^2 - y_{eq}^2) \quad (12)$$

3. SIMULATION

The simulation process in this case was fairly simple as the computational problems were not all that complex. The main problem comes down to how to solve the implicit equation as described in the method. This is done by using an external implicit equation solver from the scipy module. The script is then as implied by the use of scipy, written in python. The script used to solve the numerical part of the equations, the method used is the implicit Radau version a variety of the well known Runge-Kutta functions. This method is employed for numerical stability as a lower order method such as forward euler would not achieve the desired stability and would present us with significant numerical problems.

Thus the main thing left then is implementing the functions for finding the dark matter and extracting the desired values. The functions are then simply implemented as functions and the desired values are extracted by numerical means. Other things that should be noted about the simulation is that it is rather computational taxing for such a small script. The implicit equation is also solved for ten thousand values inbetween the x limits. The implicit equation is also solved for another ten thousand datapoints inbetween the the cross section limits. Thus leading us to a significant amount of computation time, but still the time is not too excessive.

4. RESULTS

Then for the y and y_{eq} we have the following plots as the result of the model, main take away from this is the relation between the two different y's. They turned out very similar and follow each other almost perfectly, but should be noted that they are not equal. So first a look at the model for $1 < x < 745$ as the full x = 1000 range would produce numerical issues the range was cut for numerical reasons, the result should be still valid and further development could be devised from the plot.

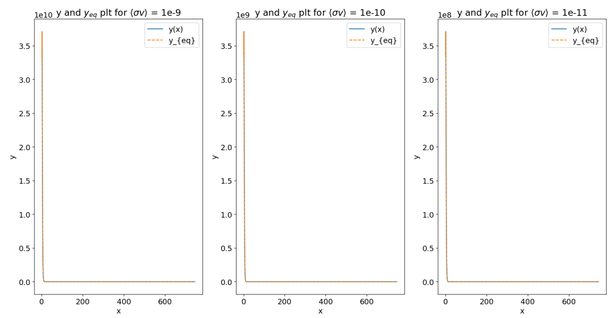

FIG. 1.— The plotting of y and y_{eq} as a function of x. As this is not a good picture of the evolution we can not tell that much from this. other than the value will converge towards zero, this becomes again more apparent if the graphs are logarithmically plotted.

Thus for a more readable plot.

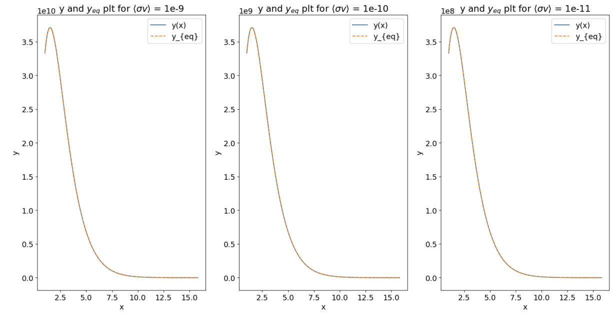

FIG. 2.— The plotting of the same curves as in figure 1 but the x range has been shortened to actually display the behaviour of the y and y_{eq} in the beginning as they converge towards zero rather quickly, at least it would seem like it, and as shall be shown by a logarithmic plot they only seem the same.

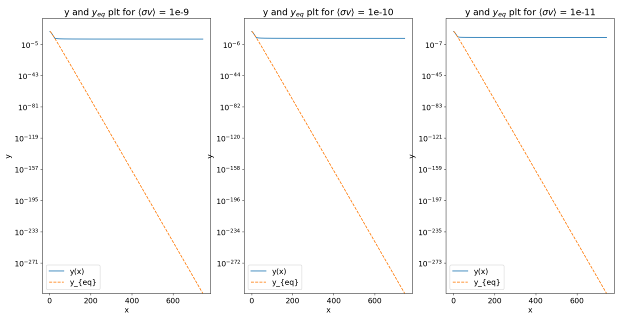

FIG. 3.— The plotting of the same curves as in figure 1 but the y axis is scaled logarithmically, and as can be seen the two values diverge significantly after a short amount of time as can be seen. The two values do as should be noted follow each other significantly close for small x values but diverge quickly for larger x.

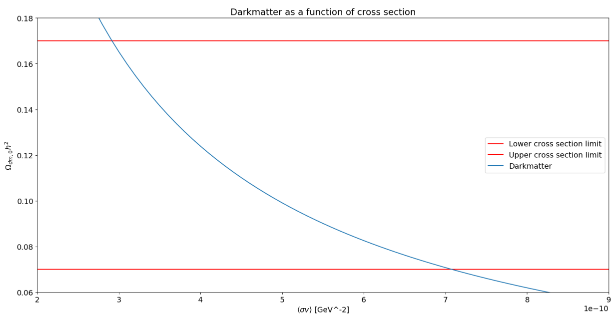

FIG. 4.— The plotting of the amount of dark matter and the limit set on the amount of dark matter. The plot has then been zoomed in on the relevant area.

For an amount of darkmatter equal to $\Omega_{dm,o}h^2 = 0.12 \pm 0.05$ we consider a sampling of reaction cross sections of $[10^{-14}, 10^{-7}]$. Thus leading us to a possible cross section interval of $[3.0 \cdot 10^{-10}, 7.0 \cdot 10^{-10}] GeV^{-2}$ This is then the model prediction based on the observed amount of dark matter in the universe.Thus with these parameters we have the following dark matter curve:

5. DISCUSSION AND CONCLUSIONS

As seen from the results the WIMP freeze out model of darkmatter the needed observed amount of darkmatter is naturally produced. The favoribility of the model becomes thus all the more desirable. Since the current understanding of dark matter is currently mostly restricted to the amount present in the universe and a coarse understanding of the structure. Thus the component structure of this unknown quantity is desirable. And as shown the theoretical amount of dark matter using a fairly reasonable sampling amount of cross sections produces a desirable amount of dark matter that is signficantly similar to the observed amount.

Although as this model produces desirable results one should be quite careful in accepting it as there exists no known particle that could fit the WIMP role. Although while this is not equivalent to the non existence of such a particle, it just has not to this point been shown to exist. Although higher energy experiments may lead to the detection of a supersymmetric particle that can fulfill the WIMP role it obviously will gain more traction and become a far more viable model. Although again even if such a particle is found that does not equate to the observed dark matter actually consisting of such a particle as there has to be a sufficient amount of such particles.

But then to conclude, the mathematics in this model is fairly sound, analogous processes are known to exist and the amount of dark matter needed is easily produced by the model, so it is obviously an interesting prospect. But as mentioned the WIMP particle is as ever illusive, but if such a particle is found to exist the behaviour and amount needed for such a particle is fairly desirable to be known. Though as always further work has to be done and further experiments to further verify the viability of the WIMP model.