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THE EVOLUTION OF THE SCALARFIELD IN A SLOW-ROLL INFLATIONARY REGIME.

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ABSTRACT

In this report he scalar field evolution shall be modeled along the lines of a slow-roll inflation model, the mathematical basis shall be described and the solution of the Friedmann equation and the scalar field equations shall be solved numerically.

Subject headings: Slow-Roll, Inflation, scalar field and potential

1. INTRODUCTION

The use of a slow-roll inflationary model is rather significant for the field of physical cosmology as it gives a way to estimate the e-foldings and the behaviour of a scalar-field in the same mathematical process. As in this model the scalar field and its collapse is the driver for the inflation process. This collapse has to be slow enough to be an approximation of a ball slowly rolling down a hill, if the collapse becomes too rapid the inflation stops and reheating begins. This then is very practical for cosmologists as this explains quite a large portion of the early universe.

A numerical solution without approximation will be employed to solve the equations describing the universe expansion. These will then be plotted and the physical implication of the solutions discussed. This is still a simple model as the model assumes an uncoupled field that is allowed to decay independently, as we know from particle physics fields are coupled and do not behave independently.

2. METHOD

In this section we will develop the mathematical basis for the numerical solutions, state all made assumptions and explain as to why these specific assumptions are made.

So for the mathematics we start off by simply making the Hubble parameter and the scalar field equation dimensionless. This we shall do by doing the following:

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + \hbar c^3\frac{dV}{d\phi}$$

$$h^2 = \frac{8\pi G}{3c^3}\left[\frac{1}{2\hbar c^3}\left(\frac{d\phi}{dt}\right)\right] \tag{1}$$

So firstly we shall begin with the first equation and we shall attempt to scale it and make it dimensionless. We shall make sure the variables used are of appropriate dimensions to make sure that the resulting equation is appropriately scaled. Thus we start with the following substitutions;

$$t = \frac{\tau}{H_i}\ \phi = \psi E_p \tag{2}$$

So if we consider these quantities, we have the following dimensions:

$$E_p = J\ \ H_i^2 = \frac{8\pi G}{3c^2}V(\phi_i)$$

Since the Planck energy obviously must have unit energy we are left with the initial Friedman:

$$H_i^2 = \frac{1}{t^2}\ \Rightarrow\ H_i = \frac{1}{t} \tag{3}$$

Thus we shall substitute these in for the equation, then by simple substitution we are left with:

$$H_i^2 E_p\frac{d^2\psi}{d\tau} + 3HHiE_p\frac{d\psi}{d\tau} + \frac{\hbar c^3}{E_p}\frac{d}{\psi}V$$

Then by dividing away the factors we are left with:

$$\frac{d^2\psi}{d^2\tau} + 3\frac{H}{H_i}\frac{d\psi}{d\tau} + \frac{dv}{d\psi} = 0 \tag{4}$$

For us to be able to state this we need the quantity to be dimensionless:

$$v = \frac{\hbar c^3}{E_p^2 H_i^2}V(\phi)$$

Thus if the potential has a dimension of J this then becomes obvious that this v has no dimension and is a valid expression for us to use.

(5)

Thus for the dimensionless Hubble parameter:

$$h^2 = \frac{H^2}{H_i^2} = \frac{8\pi}{3}\left[\frac{1}{2}\left(\frac{d\psi}{d\tau}\right)^2 + v\right]$$

So as to deal with the dimensions as we know the hubble parameter must have the unit time for this to make sense and thus as we know H_i has unit per time and then dimensionally this makes sense. Now as to how this was calculated.

$$h^2 H_i^2 = \frac{8\pi G}{3c^2}\left[\frac{E_p^2 H_i^2}{2\hbar c^3}\left(\frac{d\psi}{d\tau}\right)^2 + \frac{E_p^2 H_i^2}{\hbar c^3}v\right]$$

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$$h^2 H_i^2 = \frac{8\pi G}{3c^2}\frac{E_p^2 H_i^2}{\hbar c^3}\left[\frac{1}{2}\left(\frac{d\psi}{d\tau}\right)^2 + v\right]$$

$$h^2 = \frac{8\pi G}{3c^2}\frac{E_p^2}{\hbar c^3}\left[\frac{1}{2}\left(\frac{d\psi}{d\tau}\right)^2 + v\right]$$

Since the following is true:

$$E_p^2 = \frac{\hbar c^5}{G} \ \Rightarrow \ \left(E_p^2\right)^{-1} = \frac{G}{\hbar c^5}$$

Thus:

$$h^2 = \frac{8\pi}{3}\left[\frac{1}{2}\left(\frac{d\psi}{d\tau}\right) + v\right] \tag{6}$$

Then we shall attempt to derive a set of initial conditions, we need to know the initial value of h and the initial time derivative of the scalar field. Thus as the time is zero

$$h^2(0) = \frac{H^2(0)}{H_i^2} = 1$$

thus we may say the following:

$$\left(\frac{d\psi}{d\tau}\right)_{\tau=0} = -\frac{1}{3}\left(\frac{dv}{d\psi}\right)_{\psi=\psi_i} \tag{7}$$

This has to be true due to the slow roll assumption, the double derivative of something that rolls slowly with a constant speed has to be zero. Thus the change in the scalar field has to be equal to the change in the potential given a change in the field. This has to be true for the slow roll assumption to be valid.

So now we need only to choose an initial value for the initial field strength. This can be done simply by imposing the following the following:

$$\phi_i \gg \frac{E_p}{2\sqrt{\pi}}$$

$$\Rightarrow \psi_i \gg \frac{1}{2\sqrt{\pi}}$$

$$\psi_i \gg 0.28 \Rightarrow \psi_i \equiv 3.1 \tag{8}$$

This initial field strength also allows us to achieve a useful amount of e-foldings. This can be verified by imposing the following:

$$N = \frac{\phi_i\sqrt{2\pi}}{E_p} - \frac{1}{2}$$

As N has to be at least 60 we have the following:

$$\phi_i = \frac{11}{2\sqrt{\pi}}E_p \approx 3.1E_p \ \Rightarrow \psi = 3.1 \tag{9}$$

Thus by having found some initial conditions we now have to move on to the solution of the equation and the comparison to the slow roll approximation must be done by the use of a numerical solution. This solution shall be described in the Simulation section later on, but for now we move on to the relation between the scale factor and the initial scale factor and the relation between the energy pressure and energy density. So for the scale factor we start off with the following:

$$H = \frac{\dot{a}}{a} = \frac{1}{a}\frac{da}{dt} \ \Rightarrow \ \frac{1}{a}H_i\frac{da}{d\tau}$$

$$h = \frac{1}{a}\frac{da}{d\tau}$$

$$\frac{1}{a}da = hd\tau$$

$$\ln\left(\frac{a(\tau)}{a_i}\right) = \int_0^\tau hd\tau \tag{10}$$

This has also to be solved numerically and thus shall be dealt with at a later time. But this gives a basis for calculating the scale factor evolution and once more a way to estimate the number of e-foldings. Thus we move on to the following ratio:

$$p_\phi = \frac{1}{2\hbar c^3}\dot{\phi}^2 + V(\phi)$$

$$\rho_\phi c^2 = \frac{1}{2\hbar c^3}\dot{\phi}^2 - V(\phi)$$

$$\frac{p_\phi}{\rho_\phi c^2} = \frac{\frac{1}{2\hbar c^3}\dot{\phi}^2 + V(\phi)}{\frac{1}{2\hbar c^3}\dot{\phi}^2 - V(\phi)}$$

$$\Rightarrow \dot{\phi}^2 = E_p^2 H_i^2\left(\frac{d\psi}{d\tau}\right)^2$$

$$\Rightarrow v = \frac{\hbar c^3}{E_p^2 H_i^2}V(\phi)$$

Thus by substitution we have the following:

$$\frac{\frac{E_p^2 H_i^2}{2\hbar c^3}\left(\frac{d\psi}{d\tau}\right)^2 + \frac{E_p^2 H_i^2}{\hbar c^3}v}{\frac{E_p^2 H_i^2}{2\hbar c^3}\left(\frac{d\psi}{d\tau}\right)^2 - \frac{E_p^2 H_i^2}{\hbar c^3}v}$$

$$\Rightarrow = \frac{\frac{1}{2}\left(\frac{d\psi}{d\tau}\right)^2 + v}{\frac{1}{2}\left(\frac{d\psi}{d\tau}\right)^2 - v} \tag{11}$$

Thus to find this expression we once again must refer to the numerical solution as the time derivative of the field has to be evaluated numerically.

3. SIMULATION

The simulations were run using the python 3 programming language, the equations were evaluated for 300 units of dimensionless time and for 3000 total data points. This was deemed to be a decent resolution for the solution and a sufficient step size. The simulations ran into no numerical issues and were as expected fairly fast to run. The solution was found using a standard euler method, as since the solution required a double integration due to the second order differential equation a second order euler method was employed. The resulting results were then plotted graphically. All needed parameters for the ratios and solution of the equation were then used to produce the plots found in the results section.

4. RESULTS

So the results of the simulations clearly show the behaviour of the slow roll inflationary model. The expected inflation behaviour is as expected and the scalar field behaves linearly until it reaches approximately the ground state. And starts to oscillate about zero, slightly dipping into the negative realm.

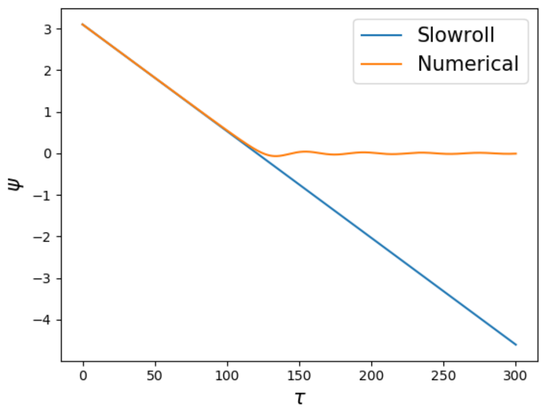

FIG. 1.— The evolution of the scalarfield as a function of dimensionless time. As can be seen the numerical solution starts to oscillate about zero after some time.

So then for the ratio between the scale factor over time and the initial scale factor. Thus also a plot of the number of e-foldings for the expansion of the universe:

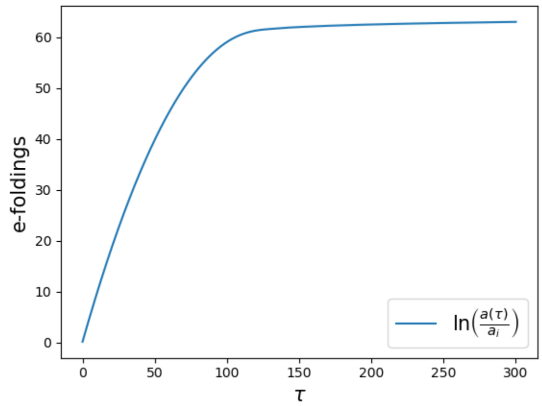

FIG. 2.— The ratio of between the initial scale factor and the time evolving one, plotted logarithmically thus also providing an estimation for the amount of e-foldings provided for the inlationary model.

So for another ratio this time between the energy pressure excerted by the scalar field and energy density, this is relevant as the ratio between these two parameters describe how the scalar field behaves whether as a regular scalar field or like a cosmological constant.

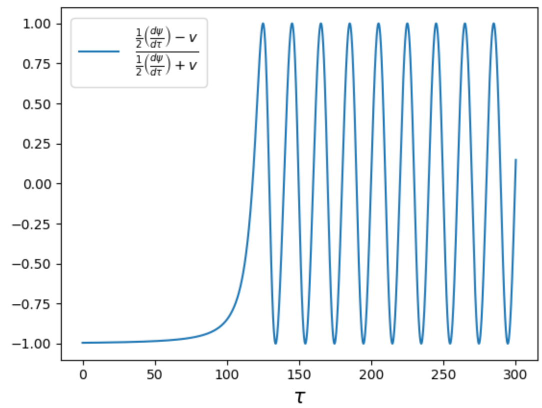

FIG. 3.— The ratio between the radiation pressure from the scalar field and the energy density as a function of dimensionless time. As can be seen when the scalar field starts to oscillate about zero so does the ratio.

These are the main results from the simulations of the slow roll model, this is then a fairly useful model of inflation and the development of the early universe as the model can be tweaked to fit the needed requirements by just simply increasing the scalar field strength or change the shape or size of the potential.

5. DISCUSSION AND CONCLUSIONS

So as to extracting something from these results, we firstly can see that all the results are in concurrence as to when and how long the inflation period takes place. Being of the period to be approximately from 0 to 100 units of dimensionless time this is important for the intrinsic integrity of the model. This then helps with the verification of the mathematical validity of the model and especially it helps for the verification of the numerical solution.

So as to the e-foldings and the scale-factor this behaves exactly as we would expect with regards to an inflating universe, there is an exponential growth for the beginning of the simulation and this growth flattens out and then continues to grow still rather fast, but not with the same extreme exponential growth. As should be remembered about figure 4 is that it is plotted logarithmically thus linear growth translates to exponential growth. Thus the inflation then stops but the expansion still continues on given this model.

So as to the ratio between the energy density and the pressure of the field we see that the ratio starts off at -1 and after the inflation period then grows slightly and then starts to oscillate between -1 and 1. This then tells us that in the beginning the scalar field behaves like a cosmological constant. As for this period the pressure is approximately equal to the energy density and thus the field behaves like a cosmological constant. So as for the behaviour of the field after the inflationary period we cant say much but we know that the field must have decayed as the potential is approximately at the end point and the field strength is at the point where it oscillates about zero and we know at this point it probably has decayed into particles. Whether this field then can affect the universe in a meaningful way is unclear.

In conclusion the model has quite a few advantages going for it, it predicts quite a few things we know about the early universe. So this mathematical model for the process of inflation has to be discussed and is rather useful for many things. But it also has its weaknesses as does most models and further work must be done especially in the coupling of other fields and a more complex universe which contains more things and thus is more complicated.

I wish to thank Torstein for great help and camaraderie in these trying times.

God save the queen.