

Humanity's Final Conjecture: Large Model Innovation Ability Evaluation

1. Introduction: The Theoretical Intersection of Deterministic Chaos and Prime Distribution

One of the greatest enigmas in number theory lies in the distribution laws of prime numbers within the sequence of natural numbers. On one hand, the definition of a prime number is extremely simple—a natural number divisible only by 1 and itself; on the other hand, their appearance on the number axis seems to follow a certain random process, exhibiting a high degree of unpredictability. From the Riemann Hypothesis, which defines the bounds of the error term in the prime-counting function, to the Cramér Model, which views primes as probabilistic events, mathematicians have long been dedicated to finding a balance between randomness and regularity. However, with the development of Nonlinear Dynamics and Chaos Theory, a completely new perspective has gradually emerged: the "randomness" of prime numbers may not be pure probabilistic randomness, but rather the chaotic manifestation of a low-dimensional deterministic nonlinear system.

This report aims to construct a detailed heuristic proof framework, exploring the feasibility of utilizing Symbolic Dynamics to isomorphically map the Sieve of Eratosthenes to a Unimodal Logistic Mapping. We do not attempt to directly provide the ultimate answer to the Twin Prime Conjecture or other number theory puzzles here; rather, we establish a rigorous logical system, core theorems, and verification paths. This framework aims to provide a theoretical "seed" for subsequent computational models, enabling them to explore a global dynamic model describing prime gap patterns through computational expansion, logical completion, and counter-example testing based on this heuristic path.

The core hypothesis is based on the observation that the periodic extinction process of the prime sieve can be encoded as a symbol sequence, and these symbol sequences, under specific conditions, constitute the "Kneading Sequence" of the Logistic mapping

$x_{n+1} = 1 - ux_n^2$. If the topological conjugacy relationship between the two can be established, static distribution problems originally belonging to the field of number theory will be transformed into problems of orbital stability and chaotic attractor structures in dynamical systems.

2. Theoretical Foundations and Tool Preparation

Before unfolding the heuristic proof, the mathematical tools used must be strictly defined. This study involves two fields that appear disparate but are deeply interconnected in structure: classical Prime Sieve Theory and modern Symbolic Dynamics.

2.1 The Dynamical Interpretation of the Prime Sieve

The Sieve of Eratosthenes is the oldest and most intuitive algorithm for generating primes. Traditionally, it is viewed as a static algorithmic process: list all integers, sequentially cross out multiples of 2, multiples of 3, multiples of 5... leaving the primes. However, in the perspective of dynamical systems, the sieve method can be re-interpreted as a discrete time evolution system.

We view the set of natural numbers N as a one-dimensional discrete space, where in the initial state, all points are marked as "Alive" (potential primes).

- **Time step** $t = 1$: Introduce operator P_2 , which acts on the space with period $T = 2$, flipping the state of all $2k$ ($k > 1$) positions to "Cleared" (composite).
- **Time step** $t = 2$: Introduce operator P_3 , it acts on the space with period $T = 3$, eliminating the $3k$ positions.
- **Time step** $t = n$: Introduce operator P_{p_n} , eliminating with period p_n .

This process reveals that the essence of prime distribution is the superposition of periodic interference waves. Although each individual wave (sieve) is strictly periodic, the superposition of infinite coprime periods leads to extreme complexity in local structure,

known as "Structural Chaos". Our goal is to find a single nonlinear operator capable of generating this complex interference pattern.

2.2 Unimodal Maps and Symbolic Dynamics

Symbolic Dynamics simplifies the analysis of dynamical systems by partitioning the continuous phase space into finite regions and transforming orbits into symbol sequences. For a Unimodal Map defined on the interval $[-1, 1]$, such as the Logistic Map $f(x) = 1 - ux^2$, its dynamic behavior is determined by the critical point $x_c = 0$ and its iteration trajectory.

2.2.1 Partition of the Symbolic Space

We divide the interval $[-1, 1]$ into two basic symbol regions:

- **L (Left):** Corresponds to the interval $x < 0$, marked as the Prime Area, denoted as L, consistent with the standard Symbolic Dynamics notation L.
- **R (Right):** Corresponds to the interval $x > 0$, marked as the Composite Area, denoted as R, also consistent with standard notation R.
- **C (Center):** The critical point $x = 0$.

We can represent the partition of the symbolic space on the bifurcation diagram of the Logistic Map (Fig.1).

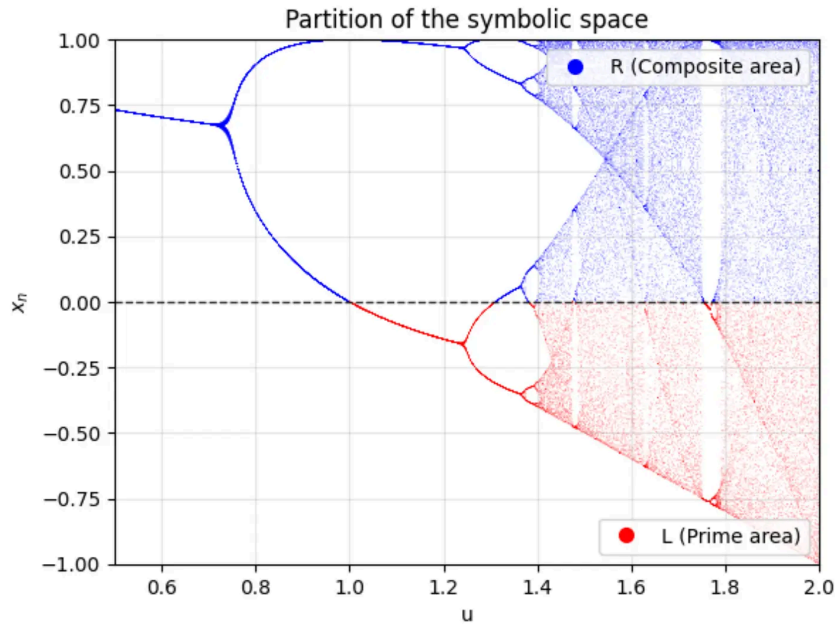


Figure.1 Symbolic region division in the bifurcation diagram of the Logistic map

For any initial point X_0 , its orbit X_0, X_1, X_2, \dots can be mapped to a symbol sequence $S = s_0 s_1 s_2 \dots$ where $s_i \in \{L, R\}$.

2.2.2 Kneading Theory

The Kneading Theory founded by Milnor and Thurston points out that the topological structure of a unimodal map is completely determined by the orbit of the critical point $x_c = 0$ (i.e., the Kneading Sequence). The Kneading Sequence $K(f)$ is a topological invariant that not only describes the system's current state but also implies the system parameters.

- **MSS Sequence:** Not any combination of L and R can constitute a legitimate Kneading Sequence. A sequence must satisfy the "Maximal Condition", meaning that the sequence must be greater than or equal to all its shift sequences under a specific shift order.
- **Monotonicity:** There exists a monotonic one-to-one correspondence between the parameter u and its corresponding Kneading Sequence. As u increases, the topological entropy of the system increases, and the Kneading Sequence also presents specific evolutionary laws in lexicographical order.

2.3 Chaos and Bifurcation Diagrams

The Logistic Map $x_{n+1} = 1 - ux_n^2$ exhibits a path to chaos via period-doubling bifurcation:

1. **Stable Fixed Point:** When u is small, the system converges to a single value.
2. **Period-Doubling Bifurcation:** As u increases, the orbit splits into 2-period, 4-period, 8-period...
3. **Chaos Outbreak:** When $u > u_\infty \approx 1.40715$, the system enters the chaotic region. Within this region, orbits no longer possess simple periodicity and are sensitive to initial conditions.
4. **Band Merging:** Inside the chaotic region, the chaotic attractor consists of multiple separated "bands" (energy bands). As u continues to increase, these bands gradually merge through inverse period-doubling bifurcations, 2^k bands merge into 2^{k-1} bands.

The core task of this report is to demonstrate that the dynamical characteristics of the prime system correspond exactly to a chaotic orbit under a specific parameter.

3. Core Construction: The Symbolic Dynamics Model of the Prime Sieve

To enable the large model to extend based on this proof, we must strictly define how to "translate" the prime sieve into symbol sequences. This is the cornerstone of the entire heuristic proof.

3.1 Symbolic Definition of Prime Characteristics

We encode the points on the natural number axis in binary.

- **L (Left):** Represents "Alive" points, i.e., primes (or numbers not yet sifted out).
- **R (Right):** Represents "Sifted" points, i.e., composite numbers.

The sieving action of each prime P can be represented as a periodic symbol sequence M_p

- **Action of Prime 2:** Prime 2 itself is retained (L), but every 2 numbers one is eliminated (R). Note that the starting phase here is crucial. In the model of this report, the basic pattern of prime 2 is defined as $M_2 = RL$. This means it has a period of 2 and has an alternating "Eliminate-Retain" structure.
 - Sequence Expansion: RL, RL, RL, \dots
- **Action of Prime 3:** Prime 3 itself is retained, and every 3 numbers one is eliminated. The basic pattern is defined as $M_3 = RLL$.
 - Sequence Expansion: RLL, RLL, RLL, \dots
- Action of General Prime p_i :

For the i – th prime p_i , its characteristic sequence M_{p_i} is defined as:

$$M_{p_i} = RL^{p_i-1}$$

This indicates that within a period of length p_i , there is one "elimination bit" (R), and the remaining $p_i - 1$ positions are "potential retention bits" (L).

A simple demonstration is shown in Figure 2:

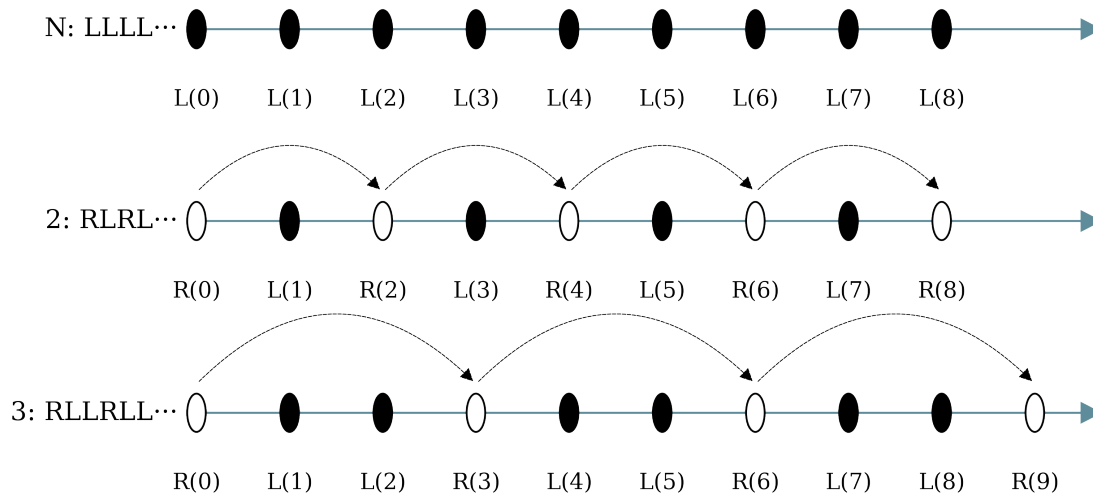


Figure.2. Symbolic sequence representation of prime numbers.

3.2 Symbol Sequence Synthesis Rule (Composition Rule)

Defined single prime characteristic sequences, we need to define how these sequences interact to simulate the "Sieve" process. This requires introducing a binary Composition Operator.

Definition 1: Sequence Composition Rule

For two symbol sequences A and B, their composition $A \cdot B$ follows the "Destruction Priority" principle. That is, as long as one sieve determines a position as composite (R), that position is composite; only when all sieves determine that position as "Alive" (L) is the position retained as L:

- $L \cdot L = L$ (Alive + Alive = Alive)
- $L \cdot R = R$ (Alive + Eliminate = Eliminate)
- $R \cdot L = R$ (Eliminate + Alive = Eliminate)
- $R \cdot R = R$ (Eliminate + Eliminate = Eliminate)

Example: The composition of M_2 and M_3

The specific calculation for the composition of M_2 and M_3 is as follows (needs to be expanded to the Least Common Multiple period 6):

- $M_2 = (RL)^\infty \rightarrow RLRLRL$
- $M_3 = (RLL)^\infty \rightarrow RLLRLL$
- Bitwise Composition $D_2 = M_2 \cdot M_3 \rightarrow RLRRRL$

An example of a composition rule is shown in Figure 3.

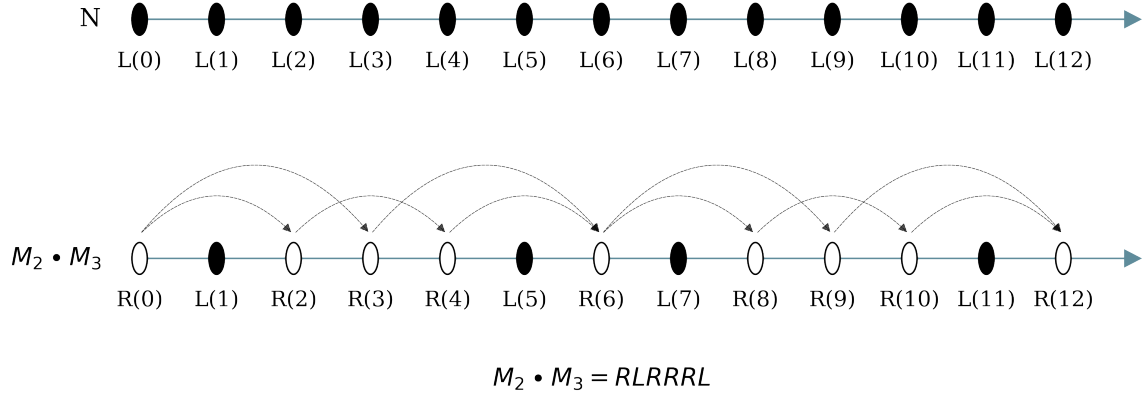


Figure.3. Example diagram of composition rules

3.3 Cumulative Dynamics Sequence D_i

We define D_i as the system state after the action of the first i prime sieves:

$$D_i = M_{p_1} \cdot M_{p_2} \cdot \dots \cdot M_{p_i}$$

As $i \rightarrow \infty$, the period T_i of sequence D_i tends to infinity, and the system transforms into a non-periodic chaotic system D_∞ .

4. Main Logic and Theorems of the Heuristic Proof

This chapter outlines the complete reasoning logic chain. To enable the large model to extend along the correct direction, we must clarify 2 Propositions (Themes) and 3 Lemmas.

4.1 Proposition 1: Logistic Mapping Hypothesis (Core Proposition)

This is the top-level hypothesis of the entire theoretical framework, pointing out the physical image of prime distribution.

Proposition (Theme 1):

The Logistic Mapping $x_{n+1} = 1 - ux_n^2$ (where $x_n \in [-1, 1]$), when the parameter $u \rightarrow 1.5437$, its chaotic orbit can describe the prime gap pattern.

- **Physical Meaning:** The parameter $u \approx 1.5437$ holds a special status in the bifurcation diagram of the Logistic Map. It is the critical point (Band Merging Point) where "Two-band Chaos" merges into "Single-band Chaos". At this point, the system just begins to exhibit ergodicity, capable of visiting the entire interval, which echoes the sparse yet global distribution of primes on the number axis.

The specific bifurcation diagram of the Logistic map is shown in Figure 4 below.

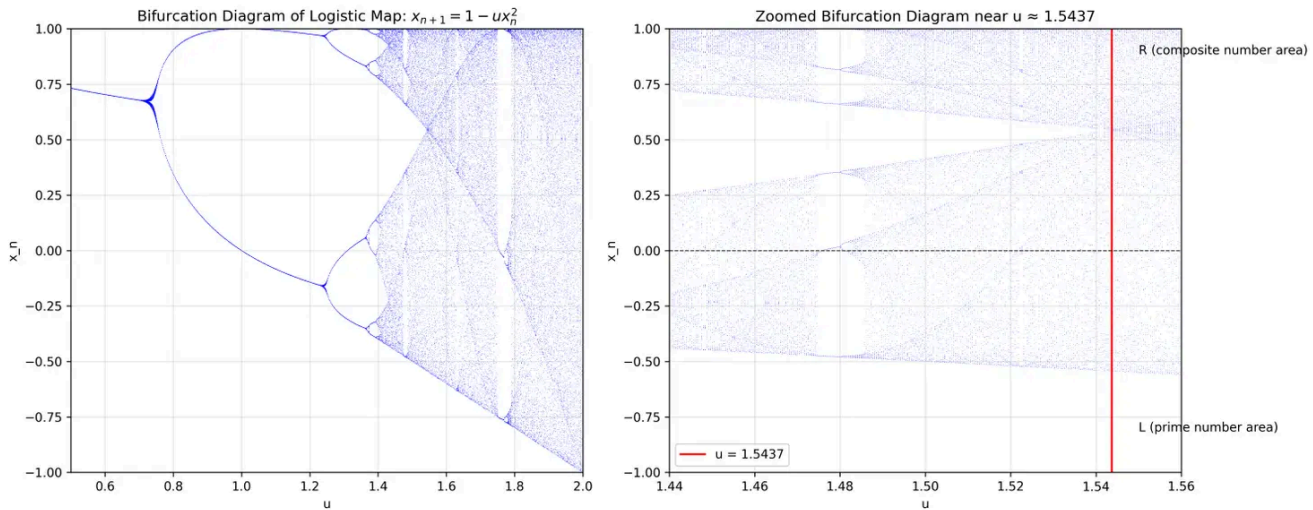


Figure 4. Bifurcation diagram of the Logistic map; the red line in the right panel corresponds to $u=1.5437$.

4.2 Proposition 2: Symbol Sequence RLR^∞ Assumption

This proposition establishes the bridge from chaotic parameters to specific symbol sequences.

Proposition 2 (Theme 2):

The gap pattern of primes can be described by the symbolic kneading sequence RLR^∞ . The dynamic characteristics of this sequence correspond precisely to the Logistic Map at $u \approx 1.5437$.

- **Analysis:** At the edge of chaos, the limit behavior of D_i tends towards some complex structure of RLR^∞ , implying that the "ultimate form" of the prime sieve is topologically equivalent to the chaotic attractor of the Logistic Map under the RLR^∞ mode.
- **Numerical Verification:** We have computed the statistics of the differences between consecutive primes, shown in the left panel of Figure 5. Meanwhile, we also computed the statistics of the differences between consecutive “primes” constructed from the chaotic orbit of the Logistic map with parameter $u \approx 1.5437$, shown in the right panel of Figure 5.

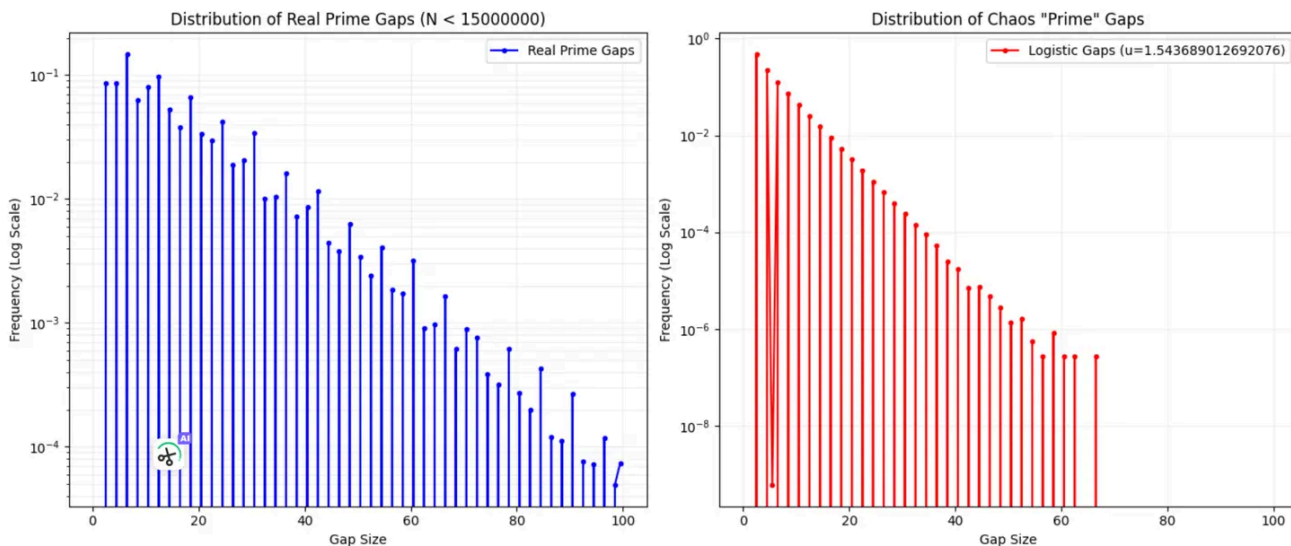


Figure 5. Statistics of the differences between consecutive primes (left panel) and statistics of the differences between “primes” constructed from the Logistic map (right panel)

From the Fig.5, it can be seen that both present an approximate linear molecular trend in logarithmic coordinates, indicating they both follow the exponential distribution law

(Poisson process characteristics). Additionally, note the details of the fluctuations; the prime gap statistics show obvious Period-3 oscillations, and the Logistic Map, although a discrete-time system, also exhibits similar periodic structures. This simple experiment verifies the core intuition of this paper: the generation process of primes behaves statistically like an orbit generated by a dynamical system at the edge of chaos.

4.3 Lemma 1: Kneading Sequence Admissibility and Truncation

To link the sieve sequence D_i in number theory with the kneading sequence in dynamics, the "legitimacy" problem must be resolved. Not any length of sieve sequence is a legitimate kneading sequence.

Lemma 1:

For the cumulative sieve sequence D_i , the subsequence formed by its first $p_i^2 + 1$ symbols is a legitimate Kneading Sequence.

- Core Hint and Number Theory Connection:

The validity of this proposition depends on the size of prime gaps. Specifically, it involves the following number theory background:

This conjecture implies that the difference between all consecutive primes before p_i^2 is not greater than p_i . If we denote $g(N)$ as the consecutive prime difference before N , we need to satisfy $g(N) < N^{1/2}$. This currently remains a conjecture (Legendre's Conjecture). The current best result regarding the upper bound of $g(N)$ is $g(N) < cN^{21/40}$. However, the precise upper bound of prime gaps has little effect on our discussion; we only need to choose a relatively short sequence fragment to ensure it is a legitimate kneading sequence.

4.4 Lemma 2: Symbolic Dynamics Monotonic Evolution

As j increases, the complexity of the system increases monotonically.

Lemma 2 (Symbol Order):

$$D_1 < D_2 < D_3 < \dots < D_i < \dots < D_\infty$$

- **Proof Idea:** Compare D_i and D_{i+1} . D_{i+1} is based on D_i , with certain positions originally being L "flipped" to R (because they are divisible by the new prime p_{i+1}). It is necessary to prove, based on the parity rules of MSS theory, that this flipping of specific positions (determined by prime multiple positions) always leads to an increase of the sequence in MSS order.

4.5 Lemma 3: Parameter Monotonic Approximation

Based on the one-to-one correspondence between parameters and sequences, Lemma 3 derives the evolutionary law of parameters.

Lemma 3 (Parameter Order):

$$u(D_1) < u(D_2) < u(D_3) < \dots < u(D_\infty)$$

This means that as the sieve process deepens, the Logistic Map parameter describing the system must constantly increase. The prime system is a dynamic process constantly approaching the edge of chaos, finally converging to $u \approx 1.5437$ mentioned in Proposition 1.

5. Extension Guidelines: Large Model Execution Path and Verification Protocols

To enable the large model to systematically expand the above heuristic proof, we divide the task into three progressive stages.

Stage 1: Basic Theory Perfection

- **Goal:** Construct a solid mathematical foundation and complete the logical gaps in the heuristic proof.

- **1.1 Complete Definition of Symbol Sequence Composition Rule:** Provide a formal definition for the composition operator \cdot for arbitrary length sequences A and B , including Period Extension (LCM) processing and bitwise Boolean operation rules.
- **1.2 Proof of Kneading Sequence Admissibility and Truncation (Lemma 1):** Based on the hint of Legendre's Conjecture, attempt to provide a complete proof or strong argument for Lemma 1. Define the truncation function $Trunc(D_i, L)$ and determine the optimal value range of L (related to p_i^2) to ensure the truncated sequence is "Maximal" in the MSS sense.
- **1.3 Proof of Lemma 2 (Monotonic Evolution of Symbolic Dynamics):** Detailed proof of $D_i < D_{i+1}$. This requires analyzing where $L \rightarrow R$ flips occur in D_{i+1} relative to D_i , and proving how the number (parity) of R's in the prefix before the first flip position affects the size comparison of the sequences.
- **1.4 Derivation of Lemma 3 (Parameter Monotonicity):** Based on the Milnor-Thurston Monotonicity Theorem, prove that if $D_i < D_{i+1}$ and both are legitimate Kneading Sequences, then the corresponding Logistic parameters $u(D_i) < u(D_{i+1})$.

Stage 2: Numerical Verification and Heuristic

- **Goal:** Provide empirical support for key theorems through computational experiments and explore limit behaviors. Some possible methods are as follows:
 - **2.1 Limit Behavior and Chaos Feature Analysis:** Calculate the convergence behavior of the parameter $u(D_i)$ as i increases. Verify if its limit tends to 1.5437. Analyze the Lyapunov exponent in the limit state, comparing the exponent calculated from the real prime gap sequence with the theoretical value (0.3406) of the Logistic Map at $u \approx 1.5437$.
 - **2.2 Numerical Verification Methods for Key Theorems:** Design algorithms to generate D_N for the first N primes and write programs to automatically verify its MSS Maximality condition. Plot the curve of $u(D_i)$ varying with i and observe its rate of approach to the edge of chaos (Does it conform to the Feigenbaum scaling law?).
 - **2.3 Analysis of Verification Conclusions:** Based on numerical results, evaluate the confidence of the heuristic proof. Point out which parts match well and which parts have deviations (e.g., outliers in large prime gaps).

Stage 3: Extension of Proof and Theoretical Correction

- **Goal:** Conduct open-ended theoretical exploration on top of the basic proof. Some possible extensions are:
 - **3.1 Number Theoretic Significance of Band Merging:** Deeply explain what "Two-band merging into Single-band" in the Logistic Map corresponds to in number theory (e.g., the homogenization of primes in modular arithmetic classes).
 - **3.2 Dynamic Prediction of Twin Prime Density:** Attempt to derive the Twin Prime Constant using the Invariant Density of the Logistic Map.
 - **3.3 Theoretical Correction:** If the decay characteristic of prime density $1/\ln N$ is found to be inconsistent with standard chaotic attractors, propose a corrected model (such as a Non-autonomous Dynamical System).
 - **3.4 Prime Distribution Problem:** When parameter $u \rightarrow 1.5437$, study the ergodicity of the corresponding Logistic Map orbit, thereby obtaining new ideas for solving problems like Twin Primes.

6. Data and Structured Presentation

To auxiliary the large model's understanding, the following table summarizes the key mapping relationships.

Table 1: Prime Sieve and Dynamic Parameter Correspondence Evolution

Sieve Stage (i)	Introduced Prime (pi)	Symbol Sequence Feature (Di)	Estimated Parameter (u)	Dynamical State
1	2	RL (Oscillation)	~2.00	2-Period
2	3	(Complication)	< 2.00	Higher Order Period Window
...
∞	-	RLR^{∞} (Quasi-Limit)	1.5437	Chaos (Band Merging)

7. Conclusion and Outlook

This report constructs a bridge connecting Number Theory and Chaos Theory. For the Large Model undertaking this task, this is not merely a computational task, but a process of theoretical exploration. Based on the core settings of Propositions 1-2 and the logical chain of Lemmas 1-3, please boldly conduct numerical experiments and theoretical deductions to finally provide a generative model capable of describing the fine structure of prime gaps.