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A REPORT ON THE SIMULATION OF RELATIVE ENERGY PRODUCTION IN THE STELLAR CORE AT VARYING TEMPERATURE.

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ABSTRACT

The stellar engine as a function of the energy production in the Proton-Proton (PP) chain and the Carbon-Nitrogen-Oxygen (CNO) chain at a temperature range of $[10^4, 10^9]$ Kelvin. The relative energy production per unit mass is the only thing being calculated in this simulation. To verify the results and make sure the relative energies gives meaning a sanity check is implemented to simulate the reactionary conditions in the stellar core of the sun. Thus giving the simulations in the extended temperature range more credibility. As the simulations show the CNO chain comes to dominate the energy production in the higher temperature range, this is to be expected as the triple alpha process produces more carbon to fuel the CNO chain. As the amount of ^{14}N increases the proton capture rate will thus also increase.

Subject headings:

1. INTRODUCTION

In this report the stellar mechanics and energy production rates will be thoroughly explained and results from the simulations will be explained. The simulations are based on the known PP and CNO chains, reaction cycles that have been known to exist and be possible in stellar engines for many decades. As it is interesting to compare simulated results to observational results in later cases, in this case only the actual results of the simulations will be dealt with, this though does not make them any less interesting as a discussion topic.

The model assumes a fully ionized core, this is quite reasonable at the temperature ranges of 10000K and greater. As most elements will be fully ionized at these temperatures. Anyway for the way this model is implemented as a function of temperature, if any elements are not ionized this will be reflected in how the reaction rates evolve. As the reaction coefficients depend on the temperature this is taken care of.

The model itself will be run on a regular computer as a high performing super computer is not necessary for the purposes of this case. The amount of calculation can easily be handled by a personal computer. Though for more advanced models that are not snapshots at a certain time, but evolving this will probably have to be changed. But for a snapshot in time a PC is quite enough.

2. METHOD

The mathematical background of the simulations and the strategy of developing them will be explained in this section. The equations that are need to explain the reactions in the stellar core, need to explain how the elements evolve, how they react and how much energy they produce. So the simplest is to begin with the energy production. The energy released in a fusion reaction where to heavier elements fuse into a lighter one. Thus the energy production of the elemental fusing can be described by:

$$E = (m_1 - m_2)c^2 \tag{1}$$

This is assuming that the reaction has a combined mass of all particles before fusing of m_1 and a mass after the reaction of m_2. This formula also works for all cases of fusion. So this equation is general for the calculation of energy production in fusion. The mass difference is extrapolated from tables, the mass is known due to numerous experimental results. Should be noted this formula also produces the correct result in the case that the reaction is endothermic. Thus this is the formula that is the foundation for the energy produced by the different reactions.

The amount of each element by volume can be described as the following:

$$N_Z = \frac{\rho \cdot X}{A \cdot m_u} \tag{2}$$

Here the Z is denoting the element type, A is the number of nucleons in the nucleus of the element, X denotes the fractional elemental abundance in the stellar core. ρ is the density of the stellar core. This relation is useful later when the reaction rates shall be calculated and the elemental composition of most main sequence stars can be calculated by observing the spectra of the star. Thus these abundances can easily be tweaked. Thus the number density is given as $\frac{1}{m^3}$

The Reaction rates when the fractional mass difference is known can be expressed as the following:

$$r_{ik} = \frac{n_i \cdot n_k}{\rho \cdot \delta_{ik}} \cdot \lambda_{ik} \tag{3}$$

Here the n denotes the element number density and the i and k denotes the isotope of the element, thus if the elements and isotope match the Kroeneker delta is equal to 1. λ_{ik} is a constant at which has the unit of volume

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per time. or $\frac{cm^3}{s}$. Thus leading us to the reaction rate having a unit defined as $\frac{1}{kgs}$ These λ values are from Gudiksen (2020) Page 31

The energy itself is calculated from the following equation:

$$\epsilon = r_{ik} \cdot Q_{ik} \tag{4}$$

Here the Q denotes the energy released in the reaction and r is still the reaction rate as denoted in equation 3. This rate calculation is used as a baseline for all reaction energy production, this produces an ϵ with the following dimensions. $\frac{E}{kg \cdot s}$. Thus giving us energy produced per mass per second

For reactions that depend on the rates of the previous reaction. The greatest example of this is the PP3 chain where all other reactions except the first is a decay of elements produced in the previous reaction. Thus a way to implement the update and distribution of rates must be devised. So for the case of the first reaction in commom for all PP branches this has to occur atleast 3 times for the PP1, PP2 and 3 common branch can happen. Thus a distribution has to be devised that will deal with the fact that the distribution of the Helium produced in the common branch is unknown. Thus a fair assumption is that we must use the previous known rates. Thus a relation of the rates should be defined as

$$r_{i'k'} = r_{i'k'} \cdot r_{r_i * k*} \cdot \frac{1}{(n \cdot r_{i'k'} + m \cdot r_{i"k"})} \tag{5}$$

This equation thus describes how a reaction rate r_ik is being updated in regard to a rate that is limiting the rates of other reactions. Thus it becomes obvious that this rate has to be checked and updated continuously. This will be discussed further in the simulations section. The mathematical relation was defined here for continuity reasons. Thus to explain the system using rates. The PP common branch produces He3 which is consumed in PP1 and PP2/3 common. This production has to be distributed in a proper way. Reaction rates will be described as RPPXY where X denotes branch and Y denotes reaction in that branch. Example: second last PP3 reaction will be RPP32, 0 denotes commonality branches. Thus we use the following system for distributing rates; if RPP0 ¡ 2 RPP1 + RPP20 then RPP1 is defined to be $RPP1 = RPP1 \cdot \frac{RPP0}{2 \cdot RPP1 + RPP20}$. This system will allow the distribution of rates in an unknown system.

2.1. *The CNO and PP-chain*

The PP-chain is an incremental nucleosynthesis process, the chain has been known to exist for a long time and dominates the low temperature range of the main sequence. The CNO chain is more dominant in higher range stars. The PP chain has 3 branches that all are energy producing branches. The resulting element for all of the branches is the extremely stable and energy efficient ^4_2He. Since all reactions in the PP chain is either a decay or an energy producing fusion reaction, in either case energy is released.

Nuclear reactions of the PP chain.

Branch	Reaction	Q' [MeV]	Q_ν [MeV]	Rate symbol
all	$^1_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^2_1\mathrm{D} + \mathrm{e}^+ + \nu_\mathrm{e}$	1.177	0.265	λ_{pp}
	$^2_1\mathrm{D} + {}^1_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + \gamma$	5.494		λ_{pd}
I	$^3_2\mathrm{He} + {}^3_2\mathrm{He} \rightarrow {}^4_2\mathrm{He} + 2\ {}^1_1\mathrm{H}$	12.860		λ_{33}
II & III	$^3_2\mathrm{He} + {}^4_2\mathrm{He} \rightarrow {}^7_4\mathrm{Be} + \gamma$	1.586		λ_{34}
II	$^7_4\mathrm{Be} + \mathrm{e}^- \rightarrow {}^7_3\mathrm{Li} + \nu_\mathrm{e} + \gamma$	0.049	0.815	$\lambda_{\mathrm{e}7}$
	$^7_3\mathrm{Li} + {}^1_1\mathrm{H} \rightarrow 2\ {}^4_2\mathrm{He}$	17.346		λ'_{17}
III	$^7_4\mathrm{Be} + {}^1_1\mathrm{H} \rightarrow {}^8_5\mathrm{B} + \gamma$	0.137		λ_{17}
	$^8_5\mathrm{B} \rightarrow {}^8_4\mathrm{Be} + \mathrm{e}^+ + \nu_\mathrm{e}$	8.367	6.711	λ_8
	$^8_4\mathrm{Be} \rightarrow 2\ {}^4_2\mathrm{He}$	2.995		λ'_8

Table 2.1: a summary of the nuclear reactions present in the PP chain sequence and a summary of the energy released by each reaction. Here the Q' denotes energy released that is made available to the thermal bath of the stellar core. Q_ν is the energy released by escaping neutrinos. λ is used to determine the reaction rate that is defined in equation 3. Table from Gudiksen (2020) Page 32

The neutrinos that are produced in this chain escape the stellar interior and thus deposit no energy in the thermal bath. The neutrinos were for a long time a problem as the amount produced in the stellar core was measured to be too low. These neutrinos are too low energy to be detected at earth. Thus the rate at which this reaction happens is hard to determine from observations.

The CNO chain is a chain based on the proton capture and decay, this chain is dominated by a nitrogen proton capture and its energy output is dominated by the slow reaction rate of this capture all later reaction rates would be identical to this limit. Thus the entire chain must be restricted by:

$$^{14}_7N + ^1_1H \rightarrow ^{15}_8O + \gamma \tag{6}$$

Since this reaction is extremely slow, all rates can safely be assumed to be equal to this reaction rate.

Nuclear reactions of the CNO cycle.

Reaction	Q' [MeV]	Q_ν [MeV]	Rate symbol
$^{12}_6\mathrm{C} + {}^1_1\mathrm{H} \rightarrow {}^{13}_7\mathrm{N} + \gamma$	1.944		λ_{p12}
$^{13}_7\mathrm{N} \rightarrow {}^{13}_6\mathrm{C} + \mathrm{e}^+ + \nu_e$	1.513	0.707	λ_{13}
$^{13}_6\mathrm{C} + {}^1_1\mathrm{H} \rightarrow {}^{14}_7\mathrm{N} + \gamma$	7.551		λ_{p13}
$^{14}_7\mathrm{N} + {}^1_1\mathrm{H} \rightarrow {}^{15}_8\mathrm{O} + \gamma$	7.297		λ_{p14}
$^{15}_8\mathrm{O} \rightarrow {}^{15}_7\mathrm{N} + \mathrm{e}^+ + \nu_e$	1.757	0.997	λ_{15}
$^{15}_7\mathrm{N} + {}^1_1\mathrm{H} \rightarrow {}^{12}_6\mathrm{C} + {}^4_2\mathrm{He}$	4.966		λ_{p15}

Table 2.1: this table summarizes the CNO chain, the notation is equivalent to that of table 2.1. Table from Gudiksen (2020) Page 32

The CNO chain is per cycle less energetic than the PP chain and thus requires alot more cycles than the PP chain to produce the same energy levels. Thus the CNO chain will for lower temperatures be less energetic than the fusion reactions of the PP chain. But once the temperature rises to sufficient levels the CNO chain becomes the dominating factor as the triple alpha process is able to produce more carbon for the start of the reaction chain. Once this is achieved the CNO chain will be

able to out produce the PP branches. In the low temperature limit the CNO chain is less energetic as the chance of nitrogen proton capture is low, but it should also be noted that in this case the CNO chain almost entirely depends on the natural abundance of C12. Thus reducing its available fuel. Common for the CNO and PP chain all positrons are assumed to be instantly annihilated. The neutrino produced in this cycle can be detected at earth although its properties is hard to determine due to the elusive properties of the neutrino.

3. SIMULATION

The stellar core was simulated using python, the simulation was performed for 1000 temperatures logarithmically scaled. All of those temperatures are then run through the simulation and rates for the relevant temperatures are updated for the temperature in question. Thus parsing through the relevant temperatures has to be done incrementally and cannot easily be vectorized. Although to verify these results and make sure that the simulated results are in the realms of sanity as energy production on the scale of the stellar core can easily get out of hand, a test function to compare the energies of the solar core to the simulated values is devised.

The relevant Q values was also verified for the simulations and the results are the following, in this the given neutrino energy was used, thus one may question the wisdom of the use of given values in the verification of said values, but if the total energy in the verfication matches the given case it strengthens the given case.

TABLE 1
VERIFIED QS AND GIVEN QS.

Reaction number	Given Q	Calculated Q
PP Common 1	1.177 MeV	1.177 MeV
PP Common 2	5.494 MeV	5.493 Mev
PP1	12.860 MeV	12.860 MeV
PP2 & 3 Common	1.586 MeV	1.587 MeV
PP2,1	0.049 MeV	0.046 MeV
PP2,2	17.346 MeV	17.346 MeV
PP3,1	0.137 MeV	0.136 MeV
PP3,2	8.367 MeV	10.75 MeV
PP3,3	2.995 MeV	0.0918 MeV
CNO 1	1.944 MeV	1.943 MeV
CNO 2	1.513 MeV	1.513 MeV
CNO 3	7.551 MeV	7.550 MeV
CNO 4	7.297 MeV	7.296 MeV
CNO 5	1.757 MeV	1.757 MeV
CNO 6	4.966 MeV	4.965 MeV

Overview of the known Q values and their verification values. This is to verify and make sure the sanity of the given Q values, just like the sanity check described in section 3

As should be noted a few of the reactions, notably the PP32 and PP33 reactions given and verified values don't match. This can mostly be explained by either error in calculations or the excitations of the nucleus. As should be noted the ^8Be has an excited state where the spin-parity is set to 2^+, although this does not fully explain the discrepancy. Still the given values were used in the simulations as the effect of using the self calculated values is negligible as they will only slightly reduce the effect of the branch in question. The simulations were run with both set of values, as shall be discussed later on.

4. RESULTS

The simulations were focused on mainly two things, the relative energy production deposited into the thermal bath of the stellar core and the energy lost to neutrino production. The script as described in the simulations section, produces a plot, but it also produces an overview of the energy lost in the production of neutrinos. As the relation of the neutrino energy released per cycle is known we can get a picture of the total neutrino production.

TABLE 2
NEUTRINO ENERGY PRODUCTION

Branch	Neutrino energy per cycle	Branch percentage neutrino energy
PP Common	0.265 MeV	N/A
PP2	0.815 MeV	3.0%
PP3	6.711 MeV	4.0%
CNO	1.704 MeV	6.37%

The energy production of each of the neutrino producing branches and the energy of the neutrinos produced and thus lost in the reaction branch.

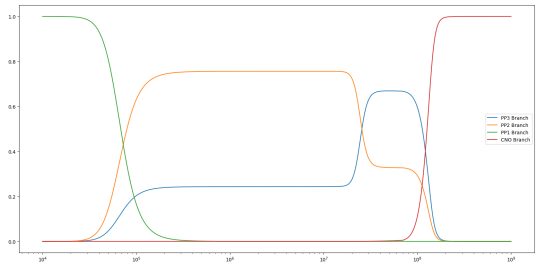

Figure 4: The relative energy production of the PP and CNO cycles as a function of the temperature. Here the temperature varies from 10^4K to 10^9K, the energy is relative so the graph only represents a percentage of the energy production not the production itself. The green line corresponds to the PP1 branch, the orange line to the PP2 branch and the blue to the PP3, the red line corresponds to CNO energy production.

The energies produced in the reactions were also tested and run in the simulations. These can be found in the simulations part as these results also were relevant for the simulations and how they were run. The use of the differing Q values does not produce an overwhelming difference, the relative production of the PP3 chain diminishes slightly but not enough to make a significant difference, the shape of the graphs does not change only the scale of the PP3 chain.

5. DISCUSSION AND CONCLUSIONS

The results are in concurrence with expected results, the domination of the CNO chain in the 8MK¡ is to be expected as the temperature and the abundance of

carbon available to fuse into nitrogen becomes more available. It is a slight surprise that CNO becomes so dominant so early, as the triple alpha process is not as active for these temperatures. Although a modelling of this cycle has to be done to complete the picture of how the stellar core behaves. Although at the lower temperature range this process is energy production wise negligible. While we could safely ignore it in these simulations further exploratory work has to done to fully understand how the CNO and energy production evolves, although this is outside the scope of this report it should be pointed out.

The discrepancy in the given and verified Q values is either a source of miscalculation or other nuclear physical phenomenon. This probably could be attributed to the excitation of the beryllium nucleus, as a spin-parity state of 2^+ exists and has an excitation value of approximately 3MeV, this would explain the energy difference. Although this might not be the actual case. Paul And Lieb (1964) As it is possible for Boron 8 to decay on both alpha and beta modes this also explains the discrepancy here and thus implies a distribution between the two modes. Thus packing both modes into one Q value that is the average is probably an advantage, although this may not be a definitive answer to the problem.

The results should be compared to observations of actual stars to verify that the model predicts a reasonable energy level, even though the model produced a reasonable result for the sanity check that uses solar observations. Though a comparison with a higher and lower temperature star should be made to verify the further validity of the model. Obviously for more energetic cores the higher energy focused cycles must be included in the model. But the model at hand is probably quite reasonable for the temperatures in the simulation.

In conclusion, the model seems to be a fair representation of the relative energy productions in the stellar core. Though as mentioned further improvements and tests has to be done. The model predicts the domination of the CNO in the higher temperature and the dominance of the PP chain in the lower temperature range.

REFERENCES

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