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EXPLORING EXTRA SPATIAL DIMENSIONS IN A COLD DARK MATTER UNIVERSE WITH NO RADIATION AND NON RELATIVISTIC MATTER.

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Subject headings:

1. INTRODUCTION

Exploring an extra spatial dimension could be beneficial in the exploration in the search for other cosmological models to fit current data, the mathematical groundwork and the attempt at a simulation will be attempted to be described in this article. Although not everything will go to plan, the groundwork should be there and the work still has value.

2. METHOD

The mathematical background of the simulations and the strategy of developing them will be explained in this section. The equations that are need to explain the behaviour of scalar fields in the spatial multidimensional case and how this behaviour manifests itself mathematically. First we will begin by looking at how a massless scalar field behaves in three spatial dimensions. So we begin by looking at how the electric field from a point charge evolves. Thus we have the following by Gauss law.

$$\Phi = \int \rho_f dA \tag{1}$$

3. SIMULATION

Thus, since the surface area A is a sphere in three dimensions we have the from the following.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\rho} \tag{2}$$

$$\rightarrow \Phi 4\pi r^2 = \frac{Q}{\epsilon_0}\frac{r^3}{R^3} \tag{3}$$

$$\rightarrow E = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3} \tag{4}$$

Since we are only interested in the strength of the electric field at a given distance, and not the direction of it we have that r cancels with R thus. $E \propto \frac{1}{r^2}$

Then we need to prove that the following equation has a solution where S = 0.

$$\Box^2\phi = (\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2})\phi = S(\mathbf{r};t) \tag{5}$$

Thus we begin with assuming a solution can be written as the following:

$$\phi(\mathbf{r};\mathbf{k},t) = f(\mathbf{r};\mathbf{k})g(t) \tag{6}$$

Then we have that the solution to the equation can be rewritten as:

$$\frac{1}{g(t)}\frac{\partial^2 g(t)}{\partial t^2} - \frac{1}{f(\mathbf{r};\mathbf{k})\nabla f(\mathbf{r};\mathbf{k})} = 0 \tag{7}$$

The only way this is possible is if it is equal to a constant so we choose $-\omega^2$ as that constant for simplicity's sake. This leaves us with to differential equations we can solve.

$$\frac{\partial^2 g(t)}{\partial t^2} = -\omega^2 g(t)$$

Thus ths solution has the following form:

$$g(t) = A_1 e^{-i\omega t} \tag{8}$$

For the spatial component:

$$\nabla f(\mathbf{r};\mathbf{k}) = -\omega f(\mathbf{r};\mathbf{k}) \tag{9}$$

Thus this also has a solution on the same form

$$f(r;k) = A_2 e^{i\mathbf{k}\cdot\mathbf{r}}$$

Here k is equal to $k_x\vec{x} + k_y\vec{y} + k_z\vec{z}$ and $\omega = k$ Thus we also have going forward:

$$\phi(\mathbf{r};\mathbf{k},t) = Ae^{i\mathbf{k}\cdot\mathbf{r}}e^{-i\omega t} \tag{10}$$

$A = A_1 \cdot A_2$ Thus finding the constant A is trivial and has to be found given a set of initial conditions, but this is not relevant for the moment.

Furthermore we consider the fourier transform:

$$\int_{-\infty}^{\infty} \frac{dp}{2\pi}\tilde{g}(p)$$

We need this to be a constant and that this constant is the fourier transform of:

$$g(\omega)\delta(\omega)$$

Thus we have the following:

$$\mathcal{F}[g(\omega)\delta(\omega)] = \mathcal{F}[g(\omega)] + \mathcal{F}[\delta(\omega)]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{g}(p)\tilde{\delta}(\omega - p)dp$$

$$\mathcal{F}[\delta(\omega)] = \int_{-\infty}^{\infty} \delta(\omega)e^{-2\pi i\omega p}d\omega = 1 = \tilde{\delta}(p)$$

$$\implies \mathcal{F}[g(\omega)\delta(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{g}(p)\cdot 1dp = \int_{-\infty}^{\infty}\frac{dp}{2\pi}\tilde{g}(p) \tag{11}$$

Now to rewrite the following equation in fourier space.

$$(\Box^2 - \frac{\partial^2}{\partial\omega^2})\phi + l\delta(\omega)\Box\phi = s(\vec{r},\omega,t)$$

We use the results we have from equation 11 to rewrite the equation above. Then moving on from the equation above:

$$= (\frac{\partial^2}{\partial t^2} - \nabla^2 - \frac{\partial}{\partial\omega^2})\phi + l\delta(\omega)(\frac{\partial^2}{\partial t^2} - \nabla^2)\phi$$

Since we only consider a static point source we have no variation in time thus $\frac{\partial}{\partial t}\phi = 0$

$$\implies (-\nabla^2 - \frac{\partial}{\partial\omega^2})\phi + l\delta(\omega)(-\nabla^2)\phi$$

$$-[\nabla^2\phi - \frac{\partial\phi}{\partial\omega^2} + l\delta(\omega)\nabla^2\phi] = s(\vec{r},\omega,t) \to \lambda\delta(\vec{r})\delta(\omega)$$

$$-\mathcal{F}[\nabla^2\phi - \frac{\partial\phi}{\partial\omega^2} + l\delta(\omega)\nabla^2\phi] = \mathcal{F}[\lambda\delta(\vec{r})\delta(\omega)]$$

This then solves to be

$$\tilde{\phi}(\vec{k},p) + \frac{(k^2\tilde{f}(\vec{k}))}{k^2+p^2} = \frac{\lambda}{k^2+p^2} \tag{12}$$

Then to determine $\tilde{f}$, we have:

$$-\mathcal{F}[\nabla^2\phi - \frac{\partial\phi}{\partial\omega^2} + l\delta(\omega)\nabla^2\phi] = \mathcal{F}[\lambda\delta(\vec{r})\delta(\omega)]$$

Where:

$$2\pi\tilde{f}(\vec{k}) = \int_{-\infty}^{\infty}\frac{dp}{2\pi}\tilde{\phi}(\vec{k},p)$$

Thusly we have the follwoing integral:

$$\int_{-\infty}^{\infty}[\tilde{\phi}(\vec{k},p) + \frac{(k^2\tilde{f}(\vec{k}))}{k^2+p^2}]dp = \int_{-\infty}^{\infty}\frac{\lambda}{k^2+p^2}dp$$

This integral then shortens out with substitutions to be:

$$-\frac{1}{k}[\arctan(u)]_{-\infty}^{\infty} = \frac{1}{k}[\frac{\pi}{2} - (-\frac{\pi}{2})] = \frac{\pi}{k}$$

Thus we return to:

$$\int_{-\infty}^{\infty}\tilde{\phi}(\vec{k},p)dp + k^2 l\tilde{f}(\vec{k})\int_{-\infty}^{\infty}\frac{1}{k^2+p^2}$$

$$2\pi\tilde{f}(\vec{k}) + k^2(\tilde{f}(\vec{k}))\frac{\pi}{k} = \lambda\frac{\pi}{k}$$

Thus

$$\tilde{f}(\vec{k}) = \frac{\lambda}{2k}\frac{1}{1+\frac{1}{2}kl} \tag{13}$$

Then this is all we need to know to find $\phi(\vec{k},0)$ This can be shown to be true:

$$\phi(\vec{r},\omega) = \mathcal{F}[\tilde{\phi}(\vec{k},p)]$$

$$= \int_{-\infty}^{\infty}\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{2\pi}\tilde{\phi}(\vec{k},p)e^{i\vec{k}\cdot\vec{r}}e^{ip\omega}d\vec{k}dp$$

$$= \int_{-\infty}^{\infty}e^{i\vec{k}\vec{r}}\int_{-\infty}^{\infty}\frac{1}{2\pi}$$

Rememembering that for this the $\omega = 0$ and solving the equation we are led to:

$$\phi(\vec{r},0) = \int_{-\infty}^{\infty}\frac{d\vec{k}}{(2\pi)^3}\tilde{f}(\vec{k}e^{i\vec{k}\cdot\vec{r}}) \tag{14}$$

Then performing the angular integration:

$$\phi(\vec{r},0) = \int_{-\infty}^{\infty}\frac{d\vec{k}}{(2\pi)^3}\tilde{f}(\vec{k})e^{i\vec{k}\cdot\vec{r}}$$

$$\implies \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\lambda}{2k}\frac{1}{(1+\frac{1}{2}k)}e^{i\vec{k}\cdot\vec{r}}d\vec{k}$$

$$= \frac{\lambda}{2^4\pi^3}\int_{-\infty}^{\infty}\int_0^{2\pi}\int_0^{\pi}\frac{1}{k(1+\frac{1}{2}k)}\cdot e^{ikr\cos(\theta)}k^2\sin(\theta)d\theta d\varphi dk$$

$$= \frac{\lambda}{2^4\pi^3}\int_0^{2\pi}d\varphi\int_{-\infty}^{\infty}\frac{k}{1+\frac{1}{2}lk}dk\int_0^{\pi}\frac{1}{k(1+\frac{1}{2}k)}\cdot e^{ikr\cos(\theta)}k^2\sin(\theta)$$

$$\frac{\lambda}{2^3\pi^2}\int_0^{\infty}\frac{k}{1+\frac{1}{2}k}\int_1^{-1}e^{ikru}dudk$$

$$\frac{\lambda}{2^3\pi^2}\int_0^{\infty}\frac{k}{1+\frac{1}{2}k}\frac{1}{ikr}[e^{ikru}]_{-1}^{1}dk$$

$$= \frac{\lambda}{4\pi r^2}\int_0^{\infty}\frac{\sin(kr)}{1+\frac{1}{2}k}$$

As to why all this is relevant for a cosmological model, it can be summed up in three words; the hierarchy problem, the hierarchy problem is defined to be as follows. When a model for something, say the universe, depends on a couple of parameters, let say they take on the values of 1, 1.3, 0.9 and 10^{-23}. This is the hierarchy problem, when a paramater differs significantly from the others without an apparent good reason to. This problem can be found in our current understanding of the universe. This being that gravity is so much weaker than the other fundamental forces of nature. Now as this is all to explore the possibilities of a universe with extra spacial dimensions this is to explore whether it is possible for gravity to manifest itself in this extra spatial dimension.

Thus we are finally left for the new sum rule for the new friedmann equation. Which we will derive from the following equation:

$$\frac{H^2(z)}{H_0^2} = (\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{rc}} + \sqrt{\Omega_{rc}})^2 + \Omega_{k0}(1+z)^2 \tag{15}$$

Thus to find the sum rule we set z = 0 and solve for Ω_{k0} thus we are left with the following expression:

$$\frac{H^2(z)}{H_0^2} = (\sqrt{\Omega_{m0} + \Omega_{rc}} + \sqrt{\Omega_{rc}})^2 + \Omega_{k0} = 1 \tag{16}$$

Thus we are finally left with:

$$\Omega_{k0} = 1 - (\sqrt{\Omega_{m0} + \Omega_{rc}} + \sqrt{\Omega_{rc}})^2 \tag{17}$$

Thus the mathematical groundwork for the cosmological model has been laid, in the next section we will discuss the computational aspects of this project and how exactly this mathematical model has been treated and implemented and analysed. The simulation of luminosity distance was used to simulate the extra spatial dimension and how it manifests itself to the effect of a massless scalarfield, however, to actually assess whether this model is to be preferred we will also use statistical analysis to try and determine whether the DGP model is to be preferred over the more widely accepted ΛCDM model. Thus firstly we will consider the regular Friedmann equation. So we will consider the following

$$\frac{H^2(z)}{H_0^2} = \Omega_{rc} + \Omega(1+z)^3 + (1+z)^4 + \Omega_{k0}(1+z)^2 + \Omega_\Lambda$$

This will become important at later stages. Thus when we consider the luminosity distance we will have the following:

$$dL = a_0(1+z)r = \frac{c(1+z)}{H_0\sqrt{|\Omega_{k0}|}} \mathcal{S}_k\left(\int_0^z \frac{dz'}{H(z')/H_0}\right) \tag{18}$$

Since we don't know what the galactic scale factor is at the moment we are only interested in the rightmost part of the equation above. So, how is this equation handled numerically, firstly a set of tests is implemented to test what form $\mathcal{S}_k$ takes, then the integral has to be evaluated for all z values the model is run for. Then again it should be noted that the entire point of this simulation is to evaluate different paramaters to the real life data. These being Ω_{m0} and Ω_Λ for the ΛCDM model, and Ω_{m0} and Ω_{r0} for the DGP model, here the Λ is omitted as this is a universe without a cosmological constant but an extra spatial dimension manifesting itself in the r0 paramater.

Further a χ^2 likelihood analysis is done, by minimizing the χ^2 the likelihood of the model is maximized. Thus the bayesian theorem is maximised. The result of the χ^2 is then plotted as a contour plot. This then represents the most probable values for the estimated parameter values, this then tells us the most likely value although not the true value, as this cannot be found by statistical analysis, all this can tell us is the most probable value.

Then the values that are the most likely (> 95%) are marked in the contour as a separate ring. This is to visualize the most probable values and to be evaluated at later points. The analysis is conducted for the DGP and ΛCDM model, although it has to be noted that the friedmann equation has to be changed to that equal to that of equation 15 Then the simulation is run fairly equally and the model is run for several parameter values.

4. RESULTS

The results are at the time inconclusive, see the script and last section for more info.

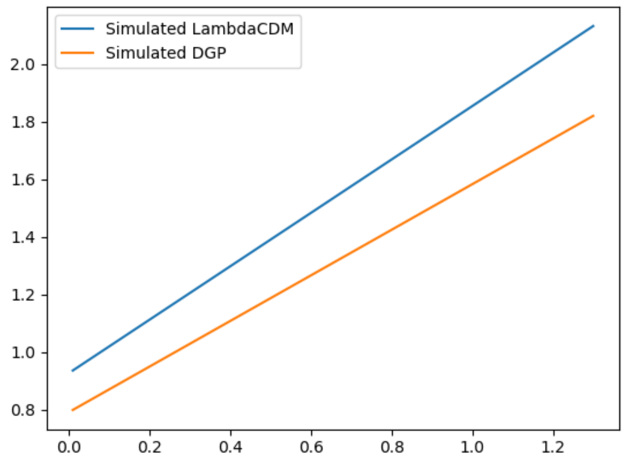

Figure 1: The luminosity distance plotted as a function of redshift in units of $\frac{c}{H_0}$. These distances is for a chosen set of parameters.

5. DISCUSSION AND CONCLUSIONS

Thus the analysis is fairly inconclusive in preferring either one of the models, the statistical results is not conclusive in excluding either model and both are valid for explaining the real life data. Since the analysis is inconclusive one has to test them for other predictions, although this may be difficult, for example an extra dimension is difficult to detect. So the other effects of these parameters and models must be considered to form a clearer picture, although a possibility is that a totally different model is preferable even though the models show that they are indeed fairly good representations according to the data given.

To conclude, other simulations has to be run and compared to other data, maybe even another set of redshift data. Although these models predictions of luminosity distance as a function of redshift, makes sense and is fairly reasonable as can be seen from the contour plots as they have a rather small χ^2 value.

6. ACKNOWLEDGMENTS AND EXPLANATION

As should be noted all the plots are not useful and are blank, this is due to some bug in the coding that i cannot figure out. As can be seen from the code it produces a reasonable luminosity distance as a function of redshift although i cannot tell whether all is as it should be, should be noted that this report has been written as if the results were present and the script worked. I dont have the time left to complete the debugging. As should be noted the script has been made so that it should make reasonable results as should be discovered by going through it. The code has been setup so that one may run different plots by removing hashed out terms.